Laser and Oscillator noise couplings to DC readout

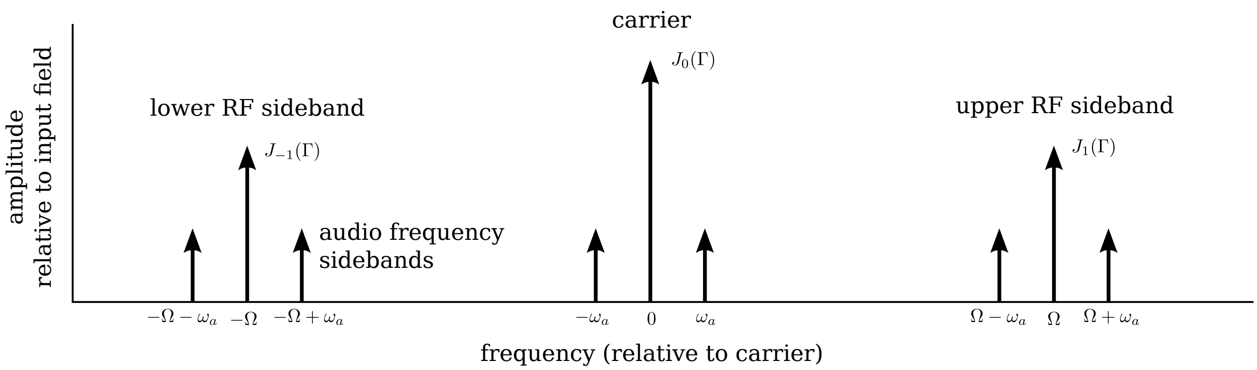
The four main noises whose transfer functions are changed by DC readout are laser amplitude and frequency noise, and RF oscillator amplitude and phase noise. The canonical reference describing the couplings of these noises is the indispensible "Frequency response of the LIGO interferometer" [2], incorporating materials from Jordan Camp's note [1]. Kentaro's paper [3] includes comprehensive discussion of laser noise couplings in DC and RF readouts both for power- and dual-recycled interferometers. Rob Ward's paper [4] and thesis [5] also provide discussion.

Conventions

The primary modulation is done by a cosine. The sine quadrature is "Q".

A periodic table of the sidebands

Here is a little doodle showing the fields under consideration. They consist of three primary fields: the carrier and two phase-modulation sidebands; and audio frequency sidebands on either side of each of the three primary sidebands:

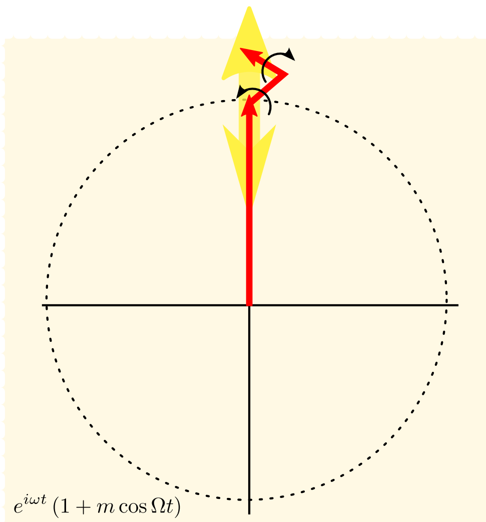


Each successive stage of modulation produces a pair of sidebands around every existing sideband. We begin with a single field, the carrier. Phase modulation produces sidebands around the carrier. Amplitude or phase modulation of the input laser field produces audio frequency sidebands around all of these existing sidebands.

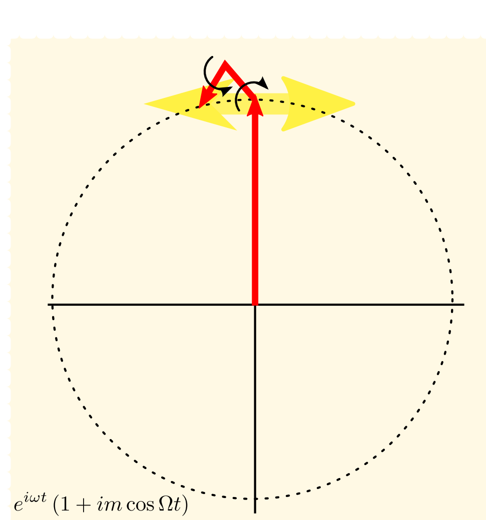
The differences between laser amplitude noise, laser frequency noise, oscillator amplitude noise, and oscillator phase noise lies in the relative phases of the audio-frequency sidebands these processes produce.

		near lower SB			near carrier			near upper SB		
		$-\Omega-\omega_a$	$-\Omega$	$-\Omega+\omega_a$	$-\omega_a$	0	$+\omega_a$	$\Omega-\omega_a$	Ω	$\Omega-\omega_a$
carrier						1				
phase modulation	$J_1(\Gamma)$		i						i	
laser AM		i		i	1		1	i		i
laser FM		$-i$		i	-1		1	$-i$		i
oscillator AM		i		i				i		i
oscillator ϕM		1		1				-1		-1
gw signal					1		1			
		phase relative to carrier								

The RF sidebands have a phase of i relative to the carrier; this is simply from the convention that the RF modulation is done with a cosine waveform. It's not physically significant, since the difference between cosine modulation and sine modulation is the origin of time.



amplitude modulation



phase modulation

Audio-frequency signals (those relevant in DC readout) are made when secondary, audio-frequency sidebands beat against their parent RF sideband.

Laser AM

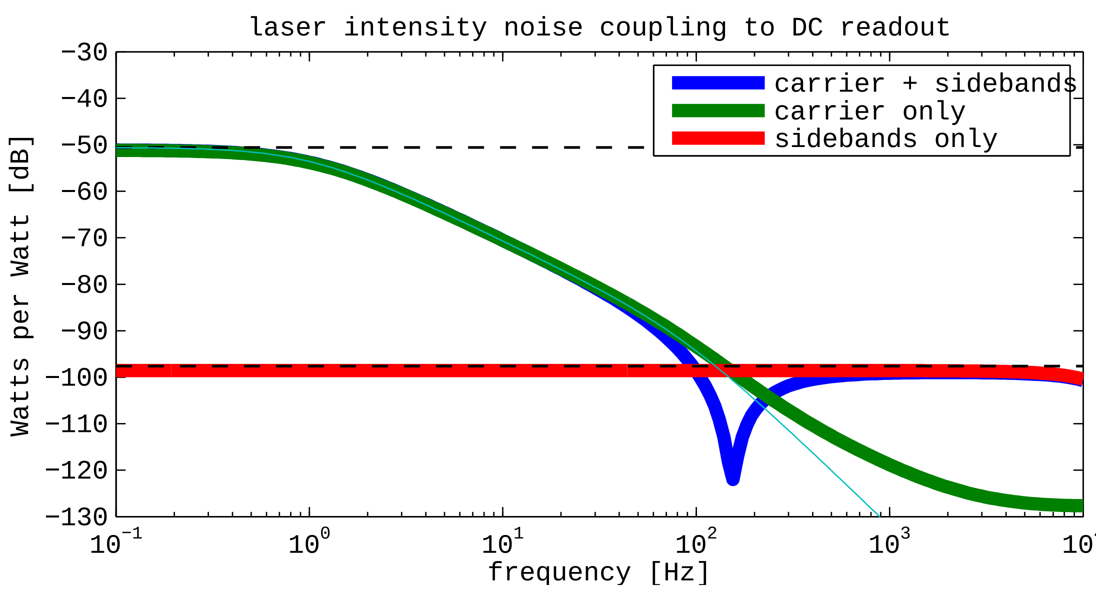
The laser AM noise coupling is the easiest to understand. In DC readout, we introduce a static DARM offset to allow some laser carrier to be transmitted to the 'dark' port. Any amplitude modulation of the carrier below the coupled-cavity pole will be transmitted too. The coupled-cavity pole kicks in near 1 Hz, attenuating the transmission of laser AM above that frequency by $\sim 1/f$.

Laser AM also produces amplitude modulation of the RF sidebands. These are not filtered by the coupled-cavity pole, and the transmission of the sidebands to the dark port is high. AM on the RF sidebands gets all the way to the output port, where they are stopped by the Output Mode Cleaner.

The transmission of SB AM to the AS port is essentially flat, while the transmission of carrier AM falls at $1/f$ above the cc pole. At some point, these curves will cross. Here the contributions have an opportunity to destructively interfere, creating a notch.

- What is the PRC cavity pole seen by the sidebands?
- What about higher order modes?

AM noise also creates technical radiation pressure noise. We ignore that here, but consider it below, as a displacement noise.



Laser frequency noise

Frequency noise (or phase noise, which is the same thing), is not seen by a photodiode, which sees only power. For frequency noise to couple to DC readout, it must be converted to AM by the interferometer–there must be some dispersion.

Oscillator AM

Amplitude modulation of the signal from the RF oscillator produces a time-varying modulation depth in the EOM. The amount of power that is transferred from the carrier into the RF sidebands becomes time-varying.

If the beam out of the EOM were directly incident on a photodiode, the photodiode would, of course, see no power variations–the an increase in sideband power is directly compensated by a decrease in carrier. This is the nature of phase modulation.

The interferometer passes the carrier and sideband field differently to the output port. The carrier sees the double-cavity pole, while the sideband sees a large attenuation by the OMC. This destroys the perfect cancellation and creates a coupling from oscillator AM to power seen at the OMC photodiode.

There is a cross-over around 100 Hz. Below this, carrier AM dominates over sideband AM at the OMC PD; above, the residual sideband field is the dominant effect.

Relation to laser AM

Given this mechanism, it would seem simple to relate the Oscillator AM coupling to the Laser AM coupling.

The power in the carrier is given by $J_0\left(\Gamma\right)^2$. Take the derivative to see how carrier power changes with respect to changes in modulation depth: $\frac{\partial}{\partial\Gamma}J_0\left(\Gamma\right)^2 = 2J_0\left(\Gamma\right)\frac{\partial}{\partial\Gamma}J_0\left(\Gamma\right) = -2J_0J_1$. To convert this to "relative intensity noise", we have to divide by the DC power in the field, which is J_0^2 for the carrier and J_1^2 for each RF sideband. Suppose our AM has the form $\Gamma = \Gamma_0\left(1+m\right)$. Then the "RIN per AM" is $-2J_1/J_0$ for the carrier and $2J_0/J_1$ for each sidebands. Divide by two to get "AM per Oscillator AM".

Oscillator AM transfers power back and forth between the carrier and the sidebands. Because the carrier and the sidebands don't have equal power to begin with, the effective "AM" induced on the sideband and carrier are different.

Oscillator ΦM: Phase modulation of phase modulation

Phase modulation on the RF oscillator source leads to "phase modulation of phase modulation." This has no effect on the laser carrier, i.e. pm of pm does not produce any audio-frequency sidebands around the carrier. Instead, it produces phase modulation around the RF sidebands. Thus, direct coupling of oscillator PM must come via the RF sidebands. Moreover, we need some asymmetry or dispersion that will turn the phase modulation into amplitude modulation for it to be seen by the photodiode.

This dispersion could come from the OMC or from reflection off of the arm cavities. Reflection off of the arm cavities is the dominant effect, since the arm's FSR is so much smaller than the OMC's.

BUT, assuming the RF sidebands are anti-resonant in the arms, any phase dispersion on AF pm sidebands of the RF sidebands will be equal and opposite, which does nothing.

There is a zero at zero; looks like a pole at 0.7 MHz.

The phase noise coupling is approximately the same for a PRMI, so the arms are not involved. Is it the Schnupp assy, then? Or the PRC?

The PRC should be perfectly on resonance for the sideband. So there should be no common-mode phase rotation for the sideband. Unless it isn't. How to check that the PRC resonates?

Nope, it's still there even without the RM. So that leaves the Schnupp assy.

Other noise couplings that are modified by the static DARM offset

Cross-couplings

We have not yet considered the effect of control loops. The DARM loop acts to suppress any variation in the power at the interferometer's output port by pushing on the DARM degree of freedom. The DARM motion produces signals at not just the AS port, but other sensing ports as well. Servos producing error signals from these ports push on other interferometer degrees of freedom in order to suppress the observed signal. Eventually this comes around to the output port again.

Cross couplings are a result of

1. non-diagonal sensing
2. non-diagonal actuation

Some things to keep in mind:

1. Cross-couplings go away above the bandwidth of the servo loops(?)

Radiation pressure noise is a displacement noise

The above discussion has not considered the presence of radiation pressure. AM fields produce modulated radiation pressure on the optics. The resulting motion of the optics modifies the fields in the interferometer. So we will handle the radiation pressure coupling in two steps: find the coupling from a noise input to radiation pressure in the arms, and then use our known DARM optical gain to find out what signal this produces at the output port.

With arm cavities held on resonance (as in RF readout), the displacements induced by radiation pressure in the arms are entirely common-mode. Thus there is a small AM $\rightarrow$ radiation pressure $\rightarrow$ CARM $\rightarrow$ ϕM coupling path. The differential arm detunings used in DC readout open up a AM$\rightarrow$radiation pressure$\rightarrow$DARM coupling path.

References

[1] Jordan Camp. Frequency, Intensity and Oscillator Noise in the LIGO. (T960019), February 1996. Available from: `http://docuserv.ligo.caltech.edu/docs/public/T/T960019-00.pdf`.

[2] Daniel Sigg and others. Frequency response of the LIGO interferometer. Technical Report T970084-00, LIGO Laboratory, February 1997. Available from: `http://www.ligo.caltech.edu/docs/T/T970084-00.pdf`.

[3] K. Somiya, Y. Chen, S. Kawamura, and N. Mio. Frequency noise and intensity noise of next-generation gravitational-wave detectors with RF/DC readout schemes. *Physical Review D*, 73(12):122005+, June 2006. Available from: `http://dx.doi.org/10.1103/PhysRevD.73.122005`.

[4] R. L. Ward, R. Adhikari, B. Abbott, R. Abbott, D. Barron, R. Bork, T. Fricke, V. Frolov, J. Heefner, A. Ivanov, O. Miyakawa, K. McKenzie, B. Slagmolen, M. Smith, R. Taylor, S. Vass, S. Waldman, and A. Weinstein. DC readout experiment at the Caltech 40m prototype interferometer. *Classical and Quantum Gravity*, 25(11):114030+, 2008. Available from: `http://dx.doi.org/10.1088/0264-9381/25/11/114030`.

[5] Robert L. Ward. *Length Sensing and Control of an Advanced Prototype Interferometric Gravitational Wave Detector*. PhD thesis, California Institute of Technology, Pasadena, CA, February 2010. Available from: `https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=9237`.