

# TikZ Style Library Test

## Fusion Category Diagrams for Mobile Anyons

Test Document

December 25, 2025

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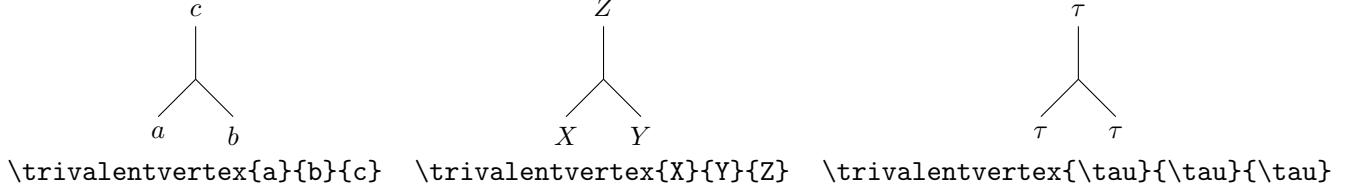
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# 1 Basic Fusion Category Diagrams

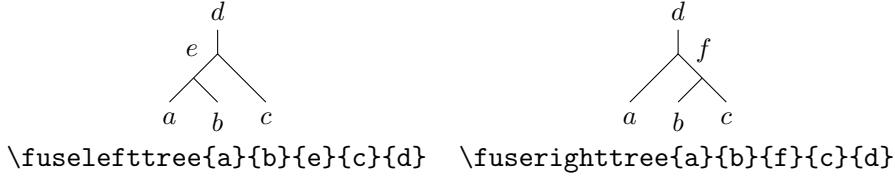
## 1.1 Trivalent Vertices

The basic trivalent vertex represents fusion/splitting:



## 1.2 Fusion Trees

Left-associative and right-associative fusion trees:



## 1.3 F-Move (Associator)

The F-move relates different fusion orders:

The diagram shows two fusion trees connected by an equals sign. The left tree has nodes  $a$ ,  $b$ ,  $c$  at the top, followed by  $e$  and  $d$ . The right tree has nodes  $a$ ,  $b$ ,  $c$  at the top, followed by  $f$  and  $d$ . Between them is the equation  $= \sum_f (F_d^{abc})_{fe}$ . Below the trees is the LaTeX code:  $\backslash Fmoveequation\{a\}\{b\}\{c\}\{d\}\{e\}\{f\}$ .

# 2 Duality: Cups and Caps

## 2.1 Evaluation and Coevaluation

Evaluation (Cup)    Coevaluation (Cap)

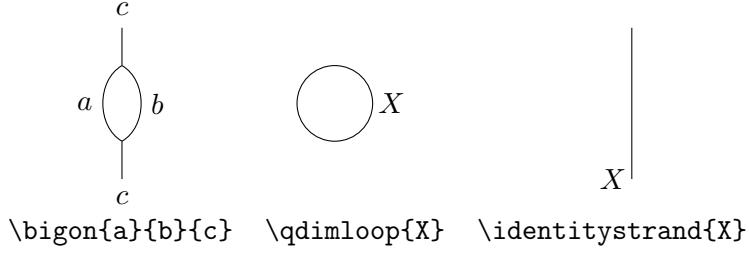
The diagram shows two dual morphisms. The left morphism, labeled  $\backslash evalcup\{X\}$ , is a cup shape with a curved arrow from  $X^*$  to  $X$ . The right morphism, labeled  $\backslash coevalcap\{X\}$ , is a cap shape with a curved arrow from  $X$  to  $X^*$ .

## 2.2 Zigzag Identities (Snake Equations)

The zigzag identities express that cups and caps are inverse:

The diagram shows two commutative square equations. The left square, labeled  $\backslash leftzigzag\{X\}$ , has nodes  $X$  at the top and bottom, and  $X^*$  at the left and right. The right square, labeled  $\backslash rightzigzag\{X\}$ , has nodes  $X$  at the top and bottom, and  $X^*$  at the left and right.

## 2.3 Bigon and Quantum Dimension

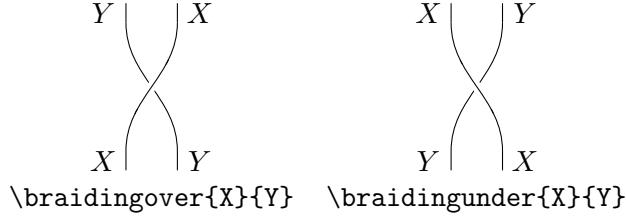


## 3 Braiding and R-Moves

### 3.1 Braiding Crossings

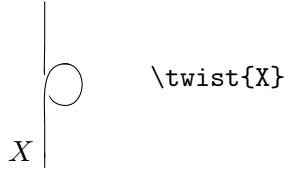
Over-crossing (positive) and under-crossing (negative):

Over-crossing  $c_{X,Y}$       Under-crossing  $c_{X,Y}^{-1}$



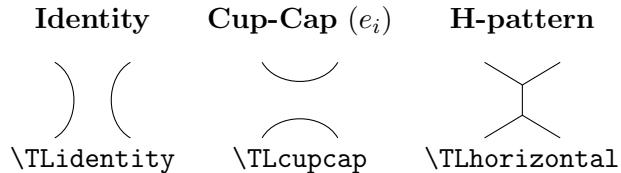
### 3.2 Twist (Ribbon Element)

The twist encodes the topological spin:



## 4 Temperley-Lieb Patterns

The Temperley-Lieb algebra generators for 2 strands:



## 5 Trivalent Category Relations

For categories generated by a rotationally invariant trivalent vertex with parameters  $(d, b, t)$ .

### 5.1 Loop Relation

A closed loop evaluates to the quantum dimension  $d$ :

$$\text{loop} = d \qquad \text{trivloopeq}$$

Or just the loop:  = \trivloop

## 5.2 Lollipop (Forbidden Diagram)

In trivalent categories, lollipops vanish:

$$\text{Diagram: } \text{\trivloop} = 0 \quad \text{\trivlollipop}$$

## 5.3 Bigon Relation

The bigon simplifies to  $b$  times the identity:

$$\text{Diagram: } \text{\trivbigoneq}$$

## 5.4 Triangle Relation

A triangle with external legs equals  $t$  times the trivalent vertex:

$$\text{Diagram: } \text{\trivtriangleeq}$$

## 5.5 Square Diagram

The square face with four external legs:

$$\text{Diagram: } \text{\trivsquare}$$

# 6 $\mathfrak{C}_4$ Basis Diagrams

In a cubic trivalent category, these four diagrams form a basis for  $\text{Hom}(\mathbf{1}, X^{\otimes 4})$ :

$w_1$	$w_2$	$w_3$	$w_4$
			
\Cfourone	\Cfourtwo	\Cfourthree	\Cfourfour

## 6.1 Square Decomposition

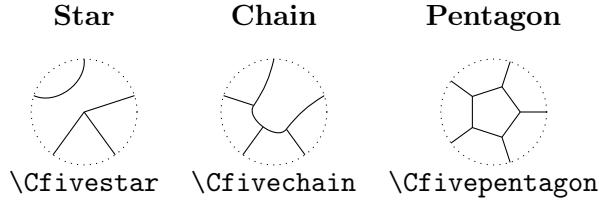
The square decomposes as a linear combination of basis elements:

$$\text{Diagram: } \text{\trivsquare} = \alpha \left( \text{\trivloop} + \text{\trivbigoneq} \right) + \beta \left( \text{\trivtriangleeq} + \text{\trivlollipop} \right)$$

where  $\alpha = \frac{b(b^2+bt-t^2)}{bd+t+dt}$  and  $\beta = \frac{t^2(d+1)-b^2}{bd+t+dt}$ .

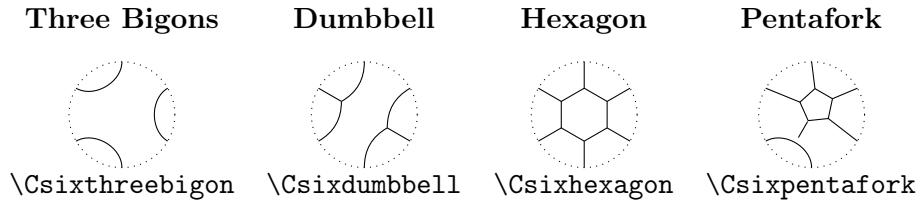
## 7 $\mathfrak{C}_5$ Basis Diagrams

Representative patterns for 5 boundary points:



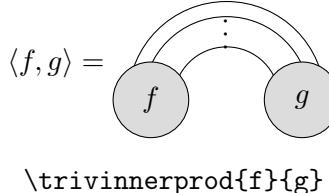
## 8 $\mathfrak{C}_6$ Basis Diagrams

Representative patterns for 6 boundary points:



## 9 Inner Product Pairing

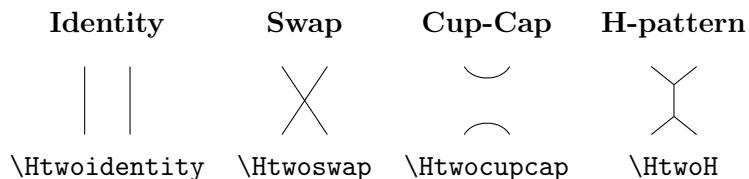
The bilinear inner product of two trivalent graphs  $f, g$  with  $n$  boundary points:



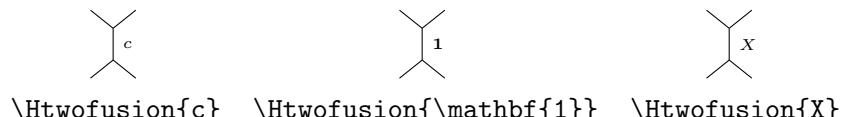
## 10 2-Local Hamiltonian Diagrams

These diagrams are useful for constructing nearest-neighbor Hamiltonians on anyon chains.

### 10.1 Basic 2-Site Operators



### 10.2 Fusion with Intermediate Channel



### 10.3 Local Operator in Chain Context



## 11 Anyon Chain

A simple anyon chain template:



`\anyonchain{a}`

## 12 Utility Macros

### 12.1 Circled Numbers



`\circlednum{1} \circlednum{2} \circlednum{3} \circlednumcolor{red}{4} \circlednumcolor{blue}{5}`

### 12.2 F-Symbol Diamond

For labeling F-symbol indices:  $\diamondsuit^\alpha$ ,  $\diamondsuit^\beta$

`\Fdiamond{\alpha}, \Fdiamond{\beta}`

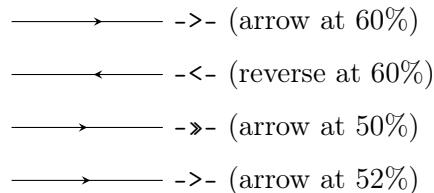
## 13 Color Definitions

The following colors are defined for use in diagrams:

<code>AnyonRed</code>	Primary anyon color
<code>AnyonBlue</code>	Secondary anyon color
<code>AnyonGreen</code>	Tertiary anyon color
<code>AnyonOrange</code>	Quaternary anyon color
<code>LightGray</code>	Background for boxes
<code>MediumGray</code>	Borders and accents

## 14 TikZ Styles Reference

### 14.1 Arrow Styles



### 14.2 Box and Vertex Styles



`morphism boxmorphism box smallmorphism box largefusion fusion vertex empty`

## 15 Example: Golden Chain Hamiltonian

As an example, here is how to write a 2-local Hamiltonian term for the golden chain using these macros:

$$H = - \sum_i \left( \text{Diagram}_1 + \phi^{-1} \text{Diagram}_\tau \right)_i$$

where  $\phi = \frac{1+\sqrt{5}}{2}$  is the golden ratio.

The projector onto the trivial fusion channel at sites  $i, i+1$  is:

$$P_{i,i+1}^{(1)} = \frac{1}{d} \text{Diagram}$$

## 16 Example: Trivalent Category Calculation

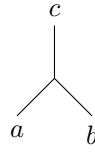
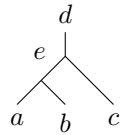
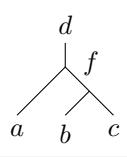
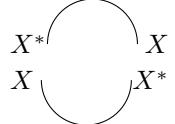
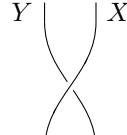
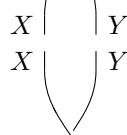
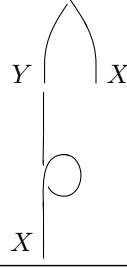
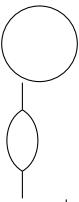
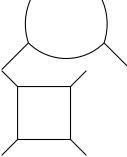
In the Fibonacci category with  $d = \phi$ ,  $b = 1$ ,  $t = \frac{d-2}{d-1}$ :

$$\begin{aligned} \text{Diagram} &= d = \phi \approx 1.618 \\ \text{Diagram} &= b \cdot | = 1 \cdot | \end{aligned}$$

The four basis diagrams in  $\mathfrak{C}_4$  satisfy:

$$\text{Diagram} - \text{Diagram} + \frac{1}{d+1} \left( + \frac{1}{d-1} \text{Diagram} \right) = 0$$

## A Quick Reference Card

Category	Macro	Output
Vertices	<code>\trivalentvertex{a}{b}{c}</code>	
	<code>\fuselefttree{a}{b}{e}{c}{d}</code>	
	<code>\fuserighttree{a}{b}{f}{c}{d}</code>	
Duality	<code>\evalcup{X}</code>	
	<code>\coevalcap{X}</code>	
Braiding	<code>\braidingover{X}{Y}</code>	
	<code>\braidingunder{X}{Y}</code>	
	<code>\twist{X}</code>	
Trivalent	<code>\trivloop</code>	
	<code>\trivbigon</code>	
$\mathfrak{C}_4$ Basis	<code>\trivtriangle</code>	
	<code>\trivsquare</code>	
$\mathfrak{C}_4$ Basis	<code>\Cfourone</code>	
	<code>\Cfourtwo</code>	
	<code>\Cfourthree</code>	
	<code>\Cfourfour</code>	
9		
<code>\Htwoidentity</code>		