Shared Area

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1 Shared area

Novel to this thesis is the analytical method of deriving frequency offsets, in terms of the observed power spectra, called shared area. Not too dissimilar to the convolution between two functions f and g

$$f(t) * g(t) = \int_0^t f(\tau)g(t-\tau)\mathrm{d}\tau' \,, \tag{1}$$

shared area adopts two boolean arrays, B_f and B_g, which are defined by the two of the lower bounding functions

$$A(\tau) = \int_0^T \left[B_f(t,\tau)f(t) + B_g(t,\tau)g(t-\tau) \right] \mathrm{d}t \,. \tag{2}$$

For example, if $f(t) < g(t-\tau)$, then $B_f(t,\tau) = 1$ and $B_g(t,\tau) = 0$, so the integral collapses to just over the curve $f(t)$ across t. Whereas, if $f(t) > g(t-\tau)$, the inverse is true, so the integral collapses to over the shifted function $g(t-\tau)$. This returns an array, A, which represents the area as a function of offset, τ, between both functions, hence the dependency of the boolean arrays on this offset. The area is maximised for the minimum offset between each function, i.e. the distance between two curves "centres". Because B_g is just the opposite of B_f, we can even choose to re-write this using $B_g(t,\tau) = |1-B_f(t,\tau)|$. Examples of its use for two Gaussians centred on $t = 1$ and $t = 6$ are given in Fig. 1. Here the top row shows the two Gaussians where one curve (blue) slides through the other (black) between $\pm T$ where T is the end value on the x-axis, and the bottom row is the area as a function of offset, maximised for $\tau = 5$ which is the difference between each Gaussian centre.

This method works for positive real-valued functions with defined peaks. If the two Gaussian curves have different amplitudes then the area A plateaus and the mid-point is chosen to represent the offset.

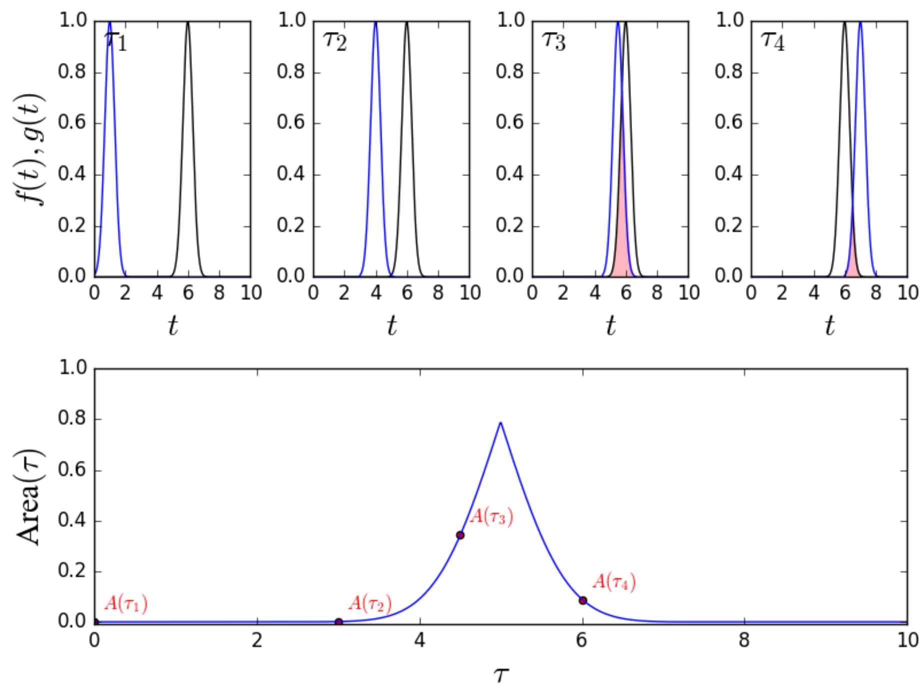

Figure 1: *Top row*: two Gaussians, $f(t)$ and $g(t)$, which are centred on $t = 6$ and $t = 1$ respectively. The sliding of one function over the other is periodic and between $\pm T$ where T is the limit on the x-axis. *Bottom row*: shared area (shown as top row red area) as a function of x-axis offset of $g(t - \tau)$. Each area of the top row offsets, τ_i, are represented as scatter points along this curve.

2 Shared-area in two dimensions

$$\tilde{A}(\delta_x,\delta_y) = \int_X \int_Y \mathrm{d}x\mathrm{d}y \Big[B_f(\delta_x,\delta_y,x,y) f(x,y) + B_g(\delta_x,\delta_y,x,y) g(x-\delta_x,y-\delta_y) \Big] \tag{3}$$

3 Generalised shared-area method for N dimensions

$$\tilde{A}(\boldsymbol{\delta},\mathbf{r}) = \int_N \mathrm{d}^N\mathbf{r} \Big[B_f(\boldsymbol{\delta},\mathbf{r}) f(\mathbf{r}) + B_g(\boldsymbol{\delta},\mathbf{r}) g(\mathbf{r}-\boldsymbol{\delta}) \Big] \tag{4}$$