Dither locking

Dither locking is a very handy technique to control some parameter such that some output signal is maximized or minimized. The technique is simple: the parameter is 'dithered' by adding a small sinusoidal excitation, and the system output is synchronously demodulated at the same frequency. This produces a linear error signal.

We can think of dither locking as sensing the first derivative of a function. Suppose we have some function $f(x)$. The parameter x is dithered around its nominal value: $x = x_0 + m \sin \Omega t$. Expand the function as a Taylor series:

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \cdots$$
$$= f(x_0) + (m \sin \Omega t) f'(x_0)$$

Now we demodulate by multiplying by $\sin \Omega t$:

$$\epsilon(x_0) = f(x_0) \sin \Omega t + m (\sin \Omega t)^2 f'(x_0) + \cdots$$

write $\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos(2x)$:

$$\epsilon(x_0) = \frac{m}{2} f'(x_0) + f(x_0) \sin \Omega t + \frac{m}{2} (\cos 2\Omega t) f'(x_0) + \cdots$$

If we low-pass this signal, we recover a measurement of the first derivative of the thing we want to maximize or minimize. This is an ideal error signal for a control system.