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Seminar thesis

Robust Linear Forecasting with Crash Risk Measures

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Contents

Abbreviations

OLS ordinary least squares

GLS generalized least squares

WLS-EV least squares estimates weighted by ex-ante return variance

1 Introduction

Linear models are the bread and butter of the financial industry. Most of these models are estimated using ordinary least squares (OLS), which is easy to use and the most efficient linear unbiased estimator when error terms ϵ_{t+1} are homoscedastic and without autocorrelation (Johnson 2018). However, these assumptions rarely hold in practice when working with financial data, resulting in model estimations with more error than necessary. When OLS is applied to heteroscedastic data, out-of-sample estimates perform poorly, because highly volatile observations with low signal-to-noise ratio are given full weight.

Standard procedure when working with heteroscedastic data is an estimation with OLS and calculation of robust standard errors following Newey and West (1987). The Newey and West (1987) approach calculates standard errors for a generalized least squares (GLS) estimator which are robust to heteroscedasticity and autocorrelation. Technically speaking, such an estimation has OLS coefficient estimates with GLS standard errors. Although the OLS coefficient estimations remain unbiased when applied to heteroscedastic data, they become inefficient. Efficiency in this context describes how fast the estimated coefficient converges to the real data generating coefficient. The coefficient estimates can be inefficiently high (low), if highly volatile observations in the data tend to be positive (negative). Coefficient i's t-statistic is calculated by $ts\hat{t}at_i = \frac{\hat{\beta}_i}{s\hat{e}_i}$, where se_i is the robust standard error following Newey and West (1987) of coefficient i. Since standard errors are robust, inefficiently high (low) OLS coefficient estimates directly result in false positive (negative) significance conclusions (Johnson 2018).

Johnson (2018) points out that GLS is the most efficient linear unbiased estimator if we know the covariance matrix of ϵ_{t+1}, which is mostly unobservable and difficult to estimate. In order to sidestep the necessity of specificating the full covariance matrix, Johnson (2018) provides a GLS method which uses ex-ante return variance as weights. This is, in the case of return predictability regressions, easy to use and results in large efficiency gains. It is called WLS-EV. Efficiency gains come from scaling regression residuals by an ex ante variance estimate, which makes different volatile observations comparable in terms of signal versus noise.

Johnson (2018) finds that with WLS-EV, traditional variables like dividend yield and cay are indeed significant return predictors, while an OLS approach often results in false negative significance (Welch and Goyal 2007). The other way around, WLS-EV does not support the evidence of a linear relationship between variance risk premium and market returns, as concluded in both Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2010). Significance from OLS estimation relies on a few observations with high ex-ante variance, which means low signal and high noise.

Johnson (2018) is not the first one to incorporate ex-ante variance information in return predictability regressions. French, Schwert, and Stambaugh (1987) scale a risk-return tradeoff regression $r_{t+1} = a + b * \sigma_t^2 + \epsilon_{t+1}$ by ex-ante return variance. Engle, Lilien, and Robins (1987), Glosten, Jagannathan, and Runkle (1993), Brenner, Harjes, and Kroner (1996) and Ghysels, Santa-Clara, and Valkanov (2005) use structural models to estimate risk-return tradeoffs while incorporating ex-ante variance. Johannes, Korteweg, and Polson (2014) uses the same concept in their stochastic volatility model.

The goal of this seminar thesis is to implement a python library which provides parameter and standard error estimates of WLS-EV. The library will then be applied to financial data. Our results are compared to the conclusions of Johnson (2018) and further applications, especially the transfer from return to non-return linear relationships, are discussed.

The thesis is structured as follows: First, theroretical principles and the intuition of WLS-EV are presented in section 2. Section 3 illustrates our algorithmic design approach and the resulting implementation. In section

4.2, analysis results are presented before the seminar is concluded in section 5.

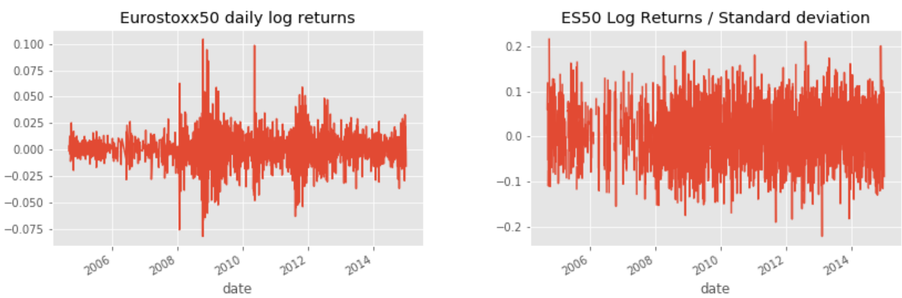
2 Weighted least squares with ex-ante return variance

This section explains the theoretical principles and calculus for WLS-EV. WLS-EV from Johnson (2018) estimates the linear regression shown in equation 1.

$$r_{t+1} = X_t * \beta + \epsilon_{t+1} \tag{1}$$

The returns r_{t+1} can be non-overlapping or overlapping over more than one period, and can be adjusted for the risk-free rate. X_t can contain multiple predictors as well as an optional constant. The basic idea is to standardize regression errors ϵ_{t+1} by ex-ante standard deviation $\hat{\sigma}_t$, which is a down-weighting of highly volatile observations with low signal but high noise.

Two Steps are necessary to estimate β: First, the conditional variance, $\hat{\sigma}_t^2$, of next-period unexpected returns ϵ_{t+1} is estimated, which is shown in section 2.1. In the second step the β is estimated. Depending on the forecast horizon, whether the predicted returns are overlapping or not. The different estimation approaches are shown in section 2.2.1 and 2.2.2.



(a) (b)

Figure 1: Update description and reference in text

2.1 Variance estimation

WLS-EV uses ex-ante variance information to scale regression residuals. Why not ex-post variance? Johnson (2018) argues that weighting with realized (ex-post) variance introduces a look-ahead bias into coefficient estimates. Realized variance σ_{t+1}^2 is strongly correlated with realized error terms e_{t+1}. Now, because negative returns are more volatile, thus have a larger variance, negative return observations get smaller weights as positive return observations. If the predictor X_t is positive correlated with return variance, coefficient estimates will be biased upwards, otherwise with negative correlation biased downwards. Using ex-ante information for variance estimates avoids this bias. The following subsections present our approximation for return and non-return variance as well as the model used for ex-ante estimation.

2.1.1 Realized return variance

As a well known proxy for daily variance we use the sum of squared intraday returns with a frequency of 5 minutes, shown in equation 2, where n is the number of intraday returns available in day t (Amaya et al. 2015).

$$\hat{RV}_t^d = \sum_{i=1}^{n} r_{t,i}^2 \tag{2}$$

2.1.2 Realized non-return variance

In section 4 we will transfer WLS-EV from return data to non return data. In particular, we want to regress P-moments on Q-moments, and Fama French Factors on Q-moments. In order to be able to transfer WLS-EV analogous, we need proxies for realized P-moments and Fama French Factors. Daily variance is defined in equation 2. Equation 3 and 4 show daily skewness and kurtosis following Amaya et al. (2015). Equation 2, 3 and 4 assume mean zero five-minute returns. This is common practice with high-frequency returns and validated in a robustness check of higher moments in Amaya et al. (2015), section 4.

$$R\hat{Skew}_t^d = \frac{\sqrt{n}\sum_{i=1}^{n} r_{t,i}^3}{(\hat{RV}_t^d)^{3/2}} \tag{3}$$

$$R\hat{Kurt}_t^d = \frac{n\sum_{i=1}^{n} r_{t,i}^4}{(\hat{RV}_t^d)^{2}} \tag{4}$$

In order to estimate WLS-EV for overlapping returns, described in section 2.2.2, we need dependend variables to be additive. As skewness and kurtosis are not additive, our work around is to only take third and fourth moments from the numerators, which are additive. The third moment is $\sum_{i=1}^{n} r_{t,i}^3$ and the fourth moment is $\sum_{i=1}^{n} r_{t,i}^4$. Now, we need to estimate the variance of the respective P-Moments. The second term of the variance formula $Var_t(x) = E_t[x^2] - E_t^2[x]$, $E_t^2[x]$, can here be non-zero compared to returns. Since we do not have the conditional expected value as of t, $E_t^2[x]$, we use the unconditional value $E^2[x] = (\frac{1}{N}\sum_{i=1}^{N} x_i)^2$. We set x to the respective P-moments and calculate the variance.

Fama French Factors are excess returns, so we can take the logarithm and proceed WLS-EV analogously to market log returns, which means we take the squared daily log factor as a variance proxy.

2.1.3 Ex-ante estimation

The ex-ante variance in WLS-EV can generally be estimated with any kind of model. In this seminar, we stay close to Corsi (2009), who provides a simple regression model to estimate ex-ante variance. Tomorrow's daily variance is here projected on a lagged daily term, weekly and monthly rolling variance windows. This model is shown in equation 5.

$$\hat{RV}_t^d = c + \beta_d * RV_{t-1}^d + \beta_w * RV_{t-1}^w + \beta_m * RV_{t-1}^m + \epsilon_t \tag{5}$$

There is also the possibility to extend the model with a lagged implied volatility term, IV_{t-1}^d, shown in equation 6. The implied volatility in our work is extracted from options with a maturity of thirty days.

$$\hat{RV}_t^d = c + \beta_d^R * RV_{t-1}^d + \beta_d^I * IV_{t-1}^d + \beta_w * RV_{t-1}^w + \beta_m * RV_{t-1}^m + \epsilon_t \quad (6)$$

The models in both equations 5 and 6 have the disadvantage of possible negative ex-ante variance estimations. Johnson (2018) works around this problem with constraint regressions, but we choose the easy way of using logarithmic transformations. We therefore slightly adjust the Corsi (2009) model, shown in equation 7. We then transform the log variance back to realized variance, shown in equation 8.

$$\log(\hat{RV}_t^d) = c + \beta_d * \log(RV_{t-1}^d) + \beta_w * \log(RV_{t-1}^w) + \beta_m * \log(RV_{t-1}^m) + \epsilon_t \quad (7)$$

$$\hat{RV}_t^d = \exp(\log(\hat{RV}_t^d)) \quad (8)$$

Possible problem with logs: implied variances smaller than 0, introduces bias to wlsev estimators as variance is not estimated correctly anymore

2.2 Coefficient estimation

Coefficient estimation in WLS-EV distinguishes two scenarios. If working with non overlapping daily returns or non-returns, WLS-EV is rather simple and described in section 2.2.1. Economic reasoning and empirical evidence suggest that there is little to no autocorrelation in non-overlapping returns, so the OLS and thus WLS-EV assumption of no autocorrelation is met (Johnson 2018). However, in finance there is often the goal to predict or fit cummulative, overlapping returns over several days or weeks. In this case, error terms exhibit significant autocorrelation which would introduce a bias to OLS and WLS-EV estimators. A work around is provided by a transformation of the regression model, introduced by Hodrick (1992). This estimation approach is covered in section 2.2.2.

2.2.1 Non-overlapping returns

The non-overlapping $\hat{\beta}_{WLS-EV}$ is estimated by equation 9, which can be implemented using a standard OLS package, regressing $\frac{r_{t+1}}{\hat{\sigma}_t}$ on $\frac{X_t}{\hat{\sigma}_t}$. r_{t+1} are next period returns, X_t is a matrix of constant and regressors, and σ_t is the estimated ex-ante variance.

$$\hat{\beta}_{WLS-EV} = \arg\min_{\beta} \sum_{t=1}^{T} (\frac{r_{t+1} - X_t * \beta}{\sigma_t})^2 \tag{9}$$

Standard errors should be adjusted for remaining heteroscedasticity and autocorrelation, which is done by using Newey and West (1987).

2.2.2 Overlapping returns

An overlapping regression is shown in equation 10. Error terms $\epsilon_{t+1,t+h}$ in such a setting exhibit significant autocorrelation. One could use the standard approach from section 2.2.1 by weighting overlapping residuals with conditional next h-period variance and use Newey and West (1987) standard errors. However, Johnson (2018) points out several efficiency and bias problems this approach would suffer from. As a work around, WLS-EV uses the Hodrick (1992) procedure. Here, the overlapping regression in equation 10 is transformed to the non-overlapping regression in equation 15. A prerequisite for the transformation is the additivity of dependent variables, $r_{t+1,t+h} = \sum_{s=1}^{h} r_{t+s}$.

Technical details following Hodrick (1992) and Johnson (2018) are described below. The overlapping regression is shown in equation 10, where h is the forecast horizon and $r_{t+1,t+h}$ is the cummulative log return from period $t+1$ to $t+h$.

$$r_{t+1,t+h} = X_t * \beta + \epsilon_{t+1,t+h} \tag{10}$$

Equation 11 shows the respective closed form OLS solution, where $\mathbb{E}_T$ represents the sample average.

$$\hat{\beta}_{OLS} = \mathbb{E}_T(X_t^T X_t)^{-1}\,\mathbb{E}_T(X_t^T r_{t+1,t+h}) \tag{11}$$

Substituting $r_{t+1,t+h} = \sum_{s=1}^{h} r_{t+s}$ leads to equation 12, 13, and 14.

$$\hat{\beta}_{OLS} = \mathbb{E}_T(X_t^T X_t)^{-1}\,\mathbb{E}_T(\sum_{s=1}^{h} X_t^T r_{t+s}) = \mathbb{E}_T(X_t^T X_t)^{-1}\,\mathbb{E}_T(\bar{X}_t^T \bar{X}_t)\hat{\beta}_{OLS}^{roll} \tag{12}$$

$$\hat{\beta}_{OLS}^{roll} = \mathbb{E}_T(\bar{X}_t{}^T \bar{X}_t)^{-1}\,\mathbb{E}_T(\bar{X}_t{}^T r_{t+1}) \tag{13}$$

$$\bar{X}_t = \sum_{s=0}^{h-1} X_{t-s} \tag{14}$$

In words, the coefficients of the overlapping regression in equation 10 are the same as the scaled coefficients of the transformed, non-overlapping regression in equation 15. The coefficients are scaled by $\mathbb{E}_T(X_t^T X_t)^{-1}\,\mathbb{E}_T(\bar{X}_t^T \bar{X}_t)$, which is a ratio of the regressor's variances.

$$r_{t+1} = (\sum_{s=0}^{h-1} X_{t-s}) * \beta_{OLS}^{roll} + \epsilon_{t+1} \tag{15}$$

After the Hodrick (1992) procedure, standard errors are adjusted for remaining heteroscedasticity and autocorrelation using Newey and West (1987). Applying the Hodrick (1992) transformation ensures that WLS-EV in overlapping regressions is the most efficient and unbiased estimator.

To estimate overlapping regressions with WLS-EV in practice, the non-overlapping regression in equation 15 is first estimated and standard errors adjusted with Newey and West (1987). Its resulting coefficients and standard errors are then scaled by $\mathbb{E}_T(X_t^T X_t)^{-1}\,\mathbb{E}_T(\bar{X}_t^T \bar{X}_t)$, resulting in coefficients and standard errors of the initial, overlapping regression in equation 10.

3 Algorithmic Design and Implementation

In this section, our implementation approach and the resulting python library are first broadly presented. Subsection **??** the nexplains how the algorithm is validated.

3.1 Python library

This section gives a broad overview over the python library and the code. The code is extensively commented for a detailed translation of section 2 into code, which is why the project is only broadly presented below. The WLS-EV approach is implemented in a object-oriented and generic way, which makes it easy to use. The following directory tree gives an overview over the library with its packages and files.

```
CRAM_WLS-EV
├─ data
├─ tests
│  └─ tests.py
├─ tools
│  ├─ helper_functions.py
│  └─ visualisation.py
├─ wlsev
│  ├─ wlsev_model.py
│  ├─ ols_model.py
│  ├─ simon_ols_model.py
│  └─ variance_estimation.py
├─ wlsev_analysis.py
├─ README.MD
└─ .gitignore
```

The directory *data* contains all the data in .csv format. The *tests.py* script in the directory *tests* compares WLS-EV and OLS results in a constant variance setting, to confirm the algorithm is working correctly. The directory *wlsev* contains the main functionality, including the variance estimation and model classes. As previously mentioned, a detailed description of the logic, classes and functions can be found in the extensively documented code.

4 Analysis

In this section we present the data sets utilized and conduct the regression analysis. After presenting the data sets, necessary preprocessing steps are explained. In the analysis section, we analyze the regression relationships and compare the results with the findings of Johnson (2018). The application of WLS-EV is tested in several regressions. First, we regress log returns on log returns. Second, corresponding to Drechsler and Yaron (2010) and Bollerslev, Tauchen, and Zhou (2009), we regress log returns on variance risk premium. To transfer WLS-EV to non return data, we also regress P moments on Q moments as well as Fama French Factors on Q moments.

4.1 Data

In this section the data and preprocessing are presented. First, we present the data sets with their source and structure. Then necessary preprocessing steps, for example how to calculate moments out of raw high-frequency data, are explained.

4.1.1 Datasets

We use price data from the Euro Stoxx 50 index. We use intraday, high-frequency 5 minute prices from 29.6.2000 to 29.9.2017. End of day prices are from 3.1.2002 to 1.6.2015. Corresponding daily volatility data is from 29.6.2000 to 30.6.2015. Daily implied volatility is extracted from options with a thirty day maturity and is from to 9.6.2004 to 30.12.2014. Variance risk premium data is from 3.1.2002 to 1.6.2015, and daily Fama French SMB HML, and Momentum factor are from 1.1.1991 to 31.5.2017. We also use risk neutral Q moments with maturities 7, 30, 60, 91, 182, and 365 days from

3.1.2002 to 30.6.2015. The riskfree rate is with a maturity of 7 days and is from 2.1.2002 to 30.6.2015.

4.1.2 Data Preprocessing

The data is generally of high quality with no missing values, but before we fit WLS-EV to the data, some preprocessing steps are necessary. First, we calculate daily P moments. Recalling section 2.1.2, we use the intraday 5min returns to calculate daily third moments, $\sum_{i=1}^{n} r_{t,i}^3$, and forth moments, $\sum_{i=1}^{n} r_{t,i}^4$. Then we combine the respective data for a linear relationship in a dataframe to match the time series lengths. We have three final dataframes for the analysis. The first is only log returns with a length of 3793, the second combines log returns and variance risk premium with a length of 2962, and the third combines p and q moments with a length of 2691.

4.2 Results

In this section we present and compare the WLS-EV results with respective OLS results. First, we will take a look at the first WLS-EV step, the ex-ante variance estimation results, before discussing the regression results.

4.3 Variance estimation results

Recalling section 2.1, variance forecasts in WLS-EV are estimated with simple OLS. Johnson (2018) finds that with OLS, estimated ex ante variance explaines a significant portion of daily realized variance. His estimations have a r-squared between 0.25 and 0.5. This suggests WLS-EV provides substantial efficiency gains compared to OLS, as a large part of next day's variance is caputered ex ante.

This coincides with our variance estimation results. Table 1, 2, and 3 show our results. For log return variance, we have an even higher r-squared of 0.673. For non returns, in this case third and fourth P moments, ex ante variance regressions have a r-squared of 0.326 and 0.302, respectively. Now, one could improve these forecasts with for example including implied volatility data or use more sophisticated models like garch forecasts. However, since the goal of this seminar thesis is to analyse if and to what extend WLS-EV brings efficiency gains, we are content with the simple OLS variance estimation. For further analysis, it might be insightful to what extend improved variance estimation influences the weighting and hence the regression results in WLS-EV.

OLS	r-squared		
log return variance	0.673		
	coeff	std error	t-stat
Intercept	-0.2800	0.039	-7.131
var daily	0.2837	0.019	14.699
var weekly	0.4201	0.031	13.588
var monthly	0.2416	0.026	9.225

Table 1: Variance estimation results of log retun ex ante variance. R squared is 0.673. As seen in the t-statistics, all daily, weekly, and monthly variance are significant predictors for next day's variance

4.4 WLS-EV Results

We evaluate WLS-EV results with the same approach as in Johnson (2018). We are mainly interested in in-sample and out-of-sample r-squared, as well as coefficient t-statistics to assess a independent variable's significance in the regression. Johnson (2018) finds that WLS-EV results are more pessimistic than OLS. In the model estimations, this can be seen in insignificant t-statistics and lower WLS-EV in-sample r-squared than in OLS. OLS might falsely indicate significance, because estimators can be wrongly influenced

OLS	r-squared		
Third P moment variance	0.326		
	coeff	std error	t-stat
Intercept	-10.0578	0.735	-13.688
var daily	0.2267	0.022	10.387
var weekly	0.2479	0.032	7.818
var monthly	0.2869	0.032	8.918

Table 2: Variance estimation results of third P moment ex ante variance. R squared is 0.326. As seen in the t-statistics, all daily, weekly, and monthly variance are significant predictors for next day's variance

OLS	r-squared		
Fourth P moment variance	0.302		
	coeff	std error	t-stat
Intercept	-14.3247	0.967	-14.812
var daily	0.2170	0.022	10.010
var weekly	0.2466	0.031	7.834
var monthly	0.2790	0.032	8.651

Table 3: Variance estimation results of fourth P moment ex ante variance. R squared is 0.673. As seen in the t-statistics, all daily, weekly, and monthly variance are significant predictors for next day's variance

by high volatile observations with low signal but much noise. Giving such observations full weight maximizes in-sample r-squared, but neglect the true data generating process. Consequently, adjusting for heteroscedasticity with WLS-EV results in lower t-statistics, meaning lower significance, and lower in-sample r-squared as the point errors at highly volatile observations get higher. For a clear understanding, WLS-EV is estimated with weighted data, but for evaluated with the initial, unweighted data as this is the time series we want to evaluate and forecast. As less noise and more signal is caputered in the model, out-of-sample r-squared statistics from Johnson (2018) are typically higher in WLS-EV than in OLS.

Let us now take a look at our analysis results. In this seminar, we analyse four linear relationships. Equation 16 shows the regression of log returns on log returns. Equation 17 shows the linar relationship of log returns and variance risk premium. Both relationships are analysed with different forecast horizons, 1, 7, 14, 22, 44, 66, and 88 days. the results are presented in table 4 and 5. We can confirm the results of Johnson (2018) for the return on return relationship. In-sample r-squared are lower with WLS-EV than with OLS, and out-of-sample r-squared are higher with WLS-EV than with OLS. Also the t-statistics are less significant or not significant, meaning that OLS leads to false positive significance by giving highly volatile observations full weight, or in other words, treat noise as signal.

With WLS-EV, it is possible that in-sample r-squared gets negative. As the model is fitted with weighted data, but evaluated with the initial, unweighted data, large point errors occur at highly volatile observations. Especially, if the algebraic sign of the coefficient is different in WLS-EV than in OLS, which translates to a lot of noise in either positive or negative direction, the point errors get very large. This can result in negative in-sample r-squared. We treat this as a nice insight, but not as a problem as the out-of-sample predictability with WLS-EV rises.

Let us take a look at the linear relationship between returns and variance risk premium. While OLS results indicate positive significance, WLS-EV indicates that these might be false positive significance, as the t-statistics are almost alway insignificant. This is a huge benefit of WLS-EV, which is able to differentiate noise from signal and is hence able to give us conclusions which are closer to the true data generating process.

$$r_{t+1,t+h} = r_t * \beta + \epsilon_{t+1,t+h} \tag{16}$$

$$r_{t+1,t+h} = vrp_t * \beta + \epsilon_{t+1,t+h} \tag{17}$$

Johnson (2018) studied the impact of WLS-EV on return predictability regressions. We also asked ourselves if we can use WLS-EV with other, non-return data, which is why we analyze two additional relationships. First, we regress third P moments on third Q moments, as in equation 18. A detailed description of our non-return data is in section 2.1.2.

$$thirdmoment^P_{t+1,t+h} = thirdmoment^Q_{t+1,t+h} * \beta + \epsilon_{t+1,t+h} \tag{18}$$

Forecast horizons are also 1, 7, 30, 60, 91, 182, and 365. We match the cumulative x-day P moment with the respective Q moment, which is therefore extracted from options with a maturity of x-day. WLS-EV results are that in-sample r-squared are lower than in OLS, and often negative, as few very highly volatile observations have a large influence on OLS. WLS-EV out-of-sample r-squared are higher than in OLS and get higher with larger forecast horizon, which results from the converging relationship between implied and realized variance with growing maturity. Besides the good out-of-sample results, third moment coefficients are mainly insignificant with WLS-EV, while OLS has some significance.

Then, we regress fourth P on fourth Q moments, as in equation 19.

$$fourthmoment^P_{t+1,t+h} = fourthmoment^Q_{t+1,t+h} * \beta + \epsilon_{t+1,t+h} \tag{19}$$

For fourth moments, low forecast horizons have positive significance with WLS-EV, but almost always significance with OLS. As the forecast horizon rises, WLS-EV estimators get insignificant with WLS-EV.

We think that WLS-EV offers valuable insights for non-return data. As significance and predictability conclusions are different than with OLS, it will be topic for further research to analyse non-return relationships more detailed. This would go beyond the scope of this seminar.

5 Conclusion

The goal of this seminar was to implement WLS-EV in a python library and test it with financial data. This goal has been archieved and furthermore the application to non-return data analyzed.

In general, WLS-EV return regression results are more pessimistic than OLS results in terms of positive linear relationships. Heteroscedastic and autocorrelated data might lead to false significance conclusions when using OLS. Using WLS-EV archieves lower in-sample predictability, lower significance in t-statitics, and higher out-of-sample-predictability. We advise to be careful when interpreting OLS results of heteroscedastic and autocorrelated data, as OLS assumptions which are not met are often a problem, as this seminar shows. A prominent example of false OLS conclusions might be the work of Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2010), which find variance risk premium as a weak predictor of equity returns. WLS-EV disagrees with this conclusions.

However, this seminar also leads to several open issues. First, it might be insightful to study what impact an improved variance estimation has on WLS-EV. Using more sophisticated models might make WLS-EV more complex to implement and use, but might also bring better results. Second, the application of WLS-EV to non-returns needs to be further analysed. Especially the variance estimation part needs to be revisited.

As a conclusion, we think that WLS-EV is a model which is easy to implement and understand and delivers great insights. It shows that one needs to be careful when analysing linear relationships of heteroscedastic and autocorrelated data, and that common OLS conclusions might be false.

6 Erklärung

Ich versichere wahrheitsgemäß, die Arbeit selbstständig angefertigt und alle benutzten Hilfsmittel und Quellen vollständig angegeben zu haben, die wörtlich oder inhaltlich übernommenen Stellen als solche kenntlich gemacht zu haben und die Satzung des KIT zur Sicherung guter wissenschaftlicher Praxis beachtet zu haben.

7.4.2018	Tobias [illegible]
Datum	*Name*

Appendices

A Results

Forecast horizon	1		7		14		22		44		66		88	
	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV
IS R2	0.0014	0.0013	0.0044	0.0041	0.0018	0.0012	0.0015	0.0006	0.0008	0.0004	-0.4738	-0.0005	0.0	-0.0008
OOS R2	0.0033	0.0032	0.0033	0.0048	0.0022	0.0035	0.0005	0.0047	0.0007	0.0138	0.0001	0.0195	0.0001	0.023
Intercept t-stat	-0.4643	-0.7566	-0.5341	-0.632	-0.7301	-0.6319	-0.8879	-0.6006	-0.0043	-0.5187	-1.3109	-0.4738	-1.4655	-0.4536
variable t-stat	-1.8234	-2.2707	-3.1294	-2.5811	-2.088	-1.1032	-2.1506	-0.6338	-0.1651	-0.8207	-1.0851	-0.4738	-0.3793	0.4333

Table 4: Return on return regression

Forecast horizon	1		7		14		22		44		66		88	
	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV
IS R2	0.0005	0.0002	0.0024	0.0004	0.0015	-0.0025	0.0032	-0.0034	0.0018	-0.0138	0.0033	-0.0177	0.0097	-0.0143
OOS R2	0.002	-0.0017	0.0025	-0.0033	0.0002	-0.0098	0.0002	-0.005	-0.0048	-0.0152	-0.0152	0.0102	-0.0412	0.0847
Intercept t-stat	-1.3762	-0.779	-1.4331	-0.3914	-1.2796	-0.2889	-1.6162	-0.2634	-1.6044	-0.2113	-1.9427	-0.1246	-2.5233	-0.0279
variable t-stat	-0.78	-0.2278	-0.9922	-0.0895	-0.6931	0.2843	-1.096	0.2239	-0.9214	0.5551	-1.1861	0.4854	-2.6395	0.2592

Table 5: return on vrp regression

Forecast horizon	1		7		30		60		91		182		365	
	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV
IS R2	0.0	-0.0005	0.0006	-0.0042	0.0044	-0.0135	0.0234	-0.0301	0.0238	-0.0474	0.0256	-0.1142	0.0012	-0.3196
OOS R2	-0.0006	0.0021	0.0099	0.0803	0.0552	0.401	0.1929	0.689	0.4414	0.7859	0.371	0.889	-0.2988	0.9711
Intercept t-stat	-0.6325	0.9871	-1.1629	0.2184	-1.7097	0.3499	-2.5945	0.9561	-3.0697	0.6081	-3.7199	-0.5442	-3.8222	-0.8805
variable t-stat	0.1197	1.887	-0.8197	1.9251	-1.405	1.4476	-2.5945	1.5309	-2.6552	1.1982	-2.9723	-0.3087	-0.811	-0.6694

Table 6: Third P moment on third Q moment regression

Forecast horizon	1		7		30		60		91		182		365	
	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV	OLS	WLSEV
IS R2	0.0011	-0.0037	0.0046	-0.0163	0.0138	-0.0401	0.0448	-0.063	0.0549	-0.0871	0.1165	-0.1588	0.0644	-0.3368
OOS R2	-0.0857	0.159	-0.3837	0.6113	0.023	0.8392	0.0732	0.923	0.4387	0.9566	0.2892	0.9763	0.441	0.9873
Intercept t-stat	2.4131	8.474	2.679	3.0076	3.5656	2.0413	4.1926	1.3837	4.963	1.2179	6.829	1.1285	8.2119	0.7475
variable t-stat	-2.1056	-3.4441	2.679	15.5086	-3.0147	3.5544	-3.7245	2.0164	-4.4019	1.4467	-6.2353	1.6687	-6.0323	0.7963

Table 7: Fourth P moment on fourth Q moment regression

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