Exercise – *The Pied Piper of Hamelin*

"Der Rattenfänger zog demnach ein Pfeifchen heraus und pfiff, da kamen alsobald die Ratten und Mäuse aus allen Häusern hervorgekrochen und sammelten sich um ihn herum."

"The rat-catcher produced a small pipe and whistled. Soon, rats and mice came creeping out from all the houses and gathered around him."

Deutsche Sagen *by Jacob Grimm (1785–1863) and Wilhelm Grimm (1786–1859)*

When word of the dire rat infestation plaguing the city of Hamelin reached the ears of the Pied Piper, he heeded the call for aid without hesitation. The prospect of the bountiful reward of one gold penny per rat eliminated, offered by the honorable mayor, proved sufficient inducement for the benevolent Piper to put his musical talent to use to lure the vermin from their lair.

Upon his arrival at the city gate, he is greeted by the mayor, who lays before the Piper a map of Hamelin in intricate detail. The map shows a total of n public squares, numbered from 0 to $n-1$, scattered across the city. The public square at the entrance of the city, where the Piper currently stands, is public square 0. The map also indicates that there are m one-way streets in Hamelin, each of which leads from some public square to another and has a number of rats residing on it. A sequence of public squares $s_0, \ldots, s_\ell$ and a sequence of streets $r_0, \ldots, r_{\ell-1}$ such that r_i leads from s_i to s_{i+1} for each $i \in \{0, \ldots, \ell-1\}$, is called a *plan* for the Pied Piper's route through the city. In order to maximize the bounty that the Piper can collect, he wants to find a plan that maximizes the total number of rats on the streets included in the plan.

However, the mayor gives a few restrictions on the kind of plan that the Piper may follow. The plan must start and end at public square 0, it must contain the main public square, $n-1$, and shouldn't contain any public square more than once, except for 0. Finally, to not disturb the citizens, the plan must not change directions more than once, defined as follows. We say that a plan $s_0, \ldots, s_\ell$ is a *monotonically increasing path* if for every $i \in \{0, \ldots, \ell-1\}$, it holds that $s_i < s_{i+1}$. Similarly, a plan $s_0, \ldots, s_\ell$ is called a *monotonically decreasing path* if for every $i \in \{0, \ldots, \ell-1\}$, we have $s_i > s_{i+1}$. The mayor requires the plan $s_0 = 0, s_1, \ldots, s_{j-1}, s_j = n-1, s_{j+1}, \ldots, s_{\ell-1}, s_\ell = 0$ followed by the Piper to be a concatenation of a monotonically increasing path and a monotonically decreasing path. That is, $s_0 = 0, s_1, \ldots, s_{j-1}, s_j = n-1$ should be a monotonically increasing path, and $s_j = n-1, s_{j+1}, \ldots, s_{\ell-1}, s_\ell = 0$ should be a monotonically decreasing path. A plan is called *acceptable* if it satisfies all the requirements of the mayor.

The Pied Piper would like to find the maximum number of rats on the streets in any acceptable plan. It is guaranteed that an acceptable plan always exists.

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains two integers `n m`, separated by a space. They denote
  - n, the number of public squares ($4 \leq n \leq 500$); and

- m, the number of one-way streets connecting two public squares ($4 \leq m \leq 4 \cdot 10^4$).

- The following m lines define the streets and the number of rats on them. Each line consists of three integers `u v f` separated by a space, such that $u, v \in \{0, \ldots, n-1\}$ with $u \neq v$, and $1 \leq f \leq 10^5$, indicating that there exists a street leading from public square u to public square v with f rats on it. It is guaranteed that there is no street from 0 to $n-1$ and there is no street from $n-1$ to 0. Furthermore, for each $u, v \in \{0, \ldots, n-1\}$, there is at most one street from u to v.

Output The output for each test case consists of a separate line containing a single integer that denotes the maximum number of rats on the streets in an acceptable plan.

Points We say that the city of Hamelin is *layered* (see Figure **??**) if there exist k non-empty sets $S_0, \ldots, S_{k-1}$ of public squares, such that:

- every public square $u \in \{0, \ldots, n-1\}$ belongs to precisely one set S_i with $i \in \{0, \ldots, k-1\}$;
- for every street leading from u to v there is $i \in \{0, \ldots, k-2\}$ such that either $u \in S_i$ and $v \in S_{i+1}$, or $v \in S_i$ and $u \in S_{i+1}$; and
- for every $i \in \{0, \ldots, k-2\}$ and each $u \in S_i$ and $v \in S_{i+1}$, it holds that $u < v$.

The sets $S_0, \ldots, S_{k-1}$ are called a *layering* of Hamelin.

There are four groups of test sets. For each group there is also a corresponding hidden test set, each worth 5 points. Overall, you can achieve 100 points.

1. For the first group of test sets, worth 25 points, you may assume that the city of Hamelin is layered and there is a layering $S_0, \ldots, S_{k-1}$ such that for each $i \in \{0, \ldots, k-1\}$, we have $|S_i| \leq 10$. Additionally, you may assume that for every monotonically increasing path from 0 to $n-1$, there exists a monotonically decreasing path from $n-1$ to 0 with the same total number of rats on its streets, and the public squares these two paths visit are disjoint, apart from 0 and $n-1$.
2. For the second group of test sets, worth 25 points, you may assume that the city of Hamelin is layered and there is a layering $S_0, \ldots, S_{k-1}$ such that for each $i \in \{0, \ldots, k-1\}$, we have $|S_i| \leq 10$.
3. For the third group of test sets, worth 15 points, you may assume that the city of Hamelin is layered.
4. For the fourth group of test sets, worth 15 points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`*i*`.in/out`, for $i \in \{1, 2, 3, 4\}$.

Sample Input

```
2
6 6
1 0 1
0 2 1
3 1 1
2 3 1
3 5 1
5 1 10
6 6
0 1 9
0 2 4
3 0 1
5 3 1
2 5 99
1 5 1
```

Sample Output

```
14
105
```

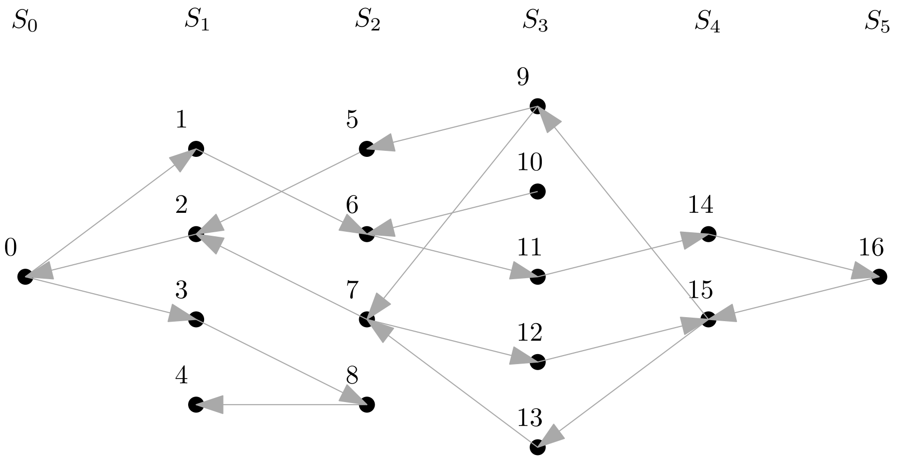

Figure 1: An example of a map of a layered city of Hamelin with $n = 17$ and $k = 6$.