University of Applied Sciences and Arts Northwestern Switzerland
School of Business

Small Area Estimation (Part II)

Short Course – Institute of Statistics, Republic of Albania

Tobias Schoch, Feburary 17 and 18, 2022

About

This slide deck has been prepared for the “Short Course on Small Area Estimation with R” at the Institute of Statistics of the Republic of Albania on February 17 and 18, 2022.

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Outline

1 Introduction

Overview of the slide deck

This part of the slide deck (i.e., Part II) is dedicated to the R packages and applications

- **R package overviews**
  - CRAN Task View OfficialStatistics (Link or Link)
  - GitHub Awesome Official Statistics Software (Link)
- **Relevant files for this part in the archive "SAEcourse.zip"**
  - `/lecture/snippets.R` and `data.R` (code snippets and data)
  - `/software/methods.R` (utility functions that extend the R package `sae`; useful but not necessary)
  - `/application/application.pdf` (exercises and applications)

[In Part I of the slide deck (introduction), you can find instructions how to download the archive]

2 R Packages for SAE

sae – Small Area Estimation (Molina & Marhuenda, 2020)

- Models
  - Area-level model (Fay & Herriot and with spatial/ temporal correlations)
  - Unit-level model (Battese et al., 1988)
- Estimation methods (variance)
  - Maximum likelihood
  - Restricted maximum likelihood
  - Fay & Herriot
- MSE estimation
  - Analytic second-order approximation
  - Parametric bootstrap (and non-parametric bootstrap for spatial FH model)

The package is rather basic; there are only a few utility function

2 R Packages for SAE (ctd.)

`emdi` – Estimating and Mapping Disaggregated Indicators

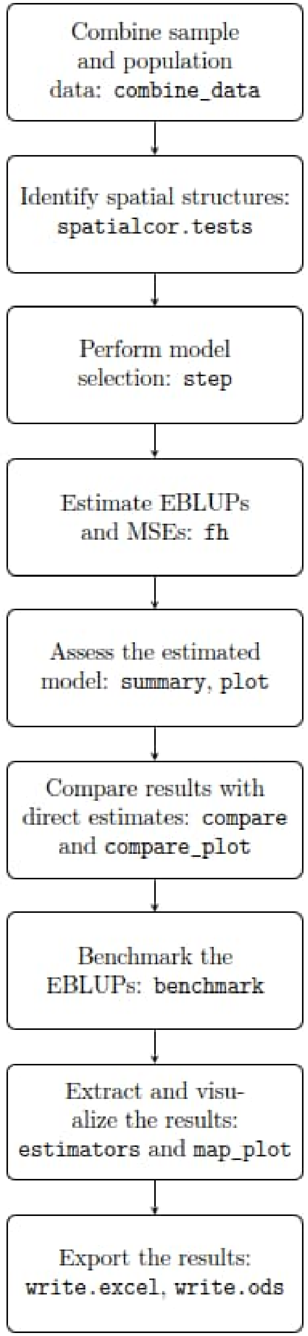
(Harmening et al., 2022; Kreutzmann et al., 2019)

- "A Framework for Producing Small Area Estimates Based on Area-Level Models in R"
  - Production framework
  - Wrapper (combines other packages)
  - Original package supported only the area-level model; since version 1.1.7 (March 2020) also unit-level model
- Focus on estimating indicators (not only mean and total)
- Methods
  - Basic models + spatial correlation + outlier robustness (FH model)
  - Transformations (log, arcsin, etc.) ⇨ important for handling indicators
  - Measurement errors (in the explanatory variables)

2 R Packages for SAE (ctd.)

emdi (ctd.)

- Package implements the entire "production chain": estimation, summary, diagnostic, prediction, benchmarks, maps, and export (to Microsoft Excel)
- Inspired by GSBPM: Generic Statistical Business Process Model (but does not follow GSBPM)
- Disadvantage
  - Large and rather convoluted package
  - It imports dozens of packages (and their dependencies)
  - For the accompanying article (J Stat Software), the authors relied on 136 packages (in my humble opinion: this is way too much)



2 R Packages for SAE

`rsae` – Robust Small Area Estimation (Schoch, 2014)

- Models
  - Unit-level model (Battese et al., 1988)
- Estimation methods (variance and regression)
  - Maximum likelihood
  - Huber M-estimator
- MSE estimation
  - Parametric bootstrap
- Unkept promise
  - "Robust methods for area-level model will be implemented" (claim) …
  - This was not case (yet!). The code base is ready but not tested.

3 SAIPE – Our Application – Brief Brush-Up

- **Goal:** Estimates of child poverty rates for the 51 US states (2005)
- CPS data (survey) ⇨ direct estimate of poverty rate (rather unreliable)
- Administrative data (tax, census) ⇨ auxiliary information (model)

```
> source("saipe.R") # data are stored as code
> head(dat, 3)
    prIRS     nfIRS   prCensus        yi       vi     state
1 23.1582   14.6657    11.8464   19.4400   3.0371        AL
2 15.1852   10.9282     7.6478   11.0042   2.2330        AK
3 19.5926   19.3337     9.0293   17.4417   2.6003        AZ
```

Explanatory variables

prIRS	poverty rate (tax data)
nfIRS	non filer rate (tax data)
prCensus	poverty rate (census)

Survey estimates (CPS data)

yi	direct estimator of poverty rate
vi	variance of direct estimator

3 SAIPE – Our Application (ctd.)

- **Model**

  Fay-Herriot model (for $i = 1, \dots, 51$ states)

$$y_i = \beta_0 + \beta_1 \,\mathrm{prIRS}_i + \beta_2 \,\mathrm{nfIRS}_i + \beta_3 \,\mathrm{prCensus}_i + u_i + e_i,$$

  where

  – $u_i \sim N(0, v_i)$, the v_i's (variance of direct estimator) are known

  – $e_i \sim N(0, \sigma^2)$, the variance σ^2 is unknown

- **Estimators**

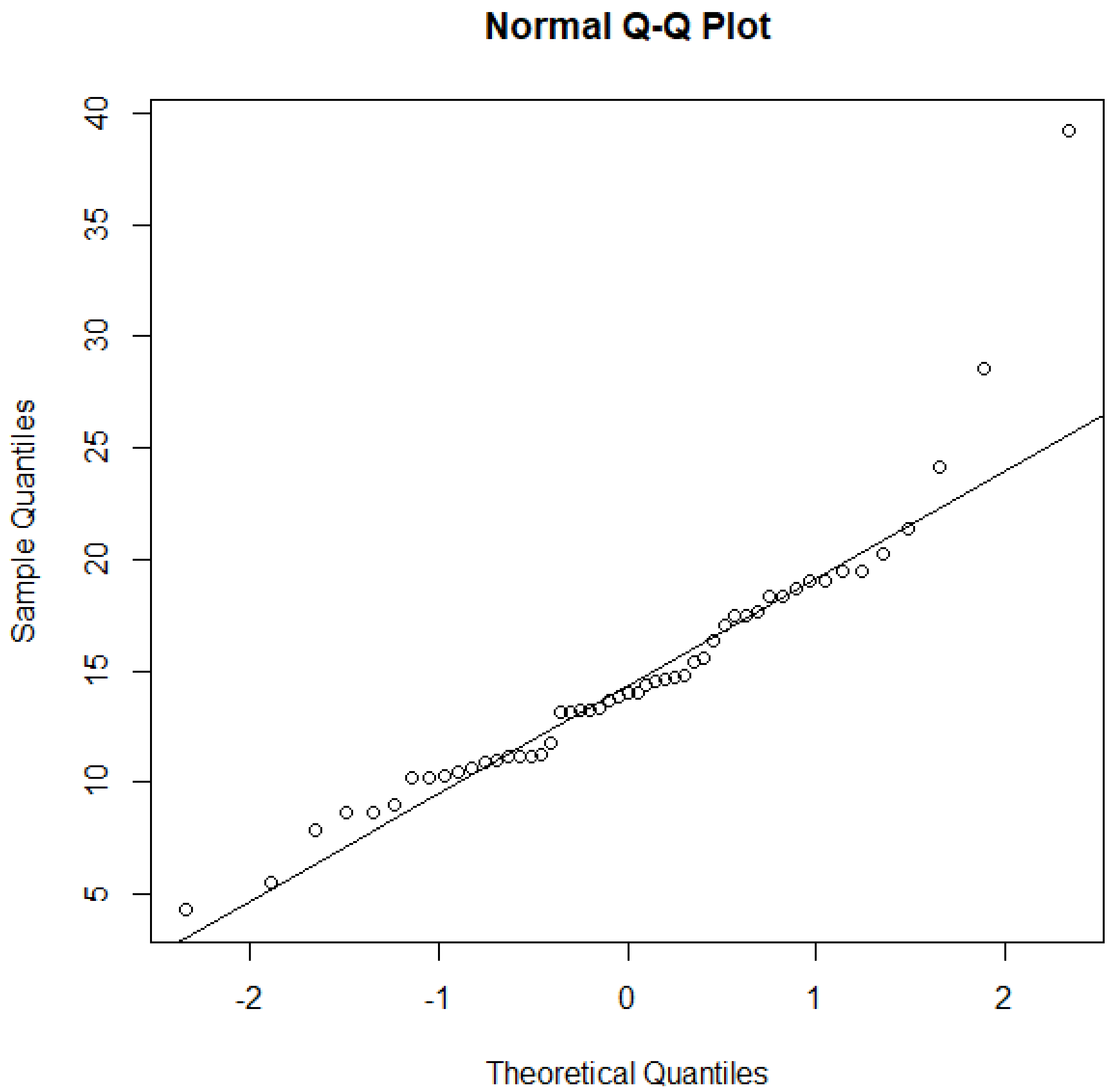
  – $\beta = (\beta_0, \dots, \beta_3)^T$ is estimated by weighted least squares

  – σ^2 is estimated by REML

3 SAIPE – Our Application (ctd.)

Normality assumption

- $u_i \sim N(0, v_i)$ cannot be checked (it is plausible by the central limit theorem)
- $e_i \sim N(0, \sigma^2)$ can be checked having fitted the model (⇨ diagnostics, later)
- At this stage, we can study the distribution of the direct estimators y_i `qqnorm(dat$yi)` and `qqline(dat$yi)`



3 SAIPE – Our Application (ctd.)

Some general remarks

- We are grateful to **William R. Bell (US Census Bureau)** for making the 2005 SAIPE data available.
- For more current datasets see https://www.census.gov/programs-surveys/saipe.html
- The SAIPE data are **"ready to use"** because the US Census Bureau prepared and compiled the data.
  - Computation of the direct estimator and variances (CPS survey)
  - The variances have been processed (generalized variance functions)
  - The US Census Bureau selected the auxiliary datasets (Census, IRS tax data) and the relevant variables
- **In practice, we need to select the potential datasets and variables** (and study them in the context of our model)

4 R Package sae

Step 0. Load the package

```
> library(sae)
```

Step 1. Fit the Fay-Herriot model

```
> m <- eblupFH(yi ~ prIRS + nfIRS + prCensus, vardir = vi,
               data = dat)
```

- By default, `method = "REML"` (alternatives: `"ML"` or `"FH"`)
- Argument `vardir` specifies the variance of the direct estimator (the v_i's)
- The function returns a list with slots
  - `eblup` (matrix with estimates $\hat{\theta}_{\mathrm{i}}^{\mathrm{EBLUP}}$ for $i = 1, \ldots, n$)
  - `fit` (list with entries `method`, `convergence`, `iterations`, `estcoef`, `refvar`, `goodness`)

4 R Package sae (ctd.)

Step 1. Fit the Fay-Herriot model (ctd.)

```
> m$fit
```

Part 1

```
$method
[1] "REML"

$convergence
[1] TRUE

$iterations
[1] 6
```

Part 2

```
$estcoef
                  beta std.error   tvalue     pvalue
(Intercept) -4.156450 1.5339727 -2.70999 6.7364e-03
prIRS        0.226095 0.1512419  1.49426 1.3493e-01
nfIRS        0.870038 0.1435364  6.06150 1.3489e-09
prCensus     0.436531 0.1817465  2.40170 1.6311e-02
```

(estimated coefficients $\hat{\beta}$)

```
$refvar
[1] 3.922967
```

(estimated variance $\hat{\sigma}^2$)

```
$goodness
  loglike       AIC       BIC       KIC
-118.1490  246.2980  255.9571  251.2980
```

4 R Package sae (ctd.)

Step 1. Fit the Fay-Herriot model (ctd.)

```
> mysummary(m)

              Estimate   Std.Err  t value   Pr(>|t|)
(Intercept) -4.15645   1.53397  -2.7096   0.006736 **
prIRS        0.22610   0.15124   1.4949   0.134934
nfIRS        0.87004   0.14354   6.0615  1.349e-09 ***
prCensus     0.43653   0.18175   2.4019   0.016312 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The `mysummary()` function is not part of the `sae` package; see `methods.R`

4 R Package sae (ctd.)

Step 2. Model selection: Information criteria

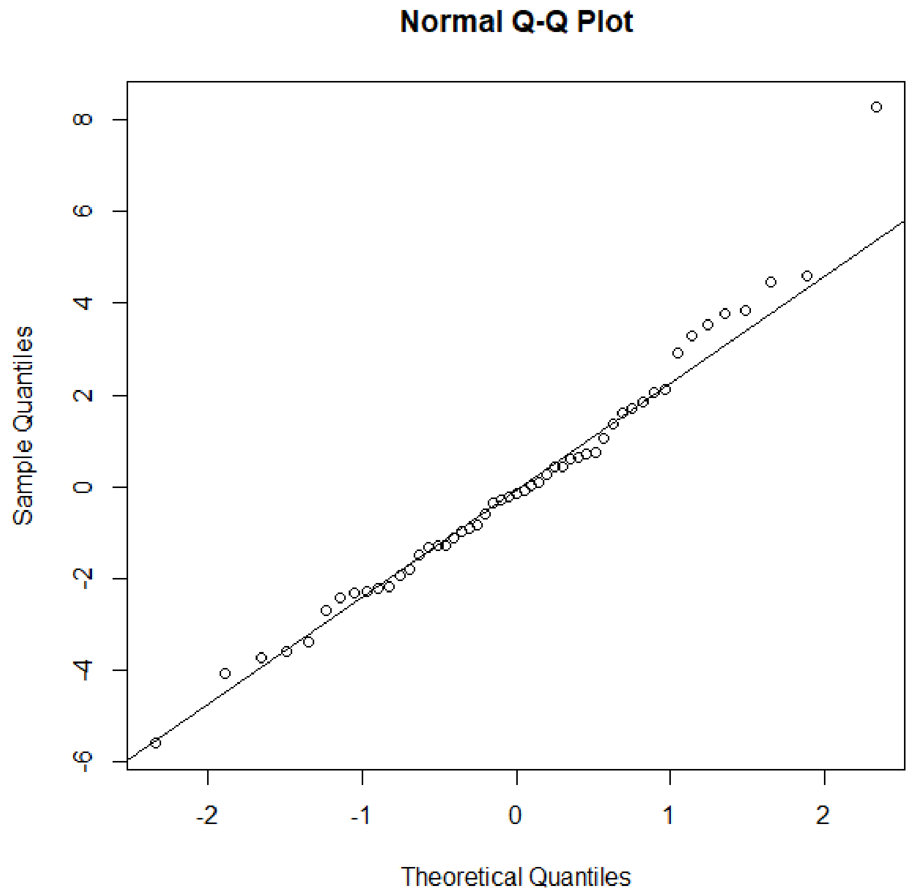
```
> m$fit$goodness                                          # main model
  loglike         AIC         BIC         KIC
-118.1490    246.2980    255.9571    251.2980

> eblupFH(yi ~ nfIRS + prCensus, vardir = vi,             # 2nd model
         data = dat)$fit$goodness
  loglike         AIC         BIC         KIC
-119.3031    246.6062    254.3335    250.6062
```

- In terms of AIC, the main model is superior. Regarding BIC and KIC, the 2nd model is better ⇨ not conclusive
- See Marhuenda et al. (2014) for more on information criteria

4 R Package sae (ctd.)

Step 3. Rudimentary diagnostics

```
> tmp <- myFH(yi ~ prIRS + nfIRS + prCensus, vardir = vi,
        data = dat)
> qqnorm(tmp$fit$residuals)
> qqline(tmp$fit$residuals)
```

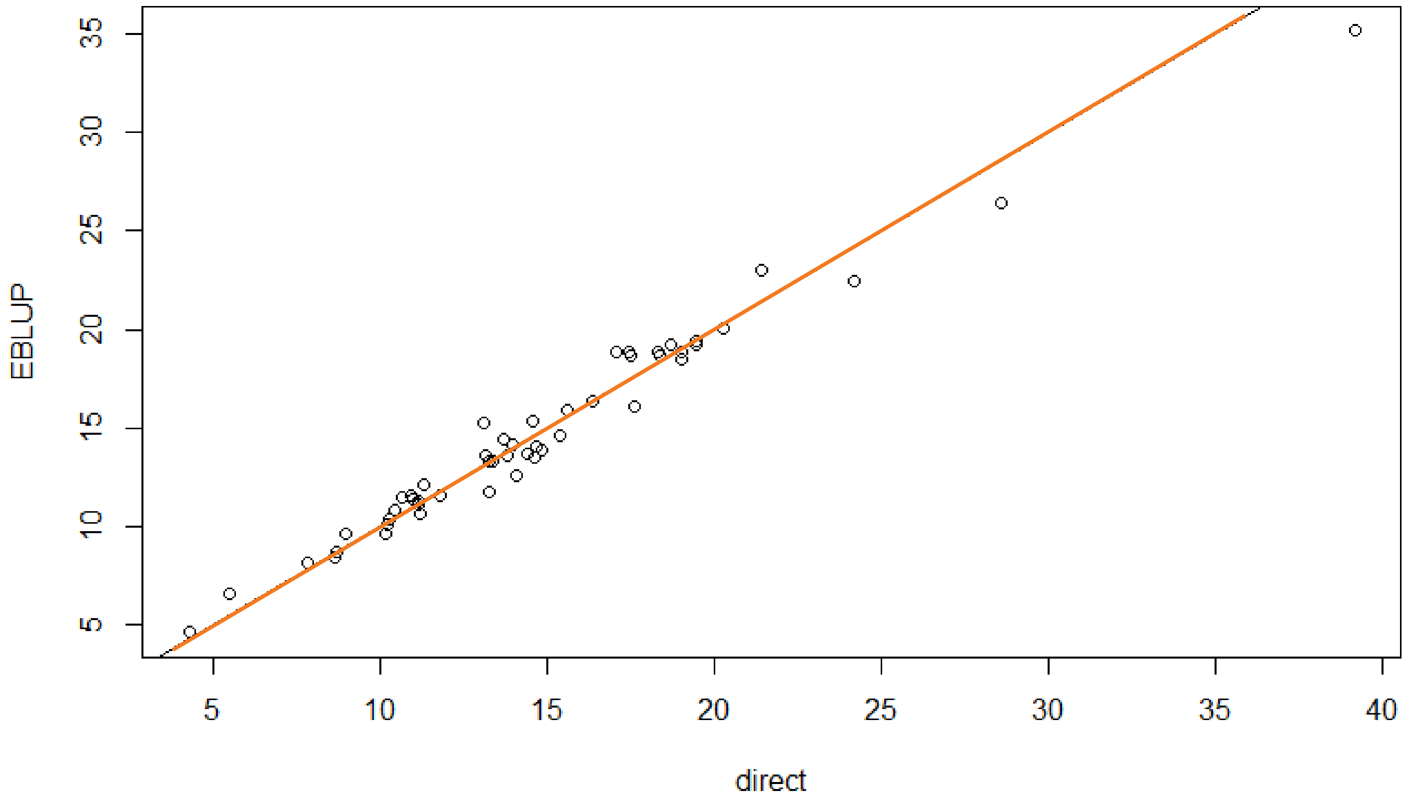


- QQ-plot of the standardized residuals
- **NOTE!** We use function `myFH()` in place of `eblupFH()`; thus, we can extract the residuals; the function `myFH()` is not part of the `sae` Package; see `methods.R`

4 R Package sae (ctd.)

Step 3. Rudimentary diagnostics (ctd.)

```
> plot(dat$yi, m$eblup[,1], xlab = "direct", ylab = "EBLUP")
> abline(0, 1)  # 45-degree line
```



This plot is also available for the `eblupFH()` function

4 R Package sae (ctd.)

Step 4. MSE estimation

```
> mse_analytic <- mseFH(yi ~ prIRS + nfIRS + prCensus,
                       vardir = vi, data = dat)
```

- By default, `B = 0` (number of bootstrap replicates) ⇨ analytical MSE
- If `B > 0`, then `B` bootstrap replicates are used to estimate the MSE

```
> mse_bootstrap <- mseFH(yi ~ prIRS + nfIRS + prCensus,
                        vardir = vi, B = 500, data = dat)
```

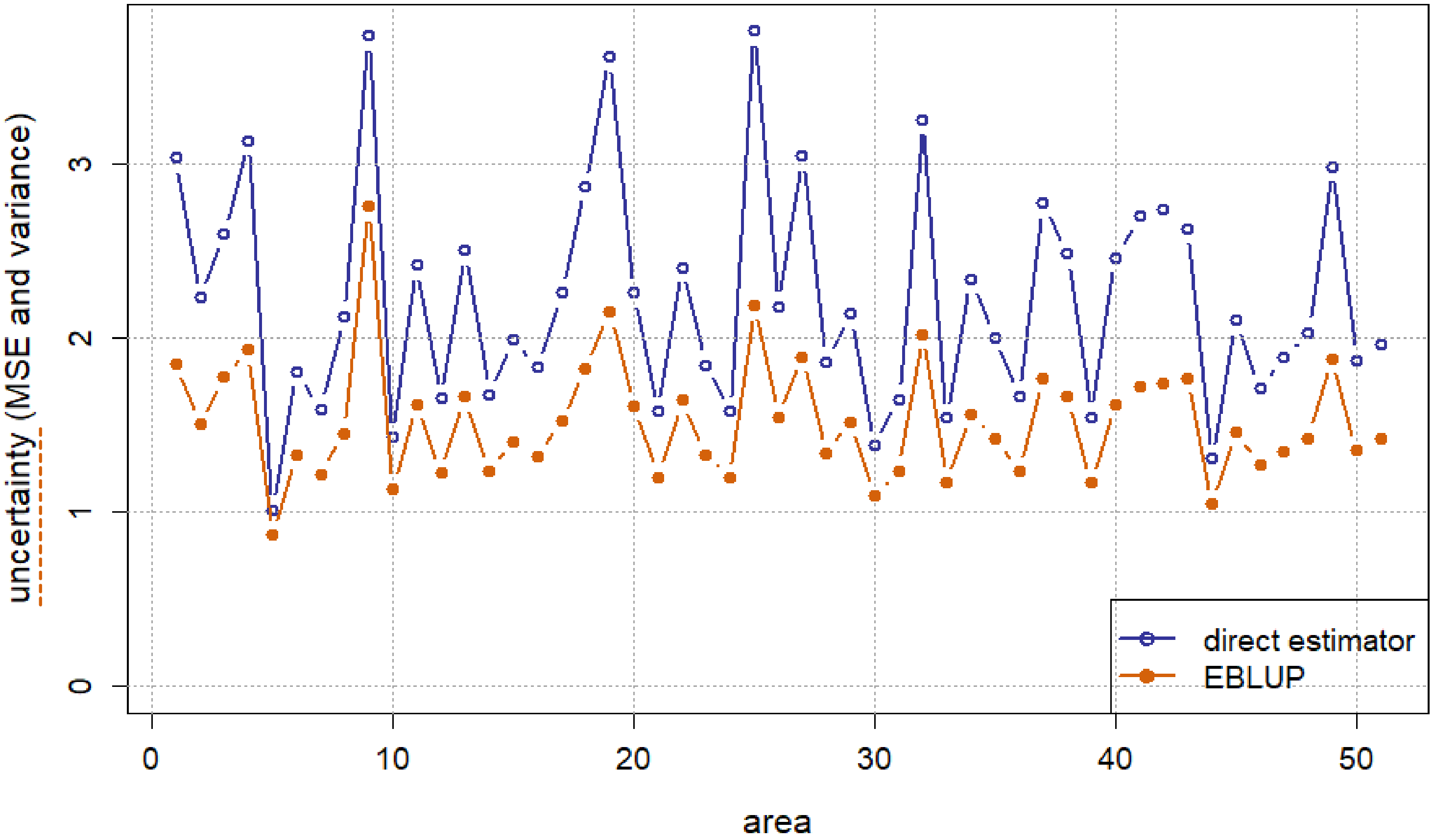
- `B = 500` bootstrap replicates is about the minimum we should use

4 R Package sae (ctd.)

Step 4. MSE estimation (ctd.)

```
> plot(dat$vi)
> lines(mse_analytic$mse)
```

Comparing MSE with variance!



4 R Package sae (ctd.)

Step 5. Confidence intervals

Normal-theory 95% confidence intervals of the EBLUP (using estimated MSE)

```
> est <- data.frame(yi = dat$yi, EBLUP = m$eblup[, 1])
> alpha <- 0.05 # 5% level of significance
> est$ci_low <- est$EBLUP - sqrt(mse_analytic$mse) *
                qnorm(1 - alpha/2)
> est$ci_high <- est$EBLUP + sqrt(mse_analytic$mse) *
                 qnorm(1 - alpha/2)
> head(est, 3)

       yi    EBLUP    ci_low   ci_high
1 19.4400 19.25261 16.583505 21.92172
2 11.0042 11.41015  9.005141 13.81515
3 17.4417 18.87445 16.261264 21.48764
```

5 R Package emdi

Package installation (Vers. 2.11): 88 dependencies...

```
package ‘bit’ successfully unpacked and MD5 sums checked
package ‘prettyunits’ successfully unpacked and MD5 sums checked
package ‘rprojroot’ successfully unpacked and MD5 sums checked
package ‘rstudioapi’ successfully unpacked and MD5 sums checked
package ‘colorspace’ successfully unpacked and MD5 sums checked
package ‘utf8’ successfully unpacked and MD5 sums checked
package ‘bit64’ successfully unpacked and MD5 sums checked
package ‘progress’ successfully unpacked and MD5 sums checked
package ‘brew’ successfully unpacked and MD5 sums checked
package ‘commonmark’ successfully unpacked and MD5 sums checked
...
package ‘emdi’ successfully unpacked and MD5 sums checked
```

5 R Package emdi (ctd.)

Step 0. Load the package

```
> library(emdi)
```

Step 1. Fit the Fay-Herriot model

```
> m <- fh(yi ~ prIRS + nfIRS + prCensus, vardir = "vi",
       combined_data = dat, method = "reml")
```

```
Empirical Best Linear Unbiased Prediction (Fay-Herriot)

Out-of-sample domains:  0
In-sample domains:  51

Variance and MSE estimation:
Variance estimation method:  reml
Variance of random effects:  3.922974
MSE method:  no mse estimated

Transformation: No transformation
```

5 R Package emdi (ctd.)

Step 1. Fit the Fay-Herriot model (ctd.)

```
> head(estimators(m), 3)
   Domain    Direct         FH  Out
1       1   19.4400   19.25261    0
2       2   11.0042   11.41015    0
3       3   17.4417   18.87445    0

> summary(m)
Call:
fh(fixed = yi ~ prIRS + nfIRS + prCensus, vardir = "vi",
    combined_data = dat, method = "reml")
```

5 R Package emdi (ctd.)

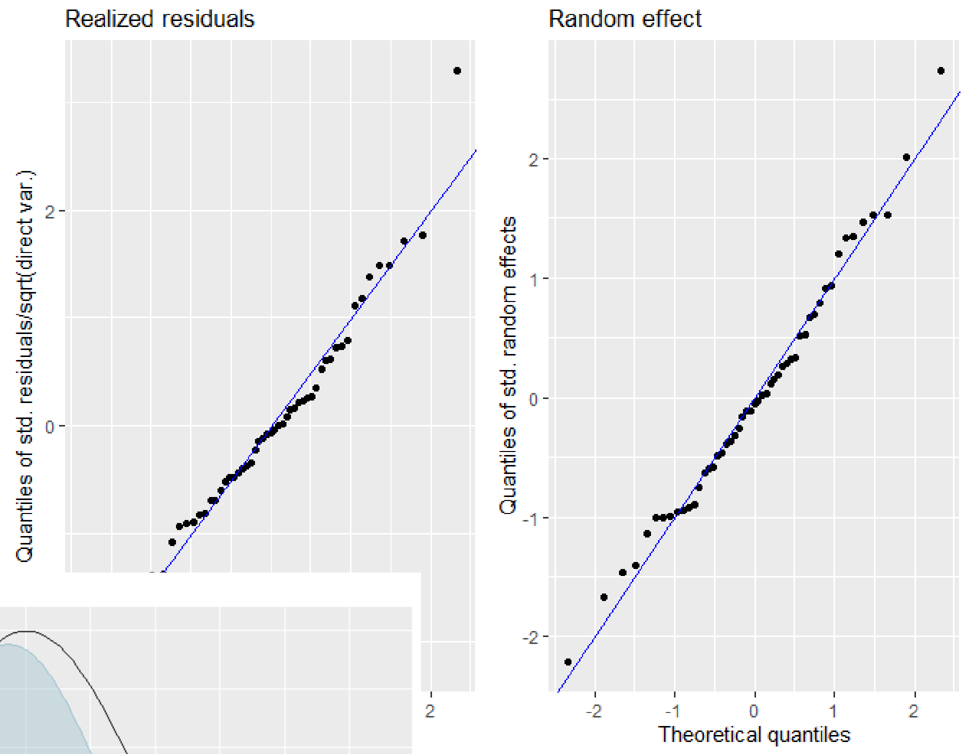
```
# output of summary (ctd.)

Out-of-sample domains:  0
In-sample domains:  51

Variance and MSE estimation:
Variance estimation method:  reml
Estimated variance component(s):  3.922974
MSE method:  no mse estimated

Coefficients:
            coefficients std.error t.value   p.value
(Intercept)     -4.15645   1.53397 -2.7096  0.006736 **
prIRS            0.22610   0.15124  1.4949  0.134934
nfIRS            0.87004   0.14354  6.0614 1.349e-09 ***
prCensus         0.43653   0.18175  2.4019  0.016312 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

5 R Package emdi (ctd.)

```
# output of summary (ctd.)

Explanatory measures:
   loglike      AIC      BIC     KIC        R2     AdjR2
1 -118.149 246.298 255.9571 251.298 0.7880858 0.8419033

Residual diagnostics:
                        Skewness Kurtosis Shapiro_W Shapiro_p
Standardized_Residuals 0.6342088 4.100074 0.9718588 0.2637918
Random_effects         0.3664708 3.035362 0.9869228 0.8424495

Transformation: No transformation
```

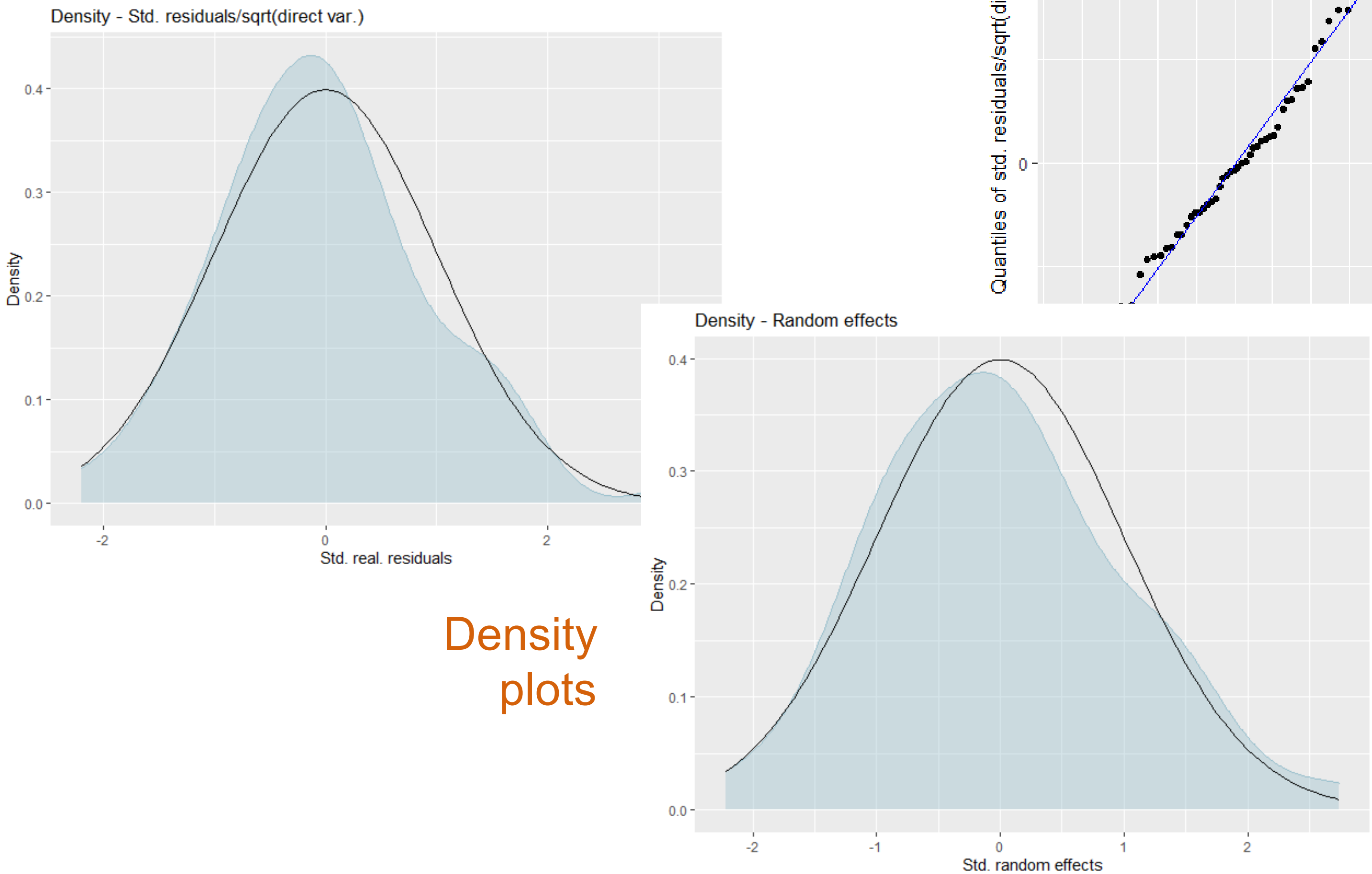
5 R Package `emdi` (ctd.)

Residuals



Step 3. Diagnostics

```
> plot(m)
```



Density plots

5 R Package emdi (ctd.)

Step 4. MSE estimation

```
> m <- fh(fixed = yi ~ prIRS + nfIRS + prCensus, vardir = "vi",
      combined_data = dat, method = "reml"
      MSE = TRUE, mse_type = "analytical")
```

Alternative MSE estimators

- `mse_type = "jackknife"`
- `mse_type = "weighted_jackknife"`
- `mse_type = "boot"`
- …

Plot method (shows direct vs EBLUP and MSE by area)

```
> compare_plot(m)     # not shown
```

5 R Package emdi (ctd.)

Step 5. Miscellaneous

Maps

```
> […]              # load maps; not shown
> map_plot(m)      # not shown
```

Output to Excel

```
> write.excel(m)    # not shown
```

Literature

Harmening, Kreutzmann, Pannier, Skarke, Rojas-Perilla, Salvati, Schmid, Templ, Tzavidis & Würz (2021) emdi: Estimating and Mapping Disaggregated Indicators. R Package version 2.1.1. URL https://CRAN.R-project.org/package=emdi

Kreutzmann, Pannier, Rojas-Perilla, Schmid, Templ & Tzavidis (2019) The R Package emdi for Estimating and Mapping Regionally Disaggregated Indicators, Journal of Statistical Software 91, p. 1-33. DOI 10.18637/jss.v091.i07

Molina & Marhuenda (2020) sae: Small Area Estimation. R Package version 1.3, URL https://CRAN.R-project.org/package=sae

Marhuenda, Morales & Pardo (2014) Information criteria for Fay-Herriot model selection. Computational Statistics and Data Analysis 70, p. 268-280.

Molina & Marhuenda (2014) sae: An R Package for Small Area. The R Journal 7, p. 81-98.

Schoch (2014) rsae: Robust Small Area Estimation. R Package version 0.1-5. URL https://CRAN.R-project.org/package=rsae