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Disturbance Rejection Using the Internal Model Principle

Persistently acting disturbances are not compensated asymptotically by a constant state feedback controller. Therefore, the design of disturbance rejecting controllers requires additional measures. If the disturbances can be modelled by a suitable signal process (*i.e.*, the disturbances can be represented as the solutions of linear differential equations with constant coefficients) two approaches exist for their rejection.

The first one was suggested by Johnson in [35]. By a feedforward of the states of the disturbance signal process its eigenvalues are rendered unobservable in the controlled outputs. This achieves an asymptotic rejection of the disturbances. As the states of the signal process are not measurable in general, an observer is required for their reconstruction. However, due to the feedforward control this approach is neither robust to changing input locations of the disturbances nor to uncertain parameters of the system.

The second approach to disturbance rejection was suggested by Davison (see [10, 11]). It constitutes a generalization of the classical controller with integral action, which only rejects constant disturbances, to general forms of disturbance signals. A model of the disturbances is added to the plant and this disturbance model is driven by the difference between the tracking signal and the corresponding output of the system. If this augmented plant is stabilized by a feedback controller the tracking error does not contain the modelled signal forms in the steady state. If it did, resonance would cause increasing states of the disturbance model, thus contradicting the stability of the closed-loop system. Therefore, independent of where they enter the system, the disturbances are rejected asymptotically. Asymptotic disturbance rejection is also assured for modelling errors that do not endanger closed-loop stability. Consequently, Davison's approach assures robust disturbance rejection. It requires, however, that the controlled variables are part of the measurements, which is not necessary when using Johnson's approach. Since the disturbance model is a part of the closed-loop system Davison's approach is called the *internal model principle*. Davison's approach not only guarantees the asymptotic rejection of all modelled disturbance signals, but also the asymptotic tracking of such reference signals. This can be a drawback, because asymptotic tracking and disturbance rejection are rarely required for the same type of signals and it may entail undesired side effects.

In Section 7.1 an extension of Davison's approach is presented in the time domain. To circumvent the possibly existing negative side effects of a joint disturbance and reference model, the signal model is not driven by the tracking error but by the controlled outputs and a suitable feedforward of the reference inputs. This additional degree of freedom is used to formulate the driven signal model as an observer. This has the effect that all modelled disturbance signals are asymptotically rejected in the controlled outputs, but the signal model does not influence the tracking behaviour. After the initial observer errors have vanished, the tracking behaviour is the same as if constant state feedback had been applied. This also allows a simple prevention of controller windup due to input saturation (see Section 4.5). In Section 7.2 the frequency-domain parameterization of the stabilizing state feedback of the augmented system is presented. Section 7.3 contains the frequency-domain parameterization of the observer for the non-augmented system, which provides sufficient information as the states of the signal model are directly measurable. The frequency-domain design of observer-based compensators with signal models for disturbance rejection can be found in Section 7.4.

7.1 Time-domain Approach to Disturbance Rejection

Considered are systems with the state equations

$$\dot{x}(t) = Ax(t) + Bu(t) + B_d d(t), \tag{7.1}$$

$$y(t) = Cx(t) + D_d d(t), \tag{7.2}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ is the input, $y \in \mathbb{R}^m$ with $m \geq p$ is the measurement, and $d \in \mathbb{R}^\rho$ is an unmeasurable disturbance. It is assumed that the pair (A, B) is controllable and the pair (C, A) is observable. In view of designing reduced-order observers of the order $n_O = n - \kappa$, $0 \leq \kappa \leq m$, the output vector y of these systems is arranged according to

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(t) + \begin{bmatrix} D_{d1} \\ D_{d2} \end{bmatrix} d(t). \tag{7.3}$$

Here, $y_2 \in \mathbb{R}^\kappa$ with $0 \leq \kappa \leq m$ contains the measurements directly used in the construction of the estimate $\hat{x}$ and $y_1 \in \mathbb{R}^{m-\kappa}$ contains the remaining $m - \kappa$ measurements (see Section 3.1). As in Chapter 2 the controlled output $y_c \in \mathbb{R}^p$ is assumed to be measurable. This is a necessity in Davison's approach because the model of the assumed signal process is driven by y_c. Therefore, a $p \times m$ selection matrix Ξ (see (2.3)) exists, so that

$$y_c(t) = \Xi y(t) = \Xi Cx(t) + \Xi D_d d(t) = C_c x(t) + D_{cd} d(t). \tag{7.4}$$