TikZ Style Library Test

Fusion Category Diagrams for Mobile Anyons

Test Document

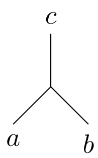
December 25, 2025

Contents

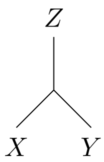
1 Basic Fusion Category Diagrams

1.1 Trivalent Vertices

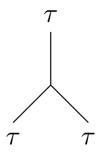
The basic trivalent vertex represents fusion/splitting:



\trivalentvertex{a}{b}{c}



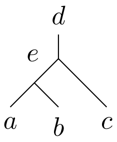
\trivalentvertex{X}{Y}{Z}



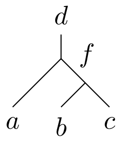
\trivalentvertex{\tau}{\tau}{\tau}

1.2 Fusion Trees

Left-associative and right-associative fusion trees:



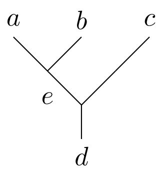
\fuselefttree{a}{b}{e}{c}{d}



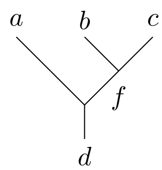
\fuserighttree{a}{b}{f}{c}{d}

1.3 F-Move (Associator)

The F-move relates different fusion orders:



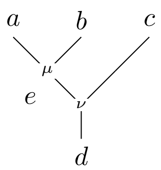
$$= \sum_f \left(F_d^{abc}\right)_{fe}$$



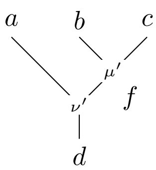
\Fmoveequation{a}{b}{c}{d}{e}{f}

1.4 F-Move with Multiplicity Indices

For categories with $N_{ab}^c > 1$, vertices carry multiplicity labels μ, ν:

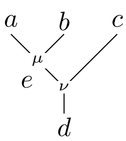


$$= \sum_{f,\mu',\nu'} \left(F_d^{abc}\right)_{ef}^{\mu\nu,\mu'\nu'}$$

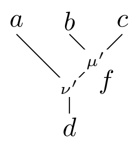


\Fmovemult{a}{b}{c}{d}{e}{f}{\mu}{\nu}

Individual trees with multiplicities for inline use:



\Ftreeleft{a}{b}{c}{d}{e}{\mu}{\nu}

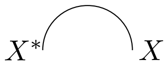


\Ftreeright{a}{b}{c}{d}{f}{\mu'}{\nu'}

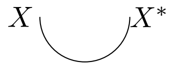
2 Duality: Cups and Caps

2.1 Evaluation and Coevaluation

Evaluation (Cup) **Coevaluation (Cap)**



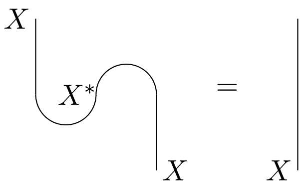
`\evalcup{X}`



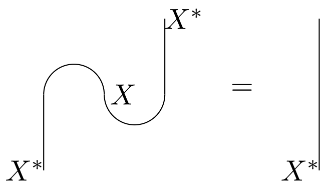
`\coevalcap{X}`

2.2 Zigzag Identities (Snake Equations)

The zigzag identities express that cups and caps are inverse:

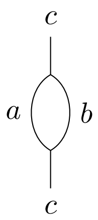


`\leftzigzag{X}`

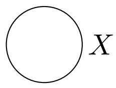


`\rightzigzag{X}`

2.3 Bigon and Quantum Dimension



`\bigon{a}{b}{c}`



`\qdimloop{X}`

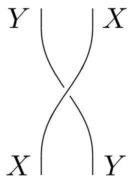
`\identitystrand{X}`

3 Braiding and R-Moves

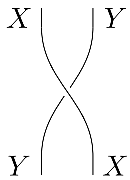
3.1 Braiding Crossings

Over-crossing (positive) and under-crossing (negative):

Over-crossing $c_{X,Y}$ **Under-crossing** $c^{-1}_{X,Y}$



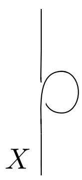
`\braidingover{X}{Y}`



`\braidingunder{X}{Y}`

3.2 Twist (Ribbon Element)

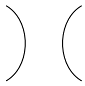
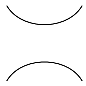
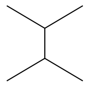
The twist encodes the topological spin:



`\twist{X}`

4 Temperley-Lieb Patterns

The Temperley-Lieb algebra generators for 2 strands:

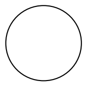
Identity	**Cup-Cap** (e_i)	**H-pattern**
		
`\TLidentity`	`\TLcupcap`	`\TLhorizontal`

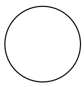
5 Trivalent Category Relations

For categories generated by a rotationally invariant trivalent vertex with parameters (d, b, t).

5.1 Loop Relation

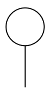
A closed loop evaluates to the quantum dimension d:

 $= d$ `\trivloopeq`

Or just the loop:  = `\trivloop`

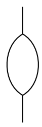
5.2 Lollipop (Forbidden Diagram)

In trivalent categories, lollipops vanish:

 $= 0$ `\trivlollipop`

5.3 Bigon Relation

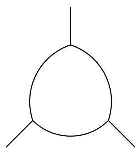
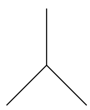
The bigon simplifies to b times the identity:

 $= b \cdot$

`\trivbigoneq`

5.4 Triangle Relation

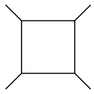
A triangle with external legs equals t times the trivalent vertex:

 $= t \cdot$ 

`\trivtriangleeq`

5.5 Square Diagram

The square face with four external legs:



`\trivsquare`

6 $\mathfrak{C}_4$ Basis Diagrams

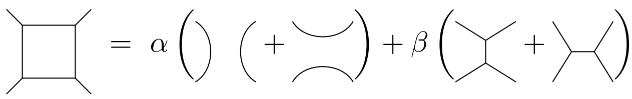
In a cubic trivalent category, these four diagrams form a basis for $\mathrm{Hom}(\mathbf{1}, X^{\otimes 4})$:

w_1 w_2 w_3 w_4

`\Cfourone` `\Cfourtwo` `\Cfourthree` `\Cfourfour`

6.1 Square Decomposition

The square decomposes as a linear combination of basis elements:

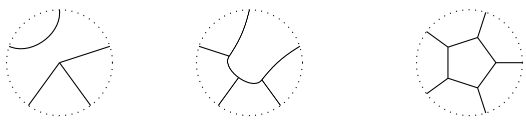


where $\alpha = \frac{b(b^2+bt-t^2)}{bd+t+dt}$ and $\beta = \frac{t^2(d+1)-b^2}{bd+t+dt}$.

7 $\mathfrak{C}_5$ Basis Diagrams

Representative patterns for 5 boundary points:

Star **Chain** **Pentagon**



`\Cfivestar` `\Cfivechain` `\Cfivepentagon`

8 $\mathfrak{C}_6$ Basis Diagrams

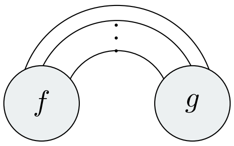
Representative patterns for 6 boundary points:

Three Bigons **Dumbbell** **Hexagon** **Pentafork**

`\Csixthreebigon` `\Csixdumbbell` `\Csixhexagon` `\Csixpentafork`

9 Inner Product Pairing

The bilinear inner product of two trivalent graphs f, g with n boundary points:

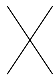
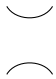
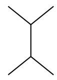
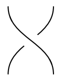
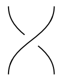
$\langle f, g \rangle =$ 

`\trivinnerprod{f}{g}`

10 2-Local Hamiltonian Diagrams

These diagrams are useful for constructing nearest-neighbor Hamiltonians on anyon chains.

10.1 Basic 2-Site Operators

Identity	**Swap**	**Cup-Cap**	**H-pattern**	**Braid**	**Braid^{-1}**
					
`\Htwoidentity`	`\Htwoswap`	`\Htwocupcap`	`\HtwoH`	`\Htwobraid`	`\Htwobraidinv`

10.2 Hopping Operators (Braid with Vacuum)

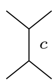
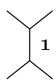
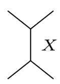
Hop operators have one strand as vacuum (gray) representing anyon hopping:

Hop Right	**Hop Left**	**Hop Right^{-1}**	**Hop Left^{-1}**
			
`\Htwohopright`	`\Htwohopleft`	`\Htwohoprghtinv`	`\Htwohopleftinv`

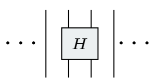
Example hopping Hamiltonian:

$H_{\text{hop}} = -t \sum_i \Big($  $+$  $\Big)_i$

10.3 Fusion with Intermediate Channel

		
`\Htwofusion{c}`	`\Htwofusion{\mathbf{1}}`	`\Htwofusion{X}`

10.4 Local Operator in Chain Context

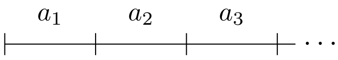


`\chainlocal{H}`

11 Anyon Chain

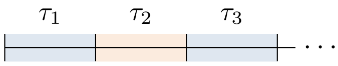
Anyons live in **intervals** (between lattice sites), not at vertices. Vertical tick marks indicate lattice sites:

11.1 Standard Chain (with tick marks)



`\anyonchain{a}`

11.2 Shaded Chain (intervals highlighted)



`\anyonchainshaded{\tau}`

12 Utility Macros

12.1 Circled Numbers

1	2	3	4	5
`\circlednum{1}`	`\circlednum{2}`	`\circlednum{3}`	`\circlednumcolor{red}{4}`	`\circlednumcolor{b`

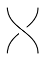
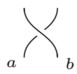
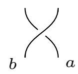
12.2 F-Symbol Diamond

For labeling F-symbol indices: α, β

`\Fdiamond{\alpha}`, `\Fdiamond{\beta}`

13 Compact Braiding for Hamiltonians

Compact (squatter) braids for use inside brackets:

Compact	**Compact Inv**	**Labeled**	**Labeled Inv**
			
`\braidcompact`	`\braidcompactinv`	`\braidlabeled{a}{b}`	`\braidlabeledinv{a}{b}`

Example in a Hamiltonian expression:

$$H = -\sum_i \left(\quad + \phi^{-1} \quad \right)_i$$

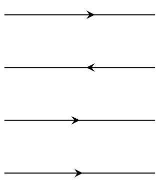
14 Color Definitions

Professional complementary color scheme (blue-orange with accents):

Name	Sample	Usage
AnyonBlue		Primary color
AnyonOrange		Complementary color
AnyonTeal		Accent color
AnyonCoral		Accent color
AnyonSlate		Dark emphasis
AnyonSilver		Subtle elements
LightGray		Backgrounds
MediumGray		Borders

15 TikZ Styles Reference

15.1 Arrow Styles



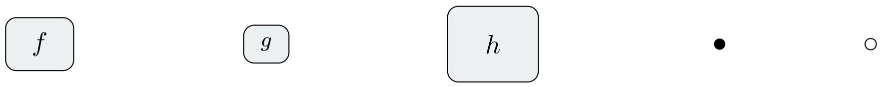
`->-` (arrow at 60%)

`-<-` (reverse at 60%)

`->>-` (arrow at 50%)

`->-` (arrow at 52%)

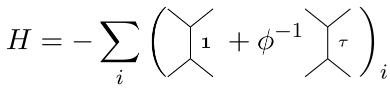
15.2 Box and Vertex Styles



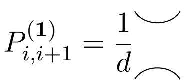
`morphism box` `morphism box small` `morphism box large` `fusion vertex` `fusion vertex empty`

16 Example: Golden Chain Hamiltonian

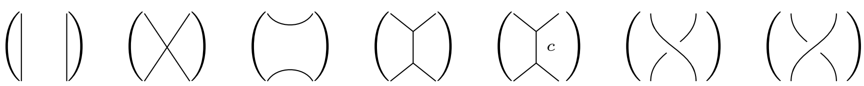
As an example, here is how to write a 2-local Hamiltonian term for the golden chain using these macros (note proper vertical centering in brackets):



where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.
The projector onto the trivial fusion channel at sites $i, i+1$ is:

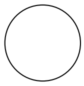


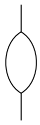
All 2-local operators side by side for comparison:



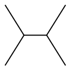
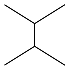
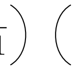
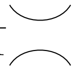
17 Example: Trivalent Category Calculation

In the Fibonacci category with $d = \phi$, $b = 1$, $t = \frac{d-2}{d-1}$:

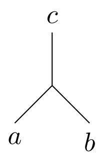
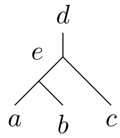
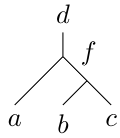
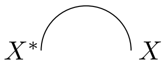
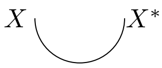
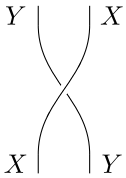
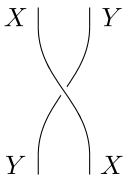
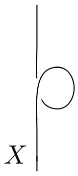
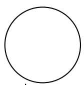
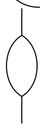
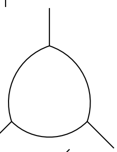
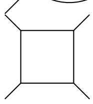
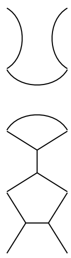
 $= d = \phi \approx 1.618$

 $= b \cdot \Big| = 1 \cdot \Big|$

The four basis diagrams in $\mathfrak{C}_4$ satisfy:

 $-$  $+ \frac{1}{d+1}$  $+ \frac{1}{d-1}$  $= 0$

A Quick Reference Card

Category	Macro	Output
Vertices	`\trivalentvertex{a}{b}{c}`	
	`\fuselefttree{a}{b}{e}{c}{d}`	
	`\fuserighttree{a}{b}{f}{c}{d}`	
Duality	`\evalcup{X}`	
	`\coevalcap{X}`	
Braiding	`\braidingover{X}{Y}`	
	`\braidingunder{X}{Y}`	
	`\twist{X}`	
Trivalent	`\trivloop`	
	`\trivbigon`	
	`\trivtriangle`	
	`\trivsquare`	
$\mathfrak{C}_4$ Basis	`\Cfourone`	
	`\Cfourtwo`	
	`\Cfourthree`	
	`\Cfourfour`	
	`\Htwoidentity`