Physics-Informed Neural Networks for Options Pricing in Stochastic Markets

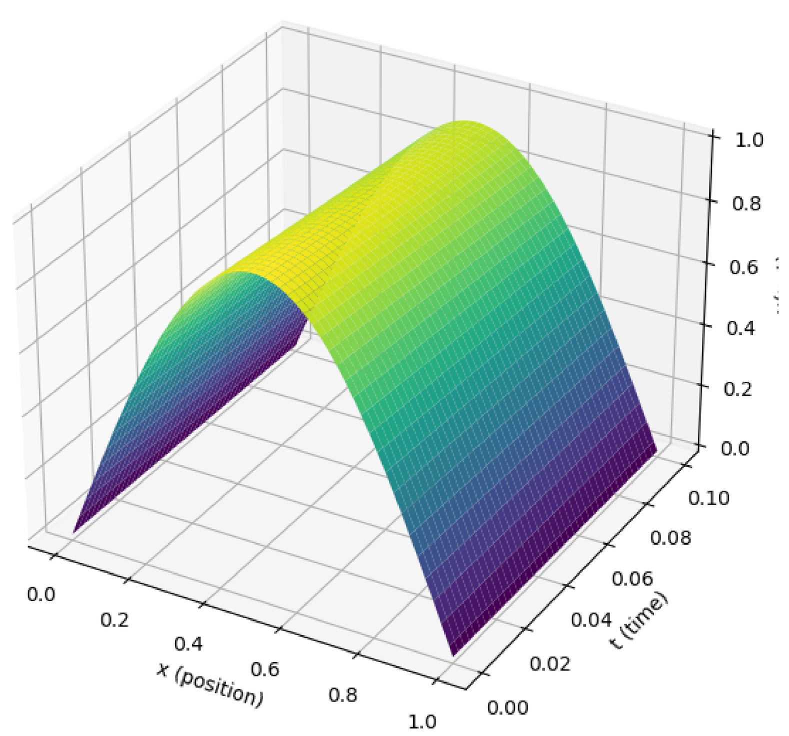
Tobias Safie, Niharika Sravan
Drexel University, Department of Physics

Drexel University Undergraduate Research & Enrichment Programs
Pennoni Honors College

STAR SCHOLARS

Introduction

- **PDEs** describe dynamics across physics and finance, from stellar mergers to market volatility.
- **Classical Solvers** (e.g., Runge–Kutta 4) are accurate but computationally costly, especially in high dimensions.
- **PINNs** embed PDEs into neural networks, enforcing both physics and boundary conditions during training.
- **Our Goal:** benchmark PINN solvers for option pricing (Black–Scholes, Heston) against numerical methods.



$$\frac{\partial q}{\partial t} = \nu \frac{\partial^2 q}{\partial x^2}$$

Figure 1.
(a) 1-D diffusion partial differential equation.
(b) 3D surface plot of its numerical solution.

Methods

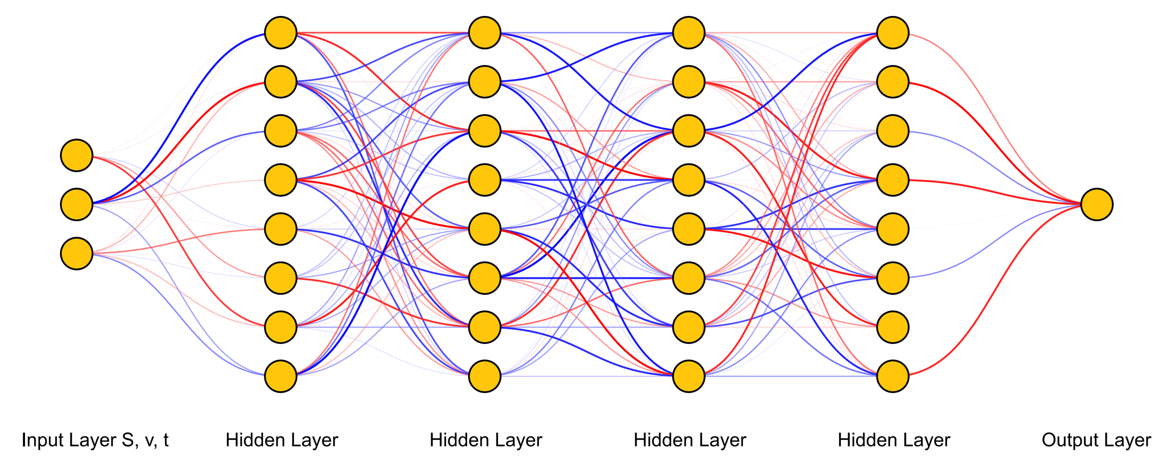


Figure 2.
Toy model of our Heston PINN.

- **Baseline:**
  - Implemented RK4 solvers for PDE test cases (diffusion; Black-Scholes)
- **PINN Design:**
  - Constructed feedforward neural nets with PDE terms hard-coded into loss function.
- **Training:**
  - Thousands of lightweight epochs; loss combined PDE, boundary, and initial conditions.
- **Hyperparameter Sweeps:**
  - 25+ sweeps per search (random + Bayesian) to optimize weights and architecture.

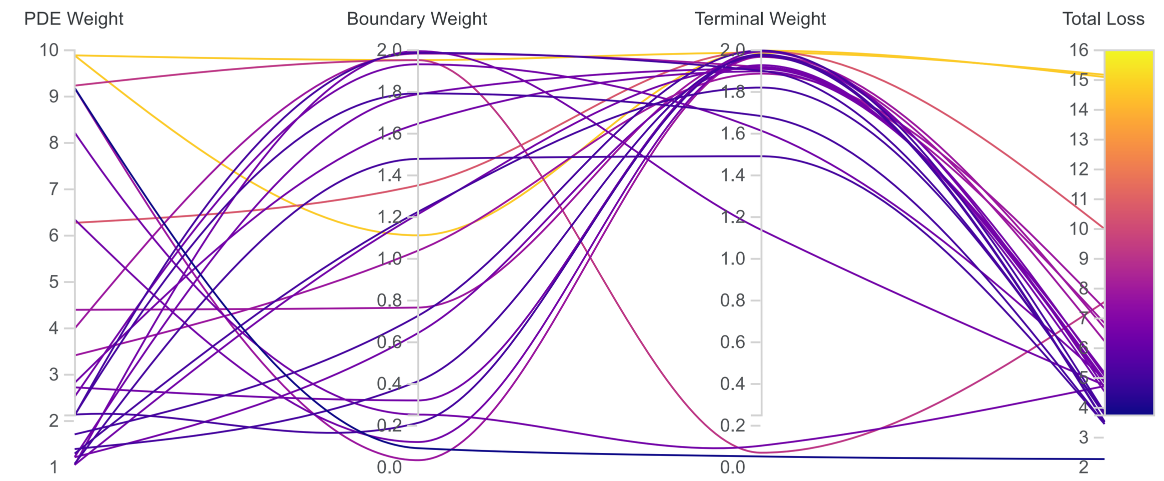


Figure 3.
Parallel coordinates - Weights & Biases sweep .

Results

- **Black-Scholes:**
  - 72x faster than RK4.
  - ~0.1% solution error.
  - Continuous resolution vs. RK4's discrete 1000 grid points.

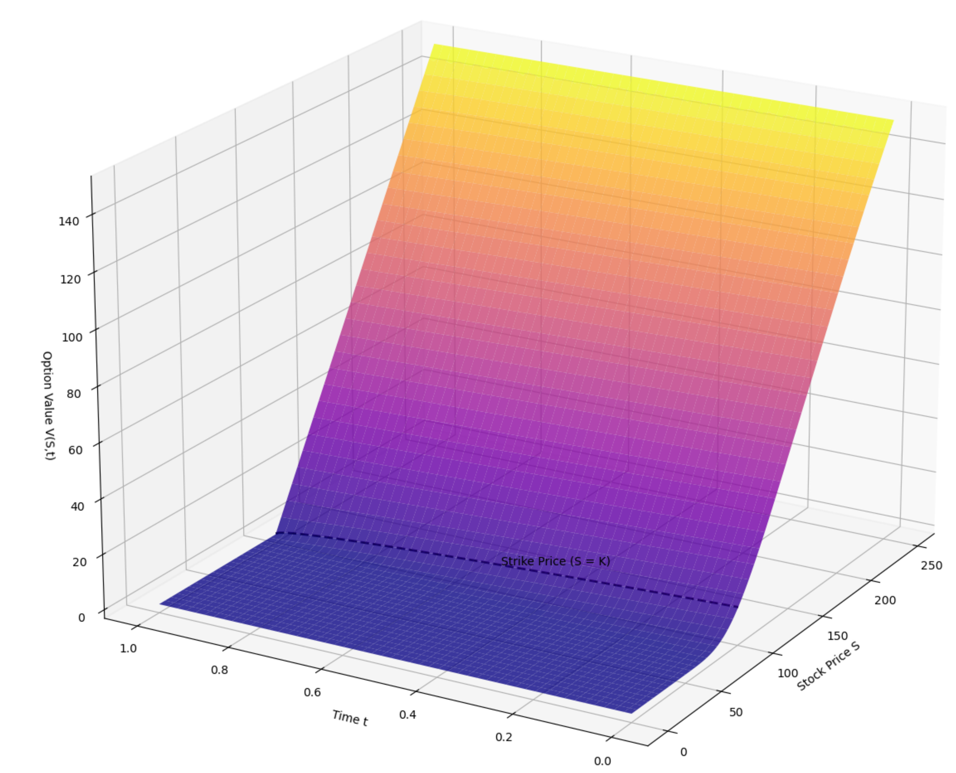


Figure 4.
Surface plot - Black-Scholes solution via a PINN.

- **Heston:**
  - Not solvable by standard integration (requires costly Fourier methods).
  - ~0.64% solution error at benchmark points (terminal, boundary, interior).
  - PINN predicted thousands of values in 0.6 sec.

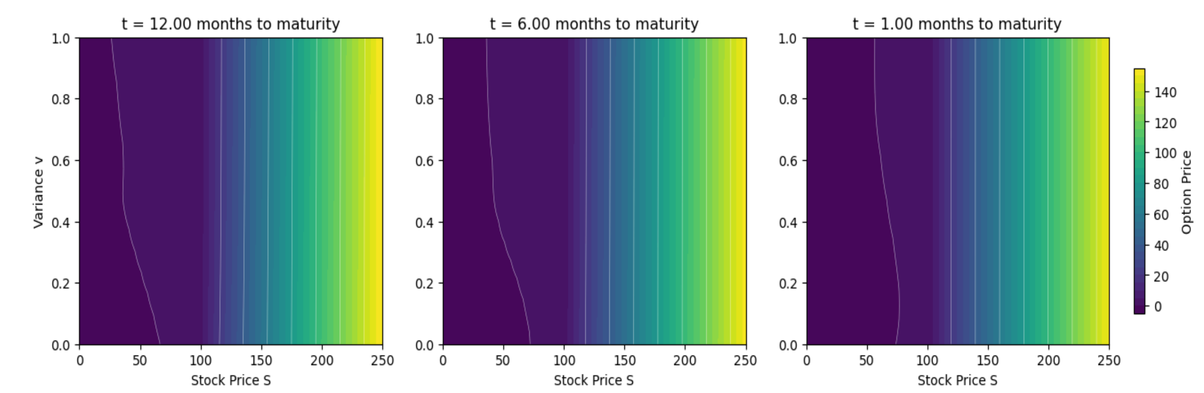


Figure 5.
Heatmaps - Heston PINN solution for different times to maturity.

Conclusion

- **PINNs** reproduce option prices with high accuracy and vastly reduced computational cost and time.
- **Continuous Resolution** makes them far faster, more flexible, and precise than fixed-grid solvers.
- **The Heston Model** shows potential for tackling high-dimensional PDEs found in physics and finance that would previously be too expensive to solve clasically or analytically.

Key PDEs

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

Figure 6.
Black-Scholes PDE.

$$\begin{aligned} 0 = {} & V_t \\ & + \tfrac{1}{2} v S^2 \, V_{SS} \\ & + \rho\sigma v S \, V_{Sv} \\ & + \tfrac{1}{2}\sigma^2 v \, V_{vv} \\ & + rS \, V_S \\ & + \kappa(\theta - v) \, V_v \\ & - rV \end{aligned}$$

Figure 7.
Heston PDE.