Algorithm Lab

Week 10: Floyd-Warshall Algorithm

There are several different well-known shortest path algorithms, e.g., A^*(best-first search /greedy, one-to-one), Dijkstra algorithm (greedy, one-to-all), Bellman-Ford algorithm (dynamic programming, one-to-all), and Floyd-Warshall algorithm (dynamic programming, all-to-all).

From above algorithm, we can find 3 kinds of shortest path algorithms: one-to-one, one-to-all, and all-to-all. The first one/all is describing start vertex and second one/all is describing target vertex. "One" means having a given vertex, and "all" means should cover every vertex. We can define an all-to-all shortest path problem as follow:

Instance: weighted directed graph $G = (V, E)$

Result: shortest path (or its cost) $\forall(s,t)$ *where* $s,t \in V$, s,t *shows start vertex and end vertex.*

Floyd-Warshall algorithm is a representative all-to-all shortest path algorithm. The main idea of Floyd-Warshall is:

Shortest path of a start-end vertices pair $f(s,t)$ of graph $G = (V,E)$, the result will be

- An edge $(s,t,w) \in E$ or
- Route through some vertex k, then the shortest path should be $f(s,k) + f(k,t)$.

If we tried all the possible internal vertices, then we should get optimal solution of all (s,t) pairs.

Description

For ease to use, we use adjacency matrix as our input format and output format.

- Represent G by a $|V| \times |V|$ matrix where we denote $(u,v,w) \in E$ by $G_{u,v} = w$ (its adjacency matrix).
- If G is not a simple graph ($(i,j) \in E$ may not be unique), leave the minimum one.
- Represent R by a $|V| \times |V|$ matrix where we store all $f(s,t)$ by $R_{s,t} = f(s,t)$.
- If there are no known paths of start-end pair (s,t), then $R_{s,t} = \infty$.

Follow the above presentations, we can rewrite Floyd-Warshall algorithm as followed:

$R \leftarrow G$

For each $k \in V$:

    For each $s \in V$:

        For each $t \in V$:

            If $R_{s,t} > R_{s,k} + R_{k,t}$ then

                $R_{s,t} \leftarrow R_{s,k} + R_{k,t}$

Questions

1 Please modify above algorithm to reconstructing correspond shortest path.

2 Please modify above algorithm to detecting negative cycles of G.

3 Please answer the differences between $R_{i,i} = 0$ and $R_{i,i} = \infty$ in Question 1.

4 Analyze space complexity, time complexity of best/worst case in Question 1 and Question 2.

5 Please solve https://oj.csie.ndhu.edu.tw/problem/ALG06