Algorithm Lab

Week 8: Interval Cover

In graph theory, an interval graph is an undirected formed from a set of intervals on the real line (From Wikipedia). For an interval graph, each vertex v is an interval that have 2 ends, left end l_v and right end r_v. Edge set E is decided by the interval set V. For an arbitrary vertices pair (u, v), $(u, v) \in E$ if and only if u and v are overlapped by each other.
There are many important greedy algorithms works on interval graphs such as interval scheduling, maximum clique finding, and minimum interval cover.
We say interval (l^*, r^*) is covered by an interval (l_i, r_i) if $l_i \leq l^*$ and $r_i \geq r^*$. Suppose we have 2 overlapped intervals u and v. The coverage of these 2 vertices equivalent to the interval $(\min(l_u, l_v), \max(r_u, r_v))$. Minimum interval cover is asking you to find the minimum vertices induced subgraph of an interval graph that can cover whole range on real line.

Instance: a set of intervals $V = \{(l_1, r_1), (l_2, r_2), \ldots, (l_n, r_n)\}$ *and target interval* (l^*, r^*).

Result: minimum $V' \subseteq V$ *that cover* (l^*, r^*).

Description

1. Set current position $p \leftarrow l^*$, answer set $V' \leftarrow \{\ \}$.
2. Find right-most (have largest right end) interval $v \in V$ that $l_v \leq p$.
3. Update $p \leftarrow r_v$, $V' \leftarrow V' \cup \{v\}$.
4. Repeat step 2 and 3 until $p \geq r^*$.

Questions

1. Analyze worst case time complexity of above algorithm.
2. Analyze worst case time complexity of above algorithm with sorted V.
3. Rewrite above algorithm to detect target range may impossible to be covered by original interval set. Analyze space complexity, time complexity of best case and worst case of modified algorithm.
4. Above algorithm is doing through left to right. Please rewrite it to a through right to left

version. Is your new algorithm always get the same result of original one? Give example(s) to explain your answer.

5. Solve https://oj.csie.ndhu.edu.tw/problem/ALG07B