Completeness Thresholds for Memory Safety of Array Traversing Programs

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Workshop on the State Of the Art in Program Analysis
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What This Work Is About

- Connection between bounded & unbounded proofs
- Ideas to increase trust in bounded model checking

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- Connection between bounded & unbounded proofs
- Ideas to increase trust in bounded model checking
- When is a bounded “proof” a proof?

Model Checking: Easy Off-by-1 Error

- WHILE language with pointer arithmetic
- Targeted property: Memory safety
- Memory assumption $\texttt{array}(a, s)$:
  $a[0] \ \ldots \ a[s{-}1]$ allocated

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for i in [0 : s-1] do
    !a[i+1]
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for i in [0 : s-1] do
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Which bounds should we choose for s?

- $s = 0$: No error
- $s = 1$: Error

Model Checking: “Harder” Off-by-N Error

Memory assumption:
$\texttt{array}(a, s)$

```
for i in [0 : s-2] do
    !a[i+2]
```

Which bounds should we choose for s?

Model Checking: “Harder” Off-by-N Error

Memory assumption: $\texttt{array}(a, s)$	for i in [0 : s-2] do !a[i+2]

Which bounds should we choose for s?

- $s = 0$: No error
- $s = 1$: No error
- $s = 2$: Error

Model Checking: No Off-by-N Error

Memory assumption:
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```
for i in [0 : s-1] do
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Which s can convince us?

Model Checking: No Off-by-N Error

Memory assumption: $\texttt{array}(a, s)$	for i in [0 : s-1] do !a[i]

Which s can convince us?

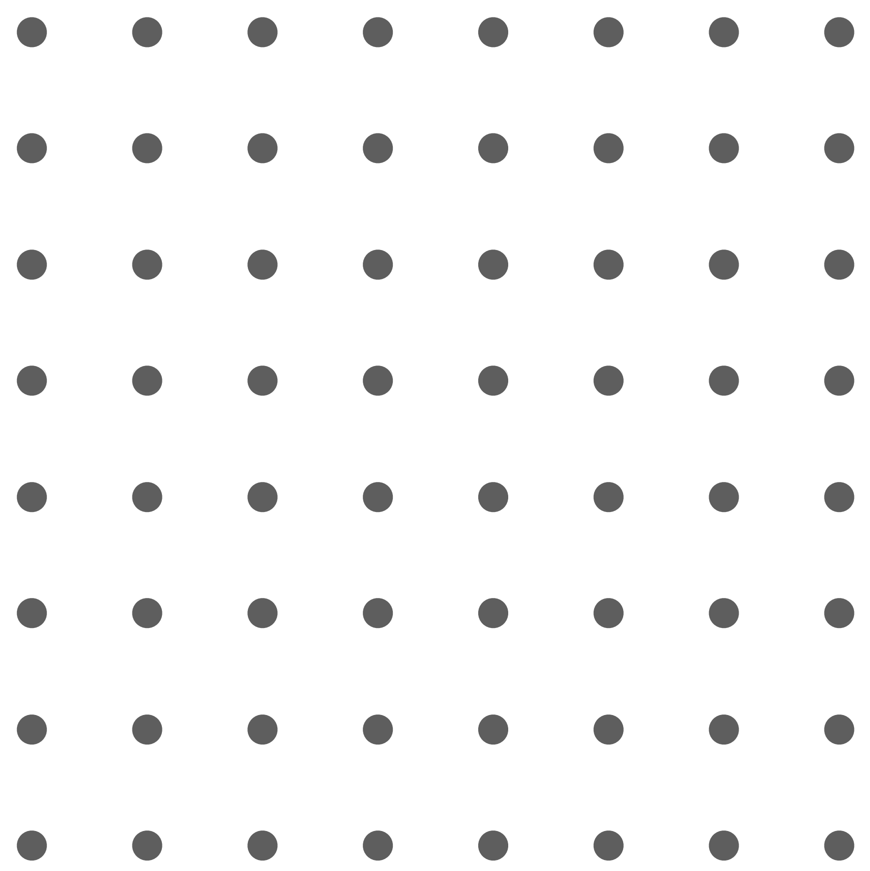
- $s = 0$: No error
- $s = 1$: No error
- $s = 2$: No error
- $s = 3$: No error

$\vdots$

$\Rightarrow$ Which size bound is large enough?

Model Checking Finite Systems

- Finite state transition system T
- Prove property Gp
  G ≈ globally ≈ p holds in every state
- Approach:
  Prove Gp for all paths up to length k
  $T \vDash_k \mathrm{G}p$



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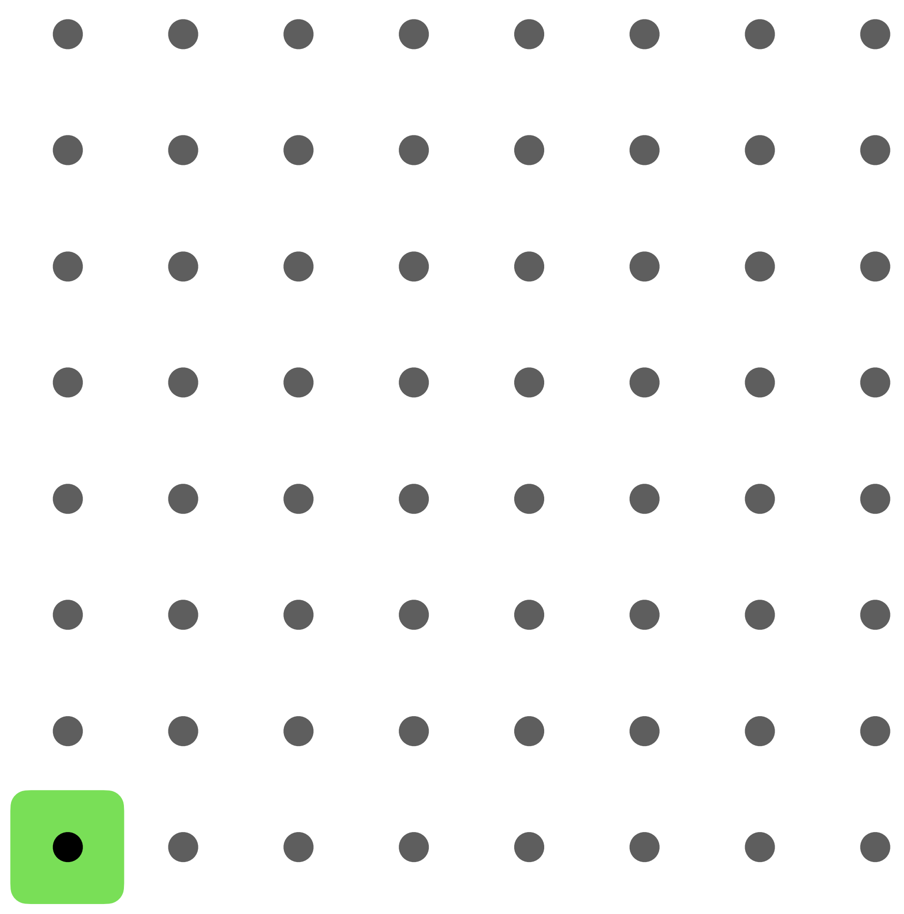

$T \vDash_0 \mathrm{G}p$

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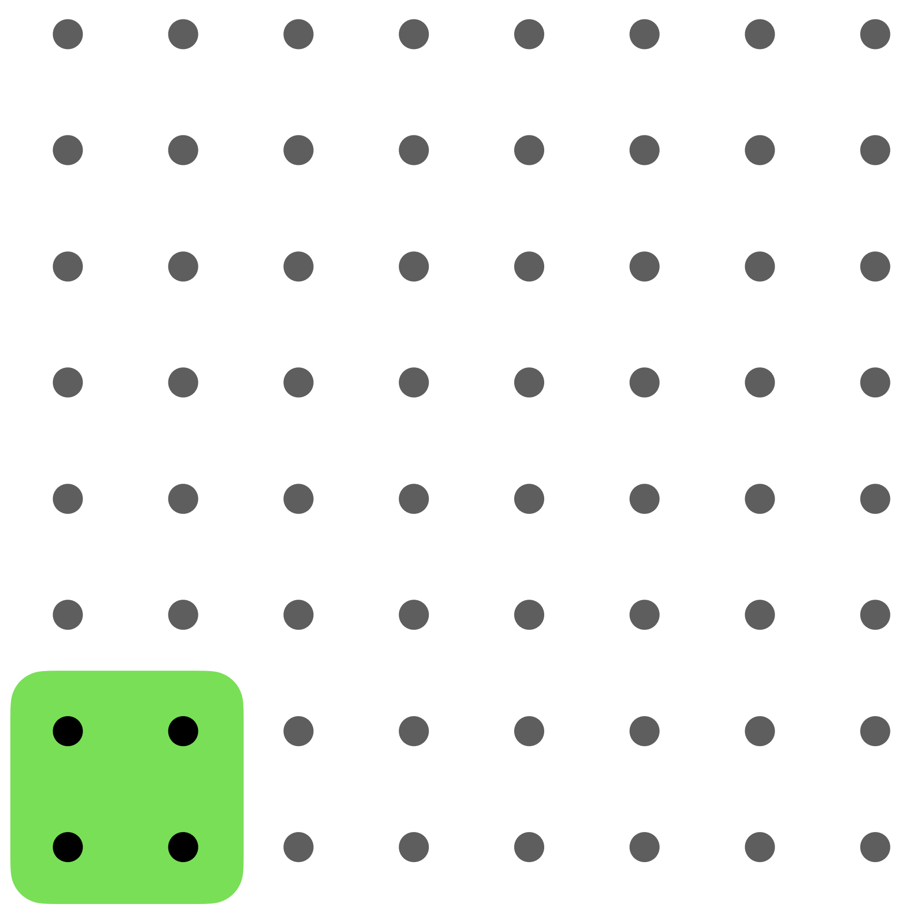

$T \vDash_1 \mathrm{G}p$

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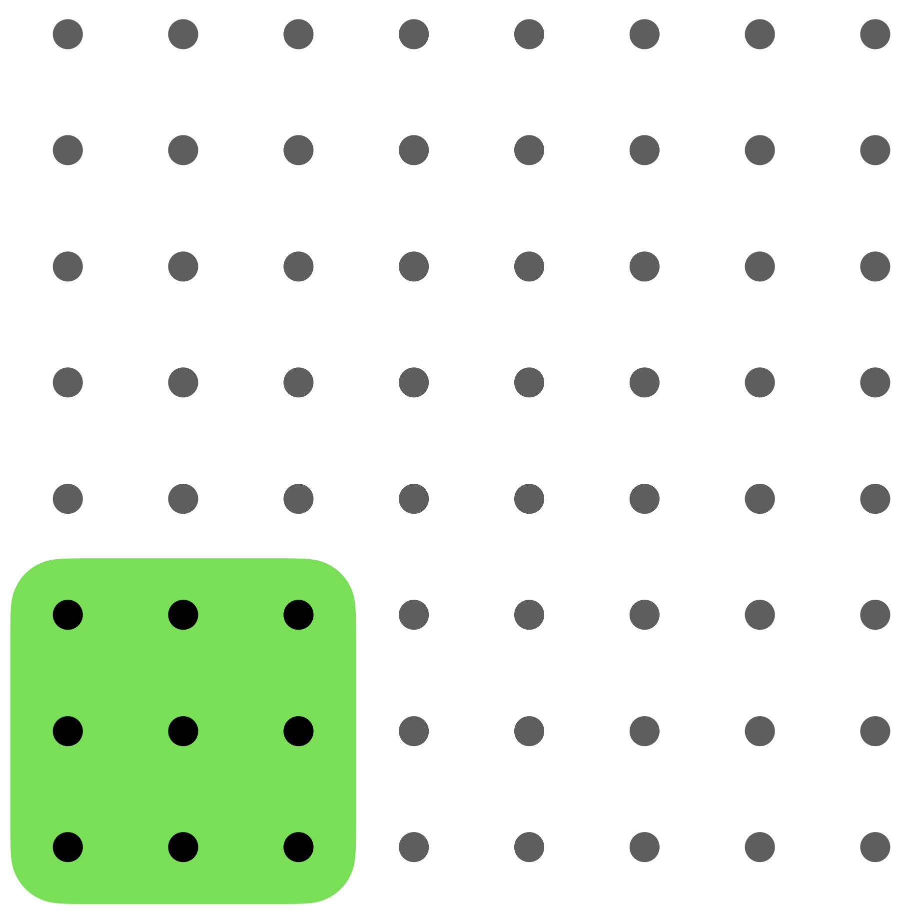


$T \vDash_2 \mathsf{G}p$

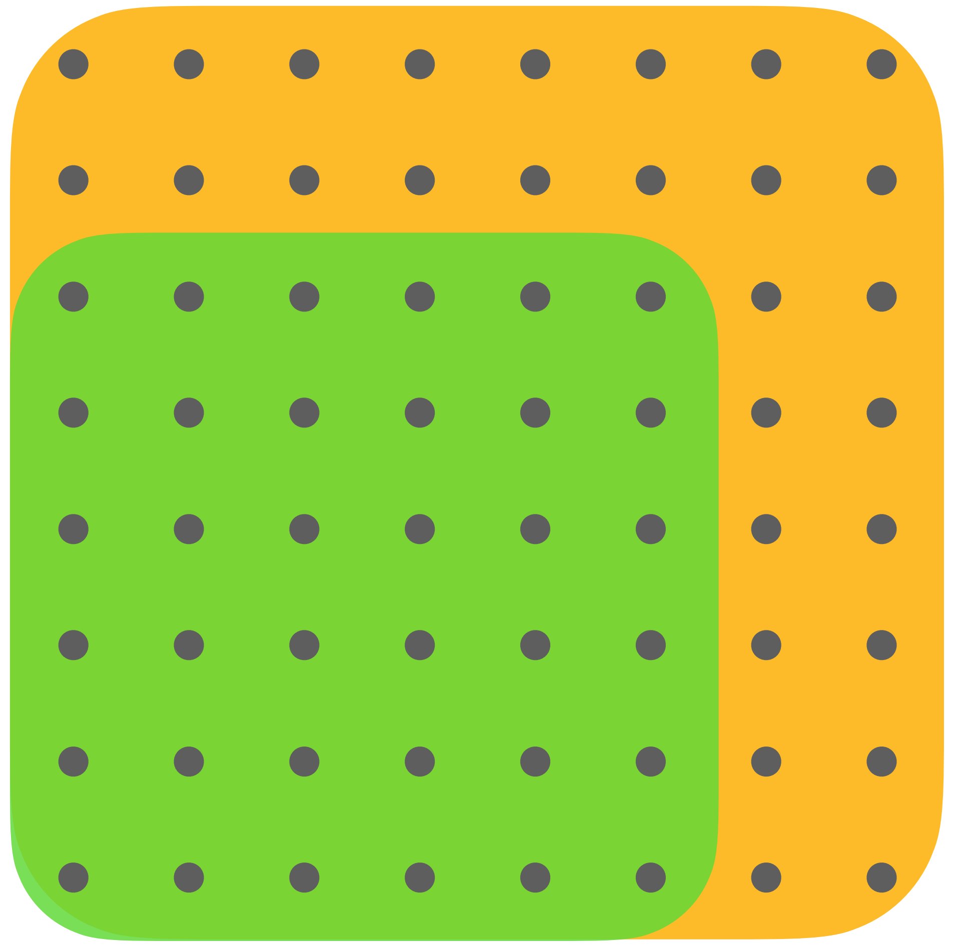
When should we stop?

Completeness Thresholds for Finite Systems

- k is completeness thresholds (CT) iff

  $$\boxed{T \models_k \phi} \Rightarrow \boxed{T \models \phi}$$

- For specific ϕ:
  Can over-approximate CT via of key props of T



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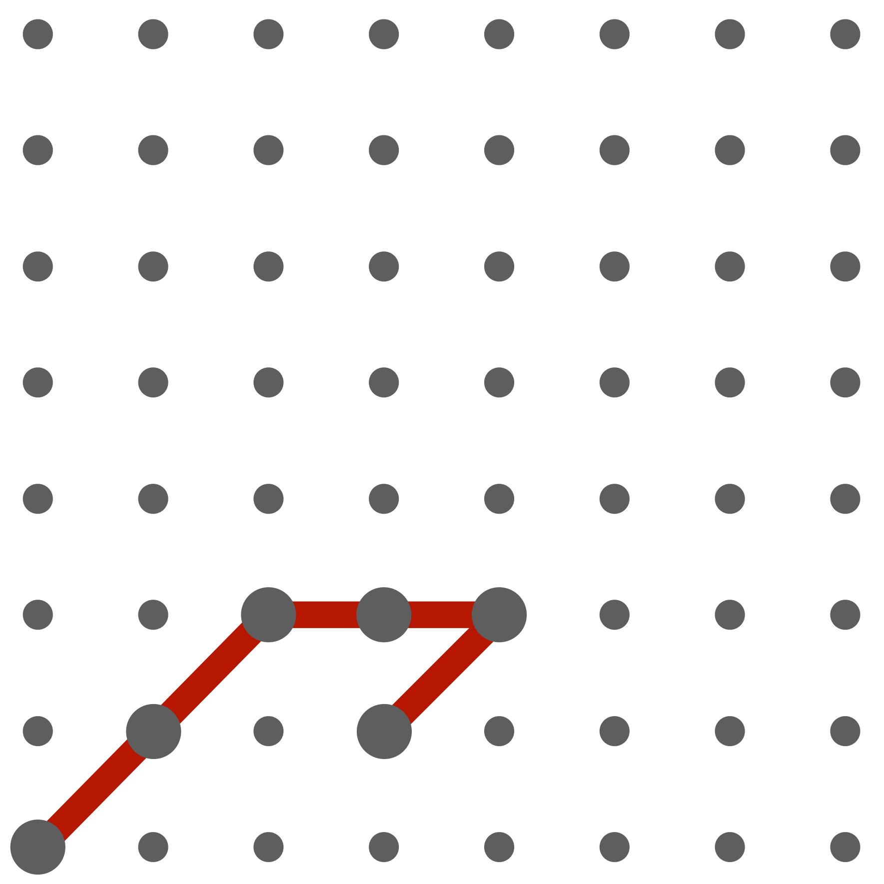
- For $\phi = \mathrm{G}p$ we know
  $\mathrm{CT}(T, \mathrm{G}p) = \text{recurrence\_diameter}(T)$
  (length of longest loop-free path)



recurrence_diameter$(T) = 5$

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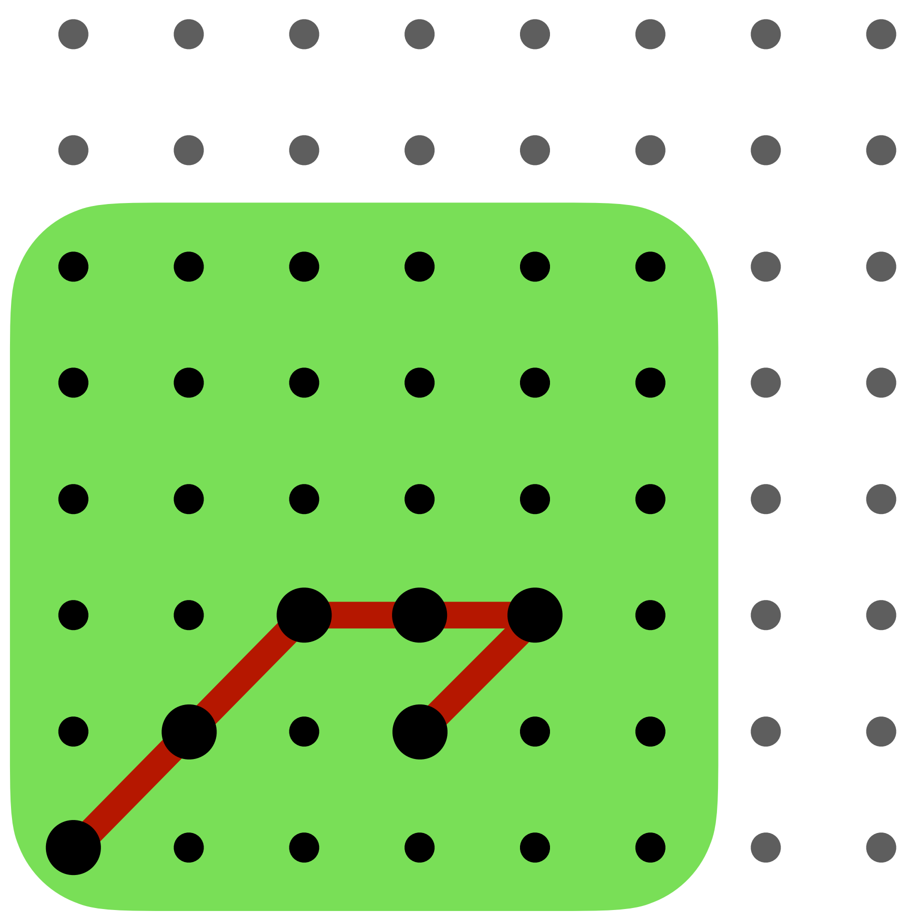

recurrence_diameter$(T) = 5$

$T \vDash_5 \mathsf{G}p$

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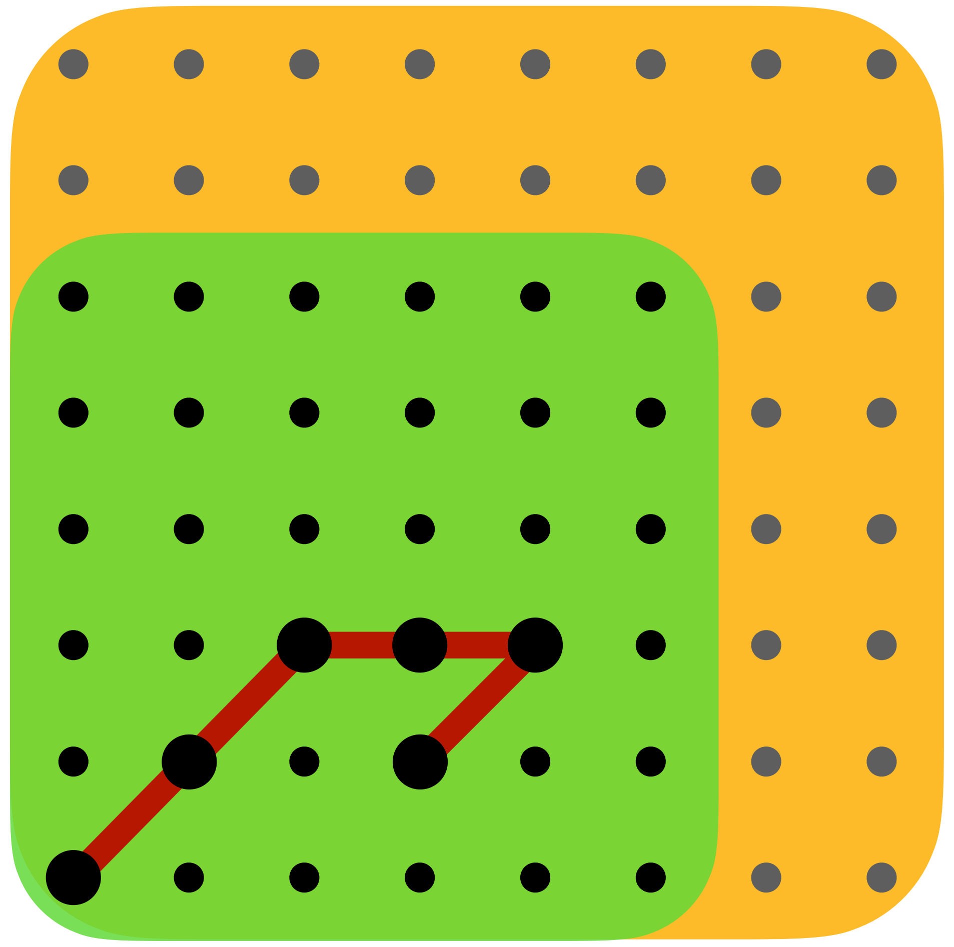


recurrence_diameter$(T) = 5$

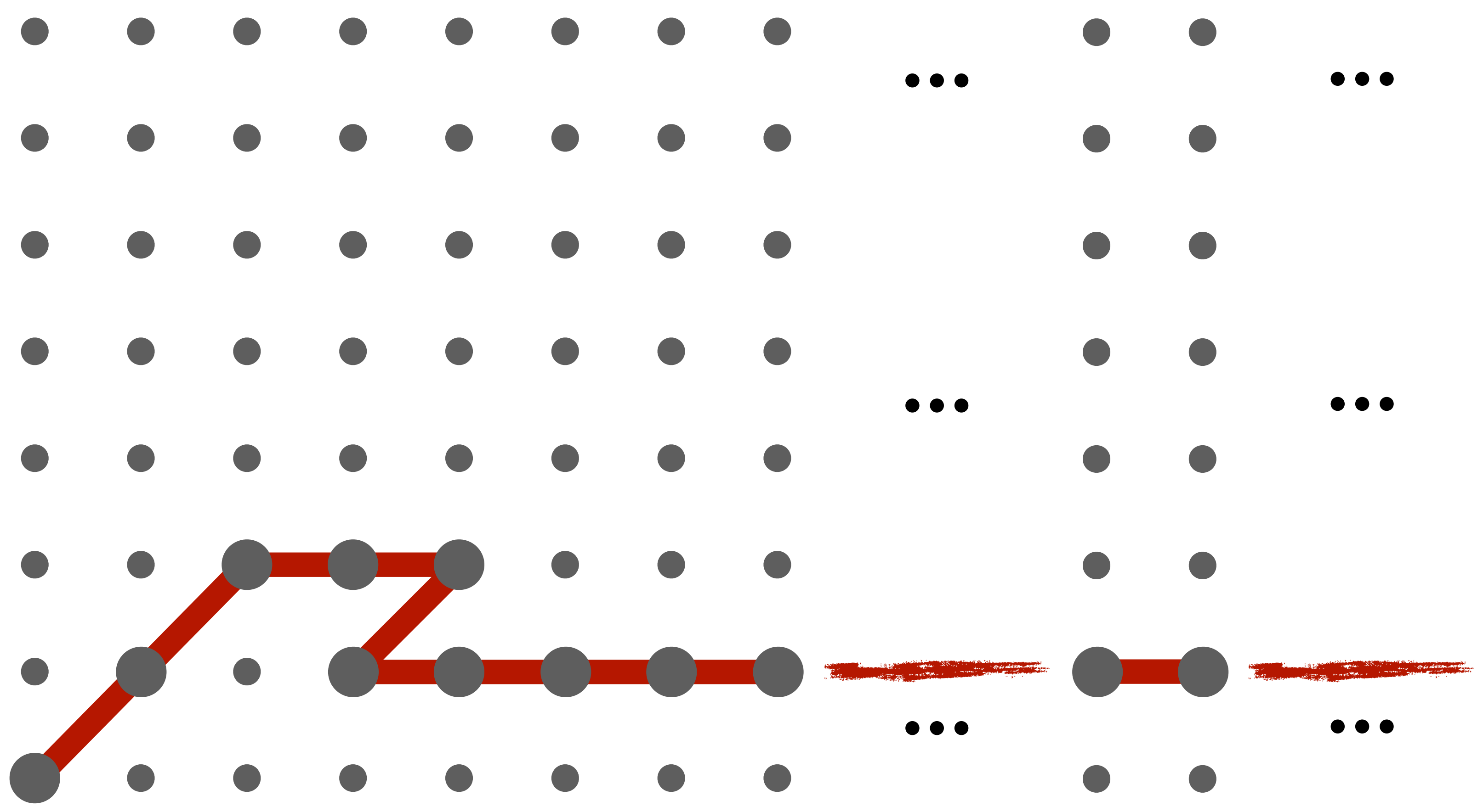
$\downarrow$

$T \models_5 \mathsf{G}p \longrightarrow T \models \mathsf{G}p$

CTs for Infinite Systems?

Problem

Key properties used to describe CTs may be ∞



recurrence_diameter$(T) = \infty$

CTs for Infinite Systems?

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Our Approach

Analyse program's *verification conditions*
instead of transition system

Verification Conditions

- Logical formula vc is VC for any spec $Spec(c)$ iff

$$\models vc \quad \Rightarrow \quad \models Spec(c)$$

- Can verify VC instead of program

- In general: VCs are over-approximations, i.e.,

  possible that $\not\models vc$ but $\models Spec(c)$

Completeness Thresholds

- Program variable x with domain X
- Specification $\forall x \in X \,.\, Spec(c)$

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- Program variable x with domain X
- Specification $\forall x \in X \,.\, Spec(c)$
- Subdomain $Q \subseteq X$ is a CT for x in $\forall x \in X \,.\, Spec(c)$ iff

$$\models \forall x \in Q \,.\; Spec(c) \quad \Rightarrow \quad \models \forall x \in X \,.\; Spec(c)$$

- For us: CT are subdomains, not depths

How to Prove CTs

- Generate VC: $Spec(c) \quad \rightsquigarrow \quad \forall x \in X \,.\, vc(x)$

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- Generate VC: $Spec(c) \rightsquigarrow \forall x \in X \,.\, vc(x)$

- Identify subdomain $Y \subseteq X$ where choice $x \in Y$ does not influence validity of $vc(x)$

$$\Big(\models vc(x) \quad \Leftrightarrow \quad \models vc' \quad \text{with } \; x \notin \text{free}(vc') \Big)$$

$$\Longrightarrow \quad \text{Found CT:} \quad (X \backslash Y) \cup \{y\} \quad \text{(for any choice of } y \in Y\text{)}$$

How Does the Array Size Affect Memory Safety?

Memory assumption:

$\texttt{array}(a, s)$

```
for i in [L : s-R] do
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Generate VC

(fully automated)

VC $vc_0 := \forall s \,.\, \texttt{array}(a, s) \rightarrow \forall i \in \{L, \ldots, s - R\} \,.\, a[i+Z]$ alloc

How Does the Array Size Affect Memory Safety?

$$\boxed{\text{VC } vc_0 := \forall s \,.\, \texttt{array}(a, s) \rightarrow \forall i \in \{L, \ldots, s - R\} \,.\, a[i{+}Z] \text{ alloc}}$$

Range L, ..., s-R empty?

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Range L, ..., s-R empty?

Yes

$s^- < L + R$

Simplify VC!

$$vc_0 \;\equiv\; \forall s^-.\; \ldots \rightarrow \forall i \in \varnothing\,.\; \ldots$$
$$\equiv\; \text{True}$$

How Does the Array Size Affect Memory Safety?

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Simplify VC!

Yes

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No need to check

No

$s^+ \geq L + R$

$$vc_0 \ \equiv \ \forall i \,.\, (L \leq i < s^+ - R) \rightarrow (0 \leq i+Z < s^+)$$

How Does the Array Size Affect Memory Safety?

$$\boxed{\text{VC } vc_0 \ := \ \forall s\,.\ \texttt{array}(a, s) \to \forall i \in \{L, \ldots, s - R\}\,.\ a[i{+}Z] \text{ alloc}}$$

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No

$s^+ \geq L + R$

$$vc_0 \ \equiv \ \forall i\,.(L \leq i < \cancel{s^+} - R) \to (0 \leq i{+}Z < \cancel{s^+})$$

$$\equiv \ \forall i\,.(L \leq i \to 0 \leq i{+}Z)$$

$$\wedge \ (i \leq -R) \to i{+}Z < 0)$$

$\Rightarrow$ Validity does not depend on size

How Does the Array Size Affect Memory Safety?

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No need to check

No

$s^+ \geq L + R$

Can check for any

$s^+ \geq L + R$

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No

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Found CT: $\{s^+\}$

Workflow: How to Find CTs

Mem spec
+ program

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$\xrightarrow{\text{Generate VC}}$ VC $\forall x \in X \,.\, vc(x)$

Workflow: How to Find CTs

Mem spec
+ program

Generate VC

VC $\forall x \in X\,.\; vc(x)$

Select subdomain

$Y \subseteq X$

Restricted VC
$vc(x)$ with $x \in Y$

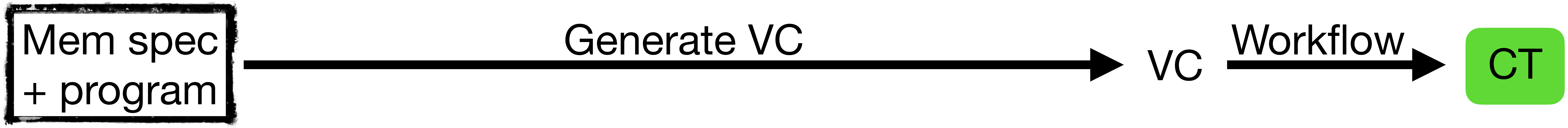
Try to eliminate x

$vc'(x)$ with $x \in Y$

Workflow: How to Find CTs

Scalability

Program Slicing



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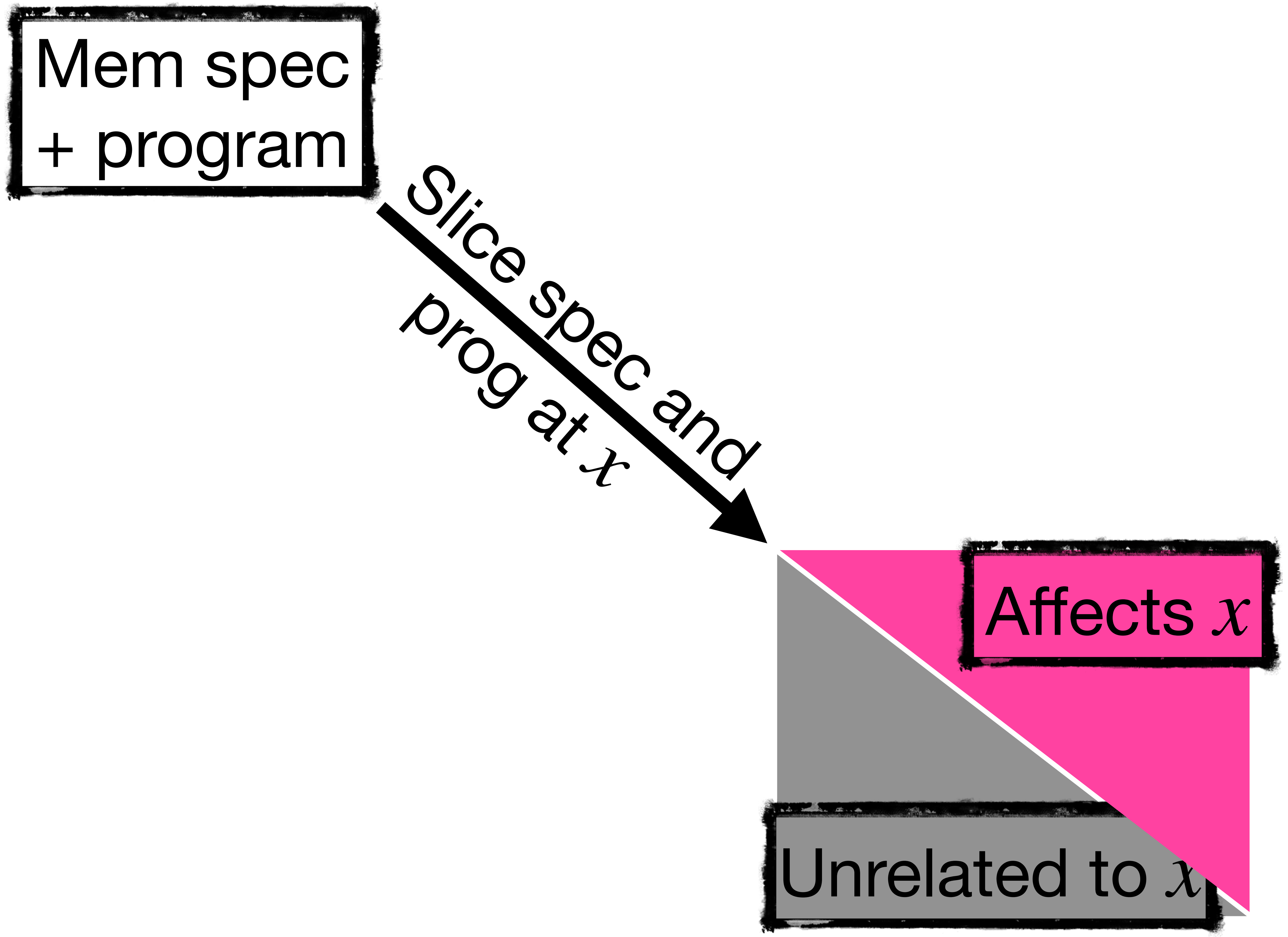

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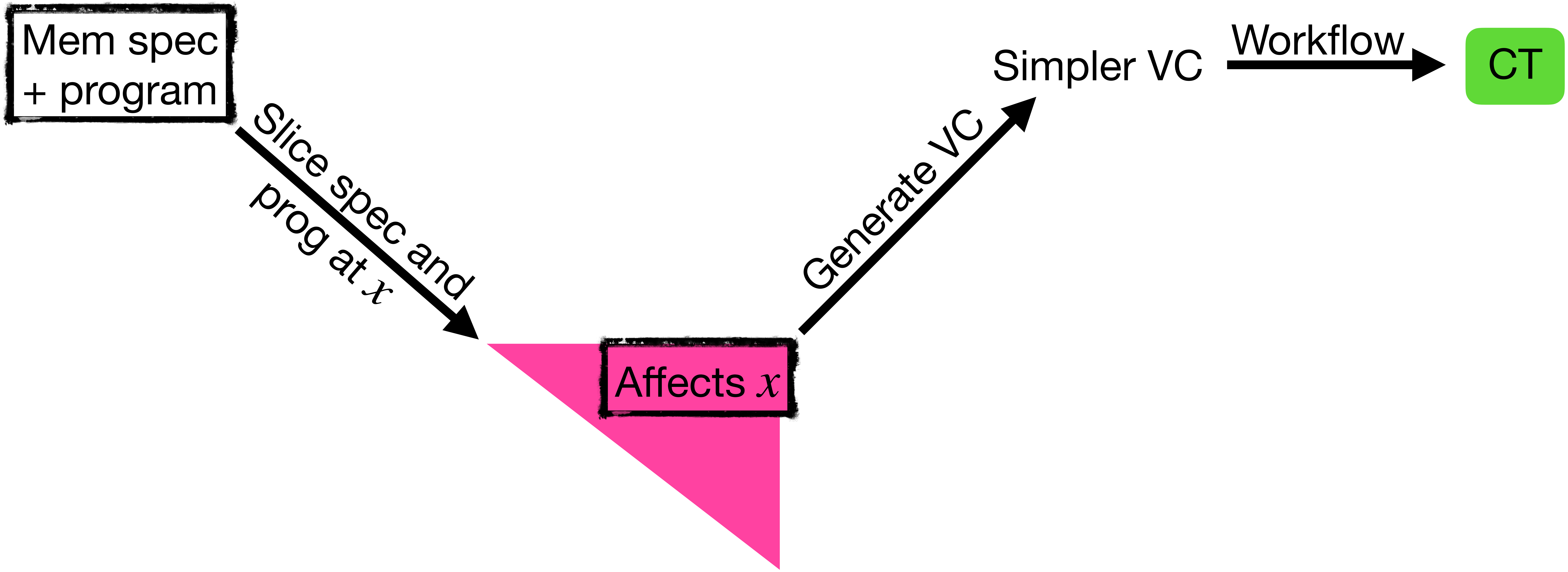

Scalability

CT Combinators

Sequencing

$c_1; c_2$ CTs Q_1, Q_2

$$\downarrow$$

$$Q = Q_1 \cup Q_2$$

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Sequencing

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Branching

if e then c_1 else c_2

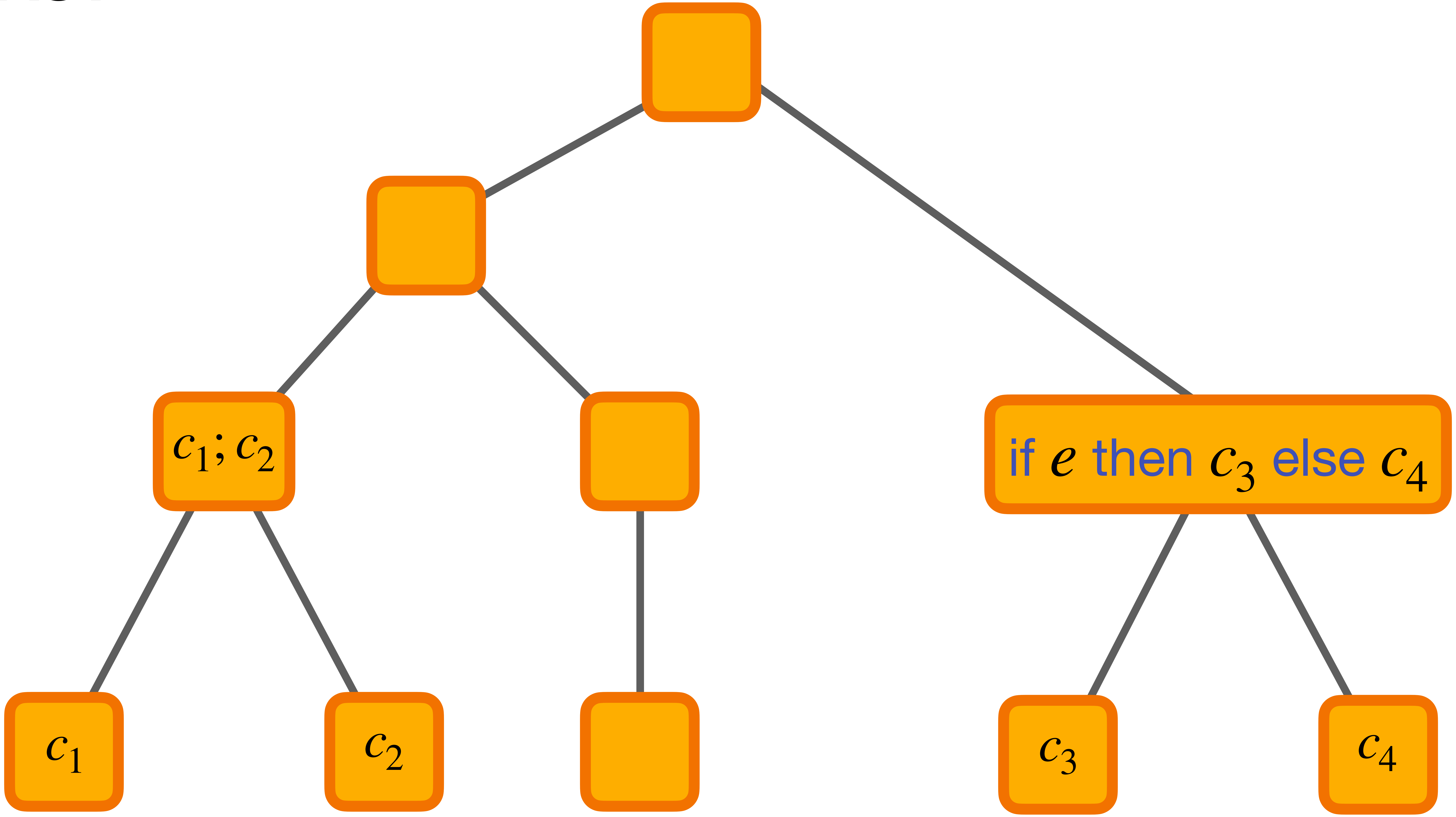
CTs as contraints

$Q_i \sim K_i$

$Q \ \sim \ (e \wedge K_1) \vee (\neg e \wedge K_2)$

Scalability

Follow AST



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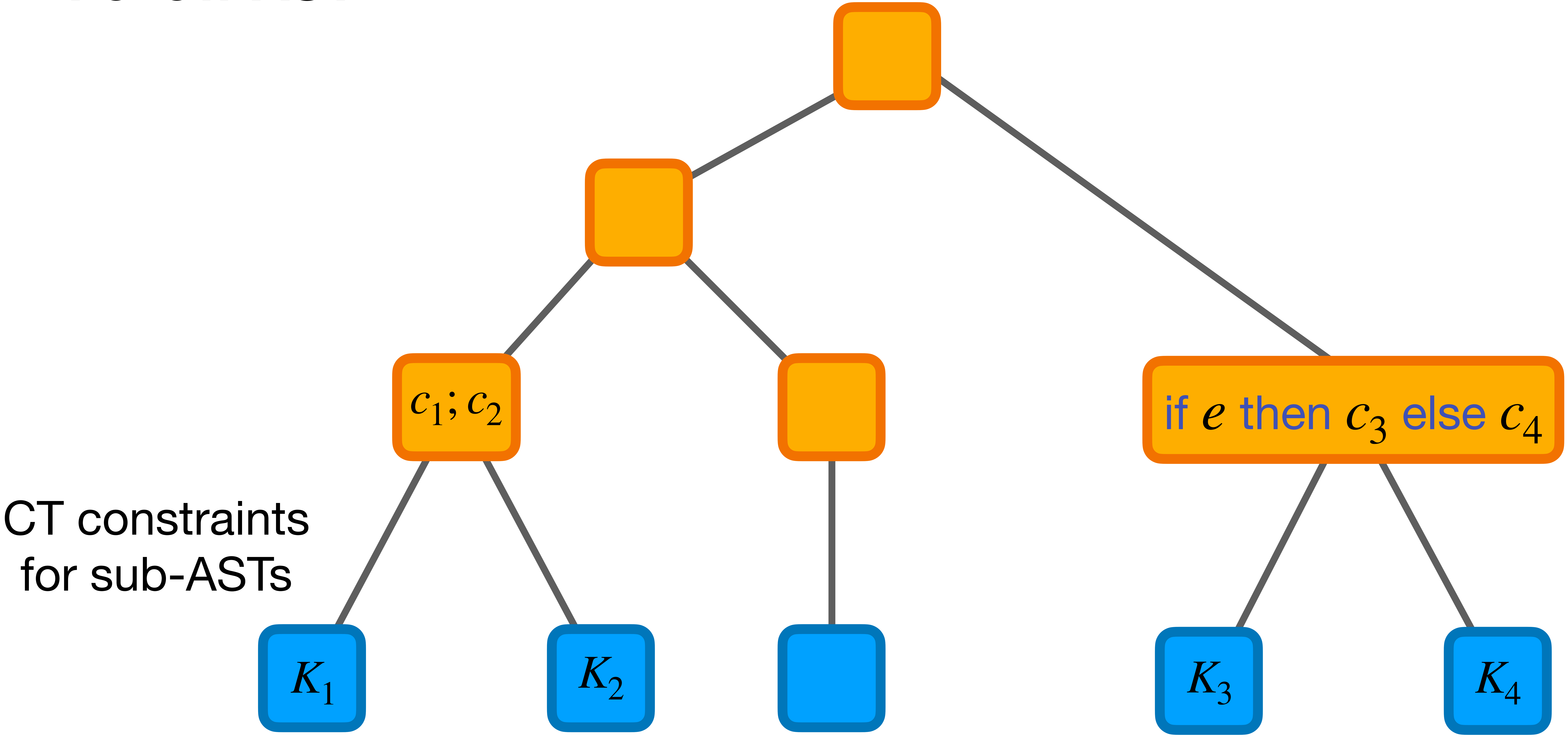

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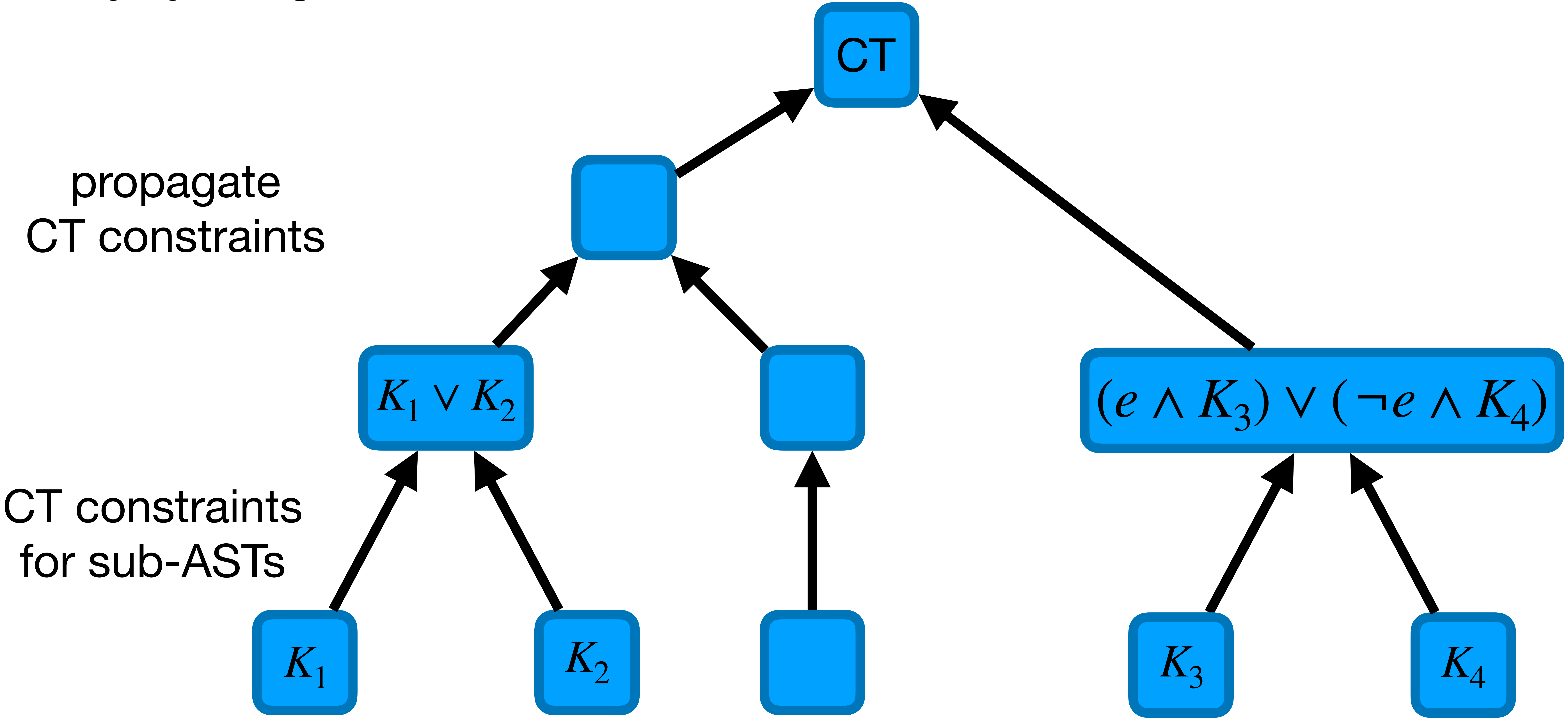

Outlook: Challenges

- Automation, e.g., automatic VC rewriting
- Demo scalability: Complex programs & data (e.g. lists, trees)

Outlook: Increase Trust in BMC

- Turn bounded into unbounded proof

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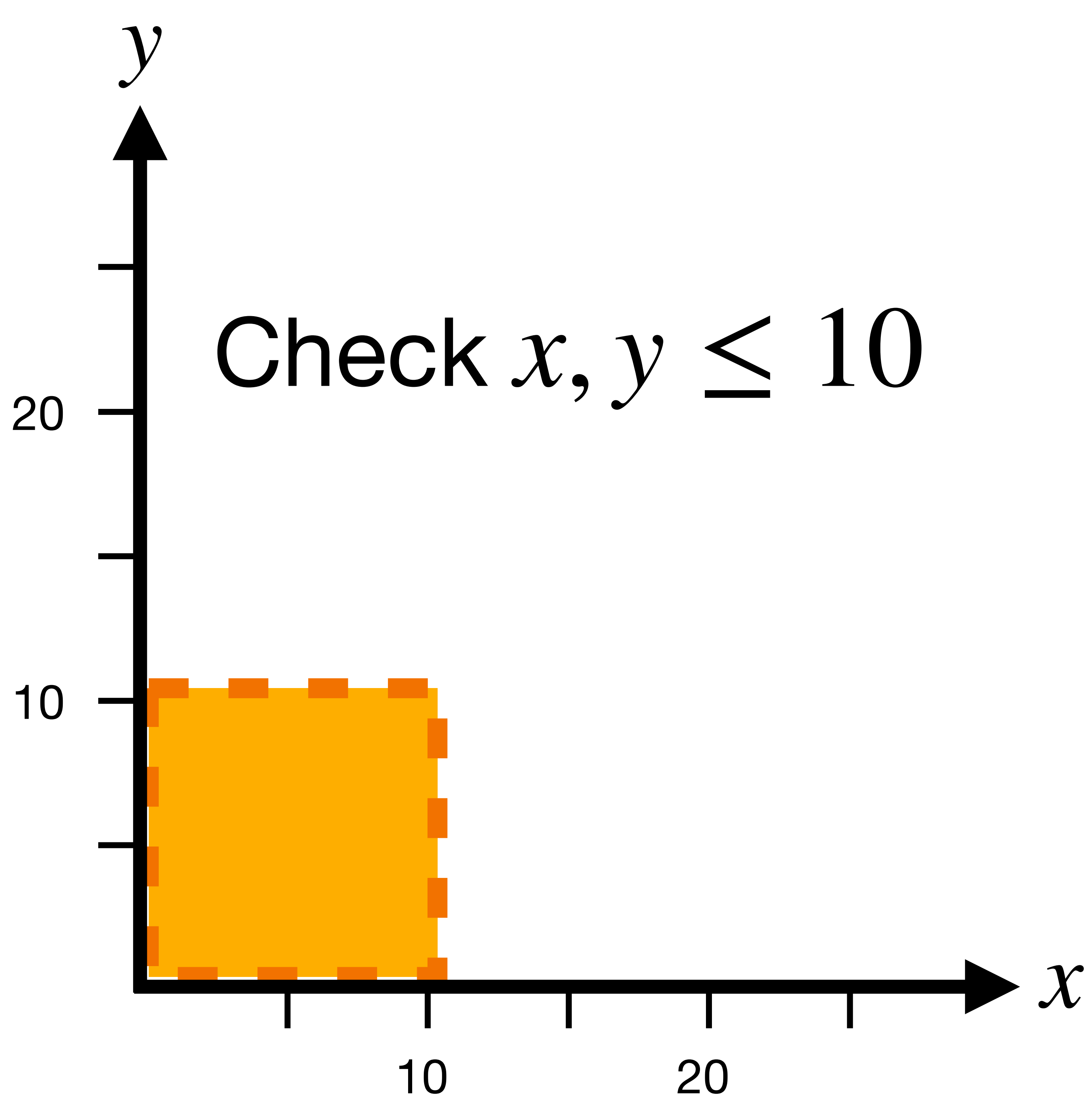
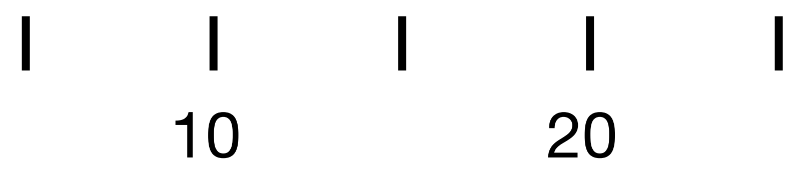
- Turn bounded into unbounded proof
- Shift resources to critical bounds





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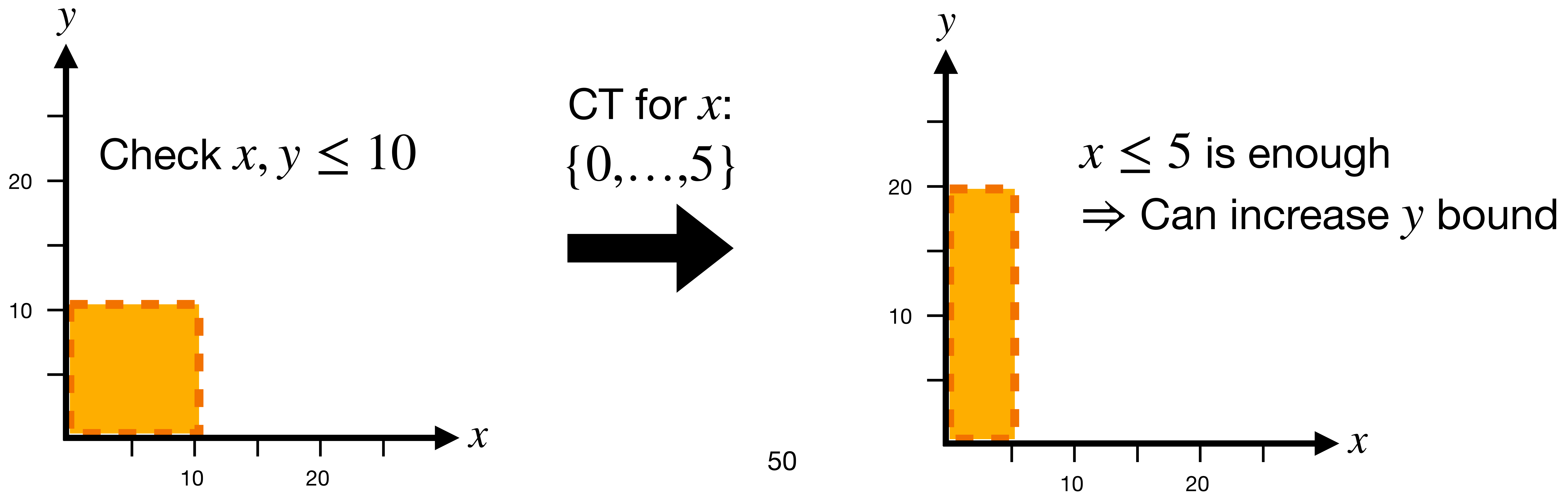

Conclusion

- First generalisation of CTs to infinite state systems
- Connection between bounded & unbounded proofs in program verification
- Foundational research but potential for integration into BMC

Backup Slides

Precise VCs

- VC vc is *precise* for x in $Spec$ iff

  $$\forall v\,.\ \left(\ \models Spec[x \mapsto v] \ \Rightarrow\ \models vc[x \mapsto v]\ \right)$$

  Intuition: vc does not over-approximate wrt. x

- Q is CT $vc\ \wedge\ vc$ is precise $\ \Rightarrow\ Q$ is CT $Spec$

Precise VCs

$$
\begin{tikzcd}
\forall x\,.\ Spec \arrow[r, "\text{Unbounded proof}"] \arrow[d] & \models \forall x\,.\ Spec \\
\forall x\,.\ vc \arrow[d] & \\
Q \text{ is CT for } vc \arrow[d, "vc \text{ precise for } x"] & \\
Q \text{ is CT for } Spec \arrow[r, "\text{Bounded proof}"] & \models \forall x \in Q\,.\ Spec \arrow[uuu, "\text{CT } Q"']
\end{tikzcd}
$$