

Frequency modulation with modulation depth  $m_{\text{FM}}$  [Hz] at a frequency  $f$  [Hz] has the form

$$y(t) = \exp \{i\omega t\} \exp \left\{ i \int_0^t 2\pi m_{\text{FM}} \Re \left\{ e^{i2\pi f t'} \right\} dt' \right\}$$

where  $\omega$  [radians/second] is the carrier frequency. We can simply do the integral and get:

$$y(t) = \exp \{i\omega t\} \exp \left\{ i \frac{1}{f} m_{\text{FM}} \Re \left\{ i e^{i2\pi f t} \right\} \right\}$$

which is just phase modulation at frequency  $f$  [Hz] with modulation depth  $m_{\text{PM}} = \left( \frac{i}{f} \right) m_{\text{FM}}$  [radians]:

$$\frac{d}{dt} \left( \frac{1}{f} m_{\text{FM}} \sin 2\pi f t \right) = 2\pi m_{\text{FM}} \cos 2\pi f t$$

The  $i$  signifies a phase shift of 90 degrees in the modulation (turning cos to sin).