Paper Results

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Trends and distributional properties

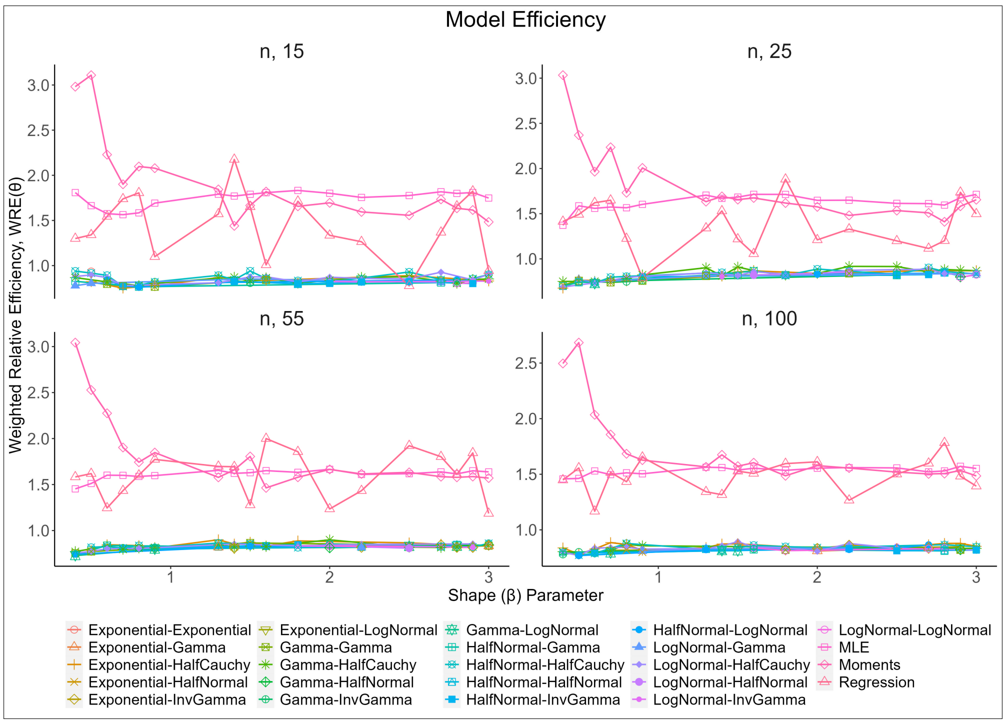

Figure 1: The trend of model efficiency for small and large samples of Weibull distributed datasets with monotone hazard rates shows that MCMC methods consistently outperform classical methods

Model selections and priorities

Table 1: A section of results showing weighted relative efficiency for a small sample size of n = 15 units. The table compares efficiency of different combinations of prior distributions for the shape and scale parameters of the Weibull distribution with the classical methods.

			Prior Distribution		Model Efficiency
n	shape	method	shape_prior	scale_prior	WRE
15	0.4	Moments	-	-	2.982
		MLE	-	-	1.808
		Regression	-	-	1.300
		MCMC	HalfNormal	HalfCauchy	0.939
			LogNormal	HalfCauchy	0.880
			Gamma	HalfCauchy	0.867
			HalfNormal	Gamma	0.826
			LogNormal	Gamma	0.777
	0.5	Moments	-	-	3.109
		MLE	-	-	1.664
		Regression	-	-	1.340
		MCMC	Exponential	Exponential	0.934
			HalfNormal	HalfCauchy	0.917
			LogNormal	HalfCauchy	0.896
			Exponential	HalfCauchy	0.823
			LogNormal	Gamma	0.803
	0.6	Moments	-	-	2.229
		MLE	-	-	1.575
		Regression	-	-	1.545
		MCMC	HalfNormal	HalfCauchy	0.892
			LogNormal	HalfCauchy	0.867
			HalfNormal	LogNormal	0.840
			Gamma	HalfCauchy	0.819
			Gamma	Gamma	0.797

Table 2: A section of results showing weighted relative efficiency for a large sample size of n = 100. The table highlights that MCMC methods are consistently more efficient than classical methods.

			Prior Distribution		Model Efficiency
n	shape	method	shape_prior	scale_prior	WRE
100	1.4	Moments	-	-	1.675
		MLE	-	-	1.561
		Regression	-	-	1.316
		MCMC	Exponential	HalfCauchy	0.873
			LogNormal	HalfCauchy	0.855
			HalfNormal	Gamma	0.821
			HalfNormal	HalfCauchy	0.817
			Gamma	LogNormal	0.814
	1.5	Moments	-	-	1.573
		MLE	-	-	1.538
		Regression	-	-	1.528
		MCMC	LogNormal	HalfCauchy	0.893
			Exponential	HalfCauchy	0.874
			Exponential	Gamma	0.859
			HalfNormal	HalfCauchy	0.838
			Gamma	LogNormal	0.812
	1.6	Moments	-	-	1.604
		MLE	-	-	1.554
		Regression	-	-	1.508
		MCMC	HalfNormal	HalfCauchy	0.862
			LogNormal	HalfCauchy	0.846
			LogNormal	LogNormal	0.838
			Gamma	InvGamma	0.828
			HalfNormal	Gamma	0.823

Table 3: A comparison of the average weighted relative efficiency of the top MCMC methods with classical methods for small sample sizes. The AWRE is the average of the weighted relative efficiency (WRE) across lifetime datasets with either decreasing or increasing hazard rate properties. On average, MCMC methods are more efficient.

			Prior Distribution		Model Efficiency
n	hazard	method	shape_prior	scale_prior	AWRE
15	DHR	Moments	-	-	2.399
		MLE	-	-	1.648
		Regression	-	-	1.471
		MCMC	Exponential	Exponential	0.934
			HalfNormal	HalfCauchy	0.853
			LogNormal	HalfCauchy	0.833
			Gamma	HalfCauchy	0.800
			HalfNormal	LogNormal	0.782
	IHR	MLE	-	-	1.791
		Moments	-	-	1.645
		Regression	-	-	1.441
		MCMC	HalfNormal	HalfCauchy	0.862
			LogNormal	HalfCauchy	0.856
			Gamma	HalfCauchy	0.845
25	DHR	Moments	-	-	2.224
		MLE	-	-	1.543
		Regression	-	-	1.365
		MCMC	Gamma	HalfCauchy	0.762
			LogNormal	HalfCauchy	0.749
			HalfNormal	HalfCauchy	0.743
			Gamma	Gamma	0.738
			LogNormal	Gamma	0.729
	IHR	MLE	-	-	1.666
		Moments	-	-	1.585
		Regression	-	-	1.360
		MCMC	Gamma	HalfCauchy	0.872
			HalfNormal	HalfCauchy	0.837
			LogNormal	HalfCauchy	0.833

Table 4: The average weighted relative efficiency (AWRE) of the top MCMC methods and the classical methods for large sample sizes. The AWRE is the average of the weighted relative efficiency (WRE) across lifetime datasets with either decreasing or increasing hazard rate properties. On average, MCMC methods are more efficient than the classical methods.

			Prior Distribution		Model Efficiency
n	hazard	method	shape_prior	scale_prior	AWRE
55	DHR	Moments	-	-	2.223
		MLE	-	-	1.559
		Regression	-	-	1.543
		MCMC	HalfNormal	HalfCauchy	0.802
			Exponential	HalfCauchy	0.795
			Gamma	HalfCauchy	0.791
			LogNormal	HalfCauchy	0.783
			HalfNormal	Gamma	0.782
	IHR	MLE	-	-	1.635
		Regression	-	-	1.630
		Moments	-	-	1.610
		MCMC	Exponential	HalfCauchy	0.848
			Gamma	HalfCauchy	0.827
			HalfNormal	HalfCauchy	0.824
100	DHR	Moments	-	-	2.064
		MLE	-	-	1.494
		Regression	-	-	1.462
		MCMC	Exponential	HalfCauchy	0.836
			LogNormal	HalfCauchy	0.819
			Gamma	HalfCauchy	0.816
			HalfNormal	HalfCauchy	0.797
			LogNormal	Gamma	0.778
	IHR	MLE	-	-	1.550
		Moments	-	-	1.549
		Regression	-	-	1.494
		MCMC	Exponential	HalfCauchy	0.845
			LogNormal	HalfCauchy	0.835
			HalfNormal	HalfCauchy	0.831

Application: Prostate Cancer Survival Data

Table 5: The best combination of prior distributions versus classic methods for accurately estimating the Weibull model's parameters for the Prostrate cancer data. The actual parameter values are ß = 1.43 and a =40.34, pooled from the results of the 27 fitted models.

		Priors		Estimated Parameters		Model Efficiency
sample_size	method	shapePrior	scalePrior	mean_shape	mean_scale	WRE
87	Regression	-	-	1.583	40.095	1.820
	MLE	-	-	1.457	40.391	1.566
	Moments	-	-	1.367	39.742	1.330
	MCMC	LogNormal	Gamma	1.433	40.360	0.854
		Gamma	HalfCauchy	1.436	40.489	0.845
		Exponential	HalfNormal	1.417	40.402	0.832
		Exponential	HalfCauchy	1.416	40.368	0.831
		HalfNormal	HalfCauchy	1.440	40.480	0.829
		Gamma	HalfNormal	1.434	40.404	0.816
		HalfNormal	LogNormal	1.436	40.366	0.808
		Exponential	InvGamma	1.418	40.240	0.805
		Exponential	LogNormal	1.413	40.232	0.798
		HalfNormal	HalfNormal	1.439	40.459	0.795

Table 6: Mean residual lifetime of top three MCMC-based and the classic Weibull model of a randomly sampled prostate cancer survival times. The Integrated model uses the actual parameter values ß = 1.43 and a =40.34, pooled from the results of all the models under the study. MCMC and Classic results represent percent deviations from the true mean residual lifetime of the integrated method

		MCMC			Classics		
time	Integrated	LogNormal-Gamma	Gamma-HalfCauchy	Exponential-HalfNormal	MLE	Regression	Moments
2	35.12	+0.06%	+0.34%	+0.40%	-0.20%	-2.36%	-0.46%
12	30.34	+0.10%	+0.36%	+0.92%	-0.73%	-5.83%	+1.25%
17	28.70		+0.35%	+1.11%	-1.01%	-7.42%	+2.02%
26	26.37	+0.11%	+0.34%	+1.48%	-1.40%	-9.97%	+3.26%
28	25.93	+0.12%	+0.35%	+1.54%	-1.47%	-10.49%	+3.51%
30	25.51	+0.16%		+1.61%	-1.53%	-10.94%	+3.80%
35	24.56		+0.37%	+1.79%	-1.75%	-12.13%	+4.40%
37	24.21	+0.17%		+1.86%	-1.82%	-12.60%	+4.63%
42	23.40		+0.34%	+2.01%	-1.97%	-13.63%	+5.17%
48	22.54	+0.13%	+0.31%	+2.13%	-2.17%	-14.82%	+5.77%

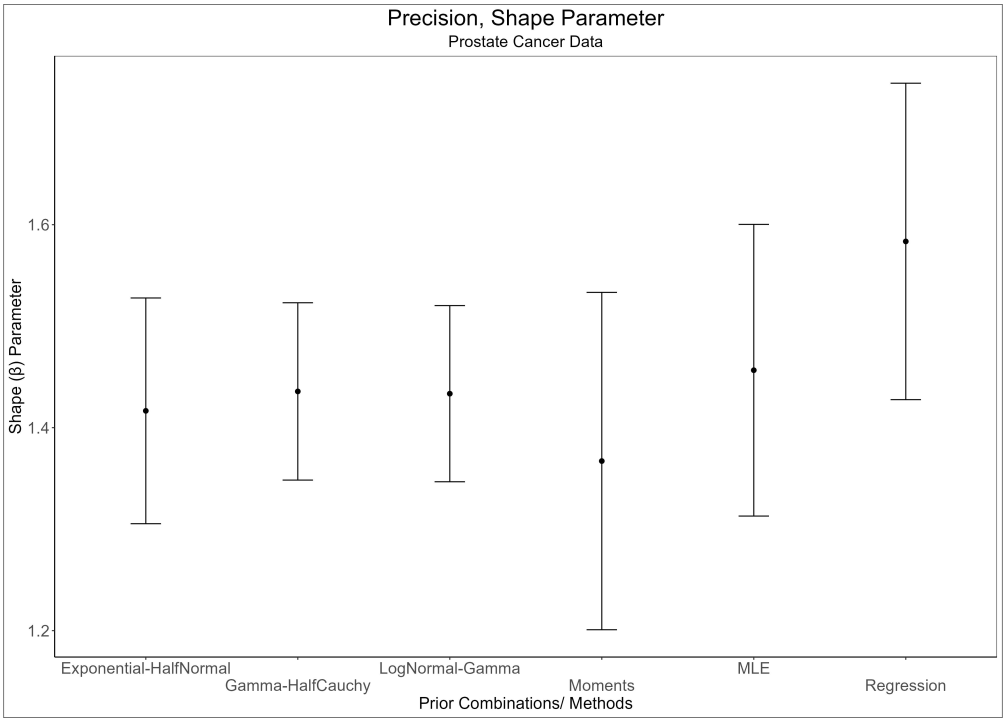

Figure 2: Uncertainty for the shape parameter of the Weibull model. We compute the interval as emprical two standard deviations from the actual parameter value obtained using an integrated approach where we pool the results from all methods under the study.

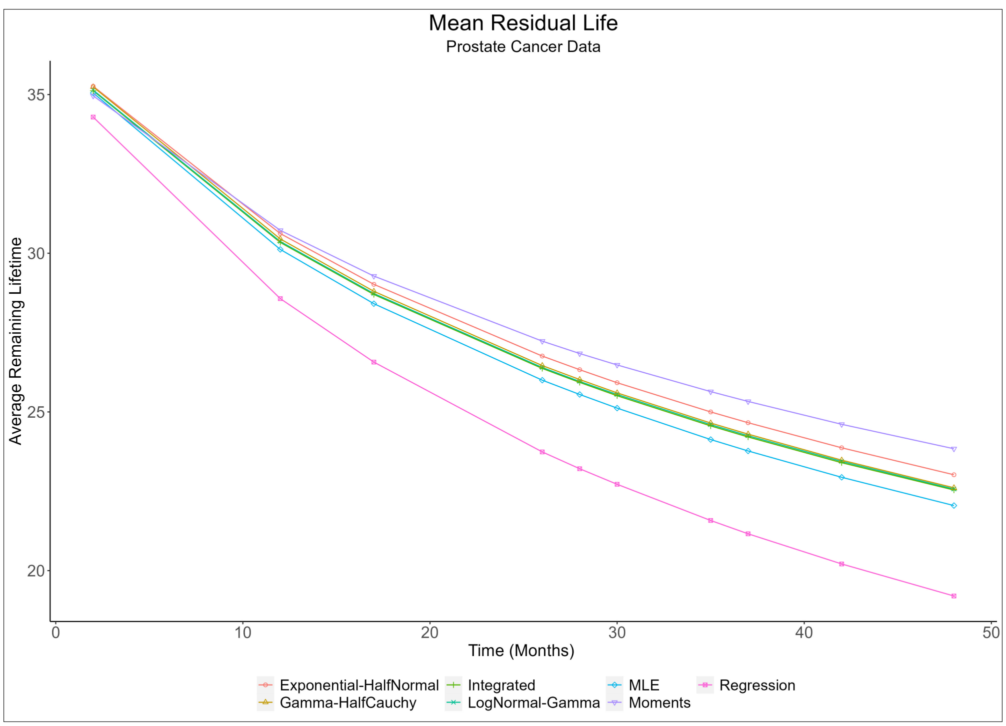

Figure 3: Mean residual life plot for Prostate cancer data. The MCMC method for calculating the mean residual lifetime is more robust than traditional methods. Regression and MLE methods underestimate the population mean remaining life, while the Moments overestimate the parameter.

Conclussion and Discussion

Table 7: A list of recommended combinations of prior distributions for small and large Weibull-distributed data sets with decreasing hazard rate properties. The priors are ranked based on the AWRE.

n	hazard	shape_prior	scale_prior	AWRE
n = (15, 25)	DHR	Exponential	Exponential	0.9340
		HalfNormal	HalfCauchy	0.7980
		LogNormal		0.7910
		HalfNormal	LogNormal	0.7820
		Gamma	HalfCauchy	0.7810
		Gamma	Gamma	0.7380
		LogNormal		0.7290
n = (55, 100)		Exponential	HalfCauchy	0.8155
		Gamma		0.8035
		LogNormal		0.8010
		HalfNormal		0.7995
		HalfNormal	Gamma	0.7820
		LogNormal		0.7780

Table 8: A list of recommended combinations of prior distributions for small and large Weibull-distributed data sets with increasing hazard rate properties. The priors are ranked based on the AWRE.

n	hazard	shape_prior	scale_prior	AWRE
n = (15, 25)	IHR	Gamma	HalfCauchy	0.8585
		HalfNormal		0.8495
		LogNormal		0.8445
n = (55, 100)		Exponential		0.8465
		LogNormal		0.8350
		HalfNormal		0.8275
		Gamma		0.8270