Completeness Thresholds for Memory Safety: Unbounded Guarantees via Bounded Proofs

Tobias Reinhard[1], Justus Fasse[2], Bart Jacobs[2]
[1] TU Darmstadt
[2] KU Leuven

Programming Technology Group
University of Oslo
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What This Work Is About

- Connection between bounded & unbounded proofs
- Ideas to increase trust in bounded model checking

What This Work Is About

- Connection between bounded & unbounded proofs
- Ideas to increase trust in bounded model checking
- When is a bounded “proof” a proof?

Focus: Traversing Programs

- Target property: Memory safety
- Programs that:
  - Traverse a data structure
  - Preserve its memory layout
- But approach & many results are very general

Model Checking: Easy Off-by-1 Error

- Imperative language with pointer arithmetic
- Memory assumption $\texttt{array}(a, s)$:
  $a[0]\ \ \ldots\ \ a[s-1]$ allocated

```
for i in [0 : s-1] do
    !a[i+1]
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for i in [0 : s-1] do
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Which bounds should we choose for s?

- $s = 0$: No error
- $s = 1$: Error

Model Checking: “Harder” Off-by-N Error

Memory assumption:
$\texttt{array}(a, s)$

```
for i in [0 : s-2] do
    !a[i+2]
```

Which bounds should we choose for s?

Model Checking: “Harder” Off-by-N Error

Memory assumption:
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```
for i in [0 : s-2] do
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```

Which bounds should we choose for s?

- $s = 0$: No error
- $s = 1$: No error
- $s = 2$: Error

Model Checking: No Off-by-N Error

Memory assumption:
$\texttt{array}(a, s)$

```
for i in [0 : s-1] do
    !a[i]
```

Which s can convince us?

Model Checking: No Off-by-N Error

Memory assumption:
$\texttt{array}(a, s)$

```
for i in [0 : s-1] do
    !a[i]
```

Which s can convince us?

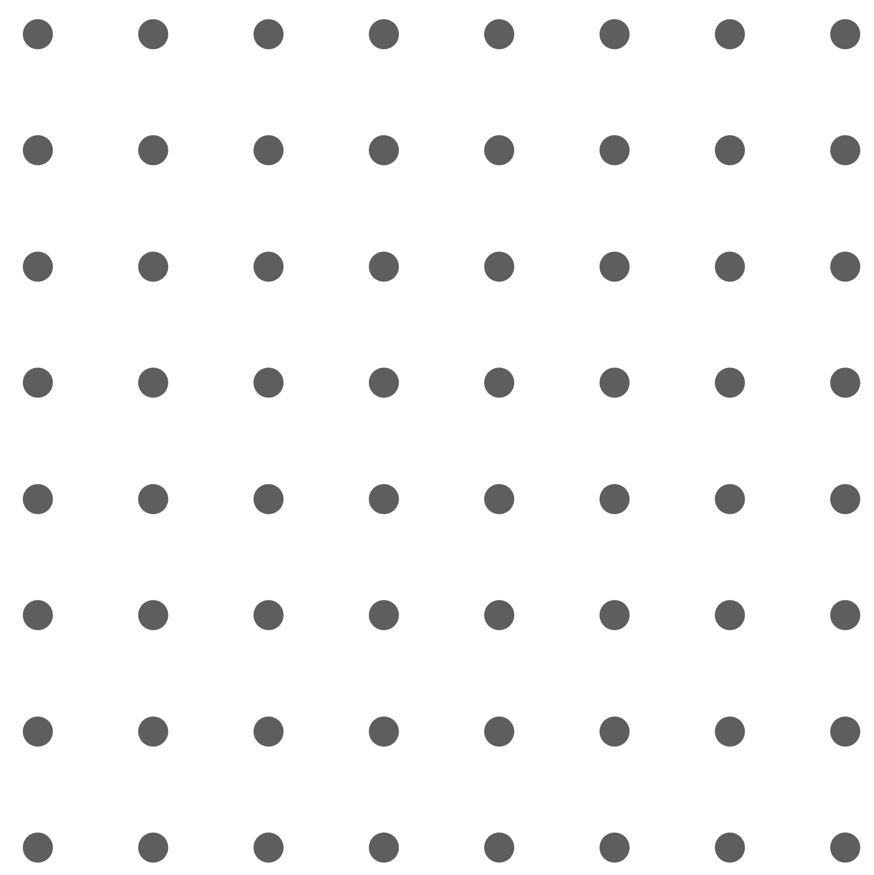
- $s = 0$: No error
- $s = 1$: No error
- $s = 2$: No error
- $s = 3$: No error

$\vdots$

$\Rightarrow$ Which size bound is large enough?

Model Checking Finite Systems

- Finite state transition system T
- Prove property Gp
  G ≈ globally ≈ p holds in every state
- Approach:
  Prove Gp for all paths up to length k
  $T \vDash_k \mathrm{G}p$



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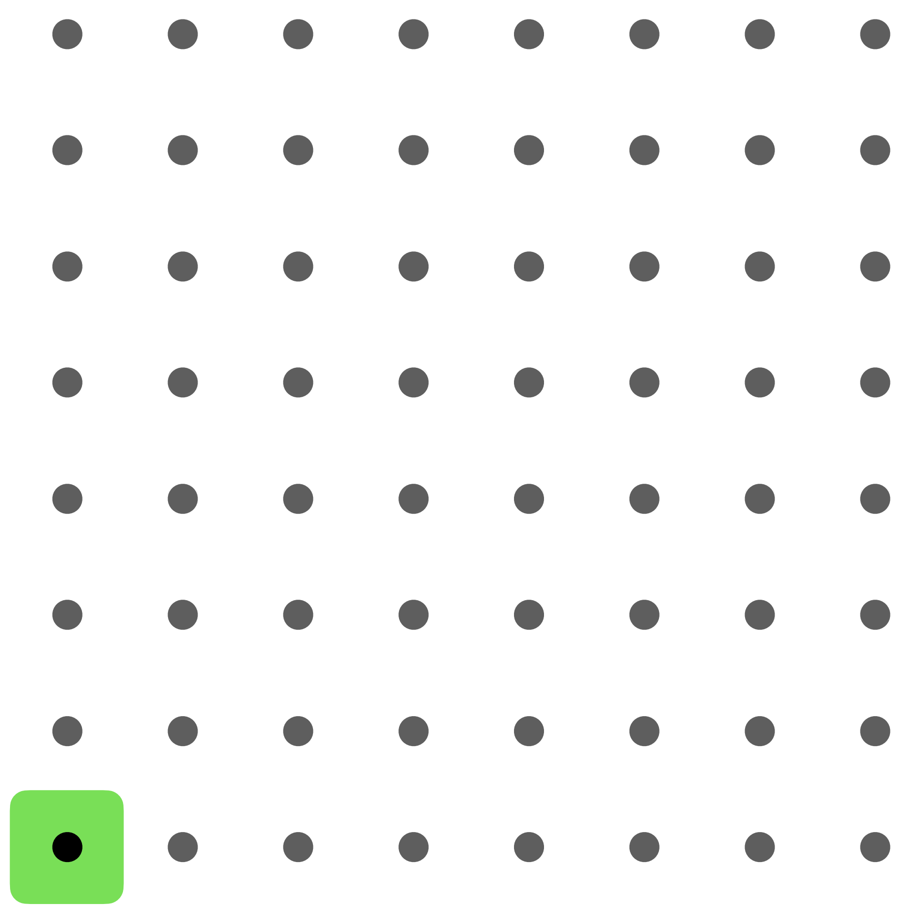

$T \vDash_0 \mathrm{G}p$

Model Checking Finite Systems

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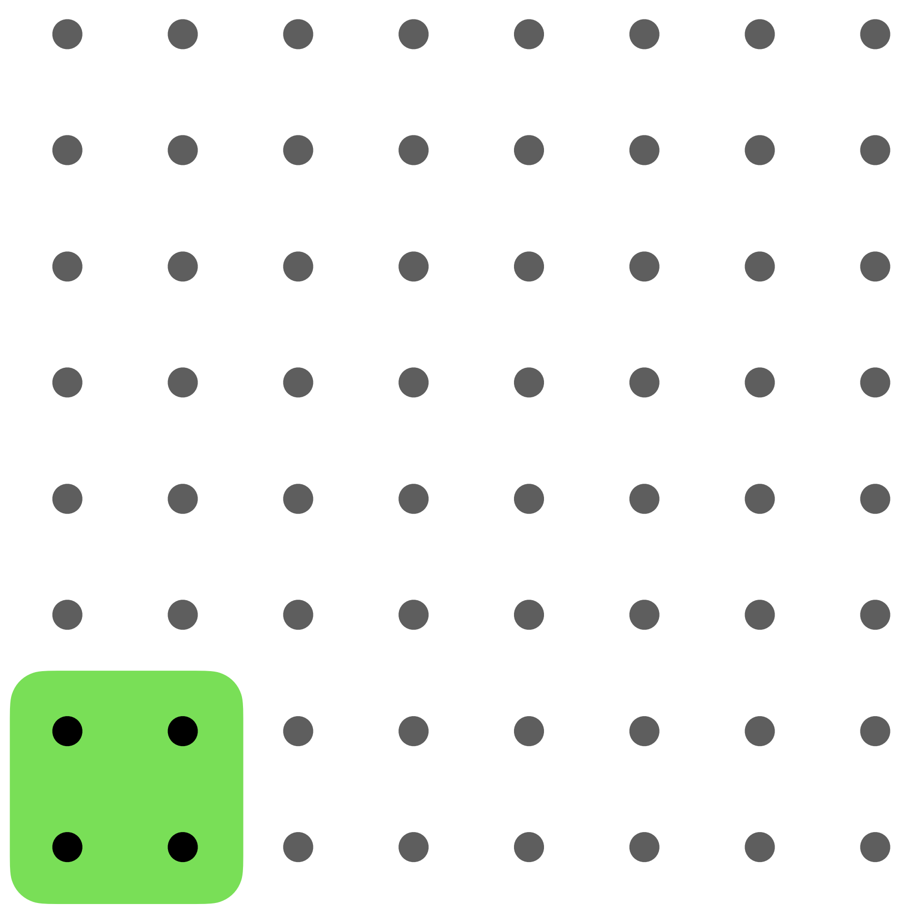

$T \vDash_1 \mathrm{G}p$

Model Checking Finite Systems

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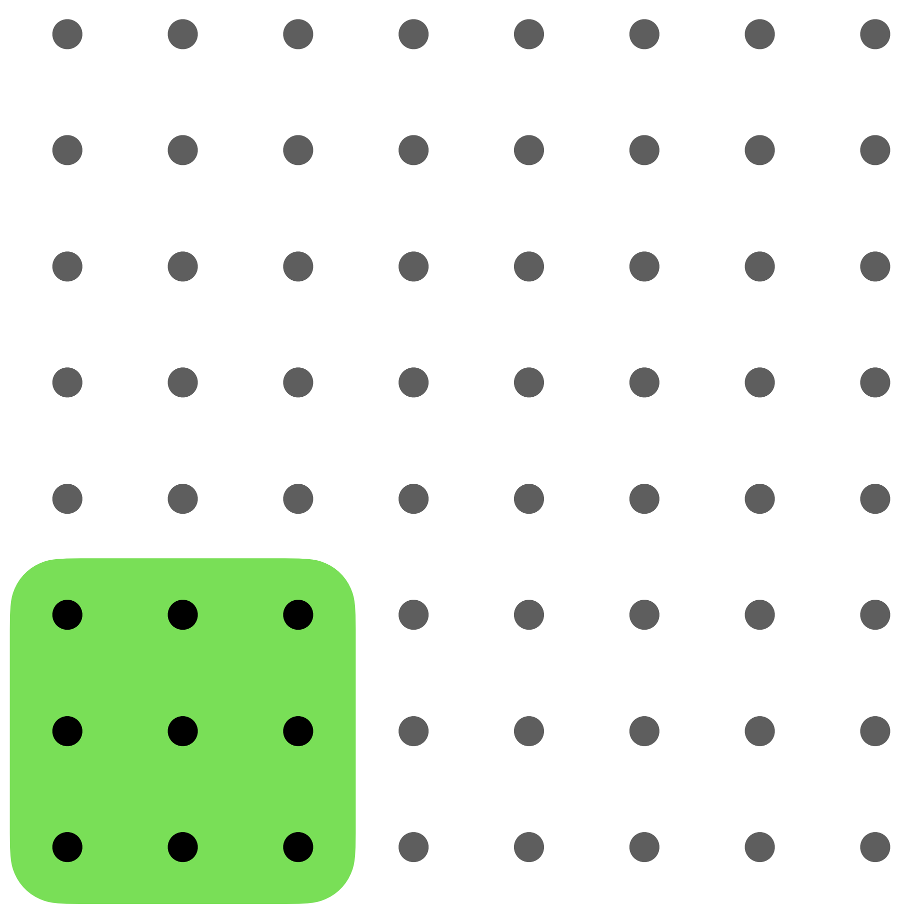


$T \vDash_2 \mathrm{G}p$

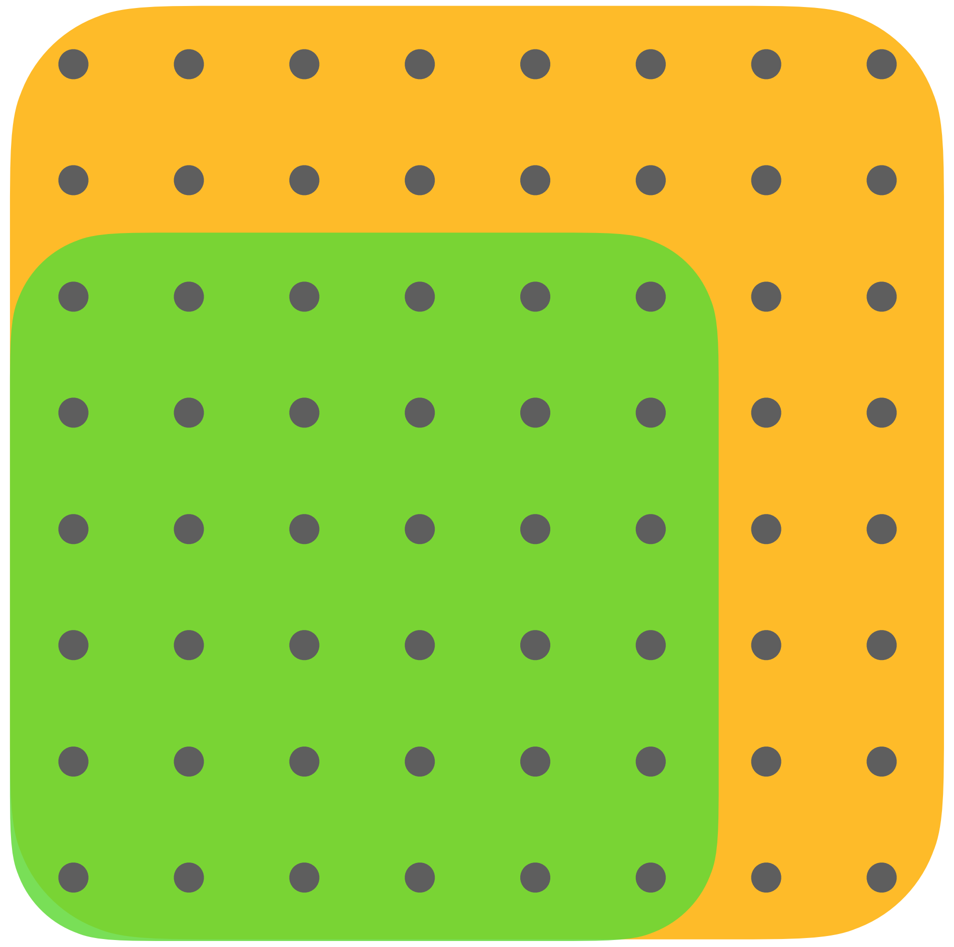
When should we stop?

Completeness Thresholds for Finite Systems

- k is completeness thresholds (CT) iff

  $$T \vDash_k \phi \Rightarrow T \vDash \phi$$

- For specific ϕ:
  Can over-approximate CT via of key props of T



Completeness Thresholds for Finite Systems

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$$T \models_k \phi \;\; \Rightarrow \;\; T \models \phi$$

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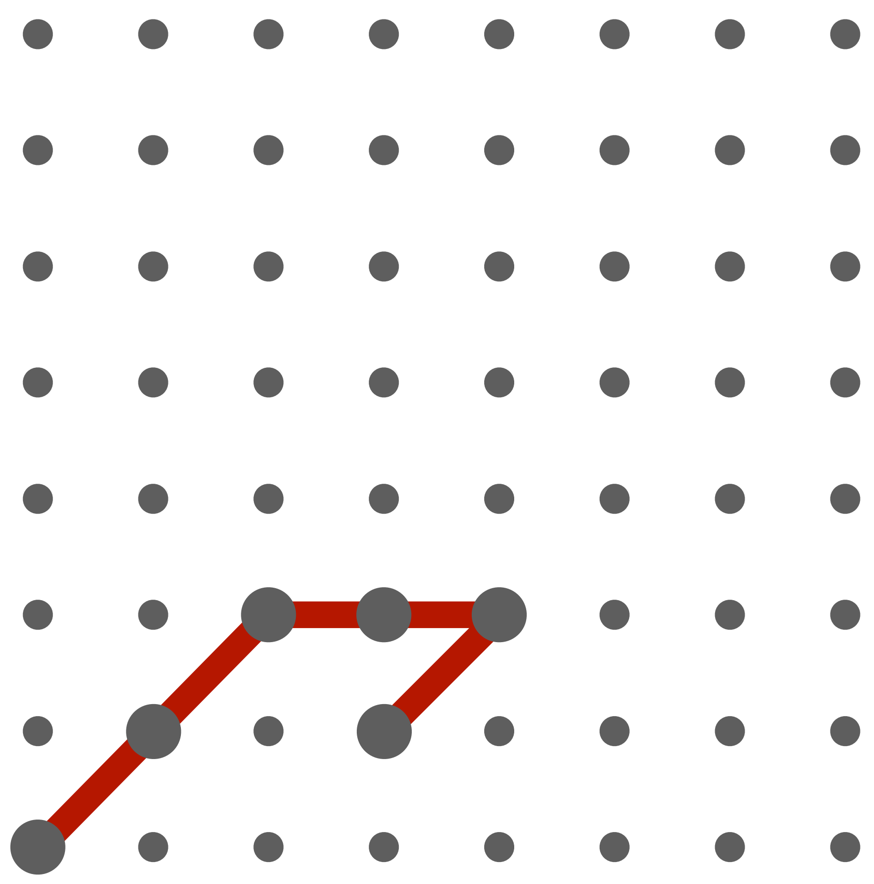
- For $\phi = \mathsf{G}p$ we know
  $\mathsf{CT}(T, \mathsf{G}p) = \text{diameter}(T)$
  (longest distance between any states)



diameter$(T) = 5$

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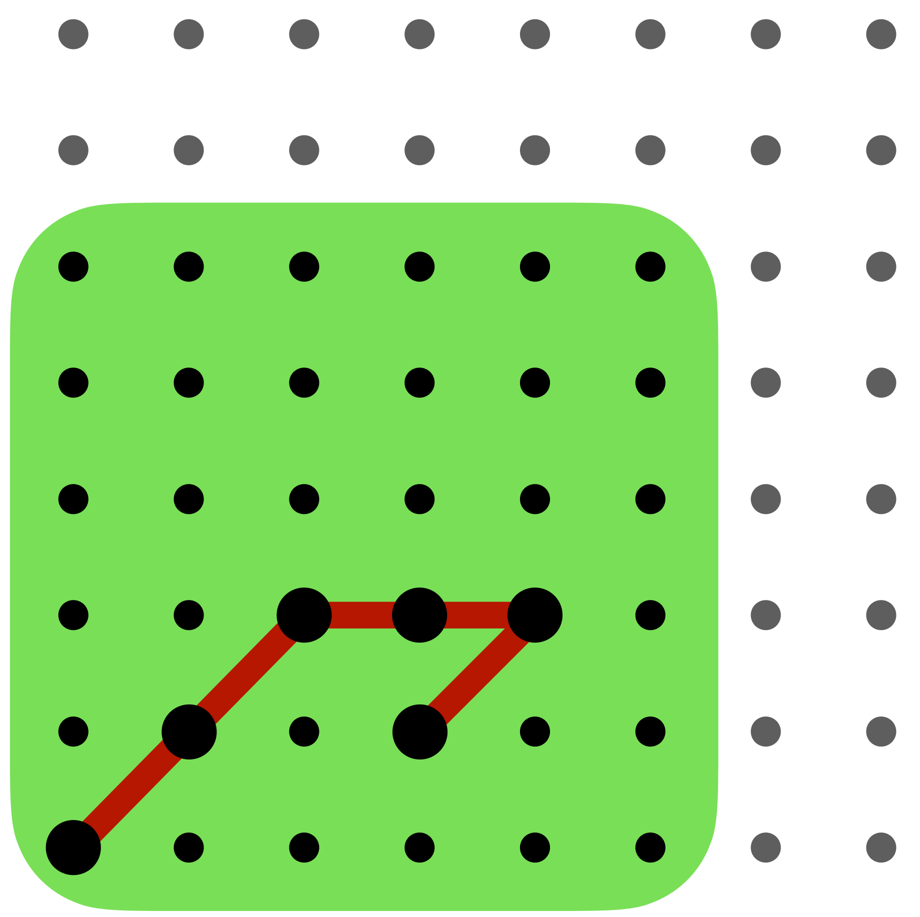

$\mathrm{diameter}(T) = 5$

$T \vDash_5 \mathrm{G}p$

Completeness Thresholds for Finite Systems

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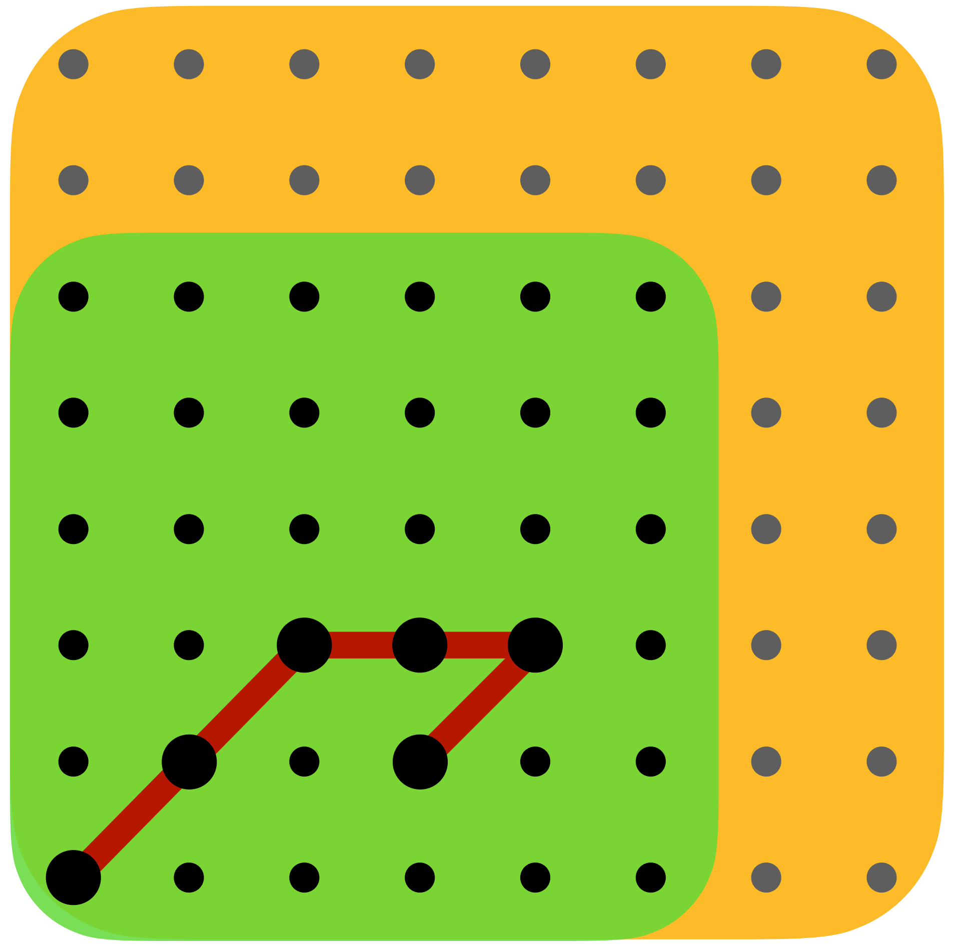


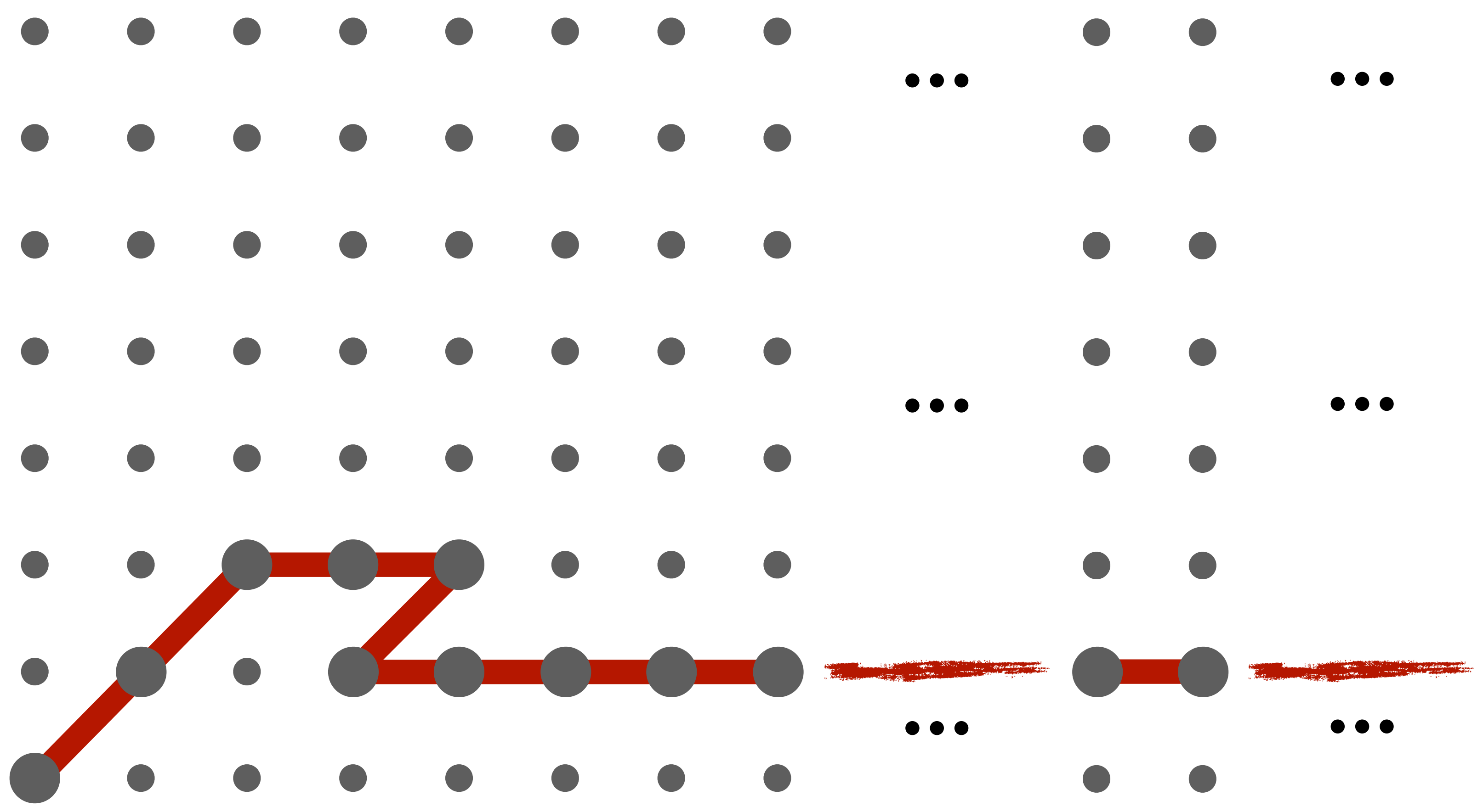
$\mathsf{diameter}(T) = 5$

$$T \vDash_5 \mathsf{G}p \longrightarrow T \vDash \mathsf{G}p$$

CTs for Infinite Systems?

Problem

Key properties used to describe CTs may be ∞



diameter$(T) = \infty$

CTs for Infinite Systems?

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Key properties used to describe CTs may be ∞

Our Approach

Analyse program's *verification conditions*
instead of transition system

How Does the Array Size Affect Memory Safety?

Memory assumption:

$\texttt{array}(a, s)$

```
for i in [L : s-R] do
    !a[i+Z]
```

Only source for memory errors

How Does the Array Size Affect Memory Safety?

Memory assumption:	for i in [L : s-R] do
$\mathtt{array}(a, s)$	!a[i+Z]

Range L, …, s-R empty?

Yes

$s^{-} < L + R$

No iteration

⇒ Memory safe

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a[L+Z] … a[s-R+Z] allocated?

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$$\iff 0 \leq L + Z \quad \wedge \quad s^+ - R + Z < s^+ \ ?$$

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a[L+Z] … a[s-R+Z] allocated?

$$\iff \quad 0 \leq L + Z \quad \wedge \quad \cancel{s^+} - R + Z < \cancel{s^+} \ ?$$

$$\iff \quad 0 \leq L + Z \quad \wedge \quad -R + Z < 0 \ ?$$

No $s^+ \Rightarrow$ Can check any $s^+ \geq L + R$

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`array`(a, s)

for i in [L : s-R] do
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Found CT: $\{s^+\}$

Completeness Thresholds

- Program variable x with domain X
- Specification $\forall x \in X \,.\, Spec(c)$

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- Program variable x with domain X
- Specification $\forall x \in X \,.\, Spec(c)$
- Subdomain $Q \subseteq X$ is a CT for x in $\forall x \in X \,.\, Spec(c)$ iff

$$\models \forall x \in Q \,.\; Spec(c) \quad \Rightarrow \quad \models \forall x \in X \,.\; Spec(c)$$

- For us: CT are subdomains, not depths

Verification Conditions

- Logical formula vc is VC for any spec $Spec(c)$ iff

$$\models vc \quad \Rightarrow \quad \models Spec(c)$$

- Can verify VC instead of program

- In general: VCs are over-approximations, i.e.,

  possible that $\not\models vc$ but $\models Spec(c)$

How to Prove CTs

- Generate VC: $Spec(c) \rightsquigarrow \forall x \in X \,.\, vc(x)$

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- Identify subdomain $Y \subseteq X$ where choice $x \in Y$ does not influence validity of $vc(x)$

$$\left(\models vc(x) \quad \Leftrightarrow \quad \models vc' \quad \text{with } \; x \notin \text{free}(vc') \right)$$

$$\implies \quad \text{Found CT:} \quad (X \backslash Y) \cup \{y\} \quad (\text{for any choice of } y \in Y)$$

Proving CT in Action

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Generate VC

(fully automated)

VC $vc_0 := \forall s\,.\,\mathtt{array}(a, s) \to \forall i \in \{L, \ldots, s - R\}\,.\,a[i+Z]$ alloc

Proving CT in Action

$$\boxed{\text{VC } vc_0 \;:=\; \forall s\,.\,\texttt{array}(a,s) \rightarrow \forall i \in \{L, \ldots, s-R\}\,.\,a[i+Z] \text{ alloc}}$$

Range L, ..., s-R empty?

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Range L, …, s-R empty?

Yes

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Simplify VC!

$$vc_0 \equiv \forall s^- . \;\ldots \to \forall i \in \varnothing . \;\ldots$$
$$\equiv \text{True}$$

Proving CT in Action

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No

$s^+ \geq L + R$

$$vc_0 \equiv \forall i\,.\,(L \leq i < s^+ - R) \rightarrow (0 \leq i{+}Z < s^+)$$

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$$vc_0 \equiv \forall i\,.\,(L \leq i < \cancel{s^+} - R) \to (0 \leq i{+}Z < \cancel{s^+})$$

$$\equiv \forall i\,.\,(L \leq i \to 0 \leq i{+}Z)$$

$$\wedge\ (i \leq -R) \to i{+}Z < 0)$$

$\Rightarrow$ Validity does not depend on size

Proving CT in Action

$$\boxed{\text{VC } vc_0 := \forall s\,.\, \texttt{array}(a, s) \rightarrow \forall i \in \{L, \ldots, s - R\}\,.\, a[i{+}Z] \text{ alloc}}$$

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Yes

$s^- < L + R$

No need to check

No

$s^+ \geq L + R$

Can check for any

$s^+ \geq L + R$

Proving CT in Action

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Workflow: How to Find CTs

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Mem spec + program $\xrightarrow{\text{Generate VC}}$ VC $\forall x \in X \,.\, vc(x)$

Workflow: How to Find CTs

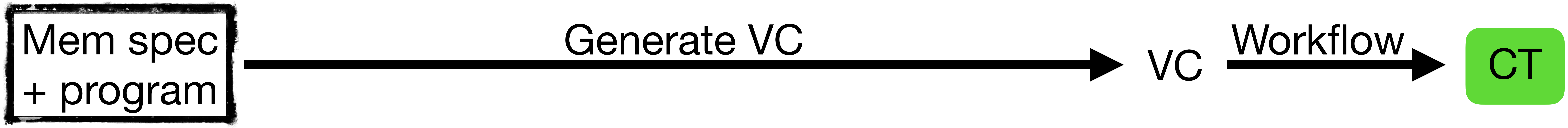
Mem spec + program $\xrightarrow{\text{Generate VC}}$ VC $\forall x \in X \,.\, vc(x)$ $\xrightarrow[Y \subseteq X]{\text{Select subdomain}}$ Restricted VC $vc(x)$ with $x \in Y$

Restricted VC $vc(x)$ with $x \in Y$ $\xrightarrow{\text{Try to eliminate } x}$ $vc'(x)$ with $x \in Y$

Workflow: How to Find CTs

Scalability

Program Slicing



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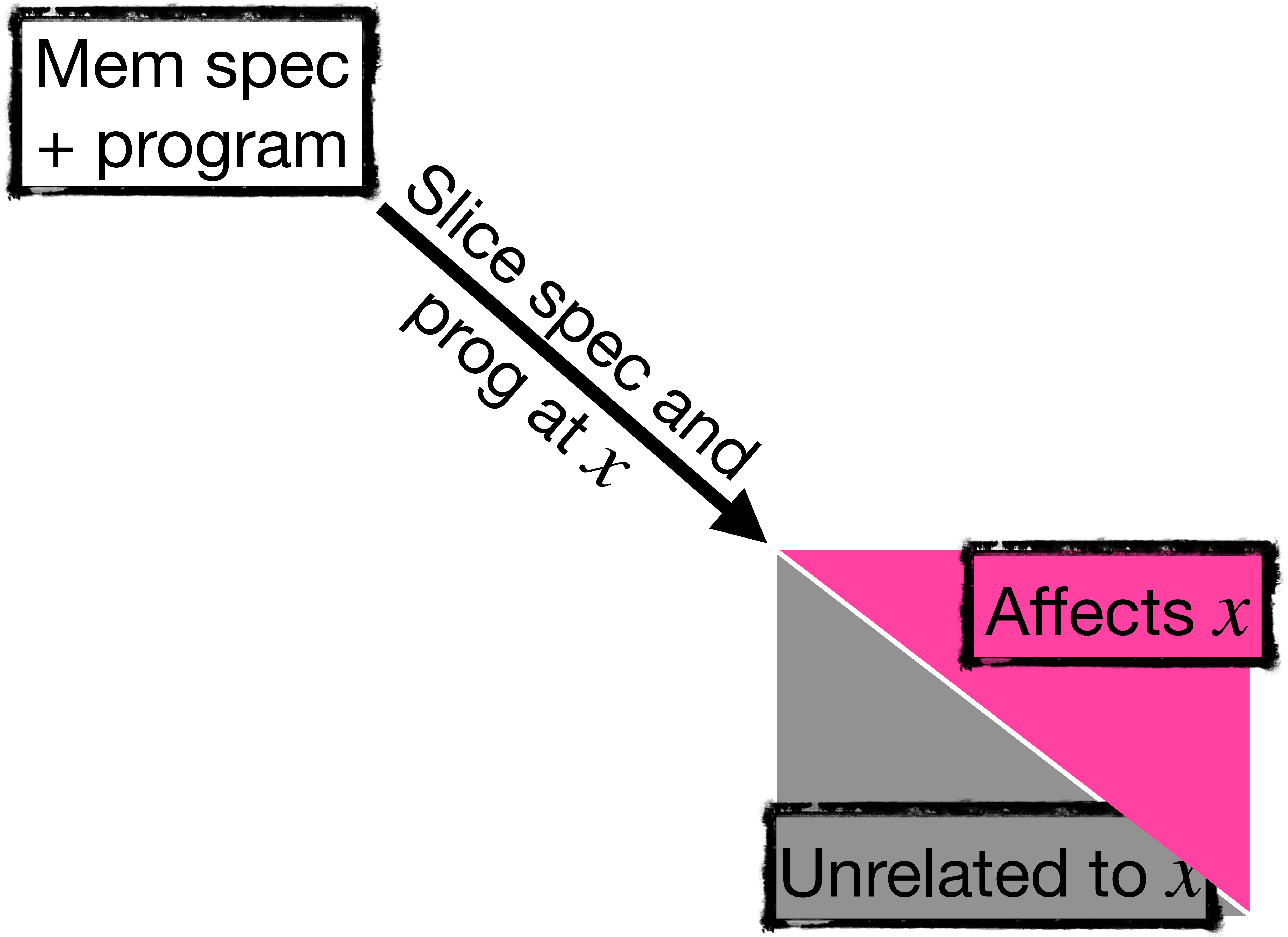
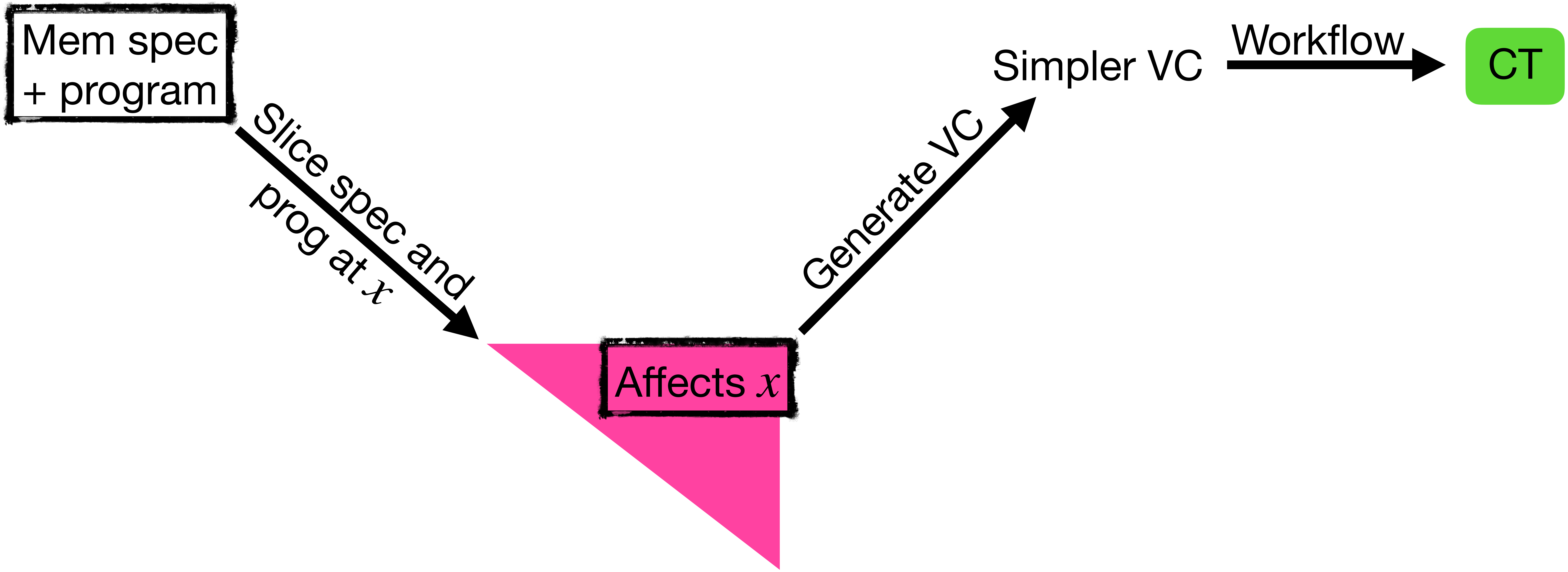


Simpler VC $\xrightarrow{\text{Workflow}}$ CT

Scalability

Program Slicing



Program Slicing In Action

```
{ array(a, s) * F }
  f;
  if s > B then
    r := a[Y]
  g
{ array(a, s) * F }
```

Precondition

Complex code not mentioning a, s

bubble_sort(a)

Select element

Complex code not mentioning a, s

Postcondition

Program Slicing In Action

```
{ array(a, s) * F }
  f;
  if s > B then
    not_sorted := true;
    while not_sorted do
      not_sorted := false;
      for i in [L : s-R] do
        if a[i+1] < a[i] then
          not_sorted := true;
          tmp := a[i];
          a[i] := a[i+1];
          a[i+1] := tmp;
    r := a[Y]
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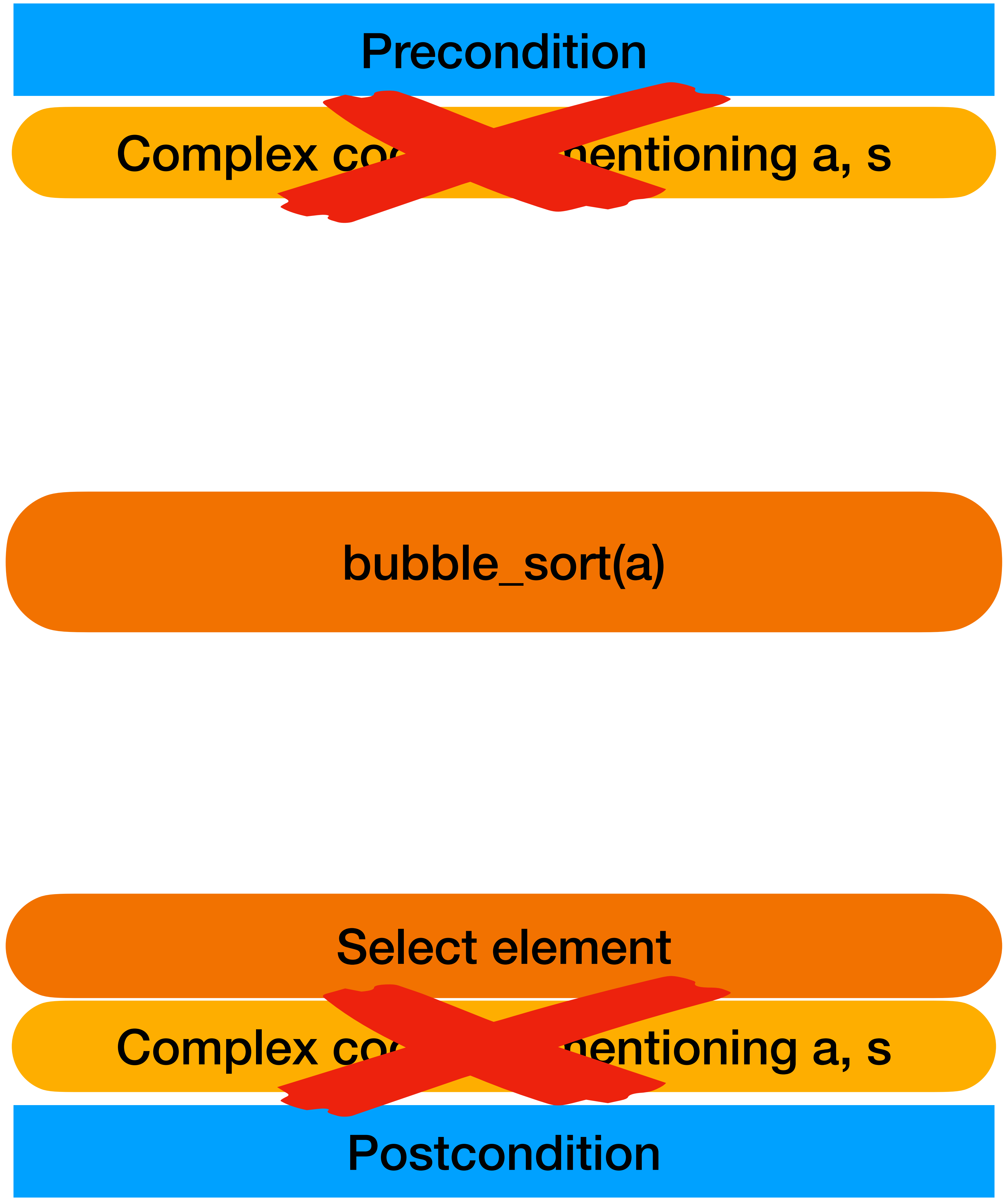
Program Slicing In Action

```
{ array(a, s) * }

  if s > B then
     not s        true;
     while        ed do
        not        alse;
        for i in [L : s-R] do
           if a[i+1] < a[i] then
              not s        rue;
              t  p := a[i];
              a[i] := a[i+1];
              a[i+1] := t  p;
   := a[Y]

{ array(a, s) * }
```



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{ array(a, s) }
  if s > B then
    for i in [L : s-R] do
      if a[i+1] < a[i] then
        a[i];
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        a[i+1];
    a[Y]
{ array(a, s) }
```

Further Refinement via Static Analysis

```
{ array(a, s) }
  if s > B then
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Comparison does not depend on s

⇒ Can ignore “if”

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Subsumed by a[i+Z]

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CT Combinators

Sequencing

$c_1; c_2$ CTs Q_1, Q_2

$$Q = Q_1 \cup Q_2$$

Scalability

CT Combinators

Sequencing

$c_1; c_2$

CTs Q_1, Q_2

$$Q = Q_1 \cup Q_2$$

Branching

if e then c_1 else c_2

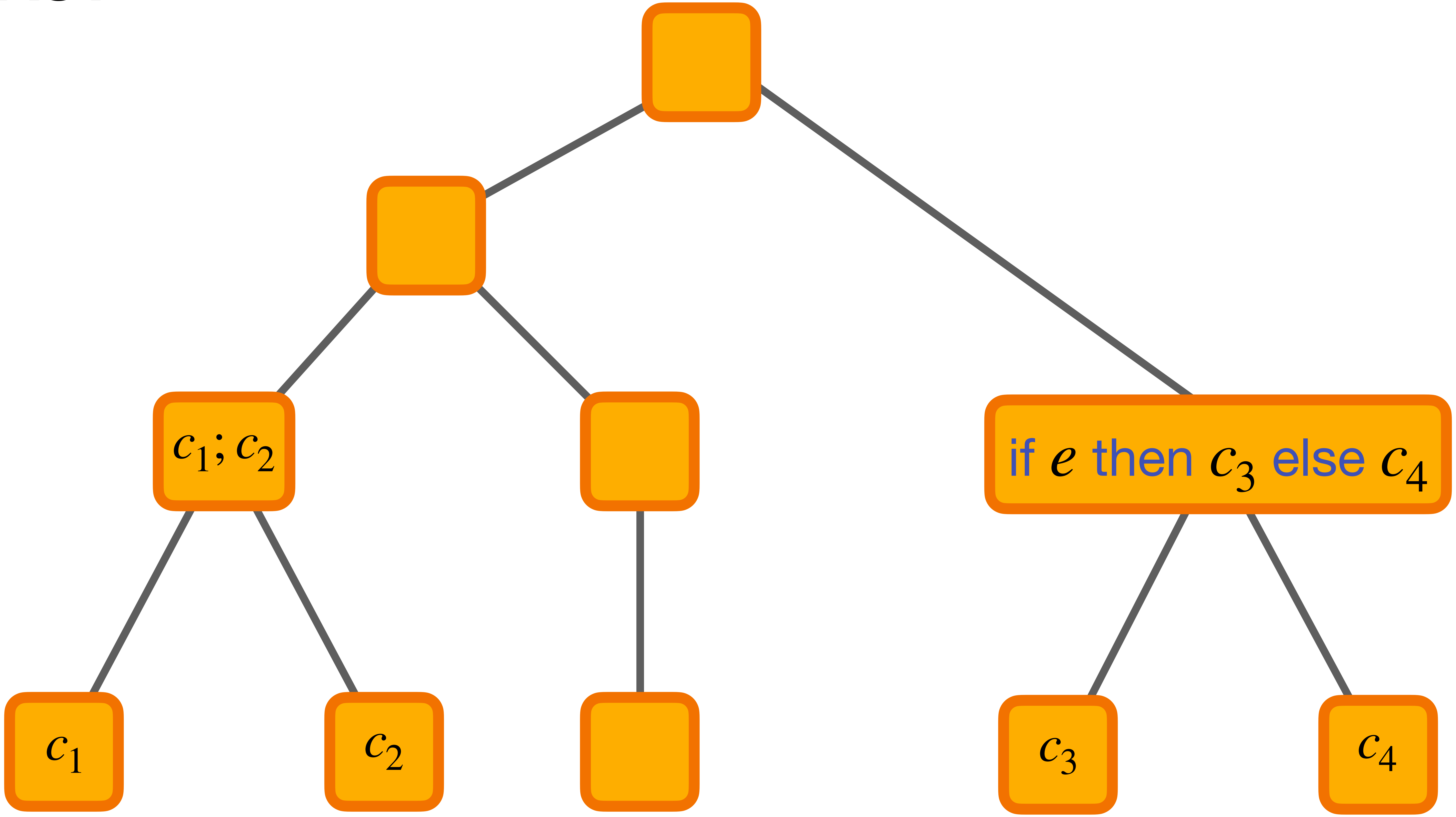
CTs as contraint sets

$Q_i \sim K_i$

$$Q \sim (e \wedge K_1) \cup (\neg e \wedge K_2)$$

Scalability

Follow AST



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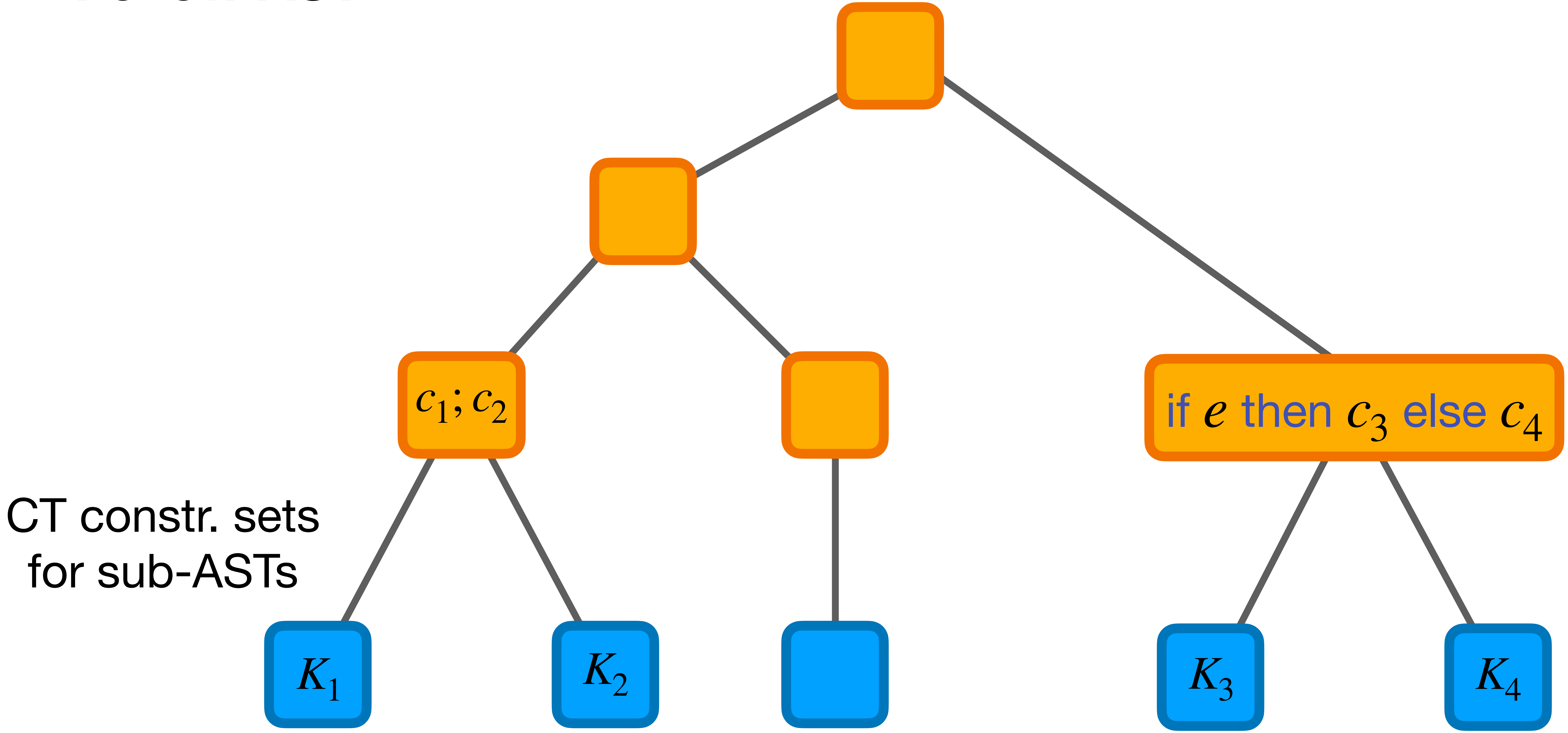

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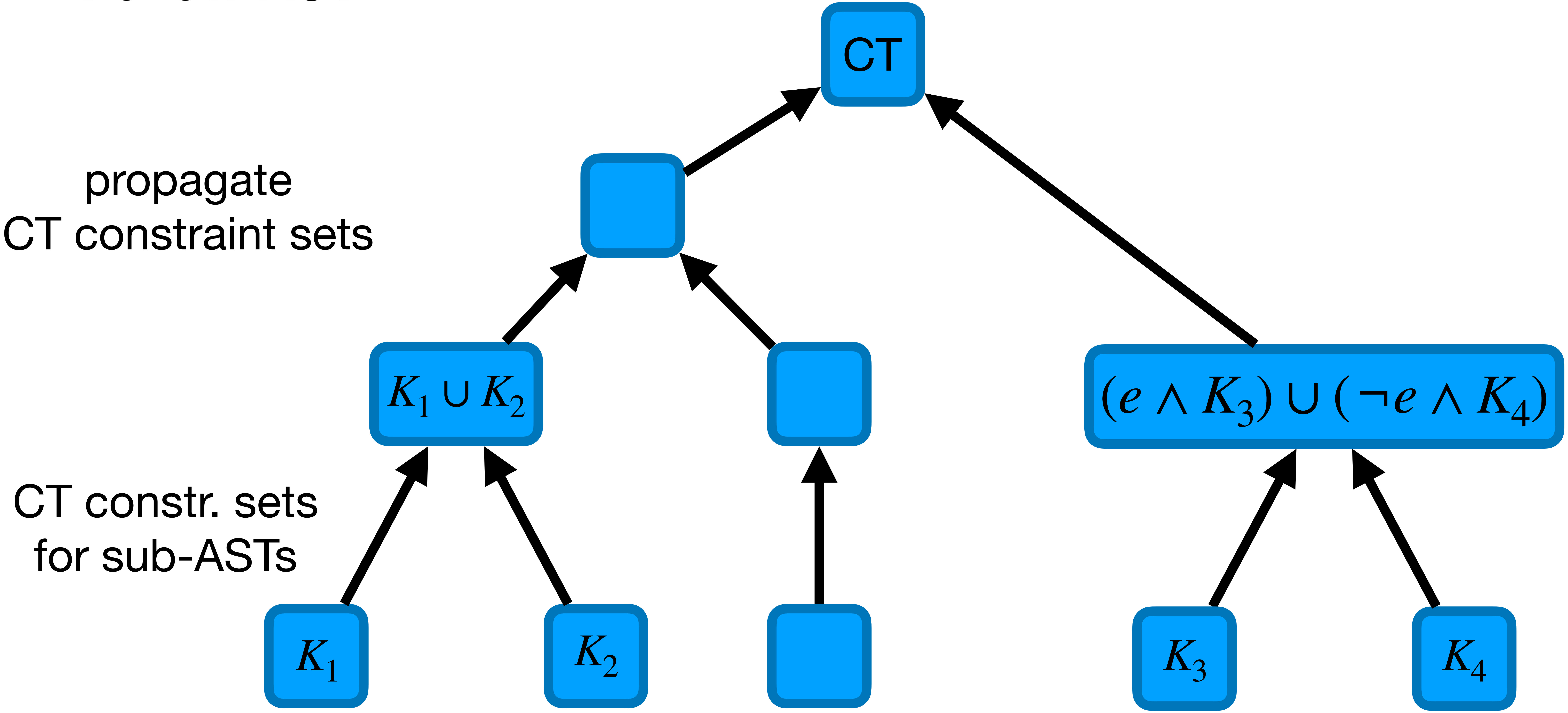

CT Propagation In Action

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{ array(a, s) }
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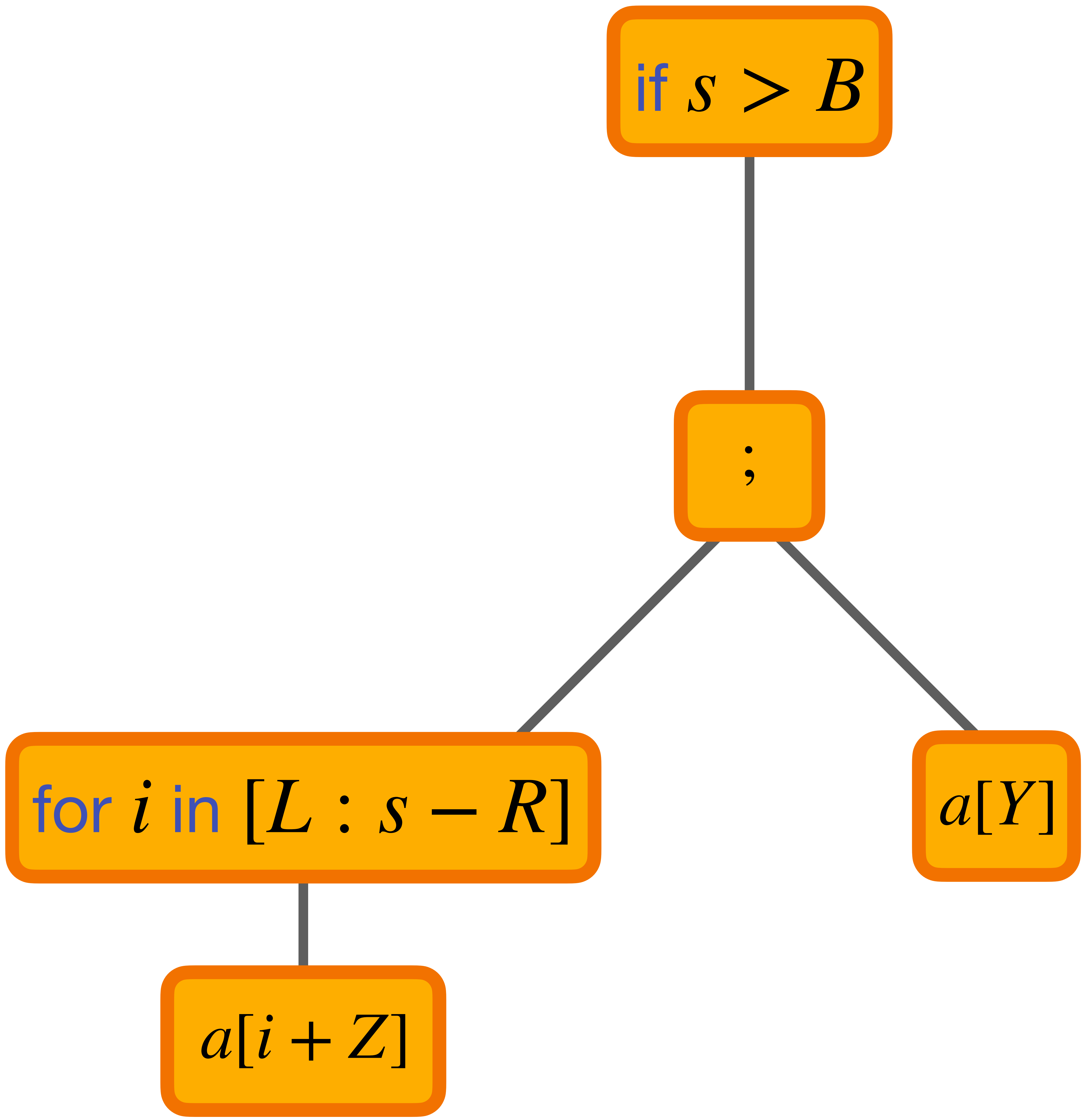

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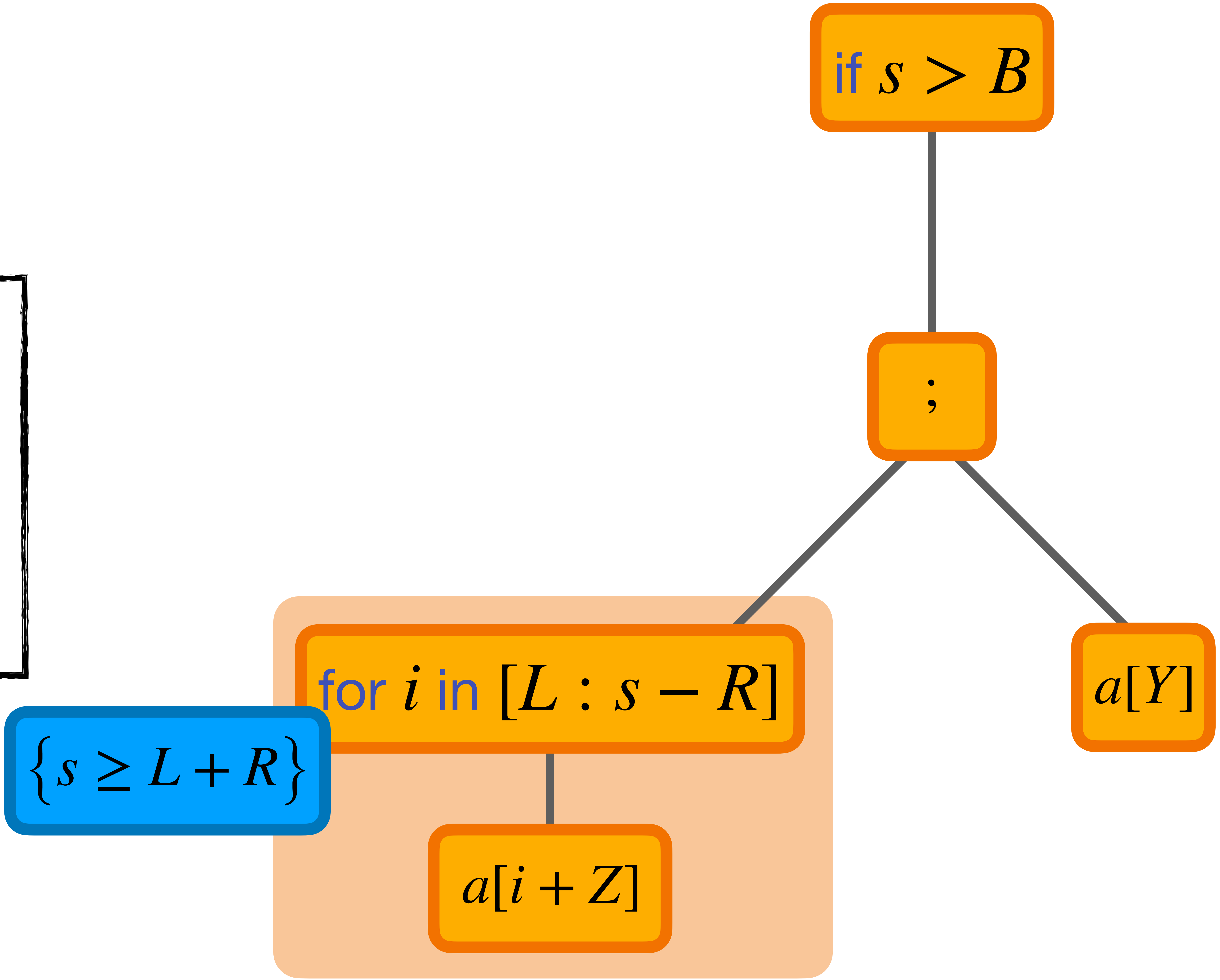

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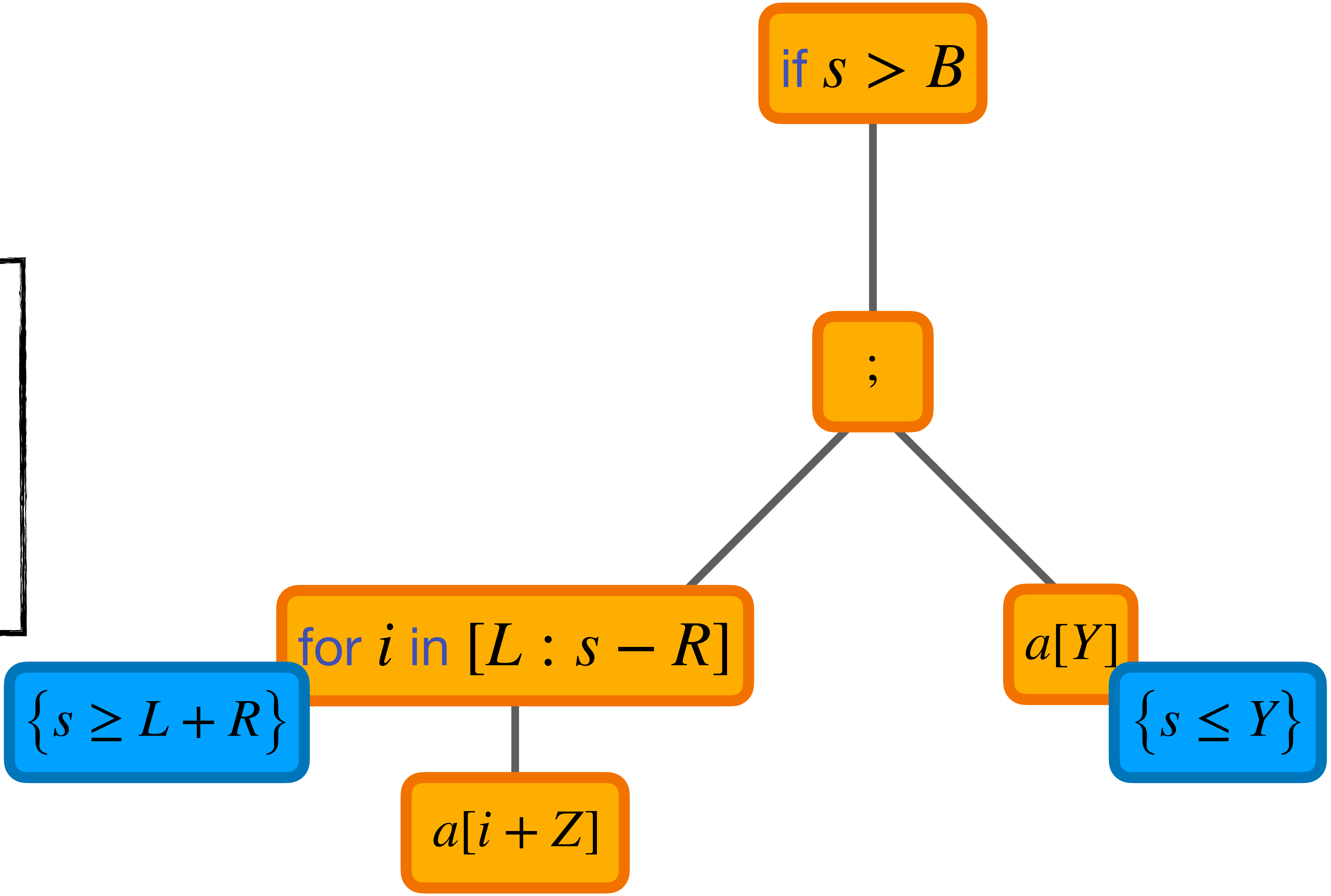

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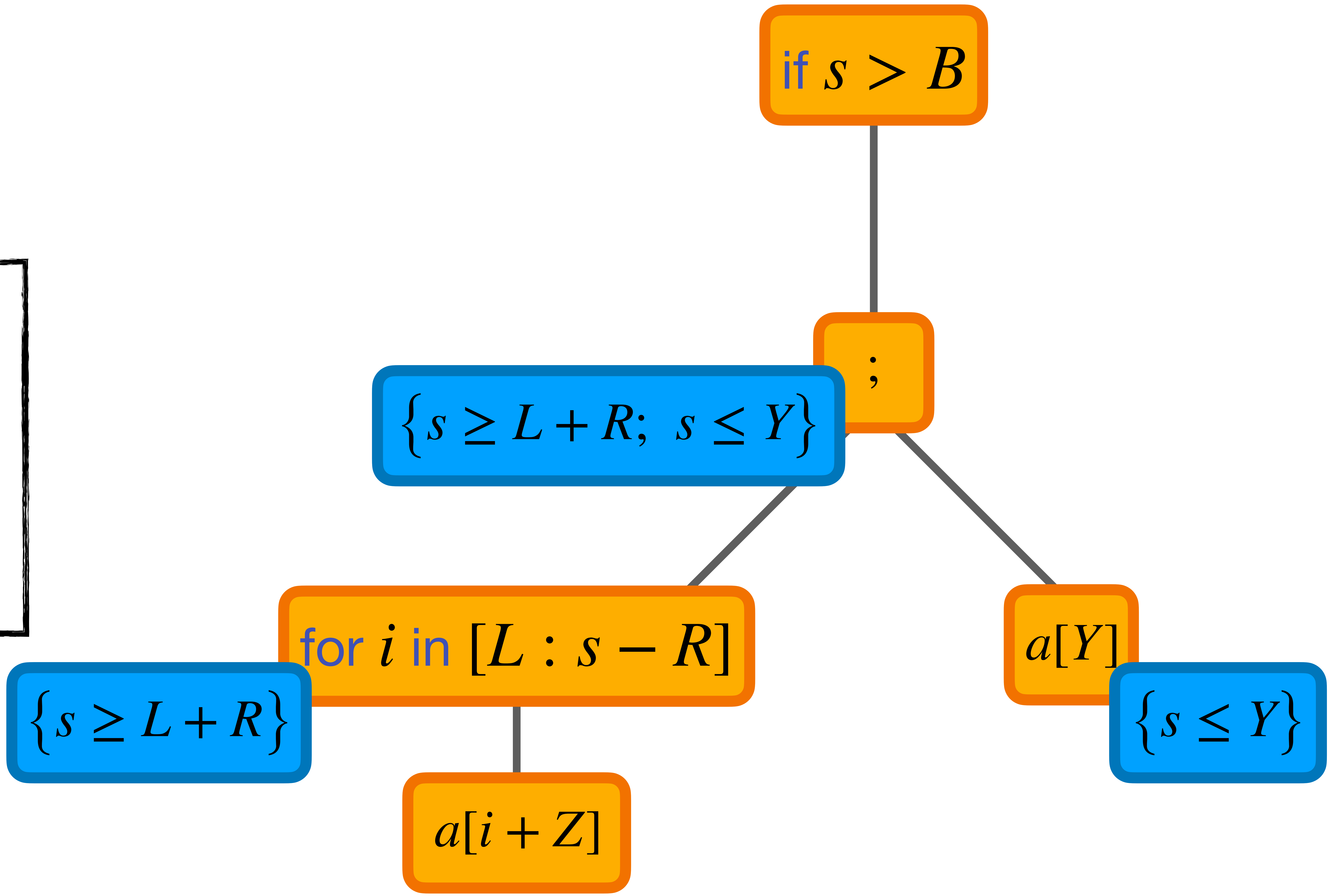

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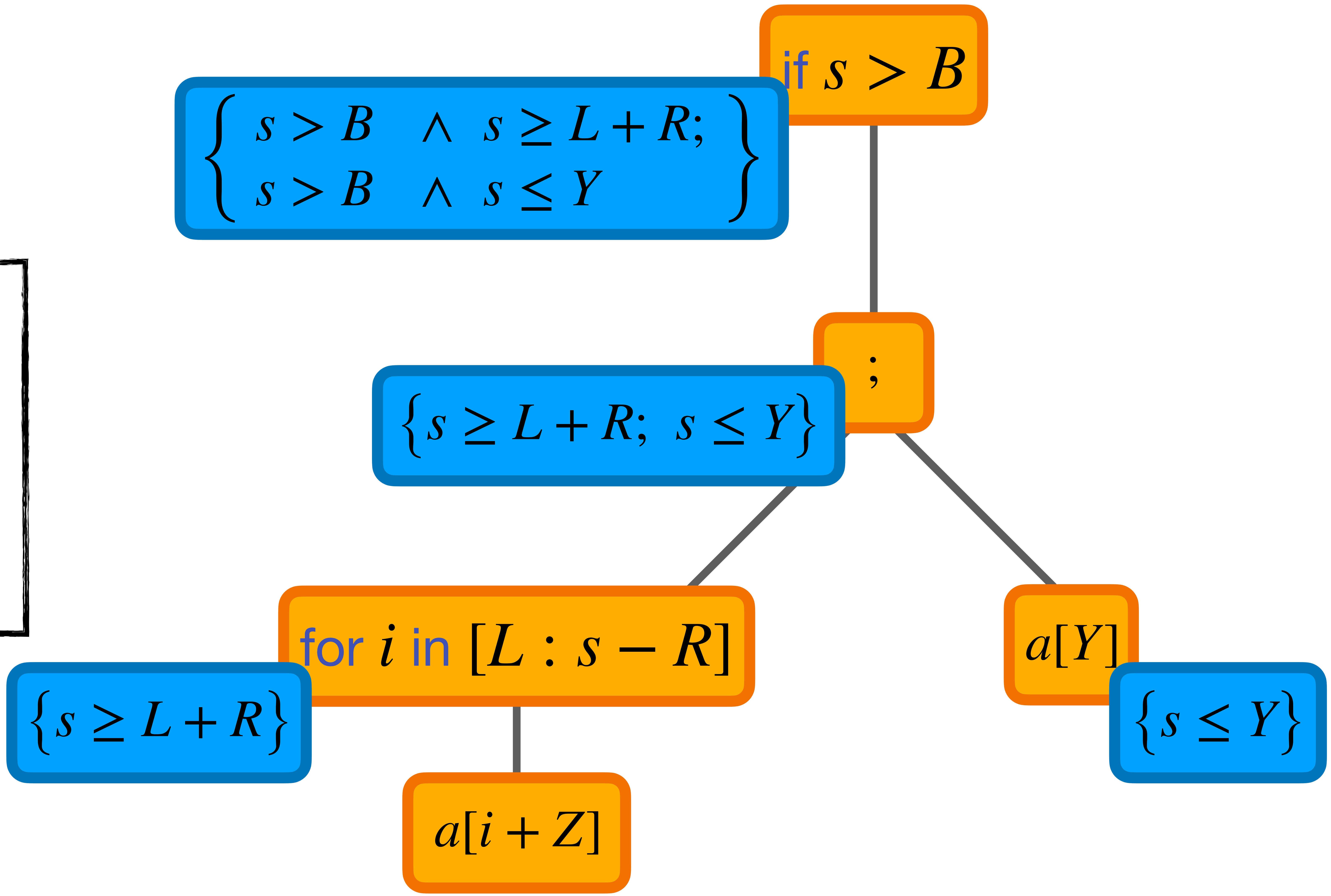
{ array(a, s) }
```



Generalisation

	Memory Safety	Arbitrary Correctness Property ϕ
Theory		
• What are CTs?	✓	✓
• Describe behaviour	✓	✓
• Extraction approach	✓	✓
Practice		
• Obtain CT	✓	
• Scale	✓	

Generalisation

	Memory Safety	Arbitrary Correctness Property ϕ
Theory		
• What are CTs?	✓	✓
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Practice		
• Obtain CT	✓	?
• Scale	✓	

ϕ-specific relationship: program↔VC

Generalisation

	Memory Safety	Arbitrary Correctness Property ϕ
Theory		
• What are CTs?	✓	✓
• Describe behaviour	✓	✓
• Extraction approach	✓	✓
Practice		Requires “local” ϕ
• Obtain CT	✓	?
• Scale	✓	?

Generalising CT Theory

- Correctness ~ arbitrary quantified predicate $\nabla x \in X \,.\, \phi$ with $\nabla \in \{\forall, \exists\}$

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- Challenge: **Soundness** (also for memory safety)

VC Over-Approximation

$$\models \nabla x \in X\,.\; vc \xrightarrow{\text{Unbounded proof}} \models \nabla x \in X\,.\; \phi$$

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✗ $\models \forall x \in \mathbb{N}\,.\ \mathit{false}$

✗ $\models \forall x \in \mathbb{N}\,.\ x = 5$

VC Over-Approximation

$\vDash \forall x \in \mathbb{N} \,.\, \mathit{false}$

$\vDash \forall x \in \mathbb{N} \,.\, x = 5$

$\{5\}$ is CT ✓

Generalising bounded proof *sound*

$\vDash \forall x \in \{5\} \,.\, \mathit{false}$

VC Over-Approximation

$\models \forall x \in \mathbb{N} \,.\, \mathit{false}$

$\{5\}$ is CT ✔

Generalising bounded proof *sound*

$\models \forall x \in \{5\} \,.\, \mathit{false}$

$\models \forall x \in \mathbb{N} \,.\, x = 5$

✘ $\{5\}$ is no CT

Generalising bounded proof ***unsound***

$\models \forall x \in \{5\} \,.\, x = 5$

VC Over-Approximation

$$\models \forall x \in \mathbb{N} \,.\, \mathit{false} \quad \longrightarrow \quad \models \forall x \in \mathbb{N} \,.\, x = 5$$

Must limit over-approximation

Limited Over-Approximation

- VC vc is *precise* for x in ϕ iff

  vc captures influence of x on correctness ϕ

- vc may over-approximate for other variables

Precise VCs

- VC vc is *precise* for x in ϕ iff

$$\forall v\,.\ \Big(\ \models \phi[x \mapsto v] \ \Rightarrow\ \models vc[x \mapsto v]\ \Big)$$

Intuition: vc does not over-approximate wrt. x

Precise VCs

- VC vc is *precise* for x in ϕ iff

  $$\forall v \,.\ \Big(\models \phi[x \mapsto v] \quad \Rightarrow \quad \models vc[x \mapsto v] \Big)$$

  Intuition: vc does not over-approximate wrt. x

- Q is CT $vc \ \wedge\ vc$ is precise $\ \Rightarrow\ $ Q is CT ϕ

Soundness

$\nabla \in \{\forall, \exists\}$

$$
\begin{tikzcd}
\nabla x .\, \phi \arrow[r, "\text{Unbounded proof}"] \arrow[d] & \models \nabla x .\, \phi \\
\nabla x .\, vc \arrow[d] & \\
Q \text{ is CT for } vc \arrow[d, "\boxed{vc \text{ precise for } x}"] & \\
Q \text{ is CT for } \phi \arrow[r, "\text{Bounded proof}"] & \models \nabla x \in Q .\, \phi \arrow[uuu, "\text{CT } Q"']
\end{tikzcd}
$$

Known Sound Setting

- ϕ : Memory safety
- Traverse (linear) data structure D, e.g., array, list
- CT for size of D
- D : stable memory layout

Outlook: Plans & Challenges

Plans

- Demo scalability: Complex programs & data, e.g., trees
- Evaluate CT’s impact on runtime:
  ⇒ Case study: FreeRTOS’ TCP stack

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Plans

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Challenge: Automation

- Program reduction: Property-specific slicing
- Pattern recognition

Outlook: Increase Trust in BMC

- Turn bounded into unbounded proof

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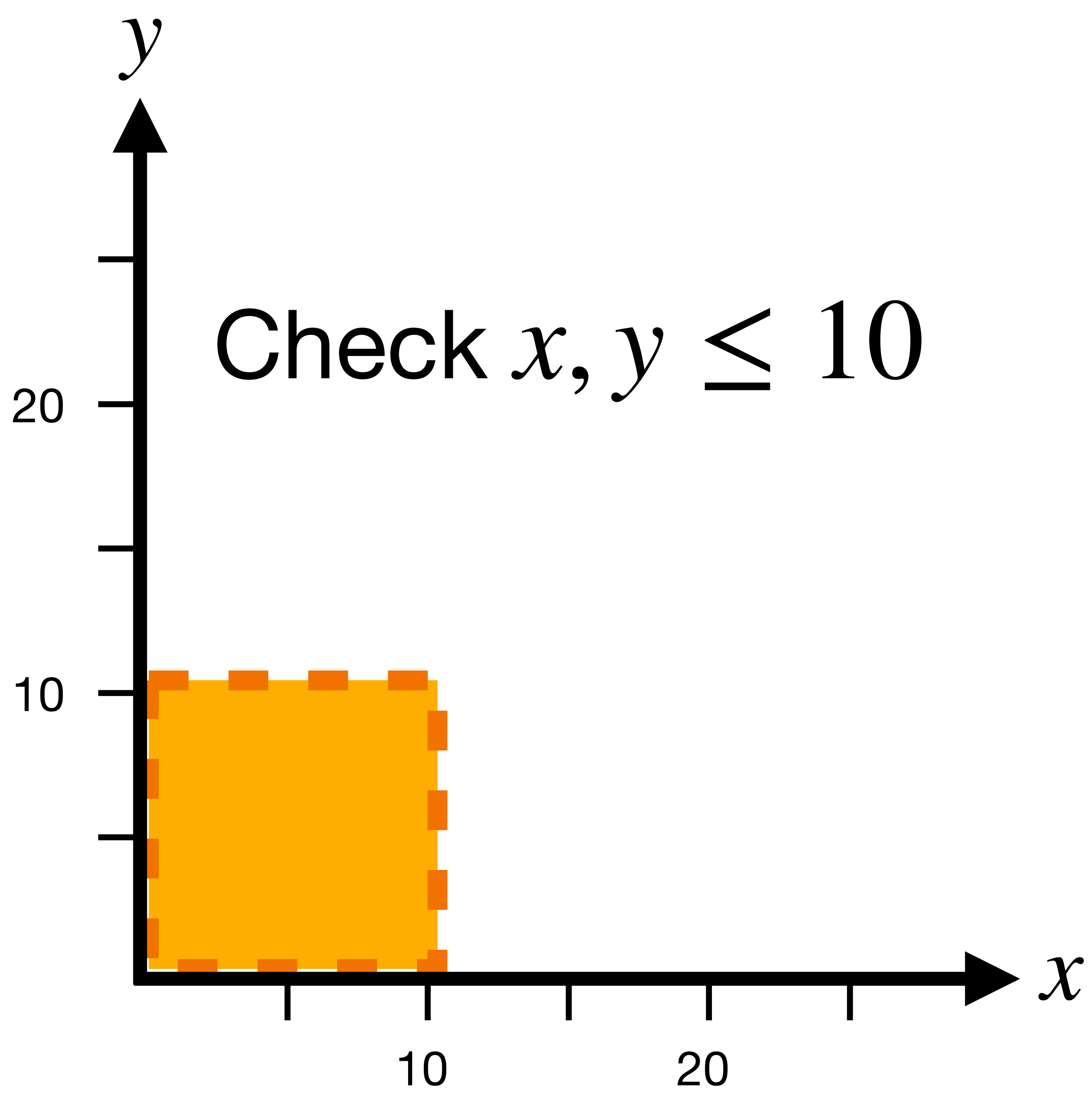
- Turn bounded into unbounded proof
- Shift resources to critical bounds



Outlook: Increase Trust in BMC

- Turn bounded into unbounded proof
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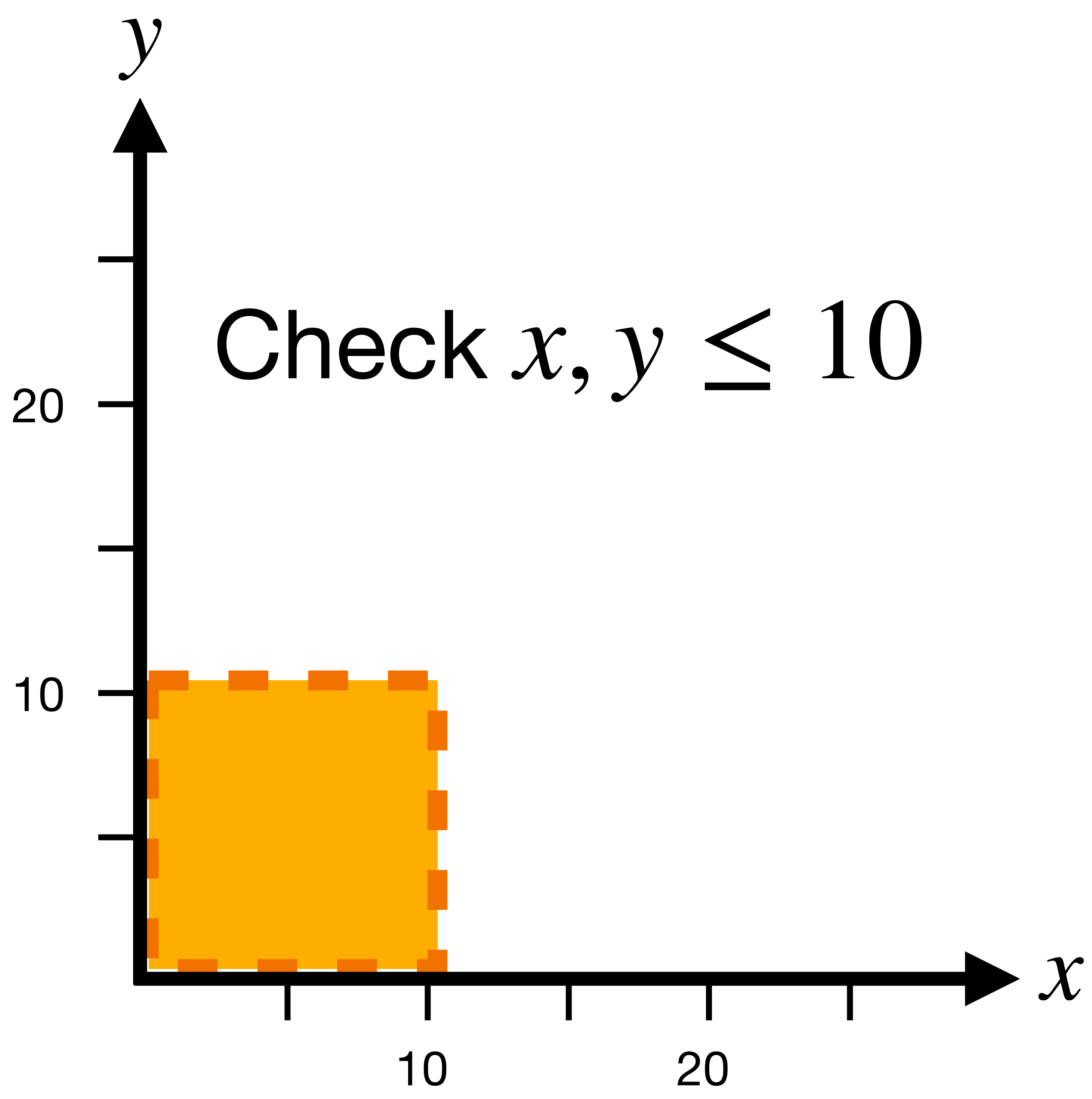
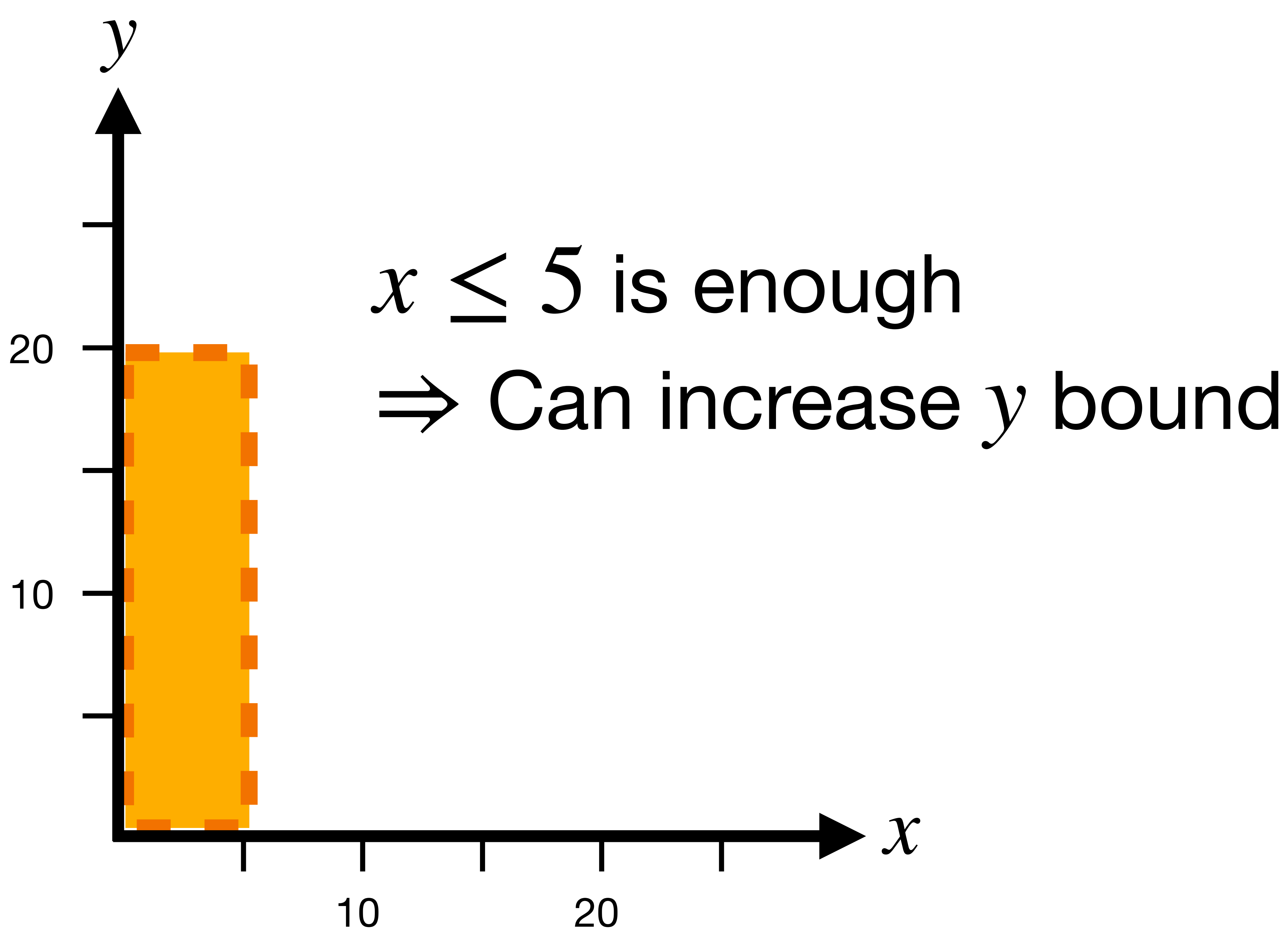


CT for x:
$\{0,\dots,5\}$



Conclusion

- First generalisation of CTs to infinite state systems
- Connection between bounded & unbounded proofs in program verification
- Foundational research but potential for integration into BMC