

Übung 1

Freitag, 1. Mai 2020 16:33

A1

$$\begin{aligned}
 D_{KL}(p, q) &= - \sum_{i=1}^m P(X=x_i) \log\left(\frac{Q(X=x_i)}{P(X=x_i)}\right) \\
 D_{KL}(P(X,Y), P(X)P(Y)) &= D_{KL}(P(X)P(Y), P(X,Y)) \\
 \sum_i \sum_j P(X,Y) \log\left(\frac{P(X)P(Y)}{P(X,Y)}\right) &= \sum_i \sum_j P(X)P(Y) \log\left(\frac{P(X,Y)}{P(X)P(Y)}\right) \\
 \sum_i \sum_j P(X,Y) \log(P(X)P(Y)) - \sum_i \sum_j P(X,Y) \log(P(X,Y)) &= \sum_i \sum_j P(X)P(Y) \log(P(X,Y)) - \sum_i \sum_j P(X)P(Y) \log(P(X)P(Y)) \\
 \sum_i \sum_j (P(X,Y) \log(P(X)P(Y)) + P(X)P(Y) \log(P(X,Y))) &= \sum_i \sum_j (P(X,Y) \log(P(X,Y)) + P(X)P(Y) \log(P(X,Y))) \\
 \sum_i \sum_j ((P(X,Y) + P(X)P(Y)) \cdot \log(P(X)P(Y))) &= \sum_i \sum_j ((P(X,Y) + P(X)P(Y)) \cdot \log(P(X,Y))) \\
 \log(P(X)P(Y)) &= \log(P(X,Y)) \quad \text{nur bei Unabhängigkeit}
 \end{aligned}$$

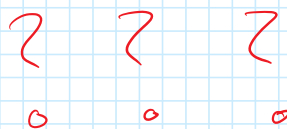
A2

Indifferenzprinzip:

unterscheidbare Ereignisse, schließen sich gegenseitig aus

→ wenn keine weiteren Infos über Ereignisse → Gleichverteilung

warum sinnvoll gewählt?:



A3

$$\begin{aligned}
 D_{KL}(P(X,Y), P(X)P(Y)) &= I(X, Y) \\
 - \sum_i \sum_j P(X,Y) \log\left(\frac{P(X)P(Y)}{P(X,Y)}\right) &= H(X) - H(X|Y) \\
 &= - \sum_i P(X) \log(P(X)) + \sum_i \sum_j P(X,Y) \log(P(X|Y)) \\
 &= - \sum_i P(X) \log(P(X)) + \sum_i \sum_j P(X,Y) \log\left(\frac{P(X,Y)}{P(Y)}\right) \\
 - \sum_i \sum_j P(X,Y) \log(P(X)P(Y)) + \sum_i \sum_j P(X,Y) \log(P(X,Y)) &= - \sum_i P(X) \log(P(X)) + \sum_i \sum_j P(X,Y) \log(P(X,Y)) - \sum_i \sum_j P(X,Y) \log(P(Y)) \\
 - \sum_i \sum_j P(X,Y) \log(P(X)P(Y)) &= - \sum_i P(X) \log(P(X)) - \sum_i \sum_j P(X,Y) \log(P(Y)) \\
 - \sum_i \sum_j (P(X,Y) \log(P(X)P(Y)) - P(X,Y) \log(P(Y))) &= - \sum_i P(X) \log(P(X)) \\
 - \sum_i \sum_j \left(P(X,Y) \cdot \log\left(\frac{P(X)P(Y)}{P(Y)}\right)\right) &= - \sum_i P(X) \log(P(X)) \\
 - \sum_i \sum_j (P(X,Y) \cdot \log(P(X))) &= \\
 \sum_i \log(P(X)) \sum_j P(X,Y) &= \sum_i P(X) \log(P(X)) \\
 \sum_i \log(P(X)) \cdot P(X) &= \sum_i P(X) \log(P(X))
 \end{aligned}$$

$$\sum_i \lg(P(x)) \sum_j P(x,y)$$

$$\sum_i \lg(P(x)) \cdot P(x)$$

$$= \sum_i P(x) \lg(P(x))$$

$$= \sum_i P(x) \lg(P(x))$$