

# TikZ Style Library Test

## Fusion Category Diagrams for Mobile Anyons

Test Document

December 25, 2025

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# 1 Basic Fusion Category Diagrams

## 1.1 Trivalent Vertices

The basic trivalent vertex represents fusion/splitting:

$$\begin{array}{ccc}
 \begin{array}{c} c \\ | \\ \swarrow \quad \searrow \\ a \quad b \end{array} & 
 \begin{array}{c} Z \\ | \\ \swarrow \quad \searrow \\ X \quad Y \end{array} & 
 \begin{array}{c} \tau \\ | \\ \swarrow \quad \searrow \\ \tau \quad \tau \end{array} \\
 \backslash \text{trivalentvertex}\{a\}\{b\}\{c\} & 
 \backslash \text{trivalentvertex}\{X\}\{Y\}\{Z\} & 
 \backslash \text{trivalentvertex}\{\tau\}\{\tau\}\{\tau\}
 \end{array}$$

## 1.2 Fusion Trees

Left-associative and right-associative fusion trees:

$$\begin{array}{cc}
 \begin{array}{c} d \\ | \\ e \\ \swarrow \quad \searrow \\ a \quad b \quad c \end{array} & 
 \begin{array}{c} d \\ | \\ f \\ \swarrow \quad \searrow \\ a \quad b \quad c \end{array} \\
 \backslash \text{fuselefttree}\{a\}\{b\}\{e\}\{c\}\{d\} & 
 \backslash \text{fuserighttree}\{a\}\{b\}\{f\}\{c\}\{d\}
 \end{array}$$

## 1.3 F-Move (Associator)

The F-move relates different fusion orders:

$$\begin{array}{ccc}
 \begin{array}{c} a \quad b \quad c \\ \swarrow \quad \downarrow \quad \searrow \\ e \quad \quad d \end{array} & = \sum_f \left( F_d^{abc} \right)_{fe} & 
 \begin{array}{c} a \quad b \quad c \\ \swarrow \quad \downarrow \quad \searrow \\ d \quad \quad f \end{array} \\
 \backslash \text{Fmoveequation}\{a\}\{b\}\{c\}\{d\}\{e\}\{f\}
 \end{array}$$

## 1.4 F-Move with Multiplicity Indices

For categories with  $N_{ab}^c > 1$ , vertices carry multiplicity labels  $\mu, \nu$ :

$$\begin{array}{ccc}
 \begin{array}{c} a \quad b \quad c \\ \swarrow \quad \downarrow \quad \searrow \\ \mu \quad \quad \nu \\ e \quad \quad d \end{array} & = \sum_{f, \mu', \nu'} \left( F_d^{abc} \right)_{ef}^{\mu\nu, \mu'\nu'} & 
 \begin{array}{c} a \quad b \quad c \\ \swarrow \quad \downarrow \quad \searrow \\ \mu' \quad \quad \nu' \\ f \quad \quad d \end{array} \\
 \backslash \text{Fmovemult}\{a\}\{b\}\{c\}\{d\}\{e\}\{f\}\{\mu\}\{\nu\}
 \end{array}$$

Individual trees with multiplicities for inline use:

$$\begin{array}{cc}
 \begin{array}{c} a \quad b \quad c \\ \swarrow \quad \downarrow \quad \searrow \\ \mu \quad \quad \nu \\ e \quad \quad d \end{array} & 
 \begin{array}{c} a \quad b \quad c \\ \swarrow \quad \downarrow \quad \searrow \\ \mu' \quad \quad \nu' \\ f \quad \quad d \end{array} \\
 \backslash \text{Ftreeleft}\{a\}\{b\}\{c\}\{d\}\{e\}\{\mu\}\{\nu\} & 
 \backslash \text{Ftreeright}\{a\}\{b\}\{c\}\{d\}\{f\}\{\mu'\}\{\nu'\}
 \end{array}$$

## 2 Duality: Cups and Caps

### 2.1 Evaluation and Coevaluation

Evaluation (Cup)      Coevaluation (Cap)

$$\begin{array}{cc} \begin{array}{c} X^* \quad \text{---} \quad X \\ \text{\texttt{\textbackslash evalcup\{X\}}} \end{array} & \begin{array}{c} X \quad \text{---} \quad X^* \\ \text{\texttt{\textbackslash coevalcap\{X\}}} \end{array} \end{array}$$

### 2.2 Zigzag Identities (Snake Equations)

The zigzag identities express that cups and caps are inverse:

$$\begin{array}{cc} \begin{array}{c} X \\ | \\ \text{\texttt{\textbackslash leftzigzag\{X\}}} \end{array} = \begin{array}{c} X \\ | \\ X \end{array} & \begin{array}{c} X^* \\ | \\ \text{\texttt{\textbackslash rightzigzag\{X\}}} \end{array} = \begin{array}{c} X^* \\ | \\ X^* \end{array} \end{array}$$

### 2.3 Bigon and Quantum Dimension

$$\begin{array}{ccc} \begin{array}{c} c \\ | \\ a \text{---} b \\ | \\ c \\ \text{\texttt{\textbackslash bigon\{a\}\{b\}\{c\}}} \end{array} & \begin{array}{c} \bigcirc \\ X \\ \text{\texttt{\textbackslash qdimloop\{X\}}} \end{array} & \begin{array}{c} | \\ X \\ \text{\texttt{\textbackslash identitystrand\{X\}}} \end{array} \end{array}$$

## 3 Braiding and R-Moves

### 3.1 Braiding Crossings

Over-crossing (positive) and under-crossing (negative):

$$\begin{array}{cc} \text{Over-crossing } c_{X,Y} & \text{Under-crossing } c_{X,Y}^{-1} \\ \begin{array}{c} Y \quad X \\ | \quad | \\ \text{\texttt{\textbackslash braidingover\{X\}\{Y\}}} \\ X \quad Y \\ | \quad | \end{array} & \begin{array}{c} X \quad Y \\ | \quad | \\ \text{\texttt{\textbackslash braidingunder\{X\}\{Y\}}} \\ Y \quad X \\ | \quad | \end{array} \end{array}$$




### 3.2 Twist (Ribbon Element)

The twist encodes the topological spin:

$$\begin{array}{c} | \\ | \\ | \\ X \quad | \\ \text{\texttt{\textbackslash twist\{X\}}} \end{array}$$

## 4 Temperley-Lieb Patterns

The Temperley-Lieb algebra generators for 2 strands:

<b>Identity</b>	<b>Cup-Cap (<math>e_i</math>)</b>	<b>H-pattern</b>
		
<code>\TLidentity</code>	<code>\TLCupcap</code>	<code>\TLhorizontal</code>

## 5 Trivalent Category Relations

For categories generated by a rotationally invariant trivalent vertex with parameters  $(d, b, t)$ .

### 5.1 Loop Relation

A closed loop evaluates to the quantum dimension  $d$ :

$$\bigcirc = d \quad \text{\code{\trivloopeq}}$$

Or just the loop:  $\bigcirc = \text{\code{\trivloop}}$

### 5.2 Lollipop (Forbidden Diagram)

In trivalent categories, lollipops vanish:

$$\bigcirc \text{---} \text{---} = 0 \quad \text{\code{\trivlollipop}}$$

### 5.3 Bigon Relation

The bigon simplifies to  $b$  times the identity:

$$\text{---} \bigcirc \text{---} = b \cdot \text{---} \quad \text{\code{\trivbigoneq}}$$

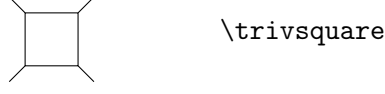
### 5.4 Triangle Relation

A triangle with external legs equals  $t$  times the trivalent vertex:

$$\text{---} \bigcirc \text{---} \text{---} = t \cdot \text{---} \text{---} \text{---} \quad \text{\code{\trivtriangleeq}}$$

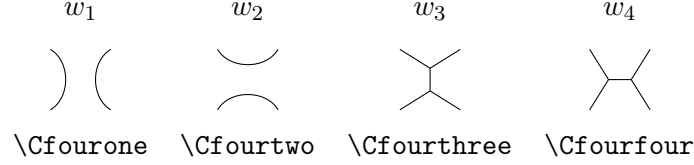
## 5.5 Square Diagram

The square face with four external legs:



## 6 $\mathfrak{C}_4$ Basis Diagrams

In a cubic trivalent category, these four diagrams form a basis for  $\text{Hom}(\mathbf{1}, X^{\otimes 4})$ :



### 6.1 Square Decomposition

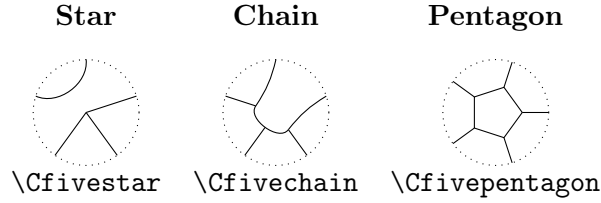
The square decomposes as a linear combination of basis elements:

$$\text{Square Diagram} = \alpha \left( \text{Diagram } w_1 + \text{Diagram } w_2 \right) + \beta \left( \text{Diagram } w_3 + \text{Diagram } w_4 \right)$$

where  $\alpha = \frac{b(b^2+bt-t^2)}{bd+t+dt}$  and  $\beta = \frac{t^2(d+1)-b^2}{bd+t+dt}$ .

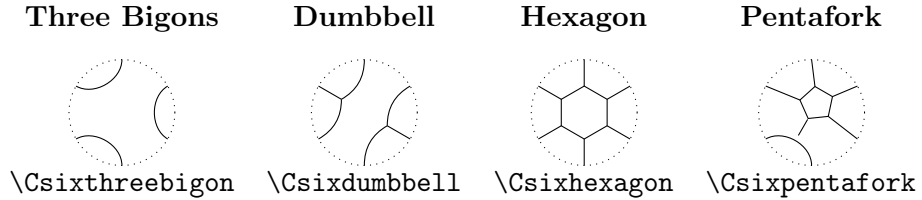
## 7 $\mathfrak{C}_5$ Basis Diagrams

Representative patterns for 5 boundary points:



## 8 $\mathfrak{C}_6$ Basis Diagrams

Representative patterns for 6 boundary points:



## 9 Inner Product Pairing

The bilinear inner product of two trivalent graphs  $f, g$  with  $n$  boundary points:

$$\langle f, g \rangle = \text{Diagram showing two circles labeled } f \text{ and } g \text{ connected by } n \text{ arcs. The top arc is labeled } \vdots \text{ indicating } n \text{ arcs.}$$

`\trivinnerprod{f}{g}`

## 10 2-Local Hamiltonian Diagrams

These diagrams are useful for constructing nearest-neighbor Hamiltonians on anyon chains.

### 10.1 Basic 2-Site Operators

Identity	Swap	Cup-Cap	H-pattern	Braid	Braid <sup>-1</sup>
<code>\Htwoidentity</code>	<code>\Htwoswap</code>	<code>\Htwocupcap</code>	<code>\HtwoH</code>	<code>\Htwobraid</code>	<code>\Htwobraidinv</code>

### 10.2 Hopping Operators (Braid with Vacuum)

Hop operators have one strand as vacuum (dashed) representing anyon hopping:

Hop Right	Hop Left	Hop Right <sup>-1</sup>	Hop Left <sup>-1</sup>
<code>\Htwohopright</code>	<code>\Htwohopleft</code>	<code>\Htwohoprghtinv</code>	<code>\Htwohopleftinv</code>

Example hopping Hamiltonian:

$$H_{\text{hop}} = -t \sum_i \left( \text{Hop Right} + \text{Hop Left} \right)_i$$

### 10.3 Fusion with Intermediate Channel

<code>\Htwofusion{c}</code>	<code>\Htwofusion{\mathbf{1}}</code>	<code>\Htwofusion{X}</code>

### 10.4 Local Operator in Chain Context

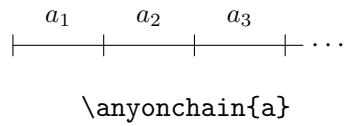
$$\dots \left| \begin{array}{c} \text{---} \\ | \\ \boxed{H} \\ | \\ \text{---} \end{array} \right| \dots \quad \text{---} \quad \text{---}$$

`\chainlocal{H}`

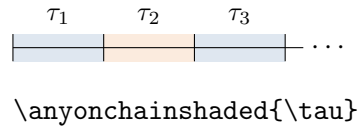
## 11 Anyon Chain

Anyons live in **intervals** (between lattice sites), not at vertices. Vertical tick marks indicate lattice sites:

## 11.1 Standard Chain (with tick marks)

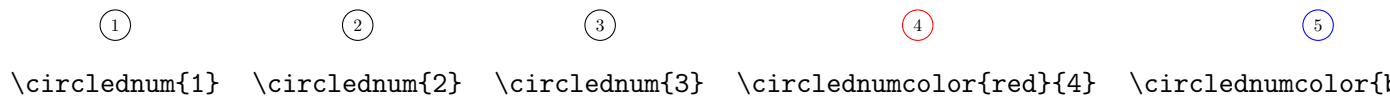


## 11.2 Shaded Chain (intervals highlighted)



# 12 Utility Macros

## 12.1 Circled Numbers



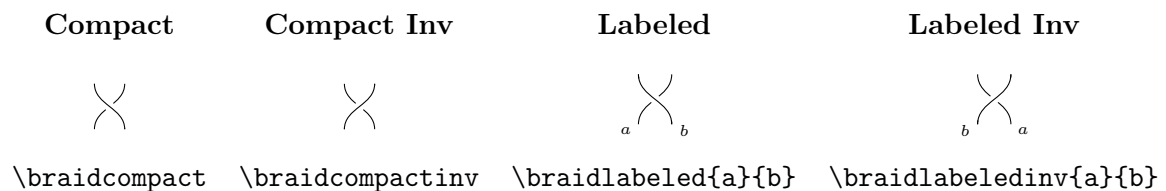
## 12.2 F-Symbol Diamond

For labeling F-symbol indices:  $\diamond_\alpha$ ,  $\diamond_\beta$

`\Fdiamond{\alpha}`, `\Fdiamond{\beta}`

# 13 Compact Braiding for Hamiltonians

Compact (squatter) braids for use inside brackets:











Example in a Hamiltonian expression:

$$H = - \sum_i \left( \text{braidcompact} + \phi^{-1} \text{braidcompactinv} \right)_i$$

# 14 Color Definitions

Professional complementary color scheme (blue-orange with accents):








Name	Sample	Usage
AnyonBlue		Primary color
AnyonOrange		Complementary color
AnyonTeal		Accent color
AnyonCoral		Accent color
AnyonSlate		Dark emphasis
AnyonSilver		Subtle elements
LightGray		Backgrounds
MediumGray		Borders

## 15 TikZ Styles Reference

### 15.1 Arrow Styles

$\longrightarrow$   $\longrightarrow$   $\rightarrow$  (arrow at 60%)  
 $\longrightarrow$   $\longleftarrow$   $\leftarrow$  (reverse at 60%)  
 $\longrightarrow$   $\longrightarrow$   $\rightarrow$  (arrow at 50%)  
 $\longrightarrow$   $\longrightarrow$   $\rightarrow$  (arrow at 52%)

### 15.2 Box and Vertex Styles

morphism box morphism box small morphism box large fusion fusion vertex empty

## 16 Example: Golden Chain Hamiltonian

As an example, here is how to write a 2-local Hamiltonian term for the golden chain using these macros (note proper vertical centering in brackets):

$$H = - \sum_i \left( \text{diag}_1 + \phi^{-1} \text{diag}_\tau \right)_i$$

where  $\phi = \frac{1+\sqrt{5}}{2}$  is the golden ratio.

The projector onto the trivial fusion channel at sites  $i, i+1$  is:

$$P_{i,i+1}^{(1)} = \frac{1}{d} \text{diag}_1$$

All 2-local operators side by side for comparison:









## 17 Example: Trivalent Category Calculation

In the Fibonacci category with  $d = \phi$ ,  $b = 1$ ,  $t = \frac{d-2}{d-1}$ :

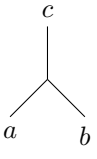
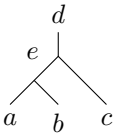
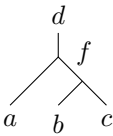


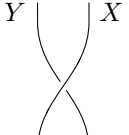
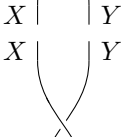
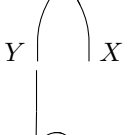
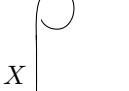


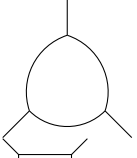
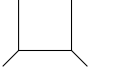




$$\bigcirc = d = \phi \approx 1.618$$

$$\begin{array}{c} | \\ \bigcirc \\ | \end{array} = b \cdot \begin{array}{c} | \\ | \\ | \end{array} = 1 \cdot \begin{array}{c} | \\ | \\ | \end{array}$$

The four basis diagrams in  $\mathfrak{C}_4$  satisfy:

$$\begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} - \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} + \frac{1}{d+1} \left( \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} + \frac{1}{d-1} \begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} \right) = 0$$

# A Quick Reference Card

Category	Macro	Output
Vertices	<code>\trivalentvertex{a}{b}{c}</code>	
	<code>\fuselefttree{a}{b}{e}{c}{d}</code>	
	<code>\fuserighttree{a}{b}{f}{c}{d}</code>	
Duality	<code>\evalcup{X}</code>	
	<code>\coevalcap{X}</code>	
Braiding	<code>\braidingover{X}{Y}</code>	
	<code>\braidingunder{X}{Y}</code>	
	<code>\twist{X}</code>	
		
Trivalent	<code>\trivloop</code>	
	<code>\trivbigon</code>	
	<code>\trivtriangle</code>	
	<code>\trivsquare</code>	
$\mathfrak{C}_4$ Basis	<code>\Cfourone</code>	
	<code>\Cfourtwo</code>	
	<code>\Cfourthree</code>	
	<code>\Cfourfour</code>	
	<code>\Htwoidentity</code>	