JACARANDA PHYSICS 11

FOR NSW | FOURTH EDITION

KAHNI BURROWS
DAN O'KEEFFE
GRAEME LOFTS
BRUCE MCKAY
RIC MORANTE
JOSEPH BARRY MOTT

jacaranda
A Wiley Brand

Fourth edition published 2018 by
John Wiley & Sons Australia, Ltd
42 McDougall Street, Milton, Qld 4064

First edition published 2002
Second edition published 2004
Third edition published 2009

Typeset in 11/14 pt Time LT Std

National Library of Australia
Cataloguing-in-publication data

Creator:	Burrows, Kahni, author.
Title:	Jacaranda physics 11 / Kahni Burrows [and five] others.
Edition:	Fourth Edition
ISBN:	978 0 7303 4777 4 (paperback)
Notes:	Includes index.
Other Authors:	Graeme Lofts, Dan O'Keeffe, Bruce McKay, Ric Morante, Joseph Barry Mott.
Target Audience:	For secondary school age.
Subjects:	Physics — Textbooks. Physics — Study and teaching (Secondary) — New South Wales.

Illustrated by various artists, diacriTech and Wiley Composition Services

Typeset in India by diacriTech

Printed in China by
Printplus Limited

10 9 8 7 6 5 4 3 2 1

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TOPIC 1
Learning to think like a physicist

1.1 Overview

1.1.1 Working scientifically

Working Scientifically is a major component of the physics course. The Working Scientifically skills are developed during practical investigations associated with both the inquiry questions in each of the modules and the depth studies.

There are seven Working Scientifically outcomes and each must be addressed by the end of the course.

This chapter deals with skills associated with experimental physics and addresses all or some of the points below, which are associated with Working Scientifically.

Planning investigations

A student:

- designs and evaluates investigations in order to obtain primary and secondary data and information PH11/12-2

Students:

- assess risks, consider ethical issues and select appropriate materials and technologies when designing and planning an investigation (ACSPH031, ACSPH097)
- justify and evaluate the use of variables and experimental controls to ensure that a valid procedure is developed that allows for the reliable collection of data (ACSPH002).

Conducting investigations

A student:

- conducts investigations to collect valid and reliable primary and secondary data and information PH11/12-3

Students:

- use appropriate technologies to ensure and evaluate accuracy.

Processing data and information

A student:

- selects and processes appropriate qualitative and quantitative data and information using a range of appropriate media PH11/12-4

Students:

- select qualitative and quantitative data and information and represent them using a range of formats, digital technologies and appropriate media (ACSPH004, ACSPH007, ACSPH064, ACSPH101)
- evaluate and improve the quality of data.

Analysing data and information

A student:

- analyses and evaluates primary and secondary data and information PH11/12-5

Students:

- derive trends, patterns and relationships in data and information
- assess error, uncertainty and limitations in data (ACSPH004, ACSPH005, ACSPH033, ACSPH099)
- assess the relevance, accuracy, validity and reliability of primary and secondary data and suggest improvements to investigations (ACSPH005).

Communicating

Students:

- select and use suitable forms of digital, visual, written and/or oral forms of communication
- select and apply appropriate scientific notations, nomenclature and scientific language to communicate in a variety of contexts (ACSPH008, ACSPH036, ACSPH067, ACSPH102).

FIGURE 1.1 This student is investigating the relationship between the strength of attraction between two bar magnets and the distance separating them.

1.2 Measurement

1.2.1 Theoretical physicists

While this first section deals mainly with measurement and aspects of experimental physics, it is important to note that not all physicists perform experiments. Theoretical physicists construct theories of nature that are then often tested by experiment, except string theory, which cannot be experimentally verified!

Sometimes theory comes before experimentation, but, on the other hand, theory often requires time to explain an observed practical phenomenon. A recent example of the latter is the Higgs boson. Peter Higgs (and others) predicted the existence of the Higgs field and the associated Higgs boson in the early 1960s, but its existence was not verified until its discovery in 2012. Of course, the discovery required the Large Hadron Collider.

Sometimes experimental discoveries come first, as in the case of superconductivity, discovered in 1911, but it was not until 1957 that an explanation of superconductivity in type-I superconductors was found (by Bardeen, Cooper and Schrieffer) and there is still no satisfactory theory for superconductivity in type-II superconductors.

It has been said that experimental physicists win Nobel prizes for making important discoveries but theoretical physicists cannot actually prove anything and, so, do not win Nobel prizes. Peter Higgs, however, shared the 2013 Nobel Prize in Physics with François Englert, who was a leader of another team that independently predicted the existence of the Higgs field and particle that now bear the name Higgs.

1.2.2 Skills of experimental physicists — making measurements

Selecting the appropriate instrument

If you were asked to measure the width of the room you are in, there are many devices available for you to use. You could use an expensive laser distance meter, which can measure up to 300 metres with an accuracy of ± 1 mm; however, a single metre ruler would give a rough but much cheaper alternative. Perhaps using a measuring tape might be the best way to go.

Your choice of instrument should be based on the *resolution* and *precision* of the instrument and the accuracy you require.

Of course, we also require that the measurements are in the correct units, and in physics this means SI Units.

FIGURE 1.2 A laser distance meter.

FIGURE 1.3 A tape measure could achieve the required level of accuracy needed in an investigation.

1.2.3 SI units

So that scientists all over the world can communicate with one another effectively, it is important that they all use the same units to measure physical quantities. In 1960, the international authority on units agreed on a standardised system called the International System of Units. They are called SI units from the French '*Système International*'.

Base units

SI units consist of seven defined base units and other derived units that are obtained by combining the base units.

TABLE 1.1 The SI base units.

Quantity	Unit	Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

*Symbols that are named after people begin with a capital letter; note, however, that the full name of such a unit begins with a small letter.

Each base unit is defined by a standard that can be reproduced in laboratories throughout the world. The standards have changed over time to make them more accurate and reproducible. For example, in 1800, the standard metre was defined as one ten-millionth of the distance from the Earth's equator to either pole. By 1900, it had changed to the distance between two notches on a bar of platinum–iridium alloy kept in Paris. In 1960, it was redefined as $1650\,763.73 \times$ the wavelength of the light emitted by the atoms of the gas krypton-86. In 1983, the definition was changed to what it is today — the distance travelled by light in a vacuum in $\frac{1}{299\,792\,458}$ of a second.

The second is defined as the time taken for 9192631770 vibrations of a caesium-133 atom.

AS A MATTER OF FACT

The kilogram is defined by a standard mass of a platinum–iridium cylinder that has been kept at the International Bureau of Weights and Measures in Paris since 1889.

It has been determined that the 'standard' kilogram lost 0.0001 g during the last century. It is very likely that the standard mass will be changed to a definition involving Planck's constant.

FIGURE 1.4 The National prototype kilogram K20 is one of two prototypes stored at the US National Institute of Standards and Technology in Gaithersburg, Maryland. Many countries have their own copies of the international prototype. This image is a replica for public display, shown as it is normally stored, under two bell jars. It is approximately 39.17 mm in diameter and height.

1.2.4 Derived units

Speed is an example of a quantity that is measured in derived SI units. The SI unit of speed is the metre per second, written as m/s or m s^{-1}. Table 1.2 shows some other commonly used derived SI units.

TABLE 1.2 Some SI derived units commonly used in physics.

Quantity	Unit	Symbol*	Unit in terms of other units
Force	newton	N	Kg m s^{-2}
Energy and work	joule	J	N m
Pressure	pascal	Pa	N m^{-2}
Power	watt	W	J s^{-1}
Electric charge	coulomb	C	A s
Voltage	volt	V	J C^{-1}
Resistance	ohm	Ω	V A^{-1}
Radiation dose equivalent	sievert	Sv	J kg^{-1}

Again, as these units are all named after people, the name of the unit has a lowercase letter, and the abbreviation of the unit is uppercase (except in the case of Ohm, where the unit is given the upper case Greek letter omega; the capital letter 'O' is not used as it could be mistaken for an extra zero being added to the number).

1.2.5 Units and negative indices

Derived units are often expressed with negative indices. For example, the unit of speed is usually expressed as m s^{-1} rather than m/s. This is because:

$$\begin{aligned} 1\text{ m/s} &= 1\text{ m} \times \frac{1}{\text{s}} \\ &= 1\text{ m} \times 1\text{ s}^{-1} \\ &= 1\text{ m s}^{-1} \end{aligned}$$

Similarly, the unit of power, joule per second or J/s, is written as J s^{-1}.

The unit of pressure, newtons per square metre, or N/m^2, is written as N m^{-2} because $\frac{1}{m^2} = m^{-2}$

1.2.6 Metric prefixes

Some SI units are too large or small for measuring some quantities. For example, it is not practical to measure the thickness of a human hair in metres. It is also inappropriate to measure the distance from Sydney to Perth in metres. The prefixes used in front of SI units allow you to use more appropriate units such as millimetres or kilometres.

TABLE 1.3 Commonly used metric prefixes.

Prefix	Symbol	Factor by which unit is multiplied	Example
tera-	T	10^{12}	TB
giga-	G	10^{9}	GW
mega-	M	10^{6}	MV
kilo-	k	10^{3}	kJ
deci-	d	10^{-1}	dm
centi-	c	10^{-2}	cm
milli-	m	10^{-3}	mA
micro-	µ	10^{-6}	µm
nano-	n	10^{-9}	nm

1.2 SAMPLE PROBLEM 1

Express 25 g in SI base units.

SOLUTION

The SI base unit for mass is the kg. Since 1 kg = 1000 g, then 1 g = 10^{-3} kg

$$\begin{aligned} 25\,\text{g} &= 25 \times \frac{1}{1000}\,\text{kg} \\ &= 2.5 \times 10^{-2}\,\text{kg} \end{aligned}$$

1.2 Exercise 1

1. Express each of the following quantities in SI base units:
   (a) 1500 mA
   (b) 750 g
   (c) 250 GW
   (d) 0.52 km
   (e) 600 nm
   (f) 150 µs
   (g) 5 cm
   (h) 50 MV
   (i) 12 dm
2. Acceleration is defined as the rate of change of velocity. Velocity has the same SI unit as speed. What is the SI unit of acceleration?
3. The size of the gravitational force $\boldsymbol{F}$ on an object of mass m is given by the formula:
   $\boldsymbol{F} = mg$ where g is the size of the gravitational field strength.
   (a) What is the SI unit of g?
   (b) Express the SI unit of g in terms of base SI units only.

1.3 Scientific notation

1.3.1 Using scientific notation

Very large and very small quantities can be more conveniently expressed in scientific notation. In scientific notation, a quantity is expressed as a number between 1 and 10 multiplied by a power of 10. For example, the average distance between the Earth and the moon is 380 000 000 m. This is more conveniently expressed as 3.8×10^8 m.

Using the power of 10 in scientific notation involves counting the number of places the decimal point in a number between 1 and 10 needs to be shifted to the right to obtain a multi-digit number.

For example, the decimal point is shifted eight places to the right to get from 3.8 to 380 000 000. The latter number is therefore expressed as 3.8×10^8.

Scientific notation can also be used to express very small quantities conveniently and concisely. To give one example, the mass of a proton is:

0.000 000 000 000 000 000 000 000 001 67 kg

In case you don't feel like counting them, there are 26 zeros after the decimal point! In scientific notation, the mass of the proton can be expressed as 1.67×10^{-27} kg. The power of 10 is obtained by counting the number of places the decimal point in the number between 1 and 10 is shifted to the *left* to obtain the small number. The expression

$$1.67 \times 10^{-27} \text{ means } \frac{1.67}{10^{27}}$$

In physics, scientific notation is generally used for numbers less than 0.01 and greater than 1000.

You can enter quantities in scientific notation into your calculator using the EXP button. For example, to enter 425 000 000 000, you would enter 4.25×10^{11} as:

4.25 EXP 11.

1.3 Exercise 1

Express the following quantities in scientific notation:
(a) the radius of the Earth, 637 000 m
(b) the speed of light in a vacuum, 300 000 000 m s^{-1}
(c) the diameter of a typical atom, 0.000 000 000 3 m.

1.4 Significant figures

1.4.1 Using significant figures

There is a degree of uncertainty in any physical measurement. The uncertainty can be due to human error or to the limitations of the measuring instrument.

Before 1964, when the first electronic quartz timing system was used in international events, stopwatches (accurate to the nearest 0.1 s) were used to measure running times. There was no point in having more accurate handheld stopwatches because the timing was dependent on human judgement and reaction time, a minimum of about 0.1 s. Any measurement of running time by a handheld timing device has an uncertainty of at least 0.2 s. The International Amateur Athletic Federation now requires that world record times in running events are measured to the nearest one-hundredth of a second.

In 1960, the women's Olympic 100 m sprint was won by Wyomia Tyus (USA) in a time of 11.0 s. In 1984, the same event was won by Evelyn Ashford (USA) in a time of 10.97 s. The 1960 event was not timed electronically. The uncertainty of the measurement of time is indicated by the number of significant figures quoted.

The Wyomia Tyus time of 11.0 s has three significant figures. There would have been no point expressing the time as 11.00 s because the nature of the timing device and human judgement and reaction time provide no degree of certainty in the second decimal place. The expression of the time as 11.0 s is consistent with the small degree of uncertainty in the last significant figure. To express the time as 11 s would suggest that the time was measured only to the nearest second.

The Evelyn Ashford time of 10.97 s has four significant figures. This is a reflection of the accuracy of the electronic timing devices and suggests that there could be a small degree of uncertainty in the last figure. The computerised timing systems used today can measure times to the nearest 0.001 s. The last figure quoted in world records therefore has no degree of uncertainty of measurement.

In most physical measurements, the last significant figure shows a small degree of uncertainty. For example, the length of an Olympic competition swimming pool is correctly expressed as 50.00 m.

The last zero has a small degree of uncertainty. A pool can still be used for Olympic competition if it is up to 3 cm too long.

AS A MATTER OF FACT

Because of this permitted error of 3 cm variation in the pool length (it is not possible to construct a pool to a finer tolerance than that), swimming events are now timed to the nearest one hundredth of a second. They were once timed to the nearest thousandth of a second but that extra significant figure in timing cannot be justified because of the possible variation in length. There was a triple dead heat for the silver medal in the 100 m butterfly at the Rio de Janeiro Olympics and many people wondered why the swimmers were not timed to one thousandth of a second, to split them.

FIGURE 1.5 Laszlo Cseh of Hungary (L), Chad le Clos of the RSA, Michael Phelps of the USA, and Joseph Schooling of Singapore during the medal ceremony after the men's 100 m butterfly in the Rio 2016 Olympics.

1.4.2 Complicated by zeros

Two simple rules can be used to help you decide if zeros are significant:

- Zeros before the decimal point are significant if they are between non-zero digits. For example, all of the zeros in the numbers 4506, 27 034 and 602 007 are significant. The numbers therefore have four, five and six significant figures respectively. The zero in the number 0.56 is not significant.
- Zeros after the decimal point are significant if they follow a non-zero digit. For example, in the number 28.00, the two zeros are significant. The number has four significant figures. However, in the number 0.0028, the two zeros are not significant. They do not follow a non-zero digit and are present only to indicate the position of the decimal point. This number therefore has only two significant figures. The number 0.00280 has three significant figures.

Sometimes, the number of significant figures in a measured quantity is not clear. For example, a length of 1500 m may have been measured to the nearest metre, the nearest 10 m or even the nearest 100 m. The two zeros are not between non-zero digits. The first rule given above, therefore, suggests that the length of 1500 m has only two significant figures. However, it could have two, three or four significant figures depending on how the length was measured. To avoid confusion, quantities such as this can be expressed in scientific notation. The length could then be expressed as 1.500×10^3 m, 1.50×10^3 m or 1.5×10^3 m, giving an indication of the uncertainty.

When scientific notation appears clumsy, as it would for numbers such as 100 or 10, it is generally assumed that the zeros are significant.

1.4.3 Calculating and significant figures

When quantities are multiplied or divided, the result should be expressed in the number of significant figures quoted in the least accurate quantity. For example, if you travelled 432 m in a car for 25 s, your average speed would be given by:

$$\begin{aligned}\text{average speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= \frac{432\text{ m}}{25\text{ s}} \\ &= 17.28\text{ m s}^{-1}\end{aligned}$$

The result should be rounded off to two significant figures to reflect the uncertainty in the data used to determine the distance and time, and therefore should be expressed as 17 m s^{-1}.

When quantities are added or subtracted, the result should be expressed to the minimum number of decimal places used in the data. For example, if you travelled three consecutive distances of 63.5 m, 12.2517 m and 32.78 m, the total distance travelled would be given by:

$$63.5 \text{ m} + 12.2517 \text{ m} + 32.78 \text{ m}$$
$$= 108.5317 \text{ m}$$

The result should be rounded off to one decimal place as the minimum number of decimal places used in the data is one: in the distance of 63.5 m.

1.4 Exercise 1

1 How many significant figures are quoted in each of the following quantities?
(a) 566.2 kJ
(b) 0.00032 m
(c) 602.5 kg
(d) 42.5300 s
(e) 5.6×10^3 W
(f) 0.00840 V
2 Calculate each of the following quantities and express them to the appropriate number of significant figures:
(a) the area of a rectangular netball court that is 30.5 m long and 15.24 m wide
(b) the perimeter of a soccer pitch that is measured to have a length of 96.3 m and a width of 72.42 m.
3 A Commonwealth Games athlete completes one lap of a circular track in a time of 46.52 s. The radius of the track is measured to be 64 m. What is the average speed of the athlete?

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1.5 Accuracy, resolution, precision, validity and reliability

1.5.1 What are accuracy, resolution, precision, validity and reliability?

Accuracy is how closely a measured or calculated value matches the true value for that quantity.

Resolution is the fineness to which an instrument can be read. However, resolution may not be significant. A good analogue stopwatch might be read to one tenth of a second. A digital stopwatch displays to one hundredth of a second so is of higher resolution.

Precision is how closely results of repeated measurements agree with one another. The measurements must be repeatable and reliable. In the case of the stopwatches, it is impossible for a person to repeat measurements of time to one hundredth of a second, so the precision of the digital stopwatch is one tenth of a second, or worse, even though it has a higher resolution.

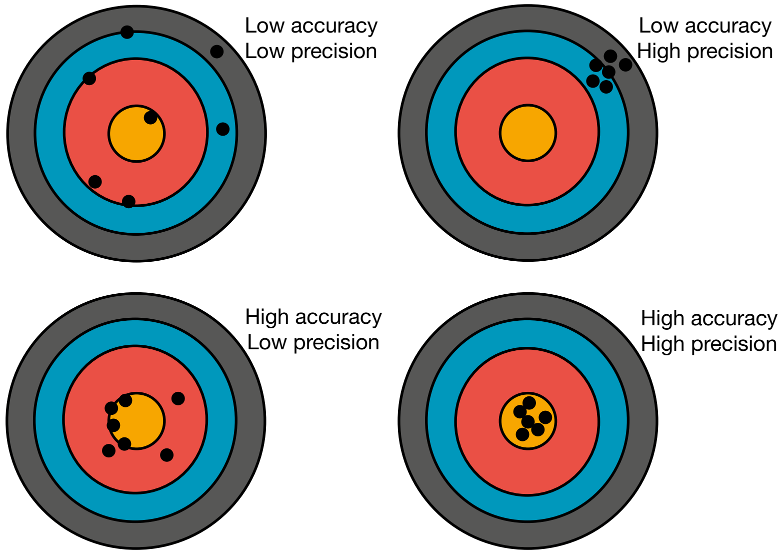
Generally, a greater number of significant figures means the measurement is more precise (provided the figures are reliable and can be repeated).

Accuracy and precision are often represented diagrammatically as bullets hitting a target, as shown in figure 1.6.

Results or measurements that are **precise** are in very good agreement.

If a student calculated the acceleration due to gravity to be 9.008, 9.009, 9.006, 9.007, 9.008 and 9.007 m s^{-2}, the results are very precise. However, they are **inaccurate**, as the accepted value is close to 9.80 m s^{-2}.

FIGURE 1.6 This diagram of bullets fired at a target shows the different combinations of precision and accuracy.



The student should then examine their experiment and consider its reliability and validity.

An experiment is **reliable** if repeating the experiment gives consistent results. (This corresponds to high precision in our target analogy.) An experiment is **valid** if it measures or enables calculation of the quantity it sets out to determine. The results given above show that the experiment is reliable but that it does not produce an accurate value for acceleration due to gravity, so it is not a valid experiment. Perhaps the experiment used a pendulum and there was a systematic error in the measurement of the length of the pendulum. This would render the experiment invalid.

1.5.2 Uncertainty in measurement

Whenever we make a reading, there is a degree of uncertainty associated with that reading. (There are other sources of error — systematic errors, random errors or actual mistakes made by the observer — but, regardless of these and how they are minimised, there will still be errors or uncertainties associated with making a reading.)

There is no such thing as a truly exact measurement. If you measure a length to be 2.3 millimetres, 2.3×10^{-3} m, that says nothing about the next decimal place. Is that 2.30×10^{-3} m? Could it be measured to the nearest micron (10^{-6} m)? If you were able to express the value in microns, what about in nanometres? It is vitally important in physics to be able to express information about how well the value has been measured.

AS A MATTER OF FACT

Should we use significant figures or decimal places?

A reading of 9.2 cm (to the nearest millimetre) made with a metre rule has two significant figures. A slightly larger value of 10.3 cm (again to the nearest millimetre) has three significant figures, but each one has been measured to the nearest millimetre so, in this case, it makes sense to match decimal places in measurements.

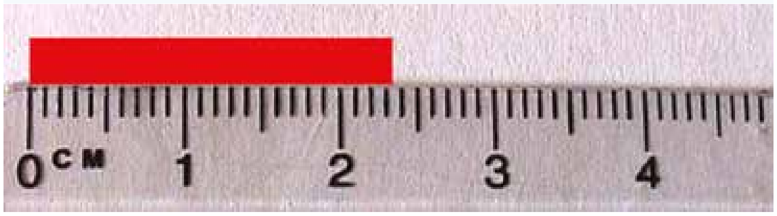
1.5.3 Making a reading on a linear scale

Some physics books look for hard and fast rules to express the uncertainty in a measurement in terms of the divisions on the scale. However, it is usually better to consider the actual scale, how clearly the lines on the scale have been marked and how fine those lines are, then write the uncertainty with a realistic range of uncertainty for that instrument.

If the instrument has a very sharply marked scale of high quality, you could try to estimate a reading to a tenth of a division. The uncertainty could realistically be ± 0.1 of the division. If the scale is not of such high quality, the uncertainty might be $\pm$ 0.3 or maybe even $\pm$ 0.5 of the division.

The best estimate for the position of the end of the red line in figure 1.7 is 2.35 cm. The position is closer to 2.35 cm than it is to either 2.30 cm or 2.40 cm. The scale is marked by lines that are about one quarter of a millimetre thick, so the scale is not of high quality. We might be tempted to say that it is possible to judge the position to the nearest quarter of a division, but to say that this is 0.25 mm is claiming that it is possible to distinguish between 0.24 mm and 0.25 mm, which is clearly impossible. It would be very unusual to give the uncertainty in a measurement to more than one significant figure.

FIGURE 1.7 What is the position of the right-hand end of this red line being measured by the plastic ruler?



The measurement could be stated as 2.35 $\pm$ 0.03 cm. This says the actual position is somewhere between 2.32 cm and 2.38 cm.

The 0.03 cm represents the tolerance or uncertainty in the measurement.

Measuring the length of the red line would require a similar determination of the position of the other end — the start of the red line. This reading would be subtracted from the 2.35 $\pm$ 0.03 cm, using the process on page 11.

1.5.4 Reading a digital instrument

With digital devices, the uncertainty would be $\pm$ 0.5 of the step between one value and the next on the digital scale. However, there is also a factor due to the inherent accuracy of the device itself. There might be a statement such as '0.09% on the 2 mV scale' on the instrument. We can assume that the instrument rounds off the number displayed and that the uncertainty is $\pm$ 0.5 of the next (unseen) digit.

The reading on the digital scale in figure 1.8 is 0.71 grams. This means the mass is neither 0.70 g nor 0.72 g. The actual mass is somewhere between 0.705 g and 0.715 g.

One way to write this mass would be: mass = 0.71 $\pm$ 0.005 g.

While theoretically this might be correct, it would be unusual to have more decimal places in the uncertainty than in the value. When on the limit of its measurement, the digital balance will often not keep a perfectly steady value. If the reading fluctuates between 0.71 and 0.72, the best way to write the mass would be: mass = 0.71 $\pm$ 0.01 g.

FIGURE 1.8 A digital scale.

1.5 SAMPLE PROBLEM 1

Record the reading on the scales below, including the tolerance.

SOLUTION

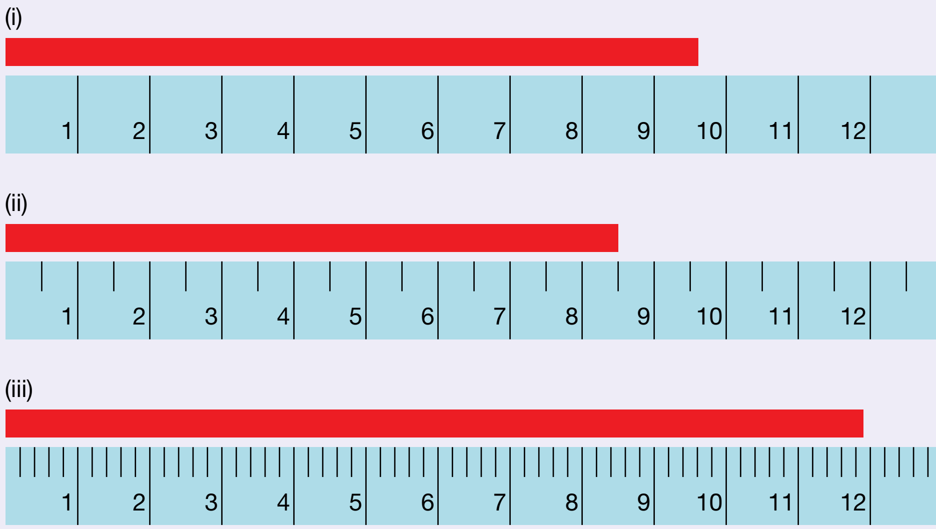
The scale shows 0.250 g, so the actual weight may be between 0.249 g and 0.251 g. The mass is written as 0.250 $\pm$ 0.001 g.

FIGURE 1.9

1.5 Exercise 1

(a) Determine the length of each line in the diagram below, showing the tolerance in each case.

FIGURE 1.10



(b) Record the reading on the scales at right, including the tolerance.

FIGURE 1.11

1.6 Calculations involving numbers with uncertainties (errors)

1.6.1 Adding or subtracting numbers with errors

When adding or subtracting numbers with errors, add the errors.

Example

(i) Adding

$20.0 \pm 0.2 + 15.3 \pm 0.4$

$= (20.0 + 15.3) \pm (0.2 + 0.4)$

$= 35.3 \pm 0.6$

(ii) Subtracting

$20.0 \pm 0.2 - 15.3 \pm 0.4$

$= (20.0 - 15.3) \pm (0.2 + 0.4)$

$= 4.7 \pm 0.6$

1.6.2 Multiplying or dividing numbers with errors

When multiplying or dividing numbers with errors, follow these steps:

1. Convert the errors to percentage errors
2. Add the percentage errors
3. Convert the percentage error back to an actual error
4. Round off the value and error in result to an appropriate number of significant figures.

Example

(i) Multiplying

$20.0 \pm 0.2 \times 3.1 \pm 0.1$

Percentage errors are:

$\frac{0.2}{20.0} \times 100 = 1.0$ and $\frac{0.1}{3.1} \times 100 = 0.235$

So, the answer is $20.0 \times 3.1 = 62.0 \pm 1.235\%$

Converting back from the percentage error

$$\frac{1.235}{100} \times 62.0 = 0.767.$$

Which gives the actual error as 62.0 ± 0.767.

If we follow the basic rule with significant figures, as there are only two significant figures in 3.1, the answer should be quoted to only two significant figures; it is pointless quoting the error in the answer to three decimal places.

The answer could be better expressed as 62 ± 1.

(ii) Dividing

$20.0 \pm 0.2 \div 3.1 \pm 0.1$

Percentage errors are again:

$\frac{0.2}{20.0} \times 100 = 1.0$ and $\frac{0.1}{3.1} \times 100 = 0.235$

So, the answer is $20.0 \div 3.1 = 6.45 \pm 1.235\%$

Converting back from the percentage error gives 6.45 ± 0.7966.

Again, reducing to two significant figures gives the answer 6.5 ± 0.8.

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Watch this eLesson: Calculating error
Searchlight ID: eles-2560

1.7 Graphs

1.7.1 The importance of graphs

A graph presents information visually in a way that we can see trends or pick up other information easily. We can see the range over which the measurements were made, perhaps see the uncertainty in each measurement, identify trends in the data collected, identify measurements that do not follow the general trend, and identify outliers where perhaps an incorrect measurement was made.

1.7.2 Plotting graphs

Anybody who has used the graphing function of a spreadsheet application will be aware of the many different types of graphs. These applications probably use the term ‘charts’ rather than ‘graphs’ and have a variety of versions of each type, including bar graphs, column graphs, pie graphs, line graphs, scatter graphs, as well as two-dimensional and three-dimensional versions of many. We will consider what might be called a scatter graph with lines (the lines may be straight or curved) that are plotted on a conventional Cartesian coordinate system (an x-y graph).

Independent and dependent variables

The quantity that we control or change is the independent variable, and the quantity that we measure, which varies as a result of our change, is the dependent variable.

While it is usual to plot the independent variable on the x-axis and the dependent variable on the y-axis, there are times in physics when it makes more sense not to do this. You could set up an experiment to measure displacement and time in such a way that the displacement was the independent variable and time the dependent variable. A displacement–time graph is a standard graph and it would not be sensible to plot a time–displacement graph.

Title, labels on axes, and units

Every graph must have a title, and each axis must be labelled with the quantity plotted on the axis along with its units. Forgetting to label either the quantity or its unit renders the graph meaningless.

Scales

The scales should be chosen to provide easy-to-read values as well as the maximum possible accuracy in the plotted points. Graph paper may have major lines 1 cm apart and minor lines with a spacing of 2 mm, so, depending on the range of the values to be plotted, you might choose a scale of say 1.0 metre per cm on the scale, or, maybe if the range is smaller, 0.5 metres per cm. It would not be sensible to choose 0.3 metres per cm on the scale as it will be far too difficult to plot a point of 0.4 metres.

The scale should be chosen so that the graph occupies the maximum possible area of the graph sheet. A good rule is that if you could double the scale and still fit the graph on the paper, the chosen scale is too small.

How to mark points

There are different ways points can be marked, but you should remember that a point has zero area and should be marked with a sharp pencil. Some people prefer a small cross, others prefer a small dot with a circle surrounding it (in case the line of best fit passes through the dot point and the point disappears into the line) and there might be other variations of these. Points in some graphs are plotted with error bars, which are small vertical and horizontal lines indicating the error associated with the data (see figure 1.12).

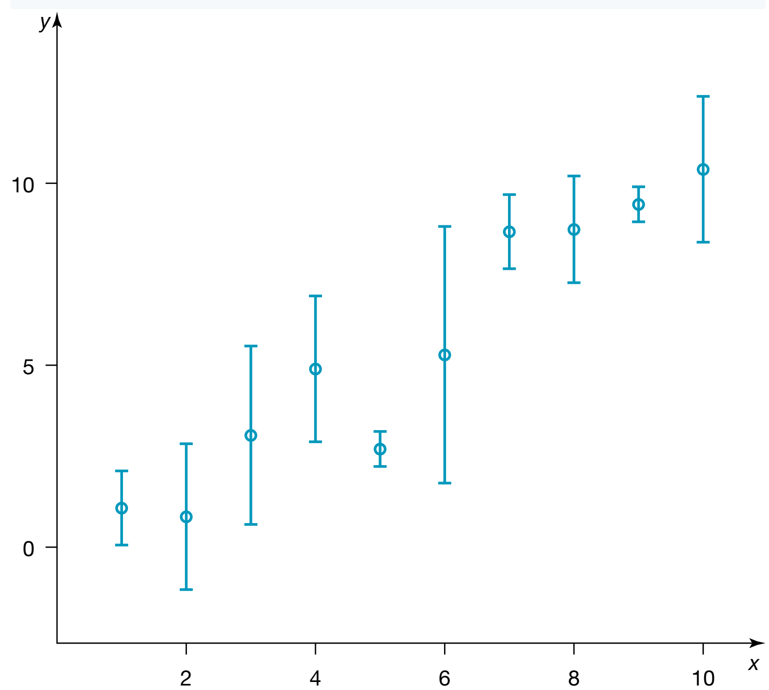
Lines, lines of best fit or join the dots?

While it might be appropriate in some disciplines to plot points and then join the dots, in physics we usually plot quantities that vary continuously, so it would be inappropriate to join dots.

We usually draw a 'line of best fit', which fits our points in the smoothest way possible. A line of best fit could be a straight line or a curved line.

It takes practice to develop the skill to draw a good-looking curved line of best fit, but, if you use your judgement to decide the line should be straight, draw it with a ruler and not freehand. A straight line of best fit is probably best drawn with a transparent ruler, which you shift until the points are scattered equally on either side of your line. Remember that each plotted point comes from a measurement that has an uncertainty associated with it; drawing the line of best fit smooths out the influence of these errors (see figure 1.13).

FIGURE 1.12 An example of error bars.



Outliers

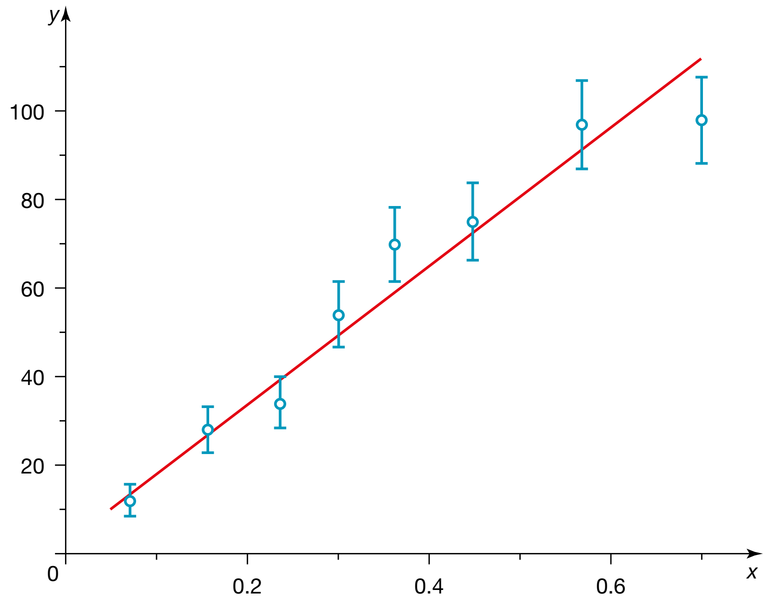
Outliers are points that do not follow the overall trend in the data. Outliers may exist for a variety of reasons, such as the occurrence of an intermittent error by the measuring instrument, the incorrect recording of data or the incorrect plotting of a point. The plotting can be checked, but it is good practice to be able to return to your equipment to check your measurement; so, as a general rule, you should not dismantle equipment until you know there are no issues with your measurements.

Determining the gradient

If your line of best fit is a straight line, you will need to determine the gradient of that line. This should be done by taking two well-spaced points on the line of best fit and using these to find the 'rise over run'. Remember that your line of best fit has smoothed out any errors, so **do not use data points to calculate the gradient**.

If your line of best fit is a curve and you wish to find the gradient at a point, draw a tangent to the curve at that point and find the gradient of the tangent.

FIGURE 1.13 A graph showing the line of best fit.



Advanced graphing techniques

If, for example, you performed an experiment to find the relationship between the time taken for a ball to roll down a slope and the distance it rolled, your plot would be a curved line when you plot distance rolled against time, as in figure 1.14.

If want to find the relationship between these quantities, you would then need to look for other graphs, because you require a straight-line graph. Plotting distance rolled against time squared will yield a straight-line graph, as in figure 1.15.

In this case, you should already know that for an object moving with uniform acceleration from rest, the equation $s = \frac{1}{2}at^2$ applies.

Your straight-line graph shows that s is proportional to t^2, so, comparing your graph with the standard form of $y = mx + b$ with the intercept $b = 0$, the gradient must represent $\frac{1}{2}a$.

FIGURE 1.14

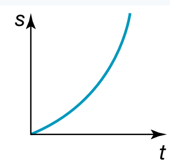
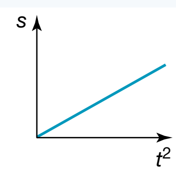


FIGURE 1.15



1.8 Other Working Scientifically skills

1.8.1 Working Scientifically outcomes

The Working Scientifically outcomes are:

QUESTIONING AND PREDICTING

Developing, proposing and evaluating inquiry questions and hypotheses challenges students to identify an issue or phenomenon that can be investigated scientifically by gathering primary and/or secondary-sourced data. Students develop inquiry question(s) that require observations, experimentation and/or research to aid in constructing a reasonable and informed hypothesis. The consideration of variables is to be included in the questioning process.

PLANNING INVESTIGATIONS

Students justify the selection of equipment, resources chosen and design of an investigation. They ensure that all risks are assessed, appropriate materials and technologies are sourced, and all ethical concerns are considered. Variables are to be identified as independent, dependent and controlled to ensure a valid procedure is developed that will allow for the reliable collection of data. Investigations should include strategies that ensure controlled variables are kept constant and an experimental control is used as appropriate.

CONDUCTING INVESTIGATIONS

Students are to select appropriate equipment, employ safe work practices and ensure that risk assessments are conducted and followed. Appropriate technologies are to be used and procedures followed when disposing of waste. The selection and criteria for collecting valid and reliable data is to be methodical and, where appropriate, secondary-sourced information referenced correctly.

PROCESSING DATA AND INFORMATION

Students use the most appropriate and meaningful methods and media to organise and analyse data and information sources, including digital technologies and the use of a variety of visual representations as appropriate. They process data from primary and secondary sources, including both qualitative and quantitative data and information.

ANALYSING DATA AND INFORMATION

Students identify trends, patterns and relationships; recognise error, uncertainty and limitations in data; and interpret scientific and media texts. They evaluate the relevance, accuracy, validity and reliability of the primary or secondary-sourced data in relation to investigations. They evaluate processes, claims and conclusions by considering the quality of available evidence, and use reasoning to construct scientific arguments. Where appropriate, mathematical models are to be applied, to demonstrate the trends and relationships that occur in data.

PROBLEM SOLVING

Students use critical thinking skills and creativity to demonstrate an understanding of scientific principles underlying the solutions to inquiry questions and problems posed in investigations. Appropriate and varied strategies are employed, including the use of models, to quantitatively and qualitatively explain and predict cause-and-effect relationships. In Working Scientifically, students synthesise and use evidence to construct and justify conclusions. To solve problems, students: interpret scientific and media texts; evaluate processes, claims and conclusions; and consider the quality of available evidence.

COMMUNICATING

Communicating all components of the Working Scientifically processes with clarity and accuracy is essential. Students use qualitative and quantitative information gained from investigations using primary and secondary sources, including digital, visual, written and/or verbal forms of communication as appropriate. They apply appropriate scientific notations and nomenclature. They also appropriately apply and use scientific language that is suitable for specific audiences and contexts.

1.9 Depth studies

1.9.1 What are depth studies?

A depth study is a type of investigation or an activity that can be completed individually or collaboratively. At least one depth study must be undertaken in both Year 11 and Year 12. Each depth study will allow the further development of one or more physics concepts. The depth study might be one investigation or activity or a series of them.

Depth studies will allow you to pursue your interests in physics while gaining a depth of understanding. You will need to take responsibility for your own learning. Depth studies allow for the demonstration of a range of Working Scientifically skills.

A depth study may be:

- a practical investigation or series of practical investigations
- a secondary-sourced investigation or series of secondary-sourced investigations

- a presentation
- a working model, an invention or a portfolio you have created
- a research assignment
- a fieldwork report
- data analysis
- the extension of concepts found within the course, either qualitatively and/or quantitatively.

1.10 Summary

- Resolution — the fineness of the scale of an instrument is the instrument's resolution.
- Precision — results are precise if repeated measurements are in close agreement.
- Accuracy — accurate results are in good agreement with the true values.
- Validity — an experiment is valid if it enables measurement of and/or calculation of the quantity it was designed to determine.
- Reliability — an experiment is reliable if it yields consistent results.
- Uncertainty in measurement — all measurements have some degree of uncertainty and, when appropriate, this should be taken into account in calculations:
  - when adding or subtracting numbers with errors, add the errors
  - when multiplying or dividing numbers with errors, add the percentage (or relative) errors.
- Graphing — choose a suitable scale, making optimum use of the graph paper, and:
  - include a title, and label axes with quantities and their units
  - draw a line of best fit (either a straight line with a ruler or a curved line, freehand)
  - if calculating a gradient, ensure it is the gradient of the line of best fit (do not use data points)
  - by comparing the standard equation $y = mx + b$ with the equation of the quantities graphed, it will be possible to relate the gradient to a physical constant.
- Depth study — this is a type of investigation or an activity that can be completed individually or collaboratively.

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TOPIC 2
Motion in a straight line

2.1 Overview

2.1.1 Module 1: Kinematics

Motion in a straight line

Inquiry question: How is the motion of an object moving in a straight line described and predicted?
Students:

- describe uniform straight-line (rectilinear) motion and uniformly accelerated motion through:
  - qualitative descriptions
  - the use of scalar and vector quantities (ACSPH060).
- conduct a practical investigation to gather data to facilitate the analysis of instantaneous and average velocity through:
  - quantitative, first-hand measurements
  - the graphical representation and interpretation of data (ACSPH061).
- calculate the relative velocity of two objects moving along the same line, using vector analysis
- conduct practical investigations, selecting from a range of technologies, to record and analyse the motion of objects in a variety of situations in one dimension in order to measure or calculate:
  - time
  - distance
  - displacement
  - speed
  - velocity
  - acceleration.
- use mathematical modelling and graphs, selected from a range of technologies, to analyse and derive relationships between time, distance, displacement, speed, velocity and acceleration in rectilinear motion, including:
  - $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
  - $\vec{v} = \vec{u} + \vec{a}t$
  - $\vec{v}^{\,2} = \vec{u}^{\,2} + 2\vec{a}\vec{s}$ (ACSPH061)

FIGURE 2.1 Whether you are riding a bike, driving a car or even skydiving, you will need to be able to describe your movement in terms of your position, speed, direction and acceleration.

2.2 Distance and displacement

2.2.1 Use of scalar and vector quantities

AS A MATTER OF FACT

Vector quantities can be described in writing or by labelled arrows. If a symbol is used to represent a vector quantity, it may have a half-arrow above it or a 'squiggly' line below it. In this text, vector quantities are represented by symbols in bold italic type.

Most people today rely on some form of transport to get to school or work and to get around on weekends or during holidays. Whether you ride, drive, fly or sail, you need to know how far you are going, in which direction and when you intend to arrive. Whether or not you arrive on time depends on how fast you move and the direction you take. Describing motion is important in planning a journey, even if it is by foot. The study of motion is called kinematics.

While the terms **distance** and **displacement** tend to be used interchangeably in the English language, they actually have very different meanings in physics.

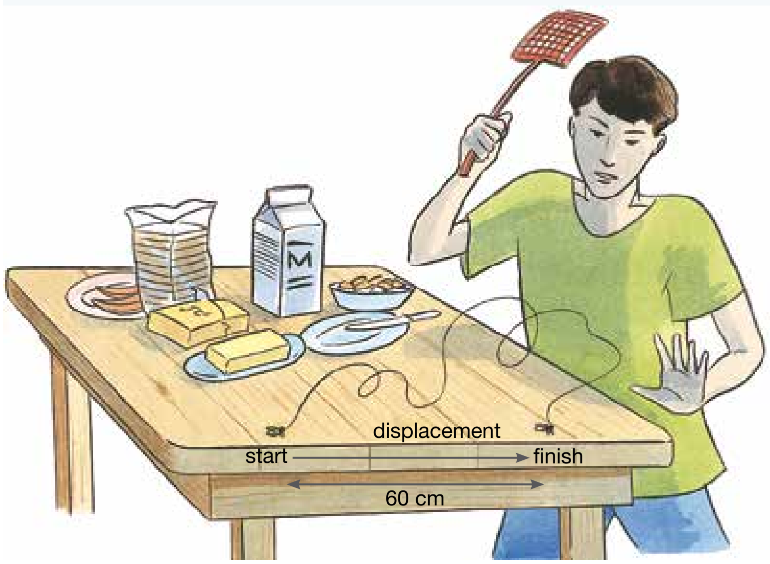
Distance describes the total length of the pathway taken between the starting point and the finishing point of a journey and is a **scalar** quantity. Scalar quantities are those that have a magnitude but do not have a direction associated with them.

Displacement, on the other hand, indicates the location of the destination relative to the journey's starting location, irrespective of the path actually taken between the two points. Displacement is a **vector** quantity, which means that it has both magnitude and direction associated with it. Displacement is usually represented by the symbol $\boldsymbol{s}$ in equations of motion, but other symbols such as $\boldsymbol{x}$ or $\boldsymbol{r}$ may also be encountered.

The path taken by the fly in figure 2.2 as it escapes the lethal swatter illustrates the difference between distance and displacement. The displacement of the fly is 60 cm to the right, while the distance travelled is well over 1 m.

In a 100 m sprint, the magnitude of the displacement is the same as the distance. However, it is the displacement that fully describes the change in position of the runner because it specifies the direction.

FIGURE 2.2 Distance and displacement are different quantities.



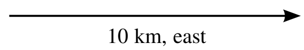
WORKING SCIENTIFICALLY 2.1

Use a mapping tool such as Google Maps to find the directions from one place to another, such as from your home to your school. Varying the route on the map will vary the distance. Use the ruler tool to measure the displacement.

2.2.2 Representing vector quantities

A vector quantity is represented graphically in the form of an arrowed line segment; its length corresponds to the magnitude of the quantity it represents, and it points in the direction in which the quantity is acting.

For example, the displacement of a car travelling 10 km in an easterly direction could be represented as shown:



When the motion of an object involves changes in direction, the object's final displacement is the vector sum of the individual vectors representing each leg of the object's journey.

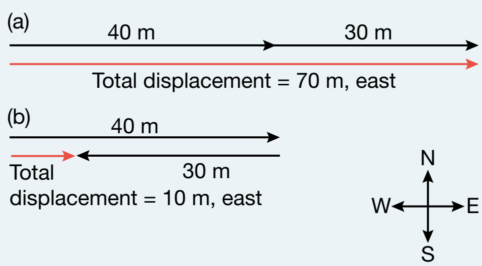
When vector quantities such as displacement are added, the labelled arrows that represent the vectors are placed 'head to tail'. The sum of the vectors is represented by the arrow drawn from the tail of the first vector and the head of the second vector. The order in which the vectors are added does not matter.

2.2 SAMPLE PROBLEM 1

Add the following pairs of vectors to find the total displacement:
(a) displacement of 40 m east, displacement of 30 m east
(b) displacement of 40 m east, displacement of 30 m west.

SOLUTION:

FIGURE 2.3



2.2 Exercise 1

1 A hare and a tortoise decide to have a race along a straight 100 m stretch of highway. They both head due north. However, at the 80 m mark, the hare notices his girlfriend back at the 20 m mark. He heads back, gives her a quick kiss on the cheek, and resumes the race, arriving at the finishing line at the same time as the tortoise. (It was a very fast tortoise!)
(a) What was the displacement of the hare during the entire race?
(b) What was the distance travelled by the hare during the race?
(c) What was the distance travelled by the tortoise during the race?
(d) What was the displacement of the hare during his return to his girlfriend?

2 A jogger heads due north from his home and runs 400 m along a straight footpath before realising that he has forgotten his sunscreen and runs straight back to get it.
(a) What distance has the jogger travelled by the time he gets back home?
(b) What was the displacement of the jogger when he started to run back home?
(c) What was his displacement when he arrived back home to pick up the sunscreen?

3 If you were to walk 400 m to the east, in what direction and how far would you need to walk to have an overall displacement of zero?

2.3 Speed and velocity

2.3.1 Measuring movement rate

Just as distance and displacement can be confusing, so too can the terms **speed** and **velocity**.

The average speed of an object over a journey is determined by dividing the total distance the object travelled by the time taken to complete the journey:

$$\text{Average speed} = \frac{\text{distance}}{\text{time}}$$

Like distance, average speed is a scalar quantity.

Average velocity (v_{av}), on the other hand, is a vector quantity and calculated by dividing an object's total displacement by its journey time:

$$\boldsymbol{v}_{av} = \frac{\boldsymbol{s}}{t}$$

The average velocity vector acts in the same direction as the displacement vector.

2.3 SAMPLE PROBLEM 1

If a champion swimmer completes 30 laps of a 50 m swimming pool, a distance of 1500 m, in a time of 15 minutes, what is:

(a) their average speed in m s^{-1}

(b) their average velocity in m s^{-1}?

SOLUTION:

(a) $$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{1500}{15 \times 60} = 1.7\ \text{m s}^{-1}$$

(b) $$\boldsymbol{v}_{av} = \frac{\boldsymbol{s}}{t} = \frac{0}{15 \times 60} = 0\ \text{m s}^{-1}$$

PHYSICS FACT

A snail would lose a race with a giant tortoise! A giant tortoise can reach a top speed of 0.37 km h^{-1}. However, its 'cruising' speed is about 0.27 km h^{-1}. The world's fastest snails cover ground at the breathtaking speed of about 0.05 km h^{-1}. However, the common garden snail is more likely to move at a speed of about 0.02 km h^{-1}. Both of these creatures are slow compared with light, which travels through the air at 1080 million km h^{-1}, and sound, which travels through the air (at sea level) at about 1200 km h^{-1}.

How long would it take the snail, giant tortoise, light and sound respectively to travel once around the equator, a distance of 40 074 km?

FIGURE 2.4 A garden snail moves at about 0.02 km h^{-1} but this is difficult to imagine. Converting this to 20 metres per hour gives you an idea of how fast they can travel.

2.3.2 Converting units of speed and velocity

While the SI units for both speed and velocity are m s^{-1}, it is common to encounter values for them given in km h^{-1}.

To convert 60 km h^{-1} to m s^{-1}, the following procedure can be followed.

$$60\text{ km h}^{-1} = \frac{60\text{ km}}{1\text{ h}}$$

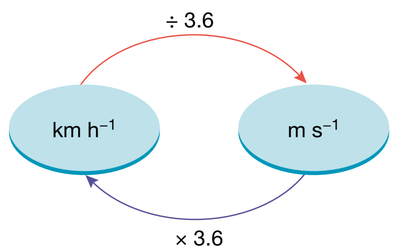
$$60\text{ km h}^{-1} = \frac{60\,000\text{ m}}{3600\text{ s}}$$

$$60\text{ km h}^{-1} = 16.7\text{ m s}^{-1}$$

In effect, the speed in km h^{-1} has been multiplied by $\frac{1000}{3600}$, or divided by 3.6.

To convert 30 m s^{-1} to km h^{-1}, a similar procedure can be followed.

$$\begin{aligned} 30\text{ m s}^{-1} &= \frac{30\text{ m}}{1\text{ s}} \\ &= \frac{0.030\text{ km}}{\frac{1}{3600}\text{ h}} \\ &= \frac{3600 \times 0.030\text{ km}}{1\text{ h}} \\ &= 108\text{ km h}^{-1} \end{aligned}$$



In effect, the speed in m s^{-1} has been multiplied by $\frac{3600}{1000}$, that is, by 3.6.

2.3 SAMPLE PROBLEM 2

A plane carrying passengers from Melbourne to Perth flies at an average speed of 250 m s^{-1}. The flight takes 3.0 hours. Use this information to determine the approximate distance by air between Melbourne and Perth.

SOLUTION:

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$\begin{aligned} \Rightarrow \text{distance travelled} &= \text{average speed} \times \text{time interval} && \text{(rearranging)} \\ &= 250\text{ m s}^{-1} \times 3.0\text{ h} \\ &= 900\text{ km h}^{-1} \times 3.0\text{ h} && (\times\ 3.6 \text{ to convert m s}^{-1} \text{ to km h}^{-1}) \\ &= 2700\text{ km} \end{aligned}$$

Alternatively, the distance could be calculated in metres and then converted to kilometres, a more appropriate unit in this case.

$$\begin{aligned} \text{distance travelled} &= \text{average speed} \times \text{time interval} && \text{(rearranging)} \\ &= 250\text{ m s}^{-1} \times 3.0\text{ h} \\ &= 250\text{ m s}^{-1} \times 10\,800\text{ s} && (\times\ 3600 \text{ to convert h to s}) \\ &= 2\,700\,000\text{ m} \\ &= 2700\text{ km} && \text{(converting m to km)} \end{aligned}$$

2.3 Exercise 1

1 A car takes 8.0 hours to travel from Canberra to Ballarat at an average speed of 25 m s^{-1}. What is the road distance from Canberra to Ballarat?

2 A jogger takes 30 minutes to cover a distance of 5.0 km. What is the jogger's average speed in:
(i) km h^{-1}
(ii) m s^{-1}?

3 How long does it take for a car travelling at 60 km h^{-1} to cover a distance of 200 m?

2.3.3 Instantaneous speed and velocity

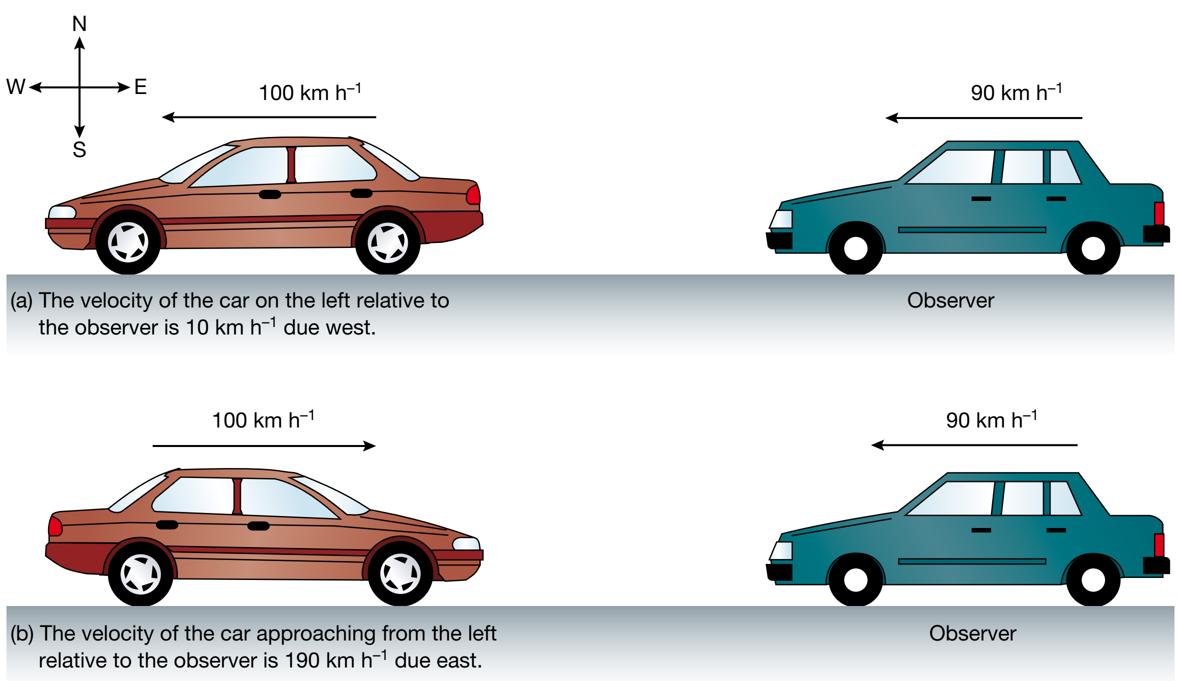
Neither the average speed nor the average velocity provides information about movement at any particular instant in time. For example, when Jamaican athlete Usain Bolt broke the 100 m world record in 2009 with a time of 9.58 s, his average speed was 10.4 m s^{-1}. However, he was not travelling at that speed throughout his run. He would have taken a short time to reach his maximum speed and would not have been able to maintain it throughout the run. His maximum speed would have been much more than 10.4 m s^{-1} .

The speed at any particular instant in time is called the **instantaneous speed**. The velocity at any particular instant in time is, not surprisingly, called the **instantaneous velocity**. If an object moves with a constant velocity during a time interval, its instantaneous velocity throughout the interval is the same as its average velocity.

2.3.4 Relative velocity

The velocity of an object measured by a moving observer is referred to as the relative velocity. The **relative velocity** is the difference between the velocity of the object relative to the ground and the velocity of the observer relative to the ground. Imagine that you are in a car travelling at a constant velocity of 90 km h^{-1} due west on a straight road. The car ahead of you is travelling at a constant speed of 100 km h^{-1} in the same direction. Although the velocity of the other car relative to the road is

FIGURE 2.5 The velocity that is measured depends on the velocity of the observer.



100 km h^{-1} due west, its velocity relative to you is 10 km h^{-1} due west. That is, the velocity of the car relative to you is equal to 100 km h^{-1} due west (velocity of car relative to the ground) minus 90 km h^{-1} due west (your velocity relative to the ground): 10 km h^{-1} due west. This is illustrated in figure 2.5a.

If another vehicle were approaching you at a speed of 100 km h^{-1} relative to the road, that is, with a velocity of 100 km h^{-1} due east relative to the road, its velocity relative to you would be the difference between 100 km h^{-1} due east and 90 km h^{-1} due west. A velocity of 90 km h^{-1} due west is the same as −90 km h^{-1} due east. The relative velocity is therefore 100 km h^{-1} due east (velocity of car relative to the ground) minus −90 km h^{-1} due east (your velocity relative to the ground): 190 km h^{-1} due east. This is illustrated in figure 2.5b.

2.3 SAMPLE PROBLEM 3

A cyclist is riding along a straight road at a constant velocity of 36 km h^{-1} (10 m s^{-1}) in an easterly direction. A car approaches the cyclist from behind and is initially 360 m behind the cyclist. If the car is travelling at a speed of 100 km h^{-1} (28 m s^{-1}), how long will it take to catch up to the cyclist?

SOLUTION:

The velocity of the car relative to the cyclist is the difference between the velocity of the car relative to the ground and the velocity of the cyclist relative to the ground. That is, 28 m s^{-1} due east minus 10 m s^{-1} due east equals 18 m s^{-1} due east. The time taken can be calculated using the formula:

$$
\begin{aligned}
v_{av} &= \frac{s}{t} \\
18\ \mathrm{m\,s^{-1}}\ \text{due east} &= \frac{360\ \text{m due east}}{t} \\
t &= \frac{360}{18} \\
&= 20\ \text{s.}
\end{aligned}
$$

PHYSICS FACT

Do you always feel like you're on the move? No wonder! When you are standing still, you are actually moving through space at a speed of about 30 km s^{-1}. That's about 110 000 km h^{-1}! This is the speed at which the Earth is hurtling through space in orbit around the Sun. An observer on the Sun could measure that speed. If you were standing still in Sydney, a person high above the South Pole would say that you were rotating with the ground around the Earth's axis at a speed of over 1300 km h^{-1}.

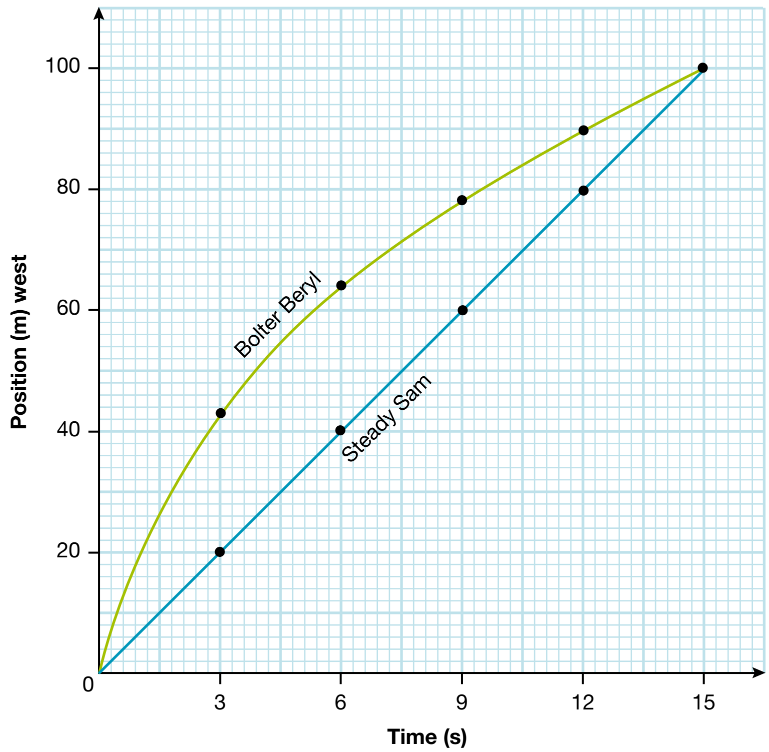
The speed you measure depends on your position, how fast you are moving and your direction of movement.

2.4 Graphing straight-line motion

2.4.1 Position versus time

Bolter Beryl and Steady Sam decide to race each other on foot over 100 m. They run due west. Timekeepers are instructed to record the position of each runner after each 3.0-second interval.

FIGURE 2.6 The graph of position versus the time the race was run provides valuable information about the way.



The points indicating Bolter Beryl's position after each 3.0 s interval are joined with a smooth curve. It is reasonable to assume that her velocity changes gradually throughout the race.

A number of observations can be made from the graph of position versus time.

TABLE 2.1 The progress of Bolter Beryl and Steady Sam.

	Position (distance from starting line) in metres	
Time (seconds)	**Bolter Beryl**	**Steady Sam**
0.0	0	0
3.0	43	20
6.0	64	40
9.0	78	60
12.0	90	80
15.0	100	100

- Both runners reach the finish at the same time. The result is a dead heat. Bolter Beryl and Steady Sam each have the same average speed and the same average velocity.
- Steady Sam, who has an exceptional talent for steady movement, maintains a constant velocity throughout the race. In fact, his instantaneous velocity at every instant throughout the race is the same as his average velocity. Steady Sam's average velocity and instantaneous velocity are both equal to the gradient of the position-versus-time graph since:

$$v_{av} = \frac{s}{t}$$

$$v_{av} = \frac{100\text{ m west}}{15\text{ s}}$$

$$v_{av} = \frac{\text{rise}}{\text{run}}$$

$$v_{av} = \text{gradient.}$$

Steady Sam's velocity throughout the race is 6.7 m s^{-1} west.

- Bolter Beryl, in her usual style, makes a flying start; however, after her initial 'burst', her instantaneous velocity decreases throughout the race as she tires. Her average velocity is also 6.7 m s^{-1} west.

  A more detailed description of Bolter Beryl's motion can be given by calculating her average velocity during each 3 s interval of the race (see table 2.2).

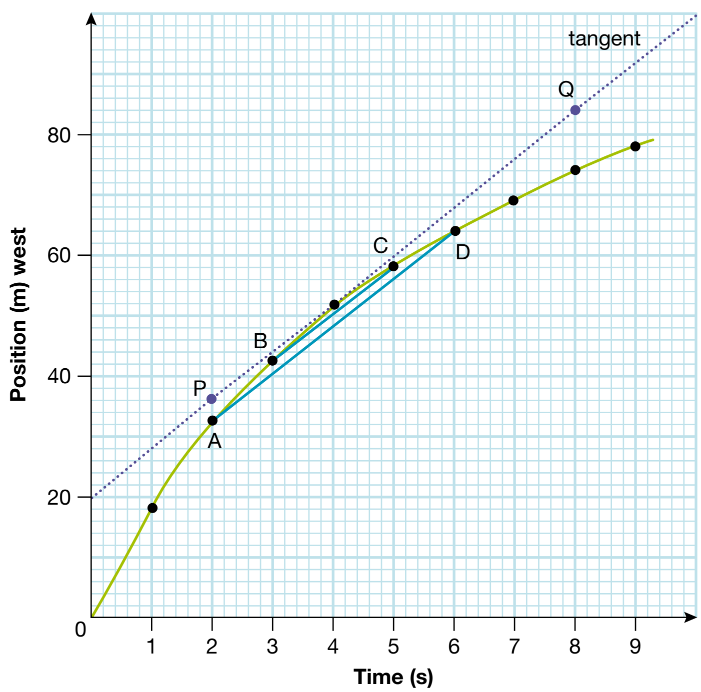
TABLE 2.2 Bolter Beryl's changing velocity.

Time interval (s)	Displacement Δs (m west)	Average velocity during interval $v_{av} = \frac{\Delta s}{\Delta t}$ (m s^{-1} west)
0.0–3.0	43 – 0 = 43	14.0
3.0–6.0	64 – 43 = 21	7.0
6.0–9.0	78 – 64 = 14	4.7
9.0–12.0	90 – 78 = 12	4.0
12.0–15.0	100 – 90 = 10	3.3

The average velocity during each interval is the same as the gradient of the straight line joining the data points representing the beginning and end of the interval. An even more detailed description of Bolter Beryl's run could be obtained if the race was divided into, say, 100 time intervals. The average velocity during each time interval (and the gradient of the line joining the data points defining it) would be a very good estimate of the instantaneous velocity in the middle of the interval. In fact, if the race is progressively divided into smaller and smaller time intervals, the average velocity during each interval would become closer and closer to the instantaneous velocity in the middle of the interval.

The graph below shows how this process of using smaller time intervals can be used to find Bolter Beryl's instantaneous velocity at exactly 4.0 seconds from the start of the race. Her instantaneous velocity is not the same as the average velocity during the 3.0 to 6.0 s time interval shown in table 2.2. However, it can be estimated by drawing the line AD and finding its gradient. The gradient of the line

FIGURE 2.7 The first 9.0 seconds of Bolter Beryl's run.



BC would provide an even better estimate of the instantaneous velocity. If you continue this process of decreasing the time interval used to estimate the instantaneous velocity, you will eventually obtain a line that is a tangent to the curve. The gradient of the tangent to the curve is equal to the instantaneous velocity at the instant represented by the point at which it meets the curve.

The gradient of the tangent to the curve at 4.0 seconds in figure 2.7 can be determined by using the points P and Q.

$$\begin{aligned}\text{gradient} &= \frac{\text{rise}}{\text{run}}\\ &= \frac{(84-36)\text{ m}}{(8.0-2.0)\text{ s}}\\ &= \frac{48\text{ m}}{6.0\text{ s}}\\ &= 8.0\text{ m s}^{-1}\end{aligned}$$

Bolter Beryl's instantaneous velocity at 4.0 seconds from the start of the race is therefore 8.0 m s^{-1} west.

Just as the gradient of a position-versus-time graph can be used to determine the velocity of an object, a graph of distance versus time can be used to determine its speed. Because Bolter Beryl and Steady Sam were running in a straight line and in one direction only, their distance from the starting point is the magnitude of their change in position. Their speed is equal to the magnitude of their velocity.

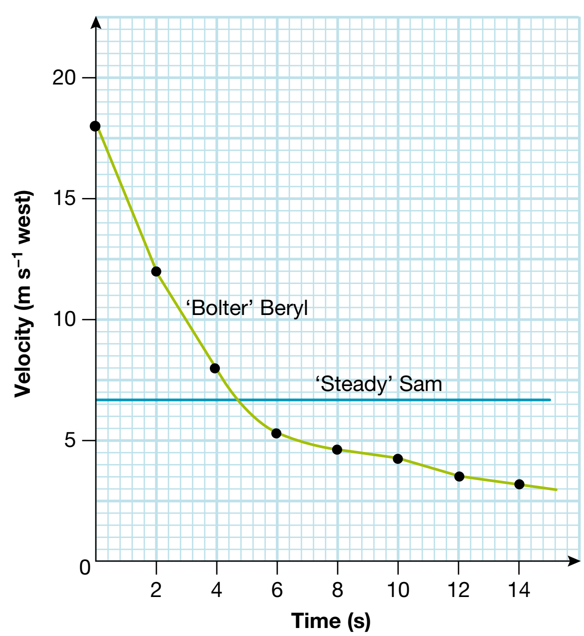
2.4.2 Graphing motion: velocity versus time

The race between Bolter Beryl and Steady Sam described by the position-versus-time graph on page 24 can also be described by a graph of velocity versus time. Steady Sam's velocity is 6.7 m s^{-1} due west throughout the race. The curve describing Bolter Beryl's motion can be plotted by determining the instantaneous velocity at various times during the race. This can be done by drawing tangents at a number of points on the position-versus-time graph on page 24. Table 2.3 shows the data obtained using this method. The velocity-versus-time graph below describes the motion of Bolter Beryl and Steady Sam.

TABLE 2.3 Beryl's velocity during the race.

Time (s)	Velocity (m s^{-1} west)
0.0	18.0
2.0	12.0
4.0	8.0
6.0	5.4
8.0	4.7
10.0	4.2
12.0	3.5
14.0	3.1

FIGURE 2.8 Graph of velocity versus time for the race.



The velocity-versus-time graph confirms what you already knew by looking at the position-versus-time graph, namely that:

- Steady Sam's velocity is constant, and equal to his average velocity
- the magnitude of Bolter Beryl's velocity is decreasing throughout the race.

The velocity-versus-time graph allows you to estimate the velocity of each runner at any time. It provides a much clearer picture of the way that Bolter Beryl's velocity changes during the race, namely that:

- the magnitude of her velocity decreases rapidly at first, but less rapidly towards the end of the race
- for most of the race, she is running more slowly than Sam. In fact, Bolter Beryl's speed (the magnitude of her velocity) drops below that of Steady Sam's after only 4.7 seconds.

2.4.3 Displacement from a velocity-versus-time graph

In the absence of a position-versus-time graph, a velocity-versus-time graph provides useful information about the change in position, or displacement, of an object. Steady Sam's constant velocity, the same as his average velocity, makes it very easy to determine his displacement during the race.

$$\boldsymbol{s} = \boldsymbol{v}_{av}\, t \left(\text{since } \boldsymbol{v}_{av} = \frac{\boldsymbol{s}}{t}\right)$$

$$\boldsymbol{s} = 6.7 \text{ m s}^{-1} \text{ west} \times 15 \text{ s}$$

$$\boldsymbol{s} = 100 \text{ m west}$$

This displacement is equal to the area of the rectangle under the graph depicting Steady Sam's motion.

$$\begin{aligned} \text{area} &= \text{length} \times \text{width} \\ &= 15 \text{ s} \times 6.7 \text{ m s}^{-1} \text{ west} \\ &= 100 \text{ m west} \end{aligned}$$

Because the race was a dead heat, Bolter Beryl's average velocity was also 6.7 m s^{-1}. Her displacement during the race can be calculated in the same way as Steady Sam's.

$$\begin{aligned} \boldsymbol{s} &= \boldsymbol{v}_{av}\, \boldsymbol{t} \\ &= 6.7 \text{ m s}^{-1} \times 15 \text{ s} \\ &= 100 \text{ m west} \end{aligned}$$

However, Bolter Beryl's displacement can also be found by calculating the area under the velocity-versus-time graph depicting her motion. This can be done by 'counting squares' or by dividing the area under the graph into rectangles and triangles as shown in the 'As a matter of fact' panel below. The area under Beryl's velocity-versus-time graph is, not surprisingly, 100 m.

In fact, the area under any part of the velocity-versus-time graph is equal to the displacement during the interval represented by that part.

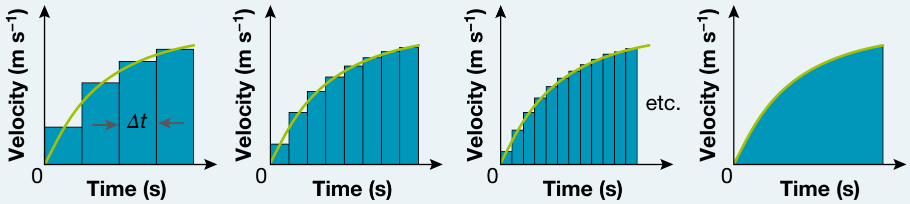
AS A MATTER OF FACT

When an object travels with a constant velocity, it is obvious that the displacement of the object is equal to the area under a velocity-versus-time graph of its motion. However, it is not so obvious when the motion is not constant. The graphs below describe the motion of an object that has an increasing velocity. The motion of the object can be approximated by dividing it into time intervals of Δt and assuming that the velocity during each time interval is constant. The approximate displacement during each time interval is equal to:

$$\boldsymbol{s} = \boldsymbol{v}_{av}\, t$$

which is the same as the area under each rectangle. The approximate total displacement is therefore equal to the total area of the rectangles.

FIGURE 2.9 By dividing the velocity-versus-time graph into rectangles representing small time intervals, the displacement can be estimated. You will discover in your study of mathematics that the integral of the function will give the area under the graph — this highlights the link between mathematics and physics.



To better approximate the displacement, the graph can be divided into smaller time intervals. The total area of the rectangles is approximately equal to the displacement. By dividing the graph into even smaller time intervals, even better estimates of the displacement can be made. In fact, by continuing the process of dividing the graph into smaller and smaller time intervals, it can be seen that the displacement is, in fact, equal to the area under the graph.

You can use Excel to analyse and graph data; to find out how to create a chart, search online for Microsoft Support, then find the Charts and shapes section.

2.4 SAMPLE PROBLEM 1

In the race between Bolter Beryl and Steady Sam, how far ahead of Steady Sam was Bolter Beryl when her speed dropped below Sam's speed?

SOLUTION:

Although it is possible to answer this question using the position-versus-time graph on page 24 (you might like to explain how you would do this!), it is easier to use the velocity-versus-time graph (see the graph on page 26). It shows that Beryl's speed (and the magnitude of her velocity) drops below Steady Sam's 4.7 s after the race starts.

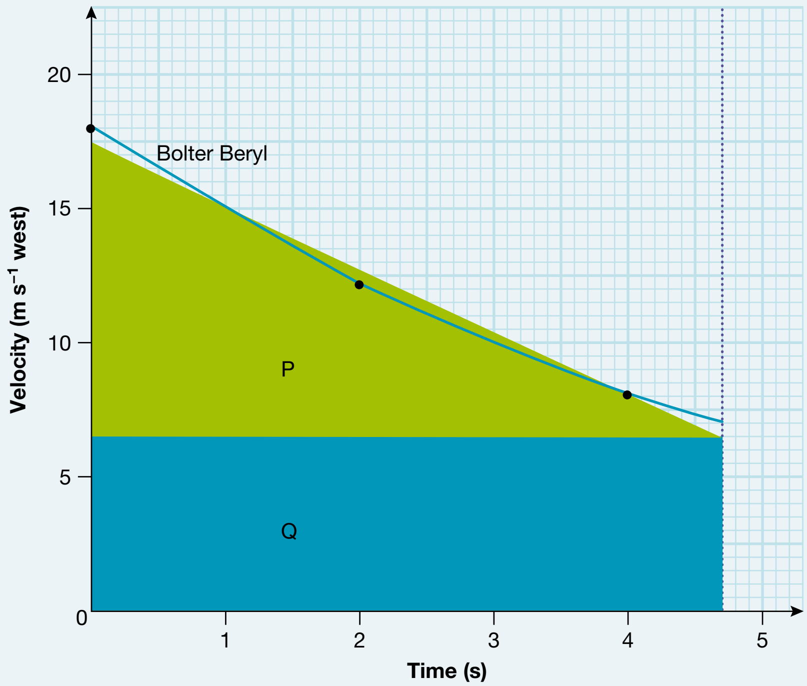
Steady Sam's displacement, after 4.7 s, is equal to the area under the line representing the first 4.3 s of his motion, that is, 4.7 s × 6.7 m s^{-1} west. Steady Sam is therefore 31 m west of the starting line after 4.7 s.

Bolter Beryl's displacement after 4.7 s equals the area under the curve representing the first 4.7 s of her motion. This area can be estimated by determining the shaded area of the triangle P and rectangle Q in figure 2.10.

$$\begin{aligned}\text{area} &= \text{area P} + \text{area Q}\\ &= \frac{1}{2} \times 4.7\ \text{s} \times 11\ \text{m s}^{-1}\ \text{west} + 4.7\ \text{s} \times 6.5\ \text{m s}^{-1}\ \text{west}\\ &= 25.85\ \text{m west} + 30.55\ \text{m west}\\ &= 56.40\ \text{m west}\end{aligned}$$

Bolter Beryl is therefore 56 m west of the starting line after 4.7 seconds. She is 25 m ahead of Steady Sam when her speed drops below his.

FIGURE 2.10 Graph of velocity versus time.



2.4 Exercise 1

1. Use the graph in figure 2.10 to estimate Bolter Beryl's displacement after 2.0 s.
2. Use the graph in figure 2.8 to determine how far ahead Bolter Beryl was 10 seconds into the race.

eBook*plus* RESOURCES

eModelling: Numerical model of motion 1: Finding speed from position–time data
Searchlight ID: doc-0048

eModelling: Numerical model of motion 2: Finding position from speed–time data
Searchlight ID: doc-0049

Complete this digital doc: Investigation 2.4: Let's play around with some graphs
Searchlight ID: doc-16181

2.5 Acceleration

2.5.1 Subtracting vectors

The rate at which an object changes its velocity is called its **acceleration**. Because velocity is a vector quantity, it follows that acceleration is also a vector quantity. The direction of the acceleration of an object is the same as the direction of its change in velocity.

The average acceleration of an object, $\boldsymbol{a}_{av}$, can be expressed as:

$$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

where

$\Delta \boldsymbol{v}$ = the change in velocity during the time interval Δt.

The change in velocity is found by subtracting the initial velocity, u, from the final velocity, v. Thus:

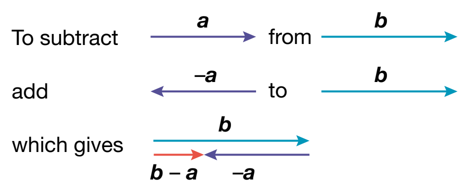
$$\boldsymbol{a}_{av} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$$

where

t is the time during which the change in velocity occurs.

In order to determine a change in velocity it is necessary to subtract the vector $\boldsymbol{u}$ from the vector $\boldsymbol{v}$. One vector can be subtracted from another by simply adding its negative. This works because subtracting a vector is the same as adding the negative vector (just as subtracting a positive number is the same as adding the negative of that number). The method of adding vectors is shown on page 48. One example of vector subtraction is shown in figure 2.11.

FIGURE 2.11 Subtracting vectors.



2.5 SAMPLE PROBLEM 1

A car starts from rest and reaches a velocity of 20 m s^{-1} due east in 5.0 s. What is its average acceleration?

SOLUTION:

$$\boldsymbol{a}_{av} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$$

$$\begin{aligned}\boldsymbol{v} - \boldsymbol{u} &= 20\,\text{m s}^{-1}\text{ due east} - 0\\ &= 20\,\text{m s}^{-1}\text{ due east}\end{aligned}$$

$$\begin{aligned}\boldsymbol{a}_{av} &= \frac{20\,\text{m s}^{-1}\text{ due east}}{5.0\,\text{s}}\\ &= 4.0\,\text{m s}^{-2}\text{ due east}\end{aligned}$$

2.5 SAMPLE PROBLEM 2

What is the average acceleration of a cyclist riding north who slows down from a speed of 8.0 m s^{-1} to a speed of 5.0 m s^{-1} in 2.0 s?

SOLUTION:

$$\boldsymbol{a}_{av} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$$

$$\begin{aligned}\boldsymbol{v} - \boldsymbol{u} &= 5.0\,\text{m s}^{-1}\text{ north} - 8.0\,\text{m s}^{-1}\text{ north}\\ &= -3.0\,\text{m s}^{-1}\text{ north}\end{aligned}$$

$$\begin{aligned}\boldsymbol{a}_{av} &= \frac{-3.0\,\text{m s}^{-1}\text{ north}}{2.0\,\text{s}}\\ &= -1.5\,\text{m s}^{-2}\text{ north}\end{aligned}$$

A negative acceleration is called a deceleration. This acceleration could also be expressed as 1.5 m s^{-2} south.

The unit of m s^{-2} used for acceleration is derived from the unit for the velocity (m s^{-1}), which is divided by the time taken (s^{-1}), which gives the unit m s^{-1} s^{-1}. This is simplified as m s^{-2}.

2.5.2 Constant acceleration formulae

When acceleration is constant (including when it is zero), the motion of an object can be described by some simple formulae. The definition of average acceleration leads to the first of these formulae. When the acceleration is constant, its value is the same as the average acceleration:

$$\boldsymbol{a} = \boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

where

$\Delta \boldsymbol{v} =$ the change in velocity during the time interval Δt.

When $t = 0$ the velocity is $\boldsymbol{u}$.

Thus:

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t} \qquad [1]$$

Where

$\boldsymbol{v}$ is the velocity at time t.

$$\boldsymbol{v} - \boldsymbol{u} = \boldsymbol{a}t$$

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

Note that this equation is a vector equation. The direction of the change in velocity ($\boldsymbol{v} - \boldsymbol{u}$) is the same as the direction of the acceleration. As long as the motion is along a straight line, the vectors can be expressed as positive or negative quantities. Vector notation is not necessary.

Thus:

$$v = u + at. \qquad [2]$$

The second of the constant acceleration equations can be found by restating the definition of average velocity

$$v_{av} = \frac{s}{t}$$

where

s = displacement from the starting position at time t.

When the acceleration is constant, the average velocity can be expressed as

$$v_{av} = \frac{(u + v)}{2}$$

Thus:

$$\frac{(u + v)}{2} = \frac{s}{t}$$

$$s = \frac{1}{2}(u + v)\,t \qquad [3]$$

Once again, vector notation is not necessary as long as the motion is along a straight line.

A third formula can be obtained by combining formulae [2] and [3]. Substituting v from formula [2] into formula [3] gives

$$s = \frac{1}{2}(u + u + at)t$$

$$s = \frac{1}{2}(2u + at)t$$

$$s = ut + \frac{1}{2}at^2 \qquad [4]$$

A final formula can be found by eliminating t from formula [3].

$s = \frac{1}{2}(u + v)t$ (formula [3])

But $t = \frac{v - u}{a}$ (rearranging formula [2])

$$s = \frac{1}{2}(u + v)\left(\frac{v - u}{a}\right)$$

$$\Rightarrow \quad = \frac{1}{2}\frac{v^2 - u^2}{a} \qquad \text{(expanding the difference of two squares)}$$

$$\Rightarrow 2as = v^2 - u^2$$

$$\Rightarrow \quad v^2 = u^2 + 2as$$

When attacking a problem involving straight-line motion, it sometimes helps to keep these steps in mind:

1. Identify all known variables. You will need to know at least three variables to find a solution using the equations we have seen so far.
2. Identify the variable you need to find.
3. Find the equation that only has the unknown variable and your known variables in it.
4. Substitute the values into the equation and solve for the unknown.

One of the biggest pitfalls is mixing up v_{av} and v in these equations. Be very careful that you only use the equation for average speed for objects that are not obviously changing their motion — that is, accelerating. Another thing to be careful of is to get the v and u the correct way around.

2.5 SAMPLE PROBLEM 3

A physics student drops a coin into a wishing well and takes 3.0 s to make a wish (for a perfect score in the next physics test!). The coin splashes into the water just as she finishes making her wish. The coin accelerates towards the water at a constant 9.8 m s^{-2}.

(a) What is the coin's velocity as it strikes the water?

(b) How far does the coin fall before hitting the water?

SOLUTION:

(a) $u = 0$

$a = 9.8$ m s^{-2}

$t = 3.0$ s

The appropriate formula here is $v = u + at$.

$v = 0 + 9.8 \times 3.0$

$= 29.4$ m s^{-1}

The coin is travelling at a velocity of 29 m s^{-1} down as it strikes the water.

(b) The appropriate formula here is $s = ut + \frac{1}{2}at^2$ because it includes the three known quantities along with the unknown quantity s.

$s = 0 + \frac{1}{2} \times 9.8 \times (3.0)^2$

$= 44.1$ m

The coin falls a distance of 44 m.

2.5 SAMPLE PROBLEM 4

The driver of a car travelling along a suburban street was forced to brake suddenly to prevent serious injury to the neighbour's cat. The car skidded in a straight line for 2.0 s, stopping just a millimetre or two away from the cat. The deceleration was constant and the length of the skid mark was 12 m.

(a) At what speed was the car travelling as it began to skid?

(b) What was the acceleration of the car?

SOLUTION:

(a) s = 12 m, t = 2.0 s, v = 0 (assigning forward as positive)

The appropriate formula is:

$$s = \frac{1}{2}(u + v)t$$

$$12 = \frac{1}{2}(u + 0)2.0$$

$$u = 12 \text{ m s}^{-1}$$

The car was travelling at a speed of 12 m s^{-1}. That's about 43 km h^{-1}.

(b) The appropriate formula is:

$$v = u + at$$

$$0 = 12 + a \times 2.0$$

$$a = \frac{-12}{2.0}$$

$$= -6.0 \text{ m s}^{-2}$$

The acceleration of the car was –6.0 m s^{-2}.

2.5 Exercise 1

1 A parked car with the handbrake off rolls down a hill in a straight line with a constant acceleration of 2.0 m s^{-2}. It stops after colliding with a brick wall at a speed of 12 m s^{-1}.
(a) For how long was the car rolling?
(b) How far did the car roll before colliding with the wall?

2 A car travelling at 24 m s^{-1} brakes to come to a stop in 1.5 s. If its acceleration (deceleration in this case) was constant, what was the car's:
(a) stopping distance
(b) acceleration?

3 A cyclist, originally travelling at 20 m s^{-1}, decelerates over a distance of 240 m until he is travelling at 3 m s^{-1}. Calculate the cyclist's average acceleration.

4 A sprinter leaving the blocks in a 100 m race accelerates from rest to 4 m s^{-1} in the first metre that she travels.
(a) What is her acceleration over this distance?
(b) How long does this acceleration take?

2.5.3 Acceleration from a velocity-versus-time graph

The graph that follows describes the motion of an elevator as it moves from the ground floor to the top floor and back down again. The elevator stops briefly at the top floor to pick up a passenger. For convenience, any upward displacement from the ground floor is defined as positive. The graph has been divided into seven sections labelled A–G.

The acceleration at any instant during the motion can be determined by calculating the gradient of the graph. This is a consequence of the definition of acceleration. The gradient of a velocity-versus-time graph is a measure of the rate of change of velocity just as the gradient of a position-versus-time graph is a measure of the rate of change of position.

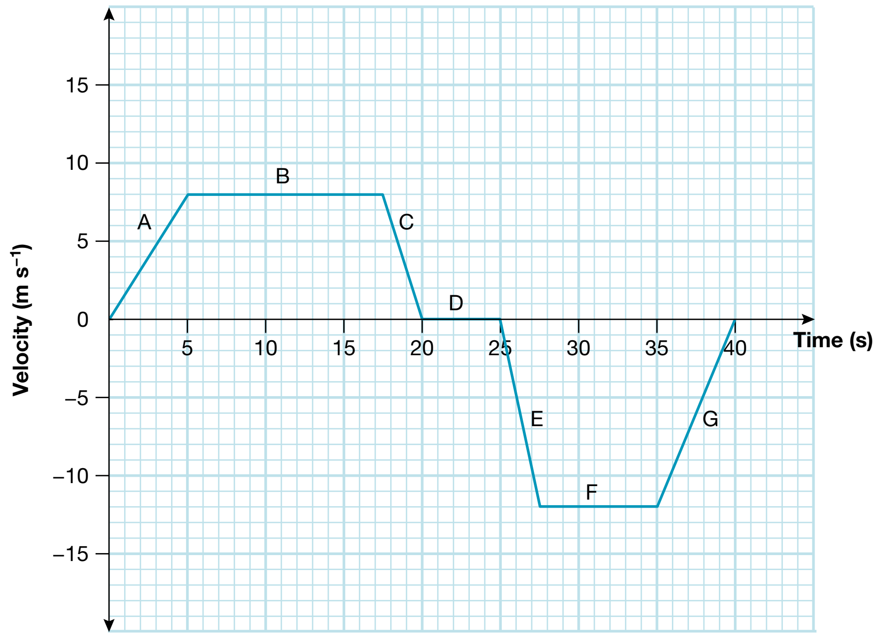
Throughout interval A (see the graph), the acceleration, a, of the elevator is:

$$a = \frac{\text{rise}}{\text{run}}$$

$$= \frac{+8.0 \text{ m s}^{-1}}{5.0 \text{ s}}$$

$$= +1.6 \text{ m s}^{-2} \text{ or } 1.6 \text{ m s}^{-2} \text{ up.}$$

FIGURE 2.12 The motion of an elevator.



During intervals B, D and F, the velocity is constant and the gradient of the graph is zero. The acceleration during each of these intervals is, therefore, zero.

Throughout interval C, the acceleration is:

$$\boldsymbol{a} = \frac{-8.0\,\text{m s}^{-1}}{2.5\,\text{s}}$$
$$= -3.2\,\text{m s}^{-2} \text{ or } 3.2\,\text{m s}^{-2} \text{ down.}$$

Throughout interval E, the acceleration is:

$$\boldsymbol{a} = \frac{-12\,\text{m s}^{-1}}{2.5\,\text{s}}$$
$$= -4.8\,\text{m s}^{-2} \text{ or } 4.8\,\text{m s}^{-2} \text{ down.}$$

Throughout interval G, the acceleration is:

$$\boldsymbol{a} = \frac{+12\,\text{m s}^{-1}}{5.0\,\text{s}}$$
$$= +2.4\,\text{m s}^{-2} \text{ or } 2.4\,\text{m s}^{-2} \text{ up.}$$

Notice that during interval G the acceleration is positive (up) while the velocity of the elevator is negative (down). The direction of the acceleration is the same as the direction of the *change* in velocity.

The area under the graph is equal to the displacement of the elevator. Dividing the area into triangles and rectangles and working from left to right yields an area of:

$$\left(\frac{1}{2} \times 5.0\,\text{s} \times 8.0\,\text{m s}^{-1}\right) + \left(12.5\,\text{s} \times 8.0\,\text{m s}^{-1}\right) + \left(\frac{1}{2} \times 2.5\,\text{s} \times 8.0\,\text{m s}^{-1}\right) +$$
$$\left(\frac{1}{2} \times 2.5\,\text{s} \times -12\,\text{m s}^{-1}\right) + \left(7.5\,\text{s} \times \; -\, 12\,\text{m s}^{-1}\right) + \left(\frac{1}{2} \times 5.0\,\text{s} \times -12\,\text{m s}^{-1}\right)$$
$$= 20\text{ m} + 100\text{ m} + 10\text{ m} - 15\text{m} - 90\text{ m} - 30\text{ m}$$
$$= -5.0\text{ m}$$

This represents a downward displacement of 5.0 m, which is consistent with the elevator finally stopping two floors below the ground floor.

2.5.4 Area under an acceleration-versus-time graph

Just as the area under a velocity-versus-time graph is equal to the change in position of an object, the area under an acceleration-versus-time graph is equal to the change in velocity of an object. The acceleration-versus-time graph of the motion of the elevator described previously is shown in the graph in figure 2.13. The area under the part of the graph representing the entire upward part of the journey is given by:

$$\begin{aligned} \text{area A} + \text{area C} &= 5.0 \text{ s} \times 1.6 \text{ m s}^{-2} + 2.5 \text{ s} \times -3.2 \text{ m s}^{-2} \\ &= +8.0 \text{ m s}^{-1} + -8.0 \text{ m s}^{-1} \\ &= 0 \end{aligned}$$

This indicates that change in velocity during the upward journey is zero. This is consistent with the fact that the elevator starts from rest and is at rest when it reaches the top floor. Similarly, the area under the whole graph is zero.

The change in velocity during intervals C, D and E is given by the sum of areas C, D and E. Thus:

$$\begin{aligned} \text{area C} + \text{area D} + \text{area E} &= 2.5 \text{ s} \times -3.2 \text{ m s}^{-2} + 0 + 2.5 \text{ s} \times -4.8 \text{ m s}^{-2} \\ &= -8.0 \text{ m s}^{-1} + -12 \text{ m s}^{-1} \\ &= -20 \text{ m s}^{-1} \end{aligned}$$

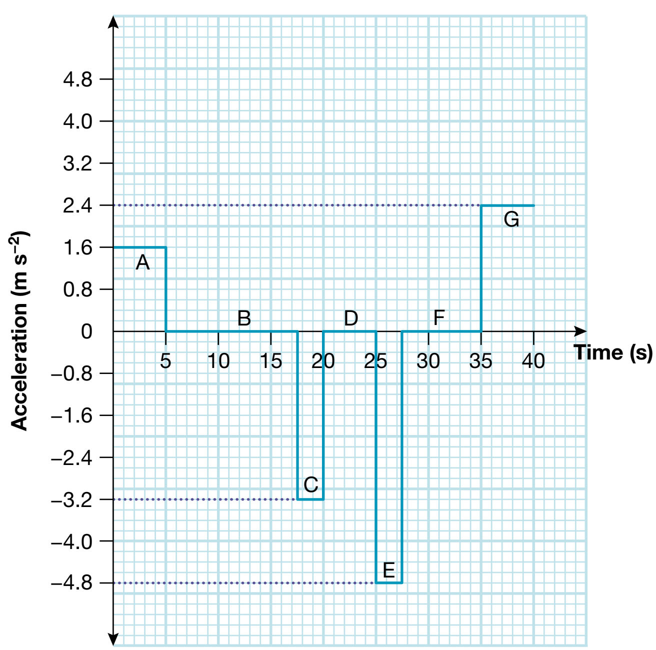
The change in velocity is –20 m s^{-1}, or 20 m s^{-1} down.

At the beginning of time interval C, the velocity was 8.0 m s^{-1} upwards. A change of velocity of –20 m s^{-1} would result in a final velocity of 12 m s^{-1} downwards. This is consistent with the description of the motion in the velocity-versus-time graph in figure 2.10. The symbol $\boldsymbol{u}$ is used to denote the initial velocity, while the symbol $\boldsymbol{v}$ is used to denote the final velocity:

In symbols, therefore:

$$\begin{aligned} \boldsymbol{v} &= \boldsymbol{u} + \Delta \boldsymbol{u} \quad (\text{since } \Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u}) \\ &= +8.0 \text{m s}^{-1} + -20 \text{m s}^{-1} \\ &= -12 \text{m s}^{-1}. \end{aligned}$$

FIGURE 2.13 An acceleration-versus-time graph for the elevator.



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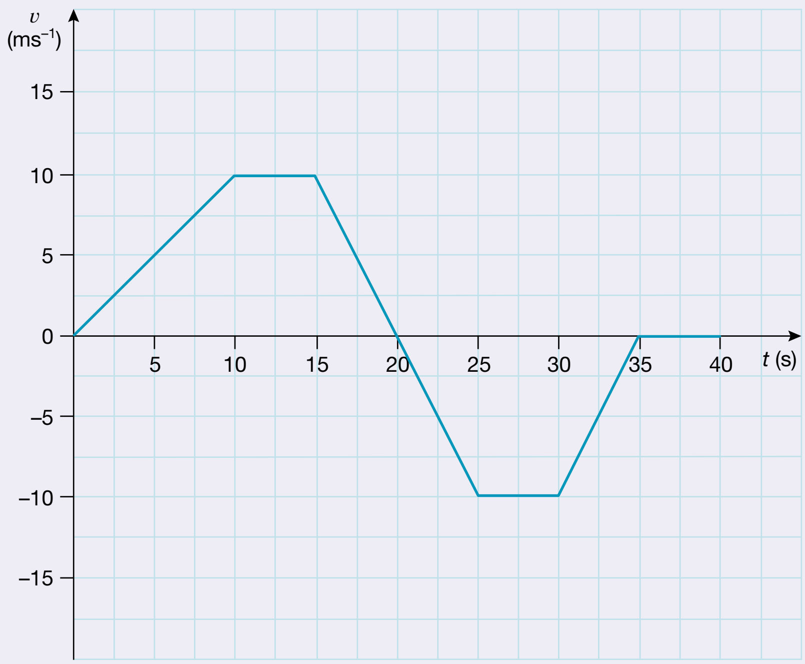
2.5 Exercise 2

The following velocity–time graph describes the motion of a car for a 40 s period of time.

FIGURE 2.14



Using the graph, determine (a) the car's acceleration during its first 5 s of motion, (b) the times when the car was stationary and (c) the car's displacement at the end of the 40 s.

2.6 Review

2.6.1 Summary

- Displacement is a measure of the change in position of an object. Displacement is a vector quantity.
- In order to fully describe any vector quantity, a direction must be specified as well as a magnitude.

- Speed is a measure of the rate at which an object moves over distance and is a scalar quantity. Velocity is the time rate of displacement and is a vector quantity.
- The velocity of an object measured by a moving observer is referred to as the relative velocity. The relative velocity is the difference between the velocity of the object relative to the ground and the velocity of the observer relative to the ground.
- Average speed $= \dfrac{\text{distance travelled}}{\text{time interval}}$
- The average velocity of an object, $\boldsymbol{v}_{av}$ during a time interval, t, can be expressed as $\boldsymbol{v}_{av} = \boldsymbol{s}/t$.
- Instantaneous speed is the speed at a particular instant of time. Instantaneous velocity is the velocity at a particular instant of time.
- Acceleration is the rate at which an object changes its velocity. Acceleration is a vector quantity. The average acceleration of an object, $\boldsymbol{a}_{av}$ can be expressed as $\boldsymbol{a}_{av} = \dfrac{\Delta \boldsymbol{v}}{\Delta t}$, where $\Delta \boldsymbol{v}$ = the change in velocity during the time interval Δt.
- When the acceleration of an object is constant, the following formulae can be used to describe its motion:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$\boldsymbol{s} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t$$
$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

- The instantaneous velocity of an object can be found from a graph of its displacement versus time by calculating the gradient of the graph. Similarly, the instantaneous speed can be found from a graph of its distance versus time by calculating the gradient of the graph.
- The displacement of an object during a time interval can be found by determining the area under its velocity-versus-time graph. Similarly, the distance travelled by an object can be found by determining the area under its speed-versus-time graph.
- The instantaneous acceleration of an object can be found from a graph of its velocity versus time by calculating the gradient of the graph.

2.6.2 Questions

1. State which of the following are vector quantities.
   (a) distance (b) displacement (c) speed (d) velocity
2. In 2004, cyclist Sarah Ulmer, of New Zealand, set a world record of 3 min 24.537 s for the 3000 m pursuit.
   (a) Calculate her average speed.
   (b) Calculate the time taken by Sarah to cycle from Sydney to Newcastle, a distance of 155 km, if she could maintain her average speed for the 3000 m pursuit for the whole distance.
   (c) Calculate the time for a car to travel from Sydney to Newcastle if its average speed is 80 km h^{-1}.
   (d) A car travels from Sydney to Newcastle and back to Sydney in 4.0 h.
      (i) Calculate its average speed.
      (ii) Calculate its average velocity.
3. Explain why you cannot ever measure the instantaneous velocity of an object with a stopwatch.
4. A holidaying physics teacher drives her old Volkswagen from Sydney to Batemans Bay, a distance of 300 km. Her average speed is 80 km h^{-1}. She trades in her Volkswagen and purchases

a brand new Toyota Corolla. She proudly drives her new car back home to Sydney at an average speed of 100 km h^{-1}.

(a) Make a quick prediction of her average speed for the whole trip.

(b) Calculate the average speed for the whole journey and explain any difference between the predicted and calculated average speed.

5. Which is larger in magnitude — the speed of a fly or the velocity of a fly? Explain your answer.
6. The police are pursuing a speeding motorist on a straight road. The speeding car is travelling at 90 km h^{-1} (25 m s^{-1}). The police car, initially 200 metres behind the speeding car, travels at a speed of 105 km h^{-1} (29 m s^{-1}) with lights flashing and siren screaming. Calculate how long it takes the police car to catch up with the speeding car.
7. Calculate the time for:
   (a) a car to accelerate on a straight road at a constant 6.0 m s^{-2} from an initial speed of 60 km h^{-1} (17 m s^{-1}) to a final speed of 100 km h^{-1} (28 m s^{-1})
   (b) a cyclist to accelerate from rest at a constant 2.0 m s^{-2} to a speed of 10 m s^{-1}.
8. Calculate (i) the change in speed and (ii) the change in velocity in each of the following situations.
   (a) The driver of a car heading north along a freeway at 100 km h^{-1} slows down to 60 km h^{-1} as the traffic gets heavier.
   (b) A fielder catches a cricket ball travelling towards him at 20 m s^{-1}.
   (c) A tennis ball travelling at 25 m s^{-1} is returned directly back to the server at a speed of 30 m s^{-1}.
9. Calculate the average acceleration of a car, starting from rest, that reaches a velocity of 20 m s^{-1} due north in 5.0 s.
10. In Acapulco, on the coast of Mexico, professional high divers plunge from a height of 36 m above the water. (The highest diving boards in Olympic diving events are 10 m above the water.) Estimate:
    (a) the length of the time interval during which the divers fall through the air
    (b) the speed with which the divers enter the water.
    Assume that throughout their dive, the divers are falling vertically from rest with an acceleration of 9.8 m s^{-2}.
11. A skateboard rider travelling down a hill notices the busy road ahead and comes to a stop in 2.0 s over a distance of 12 m. Assume a constant negative acceleration.
    (a) Calculate the initial speed of the skateboard.
    (b) Calculate the acceleration of the skateboard as it came to a stop.
12. A car is travelling at a speed of 100 km h^{-1} (27.8 m s^{-1}) when the driver sees a large fallen tree branch in front of her. At the instant that she sees the branch it is 50.0 m from the front of her car. After she applies the brakes, the car travels a distance of 48.0 m before coming to a stop.
    (a) Calculate the time taken for the car to stop once the brakes were applied.
    (b) Calculate the average acceleration of the car while it is braking.
    (c) What other information do you need in order to determine whether the car stops before it hits the tree branch? Make an estimate of the missing item of information to predict whether or not the car is able to stop in time.
13. Amy rides a toboggan down a steep snow-covered slope. Starting from rest, Amy reaches a speed of 12 m s^{-1} as she passes her brother, who is standing 19 m further down the slope from her starting position. Assume that Amy's acceleration is constant.
    (a) Calculate the time taken for Amy to reach her brother.
    (b) Calculate Amy's acceleration.
    (c) At what instant was Amy's instantaneous velocity equal to her average velocity?

14. The position-versus-time graph in figure 2.15 describes the motion of six different objects labelled A–E.
    (a) Which two objects start from the same position, but at different times?
    (b) Which two objects start at the same position at the same time?
    (c) Which two objects are travelling at the same speed as each other, but with different velocities?
    (d) Which two objects are moving towards each other?
    (e) Which object has a lower speed than all of the other objects?
15. The velocity-versus-time graph in figure 2.16 describes the motion of a car as it travels due south through an intersection. The car was stationary for 6 s while the traffic lights were red.
    (a) Calculate the displacement of the car during the time interval in which it was slowing down.
    (b) Calculate the average acceleration of the car during the time interval in which it was slowing down.
    (c) Calculate the average acceleration of the car during the first 4.0 s after the lights turned green.
    (d) Calculate the average velocity of the car during the time interval described by the graph.
16. The graph in figure 2.17 is a record of the straight-line motion of a skateboard rider during an 80 s time interval. The interval has been divided into sections A–E. The skateboarder initially moves north from the starting point.
    (a) During which section of the interval was the skateboard rider stationary?
    (b) During which sections was the skateboarder travelling north?
    (c) At what instant did the skateboard rider first move back towards the starting line?
    (d) What was the total displacement of the skateboarder after the 80 s interval?
    (e) What distance did the skateboarder travel during the 80 s interval?
    (f) During which section was the skateboard rider speeding up?
    (g) During which section was the skateboard rider slowing down?
    (h) What was the skateboarder's average speed during the entire interval?

FIGURE 2.15

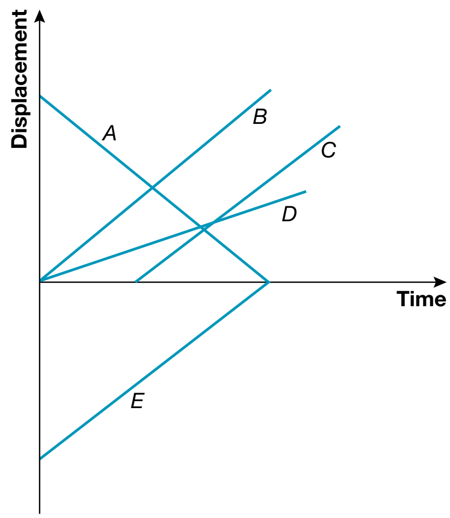


FIGURE 2.16 The straight-line motion of a car travelling through an intersection.

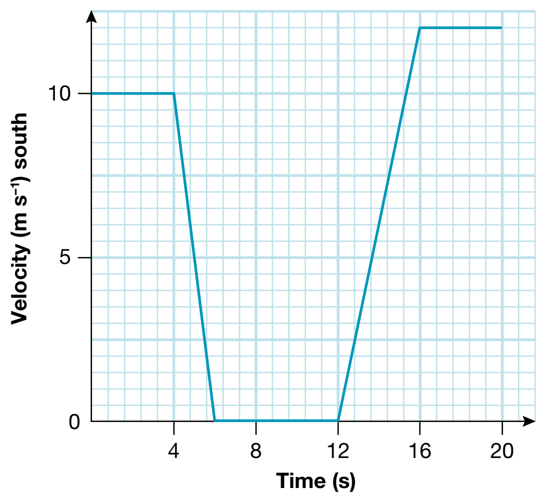
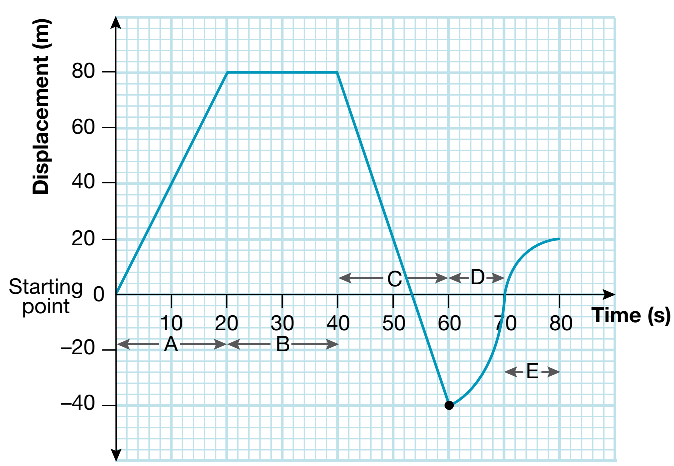


FIGURE 2.17

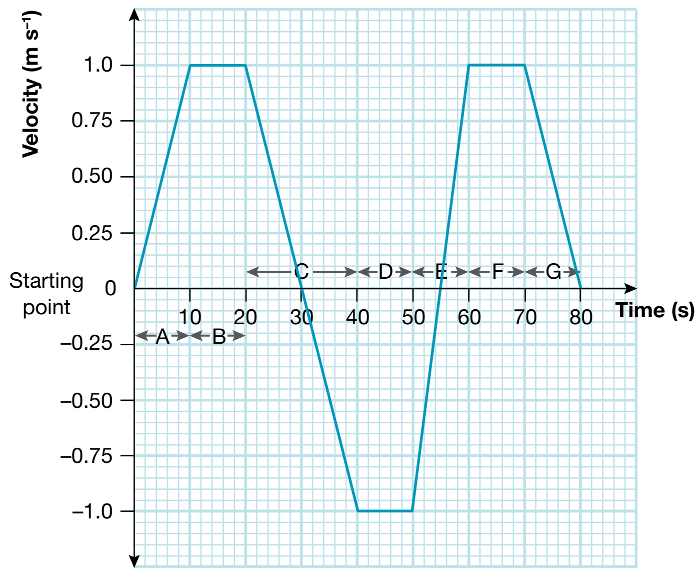


(i) What was the velocity of the skateboarder throughout section C?
(j) Estimate the velocity of the skateboarder 65 s into the interval.

17. Sketch a velocity-versus-time graph to illustrate the motion described in each of the following situations.
    (a) A bicycle is pedalled steadily along a road. The cyclist stops pedalling and allows the bicycle to come to a stop.
    (b) A ball is thrown straight up into the air and is caught at the same height from which it was thrown. The acceleration of the ball is constant and downwards.

18. The graph in figure 2.18 is a record of the motion of a remote-controlled car during an 80 s time interval. The interval has been divided into sections A–G.
    (a) During which sections is the acceleration of the car zero?
    (b) What is the total displacement of the car during the 80 s interval?
    (c) What is the average velocity of the car during the entire interval?
    (d) At what instant did the car first reverse direction?
    (e) At what instant did the car first return to its starting point?
    (f) During which sections did the car have a negative acceleration?
    (g) During which sections was the car's speed decreasing?
    (h) Explain why your answers to (f) and (g) are different from each other.
    (i) What is the acceleration of the car throughout section E?
    (j) What is the average acceleration during the first 20 s?
    (k) Describe the motion of the remote-controlled car in words.

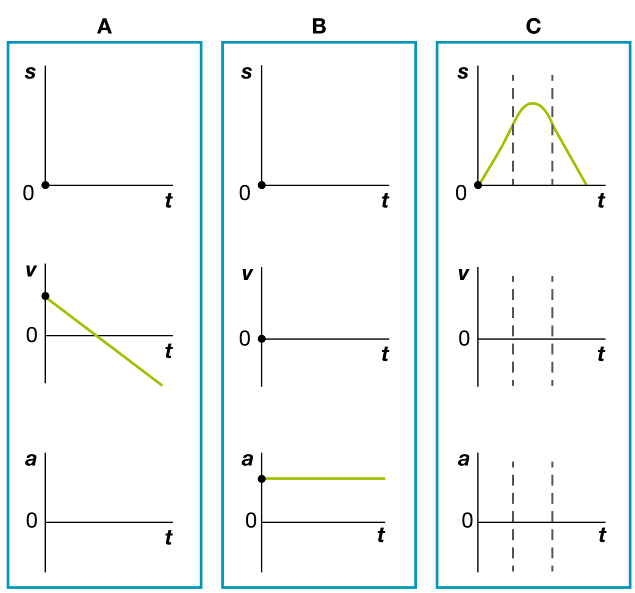
FIGURE 2.18



19. Describe in words the motion shown for each of the scenarios A, B and C in figure 2.19. Copy and complete the incomplete graphs.

20. Figure 2.20 compares the straight-line motion of a jet ski and a car as they each accelerate from an initial speed of 5.0 m s^{-1}.
    (a) Which is the first to reach a constant speed — the jet ski or the car — and when does this occur?
    (b) Calculate the final speed of:
        (i) the jet ski
        (ii) the car.
    (c) Draw a graph of speed versus time describing the motion of either the jet ski or the car.

FIGURE 2.19



21. Once upon a time, a giant tortoise had a bet with a hare that she could beat him in a foot race over a distance of 1 km. The giant tortoise can reach a speed of about 7.5 cm s^{-1}. The hare can run as fast as 20 m s^{-1}. Both animals ran at their maximum speeds during the race. However, the hare was a rather arrogant creature and decided to have a little nap along the way. How long did the hare sleep if the result was a tie?
22. A brand new Rolls-Royce rolls off the back of a truck as it is being delivered to its owner. It lands on its wheels. The truck is travelling along a straight road at a constant speed of 72 km h^{-1} (20 m s^{-1}). The Rolls-Royce slows down at a constant rate, coming to a stop over a distance of 240 m. It is a full minute before the truck driver realises that the precious load is missing. The driver brakes immediately, leaving a 25 m long skid mark on the road. The driver's reaction time (time interval between noticing the problem and depressing the brake) is 0.5 s. How far back is the Rolls-Royce when the truck stops?
23. During the filming of a new movie, a stuntman has to chase a moving bus and jump into it. The stuntman is required to stand still until the bus passes him and then start chasing. The velocity-versus-time graph in figure 2.21 opposite describes the motion of the stuntman and the bus from the instant that the bus door passes the stationary stuntman.
    (a) At what instant did the stuntman reach the same speed as the bus?
    (b) Calculate the magnitude of the acceleration of the stuntman during the first 4.0 s.
    (c) At what instant did the stuntman catch up with the bus door?
    (d) How far did the stuntman run before he reached the door of the bus?

FIGURE 2.20

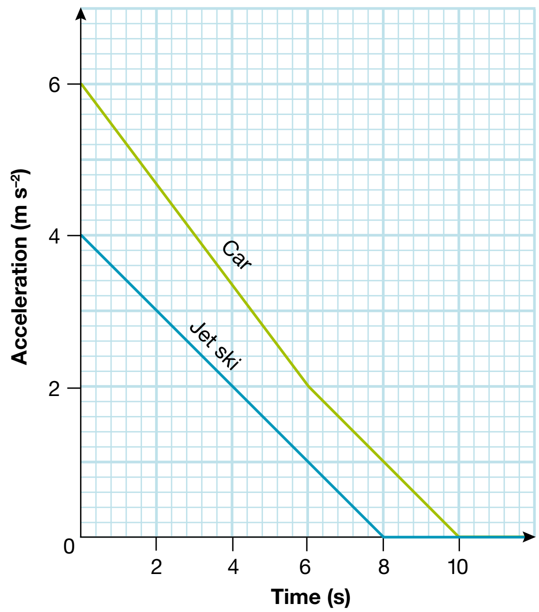
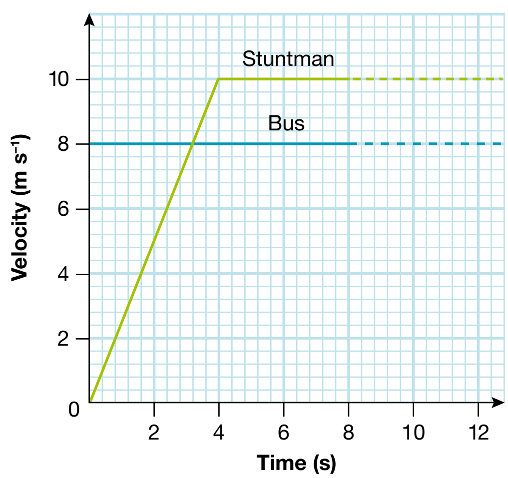


FIGURE 2.21



PRACTICAL INVESTIGATIONS

Investigation 2.1: Going home

Aim

To distinguish between scalar quantities and vector quantities

Apparatus

street directory or map

watch

Theory

Distance and speed are scalar quantities that can be fully described as a magnitude. Displacement and velocity are vector quantities that specify magnitude and direction.

Method

Draw a map to show your journey from school to home. It should occupy about half of an A4 page and be drawn to scale. An example of a map is shown in figure 2.22. Record the time taken to travel home on a typical school day.

Results

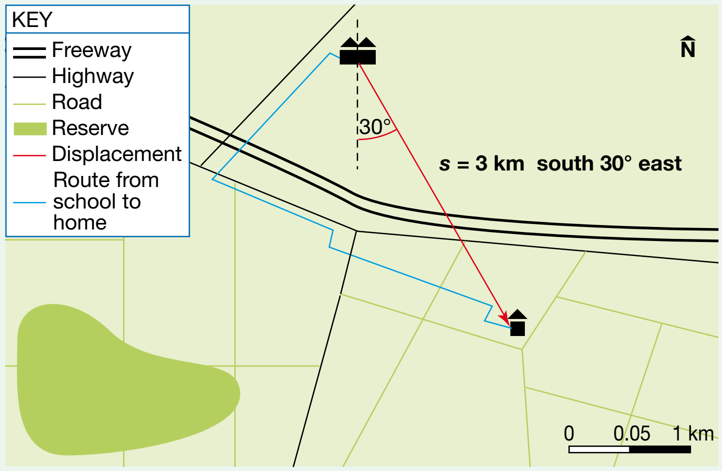
Draw and label your displacement on the map.

Analysis and questions

Determine and specify fully:

(a) your displacement
(b) your resultant average velocity during the journey home
(c) the total distance travelled
(d) your resultant average speed during the journey home.

FIGURE 2.22 A map showing a journey from school to home.



Time to travel home: 10 min
Displacement: 3 km south 30° east
Average velocity: $\frac{3000 \text{ m}}{600 \text{ s}} = 5 \text{ m s}^{-1}$ south 30° east
Total distance travelled: 4.2 km
Average speed: $\frac{4200 \text{ m}}{600 \text{ s}} = 7 \text{ m s}^{-1}$

Investigation 2.2: On your bike

Aim

To record the motion of a cyclist

Apparatus

10 stop watches
100 m measuring tape
bicycle and helmet
speedometer or phone with an app such as an exercise app to log data

Theory

The instantaneous velocity of an object can be found from a graph of its displacement versus time by calculating the gradient of the graph. For straight-line motion in one direction only the speed is the same as the magnitude of the velocity.

The instantaneous acceleration of an object can be found from a graph of its velocity versus time by calculating the gradient of the graph.

The displacement of an object during a time interval can be found by determining the area under its velocity-versus-time graph.

Method

Record the motion of a bicycle or a person walking in a straight line over a distance of 100 m. Place timekeepers at 10 m intervals along the track. The role of each timekeeper is to record the time interval between the start and the instant that the cyclist passes.

Results

Construct a table similar to the table below in which to record your results.

Time(s)	Displacement (m)
	0
	10
	20

Analysis and questions

1. What was the average speed (in m s^{-1}) of the cyclist?
   Use the table to construct a graph of displacement versus time. Use the graph to answer the following questions.
2. What information does the gradient of the displacement-versus-time graph provide?
3. At what instant did the maximum speed occur?
4. What was the maximum speed (in m s^{-1})?
5. Express the maximum speed in km h^{-1}.

Use your displacement-versus-time graph to construct a velocity-versus-time graph of the motion. Use the velocity-versus-time graph to answer the following questions.

6. How can the acceleration be determined from your velocity-versus-time graph?
7. During which time interval was the acceleration greatest?
8. Was the acceleration zero at any time during the ride? If so, at what instant, or during which time interval, was the acceleration zero?
9. During which time interval (if any) was the acceleration negative?
   Calculate the area under the velocity-versus-time graph and answer the following question.
10. Did you get the result that you expected? What does your result indicate about your graph?

Investigation 2.3: Analysing motion with a constant acceleration

Aim

To record the motion of an object down an inclined plane and use a graphical method to describe and analyse the motion

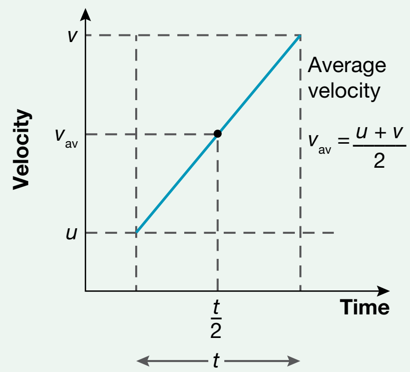
Apparatus

trolley or linear air-track glider
brick or other object (or objects) to raise one end of the plane
timing and recording device (e.g. ticker-timer, spark generator, photogates or motion detector and computer interface)
metre rule

Theory

If you are using a ticker-timer, a spark generator or photogates to record the motion, you will need to make use of the following observation. For an object moving with a constant acceleration, the instantaneous velocity midway through a time interval is equal to the average velocity during that time interval. This is shown in figure 2.23.

FIGURE 2.23 When acceleration is constant, the instantaneous velocity at time $\frac{t}{2}$ is equal to the average velocity during the time interval t.



Method

Make an inclined plane by raising one end of a laboratory bench or a linear air track. Use an angle of approximately 10° to the horizontal. Prepare the recording device and record the motion of a low-friction trolley or air-track glider as it accelerates down the inclined plane.

Results

If your data is recorded on ticker tape, find at least eleven consecutive clear dots.

Analysis

Use your data to determine the instantaneous velocity at enough instants of time to allow you to plot a graph of velocity versus time. Use a table to record time and instantaneous velocity. Include a third column in the table in which to record the acceleration.

Use your velocity-versus-time graph to determine the acceleration at a number of instants. Record the acceleration in your table and plot a graph of acceleration versus time.

Questions

1. What was the average acceleration of the trolley or glider?
2. Describe how the acceleration changes (if it does) while the trolley or glider moves along the inclined plane.
3. If the acceleration is not constant, explain why and suggest how the experiment could be improved so that it is constant.
4. What is the greatest source of error in measuring the instantaneous velocity of the trolley or glider?
5. How could the experiment be changed so that the error in measuring the instantaneous velocity of the trolley or glider is reduced?
6. Use your graph of velocity versus time to estimate the distance travelled by the trolley or glider. How does this distance compare with the distance measured with a metre rule?

WORKING SCIENTIFICALLY 2.2

1. Design and build a small cart that is fitted with a sail (you may adapt existing carts or toy cars for the purpose). The cart will then move when placed in front of a table fan or small handheld electric fan.
2. Use the fan-powered cart to investigate one of the following:
   - How does the area of the sail affect the maximum speed of the cart?
   - What is the relationship between the fan speed and the final displacement of the cart?
   - What has the greatest effect on the maximum speed of the cart — wheel radius or cart mass?
3. Write a scientific report describing your investigation and your findings. Search online to find how to structure a scientific report such as the one found at the UniLearning website of the University of Wollongong.

WORKING SCIENTIFICALLY 2.3

A dynamics cart or linear air-track glider may be accelerated by releasing it to move freely down an incline. Design and conduct an investigation to determine the relationship between the slope's incline angle and the cart's or slider's maximum acceleration.

WORKING SCIENTIFICALLY 2.4

Wheeled toys requiring a push to make them move (such as Matchbox cars) will often follow a curved path as they move rather than travelling in a straight line. Investigate what factors cause this curving and describe the conditions under which you could eliminate this effect.

WORKING SCIENTIFICALLY 2.5

Find a wind-up toy that moves linearly when released. Determine the relationship between the number of complete turns the winding key is given and the distance travelled by the toy. Take care not to overwind the toy!

TOPIC 3
Motion in a plane

3.1 Overview

3.1.1 Module 1: Kinematics

Motion on a Plane

Inquiry question: How is the motion of an object that changes its direction of movement on a plane described?

Students:

- analyse vectors in one and two dimensions to:
  - resolve a vector into two perpendicular components
  - add two perpendicular vector components to obtain a single vector (ACSPH061).
- represent the distance and displacement of objects moving on a horizontal plane using:
  - vector addition
  - resolution of components of vectors (ACSPH060).
- describe and analyse algebraically, graphically and with vector diagrams, the ways in which the motion of objects changes, including:
  - velocity
  - displacement (ACSPH060, ACSPH061).
- describe and analyse, using vector analysis, the relative positions and motions of one object relative to another object on a plane (ACSPH061)
- analyse the relative motion of objects in two dimensions in a variety of situations, for example:
  - the motion of a boat on a flowing river
  - the motion of two moving cars
  - the motion of an aeroplane in a crosswind (ACSPH060, ACSPH132).

FIGURE 3.1 The motion of objects such as cars and aircraft is described in two and three dimensions rather than rectilinearly.

3.2 Graphical treatment of vectors

3.2.1 Drawing vector diagrams to scale

Up until now, we have only examined motion in a rectilinear framework — that is, where motion occurs along a single line. While such analysis is useful when we consider motion along straight roads or railway tracks, motion in the real world is more realistically represented by vectors acting in two and three dimensions.

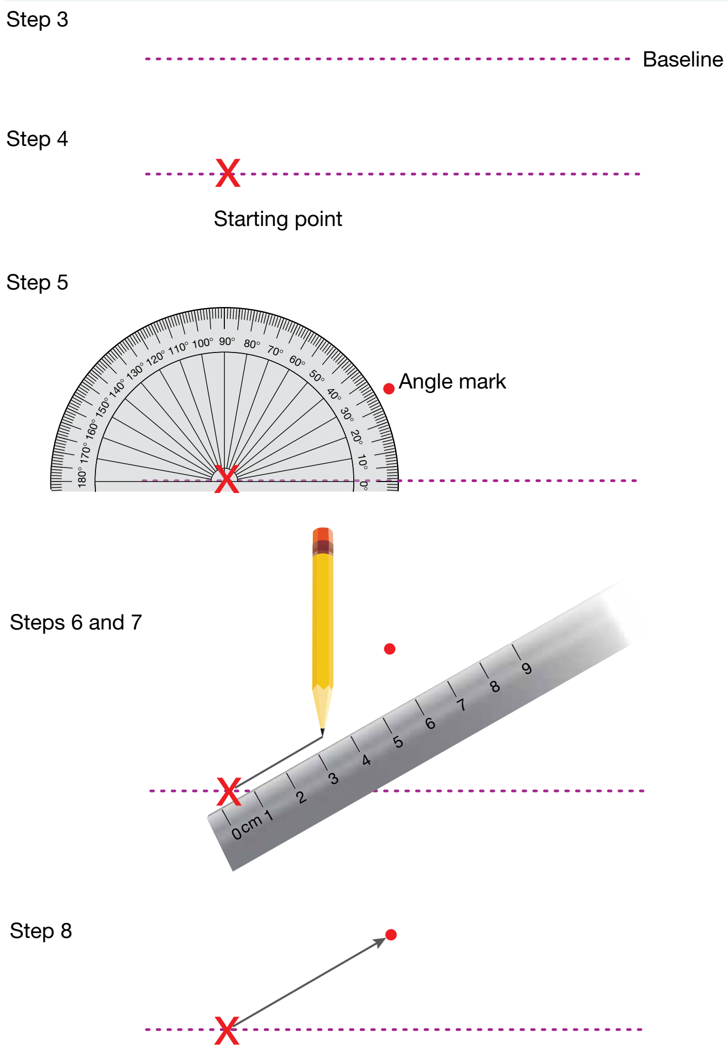
Changes in position and other vector quantities such as velocity and acceleration can be depicted by scale diagrams, and the resultant determined simply by using a ruler and protractor.

When drawing vectors, selecting an appropriate scale for their length is vitally important. After all, if you want to draw a vector that represents an object moving 200 m south, it would be unwise to choose a scale of 1 centimetre for every metre, as you would need to draw an arrow 2 m in length. On the other hand, it would be equally unwise to use 1 mm for every 100 metres, as the vector diagram would be too tiny to read.

To draw a vector diagram, the following steps should be followed:

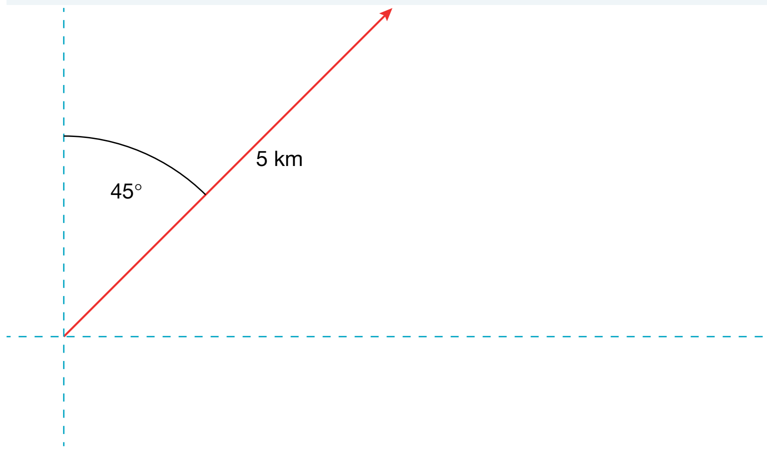
1. Select a sharp pencil, a ruler with a good edge and a protractor.
2. Choose a suitable scale for your diagram.
3. Mark a faint horizontal line on your page. This will be your baseline.
4. Mark a small point on your baseline. This is where your vector will start.
5. Place the crosshairs of your protractor on the point and mark, on your page, the position of the angle at which you need to draw your vector.
6. Place your ruler so that it connects the starting point and the angle mark.
7. Using your selected scale, draw a line from the starting point, along the ruler, for the required length.
8. Draw an arrowhead where you have ended your line.
9. Write the magnitude of the vector next to it (don't forget the units) and mark in the angles where appropriate.

FIGURE 3.2 Drawing vector diagrams.



Using this system, the displacement vector of a cyclist as he rides north-east for 5 km would be drawn like this:

FIGURE 3.3 Displacement vector representing 5 km, north-east.



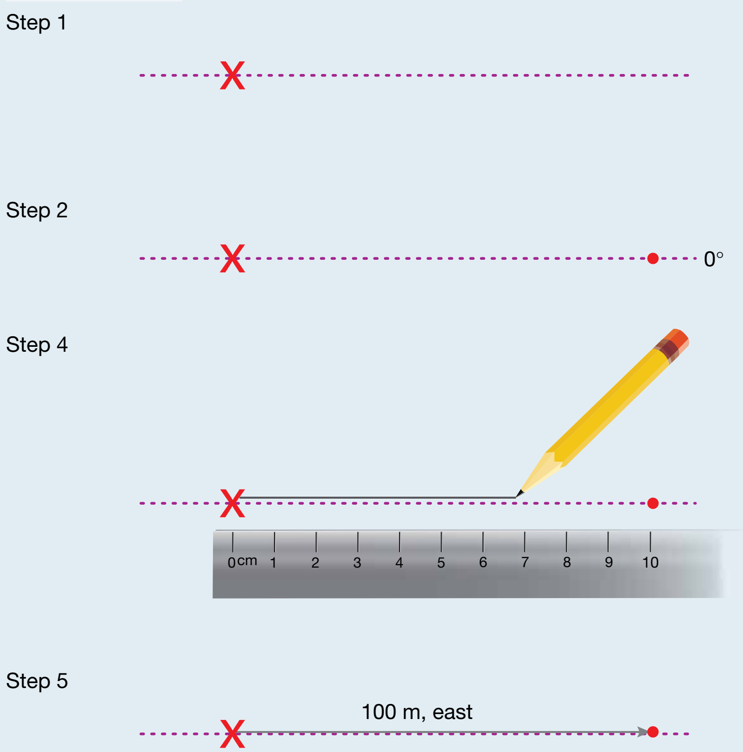
3.2 SAMPLE PROBLEM 1

Draw a vector diagram representing a displacement of 100 m to the east.

SOLUTION:

1. First draw your baseline and mark your starting point.
2. Mark your angle using your protractor. Remember that east is represented by 0° on the right-hand side of the protractor.
3. Here, a scale of 1 cm to every 10 m has been selected. This means that we will draw a 10 cm line for our vector.
4. Placing the ruler to connect the starting point and the angle mark, we draw our 10 cm line.
5. Finally, we draw an arrowhead on the end of the vector, and mark the value that it represents above it.

FIGURE 3.4

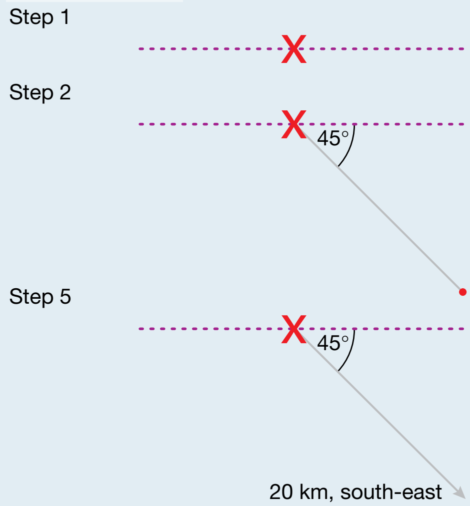


3.2 SAMPLE PROBLEM 2

Draw a vector representing the motion of a cyclist travelling 20 km south-east.

SOLUTION:

FIGURE 3.5



1. Once again, we draw our baseline and our starting point.
2. Place your protractor on the starting point and rotate it so that you are measuring 45° *underneath* the baseline. Mark the angle's position.
3. Place your ruler between the starting point and the angle mark.
4. This time we've selected a scale of 1 cm for every 5 km, so we draw our line segment from the starting point until it is 4 cm long. This represents 20 km.
5. Add the arrowhead, the magnitude and the angle.

3.2.2 Vector Addition

Vector diagrams are particularly helpful when evaluating the final displacement of an object travelling for different distances in different directions over the course of its journey.

In these cases, each 'leg' of the journey is represented by a different vector, and these vectors are 'added' sequentially by positioning the 'tail' of each vector at the 'head' of the previous vector.

3.2 SAMPLE PROBLEM 3

A plane flies due north for 50 km, turns north-east and continues for 150 km and then flies due east for 80 km.

(a) Draw a vector diagram showing the plane's journey.

(b) Use the vector diagram to determine the final displacement of the plane from its starting position.

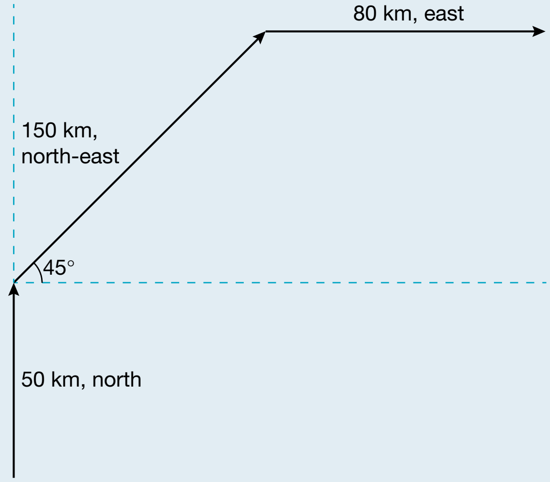
(c) What distance has the plane travelled?

(d) If the total journey took 30 minutes, determine the plane's (i) average speed and (ii) average velocity.

SOLUTION:

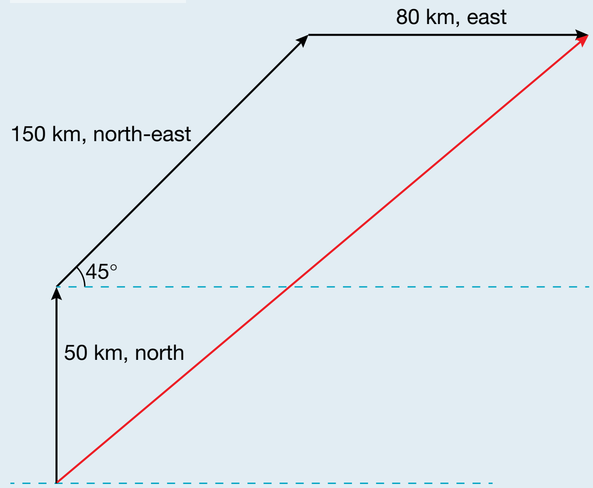
(a) Choosing a scale of 1 cm = 10 km, the three legs of the journey can be represented like this:

FIGURE 3.6



(b) To determine the plane's displacement, a vector is drawn from the plane's starting point to the plane's final position as shown:

FIGURE 3.7

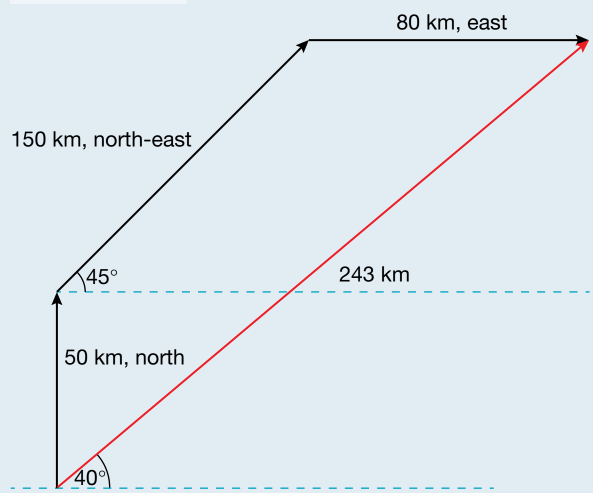


Using a ruler, the length of this displacement vector is found to be 24.3 cm long; as the scale of the diagram is 1 cm = 10 km, this means that the displacement vector has a magnitude of 24.3 cm × 10 km/cm = 243 km.

To find the direction of the displacement vector, a horizontal line is drawn through the starting point, and a protractor is used to determine the angle between the horizontal and the displacement vector:

As a result, the displacement of the plane is 243 km east 40° north (or, equally, north 50° east).

FIGURE 3.8



(c) The distance covered by the plane is simply equal to the sum of the distances covered in each individual leg of the journey: 50 km + 150 km + 80 km = 280 km.

As distance is a scalar quantity, there is no direction associated with this value.

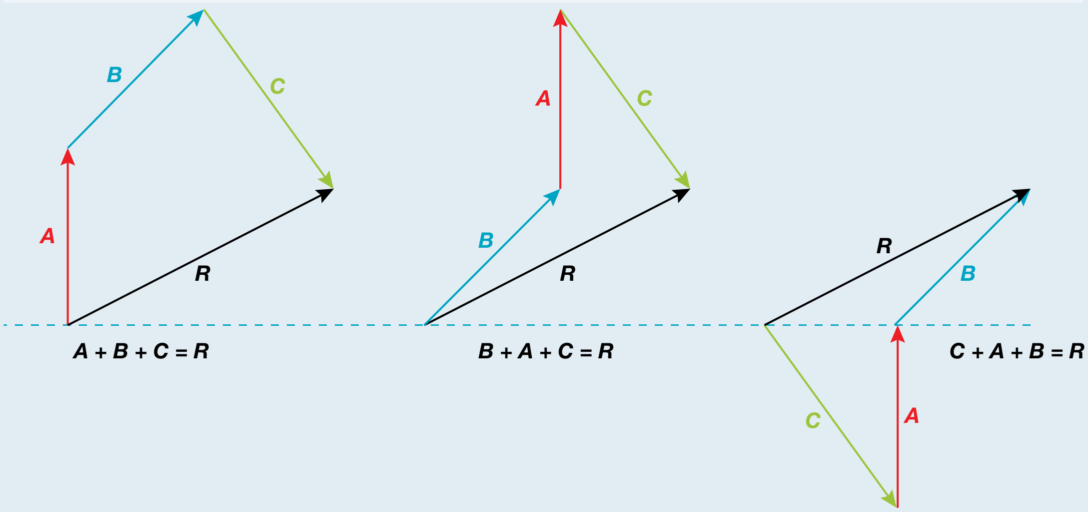
(d) (i) The average speed of the plane is determined by dividing the distance travelled by the time of flight:

$$\text{average speed} = \frac{\text{distance}}{\text{time}} = \frac{280\ \text{km}}{0.5\ \text{h}} = 560\ \text{km h}^{-1}$$

(ii) The average velocity is determined from the displacement as follows:

$$\boldsymbol{v_{av}} = \frac{\boldsymbol{s}}{\text{t}} = \frac{243\ \text{km}}{0.5\ \text{h}}\ \text{at east } 40^\circ\ \text{north} = 486\ \text{km h}^{-1}\ \text{at east } 40^\circ\ \text{north}$$

FIGURE 3.9 The resultant vector does not depend upon the order in which the contributing vectors are added.



When vectors are added together, their sum is referred to as the resultant vector (or, simply, the **resultant**).

It should be noted that, when adding vectors together, the order in which the individual vectors are placed does not affect the magnitude or direction of the resultant vector. If we define three vectors **A**, **B** and **C** as shown below, it is quite clear that

$$\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} = \boldsymbol{B} + \boldsymbol{A} + \boldsymbol{C} = \boldsymbol{C} + \boldsymbol{A} + \boldsymbol{B}$$

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Explore more with this weblink: Visually adding vectors

WORKING SCIENTIFICALLY 3.1

Design and carry out an investigation to determine the average velocity of an ant. Note that the ant is not to be harmed in any way and is to be returned to the place from where it was collected.

3.2 Exercise 1

1 Draw vectors to represent the following:
   (a) 70 m due east
   (b) 2 km due south
   (c) 600 m at 70° north of east
2 Use a vector diagram to determine the resultant velocity of the following. In each case, give velocity to 2 significant figures, and angles to the nearest whole number degree.
   (a) 20 m s^{-1} due North, 10 m s^{-1} due West, 8 m s^{-1} NW.
   (b) 100 km h^{-1} at 30° west of south, 80 km h^{-1} at 45° east of south, 60 km h^{-1} due south

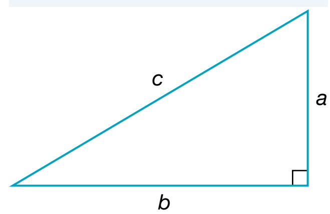
3.3 Algebraic resolution of vector addition

3.3.1 Adding two perpendicular vectors

While drawing scale diagrams helps in solving a problem involving the addition of vectors, its major shortcoming is its lack of precision. A wobbly ruler, a blunt pencil or a poor-quality protractor might combine to produce a resultant with a large degree of error. Consequently, algebraic determinations of resultant vectors are preferred.

The simplest case of vector addition occurs when two vectors are at right angles to each other. The magnitude of the resultant can be found by using Pythagoras's theorem and trigonometric methods employed to determine the resultant vector's direction.

FIGURE 3.10 Pythagoras's theorem: for any right-angled triangle with sides *a*, *b* and *c* as shown: $c^2 = a^2 + b^2$



3.3 SAMPLE PROBLEM 1

Jess cycles 4 km due east and then 7 km due north. Calculate (a) her distance covered and (b) her displacement.

SOLUTION:

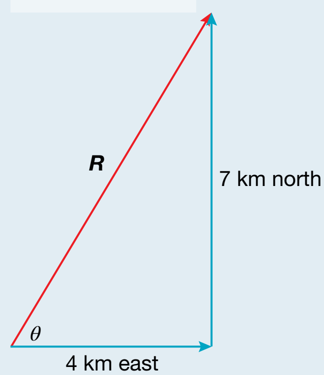
(a) The distance covered will simply be 4 km + 7 km = 11 km.

(b) By using Pythagoras's theorem, we can calculate $\boldsymbol{R}$:

$$\begin{aligned}\boldsymbol{R}^2 &= (4\text{ km})^2 + (7\text{ km})^2\\ &= 16 + 49\\ &= 65\text{ km}^2\end{aligned}$$

$$\begin{aligned}\text{So, } R &= \sqrt{65\text{ km}^2}\\ &= 8.06\text{ km}\\ \text{and } \theta &= \tan^{-1}\frac{7\text{ km}}{4\text{ km}}\\ &= 60.3°\end{aligned}$$

FIGURE 3.11



Therefore, her displacement is equal to 8.06 km at 60.3° north of east.

3.3 SAMPLE PROBLEM 2

A car travels 4.0 km north and then 6.0 km west in 10 min. Calculate (a) its average speed and (b) its average velocity.

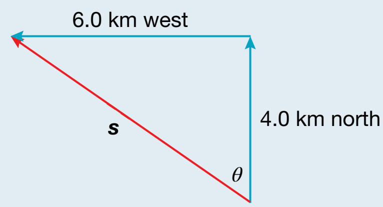
SOLUTION:

(a) distance = 4.0 km + 6.0 km = 10 km

$$\text{time} = 10 \text{ min } = \frac{10}{60} \text{ h}$$

$$\begin{aligned}\text{average speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{10 \text{ km}}{\left(\frac{10}{60}\right) \text{ h}} \\ &= 60 \text{ km h}^{-1}\end{aligned}$$

FIGURE 3.12



(b) To find $\boldsymbol{v}_{av}$, we must first find the value of the displacement $\boldsymbol{s}$.

Using Pythagoras's theorem, we find the magnitude of the displacement vector:

$$\boldsymbol{s}^2 = (4)^2 + (6)^2 = 52$$

$$\boldsymbol{s} = 7.2 \text{ km}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{6}{4}\right) \\ &= 56.3° \text{ west of north}\end{aligned}$$

$$\boldsymbol{s} = 7.2 \text{ km at } 56.3° \text{ west of north}$$

$$\begin{aligned}\boldsymbol{v}_{av} &= \frac{\boldsymbol{s}}{t} = \frac{7.2 \text{ km}}{\left(\frac{10}{60}\right) \text{ h}} \text{ at north } 56.3° \text{ west} \\ &= 43.2 \text{ km h}^{-1} \text{ at north } 56.3° \text{ west}\end{aligned}$$

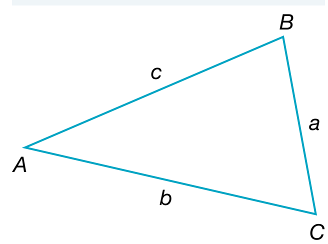
3.3.2 Adding two non-perpendicular vectors

In the case where two vectors to be added are not perpendicular to each other, the cosine rule and the sine rule can be used to determine the magnitude and angle of the resultant vector.

cosine rule: in any triangle ABC, $c^2 = a^2 + b^2 - 2ab\cos C$.

sine rule: in any triangle ABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

FIGURE 3.13 The labelling convention for a triangle when using the sine rule and the cosine rule.



3.3 SAMPLE PROBLEM 3

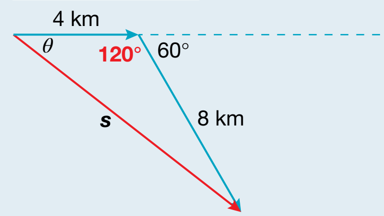
A cyclist travels due east for 4 km before turning 60° towards the south and then continuing to ride for a further 8 km. What is the cyclist's displacement?

SOLUTION:

Clearly, the interior angle between the 4 km and the 8 km vectors is equal to 120° (the complementary angle of 60°).

Using the cosine rule, the magnitude of the displacement $\boldsymbol{s}$ can be found:

FIGURE 3.14



$$\begin{aligned} s^2 &= (4)^2 + (8)^2 - 2(4)(8)\cos 120° \\ &= 80 - 64(-0.5) \\ &= 112 \\ \boldsymbol{s} &= 10.58 \text{ km} \end{aligned}$$

The sine rule is then used to find θ:

$$\frac{10.58}{\sin 120°} = \frac{8}{\sin\theta}$$

$$12.22 = \frac{8}{\sin\theta}$$

$$\theta = \sin^{-1}\left(\frac{8}{12.22}\right) = 40.9°$$

Hence, $\boldsymbol{s}$ = 10.58 km at 40.9° south of west.

WORKING SCIENTIFICALLY 3.2

Pin each end of a 15 cm length of string to a piece of foamboard using thumbtacks. Pull the string into an angle using a third thumbtack. Investigate how the size of the maximum angle subtended at the third thumbtack varies according to the separation of the thumbtacks anchoring the string. Derive an equation that describes the relationship.

3.3.3 Adding multiple vectors

All vectors in two dimensions can be considered to be made up of two components — a horizontal (or *x*-axis) component and a vertical (or *y*-axis) component. By using trigonometry, we can calculate the size of these component vectors.

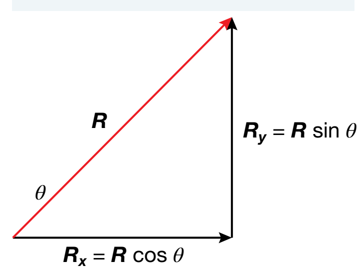
If you use polar angles, then

$\boldsymbol{R_x} = R\cos\theta$, and $\boldsymbol{R_y} = R\sin\theta$

where R is the magnitude of the vector and θ is the vector's polar angle, as shown in figure 3.15.

When a series of vectors is added, we add all the individual horizontal components to find a single resultant horizontal component and all the individual vertical components to find a single resultant vertical component. These two vectors are the independent horizontal and vertical components of the resultant vector.

FIGURE 3.15 All vectors may be described as the sum of their horizontal and vertical component vectors.



When adding vectors by perpendicular components:

1. add all horizontal components to find $\boldsymbol{R}_x$ (the horizontal component of the resultant)
2. add all vertical components to find $\boldsymbol{R}_y$ (the vertical component of the resultant)
3. use Pythagoras's theorem to find the magnitude of $\boldsymbol{R}$
4. use trigonometry to find the value of θ (the resultant angle).

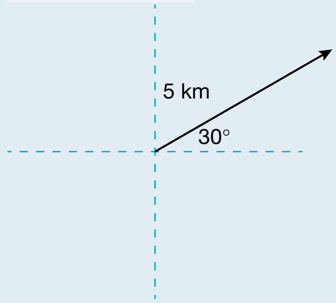
3.3 SAMPLE PROBLEM 4

A yacht navigates a passage through a reef, first sailing 5 km at 30° north of east, then 8 km south-east, 4 km south and, finally, 10 km at 20° west of south. What is the yacht's final displacement from the point at which it started to negotiate the reef?

SOLUTION:

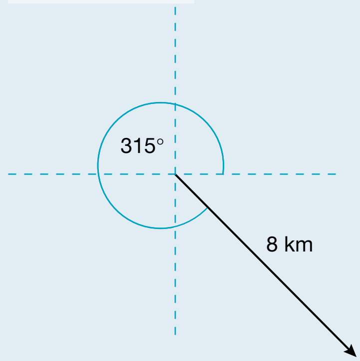
First, each position vector is resolved into its horizontal and vertical components:

FIGURE 3.16



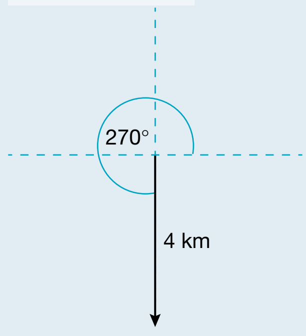
$$s_x = 5\cos 30° = 5(0.8660) = 4.33\text{ km}$$
$$s_y = 5\sin 30° = 5(0.50) = 2.5\text{ km}$$

FIGURE 3.17



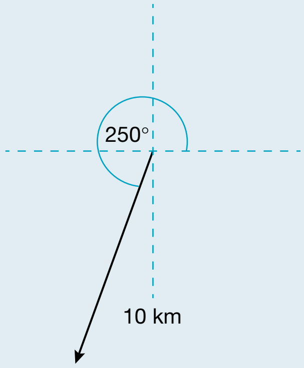
$$s_x = 8\cos 315° = 8(0.7071) = 5.7\text{ km}$$
$$s_y = 8\sin 315° = 8(-0.7071) = -5.7\text{ km}$$

FIGURE 3.18



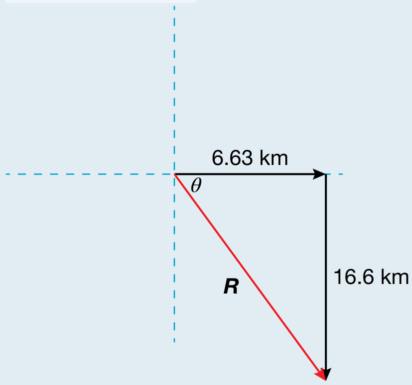
$$s_x = 4\cos 270° = 4(0) = 0$$
$$s_y = 4\sin 270° = 4(-1) = -4\text{ km}$$

FIGURE 3.19



$$s_x = 10\cos 250° = 10(-0.342) = -3.4 \text{ km}$$
$$s_y = 10\sin 250° = 10(-0.940) = -9.4 \text{ km}$$

FIGURE 3.20



Now adding the horizontal components:

$$\boldsymbol{R}_x = 4.33 \text{ km} + 5.7 \text{ km} + 0 + -3.4 \text{ km} = 6.63 \text{ km}$$

Now adding the vertical components:

$$\boldsymbol{R}_y = 2.5 \text{ km} + \; -5.7 \text{ km} + \; -4 \text{ km} + -9.4 \text{ km} = -16.6 \text{ km}$$

$$R = \sqrt{6.63^2 + 16.6^2} = 17.9 \text{ km}$$

$$\theta = \tan^{-1}\left(\frac{16.6}{6.63}\right) = 68.2°$$

Therefore, the yacht's displacement is 17.9 km at 68.2° south of east.

3.3 Exercise 1

1 (a) What is the displacement of a Volkswagen that travels west along a road for 10 km and then south for 8 km?
(b) What is the distance covered by the Volkswagen?

2 A skateboarder travels east for 500 m and then turns sharply until he is travelling 10° west of south. He travels in this direction for 300 m. If his entire journey took 45 seconds, determine his average velocity for this time period.

3 A small boat sets out from the west bank of a river, crossing to its east bank and travelling at 4 m s^{-1}. The river current flows from north to south at 5 m s^{-1}. What will be the resultant velocity of the boat?

4 Find the horizontal and vertical components of the following vectors:
(a) 300 km at 75°
(b) 10 m s^{-1} at 35° west of south
(c) 6 m at 310°
(d) 4 m s^{-2} at –60°

5 Over the course of two hours, a runner travels north for 5 km, then 4 km north-west and finally 10 km at 10° east of south. What was the runner's average velocity?

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Explore more with this weblink: Vector addition

3.4 Vector subtraction

3.4.1 Acceleration in two dimensions

As you will recall from the previous chapter, the average acceleration of an object is equal to the rate of change of the object's velocity per unit time:

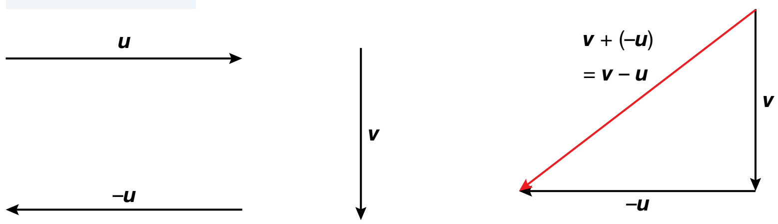
$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

This can be expressed in the equation

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$$

It can be seen that, to evaluate the average acceleration vector, the vector subtraction of the initial velocity $\boldsymbol{u}$ from the final velocity $\boldsymbol{v}$ over the time period must be resolved. This is achieved by adding the inverse vector of $\boldsymbol{u}$ (called $-\boldsymbol{u}$) to $\boldsymbol{v}$. The vector $-\boldsymbol{u}$ is equal in magnitude to $\boldsymbol{u}$ but is directed in the opposite direction.

FIGURE 3.21

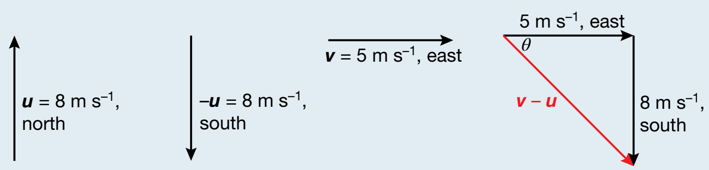


3.4 SAMPLE PROBLEM 1

What is the average acceleration of a cyclist riding north who slows down from a speed of $8\,\text{m}\,\text{s}^{-1}$ to $5\,\text{m}\,\text{s}^{-1}$ over $2.0\,\text{s}$ when turning a corner to travel east?

SOLUTION:

FIGURE 3.22



Using Pythagoras's theorem and the triangle above:

$(\boldsymbol{v} - \boldsymbol{u})^2 = (5)^2 + (8)^2 = 89$

$(\boldsymbol{v} - \boldsymbol{u}) = 9.4\text{ m s}^{-1}$

$\theta = \tan^{-1}\left(\frac{8}{5}\right) = 58°$

So $(\boldsymbol{v} - \boldsymbol{u}) = 9.4\text{ m s}^{-1}$ at 58° south of east (or south 32° east)

$$\begin{aligned}\boldsymbol{a} &= \frac{(\boldsymbol{v} - \boldsymbol{u})}{t} \\ &= \frac{9.4\text{ m s}^{-1}}{2.0\text{ s}} \text{ at south } 32°\text{ east} \\ &= 4.7\text{ m s}^{-2} \text{ at south } 32°\text{ east}\end{aligned}$$

3.4.2 Relative velocity in two dimensions

PHYSICS FACT

A non-zero acceleration does not always result from a change in speed. Consider a car travelling at 60 km h^{-1} in a northerly direction turing right and continuing in an easterly direction at the same speed. Assume that the complete turn takes 10 s. The average acceleration during the time interval of 10 s is given by:

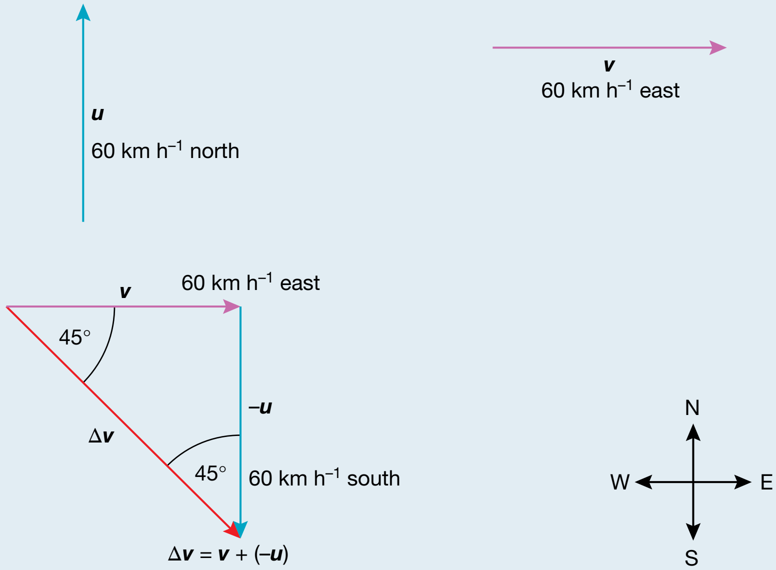
$$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}.$$

The change in velocity must be determined first. Thus,

$$\begin{aligned}\Delta \boldsymbol{v} &= \boldsymbol{v} - \boldsymbol{u} \\ &= \boldsymbol{v} + -\boldsymbol{u}.\end{aligned}$$

The vectors $\boldsymbol{v}$ and $-\boldsymbol{u}$ are added together to give the resulting change in velocity.

FIGURE 3.23 A change in velocity can occur even if there is no change in speed.



The magnitude of the change in velocity is calculated using Pythagoras's Theorem or trigonometric ratios to be 85 km h^{-1}. Alternatively, the vectors can be added using a scale drawing and then measuring the magnitude and direction of the sum. The direction of the change in velocity can be seen in figure 3.23 to be south-east.

$$\begin{aligned}\boldsymbol{a}_{av} &= \frac{\Delta \boldsymbol{v}}{\Delta t} \\ &= \frac{85}{10} \\ &= 8.5 \text{ km h}^{-1}\text{s}^{-1} \text{ south-east}\end{aligned}$$

As you will recall from the previous chapter, relative velocity is the velocity of an object as measured by a moving observer. The velocity of an object $\boldsymbol{v}_B$ relative to an observer travelling at a velocity $\boldsymbol{v}_A$ can be described by the equation

$$\boldsymbol{v}_{BrelA} = \boldsymbol{v}_B - \boldsymbol{v}_A$$

3.4 SAMPLE PROBLEM 2

Two skateboard riders, Finn and Jess, are travelling north along the same bike path. Finn is travelling at 12 m s^{-1} when Jess, 20 metres behind him, is travelling at 16 m s^{-1}.

(a) What is Finn's velocity relative to that of Jess?
(b) What is Jess's velocity relative to that of Finn?
(c) How long will it take Jess to catch up to Finn?

SOLUTION

(a) $\boldsymbol{v}_{F\,rel\,J} = \boldsymbol{v}_F - \boldsymbol{v}_J$

$= (12\ \text{m s}^{-1},\ \text{north}) - (16\ \text{m s}^{-1},\ \text{north})$

$= -4\ \text{m s}^{-1},\ \text{north}$

$= 4\ \text{m s}^{-1},\ \text{south}$

To Jess, it appears that Finn is moving back towards her at $4\ \text{m s}^{-1}$.

(b) $\boldsymbol{v}_{J\,rel\,F} = \boldsymbol{v}_J - \boldsymbol{v}_F$

$= (16\ \text{m s}^{-1},\ \text{north}) - (12\ \text{m s}^{-1},\ \text{north})$

$= 4\ \text{m s}^{-1},\ \text{north}$

To Finn, it appears as if Jess is coming up from behind him at $4\ \text{m s}^{-1}$.

(c) Finn is located 20 m north of Jess.

As $\boldsymbol{v}_{av} = \dfrac{\boldsymbol{s}}{t}$, then

$$t = \frac{\boldsymbol{s}}{\boldsymbol{v}_{av}} = \frac{20\ \text{m, north}}{4\ \text{m s}^{-1},\ \text{north}} = 5\ \text{s}$$

3.4 SAMPLE PROBLEM 3

A light plane is flying north at a speed of 200 km h^{-1} when it encounters winds. Relative to the plane, the winds are travelling at 20 km h^{-1} on a bearing of north 40° east. What would be the velocity of the winds as seen by a stationary observer on the ground?

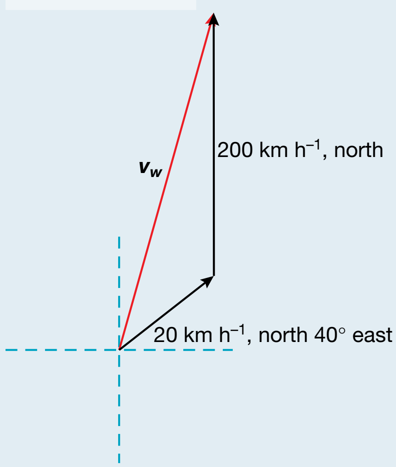
SOLUTION

$\boldsymbol{v}_{w\,rel\,p} = \boldsymbol{v}_w - \boldsymbol{v}_p$

Rearranging the equation, we get

$\boldsymbol{v}_w = \boldsymbol{v}_{w\,rel\,p} + \boldsymbol{v}_p$

FIGURE 3.24

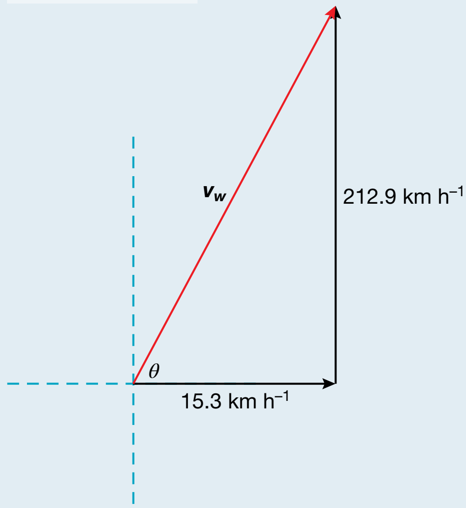


The horizontal and vertical components of $\boldsymbol{v}_w$ are found by adding the perpendicular components of $\boldsymbol{v}_{w\,rel\,p}$ and $\boldsymbol{v}_p$:

horizontal component of $\boldsymbol{v}_w = 200 \cos 90° + 20 \cos 40° = 0 + 15.3 = 15.3\ \text{km h}^{-1}$

vertical component of $\boldsymbol{v}_w = 200 \sin 90° + 20 \sin 40° = 200 + 12.9 = 212.9\ \text{km h}^{-1}$

FIGURE 3.25



$$v_w = \sqrt{(15.3)^2 + (212.9)^2}$$
$$= 213.4 \text{ km h}^{-1}$$
$$\theta = \tan^{-1}\left(\frac{212.9}{15.3}\right) = 85.9°$$

Therefore, to a stationary observer, the wind is travelling at 213.4 km h^{-1} at 85.9° north of east (or north 4.1° east)

3.4 Exercise 1

1 A car travelling at 20 m s^{-1} south slows to 15 m s^{-1}. What is the car's deceleration? (*Note:* **deceleration** is the term used for a negative value of acceleration.)

2 What is the acceleration experienced by a tennis player's racquet travelling at 10 m s^{-1} if it is swung in a 90° arc in 0.4 s to travel at 20 m s^{-1}?

3 A model car travelling 4 m s^{-1} east veers around a rock and, 4.0 seconds later, is moving at 6 m s^{-1} at 50° east of north. What was the model car's acceleration?

4 The yacht *Saucy Gibbon* is sailing due south at 6 knots when a cruise ship is spotted in the distance. The cruise ship is moving due east at 20 knots.
   (a) What is the velocity of the cruise ship relative to the yacht?
   (b) What is the velocity of the yacht relative to the cruise ship?

5 Thuy is on the west bank of a river that flows from north to south at a speed of 5 m s^{-1}. She is easily able to swim at a constant speed of 1 m s^{-1}. The river is 30 m wide.
   (a) On what bearing should Thuy swim to end up on the east bank at a point directly opposite her starting position?
   (b) How long will it take Thuy to reach the other side of the river?

WORKING SCIENTIFICALLY 3.3

Using two battery-powered cars, investigate how the velocity of one car relative to the other changes according to the angle between their directions of motion. Note that this will require you to first determine the speeds of the individual vehicles and to devise a method of ensuring that they are travelling at constant speed when placed in their starting positions.

3.5 Review

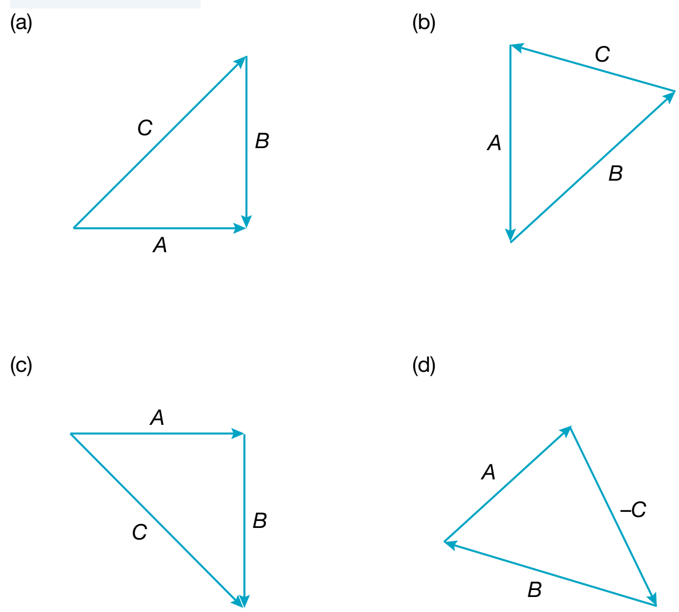
3.5.1 Summary

- A vector is represented by an arrowed line with a length that represents the magnitude of the vector quantity and that points in the direction that the quantity acts.
- Displacement, velocity and acceleration are all vector quantities and are given in terms of a magnitude and a direction.
- A vector can be resolved into two independent, perpendicular components:
  $\boldsymbol{V}_x = \boldsymbol{V} \cos \theta$, and $\boldsymbol{V}_y = \boldsymbol{V} \sin \theta$
- Two vectors are added by placing the tail of the second vector at the head of the first vector. The sum of the vectors is represented by a resultant vector, which is drawn from the tail of the first vector to the head of the last vector.
- Vectors can be added using graphical or algebraic methods.
- The horizontal and vertical components of the resultant vector are found by adding the horizontal and vertical components of the individual vectors being added.
- In vector subtraction, the resultant of $\boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B})$ where $(-\boldsymbol{B})$ has the same magnitude as $\boldsymbol{B}$ but is directed in the opposite direction.
- Relative velocity is the velocity of an object as measured by a moving observer. The velocity of A relative to $\boldsymbol{B}$ $(\boldsymbol{V}_{A\,rel\,B})$ is found from the velocities of A and B relative to a stationary frame of reference:
  $(\boldsymbol{V}_{A\,rel\,B}) = \boldsymbol{V}_A - \boldsymbol{V}_B$

3.5.2 Questions

1. Can two vectors with different magnitudes be combined to give a zero resultant? Can three vectors with different magnitudes be combined to give a zero resultant?
2. What is the largest possible resultant vector that you could make using two vectors of magnitudes 10 N and 9 N? What is the smallest you could make?
3. At what angle(s) are the horizontal and vertical components of a vector equal?
4. In which of the following diagrams does $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{C}$?

FIGURE 3.26



5. Is it possible to determine the direction of motion of an object moving in two dimensions from a velocity–time graph? Explain.
6. Does a car travelling at 60 km h^{-1} north have the same velocity as a car travelling east at 60 km h^{-1}? Explain your answer.
7. Two position vectors have lengths corresponding to 20 m and 30 m.
   (a) What is the largest magnitude their vector sum can have?
   (b) What is the smallest magnitude that their vector sum can have?
8. Determine the resultant displacement in each of the following cases:
   (a) 10 m north + 15 m north
   (b) 12 km north + 10 km south
   (c) 1 km north + 150 m east
   (d) 100 km north + 100 km north-east.
9. (a) Determine the resultant displacement in each case by drawing a scale diagram:
      (i) 7 m at 90°, 6 m at 180°
      (ii) 100 km at 110°, 140 km at 270°
      (iii) 50 km at 30°, 60 km at 90°, 50 km at 180°
      (iv) 1200 m at 300°, 800 m at 120°, 2000 m at 230°.
   (b) Use algebraic methods to determine the resultant displacement in each of (i) – (iv) above. How closely do your calculated answers match your answers in (a)?
10. A car travelling east at a speed of 100 m s^{-1} turns right to head south at the same speed. Has the car undergone an acceleration? Explain your answer with the aid of a diagram.
11. Calculate the average acceleration of:
    (a) a car, starting from rest, which reaches a velocity of 20 m s^{-1} due north in 5.0 s
    (b) a cyclist travelling due west at a speed of 15 m s^{-1}, who turns to cycle due north at a speed of 20 m s^{-1} (the change occurs in a time interval of 2.5 s)
    (c) a bus travelling due north at 8.0 m s^{-1}, which turns right to travel due east without changing speed, in a time interval of 4.0 s.
12. A motorbike travelling north at 12 m s^{-1} turns a corner and heads west without changing speed. If it took 1.5 s to make the turn, what was the motorbike's average acceleration?
13. A pilot flies 400 km in a direction 60° south of east and then 250 km in a direction 45° south of east. What was the plane's displacement from its starting position?
14. A hiker walks 20.0 km in an easterly direction and then walks for 42.0 km in a direction 50° north of west.
    (a) Draw a vector diagram of the hiker's journey.
    (b) What distance has the hiker travelled?
    (c) What is the hiker's displacement from his starting point?
    (d) If the journey takes ten hours of walking, determine the hiker's (i) average speed and (ii) average velocity.
15. A boat able to travel at 10 km h^{-1} is attempting to cross a river that flows at 6 km h^{-1} westward. If the river is 600 m wide, find (a) the time the boat takes to cross the river, and (b) the distance downstream from its starting position that the boat will land on the opposite bank of the river.
16. A cyclist is riding north at 12 km h^{-1} when it starts to rain. The rain appears to be falling towards her at an angle of 10° relative to the vertical. Deciding to return home, the cyclist turns south, riding at the same speed. Now the rain appears to be coming towards her at an angle of 6° to the vertical. What is the velocity of the rain?
17. Looking at a map, a tourist finds that the museum she wants to go to is 840 metres away on a bearing of 30° south of west. However, the region of city that she is in only has streets orientated north-south and east-west. What is the minimum distance she would need to walk to reach the museum?

18. A plane flies 480 km east from town A to town B in 45 minutes, 920 km south-east from town B to town C in 90 minutes and then 320 km south-west from town C to town D in 30 minutes. What is the plane's average velocity in flying from town A to town D?
19. Traditionally, the lead horse on a carousel is the largest and most ornately decorated one. On a particular carousel, the lead horse moves at a constant speed of 10 $m s^{-1}$ and takes five seconds to move through a complete circle. What is the magnitude of the average acceleration of the lead horse? (HINT: consider the velocity vectors for one quarter of a rotation.)
20. A car travels for 40 km with an average velocity of 60 km h^{-1} south-west. With what velocity would the car need to travel over the next 40 km if it is to have an overall average velocity of 80 km h^{-1} at 10° north of west?
21. A stationary radar operator notes a ship that is 20 km out at sea on a bearing of 250° relative to his position. Two hours later, the same ship is observed to be 10 km out at sea on a bearing of 290°. If the ship maintained a constant speed and did not change course between the two observations, what is the ship's velocity?
22. A mine shaft extends 50 m straight down into the earth. At the bottom of the mineshaft, a horizontal tunnel proceeds north for 70 m and then turns west for 30 m. What is the displacement at the end of the western tunnel from the mine entrance on the surface?

TOPIC 4 Forces

4.1 Overview

4.1.1 Module 2: Dynamics

Forces

Inquiry Questions:

1. How are forces produced between objects and what effects do forces produce?
2. How can the motion of objects be explained and analysed?

Students:

- using Newton's Laws of Motion, describe static and dynamic interactions between two or more objects and the changes that occur resulting from:
  - a contact force
  - a force mediated by fields.
- explore the concept of net force and equilibrium in one-dimensional and simple two-dimensional contexts using: (ACSPH050)
  - algebraic addition
  - vector addition
  - vector addition by resolution into components.

FIGURE 4.1 The motion of all vehicles depends on the sum of all the forces acting on them. The same statement applies to the occupants.

- solve problems or make quantitative predictions about resultant and component forces by applying the following relationships:

  $\boldsymbol{F}_{AB} = -\boldsymbol{F}_{BA}$
  $\boldsymbol{F}_x = \boldsymbol{F}\cos\theta, \boldsymbol{F}_y = \boldsymbol{F}\sin\theta$

- conduct an investigation to explain and predict the motion of objects on inclined planes (ACSPH098)
- apply Newton's first two laws of motion to a variety of everyday situations, including both static and dynamic examples, and include the role played by friction (ACSPH063)
- investigate, describe and analyse the acceleration of a single object subjected to a constant net force and relate the motion of the object to Newton's Second Law of Motion through the use of: (ACSPH062, ACSPH063)
  - qualitative descriptions
  - graphs and vectors
  - deriving relationships including $\boldsymbol{F}_{net} = \mathrm{m}\boldsymbol{a}$ and relationships of uniformly accelerated motion.

4.2 Analysing forces

4.2.1 Describing forces

A **force** is exerted by one object on another, and is an interaction that allows a change to the states of motion of the objects. In simpler terms, we can say that a force is a 'push' or a 'pull'. As force is a vector quantity, it has both magnitude and direction associated with it. When turning on a tap, pulling a chair across the floor, pushing a pencil across a page or scrunching up a piece of paper, a force is being applied to an object.

The SI unit of force is the **newton (N)**, named after Sir Isaac Newton, the English scientist whose work contributed so much to our understanding of forces in the physical world. A newton is not a particularly big measurement: a little less than 1 N of force is exerted when lifting a 100 g apple, nearly 10 N when picking up a 1 kg bag of potatoes and about 490 N when lifting a 50 kg person.

4.2.2 Contact and non-contact forces

Forces can be categorised as either **contact forces** or **non-contact forces**. A contact force is exerted by an object that is in physical contact with another object, while a non-contact force can be exerted by an object that is not touching the thing that it is influencing. The force applied in pulling out a chair, the friction that stops your shoe slipping on the ground and the buoyant force that pushes us towards the surface when we are in the water are all examples of contact forces.

The effects of gravity, magnetism and electrostatics are examples of non-contact forces. An apple falls downwards from a tree because of the gravitational force that the Earth itself exerts on it at a distance. Similarly, a steel pin will be pulled towards a magnet across a tabletop.

FIGURE 4.2 Contact and non-contact forces at work.

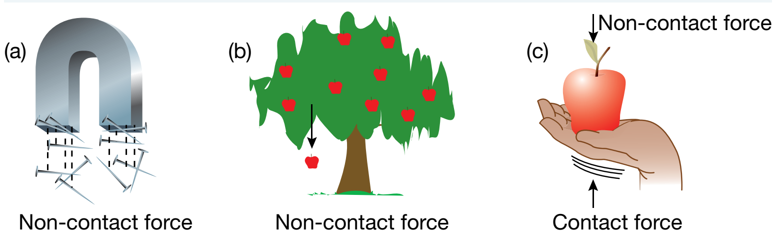


TABLE 4.1 Contact forces and non-contact forces.

Examples of contact forces	Examples of non-contact forces
applied	gravitational
friction	electromagnetic
air resistance	electrostatic
fluid drag	strong nuclear
tension	weak nuclear
normal	
buoyant	

4.3 Forces in action

4.3.1 Gravity — an attraction to Earth

The apple in figure 4.3 is attracted to the Earth by the force of gravity. Even before it falls, the force of gravity is pulling it down. However, before it falls, the tree branch is also pulling it up with a force of equal magnitude.

The force of gravity is a force of attraction that exists between any pair of objects that have mass. Gravity is such a small force that, unless at least one of the objects is as massive as a planet or a natural satellite like the Moon, it is too small to measure.

The force on an object due to the pull of gravity is called **weight** and is usually given the symbol $\boldsymbol{W}$. The magnitude of the weight of an object is directly proportional to its mass (m). Thus, $\boldsymbol{W} \propto m$.

The weight of an object also depends on where it is. For example, the weight of your body on the Moon is considerably less than it is on Earth. Your mass remains the same wherever you are because it is a measure of the amount of matter in an object or substance. The **gravitational field strength**, which is usually given the symbol $\boldsymbol{g}$, is defined as the force of gravity on a unit of mass. Gravitational field strength is a vector quantity. In symbols,

$$\boldsymbol{g} = \frac{\boldsymbol{W}}{m}$$

Thus:

$$\boldsymbol{W} = m\boldsymbol{g}$$

The gravitational field strength, $\boldsymbol{g}$, can be expressed in units of N kg^{-1}. However, $\boldsymbol{g}$ can also be expressed in units of m s^{-2} because it is also equal to the acceleration due to gravity.

The magnitude of the gravitational field strength, $\boldsymbol{g}$, at the Earth's surface is, on average, $9.81\ \text{m s}^{-2}$. Its magnitude decreases as altitude (height above sea level) increases. The magnitude of $\boldsymbol{g}$ also decreases as one moves from the poles towards the equator. Table 4.2 shows the magnitude of $\boldsymbol{g}$ at several different locations on Earth. The magnitude of the gravitational field strength at the surface of the Moon is approximately one-sixth of that at the surface of the Earth.

The magnitude of $\boldsymbol{g}$ at the Earth's surface will be taken as $9.8\ \text{m s}^{-2}$ throughout this text. At the surface of the Moon, the magnitude of $\boldsymbol{g}$ is $1.6\ \text{m s}^{-2}$.

FIGURE 4.3 Force is a vector quantity. Symbols representing vector quantities are in ***bold italic type*** in this text.

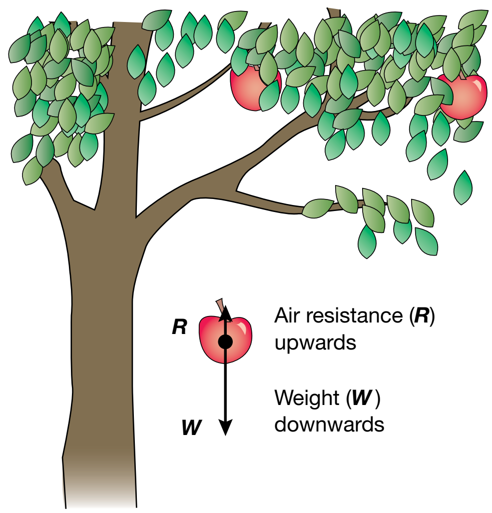


TABLE 4.2 Variation in gravitational field strength.

Location	Altitude (m)	Latitude	Magnitude of g (ms^{-2})
Equator	0	0°	9.780
Sydney	18	34°S	9.797
Melbourne	12	37°S	9.800
Denver	1609	40°N	9.796
New York	38	41°N	9.803
North Pole	0	90°N	9.832

4.3 SAMPLE PROBLEM 1

CALCULATING WEIGHT

Calculate the weight of a 50 kg student:
(a) on the Earth
(b) on the Moon.

SOLUTION:

(a) on the Earth

$$\boldsymbol{W} = mg = 50 \times 9.8 = 490\text{ N downwards}$$

(b) on the Moon

$$\boldsymbol{W} = mg = 50 \times 1.6 = 80\text{ N downwards}$$

Note that the direction must be stated to describe the weight fully as weight is a vector.

PHYSICS FACT

Bathroom scales are designed for use only on Earth. Fortunately (at this time), that's where most of us live.

If a 60 kg student stood on bathroom scales on the Moon, the reading would be only about 10 kg. Yet the mass of the student remains at 60 kg. Bathroom scales measure force, not mass.

However, scales are designed so that you can read your mass in kilograms on Earth. Otherwise, you would have to divide the measured force by 9.8 to determine your mass. The manufacturer of the bathroom scales saves you the trouble of having to do this.

The 60 kg student has a weight of about 600 N on the Earth. However, on the Moon the student's weight is only about 100 N. The reading on the Earth-manufactured scales will be 100 N divided by 9.8 m s^{-2}, giving the result of 10 kg.

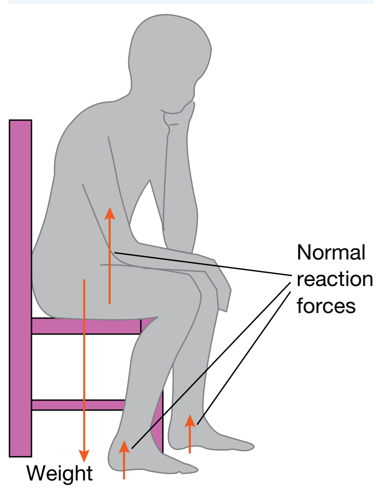
4.3.2 Normal reaction force

It is almost certain that at this very moment you are sitting on a chair with your feet on the floor. If your weight were the only force acting on you, what would happen? What stops you from falling through the floor?

Clearly, there must be at least one other force acting on you to stop you from falling through the floor. As figure 4.4 shows, the chair is pushing upwards on your body and the floor is pushing up on your feet. (You can actually control the size of each of these two upward forces by pushing down with your feet. However, that's another story.) The sum of these upward forces must be just enough to balance the pull of gravity downwards. Each one of these upward forces, or support forces, is called a **normal reaction** force. It is described as a *normal* force because it acts at right angles to the surface. It is described as a *reaction* force because it is only acting in response to the force that your body is applying to the floor.

The normal reaction force is represented by the symbol $\boldsymbol{R}$ and it is equal in magnitude to the sum of the forces exerted perpendicularly to a surface.

FIGURE 4.4 There is more than one force acting on you when you sit on a chair.



WORKING SCIENTIFICALLY 4.1

Not all chairs are comfortable to sit in; some feel as if they are tipping you forwards, while others tend to fall backwards when you get up out of them.

Design a method that would allow you to determine how weight is distributed through the bases and legs of at least four different chair designs when you are sitting on and getting up from them. Perform your investigation and use your results to determine the weight distribution characteristics of a comfortable and stable chair design.

4.3 SAMPLE PROBLEM 2

An apple with a mass of 100 grams is placed on a tabletop. What is the normal reaction force exerted by the table surface on the apple?

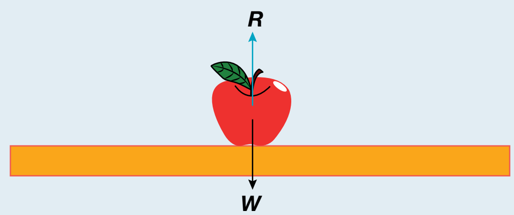
SOLUTION:

The only downward force exerted is that of the apple's weight.

$$\begin{aligned} \boldsymbol{W} &= \mathrm{m}\boldsymbol{g} \\ &= 0.100\,\text{kg} \times 9.8\,\text{m s}^{-2} \\ &= 0.98\,\text{N, downwards} \end{aligned}$$

Therefore, $\boldsymbol{R} = 0.98\,\text{N}$, upwards

FIGURE 4.5



4.3 SAMPLE PROBLEM 3

What normal force will result when a 4 kg brick is placed on a level surface and

(a) the brick is pressed down with a force of 30 N?

(b) a force of 30 N is applied to the brick at an angle of 30° to the surface?

SOLUTION:

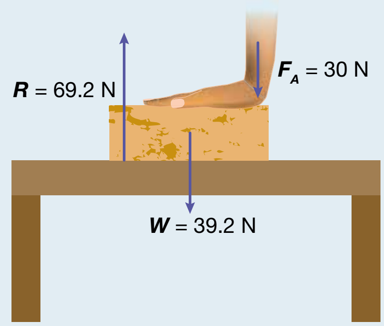
(a) There are now two forces acting downwards on the brick: gravitational force $\boldsymbol{W}$ and an applied force $\boldsymbol{F_A}$.

$$\begin{aligned} \boldsymbol{W} &= \mathrm{m}\boldsymbol{g} \\ &= 4\,\text{kg} \times 9.8\,\text{m s}^{-2} \\ &= 39.2\,\text{N, downwards} \end{aligned}$$

$$\begin{aligned} \text{The total downwards force} &= \boldsymbol{W} + \boldsymbol{F_A} \\ &= 39.2\,\text{N} + 30\,\text{N} \\ &= 69.2\,\text{N} \end{aligned}$$

As a result, $\boldsymbol{R} = 69.2\,\text{N}$, upwards.

FIGURE 4.6 Pressing the brick on the tabletop downwards increases the normal reaction force, ***R***, acting on the brick.

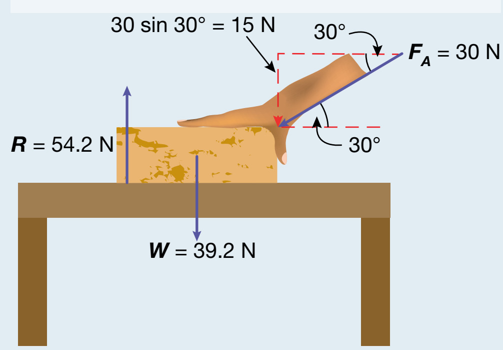


(b) Only the downward component of the applied force will affect the normal force, so

$$\begin{aligned} \text{the total downward force} &= \boldsymbol{W} + \boldsymbol{F_A} \sin 30° \\ &= 39.2 + 30 \sin 30° \\ &= 54.2\,\text{N} \end{aligned}$$

Hence, $\boldsymbol{R} = 54.2\,\text{N}$, upwards on the brick

FIGURE 4.7 Applying a force at a downward angle changes the normal reaction force.



WORKING SCIENTIFICALLY 4.2

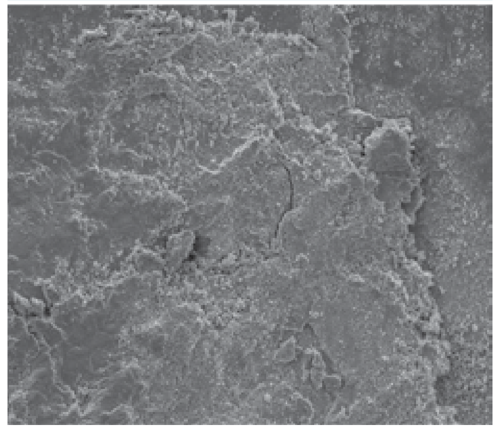
Does jumping on an object exert more force on it than simply standing on it? Design and perform an experiment to test this.

4.3.3 Friction

Friction is a force that resists the motion of one surface across another. While the normal reaction force acts perpendicularly to the surface interface, the frictional force, $\boldsymbol{F_f}$, acts parallel to the interface.

There are two main causes of friction between surfaces. First, all surfaces have irregularities. On some objects such as bricks, sandpaper or Monte Carlo biscuits, the irregularities are easy to spot. However, even the surface of highly polished glass or metal, which seems perfectly smooth to the touch, has irregularities. It is only when you look at these 'smooth' materials under a microscope that the minute bumps and ripples peppering their surfaces become visible.

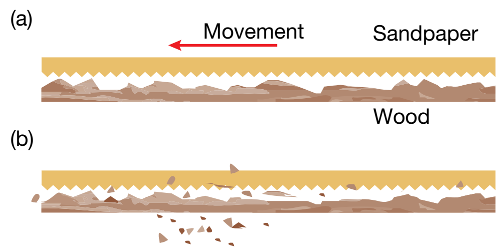
FIGURE 4.8 Cracks and rough surfaces can be seen on this worn-out metal sample in this image taken with a scanning electron microscope. The metal sample looks smooth and shiny when viewed with the naked eye.



As two surfaces come into contact, their lumps and bumps make contact and, when one surface moves across the other, the little bumps catch on each other. This makes it harder to move the surfaces relative to each other and it also causes the surfaces to heat up. You can experience this easily by rubbing your hands together. The little friction ridges that cover your hands catch on each other and you soon notice the heating effect.

Sometimes, as the bumps and ridges of one surface catch on those of another surface, some of the larger bumps are broken off. This is how sandpaper acts to smooth wood. The bumps on the sandpaper are very hard, while those in the wood are more brittle. As bump meets bump, the larger bumps of the rough wood are broken off, leaving behind smaller peaks on its surface. As a result, the wood becomes smoother.

FIGURE 4.9 As large bumps are broken off, the wood becomes smoother.



The second cause of friction is on a smaller scale than the interaction between the bumps. As the bumps and ridges of the two surfaces push against each other, bonds form between the particles of the two materials. These bonds are quite strong and are only broken when a large enough force is applied to pull them apart. As surfaces move across each other, bonds are made and broken between the particles, causing a resistance to the motion.

Static friction

Static friction is the frictional force that must be overcome as you first start to push one surface across another. When two surfaces are placed into contact, considerable interlocking occurs between the peaks of the two surfaces, and bonds form between the particles of the materials. Remember, these bonds are very strong. As you start to push one surface across the other by applying a force, the combination of particle bonds and the locking of peaks acts to exert a resisting force in the opposite direction and so the surfaces do not move. As you exert greater force, the peaks start to fracture and the bonds begin to break. Eventually, with increasing force, enough of the bonds and peaks will be damaged so that the two surfaces start to move past each other. The size of the static force at any time equals that of the force applied to the surfaces, up to a maximum value called the **limiting friction**. When the applied force is large enough to overcome this maximum value, the surfaces will start to move. The value of the limiting friction depends on the nature of the two surfaces in contact and the normal reaction force. As long as the force that you apply to the surfaces is less than the limiting friction, the static friction opposing the motion will be equal in size to that applied force.

Sliding friction

Sliding friction takes effect once the static friction has been overcome by the applied force and the materials are actually moving past each other. Sliding friction is weaker than static friction in magnitude but has a similar relationship with the nature of the surfaces and the normal reaction force.

Rolling friction

Rolling friction resists the motion of one surface rolling across another, such as in the case of a wheel rolling over a road surface. Rolling friction is much smaller than either static or sliding friction.

The coefficient of friction

The frictional force, $\boldsymbol{F_f}$, is directly proportional to the normal reaction force and the **coefficient of friction** (μ).

$$\boldsymbol{F_f} = \mu \boldsymbol{R}$$

The coefficient of friction is a measure of how easily two surfaces move across each other and varies widely depending upon the combination of surfaces being considered. The coefficient of friction for aluminium sliding across wood will differ from that for aluminium sliding across ice.

The coefficient of friction has no units as it is the ratio of the frictional force to the normal reaction force experienced by an object, and usually has a value less than 1.

TABLE 4.3 Values of static and sliding friction for surface pairs.

Surface pair	Static friction, μ_s	Sliding friction, μ_k
Rubber on dry concrete	0.9	0.7
Rubber on wet concrete	0.6	0.4
Rubber on wet ice	0.14	0.1
Wood on wood	0.25–0.6	0.3
Steel on smooth steel	0.15	0.09
Steel on oiled steel	0.03	0.03
Steel on glass	0.13	0.12
Steel on ice	0.1	0.02

4.3 SAMPLE PROBLEM 4

A 4 kg box is placed on a level piece of carpet.

(a) What is the minimum force required to move the box if $\mu_s = 0.5$?

(b) What would happen if a force of 10 N were applied sideways?

SOLUTION:

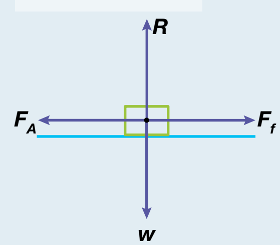
(a) Given: $m = 4\,\text{kg}$, $\mu_s = 0.5$

$$\begin{aligned} \boldsymbol{W} &= m\boldsymbol{g} \\ &= 4 \times 9.8 \\ &= 39.2\,\text{N} \\ \text{Therefore, } \boldsymbol{R} &= 39.2\,\text{N} \\ \text{Limiting friction} &= \mu_s \boldsymbol{R} \\ &= 0.5 \times 39.2 \\ &= 19.6\,\text{N} \end{aligned}$$

Therefore, at least 19.6 N will need to be applied to start the box moving.

(b) As 10 N is less than the limiting friction, the box will remain stationary.

FIGURE 4.10



4.3 Exercise 1

1 On the surface of Mars, gravitational acceleration is approximately 3.7 m s^{-2}. What would be the weight of an astronaut in full EVA suit on Mars if the total mass of the astronaut and gear was 130 kg?
2 A student stands on a set of bathroom scales in Sydney and sees that they measure his mass at 52 kg. If the student stands on these same scales in New York, would he notice a difference in the mass reading? (Assume that the scales are precise to the nearest 0.1 kg.)
3 In table 4.2, the North Pole and the Equator values for ***g*** are different although they are both recorded at the same altitude. What is the most likely cause of the difference between the values?
4 What will be the normal reaction force exerted on a 4.00 kg block by the table on which it rests?
5 Calculate the normal reaction force acting on a 1.20 kg book that is lying on a bench if you (a) press on it with a force of 20.0 N or (b) pull upwards on it with a force of 4.0 N.
6 An 8.0 kg wooden sled is pulled over the snow by means of a rope that makes an angle of 40° with the horizontal. If the rope has a tension of 70.0 N, what is the normal reaction force acting on the sled?
7 What would be the frictional force acting on a 3.0 kg box dragged over a surface with which it has a coefficient of friction of 0.4?
8 Calculate the coefficient of friction between a surface and a 2.0 kg object sliding across it if there is a frictional force of 3.5 N acting on the object.
9 A 12.0 kg pram is pulled over a footpath with a force of 130 N by means of a handle that makes an angle of 25° with the horizontal. What is the coefficient of friction if there is a frictional force of 8.0 N between the wheels of the pram and the footpath?

WORKING SCIENTIFICALLY 4.3

A variety of different materials are used for the soles of shoes with some sole materials providing more 'grip' than others. Explore the critical grip characteristics of shoes that are worn for different functions such as the shoes worn by roof tilers, dancers, tennis players and so on. In each case, consider the expected movement of the wearer and the type of surface that the shoes would typically be used on.

4.4 Newton's First Law of Motion

PHYSICS FACT

Sir Isaac Newton (1643–1727) was one of many famous scientists who were not outstanding students at school or university. He left school at 14 years of age to help his widowed mother on the family's farm. He found himself unsuited to farming, spending much of his time reading. At the age of 18, Isaac was sent to Cambridge University, where he showed no outstanding ability.

When Cambridge University was closed down in 1665 due to an outbreak of the plague, Newton went home and spent the next two years studying and writing. During this time, he developed the law of gravity, which explains the motion of the planets, and his three famous laws of motion. Newton also explained that white light consisted of many colours, and he invented calculus.

FIGURE 4.11 Sir Isaac Newton

Newton's laws of gravity and motion were not published until about twenty years later. They were published in Latin in a book entitled *Philosophia Naturalis Principia*. The cost of publishing was paid by Sir Edmond Halley, the person who discovered Halley's comet.

Newton later became a member of the British parliament, Warden of the Mint and president of the Royal Society. After his death in 1727, Newton was buried in Westminster Abbey, London, alongside many English kings, queens, political leaders and poets.

4.4.1 The net force

Most objects are acted upon by more than one force at a time. For example, an apple on a tabletop has a downward gravitational force and an upward normal reaction force acting upon it. A swimmer moving through a pool experiences the forces of gravity, buoyancy and drag as well as the force that he applies to move himself forward.

The vector sum of all the forces acting on an object is referred to as the **net force**, $\boldsymbol{F}_{net}$.

The net force acting on an object may be equal to zero, or may be a non-zero vector that causes a change in the object's state of motion — a concept that is at the heart of Newton's First Law of Motion.

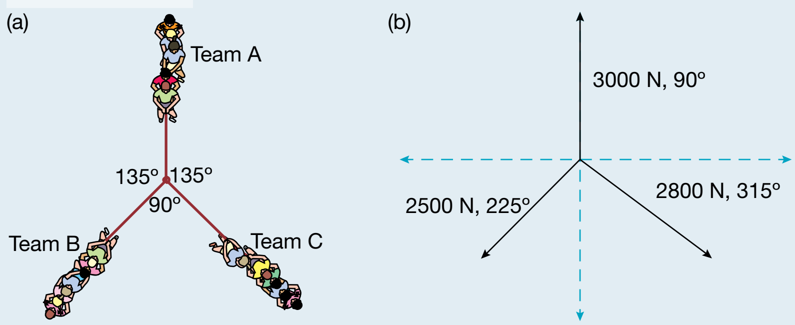
4.4 SAMPLE PROBLEM 1

In a three-way 'tug-of-war', the three teams (A, B and C) pull horizontally away from the knot joining the ropes with forces of 3000 N north, 2500 N south-west and 2800 N south-east respectively. Determine the net horizontal force exerted on the knot.

SOLUTION:

The forces acting on the central knot can be represented in the form of a vector diagram as shown:

FIGURE 4.12



Note that the vector diagram includes both the force magnitudes and their polar directions.

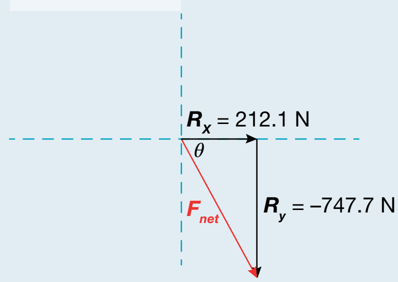
$$\begin{aligned} \boldsymbol{R}_x &= 3000\cos 90° + 2500\cos 225° + 2800\cos 315° \\ &= 0 + (-1767.8) + (1979.9) \\ &= 212.1\text{ N} \end{aligned}$$

$$\begin{aligned} \boldsymbol{R}_y &= 3000\sin 90° + 2500\sin 225° + 2800\sin 315° \\ &= (3000) + (-1767.8) + (-1979.9) \\ &= -747.7\text{ N} \end{aligned}$$

$$\begin{aligned} \boldsymbol{F}_{net} &= \sqrt{(212.1)^2 + (-747.7)^2} \\ &= 777.2\text{ N} \\ &\cong 800\text{ N (to 1 significant digit)} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}(-747.7/212.1) \\ &= -74° \\ &= 16°\text{ east of south (converted into a bearing)} \end{aligned}$$

FIGURE 4.13



Therefore, the net force acting on the knot is 800 N in a direction 16° east of south.

4.4.2 Forces in and out of balance

When the net force acting on an object is equal to zero, we say that the object is in equilibrium. An object in equilibrium keeps the same state of motion. If the object was stationary before the forces acting on it were applied, then it remains stationary afterwards. Similarly, an object travelling at constant speed in a fixed direction will not change its speed or direction of travel if the forces applied to it sum to give a net force equal to zero.

The effect of forces on the motion of objects is clearly described by Newton's First Law of Motion; that is:

Every object continues in its state of rest or uniform motion unless made to change by a non-zero net force.

Newton's First Law of Motion can also be expressed in terms of acceleration. When a non-zero net force acts on an object, it accelerates in the direction of the net force. The acceleration can take the form of a change in speed, change in direction or a change in both speed and direction.

Newton's First Law of Motion can be illustrated by flicking a coin across a tabletop. A coin flicked across a table changes its motion because the net force on it is not zero. In fact, it slows down because the direction of the net force is opposite to the direction of motion. The vertical forces, weight and the support force of the table balance each other. The only 'unbalanced' force is that of friction. The surface of the table applies a frictional force to the surface of the coin whenever there is an external force pushing the coin.

A coin pushed steadily across a table moves in a straight line at constant speed as long as the net force is zero (that is, as long as the magnitude of the pushing force is equal to the magnitude of the friction). The coin will speed up if you push horizontally with a force greater than the friction. It will slow down if the force of friction is greater than the horizontal pushing force. That is what happens when you stop pushing.

4.4.3 Inertia

Newton's First Law of Motion (described in the last section) is often referred to as the Law of Inertia. The **inertia** of an object is its tendency to resist changes to its motion. Inertia is not a force; it is a property of all objects. The inertia of an object depends only on its mass. For example, a large adult on a playground swing is more difficult to get moving than a small child on the same swing. It is also more difficult to stop or change the direction of motion of a large adult than a small child.

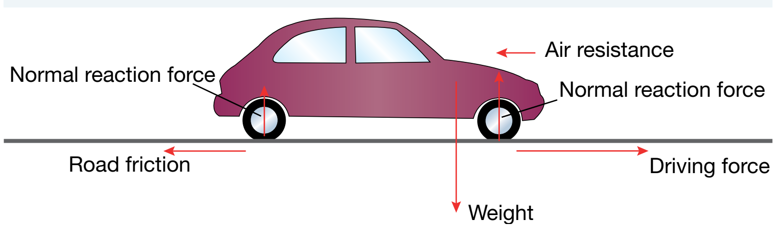
Your inertia can be a serious problem when you are in or on a fast-moving object. As a passenger in a fast-moving car that suddenly stops (e.g. in an emergency or a collision), you would continue to move at high speed until a non-zero net force stopped you. Your inertia would resist the change in motion. The car would have inertia as well. However, it would have stopped as a result of braking or colliding with another object. If you were wearing a seat belt, it would apply a force that would stop you, along with the rest of the car. Without it, you would collide with part of the car — usually the dashboard or the windscreen. If you were to crash into a solid object while riding a bicycle, you would continue to move forward after the bicycle stopped.

Your inertia is also evident when you are in a vehicle that starts rapidly or is pushed forward in a collision. Your seat pushes your body forward along with the car. However, without a properly fitted headrest, your head remains at rest until it is pulled forward by your spine. The resulting injuries are called whiplash injuries. The purpose of a headrest is to ensure that when a large net force pushes the car forward, your head is pushed forward at the same rate as the rest of your body.

Cruising along

The forces acting on a car being driven along a straight horizontal road are shown in figure 4.14 and described below.

FIGURE 4.14 The motion of a car on a horizontal road depends on the net force acting on it.



- *Weight*. A medium-sized sedan containing a driver and passenger has a weight of about 1.5×10^4 N. The weight acts through the centre of mass, or balancing point, of the car. This is normally closer to the front of the car than the back. This is because the engine at the front is the heaviest part of the car.
- *Normal reaction force*. A normal reaction force pushes up on all four wheels. Its magnitude is usually greater at the front wheels than the rear wheels. On a horizontal road, the sum of these normal reaction forces must have the same magnitude as the weight. What do you think would happen if this was not the case?
- *Driving force*. This is provided by the road and is applied to the driving wheels. The driving wheels are turned by the motor. In most cars, either the front wheels or the rear wheels are the driving wheels. The motor of a four-wheel-drive vehicle turns all four wheels. As the tyre pushes back on the road, the road pushes forward on the tyre, propelling the car forward. The forward push of the road on the tyre is a type of friction commonly referred to as traction, or grip. If the tyres do not have enough tread, or the road is icy, there is not enough friction to push the car forward and the tyre slides on the road. The wheel spins and the car skids. The car cannot be propelled forward as effectively. Skidding also occurs if the motor turns the driving wheels too fast.
- *Road friction*. The non-driving wheels of front-wheel-drive cars roll as they are pulled along the road by the moving car. In older cars, the non-driving wheels are usually at the front. They are pushed along the road by the moving car. Rolling friction acts on the non-driving wheels in a direction opposite to the direction of movement of the car. When the driving wheels are not being turned by the motor, rolling friction opposes the forward movement of all four wheels. When the brakes are applied, the wheels to which the brakes are attached are made to turn too slowly for the speed at which the car is moving. They are no longer rolling freely. This increases the road friction greatly and the car eventually stops. If the brakes are applied hard enough the wheels stop completely, or lock, and the car goes into a skid. The sliding friction that exists when the car is skidding is less than the friction that exists when the wheels are rolling just a little.
- ***Air resistance***. The drag, or air resistance, acting on the car increases as the car moves faster. Air resistance is a form of friction that can be reduced by streamlining the vehicle. This involves shaping the vehicle so that it disturbs the air less.

The net force acting on the car in figure 4.14 is zero. It is therefore moving along the road at constant speed. We know that it is moving to the right because both the air resistance and road friction act in a direction opposite to the direction of motion. If the car were stationary, neither of these forces would be acting at all.

- When the driver pushes down on the accelerator, the driving force increases. The car speeds up until the sum of the air resistance and road friction grow large enough to balance it. Then, once again, the car would be moving at a constant, although higher, speed.
- When the driver stops pushing down on the accelerator, the motor stops turning the driving wheels and the driving force becomes zero. The net force would be to the left. As the car slows down, the air resistance and road friction would gradually decrease until the car comes to a stop. The net force on the car becomes zero until the driving force is restored.

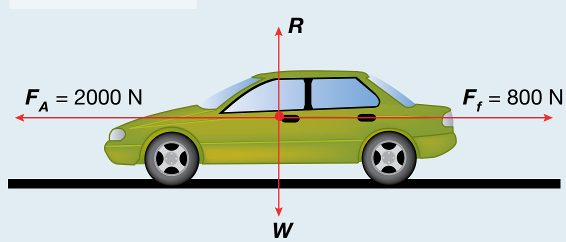
4.4 SAMPLE PROBLEM 2

What is the net force acting on a 1200 kg car if the car is acted upon by a driving force of 2000 N and a frictional force of 800 N?

SOLUTION:

$$\begin{aligned} F_{net} &= F_A - F_f \\ &= 2000 - 800 \\ &= 1200\text{ N, forwards} \end{aligned}$$

FIGURE 4.15



PHYSICS IN FOCUS

Anti-lock brake systems

When car brakes are applied too hard, as they often are when a driver panics in an emergency, the wheels lock. The car skids, steering control is lost and the car takes longer to stop than if the wheels were still rolling. Drivers are often advised to 'pump' the brakes in wet conditions to prevent locking. This involves pushing and releasing the brake pedal in quick succession until the car stops. This, however, is very difficult to do in an emergency situation.

Anti-lock brake systems (ABSs) allow the wheels to keep rolling no matter how hard the brakes are applied. A small computer attached to the braking system monitors the rotation of the wheels. If the wheels lock and rolling stops, the pressure on the brake pads (or shoes) that stops the rotation is reduced for a very short time. This action is repeated up to 15 times each second. Anti-lock brake systems are most effective on wet roads. However, even on a dry surface, braking distances can be reduced by up to 20 per cent.

Rolling downhill

A car left parked on a hill will begin to roll down the hill with increasing speed if it is left out of gear and the handbrake is off. Figure 4.16 shows the forces acting on such a car. In order to simplify the diagram, all the forces are drawn as if they were acting through the centre of mass of the car. The direction of net force acting on the car is down the hill. It is clear that the pull of gravity (the weight of the car) is a major contributor to the downhill motion of the car.

It is often useful to divide vectors into parts called **components**. Figure 4.17 shows how the weight can be broken up, or resolved into two components — one parallel to the slope and one perpendicular to the slope. Notice that the vector sum of the components is the weight. By resolving the weight into these two components, two useful observations can be made:

1. The normal reaction force is balanced by the component of weight that is perpendicular to the surface. The net force has no component perpendicular to the road surface. This must be the case because there is no change in motion perpendicular to the slope.
2. The net force is the vector sum of the component of the weight that is parallel to the surface, and the sum of road friction and air resistance.

FIGURE 4.16 A simplified diagram showing the forces acting on a car rolling down a slope.

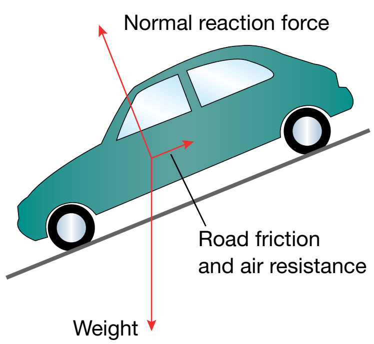
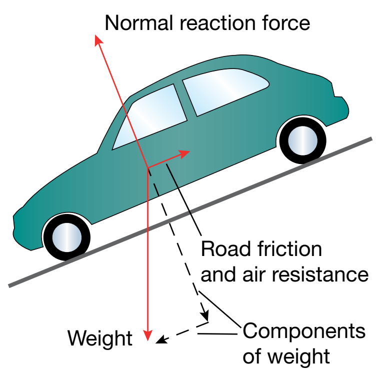


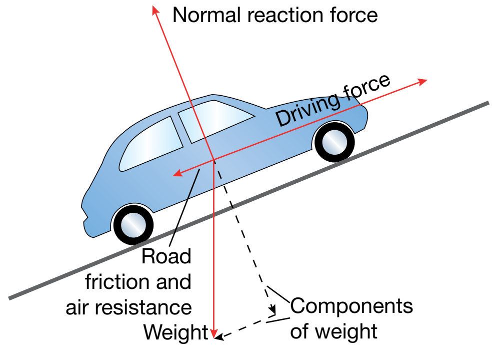
FIGURE 4.17 Vectors can be resolved into components. In this case, the weight has been resolved into two components. The net force is parallel to the slope, and the car will accelerate down the slope.



Driving uphill

When a car is driven up the slope, as shown in figure 4.18, the driving force is greater than or equal to the magnitude of the sum of the air resistance, road friction and the component of the weight that is parallel to the surface.

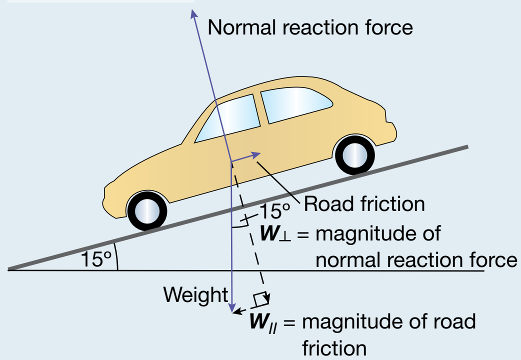
FIGURE 4.18 This diagram shows the forces acting on a car driven up a slope. In this case, the car is accelerating up the slope.



4.4 SAMPLE PROBLEM 3

A car of mass 1600 kg left parked on a steep but rough road begins to roll down the hill. After a short while it reaches a constant speed. The road is inclined at 15° to the horizontal. Its speed is sufficiently slow that the air resistance is insignificant and can be ignored. Determine the magnitude of the road friction on the car while it is rolling at constant speed.

FIGURE 4.19



SOLUTION:

Because the car is rolling at constant speed, the net force acting on it must be zero. The weight, $\boldsymbol{W}$, can be resolved into two components — one down the slope, $\boldsymbol{W}_{//}$ and one perpendicular to it, $\boldsymbol{W}_{\perp}$. The perpendicular component of the weight, $\boldsymbol{W}_{\perp}$, is balanced by the normal reaction force. The magnitude of the road friction must be equal to the magnitude of the weight component down the slope, $\boldsymbol{W}_{//}$.

In the triangle formed by the weight and its components:

$$\begin{aligned}\sin 15° &= \frac{W_{//}}{W}\\ W_{//} &= W\sin 15°\\ &= mg\sin 15°\\ &= 1600 \times 9.8 \times \sin 15° \text{ (substituting data)}\\ &= 4.1 \times 10^3\text{ N.}\end{aligned}$$

The magnitude of the road friction is therefore 4100 N while the car is rolling with a constant speed.

4.4 Exercise 1

1. A box is affected by three forces acting on it: 100 N at 30° north of east, 200 N west and 200 N south. What is the net force acting on the box?
2. What is the net force acting on a book lying on a desk? Explain.
3. A chew toy is being pulled in different directions by two dogs. One dog pulls the toy east with a force of 6 N while the other pulls it south-west with a force of 5 N. In what direction will the chew toy move?
4. A team of donkeys pulls a cart full of tourists up a 20° hill in Greece. If the cart (including tourists) has a mass of 600 kg, how much force must be exerted by the donkeys on the cart to move it up the hill at a steady speed? (Ignore friction.)

5 A 5000 kg truck is parked on a road surface inclined at an angle of 20° to the horizontal. Calculate the component of the truck's weight that is:
(a) down the slope of the road
(b) perpendicular to the slope of the road.

6 In the case of the car in 4.4 sample problem 3, what is:
(a) the component down the road surface of the normal reaction force acting on it
(b) the normal reaction force?

7 A car of mass 1400 kg is left parked on a 10° hill. Unknown to the owner, the handbrake fails and the car starts to move down the hill. What is the car's acceleration if
(a) the effect of friction and air resistance is ignored
(b) air resistance is negligible but the coefficient of friction is 0.14?

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4.5 Newton's Second Law of Motion

Casual observations indicate that the acceleration of a given object increases as the net force on the object increases. It is also clear that lighter objects change their velocity at a greater rate than heavier objects when the same force is applied.

It can be shown experimentally that the acceleration, $\boldsymbol{a}$, of an object is:

- proportional to the net force, $\boldsymbol{F}_{\text{net}}$, applied to it
- inversely proportional to the mass, m.

$$\boldsymbol{a} \propto \boldsymbol{F}_{\text{net}} \quad \boldsymbol{a} \propto \frac{1}{m}$$

Thus:

$$\boldsymbol{a} \propto \frac{\boldsymbol{F}_{\text{net}}}{m}$$

$$\Rightarrow \boldsymbol{a} = \frac{\mathrm{k}\boldsymbol{F}_{\text{net}}}{m}$$

where k = a constant of proportionality.

The SI unit of force, the newton (N), is defined such that a net force of 1 N causes a mass of 1 kg to accelerate a 1 m s^{-2}. The value of the constant, k, is 1. It has no units. Thus:

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{\text{net}}}{m}$$

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}.$$

The previous equation describes Newton's Second Law of Motion. This statement of Newton's Second Law allows you to:

- determine the net force acting on an object without knowing any of the individual forces acting on it. The net force can be deduced as long as you can measure or calculate (using formulas or graphs) the acceleration of a known mass.

- determine the mass of an object. You can do this by measuring the acceleration of an object on which a known net force is exerted.
- predict the effect of a net force on the motion of an object of known mass.

4.5 SAMPLE PROBLEM 1

A 65 kg physics teacher, starting from rest, glides gracefully down a slide in the local playground. The net force on her during the slide is a constant 350 N. How long will it take her to reach the bottom of the 8.0 m slide?

SOLUTION:

$$\begin{aligned}\boldsymbol{F}_{\text{net}} &= m\boldsymbol{a}\\ \Rightarrow 350\,\text{N} &= 65\,\text{kg} \times \boldsymbol{a} \quad \text{(substituting magnitudes)}\\ \Rightarrow \boldsymbol{a} &= \frac{350\,\text{N}}{65\,\text{kg}}\\ &= 5.4\,\text{m s}^{-2}\end{aligned}$$

$$\begin{aligned}\text{Thus, } u &= 0, a = 5.4\,\text{m s}^{-2}, s = 8.0\,\text{m and } v = ?\\ \text{apply } v^2 &= u^2 + 2as\\ \Rightarrow v^2 &= 0 + 2 \times 5.4\,\text{m s}^{-2} \times 8.0\,\text{m}\\ &= 86.4\,\text{m}^2\,\text{s}^{-2}\\ \Rightarrow v &= 9.3\,\text{m s}^{-1}\end{aligned}$$

4.5.1 Applying Newton's second law in real life

Many of the sample problems and exercises in this chapter are simplifications of the much more complicated interactions that occur when forces act on an object. For example, when a tennis ball is served, the force applied by the tennis racquet, the weight of the ball, air resistance and the compressive forces acting on the ball and the racquet strings all act to change the motion of the struck ball. However, when calculating the acceleration of this ball, it may be that only the force of the racquet on the ball is used in the calculation. The other forces are assumed to be **negligible**; that is, they are so small compared with the force of the racquet on the ball that they can be ignored while the racquet is in contact with the ball.

Surfaces might be described as 'smooth' in sample problems and exercises. This description does not imply that the surfaces are without friction (a near impossibility as we have seen in the previous section). Instead, it is included so that you would know that the force of friction is so small as to be considered negligible.

The event described in 4.5 Sample Problem 1 was also simplified. It is unlikely that the net force on the teacher gliding down the slide would be constant.

These simplifications describe an idealised environment in which we may develop an understanding of basic physics concepts and ideas. While these simplifications are often employed with minimal effect on the accuracy of the calculations made for a particular situation, caution is needed when making **idealisations**. For example, it would be unreasonable to ignore the air resistance on a tennis ball while it is soaring through the air at 150 km h^{-1}(42 m s^{-1}) after the serve was completed as, at such speeds, the force of air resistance has a significant effect on the ball's motion.

4.5 SAMPLE PROBLEM 2

When the head of an 80 kg bungee jumper is 24 m from the surface of the water below, her velocity is 16 m s^{-1} downwards and the tension in the bungee cord is 1200 N. Air resistance can be assumed to be negligible.

(a) What is her acceleration at that instant?

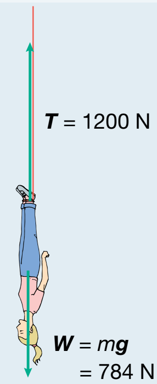
(b) If her acceleration remained constant during the rest of her fall, would she stop before hitting the water?

SOLUTION:

Firstly, a diagram must be drawn to show the forces acting on the bungee jumper (see figure 4.20). The only two forces that need to be considered are the tension ($\boldsymbol{T}$) in the cord and the jumper's weight ($\boldsymbol{W}$).

The bungee jumper's weight,
$$\begin{aligned}\boldsymbol{W} &= m\mathbf{g}\\ &= 80\,\text{kg} \times 9.8\,\text{m s}^{-2}\text{ down}\\ &= 784\,\text{N down.}\end{aligned}$$

FIGURE 4.20



(a) Apply Newton's second law to determine the acceleration. Assign up as positive for this part of the question.

$$\begin{aligned}\boldsymbol{F}_{\text{net}} &= m\boldsymbol{a}\\ \Rightarrow \boldsymbol{T} - \boldsymbol{W} &= m\boldsymbol{a} && \text{(assigning up as positive)}\\ \Rightarrow 1200\,\text{N} - 784\,\text{N} &= 80\,\text{kg} \times \boldsymbol{a} && \text{(substituting data)}\\ \Rightarrow 80\,\text{kg} \times \boldsymbol{a} &= 416\,\text{N}\\ \Rightarrow \boldsymbol{a} &= \frac{416\,\text{N}}{80\,\text{kg}}\\ \Rightarrow \boldsymbol{a} &= 5.2\,\text{m s}^{-2}\text{ in an upwards direction}\end{aligned}$$

(b) If the jumper's acceleration were constant, one of the constant acceleration formulae could be used to answer this question. Assign down as positive for this part of the question as the bungee jumper has a downwards initial velocity and displacement during the time period being considered.

$$\begin{aligned}u &= 16\,\text{m s}^{-1}, v = 0, a = -5.2\,\text{m s}^{-2}, s = ?\\ v^2 &= u^2 + 2as\\ \Rightarrow 0 &= (16\,\text{m s}^{-1})^2 + 2\,(-5.2\,\text{m s}^{-2})s && \text{(substituting data)}\\ \Rightarrow 0 &= 256\,\text{m}^2\,\text{s}^{-2} - 10.4\,\text{m s}^{-2} \times s\\ \Rightarrow 10.4\,\text{m s}^{-2}s &= 256\,\text{m}^2\,\text{s}^{-2}\\ \Rightarrow s &= 24.6\,\text{m} && \text{(dividing both sides by } 10.4\,\text{m s}^{-2}\text{)}\end{aligned}$$

Alas, the bungee jumper would not stop in time. However, do not be upset! In practice, the acceleration of the bungee jumper would not be constant. The tension in the cord would increase as she fell. Therefore, the net force on her would increase and her upwards acceleration would be greater in magnitude than the calculated value. She will therefore almost certainly come to a stop in a distance considerably less than that calculated.

4.5 SAMPLE PROBLEM 3

A waterskier of mass 80 kg, starting from rest, is pulled in a northerly direction by a horizontal rope with a constant tension of 240 N. After 6.0 s, he has reached a speed of 12 m s^{-1}.

(a) What is the net force on the skier?

(b) If the tension in the rope were the only horizontal force acting on the skier, what would his acceleration be?

(c) What is the sum of the resistance forces on the skier?

SOLUTION:

A diagram must be drawn to show the forces acting on the skier. Assign the positive direction as north as shown in figure 4.21.

(a) The net force cannot be determined by adding the individual force vectors because the resistance forces are not given, nor is there any information in the question to suggest that they can be ignored. It can, however, be calculated by applying Newton's second law if the acceleration is known.

$$u = 0, v = 12\ \text{m s}^{-1}, t = 6.0\ \text{s}, a = ?$$
$$v = u + at$$
$$12\ \text{m s}^{-1} = 0 + a \times 6.0\ \text{s} \quad \text{(substituting data)}$$
$$\Rightarrow a \times 6.0\ \text{s} = 12\ \text{m s}^{-1} \quad \text{(dividing both sides by 6.0 s)}$$
$$\Rightarrow a = 2.0\ \text{m s}^{-2}$$

Thus,

$$\begin{aligned} \boldsymbol{F}_{\text{net}} &= m\boldsymbol{a} \\ &= 80\ \text{kg} \times 2.0\ \text{m s}^{-2}\ \text{north} \\ &= 160\ \text{N north.} \end{aligned}$$

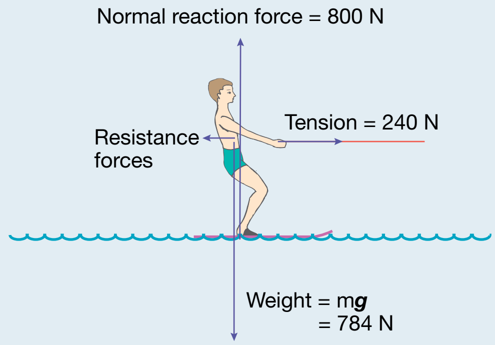
The net force on the skier is 160 N north.

(b) The net force on the skier is horizontal. If the tension were the only horizontal force acting on the skier, it would be equal to the net force since the vertical forces on the skier add to zero.

Thus, the acceleration would be given by:

$$\begin{aligned} \boldsymbol{a} &= \frac{\boldsymbol{F}_{\text{net}}}{m} \\ &= \frac{240\ \text{N north}}{80\ \text{kg}} \\ &= 3.0\ \text{m s}^{-2}\ \text{north.} \end{aligned}$$

FIGURE 4.21



(c) The sum of the resistance forces (friction caused by the water surface and air resistance) on the skier is the difference between the net force and the tension.

$$\begin{aligned} \text{sum of resistance forces} &= \boldsymbol{F}_{\text{net}} - \text{tension} \\ &= 160\ \text{N north} - 240\ \text{N north} \\ &= 80\ \text{N south} \end{aligned}$$

4.5 SAMPLE PROBLEM 4

A loaded supermarket shopping trolley with a total mass of 60 kg is left standing on a footpath that is inclined at an angle of 30° to the horizontal. As the tired shopper searches for his car keys, he fails to notice that the trolley is beginning to roll away. It rolls in a straight line down the footpath for 9.0 s before it is stopped by an alert (and very strong) supermarket employee. Find:

(a) the speed of the shopping trolley at the end of its roll

(b) the distance covered by the trolley during its roll.

Assume that the footpath exerts a constant friction force of 270 N on the runaway trolley.

SOLUTION:

A diagram must be drawn to show the three forces acting on the shopping trolley (see figure 4.22). Air resistance is not included as it is negligible. The forces should be shown as acting through the centre of mass of the loaded trolley. The components of the weight, which are parallel and perpendicular to the footpath surface, should also be shown on the diagram.

The motion of the runaway shopping trolley, originally at rest, can be described by using the information provided, along with Newton's second law, which is used to determine its acceleration.

The net force can be found by 'breaking up' the weight into two components — one parallel to the footpath surface ($\boldsymbol{W}_{/\!/}$) and the other perpendicular to the surface ($\boldsymbol{W}_{\perp}$). We know that ($\boldsymbol{W}_{\perp}$) is balanced by the normal reaction force because there is clearly no acceleration of the trolley perpendicular to the surface. The net force is therefore down the slope and has a magnitude of:

$$\begin{aligned}\boldsymbol{F}_{\text{net}} &= \boldsymbol{W}_{//} - \text{friction}\\ &= \boldsymbol{mg}\sin 30° - 270\,\text{N}\\ &= 588\,\text{N}\sin 30° - 270\,\text{N}\\ &= 294\,\text{N} - 270\,\text{N}\\ &= 24\,\text{N}.\end{aligned}$$

Newton's second law can now be applied to determine the acceleration of the trolley down the slope. Assign the positive direction as down the slope.

$$\begin{aligned}\boldsymbol{F}_{\text{net}} &= m\boldsymbol{a}\\ \Rightarrow 24\,\text{N down slope} &= 60\,\text{kg} \times \boldsymbol{a}\\ \boldsymbol{a} &= \frac{24\,\text{N down slope}}{60\,\text{kg}}\\ &= 0.40\,\text{m s}^{-2}\text{ down slope}\end{aligned}$$

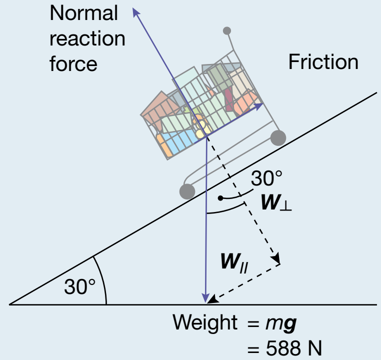
The final speed and distance travelled by the trolley can now be calculated.

$u = 0, a = 0.40\,\text{m s}^{-2}, t = 9.0\,\text{s}, v = ?, s = ?$

$$\begin{aligned}v &= u + at\\ &= 0 + 0.40\,\text{m s}^{-2} \times 9.0\,\text{s}\\ &= 3.6\,\text{m s}^{-1}\end{aligned}$$

$$\begin{aligned}s &= ut + \tfrac{1}{2}at^2\\ &= 0 \times 9.0\,\text{s} + \tfrac{1}{2} \times 0.40\,\text{m s}^{-2} \times (9.0\,\text{s})^2\\ &= \tfrac{1}{2} \times 0.40\,\text{m s}^{-2} \times 81\,\text{s}^2\\ &= 16.2\,\text{m}\end{aligned}$$

FIGURE 4.22



$W_{//}$ = Component of weight parallel to slope

$W_{\perp}$ = Component of weight perpendicular to slope

At the end of its roll, the trolley was travelling at a speed of $3.6\,\text{m s}^{-1}$ and had moved a distance of 16.2 m down the slope.

4.5 SAMPLE PROBLEM 5

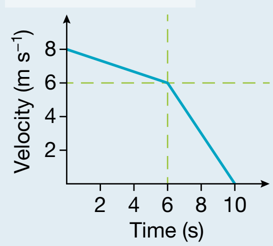
The velocity-versus-time graph in figure 4.23 describes the motion of a 45 kg girl on rollerblades as she rolls from a horizontal concrete path onto a rough horizontal gravel path.

(a) What was the magnitude of the net force on the girl on the concrete surface?

(b) If the only horizontal force acting on the blades is the friction force applied by the path, what is the value of the following ratio?

$$\frac{\text{friction force of gravel path on rollerblades}}{\text{friction force of concrete path on rollerblades}}$$

FIGURE 4.23



SOLUTION:

(a) The magnitude of acceleration of the girl while on the concrete surface can be determined from the first 6.0 s of the motion described by the graph. It is equal to the gradient of the graph.

$$\begin{aligned}a &= \frac{\text{rise}}{\text{run}}\left(= \frac{\Delta v}{\Delta t}\right)\\ &= \frac{-2.0\,\text{m s}^{-1}}{6.0\,s}\\ &= -0.33\,\text{m s}^{-2}\end{aligned}$$

The magnitude of the net force on the girl is therefore:

$$\begin{aligned} F_{net} &= m\boldsymbol{a} \\ &= 45\,\text{kg} \times 0.33\,\text{m s}^{-2} \\ &= 15\,\text{N}. \end{aligned}$$

(b) If the only horizontal force acting on the rollerblades is friction, the net force on the girl is the same as the friction force on the blades. Thus:

$$\begin{aligned} \frac{\text{friction force of gravel path on rollerblades}}{\text{friction force of concrete path on rollerblades}} &= \frac{\boldsymbol{F}_{\text{net}}\text{ on girl while on gravel}}{\boldsymbol{F}_{\text{net}}\text{ on girl while on concrete}} \\ &= \frac{m\boldsymbol{a}\text{ on gravel}}{m\boldsymbol{a}\text{ on concrete}} \\ &= \frac{\boldsymbol{a}\text{ (during last 4.0 s)}}{\boldsymbol{a}\text{ (during first 6.0 s)}} \\ &= \frac{\text{gradient (for last 4.0 s)}}{\text{gradient (for first 6.0 s)}} \\ &= \frac{\dfrac{-6.0\,\text{m s}^{-1}}{4.0\,\text{s}}}{\dfrac{-2.0\,\text{m s}^{-1}}{6.0\,\text{s}}} \\ &= 4.5. \end{aligned}$$

4.5.2 Falling down

Objects that are falling (or rising) through the air are generally subjected to two forces — weight and air resistance. The weight of the object is constant. The magnitude of the air resistance, however, is not constant. It depends on many factors, including the object's speed, surface area and density. It also depends on the density of the body of air through which the object is falling. The air resistance is always opposite to the direction of motion. The net force on a falling object of mass m and weight $\boldsymbol{W}$ can therefore be expressed as:

$$\begin{aligned} \boldsymbol{F}_{net} &= m\boldsymbol{a} \quad \text{(where } \boldsymbol{a} \text{ is the acceleration of the object)} \\ \boldsymbol{W} - \text{ air resistence} &= m\boldsymbol{a}. \end{aligned}$$

When dense objects fall through small distances near the surface of the Earth it is usually quite reasonable to assume that the air resistance is negligible. Thus:

$$\begin{aligned} \boldsymbol{W} &= \text{m}\boldsymbol{a} \\ \text{m}\boldsymbol{g} &= \text{m}\boldsymbol{a} \quad \text{(where } \boldsymbol{g} \text{ is the gravitational field strength)} \\ \boldsymbol{g} &= \boldsymbol{a}. \end{aligned}$$

The acceleration of a body in free fall in a vacuum or where air resistance is negligible is equal to the gravitational field strength. At the Earth's surface, where $\boldsymbol{g} = 9.8\,\text{N kg}^{-1}$, this acceleration is $9.8\,\text{m s}^{-2}$.

If a bowling ball, a tennis ball and a table-tennis ball were dropped at the same instant from a height of 2.0 m in a vacuum, they would all reach the ground at the same time. This is because each ball would have the same initial velocity of zero and the same acceleration.

If, however, the balls are dropped either in a classroom or outside, the table-tennis ball will reach the ground a moment later than the other two balls.

FIGURE 4.24 A bowling ball, a golf ball and a table-tennis ball dropped from a height of 2.0 m. Which one would you expect to reach the ground first?

WORKING SCIENTIFICALLY 4.4

Terminal velocity is reached by a falling object when the upward force of air resistance acting on the object is equal in magnitude to the downward force of the object's weight, causing the object to fall at a constant speed. A parachute attached to an object allows a larger drag force to be exerted so that terminal velocity can be reached more quickly.

Design, build and test a parachute system that, once attached to a 100 g mass, allows it to fall for at least 2 metres at terminal velocity. You will also need to devise a method to *prove* that this objective has been reached.

The acceleration of each of the balls is:

$$\begin{aligned} \boldsymbol{a} &= \frac{\boldsymbol{F}_{\text{net}}}{m} \\ &= \frac{mg - \boldsymbol{A}}{m} \qquad \text{(where } \boldsymbol{A} \text{ is air resistance)} \\ &= \frac{mg}{m} - \frac{\boldsymbol{A}}{m} \\ &= g - \frac{\boldsymbol{A}}{m} \end{aligned}$$

The acceleration depends on the air resistance and the mass of each ball as well as $\boldsymbol{g}$.

The term $\frac{\boldsymbol{A}}{m}$ is very small for the bowling ball and the golf ball. Even though the air resistance on the table-tennis ball is small, its mass is also small and the term $\frac{\boldsymbol{A}}{m}$ is not as small as it is for the other two balls.

WARNING: Do not drop a bowling ball. If you wish to try this experiment, replace the bowling ball with a medicine ball and keep your feet out of the way!

4.5 Exercise 1

1. What is the magnitude of the average force applied by a tennis racquet to a 58 g tennis ball during service if the average acceleration of the ball during contact with the racquet is 1.2×10^4 m s^{-2}?
2. A toy car is pulled across a smooth, polished horizontal table with a spring balance. The reading on the spring balance is 2.0 N and the acceleration of the toy car is measured to be 2.5 m s^{-2}. What is the mass of the toy car? (Note that, because the table is described as smooth and polished, friction can be ignored.)
3. A loaded sled with a mass of 60 kg is being pulled across a level snow-covered field with a horizontal rope. It accelerates from rest over a distance of 9.0 m, reaching a speed of 6.0 m s^{-1}. The tension in the rope is a constant. The frictional force on the sled is 200 N. Air resistance is negligible.
   (a) What is the acceleration of the sled?
   (b) What is the magnitude of the tension in the rope?
4. A cyclist rolls freely from rest down a slope inclined at 20° to the horizontal. The total mass of the bicycle and cyclist is 100 kg. The bicycle rolls for 12 seconds before reaching a horizontal surface. The surface exerts a constant friction force of 300 N on the bicycle.
   (a) What is the net force on the bicycle (including the cyclist)?
   (b) What is the acceleration of the bicycle?
   (c) What is the speed of the bicycle when it reaches the horizontal surface?
5. If the velocity-versus-time graph in 4.5 SAMPLE PROBLEM 5 was applied to a car of mass 1200 kg on two road surfaces, what net force (in magnitude) acts on the car during:
   (a) the first 6.0 seconds
   (b) the final 4.0 seconds?

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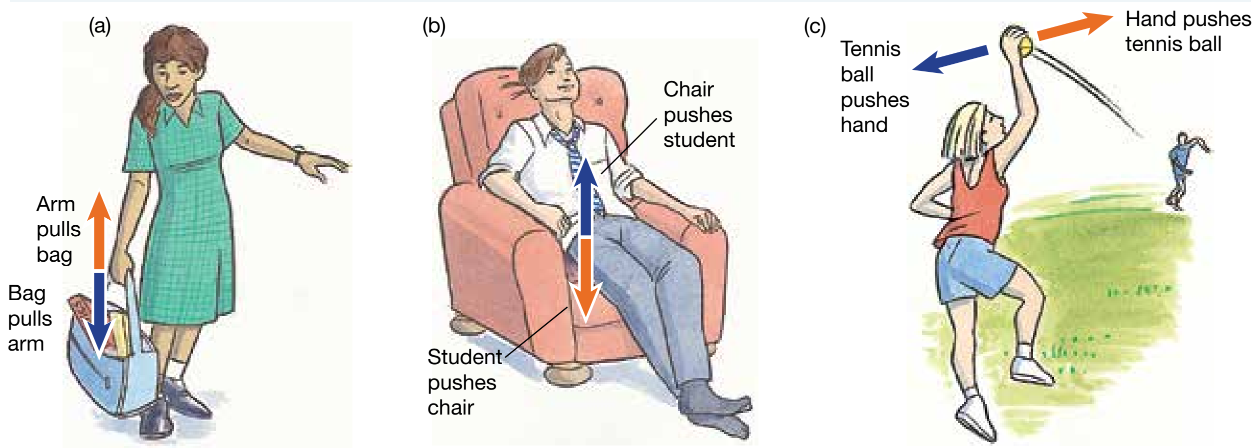
4.6 Newton's Third Law of Motion

Forces always act in pairs (see figure 4.25). When you lift a heavy schoolbag you can feel it pulling down on you. When you slump into a comfortable chair at the end of a long day at school you can feel it pushing up on you. When you catch a fast-moving ball you can feel it push on your hand as you apply the force to stop it.

FIGURE 4.25 Forces always act in pairs. (a) The arm pulls up on the bag; the bag pulls down on the arm. (b) The student pushes down and back on the chair; the chair pushes up and forward on the student. (c) The hand pushes on the ball; the ball pushes on the hand.



Sir Isaac Newton recognised that forces always acted in pairs in his Third Law of Motion, which is most commonly stated as:

For every action there is an equal and opposite reaction.

A more precise statement of the Newton's Third Law is:

Whenever an object applies a force (an action) to a second object, the second object applies an equal and opposite force (called a reaction) to the first object.

It is very important to remember that the forces that make up action–reaction pairs act on different objects. That is why it makes no sense to add them together so that they 'cancel out'. The motion of each object in figure 4.25 is determined by the net force acting on it.

The net force on the student sitting in the chair in figure 4.25b is zero because the upward push of the chair is balanced by the downward force of gravity, or weight of the student.

The tennis ball in figure 4.25c slows down because the net force on the tennis ball is not zero. The push of the hand on the ball is much larger than any of the other forces acting on the ball. The net force on the hand is zero if the hand does not change its motion during the catch. The push of the ball is balanced by the push of arm muscles on the hand.

4.6.1 Newton's Third Law of Motion in action

The rowing boat shown in figure 4.26 is propelled forward by the push of water on the oars. As the face of each oar pushes back on the water, the water pushes back with an equal and opposite force on each oar. The push on the oars, which are held tightly by the rowers, propels them and their boat forward. A greater push (or action) on the water results in a greater push (or reaction) on the oar.

In fact, none of your forward motion, whether you are on land, water or in the air, could occur without an action–reaction pair of forces.

FIGURE 4.26 This rowing team relies on a reaction force to propel itself forward.

- When you swim, you push the water backwards with your hands, arms and legs. The water pushes in the opposite direction, propelling you forwards.
- In order to walk or run, you push your feet backwards and down on the ground. The ground pushes in the opposite direction, pushing forwards and up on your feet.
- The forward driving force on the weels of a car is the result of a push back on the road by the wheels.
- A jet or a propeller-driven plane is thrust forward by air. The jet engines or propellers are designed to push air backward with a very large force. The air pushes forward on the plane with an equally large force.

4.6.2 Multiple bodies

Figure 4.27 shows a small dinghy being pulled by a larger boat. The forces acting on the larger boat are labelled in red while the forces acting on the small dinghy are labelled in green. Newton's Second Law of Motion can be applied to each of the two boats. Figure 4.28 shows only the forces acting on the system of the two boats and the rope joining them. When Newton's Second Law of Motion is applied to the whole system, the system is considered to be a single object.

The thrust that acts on the larger boat and the system is provided by the water. The propeller of the larger boat pushes back on the water and the water pushes forward on the propeller blades. The only force that can cause the small dinghy to accelerate forward is the tension in the rope. If the tension in the rope is greater than the resistance forces on the dinghy, it will accelerate. If the tension in the rope is equal to the resistance forces on the dinghy, it will move with a constant velocity. If the tension in the

FIGURE 4.27 This diagram shows the forces acting on each of the two boats.

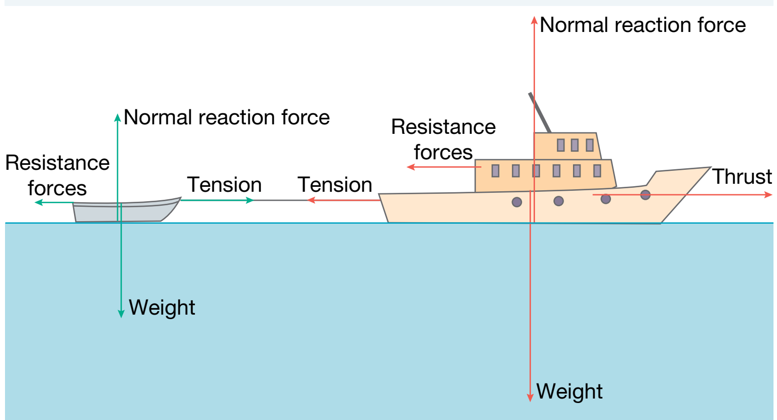
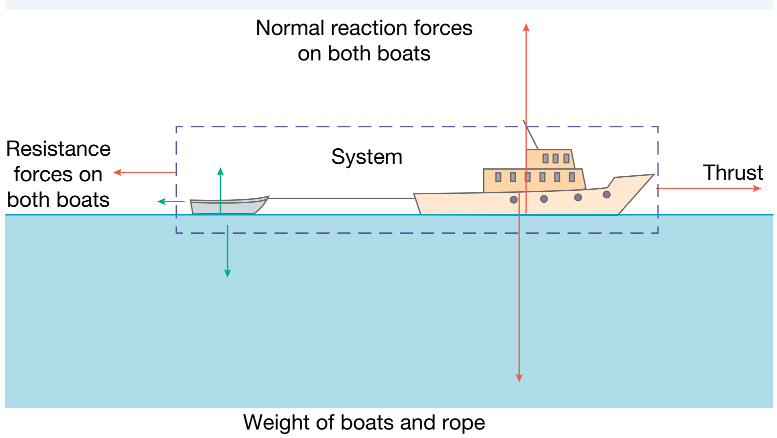


FIGURE 4.28 This diagram shows the forces acting on the system. The system consists of the two boats and the rope joining them.



rope is less than the resistance forces on the dinghy, it will slow down. That is, its acceleration will be negative.

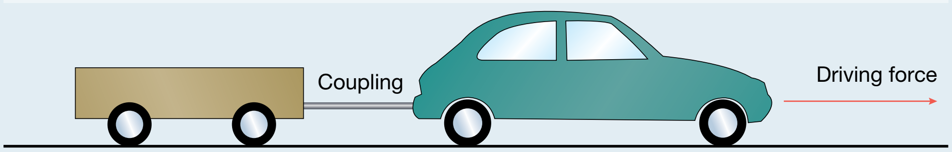
The rope pulls back on the larger boat with the same tension that it applies in a forward direction on the small dinghy. This is consistent with Newton's Third Law of Motion. Through the rope, the larger boat pulls forward on the small dinghy with a force that is equal and opposite to the force with which the smaller dinghy pulls on the larger boat.

4.6 SAMPLE PROBLEM 1

A car of mass 1600 kg tows a trailer of mass 400 kg. The coupling between the car and trailer is rigid. The driving force acting on the car as it starts from rest is 5400 N in an easterly direction. The frictional forces resisting the motion of the car and trailer are insignificant and can be ignored. Calculate:

(a) the acceleration of the car and trailer
(b) the net force acting on the trailer
(c) the force applied on the trailer by the car
(d) the force applied on the car by the trailer.

FIGURE 4.29 The car towing a trailer. The only external horizontal force is the driving force.



SOLUTION

(a) Because the coupling between the car and trailer is rigid, they have equal accelerations. Newton's Second Law of Motion can be applied to the system of the car and trailer.

$$\boldsymbol{F}_{net} = m\boldsymbol{a}$$

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{net}}{m} = \frac{5400}{2000} = 2.7\ \text{m s}^{-2}\ \text{east}$$

(b) Apply Newton's Second Law of Motion to the trailer.

$$\boldsymbol{F}_{net} = m\boldsymbol{a} = 400 \times 2.7 = 1080\ \text{N east}$$

(c) The only horizontal force acting on the trailer is the force applied by the car. The force applied on the trailer by the car is therefore 1080 N east.

(d) According to Newton's Third Law of Motion, the force applied on the car by the trailer is equal and opposite in direction to the force applied on the trailer by the car. That force is therefore 1080 N west.

4.6 Exercise 1

1 A boat of mass 2000 kg tows a small dinghy of mass 100 kg with a thick rope. The boat's propellers provide a forward thrust of 4700 N. The total resistance forces of air and water on the boat and dinghy system amount to 400 N and 100 N respectively.

(a) What is the acceleration of the boat and dinghy?
(b) What is the net force on the dinghy?
(c) What is the magnitude of the tension in the rope?

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4.7 Chapter Review

4.7.1 Summary

- Force is a vector quantity.
- Weight is a measure of the force on an object due to the pull of gravity.
- The weight of an object is directly proportional to its mass.
- The vector sum of the forces acting on an object is called the net force.
- The velocity of an object can only change if there is a non-zero net force acting on it. This statement is an expression of Newton's First Law of Motion.
- When a non-zero net force acts on an object, it accelerates in the direction of the net force.
- Acceleration occurs when there is a change in speed and/or direction.
- Inertia is the tendency of an object to resist a change in its motion.
- The forces acting on a moving vehicle are:
  - weight, downwards
  - the normal reaction force, applied perpendicular to the surface of the road
  - the driving force, applied in the direction of motion by the road
  - road friction, applied to the non-driving wheels opposite to the direction of motion
  - air resistance, applied opposite to the direction of motion.
- The motion of a vehicle depends on the net force acting on the vehicle.
- Newton's Second Law of Motion describes the relationship between the acceleration of an object, the net force acting on it, and the object's mass. It can be expressed as $\boldsymbol{F}_{net} = m\boldsymbol{a}$.
- Newton's Second Law can be applied to a single object, or a system of multiple bodies that are in contact or connected together.
- When an object applies a force (an action) to a second object, the second object applies an equal and opposite force (a reaction) to the first object. This statement is an expression of Newton's Third Law of Motion.
- The frictional force, $\boldsymbol{F}_f$, acts between pairs of surfaces such that it opposes the relative motion of one surface across the other. The frictional force always acts parallel to the surface interface. $\boldsymbol{F}_f = \mu \boldsymbol{R}$ where μ is the coefficient of friction and $\boldsymbol{R}$ is the normal force.

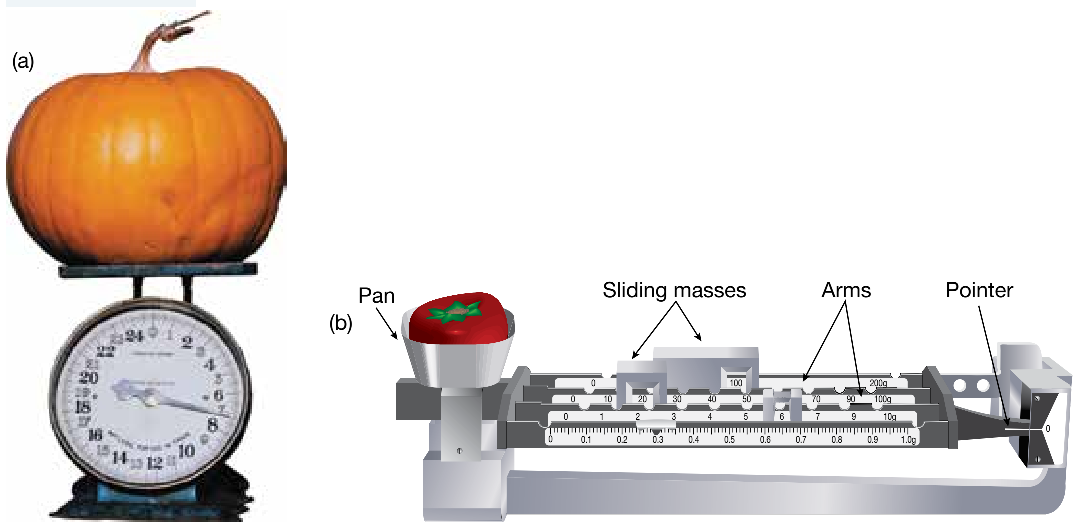
4.7.2 Questions

Assume that the magnitude of the gravitational field strength at the Earth's surface is $9.8\,\text{m}\,\text{s}^{-2}$.

1. Describe the difference between a vector quantity and a scalar quantity.
2. State which of the following are vector quantities:
   (a) mass
   (b) weight
   (c) gravitational field strength
   (d) time
   (e) acceleration.

3. A slightly overweight physics teacher steps off the bathroom scales and proudly remarks, 'My weight is down to 75 kg!' The physics teacher clearly should have known better. Rewrite the remark in two different ways so that it is correct.
4. A family sedan has a mass of 1400 kg with a full tank of petrol.
   (a) Calculate the magnitude of its weight at the surface of the Earth.
   (b) Calculate the weight of the car on the surface of Mars where the magnitude of the gravitational field strength is $3.6\ \text{m s}^{-2}$.
   (c) Calculate the mass of the car on the surface of Mars.
5. Estimate your own mass in kilograms and calculate:
   (a) the magnitude of your weight at the surface of the Earth
   (b) your weight on the surface of Mars where the magnitude of the gravitational field strength is $3.6\ \text{m s}^{-2}$
   (c) your mass on the planet Mars.
6. The set of kitchen scales in figure 4.30a is used to determine mass. As the spring inside is compressed, the pointer in front of the scale moves. The beam balance in figure 4.30b is used in many school laboratories to determine mass. Which of the two instruments would you prefer to use to measure the mass of a small rock (with a mass of less than 300 grams) on the Moon? Explain your answer.

FIGURE 4.30



7. Determine the net force in each of the situations illustrated in figure 4.31.
8. In the illustrations in figure 4.32, the net force is shown along with all but one of the contributing forces. Determine the magnitude and direction of the missing force.

FIGURE 4.31

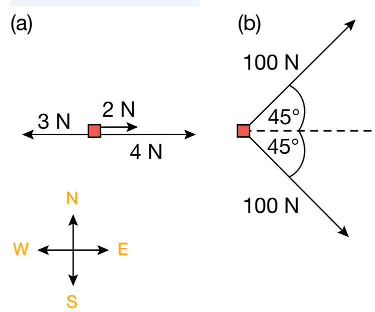
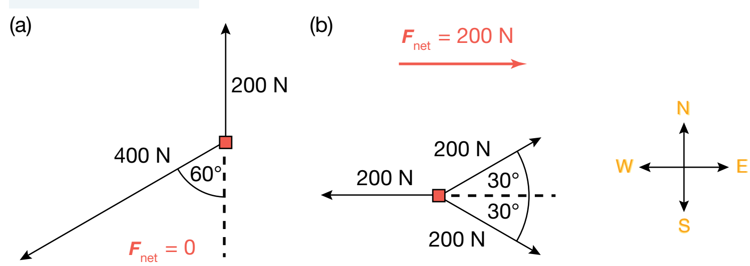
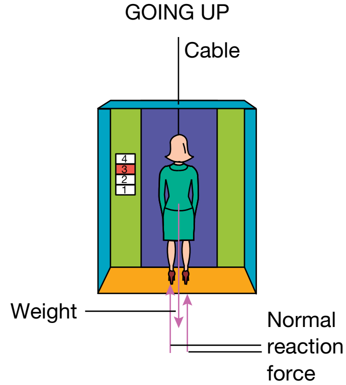


FIGURE 4.32



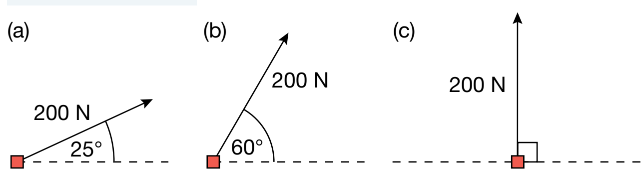
9. When you stand in an elevator there are only two significant forces acting on you — your weight and the normal reaction force. It is important to note that the tension in the cable is not pulling on you — it is pulling on the elevator. The only object that can push you upwards is the floor of the elevator.
   (a) State whether the normal reaction force is less than, equal to or greater than your weight when the elevator is:
      (i) stationary
      (ii) moving upwards with a constant speed
      (iii) speeding up on its way to the top floor
      (iv) slowing down as it approaches the top floor.
   (b) Explain how the movement of elevators in tall buildings sometimes makes you feel 'heavy' or 'light'.

FIGURE 4.33 The forces acting on you in an elevator



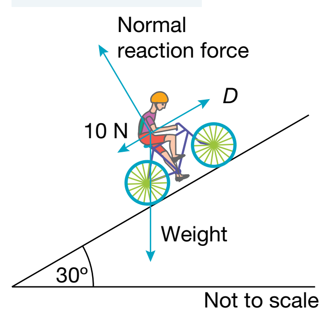
10. A car is moving north on a horizontal road at a constant speed of $60\,\text{km}\,\text{h}^{-1}$.
    (a) Draw a diagram showing all of the significant forces acting on the car. Show all of the forces as if they were acting through the centre of mass.
    (b) Calculate the net force on the car.
11. When you are standing on a bus or train that stops suddenly, you lurch forwards. Apply Newton's First Law of Motion to explain why this happens.
12. The ancient Greek philosopher Aristotle would have explained a car rolling to a stop on a horizontal road by saying that it slowed down because there was no constant force to keep it going. Propose a better explanation.
13. If the bicycle that you are riding runs into an obstacle like a large rock, you may be flung forwards over the handlebars. Explain in terms of inertia why this happens.
14. When you try to push a broken-down car with its handbrake still on, it does not move. Explain other forces that are acting on the car to produce a net force of zero.
15. Explain why a car takes longer to stop if the brakes are applied too hard.
16. Determine the magnitude of the horizontal components of each of the following forces (figure 4.34).

FIGURE 4.34



17. A car rolls freely down a hill with an increasing speed.
    (a) Draw a diagram to show all the forces acting on the car.
    (b) What is the direction of the net force on the car?
    (c) What is the largest single force acting on the car?
    (d) When the car reaches a horizontal surface it slows, eventually coming to a stop. Why does this happen?
18. A cyclist of mass 60 kg is riding at a constant speed up a hill that is inclined at 30° to the horizontal. The mass of the bicycle is 20 kg. Figure 4.35 shows the forces acting on the bicycle–cyclist system.
    (a) Calculate the net force on the bicycle–cyclist system.
    (b) The sum of the magnitudes of the road friction and air resistance on the system is 10 N. What is the magnitude of the component of the weight of the system that is parallel to the road surface?

FIGURE 4.35



(c) Calculate the magnitude of the driving force ***D***.
(d) Calculate the magnitude of the normal reaction force on the bicycle–cyclist system.

19. An experienced downhill skier with a mass of 60 kg (including skis) is moving with increasing speed down a slope inclined at 30°. She is moving in a straight line down the slope.
(a) Calculate the direction of the net force on the skier.
(b) Draw a diagram showing the forces acting on the skier. Show all the forces as if they were acting through her centre of mass.
(c) Calculate the magnitude of the component of the skier's weight that is parallel to the slope.
(d) If the sum of the forces resisting the movement of the skier down the slope is 8.0 N, calculate the magnitude of the net force on her.

20. A ball of mass 0.50 kg is thrown vertically upwards.
(a) Calculate the velocity of the ball at the top of its flight.
(b) Calculate the magnitude of its acceleration at the top of its flight.
(c) Calculate the net force on the ball at the top of its flight.

21. Calculate the magnitude of the net force on each of the following objects:
(a) a 1600 kg car while it is accelerating from 0 to 72 km h^{-1} (20 m s^{-1}) in 5.0 s
(b) a 500 tonne Manly ferry while it is cruising at a constant speed of 20 km h^{-1}
(c) a space shuttle at lift-off, when its acceleration is 3.0 m s^{-2} and its lift-off mass is 2.2×10^6 kg.

22. A car of mass 1200 kg starts from rest on a horizontal road and a forward thrust of 10 000 N is applied. The resistance to motion due to road friction and air resistance totals 2500 N.
(a) Calculate the magnitude of the net force on the car.
(b) Calculate the magnitude of the acceleration of the car.
(c) Calculate the speed of the car after 5.0 s.
(d) Calculate the distance the car has travelled after 5.0 s.

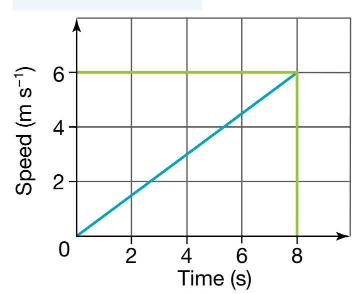
23. A train of mass 8.0×10^6 kg, travelling at a speed of 30 m s^{-1}, brakes and comes to rest in 25 s with a constant deceleration.
(a) Calculate the frictional force acting on the train while it is decelerating.
(b) Calculate the stopping distance of the train.

24. A physics teacher decides, just for fun, to use bathroom scales (calibrated in newtons) in an elevator. The scales provide a measure of the force with which they push up on the teacher. When the lift is stationary the reading on the bathroom scales is 823 N. Calculate the reading on the scales when the elevator is:
(a) moving upwards at a constant speed of 2.0 m s^{-1}
(b) accelerating downwards at 2.0 m s^{-2}
(c) accelerating upwards at 2.0 m s^{-2}.

25. A roller-coaster carriage (and its occupants), with a total mass of 400 kg, rolls freely down a straight part of the track inclined at 40° to the horizontal with a constant acceleration. The frictional force on the carriage is a constant 180 N. Assume that air resistance is insignificant. What is the magnitude of the acceleration of the carriage?

26. A skateboarder with a mass of 56 kg is rolling freely down a straight incline. The motion of the skateboarder is described in the graph in figure 4.36. Assume that air resistance is insignificant.
(a) Calculate the magnitude of the net force on the skateboarder.
(b) If the friction force resisting the motion of the skateboarder is a constant 140 N, at what angle is the slope inclined to the horizontal?

FIGURE 4.36

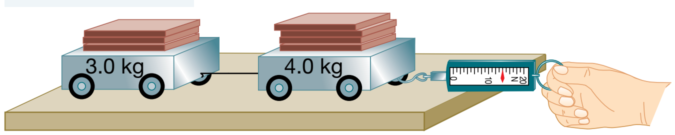


27. The magnitude of the air resistance, $\boldsymbol{R}$, on a car can be approximated by the formula:
$\boldsymbol{R} = 1.2\,v^2$
where $\boldsymbol{R}$ is measured in newtons and v is the speed of the car in $\mathrm{m\,s^{-1}}$.
(a) Design a spreadsheet to calculate the magnitude of the force of air resistance and the net force on the car for a range of speeds as it accelerates from $20\,\mathrm{km\,h^{-1}}$ to $60\,\mathrm{km\,h^{-1}}$ on a horizontal road. Assume that while accelerating, the driving force is a constant 1800 N and the road friction on the non-driving wheels is a constant 300 N.
(b) Use your spreadsheet to plot a graph of the net force versus speed for the car.
(c) Modify your spreadsheet to show how the net force on the car changes when the same acceleration (from $20\,\mathrm{km\,h^{-1}}$ to $60\,\mathrm{km\,h^{-1}}$) is undertaken while driving up a road inclined at $10°$ to the horizontal.
28. A 6 kg bowling ball and a 60 kg gold bar are dropped at the same instant from the third floor of the Leaning Tower of Pisa. Use Newton's Second Law of Motion to explain why:
(a) they both reach the ground at the same time
(b) a 6 kg doormat dropped from the same location at the same time takes longer to reach the ground.
29. Copy and complete the following table by fully describing the missing half of the action–reaction pairs.

You push on a wall with the palm of your hand.	
Your foot pushes down on a bicycle pedal.	
The ground pushes up on your feet while you are standing.	
The Earth pulls down on your body.	
You push on a broken-down car to try to get it moving.	
A hammer pushes down on a nail.	

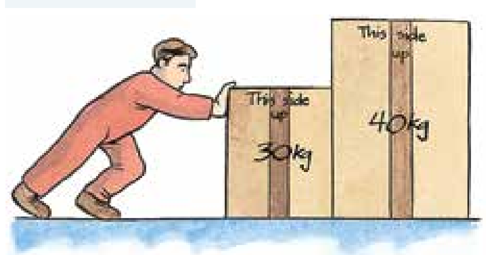
30. What force provides the forward thrust that gets you moving when you are:
(a) cycling (b) downhill skiing (c) water skiing
(d) skateboarding (e) swimming (f) rowing?
31. Two loaded trolleys of masses 3.0 kg and 4.0 kg (which are joined by a light string) are pulled by a spring balance along a smooth horizontal laboratory bench as shown in figure 4.37. The reading on the spring balance is 14 N.

FIGURE 4.37



(a) Calculate the acceleration of the trolleys.
(b) Calculate the magnitude of the tension in the light string joining the two trolleys.
(c) Calculate the net force on the 4.0 kg trolley.
(d) Calculate the acceleration of the 4.0 kg trolley if the string was cut.
32. A warehouse worker applies a force of 420 N to push two crates across the floor as shown in figure 4.38. The friction force opposing the motion of the crates is a constant 2.0 N for each kilogram.

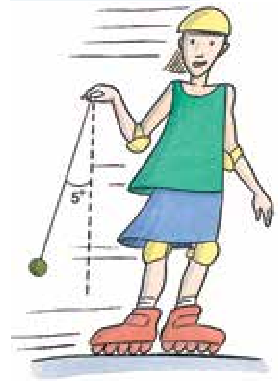
FIGURE 4.38



(a) Calculate the acceleration of the crates.
(b) Calculate the net force on the 40 kg crate.
(c) Calculate the force exerted by the 40 kg crate on the 30 kg crate.
(d) Calculate the force exerted by the 30 kg crate on the 40 kg crate.
(e) Would the worker find it any easier to give the crates the same acceleration if the positions of the two crates were reversed? Support your answer with calculations.

33. A well-coordinated in-line skater is playing with a yo-yo while accelerating on a horizontal surface. Figure 4.39 shows that when the yo-yo is at its lowest point it makes an angle of 5° with the vertical. Determine the acceleration of the in-line skater.

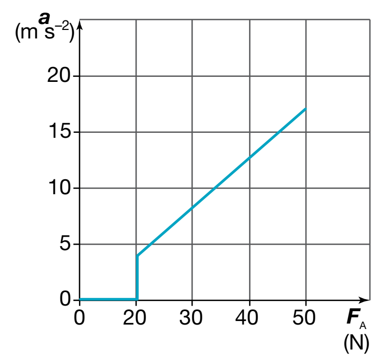
FIGURE 4.39



34. The graph in figure 4.40 shows the acceleration experienced by a wooden block placed on a concrete floor as it is pushed across the floor by a force F_A.
From this graph, determine:
(a) the limiting friction
(b) the mass of the block
(c) the sliding friction.

35. A 3 kg lantern suspended from a verandah roof by a 50 cm chain is blown by the wind so that it hangs at an angle θ to the vertical for the duration of the wind gust. If the wind blows from the east and exerts a constant force of 20 N, determine the tension, T, in the chain and the angle θ.

FIGURE 4.40



PRACTICAL INVESTIGATIONS

Investigation 4.1: Force as a vector

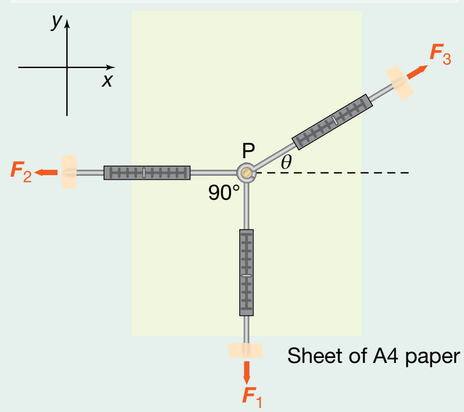
Aim

(a) To show that force is a vector and that the net force is the vector sum of all the forces acting on an object
(b) To analyse the forces acting on an object by resolving the forces into components

Apparatus

three spring balances (5 N)
slotted masses (set of nine 50 g masses and carrier)
marking pen
sheet of A4 paper
masking tape
protractor

FIGURE 4.41 Set up three spring balances horizontally as shown.



Theory

When a point is stationary, the net force acting at that point is zero. We know this because if the point is stationary, it is not changing its motion. The net force is the vector sum of all the forces acting at the point. If the net force at a point is zero, the components of the forces in any direction will add up to zero.

Method

1. Check that the spring balances are 'zeroed' and test them for accuracy by weighing known masses.
2. Using three 5 N spring balances, apply three small forces horizontally to a point, P, so that the point is in equilibrium (see figure 4.41). Use masking tape to secure the ends of the spring balances in place while maintaining the tension so that the net force at the point P is zero. The point P is the point at which the three hooks are in contact.

3. Place a sheet of A4 paper on the table beneath the point P. Use the protractor to measure the angle θ. You need to think carefully about the best way to ensure that the directions are as shown.
4. Draw a diagram of the situation, showing the spring balances and the point P, and label the angles.
5. Draw a separate vector diagram showing the point P and the three forces acting at point P.

Analysis and questions

Determine the net force acting on point P using the two methods (a) and (b) below.

(a) Vector addition method

Apply the 'head to tail' rule for vector addition to all forces. Take care when transferring vectors.

1 Label the net force clearly and state its magnitude and direction.
2 What is the expected magnitude and direction of the net force?
3 Account for any difference between your measured net force and the expected net force.

(b) Component method

1 Transfer your original vector diagram carefully onto graph paper with point P at the origin.
2 Use your graph to find the '*x*' component of each of the three forces. Add the '*x*' components to obtain the sum of the '*x*' components. Repeat the process for the '*y*' components.
3 Summarise your results in a table like the one below.

FORCE	'*X*' COMPONENT (N)	'*Y*' COMPONENT (N)
F_1		
F_2		
F_3		
SUM		

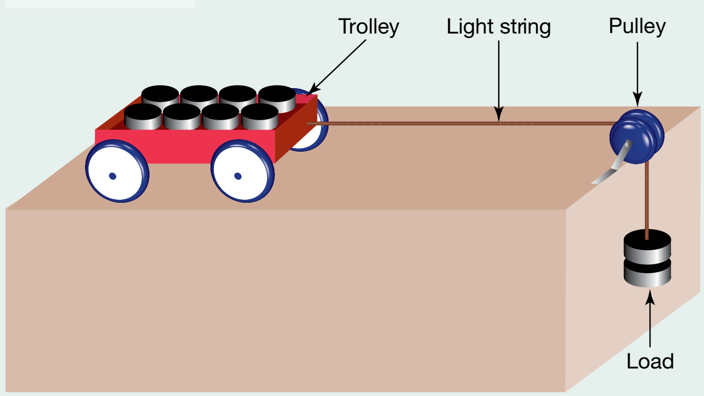
4 How does the sum of the '*x*' components of the three forces compare with the expected value of the sum?
5 How does the sum of the '*y*' components of the three forces compare with the expected value of the sum?

Investigation 4.2: Newton's Second Law of Motion

Aim

(a) To examine the relationship between the net force acting on a system, the mass of the system and its acceleration
(b) To use Newton's Second Law of Motion to determine the mass of an object

FIGURE 4.42



Apparatus

low-friction trolley
timing and recording device (e.g. ticker-timer, photogates, motion detector and computer interface)
pulley
light string
slotted masses (set of nine 50 g masses and carrier)
metre rule
balance suitable for measuring the mass of the trolley

Theory

Newton's Second Law of Motion describes the relationship between the acceleration of an object, the net force acting on it, and the object's mass. It can be expressed as $\boldsymbol{F}_{\text{net}} = ma$.

Method

1. Use the balance to measure the mass of the trolley. Record its mass.
2. Place 400 g of slotted masses on the trolley. Connect a load of 100 g to the trolley with a light string over a pulley as shown in figure 4.42. The load provides a known external force on the system of the trolley and all of the slotted masses. The magnitude of this external force is equal to the magnitude of the weight of the load.
3. Use your timing and recording device to collect data that will allow you to determine the acceleration of the trolley or glider at several instants as the load is falling.

4. Repeat this procedure for different loads by taking 100 g from the trolley and adding it to the load. That changes the load, and therefore the external force on the system, without changing the mass of the system. Continue to repeat the procedure until you have removed all of the slotted masses from the trolley.

Analysis

1. Use your data to determine the average acceleration of the system for each external force.
2. Summarise your data in a table that shows the force applied to the system by each external force and the corresponding acceleration of the system.
3. Use your table to plot a graph of external force versus acceleration.
4. Use your graph to make an estimate of the mass of the system of the slotted masses and trolley.

Questions

1. If the force applied by the load through the string was the only horizontal force acting on the trolley, where would the graph cross the vertical axis?
2. What quantity does the intercept on the vertical axis represent?
3. Using your estimate of the mass of the system, what is your estimate of the mass of the trolley?
4. How does your estimate of the mass of the trolley compare with the mass measured by the balance? Suggest reasons for differences between the estimated mass and the measured mass.

Investigation 4.3: Weight and Mass

Aim

To examine the relationship between weight and mass

Apparatus

10 × 50 g masses (usually sold as a set with a suspender base)
5-newton spring scale
retort stand
clamp and boss head

Method

1. Assemble the retort stand with clamp and boss head.
2. Calibrate the spring scale. This is usually done by pulling or pushing the metal tag at the top of the scale until the indicator is aligned with the zero mark. For some scales, you will need to twist an adjustment screw (usually also at the top of the scale).
3. Hang the calibrated scale from the clamp of the retort stand.
4. Suspend a 50 g mass from the scale hook. Read the spring scale and enter the weight registered in the table below.
5. Repeat step 4 with masses of 100 g, 150 g and so on up to 500 g.

Results

Mass (g)	Mass (kg)	Weight (N)
50		
100		
150		
200		
250		
300		
350		
400		
450		
500		

Analysis

1 On the graph section below, plot the results that you have entered in the table above, ensuring that the mass (in kg) is on the horizontal axis and weight (in N) is on the vertical axis. Draw a line of best fit through your results.

Determine the gradient of the line of best fit.

2 Weight (***W***) and mass (*m*) are related by the equation $\boldsymbol{W} = mg$, where ***g*** is the acceleration due to gravity in $m\,s^{-2}$; its value will be the gradient that you calculated above. On average, this value should be $9.81\ m\ s^{-2}$. Calculate the % error in your determination of ***g*** compared to the theoretical value of $9.81\ m\ s^{-2}$:

$$\%\,\text{error} = \left|\frac{\text{theoretical value} - \text{experimental value}}{\text{theoretical value}}\right| \times 100\,\%$$

3 What explanations can you give for any discrepancy between your investigational value and the theoretical value?

Conclusion

What have you found out about the relationship between weight and mass in this investigation?

Investigation 4.4: Static, sliding and rolling friction

Aim

To compare the relative sizes of different forms of friction

Apparatus

Wooden block in the shape of a rectangular prism
string
spring scale (newton scale)
a wooden surface (such as a floor
board, plank or benchtop)
10 × wooden dowels (each 10 cm long)

Note: Ideally, the wooden surface, dowels and block should be of the same type of wood and the same level of finish.

Method

1. Tie a loop of string tightly around the wooden block, ensuring that the largest surface area is not crossed with string.
2. Zero the spring scale, then attach the hook on the spring scale to the string loop.
3. Place the block and spring scale on the wooden surface so that the largest surface area is face down.
4. Pull the scale gently until the block *just* starts to move. Note the reading on the scale. This will be the static friction value, F_s. Enter this value into the table below.
5. Lift the block from the surface, replace it and then repeat step 4 twice more.
6. Now pull the block across the wooden surface at a uniform speed (as much as possible) and note the reading on the spring scale. This will be the sliding friction value, F_k. Enter this value into the table below. *Note:* if you are pulling with a constant speed, then the reading on the spring scale remains at the same value.
7. Replace the block at its starting position and then repeat step 6 twice more.
8. Lay the wooden dowels on the wooden surface so that they lie as close as together as possible. Run your hand lightly over them to ensure that they roll smoothly. If a dowel seems to roll unevenly, replace it with another.
9. Put the dowels back in their starting position and place the block on top of them so that the end of the block lies on the last dowel. Once again, ensure that you place the largest surface area face down.
10. Pull the spring scale and block at a constant speed until the block no longer rolls over the dowels. Note the measurement on the scale. This will be the value of rolling friction, $\boldsymbol{F_r}$. Enter this value into the table below.
11. Repeat steps 9–10 twice more.

Results

	Force of friction (N)			
Type of friction	**Trial 1**	**Trial 2**	**Trial 3**	**Average**
Static friction				
Sliding friction				
Rolling friction				

Analysis

1. What variables were controlled during this short investigation?
2. Calculate the average values obtained for each of the three types of friction and enter them into the final column in the table above.
3. According to your average values, place the friction types (static, sliding and rolling) in order from lowest to highest.
4. Is this the order that you would expect them to appear theoretically?
5. Would you expect this order to be the same if the wooden surface and dowels were replaced with steel while the block remained wood? Explain your answer.
6. Discuss at least 3 problems that you encountered in this investigation and propose possible solutions.
7. Give 2 situations in the real world where static friction is relied upon.

Conclusion

State the largest and smallest types of friction for two wooden surfaces.

TOPIC 5
Energy and work

5.1 Overview

5.1.1 Module 2: Dynamics

Forces, acceleration and energy

Inquiry question: How can the motion of objects be explained and analysed?

Students:

- apply the special case of conservation of mechanical energy to the quantitative analysis of motion involving:
  - work done and change in the kinetic energy of an object undergoing accelerated rectilinear motion in one dimension ($W = \boldsymbol{F}_{net}\boldsymbol{s}$)
  - changes in gravitational potential energy of an object in a uniform field ($\Delta U = m\boldsymbol{g}\Delta\boldsymbol{h}$).
- conduct investigations over a range of mechanical processes to analyse qualitatively and quantitatively the concept of average power ($P = \frac{\Delta E}{t}$, $P = \boldsymbol{F}\boldsymbol{v}$), including but not limited to:
  - uniformly accelerated rectilinear motion
  - objects raised against the force of gravity
  - work done against air resistance, rolling resistance and friction.

FIGURE 5.1 Jumping up and down on a trampoline involves the transformation of one energy form into another. Kinetic energy, gravitational potential energy, elastic potential energy, chemical potential energy and heat are all involved.

5.2 Describing work

5.2.1 The concept of energy

The word **energy** is often used to describe the way that you feel. For example, you might say 'I don't have a lot of energy today' or on a better day you might say 'I have enough energy to run a marathon'. The word 'energy' is also used to describe something that food has. In each of these cases, the word 'energy' is being used to describe something that provides you with the capacity to make something move. It could be a heavy object, a bicycle or even your own body. Most dictionaries and some physics textbooks define energy as the capacity to do **work**. Work is done when an object moves in the direction of a force applied to it.

The following list of some of the characteristics of energy provides some further clues as to what it really is.

- All matter possesses energy.
- Energy is a scalar quantity — it does not have a direction.
- Energy takes many different forms. It can therefore be classified. Light energy, sound energy, thermal energy, kinetic energy, gravitational potential energy, chemical energy and nuclear energy are some of the different forms of energy.
- Energy can be stored, transferred to other matter or transformed from one form into another. For example, when you hit a cricket ball with a bat, energy is transferred from the bat to the ball. When you dive into a swimming pool, gravitational potential (stored) energy is transformed into kinetic energy.
- Some energy transfers and transformations can be seen, heard, felt, smelt or even tasted.
- It is possible to measure the quantity of energy transferred or transformed.
- Energy cannot be created or destroyed. This statement is known as the Law of Conservation of Energy. The quantity of energy in the universe is a constant. However, nobody knows how much energy there is in the universe.

Transferring energy

Energy can be transferred to or from matter in several different ways.
Energy can be transferred by:

- emission or absorption of electromagnetic or nuclear radiation
- heating and cooling an object or substance as a result of a temperature difference
- the action of a force on an object resulting in movement.

The transfer of energy by the action of a force is called **mechanical energy transfer**.

5.2.2 Getting down to work

When mechanical energy is transferred to or from an object, the amount of mechanical energy transferred is called work.

The work, W, done when a force, $\boldsymbol{F}$, causes a displacement of $\boldsymbol{s}$, in the direction of the force, is defined as:

$$\text{work} = \text{force} \times \text{displacement in the direction of the force}$$
$$W = \boldsymbol{F}\boldsymbol{s}.$$

Work is a scalar quantity. The SI unit of work is the joule. One joule of work is done when a force with a magnitude of one newton causes a displacement of one metre in the same direction as the force. That is, $1\,\text{J} = 1\,\text{N} \times 1\,\text{m} = 1\,\text{N\,m}$. Because energy is a measure of the capacity to do work, the SI unit of energy is also the joule.

5.2.3 Making an effort

No work is done on a wall when a force is exerted on it (such as, for example, pushing against a solid wall that doesn't move). However, If the same force were exerted to push a shopping trolley across a floor, work is done on the trolley because the force $\boldsymbol{F}$ is applied to give the trolley a displacement $\boldsymbol{s}$ that is in the same direction as the force.

FIGURE 5.2 The work done pushing against a wall is zero.

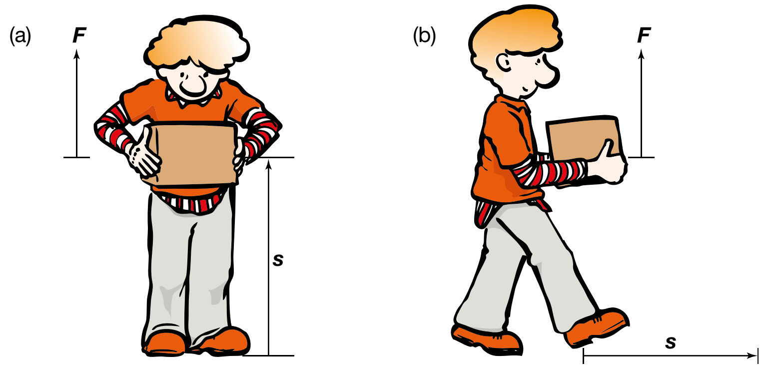
FIGURE 5.3 Work is done in applying a force to move a trolley across a floor.

But what if you were to lift a box and then carry it across the room?

In lifting the box, you are exerting a force upwards and, in moving it upwards, you have given the box a displacement that is in the same direction as the force — thus work has been done on the box against gravity. However, when you walk across the floor, you continue to apply the same upwards force to the box to hold it up, but you are now displacing the box in a direction that is at right angles to the force. As a result, no work is done on the box while it is being carried across the floor. When you put the box down on the ground again, the displacement will be in the opposite direction to the force you are applying on the box; as a result, you do negative work on the box against gravity:

$W = \boldsymbol{F}(-\boldsymbol{s})$

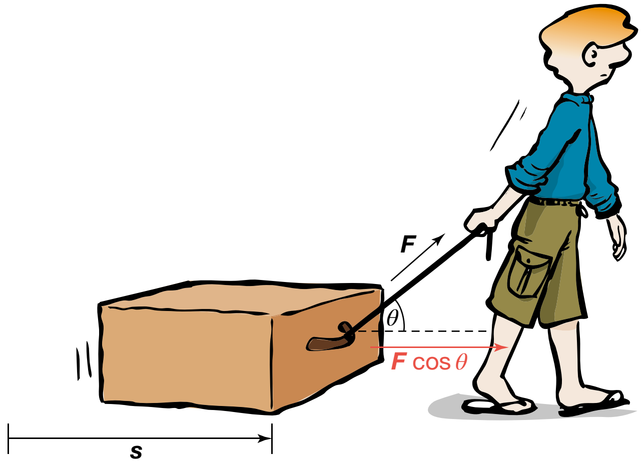
FIGURE 5.4 Work is done in (a) but not in (b).



5.2.4 Effort at an angle

Now consider a box being dragged across a smooth floor by means of a rope that makes an angle θ with the floor, as shown in figure 5.5.

FIGURE 5.5 Dragging a box across the floor by means of a rope at angle θ.



In this case, only the component of the force that is acting in the same direction as the box's displacement, $\boldsymbol{F}\cos\theta$, will contribute to the work being done.

5.2 SAMPLE PROBLEM 1

How much work is done in pushing a 5 kg box across a smooth flat surface for 4 metres by applying a horizontal force of 10 N?

SOLUTION:

$$\begin{aligned} W &= \boldsymbol{F}s \\ &= 10\,\text{N} \times 4\,\text{m} \\ &= 40\,\text{N m} \\ &= 40\,\text{J} \end{aligned}$$

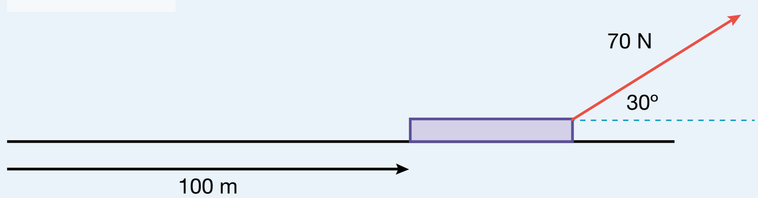
Note that, although work is the product of two vectors, it is a scalar quantity and so has no direction associated with it.

5.2 SAMPLE PROBLEM 2

A sled is dragged 100 m across ice by means of a tow rope that makes an angle of 30° to the horizontal. If a force of 70 N is applied to pull the sled, how much work is done?

SOLUTION:

FIGURE 5.6



First, the component of force acting in the direction of the displacement is found:

$$F_{//} = F\cos\theta = 70\cos 30^\circ = 60.6\ \text{N}$$

$$\begin{aligned} W &= F_{//}s = 60.6\,\text{N} \times 100\,\text{m} \\ &= 6060\,\text{N m} \\ &= 6060\ \text{J} \end{aligned}$$

5.2 SAMPLE PROBLEM 3

A 10 kg box is pushed for 5 metres at constant speed over a timber floor that has a frictional coefficient of 0.4.

(a) What work is done on the box by pushing it?
(b) What work is done on the box by the frictional force?

SOLUTION:

(a) Because the box is travelling at constant speed, the magnitude of the force applied to the box to push it must be equal to the magnitude of the frictional force opposing the motion.

$$\begin{aligned} F_f &= \mu F_N = \mu \textbf{\textit{Weight}} \\ &= \mu m g \\ &= 0.4 \times 10\ \text{kg} \times 9.8\ \text{ms}^{-2} \\ &= 39.2\ \text{N, west} \end{aligned}$$

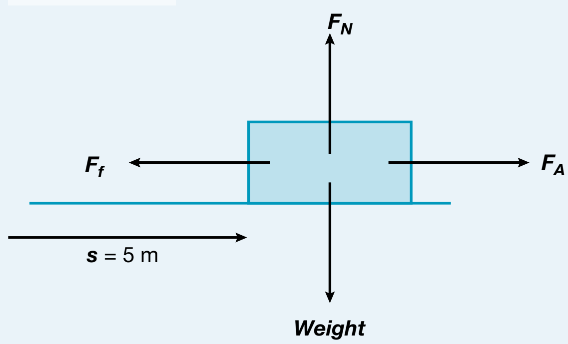
As $F_A = -F_f$,

$F_A = 39.2$ N, east

As F_A is directed in the same direction as the displacement s,

$$\begin{aligned} W &= F_A s \\ &= 39.2\ \text{N} \times 5\ \text{m} \\ &= 196\ \text{N m} \\ &= 196\ \text{J} \end{aligned}$$

FIGURE 5.7



(b) The frictional force F_f acts in the opposite direction to the displacement s, and so the work done on the box by the frictional force equals −196 J.

WORKING SCIENTIFICALLY 5.1

Design and perform an investigation to determine the relationship between the radius of the wheels on a cart and the force needed to move the cart a fixed distance.

5.2 Exercise 1

1 How much work is done on a 20 kg box in the following cases if it is:
   (a) pushed 5 m across a smooth floor with a force of 300 N?
   (b) lifted to a height of 2 m and then carried for 10 m?
   (c) pushed for 20 m around the room until it returns to its starting point? (Assume a smooth floor.)
2 (a) How much work is done by a 30 kg child who climbs to the top of a 3.2 m slippery slide?
   (b) How much work is done by the child in sliding back to the bottom?
3 A warehouse worker pushes a heavy crate a distance of 2.0 m across a horizontal concrete floor against a constant friction force of 240 N. He applies a horizontal force of 300 N on the crate. How much work is done on the crate by:
   (a) the warehouse worker
   (b) the net force?

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5.3 Kinetic energy

5.3.1 The energy of movement

Kinetic energy is the energy associated with the movement of an object. By imagining how much energy it would take to make a stationary object move, we can deduce that kinetic energy depends on the mass and speed of the object.

The change in kinetic energy of an object is equal to the work done on it by the net force acting on it. If an object initially at rest is acted on by a net force of magnitude $\boldsymbol{F}_{\text{net}}$ and moves a displacement $\boldsymbol{s}$ (which will necessarily be in the direction of the net force), its change in kinetic energy, ΔE_k, can be expressed as:

$$\Delta E_k = \boldsymbol{F}_{\text{net}}\boldsymbol{s}$$

The quantity of kinetic energy it possesses is:

$$E_k = \boldsymbol{F}_{\text{net}}\boldsymbol{s}$$

because the initial kinetic energy was zero.

Applying Newton's Second Law ($\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$) to this expression:

$$E_k = m\boldsymbol{a}\boldsymbol{s}$$

where

m is the mass of the object and $\boldsymbol{a}$ is its acceleration.

The movement of the object can also be described in terms of its final velocity $\boldsymbol{v}$ and its initial velocity $\boldsymbol{u}$. The magnitudes of the quantities $\boldsymbol{a}$, $\boldsymbol{s}$, $\boldsymbol{v}$ and $\boldsymbol{u}$ are related to each other by the equations:

$$\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$$

and $\boldsymbol{s} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t$

Substituting into the expression for kinetic energy:

$$\begin{aligned} E_k &= m\boldsymbol{a}\boldsymbol{s} \\ &= m \times \frac{(\boldsymbol{v} - \boldsymbol{u})}{t} \times \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t \\ &= \frac{1}{2} \times m \times (\boldsymbol{v}^2 - \boldsymbol{u}^2) \\ &= \frac{1}{2}m\boldsymbol{v}^2 - \frac{1}{2}m\boldsymbol{u}^2 \end{aligned}$$

Because the object was originally at rest, $\boldsymbol{u} = 0$.

The kinetic energy of an object of mass m and velocity $\boldsymbol{v}$ can therefore be expressed as:

$$E_k = \frac{1}{2}m\boldsymbol{v}^2$$

5.3 SAMPLE PROBLEM 1

DOING WORK TO CHANGE KINETIC ENERGY

A trailer is being pulled along a straight, rough, horizontal road by a car. The trailer and the car travel at a constant speed of 50 km h^{-1}. The forward force applied to the trailer by the car is 4000 N. Frictional forces oppose this force.

(a) In moving a horizontal distance of 500 metres, how much work is done on the trailer by:
  (i) the car?
  (ii) the net force?
  (iii) the force of gravity?

(b) If the force applied to the trailer by the car is increased to 5000 N and nothing else changes, how much kinetic energy is gained by the trailer over the distance of 500 metres?

SOLUTION:

(a) (i)
$$\begin{aligned} W &= \boldsymbol{F}\boldsymbol{s} \\ &= 4000 \times 500 \\ &= 4 \times 10^3 \times 5 \times 10^2 \\ &= 2.0 \times 10^6\ \text{J} \end{aligned}$$

The work done on the trailer by the car is 2.0×10^6 J.

(ii) The work done on the trailer by the net force is equal to the change in kinetic energy of the trailer. The trailer is travelling at constant speed, so there is no change in kinetic energy.

No work is done by the net force.

(iii) The work done on the trailer by the force of gravity is zero because the force of gravity has no component in the direction of motion.

(b) When the towing force was 4000 N, the net force was zero. The towing force balanced frictional forces. When the towing force is increased to 5000 N, the net force becomes 1000 N in the direction of motion of the trailer.

$$\begin{aligned} \Delta E_k &= \boldsymbol{F}_{net}\boldsymbol{s} \\ &= 1000 \times 500 \\ &= 500\,000\ \text{J} \end{aligned}$$

The kinetic energy gained is 5.0×10^5 J.

5.3 SAMPLE PROBLEM 2

KINETIC ENERGY CALCULATIONS

Compare the kinetic energy of a 100 m Olympic track athlete with that of a family car travelling through the suburbs.

Estimate the mass of the athlete to be 70 kg and the velocity of the athlete to be 10 m s^{-1}.

Estimate the total mass of the car and its passengers to be 1500 kg and the velocity of the car to be about 60 km h^{-1} (17 m s^{-1}).

SOLUTION:

For the athlete: $m = 70$ kg, $\boldsymbol{v} = 10$ m s^{-1}

$$\begin{aligned} E_k &= \frac{1}{2}m\boldsymbol{v}^2 \\ &= \frac{1}{2} \times 70 \times (10)^2 \\ &= 3.5 \times 10^3\ \text{J} \end{aligned}$$

For the car: $m = 1500$ kg, $\boldsymbol{v} = 17\ \text{m s}^{-1}$

$$E_k = \frac{1}{2} m\boldsymbol{v}^2$$
$$= \frac{1}{2} \times 1500 \times (17)^2$$
$$= 2.2 \times 10^5\ \text{J}$$

The value of the ratio $\dfrac{E_k(\text{car})}{E_k(\text{athlete})} = \dfrac{2.2 \times 10^5}{3.5 \times 10^3} = 63.$

The car has about 60 times as much kinetic energy as the athlete.

PHYSICS FACT

The truth of the advertising slogan 'Speed kills' can be appreciated by comparing the kinetic energy of a 1500 kg car travelling at 60 km h^{-1} (16.7 m s^{-1}) with the same car travelling at 120 km h^{-1} (33.3 m s^{-1}).

At 60 km h^{-1} its kinetic energy is:

$$E_k = \frac{1}{2} m\boldsymbol{v}^2$$
$$= \frac{1}{2} \times 1500 \times (16.7)^2$$
$$= 2.09 \times 10^5\ \text{J.}$$

At 120 km h^{-1} its kinetic energy is:

$$E_k = \frac{1}{2} m\boldsymbol{v}^2$$
$$= \frac{1}{2} \times 1500 \times (33.3)^2$$
$$= 8.32 \times 10^5\ \text{J.}$$

A doubling of velocity produces a fourfold increase in the kinetic energy and, therefore, a fourfold increase in the work that needs to be done on the car to stop it during a crash. It also means that four times as much energy has to be transformed into the energy of deformation, heat and sound or transferred to other objects.

WORKING SCIENTIFICALLY 5.2

When you rub your hands together, the movement of the friction ridges on your hands across one another causes some of the kinetic energy to be converted into heat. Devise a method allowing you to measure the increase in temperature of your hands. Then, use this method in an experiment to investigate one of the following:

- the relationship between hand surface area and heat increase
- the mathematical relationship between the relative speed of the hands and the increase in hand surface temperature.

5.3 Exercise 1

1 (a) Calculate the kinetic energy of a 2000 kg elephant charging at a speed of 8.0 m s^{-1}.
(b) Estimate the kinetic energy of:
(i) a cyclist riding to work
(ii) a small crawling across a footpath.

2 A gardener pushes a loaded wheelbarrow with a mass of 60 kg a distance of 4.0 m along a straight horizontal path against a constant friction force of 120 N. He applies a horizontal force of 150 N on the wheelbarrow. If the wheelbarrow is initially at rest, what is its final speed?

5.4 Potential energy

5.4.1 Stored energy

Energy that is stored is called potential energy. Objects that have potential energy have the capacity to apply forces and do work. Potential energy takes many forms.

- The food that you eat contains potential energy. Under certain conditions, the energy stored in food can be transformed into other forms of energy. Your body is able to transform the potential energy in food into internal energy so that you can maintain a constant body temperature. Your body transforms some of the food's potential energy into the kinetic energy of blood, muscles and bones so that you can stay alive and move. Some of it is transformed into electrochemical energy to operate your nervous system.
- Batteries contain potential energy. In the next chapter, you will see how the energy stored by 'separating' charges that are attracted to each other can be transformed into other forms of energy by completing a circuit.
- An object that is in a position from which it could potentially fall has **gravitational potential energy**. The gravitational potential energy of an object is 'hidden' until the object is allowed to fall. Gravitational potential energy exists because of the gravitational attraction of masses towards each other. All objects with mass near the Earth's surface are attracted towards the centre of the Earth. The further away from the Earth's surface an object is, the more gravitational potential energy it has.
- Energy can be stored in objects by compressing them, stretching them, bending them or twisting them. If the change in shape can be reversed, energy stored in this way is called **strain potential energy**. Strain potential energy can be transformed into other forms of energy by allowing the object to reverse its change in shape.

5.4.2 Gravitational potential energy

When an object is in free fall, work is done on it by the force of gravity, transforming gravitational potential energy into kinetic energy. When you lift an object, you do work on it by applying an upwards force on it greater than or equal to its weight. Although the gravitational field strength, $\boldsymbol{g}$, decreases with distance from the Earth's surface, it can be assumed to be constant near the surface. The increase in gravitational potential energy ΔU_g by an object of mass m lifted through a height $\Delta\boldsymbol{h}$ can be found by determining how much work is done on it by the force (or forces) opposing the force of gravity.

$$\begin{aligned} W &= \boldsymbol{F}\boldsymbol{s} \\ &= m\boldsymbol{g}\Delta\boldsymbol{h} \quad (\text{substituting } \boldsymbol{F} = m\boldsymbol{g} \text{ and } \boldsymbol{s} = \Delta\boldsymbol{h}) \\ \Rightarrow \Delta U_g &= m\boldsymbol{g}\Delta\boldsymbol{h} \end{aligned}$$

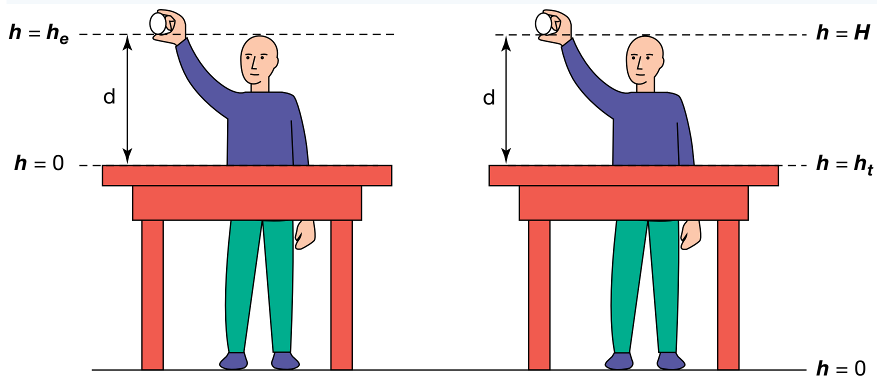
This formula only provides a way of calculating *changes* in gravitational potential energy. If the gravitational potential energy of an object is defined to be zero at a reference height, a formula for the quantity of gravitational potential energy can be found for an object at height $\boldsymbol{h}$ above the reference height.

$$\begin{aligned} \Delta U_g &= m\boldsymbol{g}\Delta\boldsymbol{h} \\ \Rightarrow U_g - 0 &= m\boldsymbol{g}(\boldsymbol{h} - 0) \\ \Rightarrow U_g &= m\boldsymbol{g}\boldsymbol{h} \end{aligned}$$

Usually the reference height is ground or floor level. Sometimes it might be more convenient to choose another reference height. However, it is the change in gravitational potential energy that is most important in investigating energy transformations. Figure 5.8 below shows that the gain in gravitational potential energy as a raw egg is lifted from the surface of a table is mgd. When the raw egg is dropped to the table, the result will be the same whether you use the height of the table or ground level as your reference height. The gravitational potential energy gained will be transformed into kinetic energy as work is done on the egg by the force of gravity.

$$\begin{aligned}\Delta U_g &= mg\Delta h \\ &= mg(h_e - 0) \\ &= mgd\end{aligned} \qquad \begin{aligned}\Delta U_g &= mg\Delta h \\ &= mg(H - h_t) \\ &= mgd\end{aligned}$$

FIGURE 5.8 The choice of reference height does not have any effect on the change in gravitational potential energy.

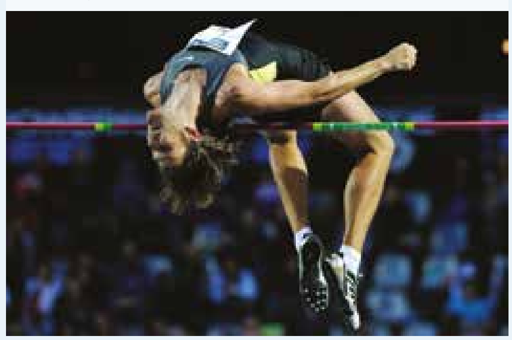


AS A MATTER OF FACT

High jumpers use a technique called the Fosbury Flop, which allows them to clear the bar while keeping their centre of mass as low as possible. The gravitational potential energy needed to clear the bar is minimised. Thus, with their maximum kinetic energy at take-off, high jumpers can clear those extra few centimetres.

Incidentally, you might like to estimate just how much energy is needed to clear the bar in the high jump. Start by working out the change in height of an athlete's centre of mass during a jump of about 2.0 m.

FIGURE 5.9 The Fosbury Flop can place a high jumper's centre of mass below the bar during the jump.



5.4 SAMPLE PROBLEM 1

A 30 kg child sits at the top of a smooth slide. The vertical distance of the slide is 3 metres. Calculate

(a) the child's gravitational potential energy at the top of the slide

(b) the child's velocity when he is 1 metre above the ground.

SOLUTION:

(a) $U_g = mgh$
$= 30\text{ kg} \times 9.8\text{ m s}^{-2} \times 3\text{ m}$
$= 882\text{ J}$

(b) By the Law of Conservation of Energy, the child's total energy will always be the sum of his gravitational potential energy and his kinetic energy, so any loss in gravitational potential energy will be equal to his gain in kinetic energy:

$$-\Delta U_g = \Delta E_k$$

$$-mg\Delta h = \frac{1}{2}mv^2$$

$$-30\text{ kg} \times 9.8\text{ m s}^{-2} \times (1\text{m} - 3\text{ m}) = \frac{1}{2} \times 30\text{ kg} \times v^2$$

$$588\text{ J} = 15v^2$$

$$v = 6.3\text{ m s}^{-1}$$

FIGURE 5.10

5.4 Exercise 1

1 A ski-lift carries a 60 kg skier to the top of a ski run that is 40 metres above the bottom of the ski run.
 (a) What is her gravitational potential energy relative to the bottom of the ski run?
 (b) What would be the skier's speed at the bottom of the ski run if she does not control her progress (sensible skiers do!) and if friction and air resistance are ignored?

2 A 1.5 kg model rocket is fired directly up into the air at a speed of 40 m s^{-1}.
 (a) What height will the rocket reach?
 (b) What will be the rocket's gravitational potential energy at the top of its flight path?

3 A 100 g weight is attached to the end of a 60 cm long wire and raised until the wire makes an angle of 45° to the vertical. What will be the speed of the weight at the bottom of its swing?

5.5 Conservation of energy

5.5.1 Energy and efficiency

Along with kinetic energy, gravitational potential energy and strain potential energy are referred to as forms of mechanical energy. Transformation to or from each of these forms of energy requires the action of a force. A single bounce of a tennis ball onto a hard surface involves the following mechanical energy transformations.

- As the ball falls, the force of gravity does work on the ball, transforming gravitational potential energy into kinetic energy.
- As soon as the bottom of the tennis ball touches the ground, the upward push of the ground does work on the tennis ball, transforming kinetic energy into strain potential energy. A small amount of gravitational potential energy is also transformed into strain potential energy. This continues until the kinetic energy of the ball is zero.

- As the ball begins to rise and remains in contact with the ground, the upward push of the ground does work on the tennis ball, transforming strain potential energy into kinetic energy and a small amount of gravitational potential energy until the ball loses contact with the ground.
- As the ball gains height, the force of gravity does work on the ball, transforming kinetic energy into gravitational potential energy.

Of course, if mechanical energy were conserved, the ball would return to the same height from which it was dropped. In fact, mechanical energy is not conserved. During each of the mechanical transformations that occur during the bounce, some of the mechanical energy of the ball is transformed. Some of the ball's mechanical energy is transformed to thermal energy of the air, ground and ball, resulting in a small temperature increase. Some mechanical energy can be lost as sound, while permanent deformation through the breaking of bonds between atoms can also lead to a loss of such energy.

Mechanical energy losses to thermal energy, sound etc. are largely permanent. It is very difficult to convert this lost energy back into mechanical energy and so it is not considered useful. The efficiency, η, of an energy transfer is calculated from the ratio:

$$\eta = \frac{\text{useful energy out}}{\text{total energy in}}$$

where η is the Greek letter eta.

5.5 SAMPLE PROBLEM 1

A ball dropped from 1.50 m rebounds to 1.20 m. What is the efficiency?

SOLUTION:

$$\eta = \frac{\text{useful energy out}}{\text{total energy in}}$$

The 'total energy in' is the gravitational potential energy of the ball at rest at a height of 1.50 m.

$$\begin{aligned} U_g &= mgh_1 \\ &= mg \times 1.50\ \text{m} \end{aligned}$$

The 'useful energy out' is the gravitational potential of the ball at its rebound height of 1.20 m.

$$\begin{aligned} U_g &= mgh_2 \\ &= mg \times 1.20\ \text{m} \\ \eta &= \frac{1.2mg}{1.5mg} \\ &= 80\% \end{aligned}$$

WORKING SCIENTIFICALLY 5.3

When a ball is dropped from a height, it bounces a number of times, with each bounce reaching a lower peak height than the one before it. By measuring the heights of at least five bounces for a tennis ball, derive a mathematical model that would allow you to predict the bounce height of the ball based on the bounce number. Use the model to determine the number of bounces after which the ball has effectively stopped bouncing (the 'bounce extinction point').

5.5.2 Conservation of total mechanical energy

While it is not possible to completely remove these mechanical energy losses, it is possible to take these mechanical energy losses into account when considering the total energy of a system.

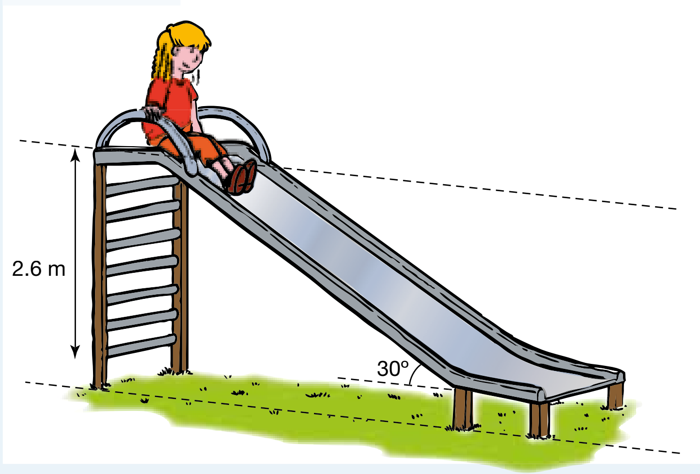
5.5 SAMPLE PROBLEM 2

A 40 kg child sits at the top of a 2.6 m high slide that is inclined at an angle of 30° to the horizontal, as shown in figure 5.11. If a frictional force of 28 N acts on the child as she comes down the slide, what will be her velocity at the bottom?

SOLUTION:

We can see that there are three things to be considered in this system: gravitational potential energy (U_g), kinetic energy (E_k) and the work done on the child by friction (W_f). As the total energy in the system is conserved, it can be seen that:

FIGURE 5.11



$$0 = \Delta U_g + \Delta E_k + W_f$$

$$0 = \mathrm{m}\boldsymbol{g}(\boldsymbol{h}_f - \boldsymbol{h}_i) + \frac{1}{2}\mathrm{m}(\boldsymbol{v}^2 - \boldsymbol{u}^2) + \boldsymbol{F}_f\boldsymbol{s}$$

The displacement over which the frictional force acts will be equal to the length of the slide and so:

$$\boldsymbol{s} = \frac{2.6\ \text{m}}{\cos 30°} = 3.0\ \text{m}$$

Substituting values into the conservation equation above:

$$0 = 40 \times 9.8 \times (0 - 2.6) + \frac{1}{2}40 \times (\boldsymbol{v}^2 - 0^2) + (28)(3.0)$$

$$0 = -1019.2 + 20\boldsymbol{v}^2 + 84.0$$

$$\boldsymbol{v}^2 = 46.76$$

$$\boldsymbol{v} = 6.8\ \text{m s}^{-1}$$

5.5 Exercise 1

1 A 100 g ball thrown vertically into the air with a speed of 8 m s^{-1} rises to a height of 2.8 m before returning to the thrower.
(a) What is the magnitude of air resistance acting on the ball on its way up?
(b) What will be the speed of the ball when it returns to the thrower?

2 A 40 kg child slides down a slippery slide that is 2.5 m high and makes an angle of 37° with the ground. If the slide provides friction equal to 12 N, what will be the child's speed:
(a) at the bottom of the slide?
(b) halfway down the slide?

3 When Susan is at the top of her path when on a swing, she is 2.5 m above the ground. If Susan has a mass of 60 kg and we ignore the effects of friction, calculate:
(a) her E_k at the top of the path
(b) her speed at the bottom
(c) her U_g when she is 1.5 m above the ground
(d) her speed when she is 1 m above the ground.

WORKING SCIENTIFICALLY 5.4

Many people jump on small trampolines as a method of burning off chemical potential energy. How effective would swinging on a playground swing be as a form of physical exercise? Design an investigation that would allow you to estimate the amount of chemical potential energy that a person needs to supply (in effort) to keep a swing coming up to the same height for a set period of time. Carry out the investigation for different swing heights and determine how energy expenditure is affected by swing height.

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5.6 Work and power

5.6.1 Defining power

Power is the rate at which energy is transferred or transformed. In the case of conversions to or from mechanical energy or between different forms of mechanical energy, power, P, can be defined as the rate at which work is done.

$$P = \frac{W}{\Delta t}$$

where
W = the work done
Δt = the time interval during which the work is done.

The SI unit of power is the watt (W), which is defined as 1 J s^{-1}.

The power delivered when a force, $\boldsymbol{F}$, is applied to an object can also be expressed in terms of the object's velocity $\boldsymbol{v}$.

$$\begin{aligned} P &= \frac{W}{\Delta t} = \frac{\boldsymbol{F}\boldsymbol{s}}{\Delta t} \\ &= \boldsymbol{F} \times \frac{\boldsymbol{s}}{\Delta t} \text{ and since speed } \boldsymbol{v} \text{ equals distance over time,} \\ &= \boldsymbol{F}\boldsymbol{v} \end{aligned}$$

5.6 SAMPLE PROBLEM 1

A student of mass 40 kg walks briskly up a flight of stairs to climb four floors of a building, a vertical distance of 12 m in a time interval of 40 s.

(a) At what rate is the student doing work against the force of gravity?

(b) If energy is transformed by the leg muscles of the student at the rate of 30 kJ every minute, what is the student's power output?

SOLUTION:

(a) The work done by the student against the force of gravity is equal to the gain in gravitational potential energy.

$W = mg\Delta \boldsymbol{h}$

The rate at which the work is done, or power (P), is:

$$P = \frac{W}{\Delta t} = \frac{40\text{ kg} \times 10\text{ N kg}^{-1} \times 12\text{ m}}{40\text{ s}}$$
$$= 120\text{ W}$$

(b) $$P = \frac{\text{energy transferred}}{\text{time taken}}$$
$$= 30\text{ kJ min}^{-1}$$
$$= \frac{30\,000\text{ J}}{60\text{ s}}$$
$$= 500\text{ W}$$

AS A MATTER OF FACT

Which is easier — riding a bike or running?

A normal bicycle being ridden at a constant speed of 4.0 m s^{-1} on a horizontal road is subjected to a rolling friction force of about 7 N and air resistance of about 6 N. The forward force applied to the bicycle by the ground must therefore be about 13 N. The mechanical power output required to push the bicycle along at this velocity is:

$$P = \boldsymbol{Fv}$$
$$= 13\text{ N} \times 4.0\text{ m s}^{-1}$$
$$= 52\text{ W}.$$

Running at a velocity of 4.0 m s^{-1} requires a mechanical power output of about 300 W. Even walking at a speed of 2.0 m s^{-1} requires a mechanical power output of about 75 W.

Riding a bicycle on a horizontal surface is less tiring than walking or running for two reasons.

FIGURE 5.12

1. Less mechanical energy is needed. The body of the rider does not rise and fall as it does while walking or running, eliminating the changes in gravitational potential energy.
2. Because the rider is seated, the muscles need to transform much less chemical energy to support body weight. The strongest muscles in the body can be used almost exclusively to turn the pedals.

Once you start riding uphill or against the wind, the mechanical power requirement increases significantly. For example, in riding along an incline that rises 1 m for every 10 m of road distance covered, the additional power needed by a 50 kg rider travelling at 4.0 m s^{-1} would be:

$$P = \frac{\Delta U_g}{\Delta t}$$
$$= \frac{m\boldsymbol{g}\Delta\boldsymbol{h}}{\Delta t}.$$

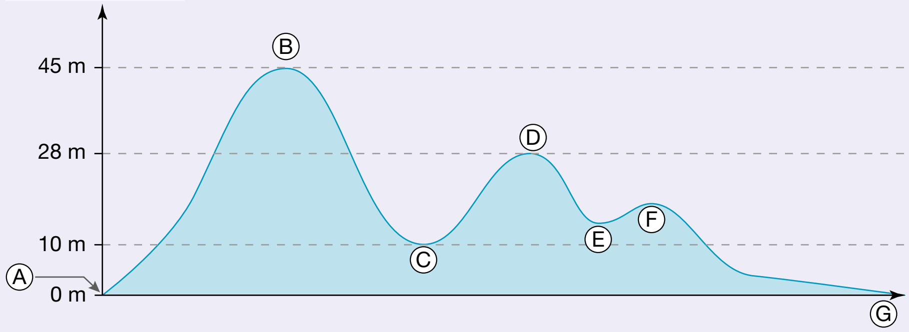
In a time interval of 1.0 s, the vertical climb is $\frac{1}{10}$ of 4.0 m = 0.4 m.

$$\Rightarrow P = \frac{50\text{ kg} \times 10\text{ N kg}^{-1} \times 0.4\text{ m}}{1.0\text{ s}}$$
$$= 200\text{ W}$$

5.6 Exercise 1

Consider the section of roller-coaster track illustrated here:

FIGURE 5.13



1 (a) Determine the amount of work done by the track motor in raising a 3200 kg roller-coaster from A to B.
(b) If the motor has a power of 12.4 kW, how long will it take to lift the roller-coaster to the crest?

2 (a) If all of the 720 J of energy stored in the hind legs of a young 50 kg kangaroo were used to jump vertically, how high could it jump?
(b) What is the kangaroo's power output if the 720 J of stored energy is transformed into kinetic energy during a 1.2 second interval?

5.7 Review

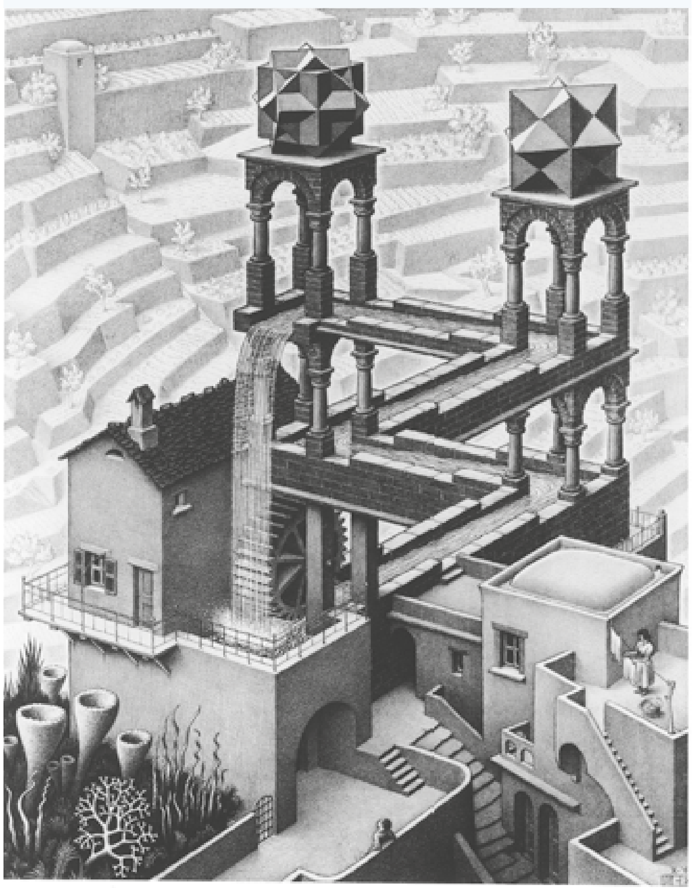
5.7.1 Summary

- The Law of Conservation of Energy states that energy cannot be created or destroyed.
- Work is done when energy is transferred to or from an object by the action of a force. The work done on an object by a force is the product of the magnitude of the force and the magnitude of the displacement in the direction of the force.
- All moving objects possess kinetic energy. The kinetic energy of an object can be expressed as $E_k = \frac{1}{2}mv^2$.
- The work done on an object by the net force is equal to the object's change in kinetic energy.
- The change in gravitational potential energy of an object near the Earth's surface can be expressed as $\Delta U_g = mg\Delta h$ where Δh is the object's change in height.
- Kinetic energy and gravitational potential energy are referred to as forms of mechanical energy. During a mechanical interaction, it is usually reasonable to assume that total mechanical energy is conserved.
- The efficiency of an energy transfer is calculated from the ratio:
  efficiency, $\eta = \dfrac{\text{useful energy out}}{\text{total energy in}}$
- Power is the rate at which energy is transferred or transformed. In mechanical interactions, power is also equal to the rate at which work is done.
- The power delivered by a force is the product of the magnitude of the force and the velocity of the object on which the force acts.

5.7.2 Questions

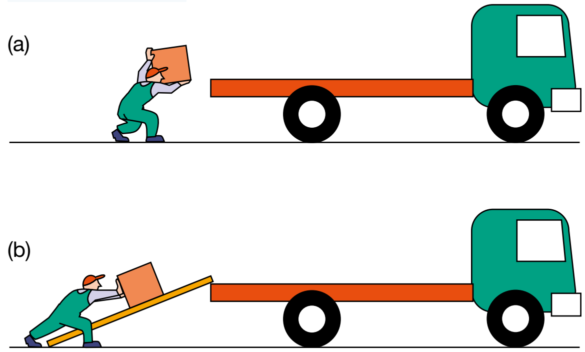
1. How are mechanical energy transfers different from other types of energy transfer?
2. Distinguish between an energy transformation and an energy transfer.
3. How much work is done on a 4.0 kg brick as it is lifted through a vertical distance of 1.5 m?
4. Imagine that you are trying to single-handedly push-start a 2000 kg truck with its handbrake on. Not surprisingly, the truck doesn't move. How much work are you doing on the truck?
5. Estimate the kinetic energy of:
   (a) a car travelling at 60 km h^{-1} (16.7 m s^{-1}) on a suburban street
   (b) a tennis ball as it is returned to the server in a Wimbledon final
   (c) a cyclist riding to work
   (d) a snail crawling across a footpath.
6. A car of mass 1200 kg is being towed by a thick rope connected to a larger car. After stopping at traffic lights, the tension in the rope is a constant 4000 N for a distance of 50 metres. The frictional force resisting the motion of the smaller car is 400 N.
   (a) Calculate how much work is done on the smaller car by the net force.
   (b) Evaluate the kinetic energy of the smaller car after the distance of 50 metres has been covered.
   (c) Calculate the velocity of the smaller car at the end of the 50 metres of towing.
7. A weightlifter holds a loaded bar above his head for three seconds. Is he doing any work on the bar during this time? Explain.
8. If you drop a book onto the floor, it comes to rest. What has happened to the gravitational potential energy that it had before you dropped it?
9. Figure 5.14 shows a drawing by the artist M.C. Escher. Explain the essential flaw underlying the motion of the water in terms of energy conservation.

FIGURE 5.14 M.C. Escher, *Waterfall*, © 2007 The M.C. Escher Company, Holland.



10. A 60 kg hiker carries a 10 kg backpack up a 10 m high hill.
    (a) How much work will be done by the hiker?
    (b) If the hiker takes 40 seconds to climb the hill, what average power did he develop? (Assume that he walked up the hill at constant speed.)
11. (a) What percentage of energy has been dissipated by a rubber ball striking the ground if it bounces to a height of 1.2 m after being dropped from a height of 2 m?
    (b) Where has this energy gone?
12. Katrina throws a 100 g ball vertically with an initial upward velocity of 12 m s^{-1}, and it rises to a height of 5 m before returning to her.
    (a) What was the average force provided by air resistance as the ball rose into the air?
    (b) Assuming that the same amount of air resistance acts on the ball as it falls back down, calculate the velocity of the ball when Katrina catches it again.
13. A toddler swings her fluffy toy by a string around in circles at a constant speed. How much work does she do on the toy in completing:
    (a) one full revolution
    (b) half of a full revolution?
14. Use the formulae for work and kinetic energy to show that their units are equivalent.
15. Estimate the amount of work done on a 58 g tennis ball by the racquet when the ball is served at a speed of 200 km h^{-1}.
16. Estimate the change in gravitational potential energy of:
    (a) a skateboarder riding down a half-pipe
    (b) a child sliding from the top to the bottom of a playground slide
    (c) you at your maximum height as you jump up from rest.
17. A truck driver wants to lift a heavy crate of books with a mass of 20 kg onto the back of a truck through a vertical distance of 1 m. The driver needs to decide whether to lift the crate straight up, or push it up along a ramp.
    (a) What is the change in gravitational potential energy of the crate of books in each case?
    (b) How much work must be done against the force of gravity in each case?
    (c) If the ramp is perfectly smooth, how much work must be done by the truck driver to push the crate of books onto the back of the truck?
    (d) In view of your answers to (b) and (c), which of the two methods is the best way to get the crate of books onto the back of the truck? Explain your answer.

FIGURE 5.15

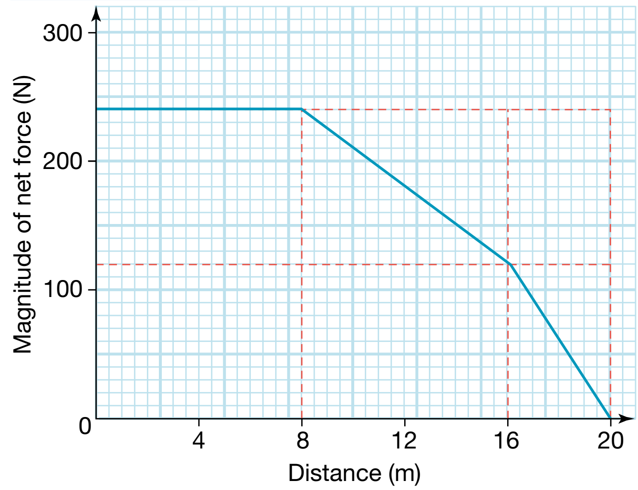


18. World-class hurdlers raise their centre of mass as little as possible when they jump over the hurdles. Why?
19. If a 160 g cricket ball is dropped from a height of 2.0 m onto a hard surface, calculate:
    (a) the kinetic energy of the ball as it hits the ground
    (b) the maximum amount of elastic potential energy stored in the ball
    (c) the height to which it will rebound.

    Assume that 32% of the kinetic energy of the cricket ball is stored in it as it bounces on a hard surface.
20. A tourist on an observation tower accidentally drops her 1.2 kg camera to the ground 20 m below.
    (a) What kinetic energy does the camera gain before shattering on the ground?
    (b) What is the velocity of the camera as it hits the ground?

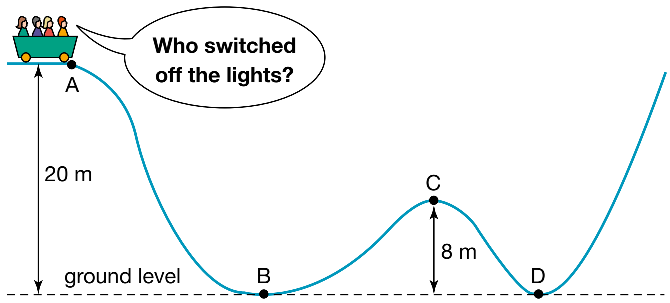
21. A girl of mass 50 kg rollerblades freely from rest down a path inclined at 30° to the horizontal. Figure 5.16 shows how the magnitude of the net force on the girl changes as she progresses down the path.
    (a) What is the kinetic energy of the girl after rolling a distance of 8.0 m?
    (b) What is the sum of the friction force and air resistance on the girl over the first 8.0 m?
    (c) What is the kinetic energy of the girl at the end of her 20 m roll?
    (d) How much gravitational potential energy has been lost by the girl during her 20 m roll?
    (e) Account for the difference between your answers to (c) and (d).

FIGURE 5.16



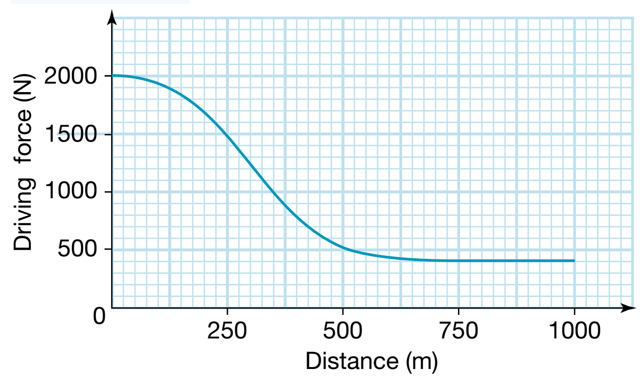
22. Figure 5.17 shows part of a roller-coaster track. As a fully loaded roller-coaster car of total mass 450 kg approaches point A with a velocity of 12 m s^{-1}, the power fails and it rolls freely down the track. The friction force on the car can be assumed to be negligible.
    (a) What is the kinetic energy of the loaded car at point A?
    (b) Determine the velocity of the loaded car at each of points B and C.
    (c) What maximum height will the car reach after passing point D?

FIGURE 5.17



23. Figure 5.18 shows how the driving force on a 1200 kg car changes as it accelerates from rest over a distance of 1 km on a horizontal road. The average force opposing the motion of the car due to air resistance and road friction is 360 N.
    (a) How much work has been done by the forward push (the driving force) on the car?
    (b) How much work has been done on the car to overcome both air resistance and road friction?
    (c) What is the velocity of the car when it has travelled 1 km?

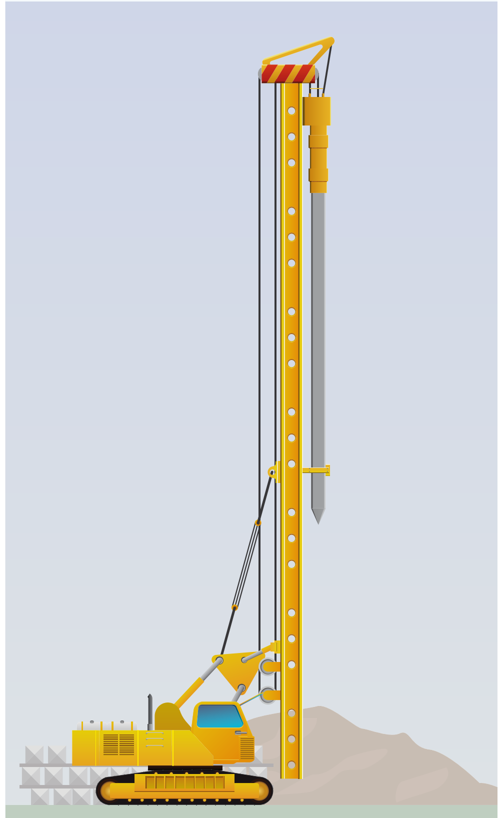
FIGURE 5.18



24. Jo and Bill are conducting an experimental investigation into the bounce of a basketball. Bill drops the ball from various heights and Jo measures the rebound height. They also use an electronic timer with thin and very light wires attached to the ball and to alfoil on the floor to measure the impact time. A top-loading balance measures the mass of the ball. What physical quantities can they calculate using these four measurements?

25. A tractor engine has a power output of 80 kW. The tractor is able to travel to the top of a 500 m hill in 4 minutes and 30 seconds. The mass of the tractor is 2.2 tonnes. What is the efficiency of the engine?
26. Human muscle has an efficiency of about 20%. Take a heavy mass, about 1–2 kg, in your hand. With your hand at your shoulder, raise and lower the mass 10 times as fast as you can. Measure the mass, your arm extension and the time taken, and calculate the amount of energy expended, your power output and your power input.
27. A pile driver has an efficiency of 80%. The hammer has a mass 500 kg and the pile a mass of 200 kg. The hammer falls through a distance of 5.0 m and drives the pile 50 mm into the ground. Calculate the average resistance force exerted by the ground.

FIGURE 5.19 A pile driver.



28. Estimate the average power delivered to a 58 g tennis ball by a racquet when the ball is served at a speed of 200 km h^{-1} and the ball is in contact with the racquet for 4.0 ms.
29. At what average rate is work done on a 4.0 kg barbell as it is lifted through a vertical distance of 1.5 m in 1.2 s?
30. In the sport of weightlifting, the clean-and-jerk involves bending down to grasp the barbell, lifting it to the shoulders while squatting and then jerking it above the head while straightening to a standing position. In 1983, Bulgarian weightlifter Stefan Topurov became the first man to clean and jerk three times his own body mass when he lifted 180 kg. Assume that he raised the barbell through a distance of 1.8 m in a time of 3.0 s.
    (a) How much work did Stefan do in overcoming the force of gravity acting on the barbell?
    (b) How much power was supplied to the barbell to raise it against the force of gravity?
    (c) How much work did Stefan do on the barbell while he was holding it stationary above his head?
31. A small car travelling at a constant speed of 20 m s^{-1} on a horizontal road is subjected to air resistance of 570 N and road friction of 150 N. What power provided by the engine of a car is used to keep it in motion at this speed?
32. While a 60 kg man is walking at a speed of 2.0 m s^{-1}, his centre of mass rises and falls 3.0 cm with each stride. At what rate is he doing work against the force of gravity if his stride length is 1.0 m?
33. A bicycle is subjected to a rolling friction force of 6.5 N and an air resistance of 5.7 N. The total mass of the bicycle and its rider is 75 kg. Its mechanical power output while being ridden at a constant speed along a horizontal road is 56 W.
    (a) At what speed is it being ridden?
    (b) If the bicycle was ridden at the same speed up a slope inclined at 30° to the horizontal, what additional mechanical power would need to be supplied to maintain the same speed? Assume that the rolling friction and air resistance are the same as on the horizontal road.
34. A roller-coaster rolling down the first hill starts to climb the next hill, which (by poor design) is the same height as the first. Sketch a graph that demonstrates the most likely motion of the roller-coaster with time, *t*, on the horizontal axis and height above ground level, *h*, on the vertical axis.
35. Engineers designing a super drop ride determine that the riders will be raised to a height of 200 m, with a braking zone starting 20 m from the bottom of the ride. Evaluate the feasibility of this design.

36. An electric motor is to power a lifting chain that raises a 1500 kg roller-coaster up a 70 m hill in 3 minutes. If the roller-coaster is moving at 4 m s^{-1} at the top of the hill, what will the minimum power of the motor need to be? (Assume that the roller-coaster is stationary at the bottom of the hill.)

PRACTICAL INVESTIGATIONS

Investigation 5.1: Climbing to the top

Aim

To calculate the work required and the power that must be developed to displace a mass vertically upwards

Apparatus

bathroom scales
builder's tape measure or laser distance meter
access to a flight of stairs
stopwatch
schoolbag with books (total mass at least 3 kg)

Theory

In running up a flight of stairs, you are doing work against gravity. The amount of work is described by the equation $W = m\,\boldsymbol{g}\,\Delta \boldsymbol{h}$, where m is the mass being moved, $\boldsymbol{g}$ is the gravitational acceleration, and $\Delta \boldsymbol{h}$ is the upwards displacement. The energy exerted in doing the work against gravity is provided by the conversion of chemical potential energy stored in the muscles.

The power developed in moving this mass upwards depends upon the rate at which work is done: that is, $P = \dfrac{W}{\Delta t}$ where W is the work done in joules and Δt is the time interval in seconds over which the work is done.

Method

Measure and record your mass in kilograms.
Measure the height of the flight of stairs in metres.
Measure and record the time taken for you to run up the flight of stairs.
After you have recovered, measure and record your mass while carrying the schoolbag full of books.
Measure and record the time taken for you to run up the flight of stairs while carrying the loaded school bag.
Enter your results in the table below.

Results

	mass, m (kilograms)	height, $\Delta \boldsymbol{h}$ (metres)	time, Δt (seconds)
without load			
with load			

Analysis

Use your results to calculate the work done against gravity and the power developed as you ran up the stairs (a) without the bag of books, and (b) while carrying the bag of books.

Questions

1. What effect (if any) did increasing the mass have on the amount of work done to travel the same vertical distance?
2. What effect did increasing the mass have on the amount of power developed?
3. If your muscles are 25% efficient, at what rate was chemical energy transformed by your body to get you up the stairs when (a) you were not carrying the bag of books, and (b) when you were carrying the bag of books?
4. Compare and comment on the difference that the extra load makes to the work done against gravity and the power developed.

TOPIC 6 Momentum, energy and simple systems

6.1 Overview

6.1.1 Module 2: Dynamics

Momentum, energy and simple systems

Inquiry question: How is the motion of objects in a simple system dependent on the interaction between the objects?

Students:

- conduct an investigation to describe and analyse one-dimensional (collinear) and two-dimensional interactions of objects in simple closed systems (ACSPH064)
- quantitatively analyse and predict, using the law of conservation of momentum ($\Sigma\, m\boldsymbol{v}_{before} = \Sigma\, m\boldsymbol{v}_{after}$) and the conservation of kinetic energy ($\Sigma\, \frac{1}{2} m\boldsymbol{v}^2_{before} = \Sigma\, \frac{1}{2} m\boldsymbol{v}^2_{after}$), the results of interactions in elastic collisions* (ACSPH066)
- investigate the relationship and analyse information obtained from graphical representations of force as a function of time
- evaluate the effects of forces involved in collisions and other interactions, and analyse the interactions quantitatively using the concept of impulse ($\Delta \boldsymbol{p} = \boldsymbol{F}\Delta t$)
- analyse and compare the momentum and kinetic energy of elastic and inelastic collisions (ACSPH066).

*Note: in the text, $\boldsymbol{v}_{\text{before}}$ (initial velocity) will be represented by $\boldsymbol{u}$, and $\boldsymbol{v}_{\text{after}}$ (final velocity) will be represented by $\boldsymbol{v}$.

FIGURE 6.1 This collision is a mechanical interaction. The motion of the car is changed as a result of the action of a force. The change in motion depends on the size of the force and the mass of the car. However, it is obvious that not just the motion of the car has changed. Some of the car's kinetic energy has been transferred to the object it has collided with — making it vibrate and even changing its shape. Some of the car's kinetic energy has been transformed into other forms of energy — for example, sound, heat, and energy stored in the deformed panels. An understanding of mechanical interactions such as these can teach us how to design safer cars, save countless lives and reduce serious injuries.

6.2 Momentum and impulse

6.2.1 A sporting example

When two objects collide, there is an interaction between them that arises from Newton's Third Law of Motion. As the two objects come into contact, they will exert equal but opposite forces on each other. For example, when a golf club hits a golf ball, the club exerts a force upon the ball that accelerates the ball into motion. However, at the same time, the golf ball will exert an equal force back upon the golf club, which will cause the club to decelerate as it acts in the opposite direction.

If we look at this mathematically, we can see that:

$$\boldsymbol{F}_{\text{ball}} = -\boldsymbol{F}_{\text{club}} \qquad [1]$$

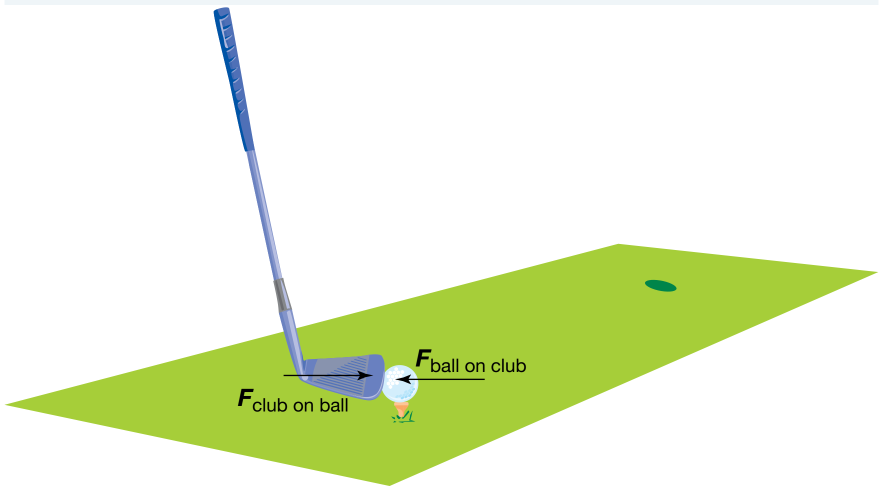
where the negative sign in front of $\boldsymbol{F}_{\text{club}}$ indicates that it is a force acting in the opposite direction to $\boldsymbol{F}_{\text{ball}}$.

In keeping with Newton's Second Law, we can give the relationship between the force exerted, mass and acceleration for both the club and the ball as:

$$\boldsymbol{F}_{\text{club}} = m_{\text{club}}\boldsymbol{a}_{\text{club}} \qquad [2]$$

$$\boldsymbol{F}_{\text{ball}} = m_{\text{ball}}\boldsymbol{a}_{\text{ball}} \qquad [3]$$

FIGURE 6.2 When a golf club hits a golf ball, each will exert an equal but opposite force on the other.



Substituting [2] and [3] into [1], we get:

$$m_{\text{ball}}\boldsymbol{a}_{\text{ball}} = -m_{\text{club}}\boldsymbol{a}_{\text{club}} \qquad [4]$$

As $\boldsymbol{a} = \dfrac{\boldsymbol{v} - \boldsymbol{u}}{\Delta t}$

we can further modify equation [4]:

$$m_{\text{ball}}\frac{(\boldsymbol{v}_{\text{ball}} - \boldsymbol{u}_{\text{ball}})}{\Delta t} = -\,m_{\text{club}}\frac{(\boldsymbol{v}_{\text{club}} - \boldsymbol{u}_{\text{club}})}{\Delta t}$$

As the time interval taken to accelerate the ball will be equal to the time interval taken to decelerate the golf club, we can cancel Δt from both sides to get:

$$m_{\text{ball}}(\boldsymbol{v}_{\text{ball}} - \boldsymbol{u}_{\text{ball}}) = -m_{\text{club}}(\boldsymbol{v}_{\text{club}} - \boldsymbol{u}_{\text{club}})$$

Using the distributive law, we end up with:

$$m_{\text{ball}}\boldsymbol{v}_{\text{ball}} - m_{\text{ball}}\boldsymbol{u}_{\text{ball}} = -(m_{\text{club}}\boldsymbol{v}_{\text{club}} - m_{\text{club}}\boldsymbol{u}_{\text{club}}) \qquad [5]$$

The product of an object's mass and its velocity is referred to as an object's **momentum**, $\boldsymbol{p}$. Momentum is a vector quantity and it is measured in either newton seconds (N s) or in kilogram metres per second (kg m s^{-1}).

Equation [5] can be modified to show the relationship between the momenta of the ball and the club before and after their collision:

$$\boldsymbol{p}_{\text{ball final}} - \boldsymbol{p}_{\text{ball initial}} = -(\boldsymbol{p}_{\text{club final}} - \boldsymbol{p}_{\text{club initial}}) \qquad [6]$$

or, more simply:

$$\Delta \boldsymbol{p}_{\text{ball}} = -\Delta \boldsymbol{p}_{\text{club}}$$

That is, the gain in momentum of the ball is equal to the loss of momentum experienced by the club.

6.2.2 The conservation of momentum

We can consider equation [6] in terms of the interaction between two colliding objects A and B as:

$$\boldsymbol{p}_{\text{A final}} - \boldsymbol{p}_{\text{A initial}} = -(\boldsymbol{p}_{\text{B final}} - \boldsymbol{p}_{\text{B initial}})$$

Rearranging, we get:

$$\boldsymbol{p}_{\text{A final}} - \boldsymbol{p}_{\text{A initial}} = -\boldsymbol{p}_{\text{B final}} + \boldsymbol{p}_{\text{B initial}}$$

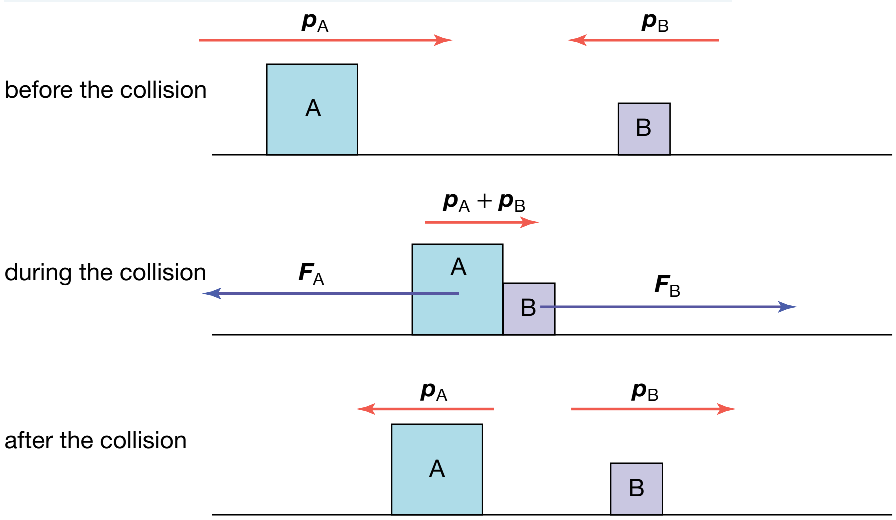
then,

$$\boldsymbol{p}_{\text{A final}} + \boldsymbol{p}_{\text{B final}} = \boldsymbol{p}_{\text{B initial}} + \boldsymbol{p}_{\text{A initial}} \qquad [7]$$

We can see that the sum of the momenta of the objects after collision is equal to the sum of the momenta of the objects before collision. This idea is embodied in the **Law of Conservation of Momentum**:

For any closed isolated system, the sum of the momenta of all objects in that system is a constant.

FIGURE 6.3 The net force on this system of two blocks is zero.



6.2 SAMPLE PROBLEM 1

Consider the collision illustrated in figure 6.3. Block A has a mass of 5.0 kg; block B has a mass of 3.0 kg; and each block has a speed of 4.0 m s^{-1} before the collision.

(a) Determine the final velocity of block B if block A rebounds to the left with a velocity of 0.50 m s^{-1}.

(b) Determine the velocity of the blocks after the collision if, instead of moving off separately, they stick together upon collision.

SOLUTION:

(a) Assign the direction to the right as positive. The expressions $\boldsymbol{u}_A$ and $\boldsymbol{u}_B$ are the velocities of blocks A and B respectively before the collision, and m_A and m_B are the respective masses of blocks A and B.

Therefore, $\Sigma \boldsymbol{p}_{\text{before}} = m_A\boldsymbol{u}_A + m_B\boldsymbol{u}_B$

$$= 5.0\text{ kg} \times 4.0\text{ m s}^{-1} + 3.0\text{ kg} \times -4.0\text{ m s}^{-1}$$
$$= 20.0\text{ kg m s}^{-1} - 12.0\text{ kg m s}^{-1}$$
$$= 8.0\text{ kg m s}^{-1}$$

According to the Law of Conservation of Momentum, the total momentum of the system before the collision is the same as the total momentum of the system after the collision.

Therefore, $\Sigma\boldsymbol{p}_{\text{after}} = \Sigma\boldsymbol{p}_{\text{before}} = 8.0\text{ kg m s}^{-1}$

Using the expressions $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ for the respective velocities of A and B after collision,

$$8.0\text{ kg m s}^{-1} = m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B$$
$$= 5.0\text{ kg} \times -0.50\text{ m s}^{-1} + 3.0\text{ kg} \times \boldsymbol{v}_B$$
$$= -2.5\text{ kg m s}^{-1} + 3.0\text{ kg} \times \boldsymbol{v}_B$$

Rearranging, we get:

$$8.0\text{ kg m s}^{-1} + 2.5\text{ kg m s}^{-1} = 3.0\text{ kg} \times \boldsymbol{v}_B$$
$$10.5\text{ kg m s}^{-1} = 3.0\text{ kg} \times \boldsymbol{v}_B$$
$$\boldsymbol{v}_B = \frac{10.5\text{ kg m s}^{-1}}{3.0\text{ kg}}$$
$$= 3.5\text{ m s}^{-1}$$

Block B moves off to the right at 3.5 m s^{-1}.

(b) In this case, the blocks A and B will move away from the collision with a combined mass of $(m_A + m_B)$ and with a common velocity that can be expressed as $\boldsymbol{v}_{AB}$.

$$\Sigma\boldsymbol{p}_{\text{after}} = (m_A + m_B)\boldsymbol{v}_{AB}$$
$$8.0\text{ kg m s}^{-1} = (5.0\text{ kg} + 3.0\text{ kg})\boldsymbol{v}_{AB}$$
$$8.0\text{ kg m s}^{-1} = 8.0\text{ kg} \times \boldsymbol{v}_{AB}$$
$$\boldsymbol{v}_{AB} = \frac{8.0\text{ kg m s}^{-1}}{8.0\text{ kg}}$$
$$= 1.0\text{ m s}^{-1}$$

The velocity of the blocks after the collision is 1.0 m s^{-1} to the right.

6.2.3 Impulse

The actual change in momentum of an object, $\Delta\boldsymbol{p}$, is also referred to as its **impulse**.

The impulse of an object can be found in terms of its mass and change in velocity:

$$\Delta\boldsymbol{p} = m\Delta\boldsymbol{v} = m(\boldsymbol{v} - \boldsymbol{u})$$

As $\boldsymbol{F} = m\boldsymbol{a}$

and $\boldsymbol{a} = \dfrac{\boldsymbol{v} - \boldsymbol{u}}{\Delta t}$

$$\boldsymbol{F} = m\frac{\boldsymbol{v} - \boldsymbol{u}}{\Delta t}$$

$$\boldsymbol{F}\Delta t = m(\boldsymbol{v} - \boldsymbol{u})$$

As a result, we can see that the impulse of an object can also be determined from the force acting on the object that changes its velocity, and the time interval over which the force acted on the object:

$$\Delta\boldsymbol{p} = \boldsymbol{F}\Delta t$$

6.2 SAMPLE PROBLEM 2

A putter exerts a force of 8 N for a time interval of 0.01 s on a golf ball at rest on the green. With what velocity will the ball leave the putter if the ball has a mass of 50 g?

SOLUTION

Given: $m = 50 \text{ g} = 0.05 \text{ kg}$; $\boldsymbol{F} = 8 \text{ N}$; $\Delta t = 0.01 \text{ s}$; $\boldsymbol{u} = 0$

To find: $\boldsymbol{v}$

$$\begin{aligned} \boldsymbol{F}\Delta t &= m(\boldsymbol{v} - \boldsymbol{u}) \\ 8 \text{ N} \times 0.01 \text{ s} &= 0.05 \text{ kg} \times (\boldsymbol{v} - 0) \text{ m s}^{-1} \\ 0.08 \text{ Ns} &= 0.05\boldsymbol{v} \text{ kg m s}^{-1} \\ \boldsymbol{v} &= \frac{0.08}{0.05} \\ &= 1.6 \text{ m s}^{-1} \end{aligned}$$

The putter gives the ball a velocity of 1.6 m s^{-1}.

6.2 SAMPLE PROBLEM 3

MOMENTUM AND IMPULSE OF A CAR

A 1200 kg car collides with a concrete wall at a speed of 15 m s^{-1} and takes 0.06 s to come to rest.

(a) What is the change in momentum of the car?
(b) What is the impulse on the car?
(c) What is the magnitude of the force exerted by the wall on the car?
(d) What would be the magnitude of the force exerted by the wall on the car if the car bounced back from the wall with a speed of 3.0 m s^{-1} after being in contact for 0.06 s?

SOLUTION

(a) Assign the initial direction of the car as positive.

$m = 1200 \text{ kg}$, $u = 15 \text{ m s}^{-1}$, $\boldsymbol{v} = -3.0 \text{ m s}^{-1}$, $\Delta t = 0.06 \text{ s}$

$$\begin{aligned} \Delta \boldsymbol{p} &= m\boldsymbol{v} - m\boldsymbol{u} \\ &= m(\boldsymbol{v} - \boldsymbol{u}) \\ &= 1200\,(0 - 15) \\ &= 1200 \times -15 \\ &= -1.8 \times 10^4 \text{ kg m s}^{-1} \end{aligned}$$

The change in momentum is $1.8 \times 10^4 \text{ kg m s}^{-1}$ in a direction opposite to the original direction of the car.

(b) Impulse on the car = change in momentum of the car

$$= -1.8 \times 10^4 \text{ kg m s}^{-1}$$

The impulse on the car is $1.8 \times 10^4 \text{ N s}$ in a direction opposite to the original direction of the car.

(c) Magnitude of impulse = $\boldsymbol{F}\Delta t$

$$\begin{aligned} 1.8 \times 10^4 &= \boldsymbol{F} \times 0.06 \\ \boldsymbol{F} &= \frac{1.8 \times 10^4}{0.06} \\ &= 3.0 \times 10^5 \text{ N} \end{aligned}$$

(d) $\text{Impulse} = m\Delta \boldsymbol{v}$

$$= 1200\,(-3 - 15)$$
$$= 1200 \times -18$$
$$= -2.16 \times 10^4 \text{ N s}$$
$$2.16 \times 10^4 = F\Delta t$$
$$2.16 \times 10^4 = F \times 0.06$$
$$F = \frac{2.16 \times 10^4}{0.06}$$
$$= 3.6 \times 10^5 \text{ N}$$

WORKING SCIENTIFICALLY 6.1

Different types of golf balls have different numbers of dimples on them. Investigate the relationship between the number and distribution of dimples on golf balls and the air resistance that they experience when dropped from a set height.

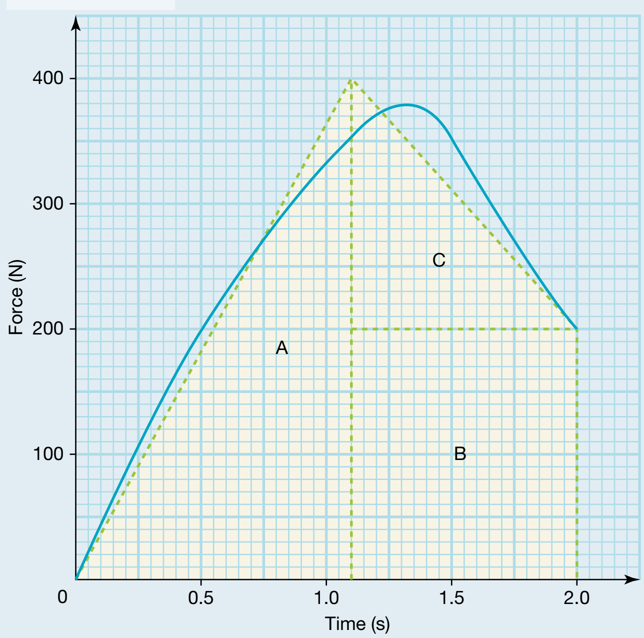
6.2.4 Investigating impulse using a graph of force as a function of time

The force that was determined in 6.2 sample problem 3 was actually the average force on the car. In fact, the force acting on the car is not constant. The impulse delivered by a changing force is given by impulse $= \boldsymbol{F}_{av}\Delta \boldsymbol{t}$. If a graph of force versus time is plotted, the impulse can be determined from the area under the graph.

6.2 SAMPLE PROBLEM 4 *SPEED OF A ROLLER SKATER*

The graph in figure 6.4 describes the changing horizontal force on a 40 kg roller skater as she begins to move from rest. Estimate her speed after 2.0 seconds.

FIGURE 6.4



SOLUTION

The magnitude of the impulse on the skater can be determined by calculating the area under the graph. This can be determined by either counting squares or by finding the shaded area.

$$\begin{aligned}\text{Magnitude of impulse} &= \text{area A} + \text{area B} + \text{area C}\\ &= \tfrac{1}{2} \times 1.1 \times 400 + 0.9 \times 200 + \tfrac{1}{2} \times 0.9 \times 200\\ &= 220 + 180 + 90\\ &= 490 \text{ N s}\end{aligned}$$

$$\begin{aligned}\text{Magnitude of change in momentum} &= m\Delta \boldsymbol{v}\\ 490 &= 40 \times \Delta \boldsymbol{v}\\ \Delta \boldsymbol{v} &= \frac{490}{40}\\ &= 12 \text{ m s}^{-1}\end{aligned}$$

As her initial speed is zero (she started from rest), her speed after 2.0 seconds is 12 m s^{-1}.

WORKING SCIENTIFICALLY 6.2

A ball dropped from a height onto a hard floor will bounce higher than one that is dropped onto a sheet of foam. Devise and conduct an experiment that will allow you to derive a mathematical relationship between the bounce height and the foam thickness.

6.2 Exercise 1

1. A 70 kg basketball player lands on the ground after a jump at a speed of 10 m s^{-1} and is brought to a stop by the ground in 0.35 s. What is the average force exerted on her by the ground?
2. A sprinter with a mass of 60 kg leaves the blocks at the start of a race by pushing off with a force of 700 N exerted over a 0.4 s interval of time. At what speed does the sprinter leave the blocks?
3. A 60 kg trampolinist jumps straight up in the air by exerting an average force of 1060 N on the trampoline bed for a time of 0.5 s.
   (a) What is the impulse of the trampolinist on the trampoline?
   (b) At what speed does he leave the trampoline?
   (c) What will be the maximum height that he reaches?
4. Consider a collision in which a model car of mass 5.0 kg travelling at 2.0 m s^{-1} in an easterly direction catches up to and collides with an identical model car travelling at 1.0 m s^{-1} in the same direction. The cars lock together after the collision. Friction can be assumed to be negligible.
   (a) What was the total momentum of the two-car system before the collision?
   (b) Calculate the velocity of the model cars as they move off together after the collision.
   (c) What is the change in momentum of the car that was travelling faster before the collision?
   (d) What is the change in momentum of the car that was travelling slower before the collision?
   (e) What was the magnitude of impulse on both cars during the collision?
   (f) How are the impulses on the two cars different from each other?

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6.3 Conservation of momentum in two dimensions

6.3.1 Components of momentum

So far, we have only really looked at the conservation of momentum in one dimension. However, it must be remembered that momentum is conserved in two and even three dimensions as well. When momentum is conserved in two dimensions, this means that the total momentum before collision is equal to the total momentum after collision in both horizontal and vertical directions. This is certainly apparent to anyone who has played pool. Let's look at a particular pool shot to make this idea a bit clearer.

A 200 g white ball strikes a stationary 6 ball, also 200 g, at a speed of 3 m s^{-1}. After the collision, the white ball and the 6 ball head off at different angles as shown in figure 6.5.

The figure shows that the white ball now moves at 2 m s^{-1} at an angle of 30° to its original motion, while the 6 ball travels at an angle of 38.2° at 1.62 m s^{-1}.

Let's see how momentum is conserved in this collision in two dimensions. In the x-direction (horizontal), the total momentum before collision, Σp_x is equal to:

$$\Sigma \boldsymbol{p}_x = (0.2 \times 3\cos 0°) + (0.2 \times 0) = 0.6 \text{ N s}$$

Now, the total momentum after collision in the x-direction, $\Sigma p_x'$, can also be found:

$$\begin{aligned}\Sigma p_x' &= (0.2 \times 2\cos 30°) + (0.2 \times 1.62\cos 321.8°)\\ &= 0.35 + 0.25\\ &= 0.6 \text{ N s}\end{aligned}$$

So, we can see that $\Sigma \boldsymbol{p}_x = \Sigma \boldsymbol{p}_x'$ which tells us that momentum has been conserved in the x-direction. We can proceed in a similar way for the y-direction.

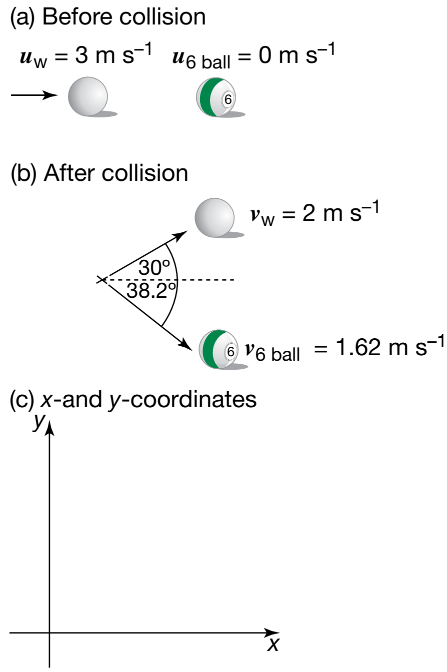
$$\Sigma p_y = (0.2 \times 3\sin 0°) + (0.2 \times 0) = 0$$

After the collision,

$$\begin{aligned}\Sigma p_y' &= (0.2 \times 2\sin 30°) + (0.2 \times 1.62\sin 321.8°)\\ &= 0.2 + -0.2\\ &= 0\end{aligned}$$

Thus, momentum is conserved in both the x-direction and the y-direction.

FIGURE 6.5 A collision between pool balls shows that momentum is conserved in two dimensions.



6.3 SAMPLE PROBLEM 1

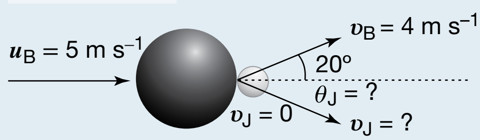
In a game of lawn bowls, a bowl is travelling at a speed of 5 m s^{-1} when it strikes the jack. After the collision, the bowl travels at an angle of 20° to its original direction at a speed of 4 m s^{-1}. If the mass of the bowl is 1.6 kg and the mass of the jack is 0.28 kg, find the speed and direction at which the jack is moving after the collision

SOLUTION

Given: $m_B = 1.6$ kg; $\boldsymbol{u}_B = 5$ m s^{-1} at 0°; $\boldsymbol{v}_B = 4$ m s^{-1} at 20°; $m_J = 0.28$ kg; $\boldsymbol{u}_J = 0$
To find: $\boldsymbol{v}_J$ and θ_J

As we know that momentum will be conserved in both x- and y-directions, we can write that $\Sigma \boldsymbol{p}_x = \Sigma \boldsymbol{p}_x'$ and $\Sigma \boldsymbol{p}_y = \Sigma \boldsymbol{p}_y'$.

FIGURE 6.6



Looking at the x-direction first:

$$\Sigma \boldsymbol{p}_x = \Sigma \boldsymbol{p}_x'$$
$$m_B\boldsymbol{u}_B + m_J\boldsymbol{u}_J = m_B\boldsymbol{v}_B + m_J\boldsymbol{v}_J$$
$$(1.6 \times 5\cos 0°) + (0.28 \times 0) = (1.6 \times 4\cos 20°) + (0.28 \times \boldsymbol{v}_J \cos\theta_J)$$
$$8 = 6.00 + 0.28\boldsymbol{v}_J \cos\theta_J$$
$$2.00 = 0.28\boldsymbol{v}_J \cos\theta_J$$
$$\boldsymbol{v}_J \cos\theta_J = 7.14 \text{ N s}$$

Now, let's look at the y-direction components:

$$m_B\boldsymbol{u}_B + m_J\boldsymbol{u}_J = m_B\boldsymbol{v}_B + m_J\boldsymbol{v}_J$$
$$(1.6 \times 5\sin 0°) + (0.28 \times 0) = (1.6 \times 4\sin 20°) + (0.28 \times \boldsymbol{v}_J \sin\theta_J)$$
$$0 = 2.19 + 0.28\boldsymbol{v}_J \sin\theta_J$$
$$-2.19 = 0.28\boldsymbol{v}_J \sin\theta_J$$
$$\boldsymbol{v}_J \sin\theta_J = -7.82 \text{ N s}$$

As the magnitude of the velocity is generally positive, the positive x-component and the negative y-component indicate that the jack is directed into the fourth quadrant. So we now have $\boldsymbol{v}_J \cos\theta_J = 7.14$ N s and $\boldsymbol{v}_J \sin\theta_J = -7.82$ N s. Although this may not seem to get us very far, don't forget that we can use the relationship for finding the tan of an angle.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

therefore,

$$\tan\theta_J = \frac{\sin\theta_J}{\cos\theta_J}$$

This in turn means that:

$$\tan\theta_J = \frac{\boldsymbol{v}_J sin\,\theta_J}{\boldsymbol{v}_J cos\,\theta_J}$$

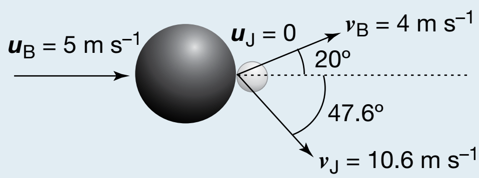
We can then substitute our numerical values to get:

$$\tan\theta_J = \frac{7.82}{7.14}$$
$$\theta_J = tan^{-1}(1.09)$$
$$= 47.6°$$

Substituting this value back in to $\boldsymbol{v}_J\, cos\,\theta_J = 7.14$:

$$\boldsymbol{v}_J\, cos\, 47.6° = 7.14$$
$$\boldsymbol{v}_J = \frac{7.14}{0.67}$$
$$= 10.6 \text{ m s}^{-1}$$

FIGURE 6.7



We find that, after the collision, the jack moves at 10.6 m s^{-1} at an angle of 312.4° to the original direction of motion of the bowl.

6.3 Exercise 1

Question

1. A 5 kg ball moving due east at 4.0 m s^{-1} collides with a 4.0 kg ball moving due west at 3.0 m s^{-1}. Just after the collision, the 5.0 kg ball has a velocity of 1.2 m s^{-1} due south.
   (a) What is the magnitude of the 4.0 kg ball's velocity just after the collision?
   (b) In what direction does it move?

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6.4 Momentum and road safety

6.4.1 Reducing the net force

In the event of a car collision, the net force applied to your body as its motion suddenly changes can be controlled in two ways:

1. By reducing your initial momentum and therefore, your change in momentum, by driving at a moderate speed. Of course, by driving at a moderate speed, you are less likely to have a collision in the first place. Low-speed zones, speed humps and strict enforcement of speed limits contribute to making accidents less likely and to reducing injuries when accidents do occur.
2. By increasing the time interval during which the change in momentum of the car, and the change in momentum of its occupants, takes place.

Cars that crumple

Cars are designed to crumple at the front and rear. This provision increases the time interval during which the momentum of the car changes in a collision, further protecting its occupants from death or serious injury. Even though the front and rear of the car crumple, the passenger compartment is surrounded by a rigid frame. The engine is also surrounded by rigid structures that prevent it from being pushed into the passenger compartment. The tendency of the roof to crush is currently being reduced by increasing the thickness of the windscreen and side windows, using stronger adhesives and strengthening the roof panel.

The inside of the passenger compartment is also designed to protect occupants. Padded dashboards, collapsible steering wheels and airbags are designed to reduce the rate of change of momentum of occupants in a collision. Interior fittings like switches, door knobs and the handbrake are sunk so that the occupants do not collide with them.

FIGURE 6.8 Crumple zones at the front and rear of cars reduce the rate of change of momentum of the car and its occupants during a collision.

FIGURE 6.9 Helmets save lives and prevent serious injury in many sports. They increase the time interval over which a change in momentum takes place.

Don’t be an egghead

In a serious bicycle accident, the head is likely to collide at high speed with the road or another vehicle. Even a simple fall from a bike can result in a collision of the head with the road at a speed of about 20 km h^{-1}. Without the protection of a helmet, concussion is likely as the skull decelerates very quickly due to the large net force on it. It will come to rest while the brain is still in motion. The brain will collide with the skull. If the net force on the skull and its subsequent deceleration is large enough, the brain can be severely bruised or torn, often resulting in permanent brain damage or death. The effect is not unlike that of dropping a soft-boiled egg onto a hard floor.

Bicycle helmets typically consist of an expanded polystyrene liner about two centimetres thick, covered in a thin, hard, polymer shell. They are designed to crush on impact. Although a helmet does not guarantee survival in a serious bicycle accident, it does reduce the net force applied to the skull, and therefore increases the chances of survival dramatically. The polystyrene liner of the helmet increases the time interval during which the skull changes its momentum.

Helmets used by motorcyclists, in horseriding, motor racing, cricket and in many other sports serve the same purpose — that is, to increase the time interval over which a change in momentum takes place.

Seatbelts and safety

In a high-speed head-on car collision, each car comes to a stop rapidly. An occupant not wearing a seatbelt continues at the original speed of the car (as described by Newton's First Law of Motion) until acted on by a non-zero net force. An unrestrained occupant therefore moves at high speed until:

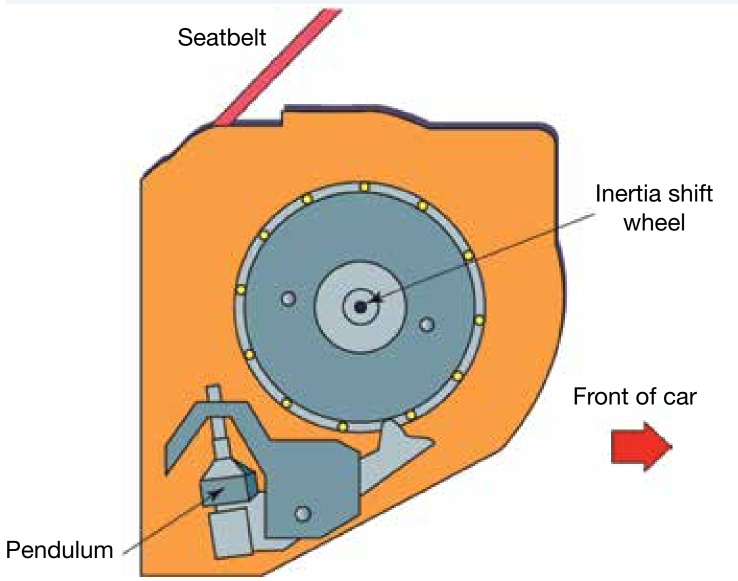
- colliding with part of the interior of the car, stopping even more rapidly than the car itself, usually over a distance of only several centimetres. (Most deaths and injuries in car crashes are caused by collisions between the occupants and the interior of the car.)
- crashing through the stationary or almost stationary windscreen into the other car or onto the road
- crashing into another occupant closer to the front of the car.

An occupant properly restrained with a seatbelt stops with the car. In a typical suburban crash, the deceleration takes place over a distance of about 50 cm. The rate of change of the momentum of a restrained occupant is much less than that of an unrestrained occupant. Therefore, the net force on a restrained occupant is less. As well as increasing the time interval over which its wearer comes to a stop, a properly fitted seatbelt spreads the force over a larger area of the body.

Inertia-reel seatbelts

Inertia-reel seatbelts allow car occupants some freedom of movement while they are worn. However, in the event of a sudden change in velocity of the car, they lock and restrain the occupant (see figure 6.10). Inertia-reel seatbelts are designed with Newton's First Law of Motion in mind. When the car stops suddenly, a pendulum continues to move forward. Part of the pendulum prevents the reel holding the belt from turning. This locks the belt into place. The name 'inertia reel' is given to these seatbelts because the inertia of the pendulum causes the belt to be locked. Another type of seatbelt uses an electronic sensor. When the sensor detects an unusually large deceleration it releases a gas propellant that causes the reel to be locked.

FIGURE 6.10 Operation of an inertia-reel seatbelt. This reel is shown in the locked position.



PHYSICS IN FOCUS

Airbag technology

Airbags are designed to increase the time interval during which the occupant's momentum decreases in a collision, reducing the net force on the occupant. Airbags inflate when the crash sensors in the car detect a large deceleration. When the sensors are activated, an electric current is used to ignite the chemical compound sodium azide (NaN_3), which is stored in a metal container at the opening of the airbag. The sodium azide burns rapidly, producing other sodium compounds and nitrogen gas. The reaction is explosive, causing a noise like the sound of gunfire. The nitrogen gas inflates the airbag to a volume of about 45 litres in only 30 milliseconds.

When the occupant's body makes contact with the airbag, nitrogen gas escapes through vents in the bag. The dust produced when an airbag is activated is a mixture of the talcum powder used to lubricate the bags and the sodium compound produced by the chemical reaction. Deflation must be rapid enough to allow a driver to see after the accident.

Before any physical testing of a new car takes place, the vehicle structure is modelled on a computer to ensure that it has adequate durability, comfort (in terms of noise and vibration for example) and accident performance. The computer modelling is then verified with the first physical testing of real vehicles. Following this, the design will progress through a number of refinements before the new model is ready for sale to the public.

FIGURE 6.11 Airbags increase the time interval during which the occupant's momentum decreases.

One interesting aspect in the development of an airbag system is the calibration of the sensor that triggers the airbags. Current 'state of the art' technology for driver and passenger airbags uses a single sensing module mounted within the passenger compartment of the vehicle. This module continually monitors the longitudinal acceleration of the car. Complex calculations and comparisons are performed by a microprocessor within the sensing module before it 'decides' whether or not to trigger the airbags.

FIGURE 6.12 Crash test with a BMW displayed at the 2010 Paris Motor Show.

Many cars are crashed on the computer and in real life during the development of the vehicle structure and airbag system. The crash events used to develop an airbag calibration include high- and low-speed collisions, full and angled frontal impacts and pole- or tree-type collisions.

6.4.2 Modelling real collisions

The Law of Conservation of Momentum makes it possible to predict the consequences of collisions between two cars or, in the case of traffic investigation forensics, estimate the speeds at which vehicles were travelling before a collision.

6.4 SAMPLE PROBLEM 1

A 2000 kg delivery van collides with a small stationary car of mass 1000 kg. The two vehicles lock together and the tangled wreck continues to move in the direction in which the van was travelling. By examining the marks left by the tangled vehicles after collision until the wreck came to rest, investigators were able to determine that they were travelling together at a speed of 20 m s^{-1} immediately after impact.

At what speed was the van travelling just before it hit the small car?

SOLUTION

$$\begin{aligned}\Sigma \boldsymbol{p}_{\text{after}} &= (m_{\text{van}} + m_{\text{car}})\, \boldsymbol{v}_{\text{van+car}} \\ &= (2000\text{ kg} + 1000\text{ kg}) \times 20\text{ m s}^{-1} \\ &= 3000\text{ kg} \times 20\text{ m s}^{-1} \\ &= 60\,000\text{ kg m s}^{-1}\end{aligned}$$

As $\Sigma \boldsymbol{p}_{\text{after}} = \Sigma p_{\text{before}} = m_{\text{van}} \boldsymbol{u}_{\text{van}} + m_{\text{car}} \boldsymbol{u}_{\text{car}}$

$$60\,000\text{ kg m s}^{-1} = 2000\text{ kg} \times \boldsymbol{u}_{\text{van}} + 1000\text{ kg} \times 0$$

$$60\,000\text{ kg m s}^{-1} = 2000\text{ kg} \times \boldsymbol{u}_{\text{van}}$$

$$\boldsymbol{u}_{\text{van}} = \frac{60\,000\text{ kg m s}^{-1}}{2000\text{ kg}}$$

$$\boldsymbol{u}_{\text{van}} = 30\text{ m s}^{-1}$$

6.4 Exercise 1

1. A 1500 kg car travelling at 12 m s^{-1} on an icy road collides with a 1200 kg car travelling at the same speed, but in the opposite direction. The cars lock together and travel at 1.3 m s^{-1} in the direction of the first car after impact. What was the speed of the second car before the collision?
2. A 3.0 kg target is balanced on a post at the end of an archery range. Michelle fires a 45 g arrow that travels at 20 m s^{-1} as it enters the target. If the arrow moves through the target and emerges on the other side with a speed of 12 m s^{-1}, what will be the speed of the target as it is knocked from the post?

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6.5 Elastic and inelastic collisions

6.5.1 Transfer of kinetic energy

In collisions, kinetic energy may be transferred between the colliding objects. When a moving billiard ball strikes one that is stationary, we would not be surprised to see that the first ball moves a bit more slowly after the collision and the stationary ball is now in motion. In this case, the first ball has transferred some of its kinetic energy to the second. As the first ball has lost kinetic energy, it moves at a lower speed. The second ball, in gaining kinetic energy, is now in motion.

In a perfect collision between two objects in which no energy is lost in the form of sound, heat or elastic potential energy, the kinetic energy lost by one object is equal to the kinetic energy gained by the other. In other words, for two colliding objects A and B:

$$\Delta E_{\text{kA}} = -\Delta E_{\text{kB}}$$

$$\frac{1}{2} m_{\text{A}} \boldsymbol{v}_{\text{A}}^2 - \frac{1}{2} m_{\text{A}} \boldsymbol{u}_{\text{A}}^2 = -\left(\frac{1}{2} m_{\text{B}} \boldsymbol{v}_{\text{B}}^2 - \frac{1}{2} m_{\text{B}} \boldsymbol{u}_{\text{B}}^2\right)$$

We can rearrange this to get:

$$\frac{1}{2}m_A\boldsymbol{u}_A^2 + \frac{1}{2}m_B\boldsymbol{u}_B^2 = \frac{1}{2}m_A\boldsymbol{v}_A^2 + \frac{1}{2}m_B\boldsymbol{v}_B^2$$

or,

$$\Sigma E_{\text{k before}} = \Sigma E_{\text{k after}}$$

In such perfect collisions, kinetic energy is conserved and the sum of kinetic energies of the objects within the system is a constant. A collision in which both momentum and kinetic energy are conserved is called an **elastic collision**.

The vast majority of collisions in the real world, however, are not elastic. Although momentum will be conserved regardless, the transfer of kinetic energy between colliding objects is usually incomplete. This is because some of the original kinetic energy is converted into other forms of energy, such as sound and heat, leaving only a fraction of the kinetic energy to be transferred. Collisions in which kinetic energy is not conserved are said to be **inelastic**.

FIGURE 6.13 Billiard balls collide in a nearly elastic collision.

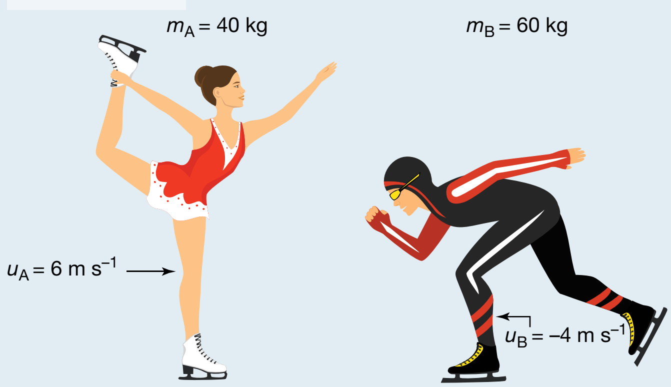
FIGURE 6.14 Humans do not collide elastically.

SAMPLE PROBLEM 6.5

A 60 kg skater travelling at 4 m s^{-1} collides with a 40 kg skater moving in the opposite direction at 6 m s^{-1}, and the two skaters are both bounced in directions opposite to the ones they had before collision. After the collision, the larger skater is moving at a speed of 1 m s^{-1} but the smaller skater has a speed of 1.5 m s^{-1}. Is this an elastic collision or an inelastic collision?

SOLUTION

FIGURE 6.15



If the collision is an elastic one, then we would expect kinetic energy to be conserved.

$$
\begin{aligned}
E_{\text{k before}} &= \frac{1}{2} m_A \boldsymbol{u}_A^2 + \frac{1}{2} m_B \boldsymbol{u}_B^2 \\
&= \frac{1}{2} 60 \text{ kg } (4 \text{ m s}^{-1})^2 + \frac{1}{2} 40 \text{ kg } (-6 \text{ m s}^{-1})^2 \\
&= 480 \text{ kg m}^2 \text{ s}^{-2} + 720 \text{ kg m}^2 \text{ s}^{-2} \\
&= 1\,200 \text{ J} \\
E_{\text{k after}} &= \frac{1}{2} m_A \boldsymbol{v}_A^2 + \frac{1}{2} m_B \boldsymbol{v}_B^2 \\
&= \frac{1}{2} 60 \text{ kg } (-1 \text{ m s}^{-1})^2 + \frac{1}{2} 40 \text{ kg } (1.5 \text{ m s}^{-1})^2 \\
&= 30 \text{ kg m}^2 \text{ s}^{-2} + 45 \text{ kg m}^2 \text{ s}^{-2} \\
&= 75 \text{ J}
\end{aligned}
$$

As you can see, $E_{\text{k before}} > E_{\text{k after}}$, therefore this is an inelastic collision.

WORKING SCIENTIFICALLY 6.3

Is there a relationship between the speed at which two balls collide and the proportion of kinetic energy lost in the collision? Design a method allowing you to investigate this. You will need to measure the speed of the balls before and after the collision, you will need to direct the balls in such a way so the collision only happens in one dimension, and you will need to change the initial speed of the balls.

6.5 Exercise 1

A 2 kg dynamics cart travelling at 2 m s^{-1} collides with a 3 kg dynamics cart travelling in the opposite direction at 4 m s^{-1}. If the 2 kg cart rebounds from the collision at 3 m s^{-1}, determine:

(a) the velocity of the 3 kg cart after collision
(b) whether this was an elastic collision.

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6.6 Review

6.6.1 Summary

- The momentum of an object is the product of its mass and its velocity.
- The impulse delivered to an object by a force is the product of the force and the time interval during which the force acts on the object.
- The impulse delivered by the net force on an object is equal to the change in momentum of the object: $\boldsymbol{F}\Delta t = m\Delta \boldsymbol{v}$.
- The impulse delivered by a force can be found by determining the area under a graph of the force versus time.
- The net force on a human body during a collision can be decreased by increasing the time interval during which its momentum changes. Vehicle safety features such as crumple zones, together with seatbelts and airbags, are designed to increase this time interval. Low-speed zones and speed humps encourage people

to drive at lower speeds and, therefore, with less momentum — reducing the likelihood of injury when a collision does occur.

- If the net force acting on a system is zero, the total momentum of the system does not change. This statement is an expression of the Law of Conservation of Momentum.
- When two objects collide, the force applied by the first object on the second is equal and opposite to the force applied by the second object on the first.
- Momentum is conserved in one-, two- and three-dimensional collisions.
- The kinetic energy, E_k, of an object is proportional to its mass and the square of its velocity. Kinetic energy is measured in joules (J).
- An inelastic collision is one in which only momentum is conserved.
- An elastic collision is one in which both kinetic energy and momentum are conserved.

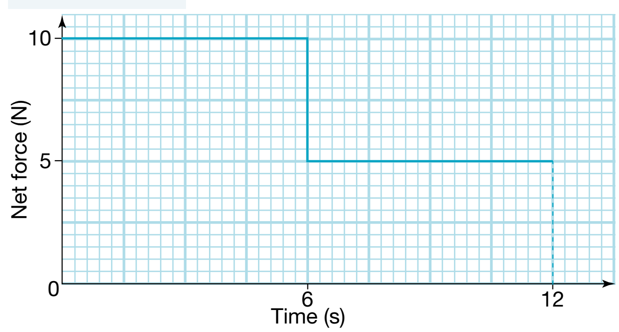
6.6.2 Questions

1. Joggers are advised to run on a soft surface such as grass rather than on hard surfaces such as bitumen or concrete to reduce knee injury. Why is this so?
2. What is the difference between an energy transfer and an energy transformation?
3. Why is a heavy bowling ball able to knock over more pins on average than a lighter ball?
4. Why does the use of boxing gloves make modern boxing safer than bare-knuckle fighting?
5. Most dance halls have what are referred to as sprung wooden floors, which are very bouncy. Why would such floors be needed in a dance hall?
6. Explain in terms of the Law of Conservation of Momentum how astronauts walking in space can change their speed or direction.

FIGURE 6.16

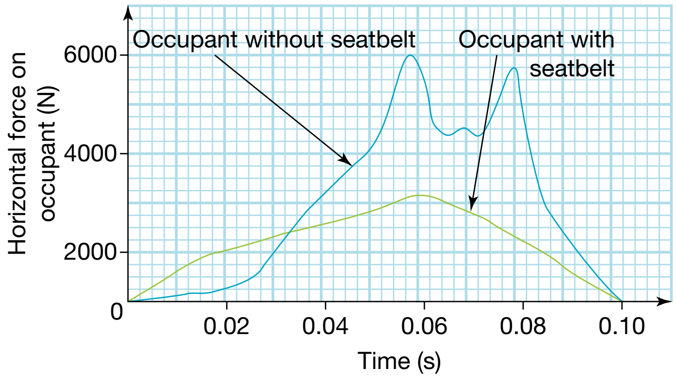
7. Make an estimate, to one significant figure, of the magnitude of each of the following:
   (a) the momentum of an Olympic athlete in the 100 m sprint
   (b) the momentum of a family car travelling at the speed limit along a suburban street
   (c) the impulse that causes a 70 kg football player running at top speed to stop abruptly as he collides with an unseen goalpost
   (d) the impulse applied to a netball by a goal shooter as she pushes it up towards the goal at a speed of 5 m s^{-1}
   (e) the change in momentum of a tennis ball as it is returned to the server in a Wimbledon final.
8. A railway cart of mass 500 kg travelling at 3.0 m s^{-1} due west comes to rest in 2.0 s when the engine pulling it stops.
   (a) Calculate the impulse that has been applied to the cart.
   (b) Calculate the change in momentum of the cart.
   (c) Calculate the magnitude of the average force acting on the cart as it comes to a stop.
9. The graph in the figure 6.17 shows how the net force on an object of mass 2.5 kg changes with time.
   (a) Calculate the impulse applied to the object during the first 6.0 s.
   (b) If the object was initially at rest, what is its momentum after 12 s?
   (c) Draw a graph of acceleration versus time for the object.

FIGURE 6.17



10. A car with a total mass of 1400 kg (including occupants), travelling at 60 km h^{-1}, hits a large tree and stops in 0.080 s.
   (a) Calculate the impulse that is applied to the car by the tree.
   (b) Calculate the force exerted by the tree on the car.
   (c) Calculate the magnitude of the deceleration of the 70 kg driver of the car if he is wearing a properly fitted seatbelt.
11. Figure 6.18 shows how the horizontal force on the upper body of each of two occupants of a car changes as a result of a head-on collision. One occupant is wearing a seatbelt while the other is not. Both occupants are stationary 0.10 s after the initial impact.

FIGURE 6.18



   (a) What is the horizontal impulse on the occupant wearing the seatbelt?
   (b) If the mass of the occupant wearing the seatbelt is 60 kg, determine the speed of the car just before the initial impact.
   (c) Is the occupant not wearing the seatbelt heavier or lighter than the other (more sensible) occupant? Write a paragraph explaining the difference in shape between the two curves on the graph.
12. A 75 kg basketballer lands vertically on the court with a speed of 3.2 m s^{-1}.
   (a) What total impulse is applied to the basketballer's feet by the ground?
   (b) If the basketballer's speed changes from 3.2 m s^{-1} to zero in 0.10 s, what total force does the ground apply to his feet?
13. Use the ideas presented in this chapter to explain why:
   (a) dashboards of cars are padded
   (b) cars are deliberately designed to crumple at the front and rear
   (c) the compulsory wearing of bicycle helmets has dramatically reduced the number of serious head injuries in bicycle accidents.

   A single answer (rather than three separate answers) is acceptable.
14. It is often said that seatbelts prevent passengers from being thrown forwards in a car collision. What is wrong with such a statement?
15. Airbags are fitted to the centre of the steering wheel of many new cars. In the event of a sudden deceleration, the airbag inflates rapidly, providing extra protection for a driver restrained by a seatbelt. Explain how airbags reduce the likelihood of serious injury or death.
16. A toy car with a mass of 2.0 kg collides with a wall at a speed of 1.0 m s^{-1} and rebounds in the opposite direction with a speed of 0.50 m s^{-1}.
   (a) What is the change in momentum of the toy car?
   (b) What is the impulse applied by the toy car to the wall? Explain how you obtained your answer without any information about the change in momentum of the wall.
   (c) Does the wall actually move as a result of the impulse applied by the toy car? Explain your answer.
17. A physics student is experimenting with a low-friction cart on a smooth horizontal surface. Predict the final velocity of the 2.0 kg cart in each of these two experiments.
   (a) The cart is travelling at a constant speed of 0.60 m s^{-1}. A suspended 2.0 kg mass is dropped onto it as it passes.
   (b) The cart is loaded with 2.0 kg of sand. As the cart moves with an initial speed of 0.60 m s^{-1} the sand is allowed to pour out through a hole behind the rear wheels.

18. Two stationary ice skaters, Denise and Lauren, are facing each other and use the palms of their hands to push each other in opposite directions. Denise, with a mass of 50 kg, moves off in a straight line with a speed of 1.2 m s^{-1}. Lauren moves off in the opposite direction with a speed of 1.5 m s^{-1}.
    (a) Calculate Lauren's mass.
    (b) Calculate the magnitude of the impulse that results in Denise's gain in speed.
    (c) Calculate the magnitude of the impulse on Lauren while the girls are pushing each other away.
    (d) What is the total momentum of the system of Denise and Lauren just after they push each other away?
    (e) Would it make any difference to their final velocities if they pushed each other harder? Explain.

FIGURE 6.19

19. Gavin and Andrew are keen rollerbladers. Gavin approaches his stationary brother at a speed of 2.0 m s^{-1} and bumps into him. As a result of the collision, Gavin, who has a mass of 60 kg, stops moving, and Andrew, who has a mass of 70 kg, moves off in a straight line. The surface on which they are 'blading' is smooth enough that friction can be ignored.
    (a) With what speed does Andrew move off?
    (b) Calculate the magnitude of the impulse on Gavin as a result of the bump.
    (c) Calculate the magnitude of Gavin's change in momentum.
    (d) Calculate the magnitude of Andrew's change in momentum.
    (e) How would the motion of each of the brothers after their interaction be different if they pushed each other instead of just bumping?
    (f) If Gavin held onto Andrew so that they moved off together, what would be their final velocity?

FIGURE 6.20

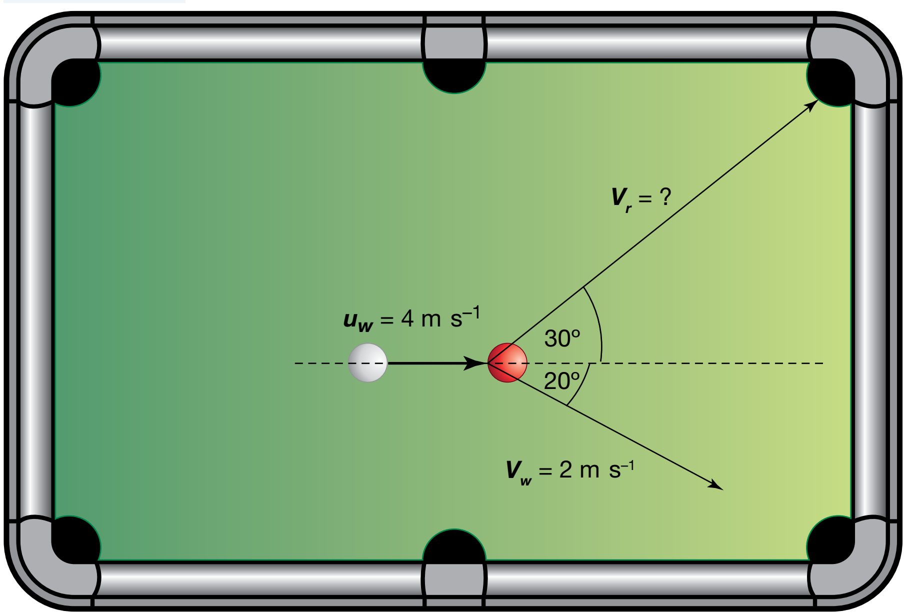
20. An unfortunate driver of mass 50 kg, travelling on an icy road in her 1200 kg car, collides with a stationary police car with a total mass (including occupants) of 1500 kg. The tangled wreck moves off after the collision with a speed of 7.0 m s^{-1}. The frictional force on both cars can be assumed to be negligible.
    (a) At what speed was the unfortunate driver travelling before her car hit the police car?
    (b) What was the impulse on the police car due to the collision?
    (c) What was the impulse on the unfortunate driver of the offending car (who was wearing a properly fitted seatbelt) due to the impact with the police car?
    (d) If the duration of the collision was 0.10 s, what average net force was applied to the police car?
21. A car of mass 1500 kg travelling due west at a speed of 20 m s^{-1} on an icy road collides with a truck of mass 2000 kg travelling at the same speed in the opposite direction. The vehicles lock together after impact.
    (a) What is the velocity of the vehicles immediately after the collision?
    (b) Which vehicle experiences the greater change in speed?
    (c) Which vehicle experiences the greater change in momentum?
    (d) Which vehicle experiences the greater force?

22. A train of mass 4.0×10^6 kg rolls freely along a horizontal track at a speed of 3.0 m s^{-1} towards a loaded coal cart. The mass of the coal cart is 5×10^5 kg and it is rolling freely at a speed of 2.0 m s^{-1} in the same direction as the train. When the train reaches the coal cart, they remain in contact and continue to roll freely. What is their common speed after contact is made?
23. In a real collision between two cars on a bitumen road on a dry day, is it reasonable to assume that the total momentum of the two cars is conserved? Explain your answer.
24. A well-meaning politician makes the suggestion that if cars were completely surrounded by rubber 'bumpers' like those on dodgem cars, they would simply bounce off each other in a collision and the passengers would be safer. Discuss the merit of this suggestion in terms of impulse, change in momentum and force.
25. In a paragraph, discuss the accuracy of the following statement. Make estimates of the physical characteristics of the car and the wall so that you can support your arguments with calculations.

    When a car collides with a solid concrete wall firmly embedded into the ground, the total momentum of the system is conserved. Therefore, the concrete wall moves, but not quickly enough to allow any measurement of the movement to be made.
26. Design a spreadsheet to model head-on collisions between two cars on an icy road. Assume that the cars are locked together after impact. Use your spreadsheet to predict the speed of the cars after the collision for a range of masses and initial speeds.
27. A 1.20 kg sports pistol discharges while lying on a highly polished tabletop, firing a 3 g bullet at 420 m s^{-1} in one direction while it recoils in the opposite direction. How far along the tabletop will the gun move before coming back to rest if the coefficient of friction between the gun and the tabletop is 0.12?
28. Two identical hockey pucks slide along the ice towards each other. At the moment that they collide elastically, one has a speed of 10 m s^{-1} and the other has a speed of 5 m s^{-1}. If they bounce off each other, what will be their speeds after collision?
29. Figure 6.21 shows a billiards shot that causes a red ball to be sunk in the corner pocket.

    If the white ball and the red ball have the same mass and the red ball was initially stationary, what speed did the red ball have after collision? (*Note:* in reality, the white ball is slightly smaller than the other balls on the table.)

FIGURE 6.21



PRACTICAL INVESTIGATIONS

Investigation 6.1: Impulse and change in momentum

Aim

To compare the change in momentum of an object with the impulse delivered by an external force

Apparatus

low-friction trolley or linear air-track glider
timing and recording device (e.g. ticker-timer, spark generator, photogates, motion detector and computer interface)
pulley
light string
load (500 g or 1.0 kg mass)
metre rule

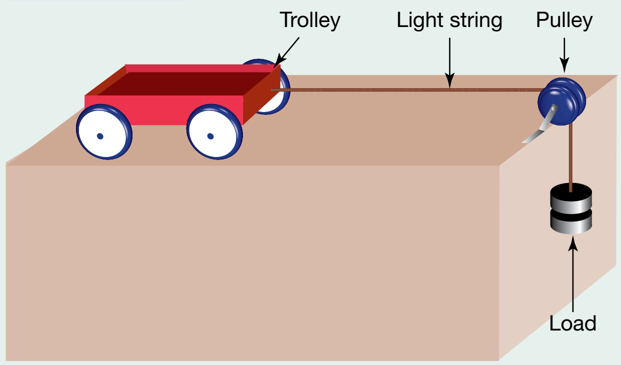
Theory

The impulse delivered to an object by a force is the product of the force and the time interval during which the force acts on the object. The impulse delivered by the net force on an object is equal to the change in momentum of the object.

Method

1. Connect a load of known mass to a dynamics trolley or linear air-track glider with a light string over a pulley, as shown in figure 6.22.
2. Use your timing and recording device to collect data that will allow you to determine the instantaneous velocity of the trolley or glider at two separate instants as the load is falling.
3. Measure and record the mass of the trolley or glider.

FIGURE 6.22



Analysis and questions

Use your record of the motion to determine the instantaneous velocity at two separate instants and hence calculate the change in velocity.

1. What is the mass of the system?
2. Calculate the change in momentum of the system.
3. What is the magnitude of the net force applied to the system?
4. Use the net force and the appropriate time interval to calculate the impulse delivered to the system by the net force.
5. Compare the impulse and change in momentum of the system, and discuss any difference between your expected results and your calculations.
6. Express the discrepancy between the change in momentum and the impulse as a percentage of the impulse.
7. Which of the measured quantities was the least accurate? Why?

Investigation 6.2: Simulating a collision

Aim

To show that momentum is conserved in a collision in which there are no unbalanced external forces acting on the system

Apparatus

low-friction trolleys or linear air-track gliders
timing and recording device (e.g. ticker-timer, spark generator, photogates, motion detector and computer interface)
brick or other weight to add to one trolley or glider
balance suitable for measuring the mass of the trolleys or gliders and the added weight
velcro, double-sided tape or plasticine
metre rule

Theory

If the net force acting on a system is zero, the total momentum of the system does not change. This statement is an expression of the Law of Conservation of Momentum. Therefore, if no external forces act on two vehicles during a collision between them, the total momentum of the system of the two vehicles remains constant. It follows that the change in momentum of the first car is equal and opposite to the change in momentum of the second car.

Method

1. Use two low-friction trolleys or linear air-track gliders to simulate a collision between a furniture truck and a medium-sized passenger car. The truck, travelling at a moderate speed, collides with the rear end of the car on an icy road. After the collision, the two vehicles lock together.
2. Attach an adhesive substance (e.g. velcro, double-sided sticky tape or plasticine) to one or both trolleys or gliders to ensure that they lock together after the collision. Record the mass of each 'vehicle' (after adhesive is attached) before setting up the collision. Place a small, light, unrestrained object on to each of the vehicles to represent loose objects.
3. Use your timing and recording device to collect data that will allow you to determine the velocity of each 'vehicle' just before and just after the collision.

Results

Record your data in a table similar to the one below.

	Furniture truck	Medium-sized car
Mass (g)		
Velocity before collision (cm s^{-1})		
Velocity after collision (cm s^{-1})		
Momentum before collision (g cm s^{-1})		
Momentum after collision (g cm s^{-1})		

Analysis and questions

1. Describe the motion of each of the loose objects after the collision. What are the implications of your observations for the drivers and passengers in each vehicle?
2. What was the total momentum of the system before the collision?
3. If there were no unbalanced external forces acting on this system, what would you expect the total momentum to be after the collision?
4. What was the total momentum of the system after the collision?
5. How do you account for the fact that momentum was not fully conserved in this collision? Mass was recorded in the tables in grams. Why is there no need to convert it to kilograms?
6. What was the impulse applied to the car during the collision?
7. What was the impulse applied to the furniture truck during the collision?
8. According to Newton's Third Law of Motion, the impulse applied to the car by the furniture truck should be equal to the impulse applied to the furniture truck by the car. Explain your answers to questions 6 and 7 in the light of this.
9. An elastic collision is one in which the total kinetic energy of the system before the collision is the same as the total kinetic energy after the collision. Is this simulated collision elastic? Show your reasoning.

TOPIC 7
Wave properties

7.1 Overview

7.1.1 Module 3: Waves and Thermodynamics

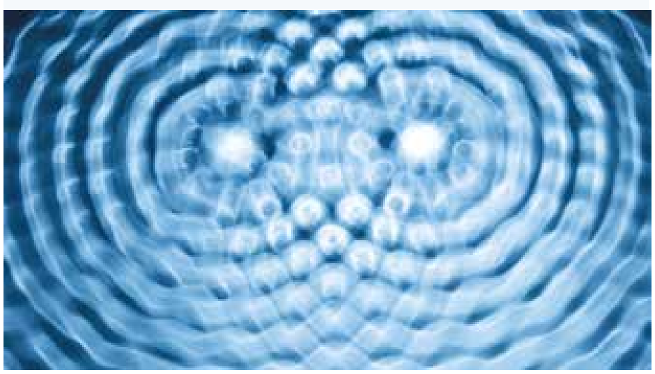
Wave properties

Inquiry question: What are the properties of all waves and wave motion?

Students:

- conduct a practical investigation involving the creation of mechanical waves in a variety of situations in order to explain:
  - the role of the medium in the propagation of mechanical waves
  - the transfer of energy involved in the propagation of mechanical waves (ACSPH067, ACSPH070).
- conduct practical investigations to explain and analyse the differences between:
  - transverse and longitudinal waves (ACSPH068)
  - mechanical and electromagnetic waves (ACSPH070, ACSPH074).
- construct and/or interpret graphs of displacement as a function of time and as a function of position of transverse and longitudinal waves, and relate the features of those graphs to the following wave characteristics:
  - velocity
  - frequency
  - period
  - wavelength
  - wave number
  - displacement and amplitude (ACSPH069).
- solve problems and/or make predictions by modelling and applying the following relationships to a variety of situations
  - $v = f\lambda$
  - $f = \dfrac{1}{T}$
  - $k = \dfrac{2\pi}{\lambda}$

FIGURE 7.1 Waves in a ripple tank.



7.2 Types of waves

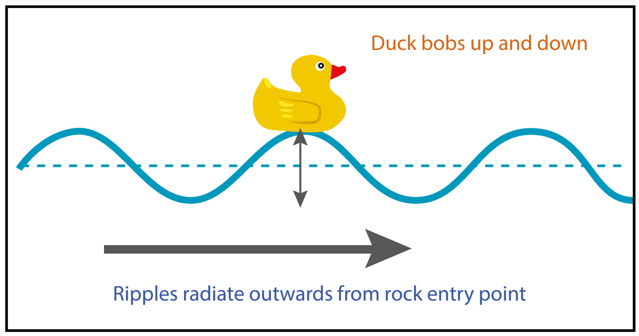
7.2.1 What is a wave?

A **wave** is a disturbance that travels through a medium from the source to the detector without any movement of matter. Waves therefore transfer energy without any net movement of particles. **Periodic waves** are disturbances that repeat themselves at regular intervals. Periodic waves propagate by the disturbance in part of a medium passing on to its neighbours. In this way, the disturbance travels, but the medium stays where it is.

When a small rock is thrown into the centre of a still pond, the kinetic energy of the rock is transferred to the water as it enters. This energy is carried through the water in the form of evenly spaced ripples, which are small waves. If there are objects on the surface of the water such as twigs or lily pads, they move up and down rather than being carried along with the waves. It is the *energy* that is travelling from the centre of the pond out to the edges in the form of the waves through the water, not the water itself.

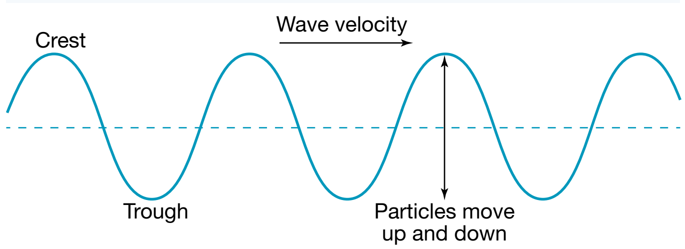
FIGURE 7.2 (a) Waves carry kinetic energy outwards from the centre of disturbance.

FIGURE 7.2 (b) A duck on the surface of the water bobs up and down in place as the ripples pass underneath it.



7.2.2 Transverse and longitudinal waves

FIGURE 7.3 A transverse wave.

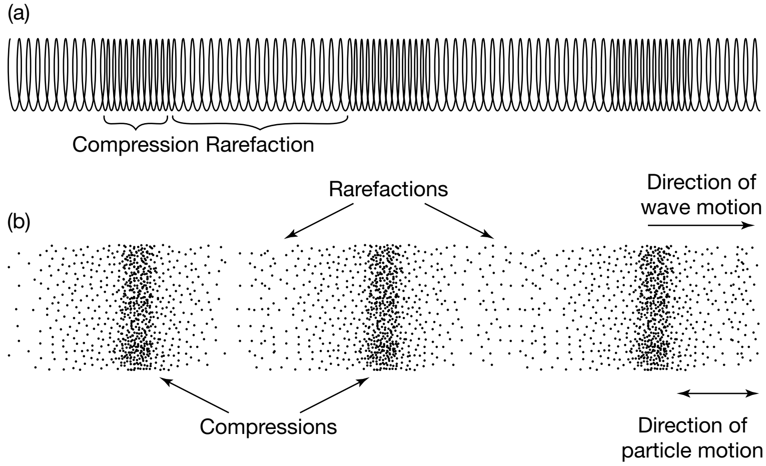


One way of categorising waves is to describe them in terms of their orientation relative to their direction of motion. A **transverse wave** is one in which the disturbance caused by the transfer of energy acts perpendicularly to the direction of the wave itself. The ripples on the surface of a pool when a rock has been dropped into it, such as in figure 7.2a, form transverse waves, with the surface of the water moving vertically up and down as the wave moves outwards horizontally parallel with the rest position of the water.

The highest points of transverse waves are called **crests** or peaks while the lowest points are called **troughs**.

A **longitudinal wave** (also called a **compression wave**) is one in which the disturbance moves in the same direction as the wave. As the disturbance moves through the particles of the medium, it alternately pushes them closer together and then pulls them further apart. The positions where the particles are crowded together the closest are called **compressions**, and those where the particles are spread furthest apart are referred to as **rarefactions**. Sound waves move as longitudinal waves, as do some of the waves produced during earthquakes. The 'push' waves created in a stretched 'slinky' spring are also longitudinal, with the compressions noticeable where the coils pack closely together, and the rarefactions noticeable where the coils spread furthest apart.

FIGURE 7.4 Longitudinal waves in (a) a slinky and (b) air.



7.2.3 Features of waves

All waves can be described in terms of their wavelength, frequency, amplitude and speed.

The **period** of a periodic wave is the time taken for a source to produce one complete wave, and it is the same as the time taken for a complete wave to pass a given point. The period is measured in seconds and is represented by the symbol T.

The **frequency, f**, of a wave is a measure of how many waves pass a location in a time interval of one second. The unit of frequency is the hertz (Hz), with $1\,\text{Hz} = 1\,\text{s}^{-1}$.

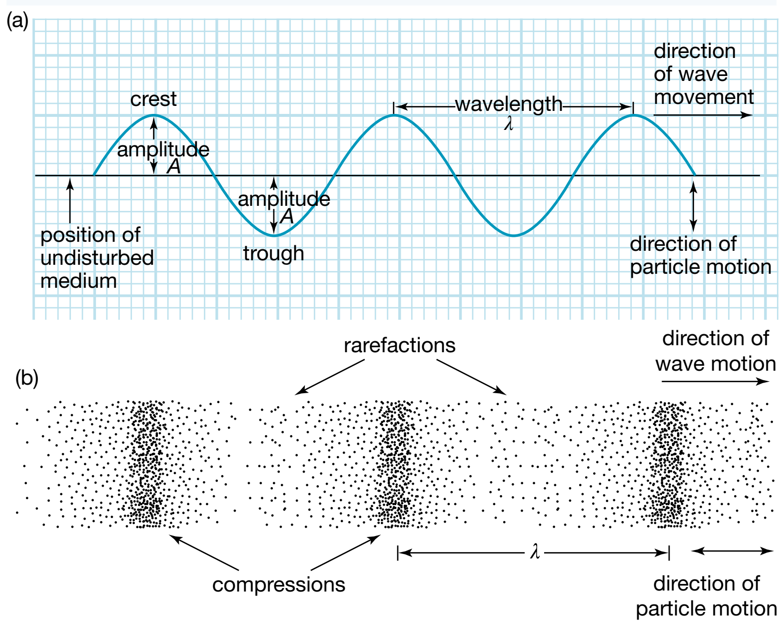
The frequency of a wave is equal to the reciprocal of its period:

$$T = \frac{1}{f}$$

For example, if a sound wave has a frequency of 256 Hz (corresponding to the pitch of middle C on a piano), then the sound source is producing 256 waves every second. The period of one of these waves (the time taken for one complete wave to be produced by the source) is equal to $\frac{1}{256\text{ s}} = 0.0039$ s or 3.9 milliseconds.

The **wavelength** of a wave series is the distance between successive waves and it is equal to the distance travelled by each wave during a time interval equal to one period. The symbol for wavelength is the Greek letter λ (called lambda and pronounced 'lam-duh'). For transverse waves, wavelength may be measured from the crest of one wave to the crest of the following wave, or between successive troughs, or between any two equivalent points on successive waves. In longitudinal waves, the wavelength is usually measured between successive compressions or successive rarefactions. As it is a measurement of length, the SI unit for wavelength is the metre.

FIGURE 7.5 Characteristics of (a) transverse waves and (b) longitudinal waves.



The **amplitude**, A, of a wave relates to the amount of energy that is associated with the wave. The bigger the amplitude of the wave, the more energy it has. The amplitude is generally defined as the difference between the maximum displacement and the normal, undisturbed position of a particle in the wave medium. The units of amplitude vary from wave type to wave type. For example, in sound waves the amplitude is measured in units of pressure, whereas the amplitude of a water wave would normally be measured in centimetres or metres. The amplitude of light is measured in terms of its intensity or, if we are considering visible light, its brightness.

The **speed** of a periodic wave (represented by the symbol $\boldsymbol{v}$) is related to the wave's wavelength and frequency by a relationship known as the **wave equation**:

$$v = f\lambda.$$

7.2 SAMPLE PROBLEM 1

What is the speed of a sound wave if it has a period of 2.0 ms and a wavelength of 68 cm?

SOLUTION:

Step 1:

Note down the known variables in their appropriate units. Time must be expressed in seconds and length in metres.

$$\begin{aligned} T &= 2.0\,\text{ms} \\ &= 2.0 \times 10^{-3}\,\text{s} \\ \lambda &= 68\,\text{cm} \\ &= 0.68\,\text{m} \end{aligned}$$

Step 2:

Choose the appropriate formula.

$$v = \frac{\lambda}{T} \left(\text{as } f = \frac{1}{T}\right)$$

Step 3:

Transpose the formula. (Not necessary in this case.)

Step 4:

Substitute values and solve.

$$\begin{aligned} v &= \frac{0.68\,\text{m}}{2.0 \times 10^{-3}\,\text{s}} \\ &= 340\ \text{m s}^{-1} \end{aligned}$$

7.2 SAMPLE PROBLEM 2

What is the wavelength of a sound of frequency 550 Hz if the speed of sound in air is 335 m s^{-1}?

SOLUTION:

$$f = 550\ \text{Hz},\ v = 335\ \text{m s}^{-1}$$

$$\begin{aligned} v &= f\lambda \\ \Rightarrow \lambda &= \frac{v}{f} \\ &= \frac{335\ \text{m s}^{-1}}{550\ \text{Hz}} \\ &= 0.609\ \text{m} \end{aligned}$$

The **wave number** (represented by the symbol k) of a periodic wave series is equal to the number of waves per unit distance. Just as frequency is equal to the reciprocal of the period, the wave number is the reciprocal of the wavelength: $k = \dfrac{2\pi}{\lambda}$

This quantity is particularly important in quantum physics as your later studies will show.

WORKING SCIENTIFICALLY 7.1

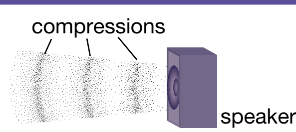
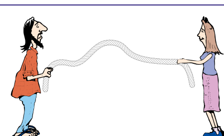
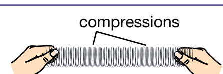
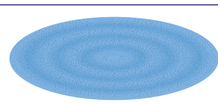
When an object is dropped into water, ripples radiate out from the position at which it entered the water. Design and perform an experiment to determine how the amplitude and velocity of the ripples produced are related to (a) the mass of the object, (b) the height from which the object is dropped, or (c) the depth of the water.

7.2.4 Propagation of waves

Waves are categorised according to how they propagate or transfer energy from place to place. There are two major groups of waves: mechanical waves and electromagnetic waves. **Mechanical waves** involve the transfer of energy through a medium by the motion of the particles of the medium itself. The particles move as oscillations or vibrations around a fixed point and, after the wave has passed, the particles move back to exactly the same places they occupied before they were disturbed. There is no bulk transfer of particles from one place to another.

However, because mechanical waves transfer energy by means of particle vibration, energy is lost, due to friction, over the course of the wave transmission through the medium. As a result, the waves have less energy and thus a smaller amplitude the further they travel from the source.

TABLE 7.1 Some examples of mechanical waves.

Wave	Source		Medium	Detector	Disturbance
Sound	Push/pull of a loudspeaker	 Sound waves	Air	Ear	Increase and decrease in air pressure
Rope	Upward flick of hand	 Pulse on a rope	Rope	Person at other end	Section of rope is lifted and falls back
Stretched spring	Push of hand	 Compressions moving along a stretched spring	Coils in the spring	Person at other end	Bunching of coils
Water	Dropped stone	 Ripples on water	Water	Bobbing cork	Water surface is lifted and drops back

Electromagnetic waves are transverse waves that consist of alternating electric and magnetic force fields positioned at ninety degrees relative to each other and to the direction of energy propagation. Unlike mechanical waves, electromagnetic waves do not need a medium to travel in. In fact, they slow down when travelling in any physical medium apart from a vacuum. As electromagnetic waves do not need the movement of any particles to propagate (as mechanical waves do), they are not subject to the same energy losses due to friction between particles. Therefore, they potentially have much greater travel ranges than mechanical waves.

All electromagnetic waves travel at the same speed in a vacuum. This is referred to as the **speed of light**, c, and it is equal to 299 792 458 m s^{-1}. This is usually approximated to 3×10^8 m s^{-1}.

While electromagnetic waves may travel at the same speed, they vary widely in wavelength and frequency. Figure 7.7 shows the range of wavelengths and frequencies for the various kinds of electromagnetic radiation.

FIGURE 7.6 A representation of an electromagnetic wave force field showing the energy propagation direction.

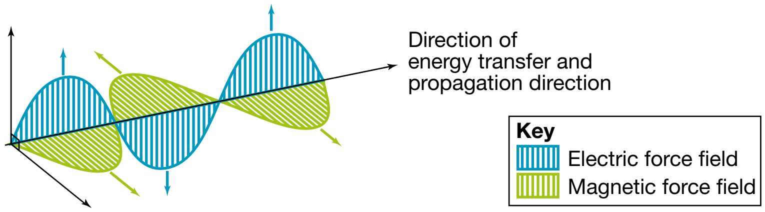
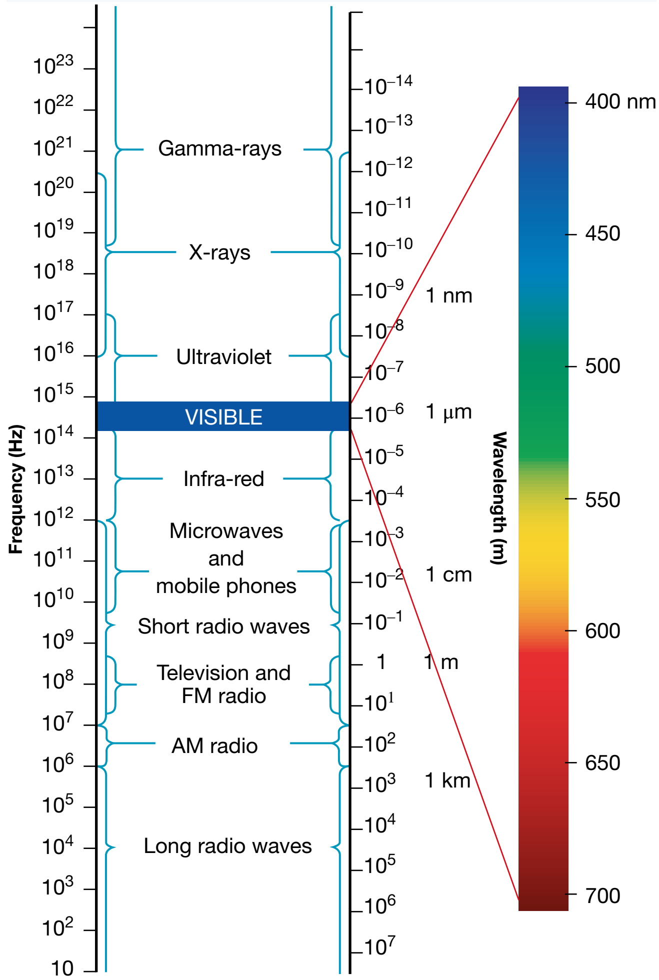


FIGURE 7.7 The electromagnetic spectrum.



7.2 Exercise 1

1. The frequency of a wave sequence observed in a vibrating string is 200 Hz. If the velocity of the waves is 30 m s^{-1}, what is the wavelength?
2. A boat on a lake bobs up and down due to ripples on the water surface. The person in the boat notices that the crests of the ripples are 6 m apart and that a ripple raises the boat every 3 s. Calculate the speed of the ripples.
3. The distance between compressions for a particular sound source is 1.3 m, and the compressions travel at a speed of 330 m s^{-1} through the air. What is the frequency with which the sound source vibrates?
4. A siren produces a sound wave with a frequency of 587 Hz. Calculate the speed of sound if the wavelength of the sound is 0.571 m.

7.3 Representing wave motion

7.3.1 Displacement-time graphs of wave motion

As a wave travels through a medium, the particles of the medium are displaced. By plotting the displacement of the medium at a particular point over time, a displacement-time graph may be produced. From such a graph, the period and amplitude of the wave may be read directly and the frequency calculated.

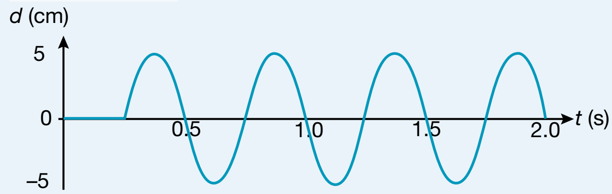
7.3 SAMPLE PROBLEM 1

A stone is dropped into a still pond, and the motion of a leaf on the surface of the water is observed as the ripples move past it. The displacement-time graph below describes the motion of the leaf over time.

Use the graph to determine the wave's:

(a) period
(b) frequency
(c) amplitude
(d) wavelength, given that the wave speed is $2\ \text{m s}^{-1}$.
(e) the distance of the leaf from the wave's point of origin.

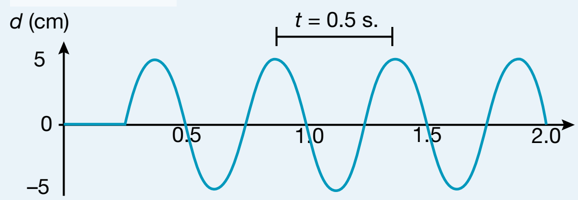
FIGURE 7.8



SOLUTION:

(a) The period is the time taken for one complete wave to be formed. From the graph, we see that the leaf makes one complete cycle (up and down again) every 0.5 seconds.

FIGURE 7.9

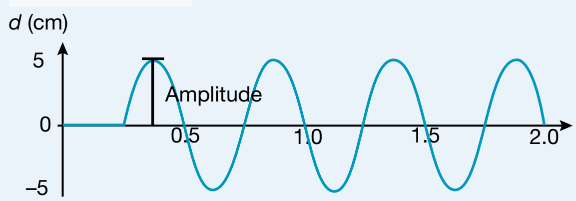


(b) The frequency, f, is equal to the reciprocal of the period, T:

$$f = \frac{1}{T} = \frac{1}{0.5\ \text{s}} = 2\ \text{Hz}$$

(c) The amplitude corresponds to the distance between the equilibrium position and the wave crest. The amplitude of the wave is 5 cm.

FIGURE 7.10



(d) Substituting values into the wave equation:

$$v = f\lambda$$
$$\Rightarrow \lambda = \frac{v}{f}$$
$$= \frac{2\ \text{m s}^{-1}}{2\ \text{Hz}}$$
$$= 1\ \text{m}$$

(e) By inspecting the graph, it can be seen that the first wave did not arrive at the leaf until $t = 0.25$ s. Given that the wave speed was $2\ \text{m s}^{-1}$, we can substitute these values into the equation for average speed:

$$s = v_{av}\,t$$
$$= 2\ \text{m s}^{-1} \times 0.25\ \text{s}$$
$$= 0.5\ \text{m}$$

That is, the leaf was 0.5 m from the wave source (presumably where the stone entered the water).

7.3.2 Displacement-position graphs of wave motion

While a displacement-time graph represents a wave as a function of time for a specific location in the medium, the displacement-position graph represents the wave as a function of position at a specific moment in time. It can almost be thought of as a 'snapshot' of the wave in the medium.

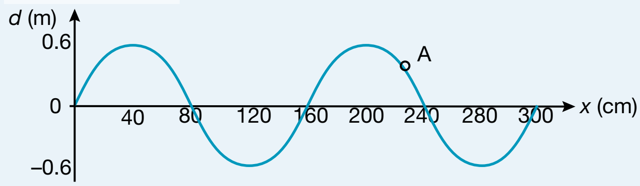
7.3 SAMPLE PROBLEM 2

Consider the displacement-position graph below, which represents a slinky spring during an experiment to model transverse waves at a particular moment in time.

(a) Use the graph to determine the wave's:
- (i) wavelength
- (ii) maximum displacement
- (iii) speed, given that waves were sent through the slinky at a rate of 2 per second.

(b) **A** represents a point on the slinky. Assuming that the wave is moving from right to left, describe whether point **A** is in the process of moving up or moving down, or will always remain in its current position.

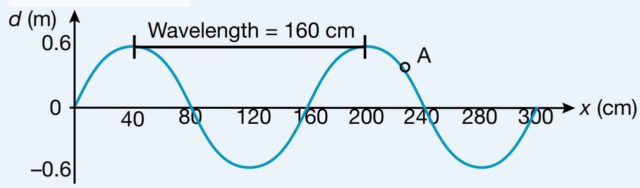
FIGURE 7.11



SOLUTION:

(a) (i) The wavelength is equal to the distance between successive peaks, so the wavelength is equal to 160 cm, or 1.6 m.

FIGURE 7.12



(ii) As with the displacement-time graph, the amplitude corresponds to the distance between the equilibrium position and the maximum displacement. Therefore, the amplitude is equal to 0.6 m.

(iii) As $f = 2\ \text{s}^{-1}$ and $\lambda = 1.6$ m (from part (i) above), we use the wave equation to determine the speed of the wave:

$$\begin{aligned} v &= f\lambda \\ &= 2 \times 1.6 \\ &= 3.2\ \text{m s}^{-1} \end{aligned}$$

(b) As the wave is moving from left to right, we can see that the displacement of point **A** from the equilibrium position will increase, so it is moving upwards at the moment in time at which the graph was made.

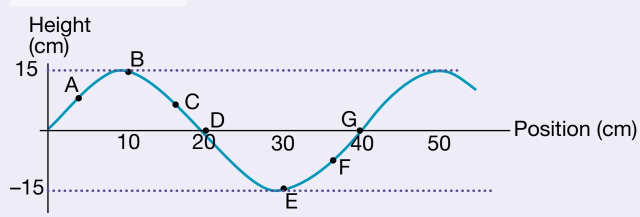
7.3 Exercise 1

Figure 7.13 shows a wave travelling from left to right in a string.

1 Which particles in the string are:
- (a) moving upwards
- (b) moving downwards
- (c) temporarily at rest?

2 Determine the wave's:
- (a) wavelength
- (b) amplitude
- (c) frequency, given that the wave is travelling at 4 m s^{-1}.

FIGURE 7.13



7.4 Review

7.4.1 Summary

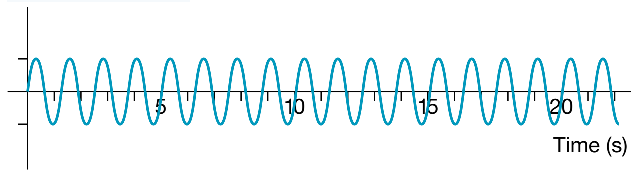
- Waves can be categorised as:
  - mechanical waves, consisting of particles with energy, which require a medium for propagation
  - electromagnetic waves, which do not require a medium for propagation.
- A wave consists of two motions:
  1. a uniform motion in the direction of wave travel; this is the direction of energy transfer
  2. a vibration of particles or fields about an equilibrium or central point.
- The vibration disturbance component of the wave may occur:
  - at right angles (90°) to the direction of propagation; these waves are called transverse waves
  - in the same direction as the direction of propagation; these waves are called longitudinal waves.
- For transverse and longitudinal waves, $v = f\lambda$ and $f = \frac{1}{T}$
- Waves transmit energy but do not transfer matter
- Properties of waves that can be measured include speed, wavelength, period, amplitude, wave number and frequency
- A displacement-time graph represents a wave as a function of time for a specific location in the medium
- A displacement-position graph represents the wave as a function of position at a specific moment in time.

7.4.2 Questions

1. Describe how each of the following observations allows you to determine that waves are carriers of energy.
   (a) On a camping trip, infra-red and visible radiation from the Sun is absorbed by a solar shower, heating the water.
   (b) Sound waves hitting the diaphragm of a microphone cause the still diaphragm to begin to vibrate.
   (c) A tsunami destroys a coastal village.
   (d) A big surf removes the sand from a 10 km stretch of beach.
   (e) Light falling on a photovoltaic cell produces stored chemical energy in a battery that is used to produce electrical energy to power a solar garden light.
2. Describe each of the following waves as propagating in one, two or three dimensions:
   (a) the light emitted from the Sun
   (b) sound from a bell
   (c) a sound wave travelling along a string telephone made from a tight string and two tin cans
   (d) a water wave produced by dropping a rock into the centre of a lake
   (e) a compression wave produced in a slinky.
3. Define the following terms as they would apply when describing the wave model:
   (a) medium
   (b) displacement
   (c) amplitude
   (d) period
   (e) compression
   (f) rarefaction
   (g) crest
   (h) trough.

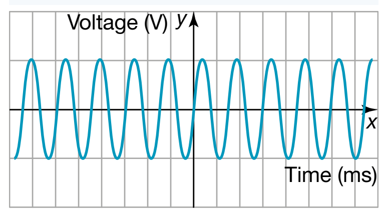
4. Draw a representation of a transverse wave, showing displacement versus time as a sine wave and label the following features:
   (a) the amplitude of the wave
   (b) a change in frequency of the wave
   (c) a wavefront
   (d) a trough
   (e) a crest.
5. In movies it is common to see a spacecraft blown up, accompanied by a large bang. With reference to the properties of mechanical waves, explain why this is impossible in space.
6. An astronomer tells you that observing the star she is showing you is like looking back in time 100 million years. Identify what this tells you about the light from the star and its nature of travel.
7. Light travels at a velocity of 3×10^8 m s^{-1}. If the light reaching Earth from a blue star has a wavelength of 410 nm (410×10^{-9} m), what is the frequency of the light?
8. Look at the transverse wave represented in figure 7.14. Calculate the frequency of the wave.

FIGURE 7.14



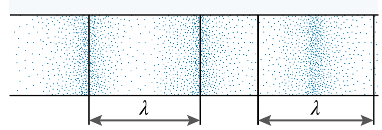
9. A cathode-ray oscilloscope (CRO) is a device that enables you to look at the electrical signal produced by a sound wave hitting the diaphragm of a microphone. The CRO acts like a sensitive voltmeter. Identify which property of the sound wave produces the sympathetic fluctuations in the voltage generated in the microphone.
10. The CRO trace in figure 7.15 is produced by a sound wave. The time base of the CRO is set at a constant value. That means every horizontal division of the figure represents a constant 0.001 s. Is the velocity of this wave constant? Explain your answer.

FIGURE 7.15 The CRO trace of a sound wave



11. Sound travels in air at a speed of 330 m s^{-1}. Calculate the wavelength of a sound wave with a frequency of 256 Hz.
12. The centre of a compression in a sound wave (longitudinal or compression wave) is equivalent to the crest of a transverse wave, and the centre of a rarefaction is equivalent to the trough of a transverse wave. Use this information to present, in a diagram, a transverse wave representation of the sound wave shown in figure 7.16. Lines close together represent high pressure zones (compressions) and lines spread far apart (rarefactions) represent low pressure zones.

FIGURE 7.16 A representation of a sound wave



13. Explain why both representations of a wavelength λ on figure 7.16 are correct.
14. Sound is travelling in a medium at 330 m s^{-1}. Using lines to represent wavefronts, present a scaled, labelled diagram to represent a 100 Hz sound wave. Include two compressions and two rarefactions in your drawing.
15. A p-type earthquake wave is a longitudinal wave, whereas an s-type earthquake wave is a transverse wave. Describe how each of these wave types would express itself in terms of earth movements underfoot as it passed. Assume that the waves you are comparing are of equal intensity and that the waves are travelling along the ground surface towards you.
16. Identify the features of sound that are wave-like.
17. Explain why it is necessary to use a wave model to explain features of the behaviour of sound and light.
18. Light is an electromagnetic wave that is considered to be transverse in nature. What feature of this wave type suggests it is a transverse wave?
19. Explain why it is not possible to have a mechanical wave in a vacuum.

20. On a CRO trace of a sound wave that looks like figure 7.15, explain what the baseline represents in terms of the sound wave.
21. What is the wavelength of a sound that has a speed of 340 m s^{-1} and a period of 3.0 ms?
22. What is the speed of a sound if the wavelength is 1.32 m and the period is 4.0×10^{-3} s?
23. The speed of sound in air is 340 m s^{-1} and a note is produced that has a frequency of 256 Hz.
    (a) What is its wavelength?
    (b) This same note is now produced in water where the speed of sound is 1.50×10^3 m s^{-1}. What is the new wavelength of the note?
24. Copy and complete table 7.2 by applying the universal wave formula.

TABLE 7.2

v(m s^{-1})	f(Hz)	λ(m)
	500	0.67
	12	25
1500		0.30
60		2.5
340	1000	
260	440	

PRACTICAL INVESTIGATIONS

Investigation 7.1: Investigating waves in a slinky spring

Aim

To observe and investigate the behaviour of waves (or pulses) travelling along a slinky spring

Apparatus

slinky spring
other pieces of apparatus as required

Theory

In a slinky spring, the velocity of the wave is quite small and it is not difficult to make observations of waves or pulses as they move along the spring. Many important properties of waves can be observed using this simple equipment.

Method

1. Stretch out a slinky spring, preferably on a smooth floor, until it is about three metres long. (Clamp one end to a fixed object.)
2. Displace the end of the spring to one side and quickly return it to its original or equilibrium position. This should cause a transverse pulse to travel along the spring.
3. Now, instead of a movement to the side to send a transverse pulse, gather up a few coils of the spring and then release them. This should produce a longitudinal or compression pulse.
4. You are now in the position to make investigations of the behaviour of pulses travelling in springs. It will be better to use transverse rather than longitudinal pulses for your experiments. You should be able to devise a series of simple experiments that can be used to investigate the following.
   (a) Do the pulses you produce really carry energy?
   Design and demonstrate a simple experiment that will show that the pulses do carry energy.
   (b) Do the pulses lose energy as they travel along?
   What observations can you make that show that the energy carried by each pulse is slowing dying away? Suggest a reason why this is happening. You may be able to think of a way to reduce the rate at which the energy is lost. This will probably involve changing the condition of the slinky.
   (c) Does the speed of a pulse depend on the condition of the slinky?
   As the tension of the slinky is changed, what happens to the speed of a pulse?
   (d) Does the speed of a pulse depend on the amplitude of the pulse?
   Is it possible to detect a change in the speed of the pulse along the slinky as the amplitude of the pulse is changed?

(e) Does the speed of a pulse depend on the length of a pulse?
Is it possible to detect a change in speed of the pulse along the slinky as the length of the pulse is changed?
(f) Do identical pulses travel at the same speed along identical slinkies?
Obtain a second slinky and lay the two side by side. Investigate how identical pulses travel in what are hopefully identical slinkies. The two 'free' ends of the slinkies could be fixed to a metre ruler (or wooden board) and movement of this could launch identical pulses simultaneously in the two slinkies.
(g) Do identical pulses travel at the same speed along slinkies that are at a different tension?
Changing the tension of one slinky by clamping a few coils to the board could enable you to observe identical pulses travelling in different media.

Extension work

(h) *Reflection from a fixed end*
You could also investigate what happens to pulses when they are reflected from the clamped end of a slinky.
(i) *Reflection from a free end*
You might repeat investigation (h) with the end tied to a length of light string so that it is free to move when the pulse reaches the end of the slinky. There will still be a reflected pulse but it should be different from the reflected pulse in (h).
(j) *Crossing boundaries*
What happens when a pulse travels from one spring to another?
If you have a different spring, connect it to a slinky and see what happens when a pulse in one spring travels into the other.

Investigation 7.2: Observing water waves

Aim

To observe a wave motion travelling in two dimensions

Apparatus

20 corks
small tank of water or a shallow, still-water pond

Theory

A simple transverse water wave is a wave travelling in two dimensions.

Method

1. Place 20 corks in a ring in the water.
2. Drop a small mass such as a stone in the centre of the ring of corks. This will make a wave in the water emanating from where the stone landed.
3. Observe what happens to all of the corks.

Analysis

Any movement of the corks outwards from the central disturbance is minor and at a very slow rate compared with the rate of energy transfer as indicated by the wavefront travelling away from the source.

Questions

1. In how many dimensions does the wave propagate?
2. What does this show about the energy of the wave motion from the central disturbance point?
3. In the previous practical activity, you observed that the energy carried by a pulse in the slinky was gradually lost. The same thing happens with the water waves. Compare the reasons for the decrease in the amplitude, and hence energy of a wave, in the slinky and in a wave spreading out on water.

Investigation 7.3: Relating frequency and amplitude

Aim

To explore the relationship between the displacement and time of constant frequency waves with varying amplitude described by the equation:

$y = n \sin ft$

where

$y =$ displacement of the wave
$n =$ amplitude of the wave
$f =$ frequency of the wave
$t =$ time

Apparatus

access to a graphics calculator or a graphing program for the computer. Some graphing programs can be downloaded from the internet.

access to a printer

Method

1. Plot the equation given under 'Aim' above into a graphics calculator or a graphing program on the computer.
2. Plot graphs with the following variables and if a printer is available, print the graphs out.

Frequency (Hz)	Amplitude (units)
1	1
2	2
0.5	1
4	1
1	4

Analysis

Study the graphs to ensure that you can identify the features of amplitude and frequency.

TOPIC 8
Wave behaviour

8.1 Overview

8.1.1 Module 3: Waves and Thermodynamics

Wave behaviour

Inquiry question: How do waves behave?

Students:

- explain the behaviour of waves in a variety of situations by investigating the phenomena of:
  - reflection
  - refraction
  - diffraction
  - wave superposition (ACSPH071, ACSPH072).
- conduct an investigation to distinguish between progressive and standing waves (ACSPH072)
- conduct an investigation to explore resonance in mechanical systems and the relationships between:
  - driving frequency
  - natural frequency of the oscillating system
  - amplitude of motion
  - transfer/transformation of energy within the system (ACSPH073).

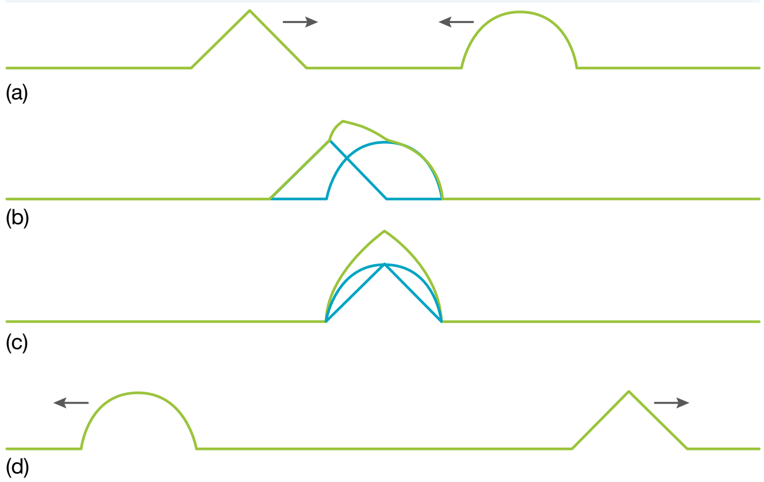
FIGURE 8.1 Ocean waves entering a gap in a barrier will experience diffraction — a behaviour exhibited by all waves under the right circumstances.

8.2 Interference of waves

8.2.1 Wave superposition

While waves pass through each other without affecting each other's frequency or wavelength, effects arise when two or more waves arrive at the same place at the same time. Depending on the relative positions of the crests and troughs, the waves will combine to either reinforce each other, or partially or completely cancel each other out. This is called **interference**. Interference can occur between periodic waves or between pulses (a single disturbance) in a medium.

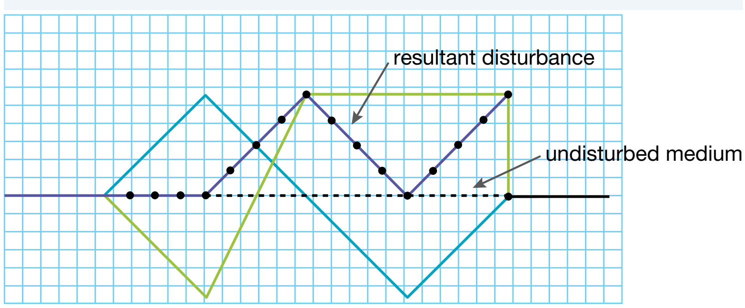
FIGURE 8.2 (a) Two pulses of different shapes approach each other on a spring. **(b)** The pulses begin to pass through each other. **(c)** As the pulses pass through each other, the amplitudes of the individual pulses add together to give a resultant disturbance. **(d)** After passing through each other, the pulses continue on undisturbed.



When waves in a medium interfere with each other, the amplitudes of the individual wave pulses add together to give the amplitude of the total disturbance of the medium. This is called **superposition.**

The shape of the resultant disturbance can be found by applying the superposition principle: *'The resultant wave is the sum of the individual waves'*. For convenience, we can add the individual displacements of the medium at regular intervals where the pulses overlap to get the approximate shape of the resultant wave. Displacements above the position of the undisturbed medium are considered to be positive and those below the position of the undisturbed medium are considered to be negative. This is illustrated in figure 8.3, in which two pulses have been drawn in green and blue with a background grid. The sum of the displacements on each vertical grid line is shown with a dot, and the resultant disturbance, drawn in black, is obtained by drawing a smooth line through the dots.

FIGURE 8.3 How to obtain the shape of a resultant disturbance.

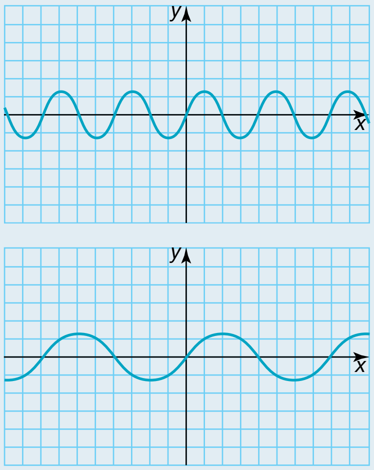


8.2 SAMPLE PROBLEM 1

ADDING WAVES

Add the two waves graphed in figure 8.4.

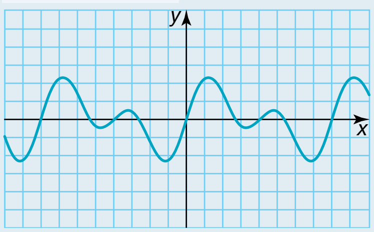
FIGURE 8.4



SOLUTION:

The resulting graph is shown in figure 8.5.

FIGURE 8.5

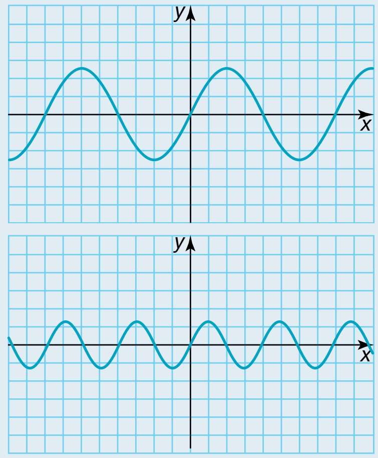


8.2 SAMPLE PROBLEM 2

ADDING WAVES

Add the two waves graphed in figure 8.6.

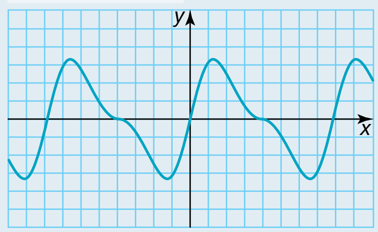
FIGURE 8.6



SOLUTION:

The resulting graph is shown in figure 8.7. As you can see, the addition of rather simple wave shapes can form a complex wave.

FIGURE 8.7

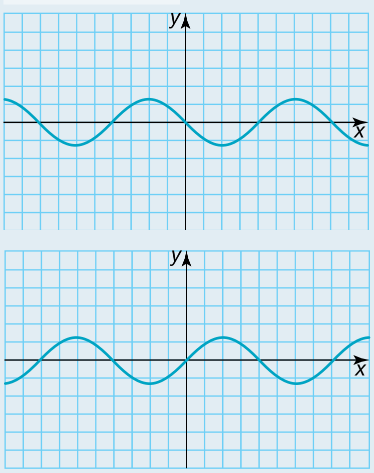


8.2 SAMPLE PROBLEM 3

ADDING WAVES

Add the two waves graphed in figure 8.8.

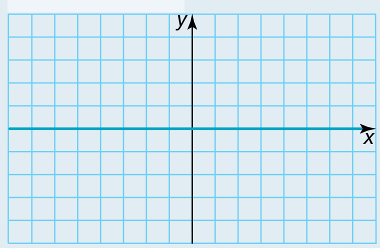
FIGURE 8.8



SOLUTION:

The resulting graph is shown in figure 8.9. This wave shows annulment of the waves. The two waves added were out of phase by 180°.

FIGURE 8.9



8.2.2 Constructive and destructive Interference

Waves are said to be in phase if they have the same frequency and their crests and troughs (or, in the case of longitudinal waves, compressions and rarefactions) occur simultaneously.

When two waves in phase with each other interfere and the sum of their amplitudes is greater than that of the individual waves alone, we say that **constructive interference** occurs.

Conversely, when two waves combine such that the sum of their amplitudes is less than the amplitude of either of the individual waves, we say that **destructive interference** has occurred. Complete destructive interference occurs when two waves completely cancel each other out so that the amplitude of the resultant disturbance is equal to zero. This occurs when the two waves have the same frequency and amplitude but have opposite phase, with one wave's peaks coinciding with the other wave's troughs.

FIGURE 8.10 The green and blue waves in **(a)** are in phase while those in **(b)** are completely out of phase.

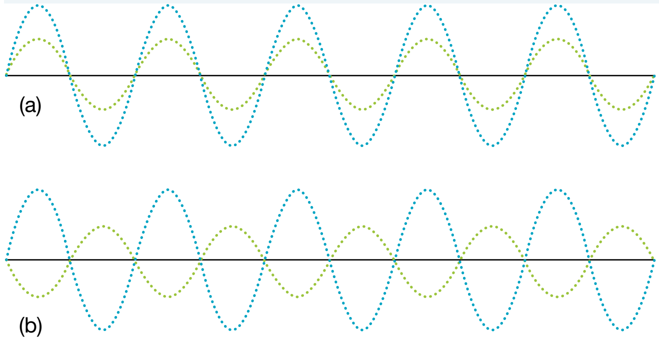
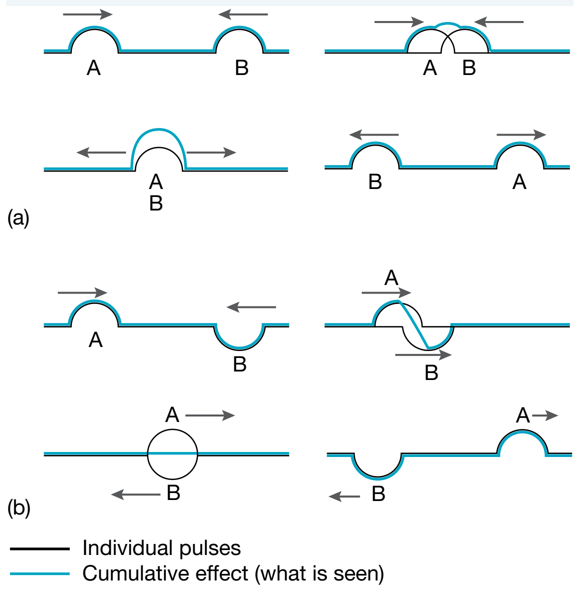


FIGURE 8.11 Where two wave pulses in a medium meet, they will interfere. This can be in the form of **(a)** constructive interference or **(b)** destructive interference.

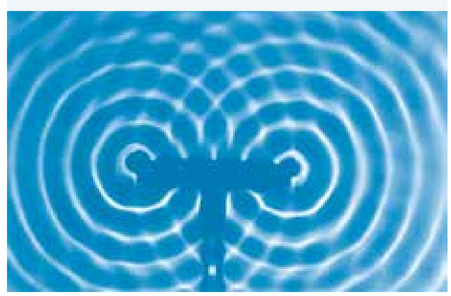


8.2.3 Interference of waves in two dimensions

Interference of waves is best observed in a ripple tank. When two point sources emit continuous waves with the same frequency and amplitude, the waves from each source interfere as they travel away from their respective sources. If the two sources are in phase (producing crests and troughs at the same time as each other), an interference pattern similar to that shown in figure 8.12 is obtained.

Lines are seen on the surface of the water where there is no displacement of the water surface. These lines are called **nodal lines**. They are caused by destructive interference between the two sets of waves. At any point on a nodal line, a crest from one source arrives at the same time as a trough from the other source, and vice versa. Any point on a nodal line is sometimes called a local minimum, because of the minimum disturbance that occurs there.

FIGURE 8.12 An interference pattern obtained in water by using two point sources that are in phase.

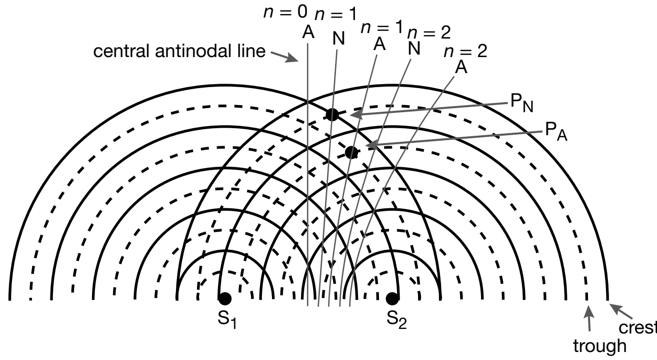


Between the nodal lines are regions where constructive interference occurs. The centres of these regions are called **antinodal lines**. At any point on an antinodal line, a crest from one source arrives at the same time as a crest from the other source, or a trough from one source arrives at the same time as a trough from the other source, and so on. Any point on an antinodal line is sometimes called a local maximum, because of the maximum disturbance that occurs there.

When the two sources are in phase, as shown in figure 8.12, the interference pattern produced is symmetrical with a central antinodal line. Note that any point on the central antinodal line is an equal distance from each source. Since the sources produce crests at the same time, crests from the two sources will arrive at any point on the central antinodal line at the same time.

Similar analysis will show that, for any point on the first antinodal line on either side of the centre of the pattern, waves from one source have travelled exactly one wavelength further from one source than from the other. This means that crests from one source still coincide with crests from the other, although they were not produced at the same time (see figure 8.13). Point P_A is on the first antinodal line from the centre of the pattern. It can be seen that P_A is 4.5 wavelengths from S_1 and 3.5 wavelengths from S_2.

FIGURE 8.13 Interference pattern produced by two sources in phase.



A way to establish whether a point is a local maximum or not is to look at the distance it is from both of the two sources. If the distance that the point is from one source is zero or a whole number multiple of the wavelength further than the distance it is from the other source, then that point is a local maximum. This 'rule' can be re-expressed as: '*If the path difference at a point is nλ, the point is a local maximum*'.

Therefore, for a point to be an antinode:

$$d(PS_1) - d(PS_2) = n\lambda \quad n = 0, 1, 2, 3, 4,\ldots$$

where

n is the number of the antinodal line from the centre of the pattern

P is the point in question

S_1 and S_2 are the sources of the waves

$d(PS_1)$ is the distance from P to S_1.

Similar analysis shows that, for a point on a nodal line, the difference in distance from the point to the two sources is $\frac{1}{2}\lambda$ or $1\frac{1}{2}\lambda$ or $2\frac{1}{2}\lambda$ and so on. This means that a crest from one source will coincide with a trough from the other source, and vice versa. Point P_N in the figure is 5 wavelengths from S_1 and 4.5 wavelengths from S_2.

For a node:

$$d(PS_1) - d(PS_2) = (n - \tfrac{1}{2})\lambda \quad n = 1, 2, 3, 4,\ldots$$

where

n is the number of the nodal line obtained by counting outward from the central antinodal line.

The same formulas that were used for water waves can be used to determine whether a point is part of a nodal or antinodal region.

8.2 SAMPLE PROBLEM 4

Two point sources S_1 and S_2 emit waves in phase in a swimming pool. The wavelength of the waves is 1.00 m. P is a point that is 10.00 m from S_1 and P is closer to S_2 than to S_1. How far is P from S_2 if:

(a) P is on the first antinodal line from the central antinodal line?

(b) P is on the first nodal line from the central antinodal line?

SOLUTION

(a) $d(PS_1)$ is greater than $d(PS_2)$; $d(PS_1) = 10.00$ m, $\lambda = 1.00$ m

If P is on the first antinodal line from the central antinodal line, then:

$$d(PS_1) - d(PS_2) = \lambda.$$

Therefore,

$$\begin{aligned} d(PS_2) &= d(PS_1) - \lambda \\ &= 10.00\text{ m} - 1.00\text{ m} \\ &= 9.00\text{ m}. \end{aligned}$$

(b) $d(PS_1)$ is greater than $d(PS_2)$; $d(PS_1) = 10.00$ m, $\lambda = 1.00$ m

If P is on the first nodal line from the central antinodal line, then:

$$d(PS_1) - d(PS_2) = \frac{1}{2}\lambda.$$

Therefore,

$$\begin{aligned} d(PS_2) &= d(PS_1) - \frac{1}{2}\lambda \\ &= 10.00\text{ m} - 0.50\text{ m} \\ &= 9.50\text{ m.} \end{aligned}$$

WORKING SCIENTIFICALLY 8.1

Draw a plan of your school assembly hall and mark the location of the speakers. Draw nodal and antinodal lines on your plan of the speaker system for sound waves with a frequency of 200 Hz (the mid-range of human speech) to determine where constructive and destructive interference occurs in your hall.

AS A MATTER OF FACT

Complete destructive interference rarely occurs as the sounds produced from each source are usually not of equal intensity, due to the different distances travelled by the individual waves and the inverse square law that describes this variation in intensity with distance from the source.

8.2 SAMPLE PROBLEM 5

A student arranges two loudspeakers, A and B, so that they are connected in phase to an audio amplifier. The speakers are then placed 2.00 m apart and they emit sound that has a wavelength of 0.26 m.

Another student stands at a point P, which is 15.00 m directly in front of speaker B. The situation representing this arrangement is shown in figure 8.14. Describe what the student standing at point P will hear from this position.

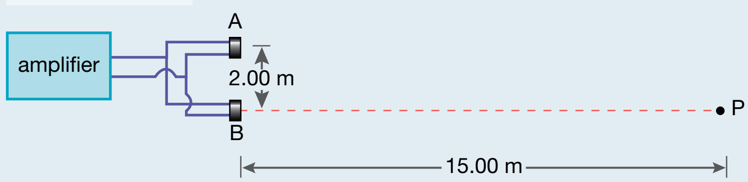
SOLUTION:

In this type of question, it is important to determine whether the point is a node or antinode.

This is done by determining the path difference and then comparing this to the wavelength.

$\lambda = 0.26$ m, $d(PB) = 15.00$ m

FIGURE 8.14



$d(PA)$ can be found by applying Pythagoras's theorem.

$$\begin{aligned} d(PA^2) &= 15.00\text{ m}^2 + 2.00\text{ m}^2 \\ &= 229\text{ m}^2 \end{aligned}$$

So $d(PA) = 15.13$ m

$$\begin{aligned} d(PA) - d(PB) &= 15.13\text{ m} - 15.00\text{ m} \\ &= 0.13\text{ m.} \end{aligned}$$

$$0.13\text{ m} = \frac{1}{2}\lambda$$

Therefore, as P corresponds to a local node, the sound intensity here will be a minimum.

8.2 Exercise 1

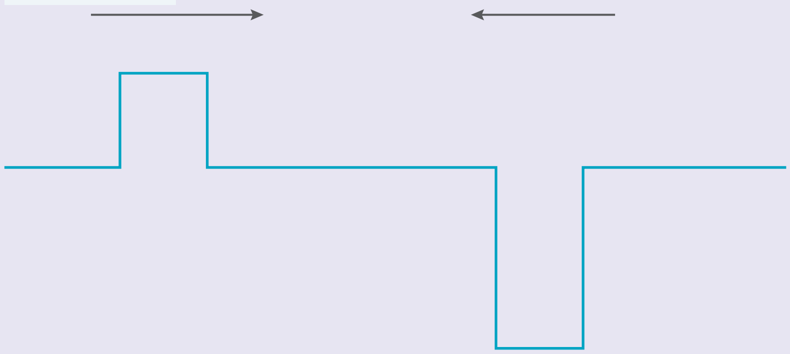
In each case, sketch the shape of the resultant disturbance created when the pulses are superimposed.

1

FIGURE 8.15

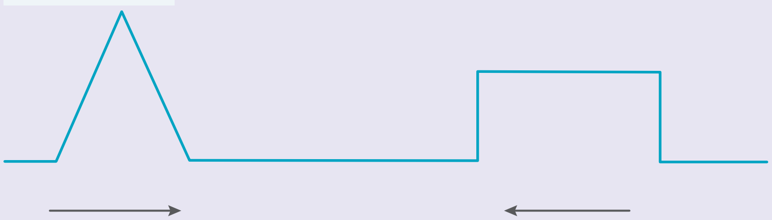
2

FIGURE 8.16



3

FIGURE 8.17



4 Two wave sources are in phase and placed 20 cm apart and send out 8 waves per second. If the waves have a speed of 40 cm s^{-1}, how many nodal lines will be produced between the two sources?

5 Owen is sitting between two speakers that are in phase and producing a signal that has a wavelength of 2 m. If he is 4 m from one speaker and 9 m from the other, will he hear a minimum or a maximum of sound intensity?

8.3 Standing waves

8.3.1 Reflection of waves

When waves arrive at a barrier, reflection occurs. Reflection is the returning of a wave into the medium in which it was originally travelling. When a wave strikes a barrier, or comes to the end of the medium in which it is travelling, at least part of the wave is reflected.

A wave's speed depends only on the medium, so the speed of the reflected wave is the same as for the original (incident) wave. The wavelength and frequency of the reflected wave will also be the same as for the incident wave.

When a string has one end fixed so that it is unable to move (for example, when it is tied to a wall or is held tightly to the 'nut' at the end of a stringed instrument), the reflected wave will be inverted. This is called a **change of phase**. If the end is free to move, the wave is reflected upright and unchanged, so there is no change of phase. These situations are illustrated in figure 8.18.

8.3.2 Standing waves

Most of the waves that have been examined so far in this topic have been examples of **progressive** (or **travelling**) **waves**. These are waves that move freely through a medium until a boundary is met. For example, ocean waves travel freely through water until they meet land or a barrier of some kind.

However, under some circumstances, waves can appear to stand still. If a guitar string such as that in figure 8.18 is plucked, wave pulses are sent travelling down the length of the string. When the pulses reach the fixed end of the string at the fret nut, they are reflected back along the string. These reflected pulses have the same wavelength and speed as the incident pulses but are inverted. Where the incident waves and the reflected waves coincide, interference between the waves occurs. When the incident pulses are produced in the string with particular frequencies, the incident and reflected waves will combine in such a way that the positions at which the waves interfere constructively and at which they interfere destructively are evenly spaced along the string. This makes the resultant superimposed wave appear to be fixed in position. As a result, waves that are produced in these circumstances are referred to as **standing waves**.

FIGURE 8.18 **(a)** and **(b)** Reflection of a transverse pulse on a string when the end of the string is fixed (as in a guitar), and **(c)** and **(d)** when the end of the string is free to move (as with a loop supported by a retort stand).

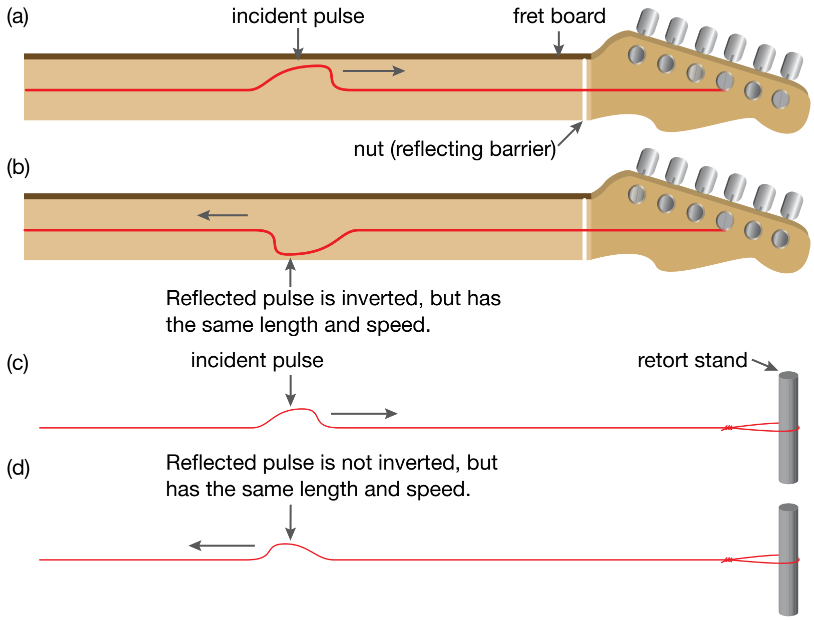
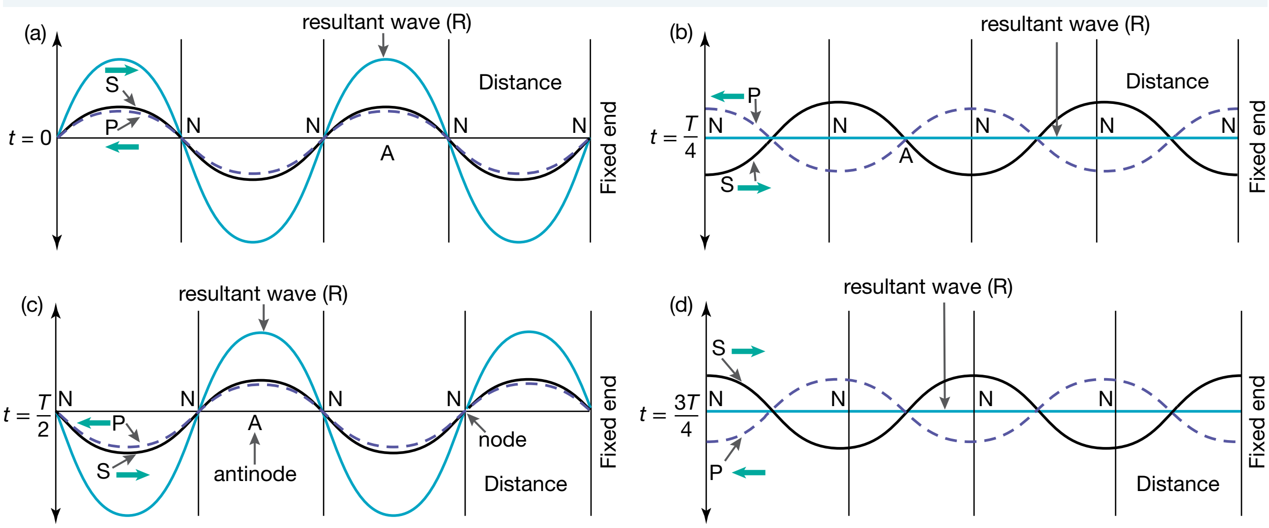


FIGURE 8.19 The formation of a standing wave.



Standing waves occur at wave frequencies where there is interference between the initially generated waves and the reflected waves. Where an incident wave coincides with a reflected wave that is opposite in phase, the two waves will essentially cancel each other out due to destructive interference, leaving the medium at that location undisturbed. In a standing wave, these undisturbed points (called **nodes**) are evenly spaced.

At locations where an incident wave coincides with a reflected wave that is equal in phase, constructive interference occurs and the amplitudes of the two waves reinforce each other both as peaks and as troughs. The points where the medium is disturbed the most are called **antinodes**.

Figure 8.19 shows how standing waves are formed in a string by an incident wave series P and the reflected waves S. It is important to note that the wavelength of the waves involved in the standing wave is twice the distance between adjacent nodes (or adjacent antinodes).

Figure 8.20 shows the motion of a spring as it carries a standing wave. It shows the shape of the spring as it completes one cycle. The time taken to do this is one period (T). Note that (i) at $t = \frac{T}{4}$ and at $t = \frac{3T}{4}$ the medium is momentarily undisturbed at all points, and (ii) that adjacent antinodes are opposite in phase — when one antinode is a crest, those next to it are troughs.

FIGURE 8.20 A standing wave over one cycle.

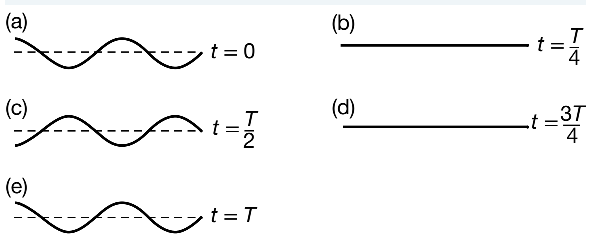
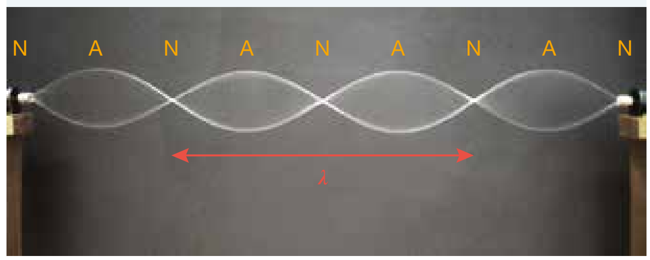


FIGURE 8.21 A standing wave produced in a string.



8.3 SAMPLE PROBLEM 1

Two students have created a standing wave in a string, as depicted in figure 8.20.
(a) How many nodes are there in the standing wave?
(b) How many antinodes are there?
(c) If the students are 8.0 m apart, what is the wavelength of the wave?
(d) If the student at the left-hand end of the string is moving her hand at a frequency of 4.0 Hz, what is the speed of the wave?
(e) At what frequency would the student need to move her hand to have only one antinode?

SOLUTION:
(a) There are four nodes, three in the picture and one at the elbow.
(b) There are three antinodes.
(c) The distance between nodes is given by $\frac{8.0}{3.5}$. The wavelength is twice this distance and equal to:

$$\frac{2 \times 8.0}{3.5} = 4.6 \text{ m}.$$

(d) Using $v = f\lambda$, speed $= 4.0 \times 4.6 = 18.4$ m s^{-1} $= 18$ m s^{-1}.
(e) The speed is unchanged at 18 m s^{-1} and the wavelength is now 16 m, so the

$$\text{frequency} = \frac{18}{16} = 1.1 \text{ Hz}.$$

WORKING SCIENTIFICALLY 8.2

Design and perform an investigation to determine the relationship between the extension given to a slinky spring, the spring constant and the speed of the compressional wave produced in it.

8.3 Exercise 1

1 What is the longest wavelength of a standing wave that can be produced on a 30 cm string fixed at both ends?

2 Consider the wave formed in figure 8.21.
 (a) If the distance between the first and last nodes is 240 cm, what is the wavelength of the incident wave in the string?
 (b) What is the speed of the incident wave in the string if the waves are produced with a frequency of 200 Hz?
 (c) At what frequency would the wave need to be produced for only 2 antinodes to be seen?

3 Which of these conditions must be fulfilled by the incident and reflected waves for a standing wave to be produced in a string fixed at both ends?
 (i) have the same amplitude
 (ii) have different amplitudes
 (iii) travel in the same direction
 (iv) travel in opposing directions
 (v) have the same frequency
 (vi) have different frequencies
 (vii) have equal wavelengths
 (viii) have different wavelengths
 (ix) are in phase
 (x) are out of phase

8.4 Bending waves

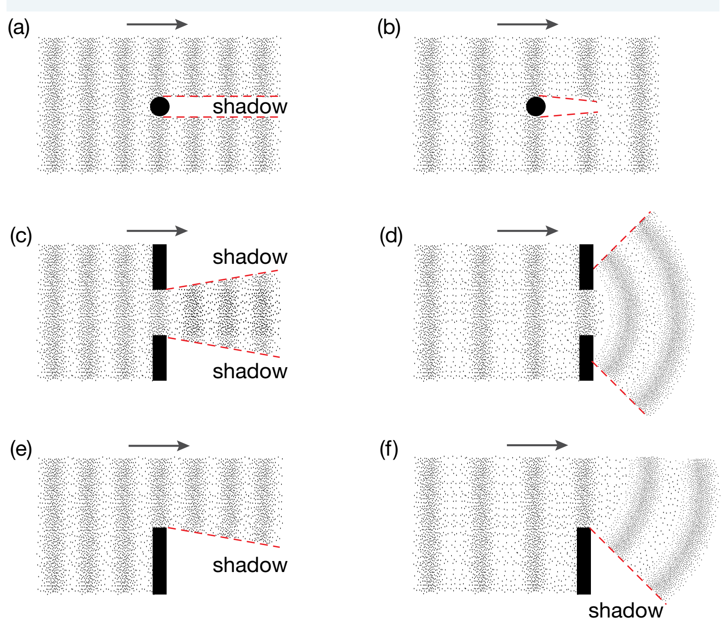
8.4.1 Diffraction

Waves spread out as they pass objects or travel through gaps in barriers. This is readily observable in sound and water waves. For example, you can hear someone speaking in the next room if the door is open, even though there is not a direct straight line between the person and your ears.

Diffraction is the directional spread of waves as they pass through gaps or pass around objects. The amount of diffraction depends on the wavelength of the wave and the width of the gap or the size of the obstacle.

For example, the spreading out of sound from loudspeakers is the result of diffraction. The sound waves spread out as they pass through the opening in the front of the loudspeaker. Without diffraction, hardly any sound would be heard other than from directly in front of the speaker cone.

FIGURE 8.22 Diffraction of water waves: (a) short wavelength around an object, (b) long wavelength around the same object, (c) short wavelength through a gap, (d) long wavelength through the same gap, (e) short wavelength around the edge of a barrier and (f) long wavelength around the edge of the same barrier.



Diffraction of water waves

The diffraction of waves in general can be modelled with water waves in a ripple tank. Figure 8.22 shows the way water waves diffract in various situations. The diagrams apply equally well to the diffraction of both transverse and longitudinal waves.

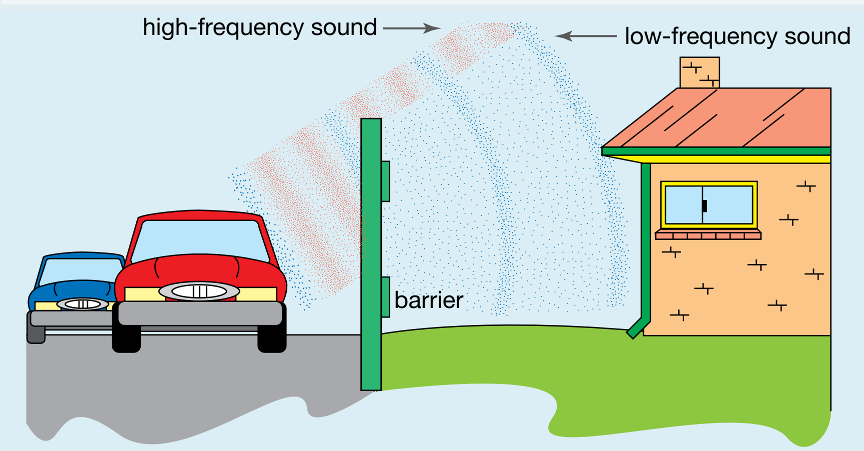
The region where no waves travel is called a shadow. The amount of diffraction that occurs depends on the wavelength of the waves. The longer the wavelength, the more diffraction occurs. As a general rule, if the wavelength is less than the size of the object, there will be a significant shadow region.

When waves diffract through a gap of width w in a barrier, the ratio $\frac{\lambda}{\omega}$ is important. As the value of this ratio increases, so, too, does the amount of diffraction that occurs.

AS A MATTER OF FACT

Barriers built next to freeways are effective in protecting nearby residents from high-frequency sounds as these have a short wavelength and undergo little diffraction. The low-frequency sounds from motors and tyres, however, diffract around the barriers because of their longer wavelengths.

FIGURE 8.23 The diffraction of low and high frequencies around a freeway barrier



8.4 SAMPLE PROBLEM 1

Two sirens are used to produce frequencies of 200 Hz and 10 000 Hz. Describe the spread of the two sounds as they pass through a window in a wall. The window has a width of 35 cm. Assume that the speed of sound in air is 330 m s^{-1}.

SOLUTION:

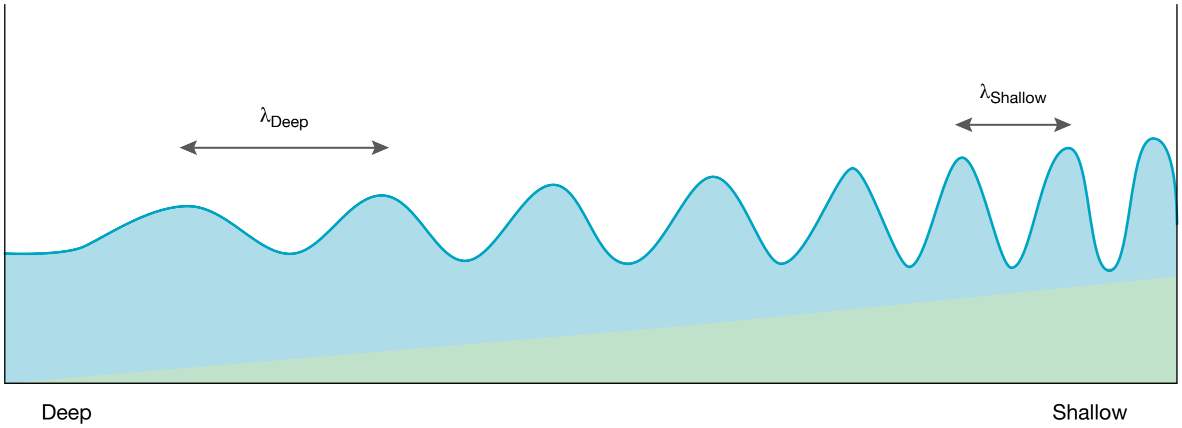
First calculate the wavelengths of the sounds using the formula $v = f\lambda$. These calculate to 165 cm and 3.3 cm respectively. There will be a very small diffraction spread for the sound of wavelength 3.3 cm because the wavelength is small compared with the opening. There will be a large diffraction spread for the sound of wavelength 165 cm because the wavelength is large compared with the opening.

8.4.2 Refraction

Refraction of waves describes the change in the direction of travel that occurs when the waves enter a medium through which they travel at a different speed.

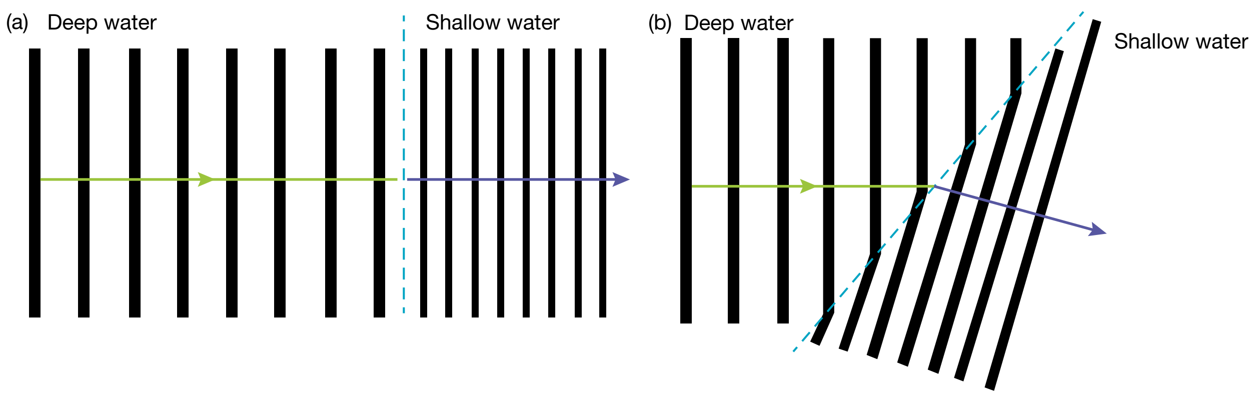
As an example, consider the effect of water waves moving from deep water into shallow water. The waves travel faster in deeper water than they do in shallow water but their frequency remains unchanged. As a result, when the waves enter a shallow region, their wavelength gets shorter and they appear to ‘bunch up’.

FIGURE 8.24 As the waves travel from deep water into shallower water, their decrease in speed causes the waves to 'bunch up'.



When the interface between deep water and shallow water is sharply defined, waves striking that interface at right angles will continue to move in the same direction, although at a slower speed and with a shorter wavelength. However, if the waves encounter the interface at an angle, then the part of each wave that enters the shallow region first will slow down before the rest of the wave. This causes the wave to bend (refract).

FIGURE 8.25 **(a)** When a wave meets a boundary between media at right angles, the direction of propagation of the wave remains unchanged. **(b)** If the wave meets the boundary at an angle, then the direction of wave propagation changes and the wave is said to be refracted.



8.4 SAMPLE PROBLEM 2

A sound wave with a wavelength of 1.1 m travels through air at a speed of 330 m s^{-1}. When it enters water, it travels at 1500 m s^{-1}. What is (a) the frequency and (b) the wavelength of the sound wave when it travels in water?

SOLUTION:

(a) The frequency of the sound wave in water will be the same as its frequency in air.
Using the wave equation:

$$v = f\lambda$$

$$\Rightarrow f = \frac{v}{\lambda}$$

$$= \frac{330 \text{ m s}^{-1}}{1.1 \text{ m}}$$

$$= 300 \text{ Hz}$$

(b) Again, we rearrange the wave equation and substitute the values for frequency and speed in water:

$$v = f\lambda$$

$$\Rightarrow \lambda = \frac{v}{f}$$

$$= \frac{1500\ \text{m s}^{-1}}{300\ \text{Hz}}$$

$$= 5\ \text{m}$$

WORKING SCIENTIFICALLY 8.3

Use Google Earth to locate coastal regions where ocean waves can be observed. Use observations of changing wavelength to locate sudden changes in ocean depth. Correlate the wavelength changes with ocean floor profile maps for the regions to develop a general equation relating wavelength change with depth change.

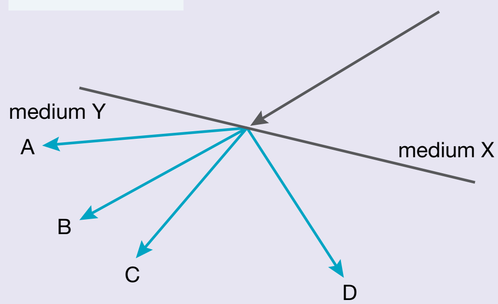
8.4 Exercise 1

1 Water waves approach a boundary at which the wave speed changes from 40 cm s^{-1} to 24 cm s^{-1}. If the waves initially had a wavelength of 10 cm, what will be their wavelength after they have crossed the boundary?

2 In figure 8.26, the direction of propagation of a wave travelling through medium X and then into medium Y is shown. If the wave travels at a higher speed in medium Y, which line best represents the direction of propagation of the wave when it enters the new medium?

3 A water wave with a wavelength of 4 cm approaches a 2 m gap in a barrier. Draw a diagram representing the observed diffraction of the wave.

FIGURE 8.26

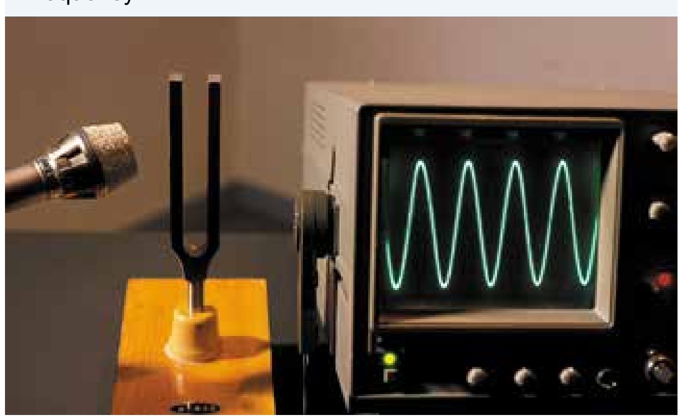


8.5 Resonance

8.5.1 Natural and forced vibration

When you pull a pendulum back and let it go, it will swing backwards and forwards at a rate dictated by the length of the string. In the same way, if you blow across the top of an empty bottle, the sound that you hear is a function of the bottle's size, shape and the material it is made from. This sound arises from the bottle's natural vibration.

FIGURE 8.27 A tuning fork vibrates at its natural frequency.



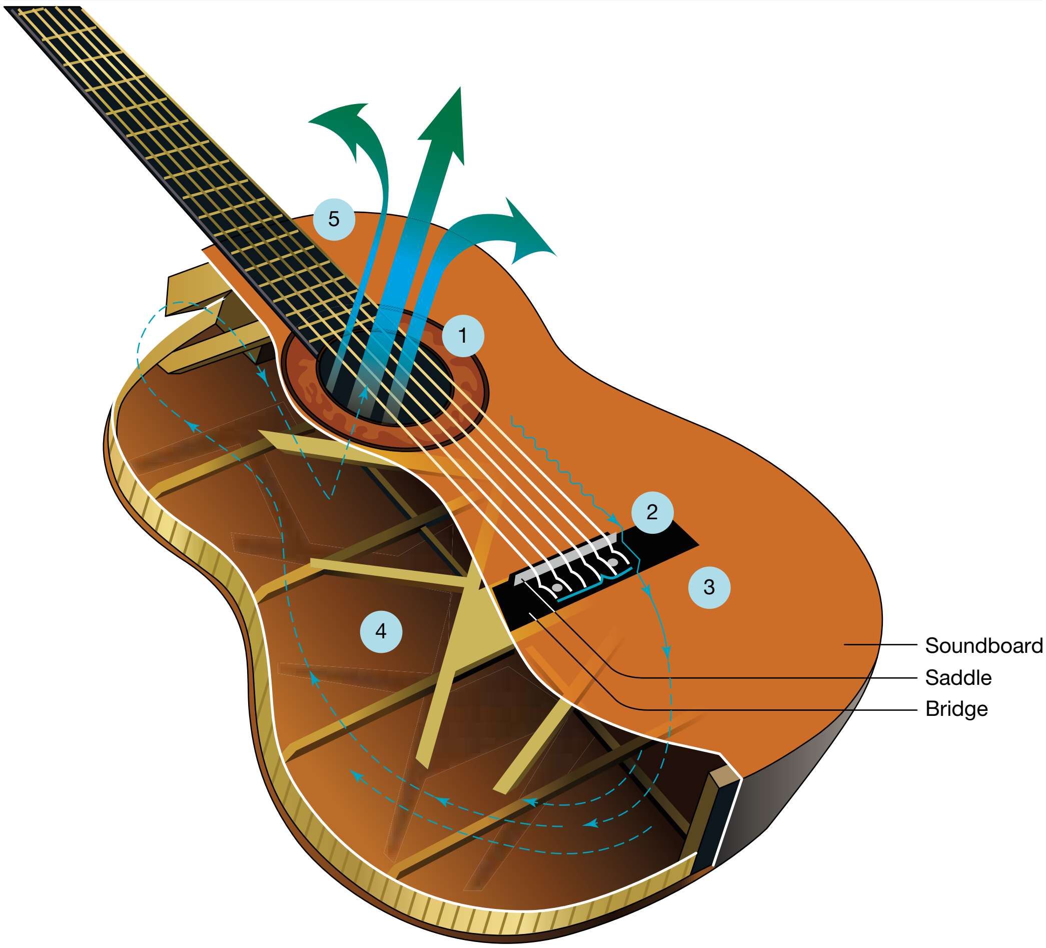
The natural vibration of an object is the rate at which it oscillates once set into motion, and it is inherent to the object's structure. A tuning fork, when struck with a rubber hammer, will vibrate at the same rate regardless of how hard it is hit, because its vibrational rate is determined by the metal it is made from, its length and the spacing of its prongs. A tuning fork tuned to the A above middle C will vibrate at 440 Hz when struck. As a result, we can say that this is its **natural frequency**.

Unlike natural vibration, **forced vibration** occurs when an object is compelled to vibrate by placing it in contact with another vibrating object.

If we pluck a guitar string, it will vibrate at its natural frequency, which is a function of the string's thickness, material, tension and length. However, the sound produced by a guitar string vibrating on its own is not very loud at all. Forced vibration is necessary for sound amplification. The vibration of the string causes the bridge, which is attached to the string, to vibrate at the same frequency and this, in turn, causes the body of the guitar and the air in the body to vibrate as well. While the string's vibration is natural, the bridge, the body and the air in the body all undergo forced vibration. The frequency of a forced vibration is referred to as the **driving frequency**.

Due to the shaping of the body of a guitar, the air and belly of the guitar are able to produce a much louder sound at the same frequency than the string alone.

FIGURE 8.28 1. When picked or strummed, each string vibrates at its natural frequency. 2. Vibration of a string causes forced vibration of the saddle and bridge. 3. Vibration of the bridge causes the forced vibration of the soundboard and guitar body. 4. This causes forced vibration of the air within the hollow body. 5. Amplified sound is then heard.



8.5.2 Resonant frequency

You have probably heard of opera singers producing high notes to shatter a wineglass. While extraordinarily difficult to do and requiring ideal circumstances and preparation, this can be done because of resonance.

Resonance occurs when an object is exposed to a driving frequency equal to the object's natural frequency. This has the effect of increasing the amplitude of the object's vibration due to constructive interference. For our opera singer, the note that they sing has the same frequency as the wine glass.

The driving frequency of the sound waves that the singer produces causes the wine glass to oscillate with an increasing amplitude until, finally, the amplitude of vibration exceeds the limits of the glass's matrix structure and the glass shatters.

Resonance can also be observed when you push a child on a swing. By pushing a child at the point where they are just about to swing forward, you will notice that, even if you push with the same force each time, the swing goes higher and higher. This happens because you have reinforced the amplitude of the swing's vibration by applying an external force at the same frequency as the natural oscillating frequency of the swing.

AS A MATTER OF FACT

An army company marching in step will always break step when crossing a rope suspension bridge. If they were to cross the bridge in step, the bridge would start to resonate at the same frequency and with increasing amplitude. Eventually, the bridge would either fail or throw them off.

8.6 Review

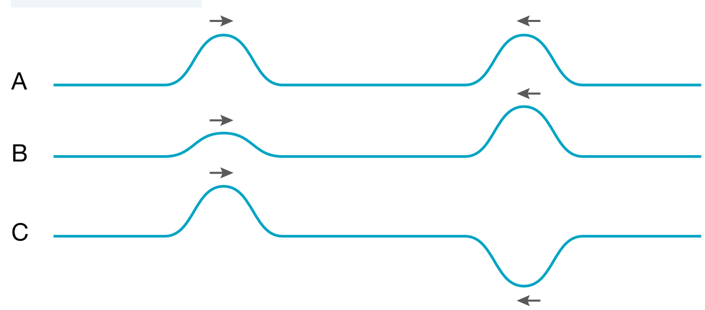
8.6.1 Summary

- Superposition is the adding together of amplitudes of two or more waves passing through the same point.
- Destructive interference is the addition of two wave disturbances to give an amplitude that is less than either of the two waves.
- Constructive interference describes the addition of two wave disturbances to give an amplitude that is greater than either of the two waves.
- Reflection is the returning of the wave into the medium in which it was originally travelling. When a wave strikes a barrier, or comes to the end of the medium in which it is travelling, at least a part of the wave is reflected.
- Standing waves are caused by the superposition of two wave trains of the same frequency travelling in opposite directions.
- Nodes are points on a standing wave that undergo the least disturbance, while antinodes form where the medium undergoes the most disturbance.
- Diffraction is the spreading out, or bending of, waves as they pass through a small opening or move past the edge of an object.
- Refraction describes the change in the direction of travel that occurs when waves enter a medium through which they travel at a different speed.
- Resonance is the condition where a medium responds to a periodic external force by vibrating with the same frequency as the force.

8.6.2 Questions

1. How is a periodic wave different from a single pulse moving along a rope?
2. In each of the diagrams in figure 8.29, two waves move towards each other. Which diagram or diagrams show waves that, as they pass through each other, could experience:
   (a) only destructive interference
   (b) only constructive interference?
3. You arrive late to an outdoor concert and have to sit 500 m from the stage. Will you hear high-frequency sounds at the same time as low-frequency sounds if they are played simultaneously? Explain your answer.

FIGURE 8.29



4. What is superposition and when does it occur?
5. What is constructive interference and when does it occur?
6. Describe the interference pattern produced when two sound sources produce sounds of equal frequency in phase. How can you determine whether a point on the interference pattern is a local maximum or local minimum?
7. Figure 8.30 shows the positions of three sets of two pulses as they pass through each other. Copy the diagram and sketch the shape of the resultant disturbances.
8. What is the wavelength of a standing wave if the nodes are separated by a distance of 0.75 m?
9. Figure 8.31 shows a standing wave in a string. At that instant ($t = 0$) all points of the string are at their maximum displacement from their rest positions.
   If the period of the standing wave is 0.40 s, sketch diagrams to show the shape of the string at the following times:
   (a) $t = 0.05$ s
   (b) $t = 0.1$ s
   (c) $t = 0.2$ s
   (d) $t = 0.4$ s.

FIGURE 8.30

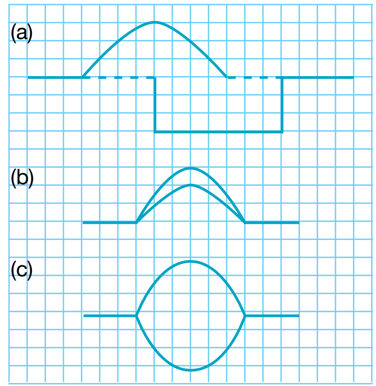


FIGURE 8.31

10. Kim and Jasmine set up two loudspeakers in accordance with the following arrangements:
   - They face each other.
   - They are 10 m apart.
   - The speakers are in phase and produce a sound with a frequency of 330 Hz.

   Jasmine uses a microphone connected to a CRO and detects a series of points between the speakers where the sound intensity is a maximum. These points are at distances of 3.5 m, 4.0 m and 4.5 m from one of the speakers.
   (a) What causes the maximum sound intensities at these points?
   (b) What is the wavelength of the sound being used?
   (c) What is the speed of sound on this occasion?
11. A standing wave is set up by sending continuous waves from opposite ends of a string. The frequency of the waves is 4.0 Hz, the wavelength is 1.2 m and the amplitude is 10 cm.
   (a) What is the speed of the waves in the string?
   (b) What is the distance between the nodes of the standing wave?
   (c) What is the maximum displacement of the string from its rest position?
   (d) What is the wavelength of the standing wave?
   (e) How many times per second is the string straight?
12. Explain what is meant by the expression 'interference pattern' when applied to two sound sources that are in phase.
13. A wave of wavelength λ passes through a gap of width w in a barrier. How will the following changes affect the amount of diffraction that occurs?
   (a) λ decreases
   (b) λ increases
   (c) w decreases
   (d) w increases.
14. Present diagrammatically (on graph paper) the following two transverse waves (that are initially in phase) and add the waves to produce a resultant wave.
   Wave 1: wavelength 2 cm, amplitude 1 cm
   Wave 2: wavelength 4 cm, amplitude 2 cm

15. Present, as diagrams on graph paper, the following two transverse waves (that are initially out of phase) and add the waves to produce a resultant wave.
    Wave 1: wavelength 2 cm, amplitude 1 cm
    Wave 2: wavelength 4 cm, amplitude 2 cm
16. Does the amplitude of a wave affect its speed through a medium?
17. The voice of a person who has inhaled helium sounds higher than normal. Why does this happen?

PRACTICAL INVESTIGATIONS

Investigation 8.1: Reflection of pulses in springs

This investigation comprises three different activities for each of which you will need to work in pairs. You will need to use a long slinky spring.

1. One person firmly holds the end of the spring at floor level. The other person then sends a short thensverse pulse down the spring. Sketch the shape and orientation of the spring before and after reflection.
2. One person loosely attaches the end of the spring to a metal bar (such as a retort stand), or uses a piece of string to support one end of the spring. This will model reflection from a free end. The other person sends a short transverse pulse down the spring. Sketch the shape and orientation of the spring before and after reflection.
3. One person holds the end of the slinky firmly at floor level. The other person sends a short longitudinal pulse down the spring. Record your observations for this activity.

Investigation 8.2: Thin soap films

For this investigation you will need the following equipment:

- glass or beaker
- soap solution.

The interference effect of light from oil on water can be observed in the kitchen. Soap bubbles often have a coloured appearance, which can be enhanced and observed in the following way.

Prepare a soapy solution. Take the glass or beaker and put the open end in the water, then take it out to see if a soap film fills the opening. If not, try again.

Hold the glass or beaker up so that the soap film is vertical. Place yourself so that there is a dark background behind the beaker and a source of light behind or near you, shining on the soap film.

You should now be able to observe the bands of colour in the soap film, like those figure 8.32. The bands will appear because the soap is flowing to the bottom of the film, making it wedge shaped — thin at the top and thick at the bottom. As the soap falls to the bottom, the thickness of the film changes and so the colours change.

As with the oil film, the light is being reflected off the front and back surfaces of the soap film. As the thickness of the film changes, the path difference changes and different colours will be reinforced or cancelled.

FIGURE 8.32

Note that at the top, where the film is very thin and there is a very small path difference, the film is black, not white as you would expect. This is because cancellation occurs in this part of the film. When the light is reflected off a material with a higher refractive index, a change of phase occurs; that is, an incoming crest is reflected as a trough. This change of phase occurs when the light enters the soap film. It does not happen at the back surface, where the light travelling in soapy water meets air on the other side. When the two reflected waves meet after a very short path difference, they cancel.

Investigation 8.3: Diffraction of waves in a ripple tank

1. Set up a ripple tank with a plane wave generator. Introduce a barrier into the ripple tank that allows waves to pass by its edge, and investigate the effects that wavelength has on the amount of diffraction that occurs. Sketch your findings, clearly showing the shadow region.
2. Now set up a barrier with a gap in its centre at right angles to the direction of wave propagation. Study the effect that varying the gap size for a constant wavelength has on the amount diffraction. Then study the effect that varying the wavelength (while keeping the gap size constant) has on the amount of diffraction that occurs.

   Summarise your results.

TOPIC 9 Sound waves

9.1 Overview

9.1.1 Module 3: Waves and thermodynamics

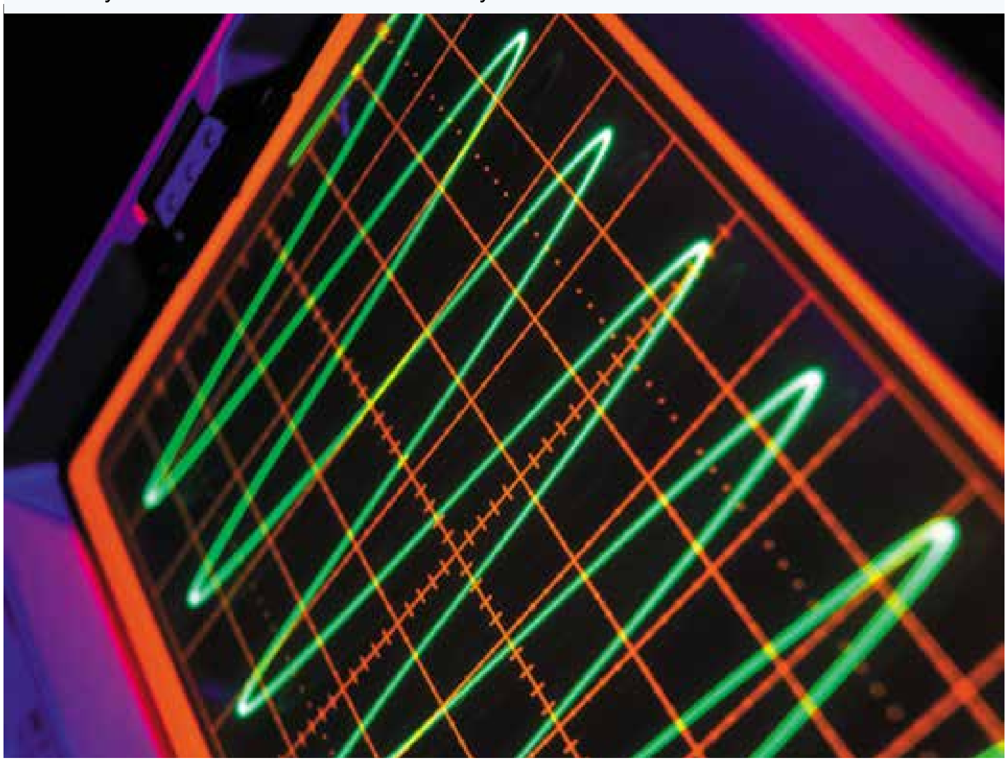
Sound waves

Inquiry question: What evidence suggests that sound is a mechanical wave?

Students:

- conduct a practical investigation to relate the pitch and loudness of a sound to its wave characteristics
- model the behaviour of sound in air as a longitudinal wave
- relate the displacement of air molecules to variations in pressure (ACSPH070)
- investigate quantitatively the relationship between distance and intensity of sound
- conduct investigations to analyse the reflection, diffraction, resonance and superposition of sound waves (ACSPH071)
- investigate and model the behaviour of standing waves on strings and/or in pipes to relate quantitatively the fundamental and harmonic frequencies of the waves that are produced to the physical characteristics (e.g. length, mass, tension, wave velocity) of the medium (ACSPH072)
- analyse qualitatively and quantitatively the relationships of the wave nature of sound to explain:
  - beats ($f_{beat} = |f_2 - f_1|$)
  - the Doppler effect $f' = f\dfrac{(v_{wave} + v_{observer})}{(v_{wave} - v_{source})}$

FIGURE 9.1 A cathode-ray oscilloscope (CRO) is an electronic device that can be used to study sound waves. The CRO enables you to see sound waves.



9.2 Sound: Vibrations in a medium

9.2.1 What is sound?

Sound is created when a vibrating object causes particles in a medium to be alternately pushed closer together (compression) and spread further apart (rarefaction). When an object such as a tuning fork, a drum membrane or a loudspeaker vibrates, it transfers some of its kinetic energy to the medium that carries the sound wave or vibrational energy away from the source in the form of longitudinal waves. The longitudinal waves move outwards from the object much as the ripples in a pond spread outwards from the point where you have dropped a stone into it.

The drum is a good example of a device acting as a source of vibrational energy. As shown in figure 9.2, the back and forth vibrations of the drum skin (note the red line on the surface of the drum skin in the figure) produce differences in air pressure. The varying air pressure produces a vibration effect in the air particles that results in zones of high air pressure (**compression**) and zones of low air pressure (**rarefaction**).

When the vibrations strike the human ear, the eardrum (under the influence of the very small increases and decreases in air pressure that make up the sound wave) will undergo forced vibration. The vibration in the eardrum, in turn, causes fine structures in the inner ear to vibrate. The neural signals sent from these receptors travel along the auditory nerve to the brain, which interprets the signals as sound.

FIGURE 9.2 Production of a sound wave in air by the vibrating skin of a drum.

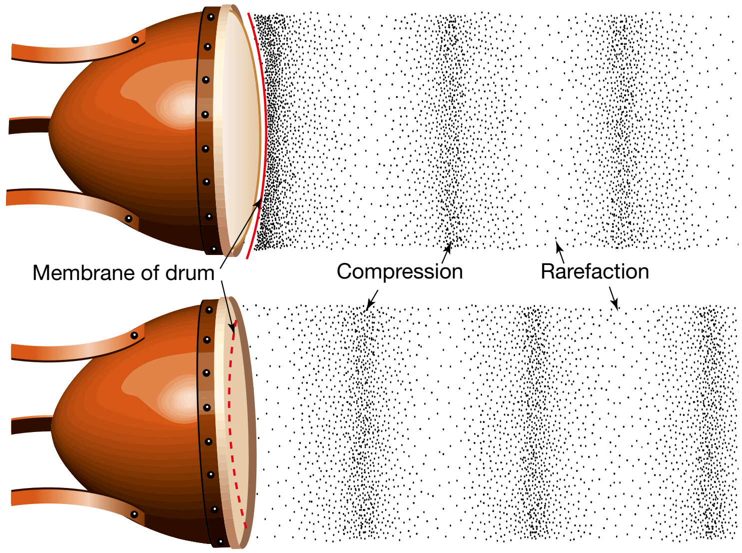
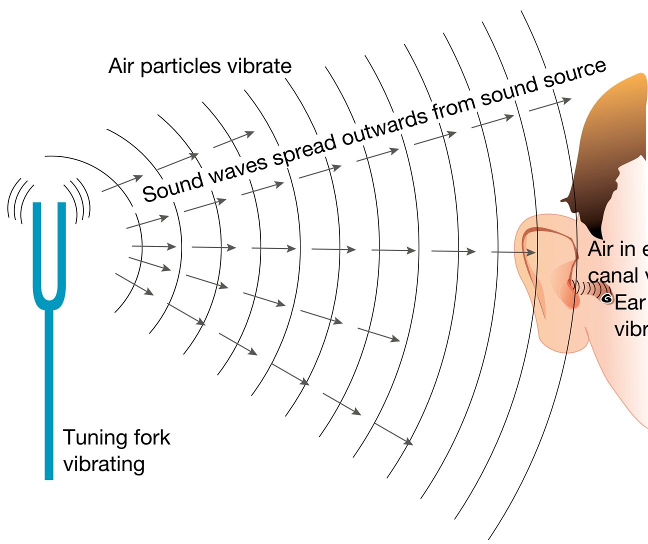


FIGURE 9.3 How we are able to hear a vibrating object.



9.2.2 Looking at sound

For ease of interpretation, sound waves are often shown diagrammatically as transverse wave traces with either time or distance from the sound source on the horizontal axis, and pressure on the vertical axis. As the compressions of longitudinal sound waves involve higher particle pressure than normal in the medium, they are represented as peaks on the transverse graph. Conversely, the rarefactions of a sound wave at which the medium pressure is lower than normal are shown as troughs on the transverse graph.

Devices such as cathode-ray oscilloscopes (CROs) and software applications such as Audacity®, Raven or Garage Band display sound waves in the form of transverse amplitude–time graphs. The amplitude of the sound wave may indicate the maximum air pressure or the maximum intensity of the sound wave. The devices and apps work in a similar way. The microphone attached to the CRO, laptop, tablet or phone converts the sound wave energy into an electrical signal. The size of the electrical voltage induced at the microphone is a function of the pressure of the air striking the microphone diaphragm. The pressure differential changes the voltage to a higher or lower value as it passes into the device. The voltage input registers on the screen as the amplitude of a waveform over time — providing the trace of the sound signal. The period is the time it takes for the signal to complete one cycle.

FIGURE 9.4 The relationship between compressions and crests and rarefactions and troughs. Notice how the crest of the transverse wave occurs at the centre of the compression where the pressure is at a maximum and the trough of the transverse wave occurs at the centre of the rarefaction where the pressure is at a minimum.

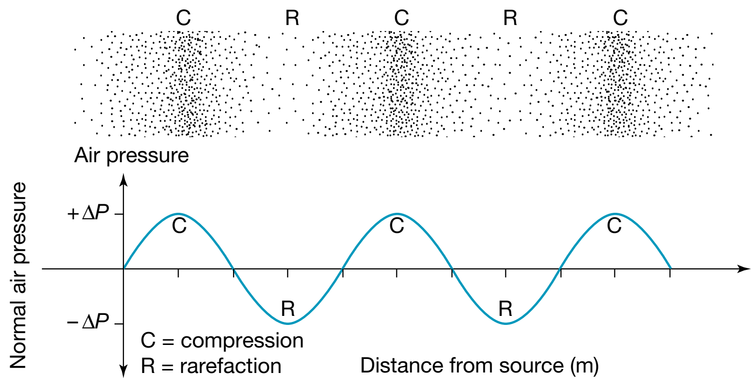
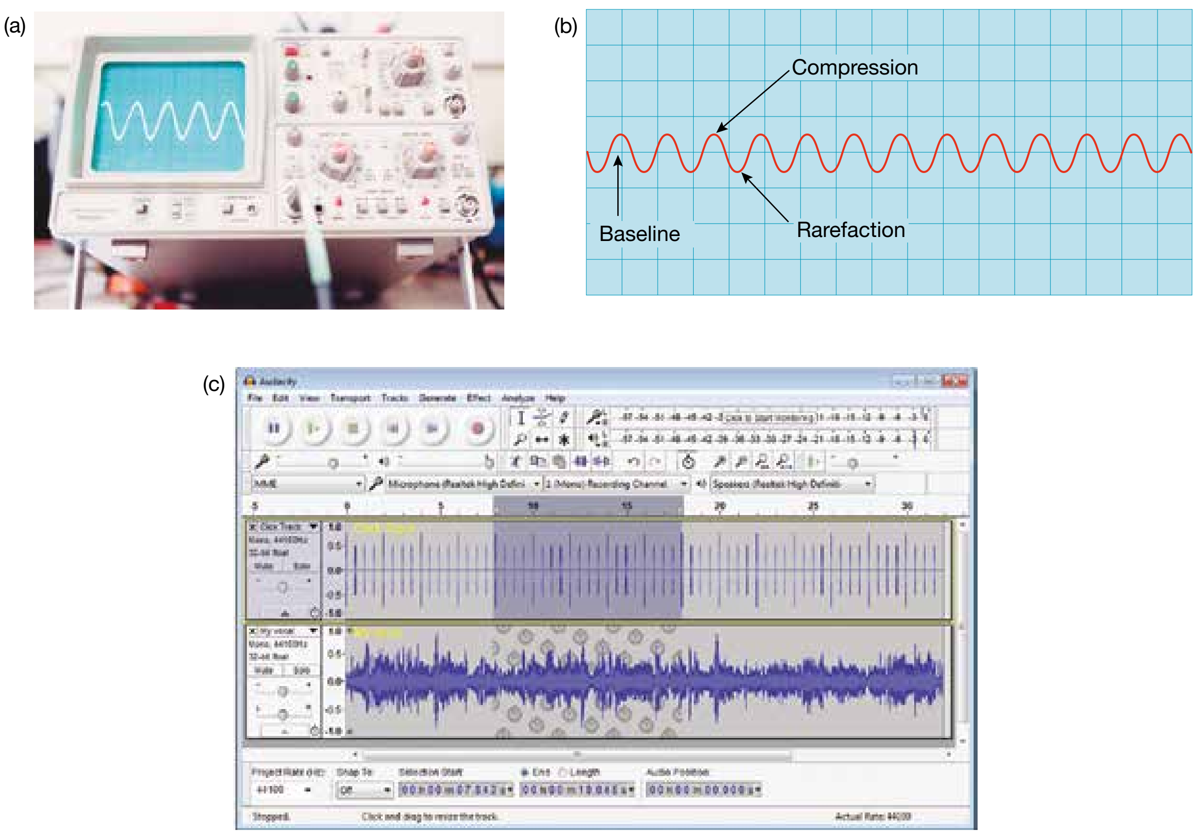
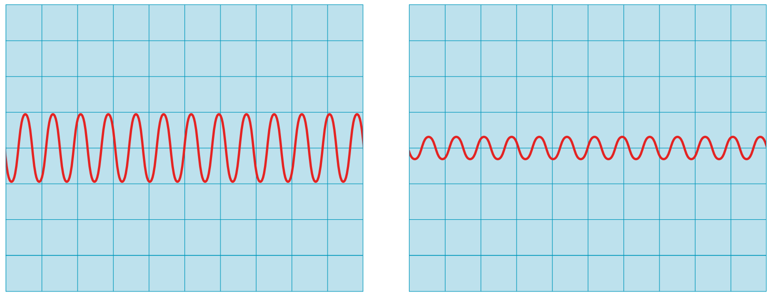


FIGURE 9.5 **(a)** A cathode-ray oscilloscope. **(b)** A CRO trace of sound from a tuning fork, which produces a sound wave of a single frequency. Each horizontal grid division represents a unit of time. **(c)** Audacity® screenshot showing a human voice; this is a complex transverse wave made up of many different frequency sounds superimposed.



If a sound wave of a particular frequency, for example, one generated by a tuning fork, is brought near the microphone and the tuning fork emits a loud sound due to a hard strike, the amplitude of the sound trace produced is much greater than if a soft sound is emitted by the tuning fork (see figure 9.6). This is because the voltage (electrical energy) induced in the microphone is much greater and in proportion with the energy of the loud sound wave. The soft sound wave would produce a much lower amplitude trace. It takes more energy to produce a large amplitude sound of the same frequency.

FIGURE 9.6 Two traces produced by sounds of the same frequency from a single tuning fork. The trace on the left was from a loud sound, hence the amplitude is quite large, whereas the sound that produced the trace on the right was much quieter and hence has a lower amplitude. The time base represented by horizontal grid divisions on the figures is the same.

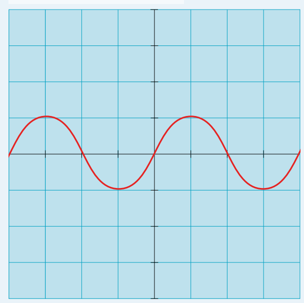


9.2 SAMPLE PROBLEM 1

Figure 9.7 shows the trace on a CRO screen produced by a microphone detecting a sound. The time scale is 1 cm equals 2 ms.

(a) What is the period of the sound?
(b) Sketch the trace produced by a sound of twice the frequency.
(c) Sketch the trace produced by a sound with the original frequency, but with twice the pressure variation.

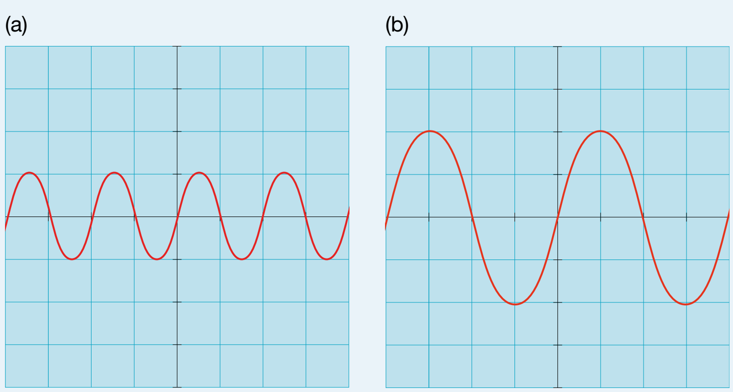
FIGURE 9.7



SOLUTION:

(a) One complete cycle is 4 cm on the screen; multiplying this by the time scale gives a period of 8 ms.
(b) Doubling the frequency halves the period, so the trace shown in figure 9.8a is obtained.
(c) Doubling the pressure variation will double the amplitude of the trace, so the trace shown in figure 9.8b is obtained.

FIGURE 9.8



9.2.3 The speed of sound and the physical characteristics of a medium

Sound travels through different media at different speeds. Generally, the speed of sound through a medium depends upon how close together the particles of that medium are and the elastic properties of that medium. The more easily that collisions between particles can occur (the process that transfers kinetic energy), the faster a sound wave can travel through the medium. As a result, sound will travel faster through solids than through liquids, and faster through liquids than through gases. Table 9.1 shows the speed at which sound travels through different media.

The temperature of the medium also affects the speed with which sound waves can travel through it. As can be seen in table 9.1, sound travels at 331 m s^{-1} through dry air at a temperature of 0 °C, but it travels at 344 m s^{-1} if the air is 20 °C. In other words, the higher the air temperature, the faster the sound will travel.

TABLE 9.1 The speed of sound in different media.

Medium	Speed of sound (m s^{-1})
Air (0 °C)	331
Air (20 °C)	344
Water (pure)	1498
Sea water	1540
Alcohol	1207
Blood (37 °C)	1570
Body tissue (37 °C)	1570
Aluminium	5100
Copper	3900
Concrete	4500
Granite	5000
Lead	1960
Glass (Pyrex)	5170
Iron	5120
Steel	4700–6000 (average 5400)
Wood	4000–5300

PHYSICS FACT

We usually speak about the speed of sound in dry air because, while an increase in air humidity causes an increase in the speed of sound, the size of that increase is so small that, for most purposes, it can be ignored.

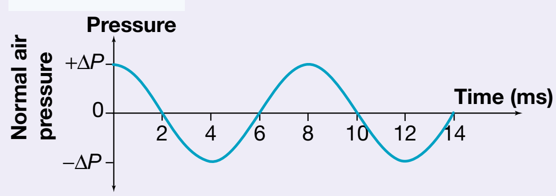
9.2 Exercise 1

1. A siren produces a sound wave with a frequency of 587 Hz in air. Calculate the speed of sound if the wavelength of the sound is 0.571 m.
2. What is the frequency of the sound depicted in Figure 9.9?
3. What will be the frequency of a sound wave that travels through copper if it has a wavelength of 6 m?
4. Sound waves produced by a tuning fork in air at 20 °C have a frequency of 512 Hz. If they then travel from air through Pyrex glass, what is:
   (a) the frequency of the sound waves
   (b) wavelength of the sound waves?
5. The speed of sound in dry air can be determined using the equation

   $$v_T = 331\sqrt{\frac{T}{273} + 1}$$

   where T is the air temperature in °C.
   Use this equation to calculate
   (a) the speed of sound in dry air at –5 °C
   (b) the temperature at which sound will travel through dry air with a speed of 350 m s^{-1}.
6. Raul hears thunder rumble nine seconds after he saw the lightning flash that caused it. How far away from Raul did the lightning strike?

FIGURE 9.9



9.3 Describing sound

9.3.1 Sound intensity

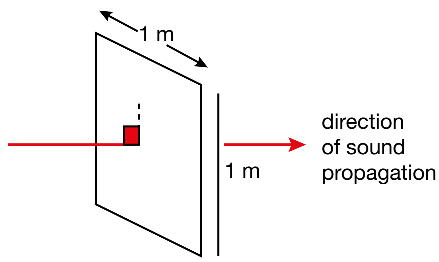
It's a common thing for someone to be playing music at what they think is a respectable volume in their bedroom only to have their parents bang on the door saying, 'It's too loud! Turn it down!'

But how do we describe the loudness of a sound? The term 'loudness' is a subjective measurement. Some people have sensitive ears and find nearly everything too loud, while others may be quite unresponsive to a large amount of noise around them. As a result, it is more useful to consider the intensity of a sound when we discuss loudness.

FIGURE 9.10 'What do you mean, "It's too loud"?'

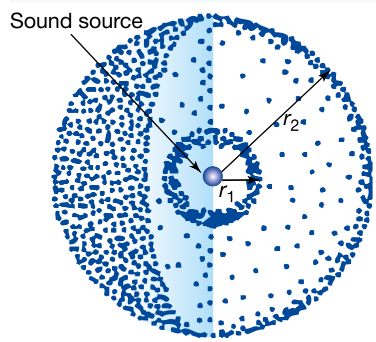
The **intensity** (I) of a sound wave is a measure of the amount of energy that it is able to transfer to a square metre of surface in a 1 second interval of time. The intensity of a sound that you experience depends upon how far away from the sound source you are. The noise of a plane flying overhead at a height of 8 kilometres is nowhere near as loud as the sound of that plane taking off from the runway when you are 400 metres away.

FIGURE 9.11 The area is at right angles to the direction of propagation.



As we have already mentioned, sound waves transfer energy from the sound source through the medium. The amount of sound energy in joules being produced by a source every second is called the **acoustic power** (P) of the source. As $P = \frac{E}{t}$, then the unit for power is the J s^{-1}. This unit is referred to as a watt (W). Sound waves travel from the source in three dimensions with the vibrational energy effectively distributed over the surface of a sphere of ever-increasing radius.

FIGURE 9.12 Sound waves travel outwards from a sound source in three dimensions.



Intensity is equal to the amount of power that falls on a 1 m^2 area of these ever-increasing spheres, so it is measured in W m^{-2}.

At 1 m away from a sound source, the intensity will be equal to the power of the source divided by the surface area of the 1 m radius sphere that the power is distributed over at that point:

$$\begin{aligned} I_1 &= \frac{P}{4\pi(1)^2} \\ &= \frac{P}{4\pi} \end{aligned}$$

At 2 m away, the same amount of power is distributed over a bigger sphere, and the intensity at 2 m is:

$$\begin{aligned} I_2 &= \frac{P}{4\pi(2)^2} \\ &= \frac{P}{16\pi} \end{aligned}$$

As you can see, the intensity of sound experienced 2 m from the source is one-quarter of the intensity at 1 m. At 3 metres away, the intensity would be one-ninth of that for 1 m, and so on. In general, the intensity of sound experienced a distance d from a sound source of power P is such that:

$$I = \frac{P}{4\pi d^2}$$

9.3 SAMPLE PROBLEM 1

What is the intensity of a sound if 6.0×10^{-3} W of acoustic power passes through an open window that has an area of $0.30\,\text{m}^2$?

SOLUTION:

$$
\begin{aligned}
I &= \frac{P}{A} \\
&= \frac{6.0 \times 10^{-3}\,\text{W}}{0.30\,\text{m}^2} \\
&= 2.0 \times 10^{-2}\,\text{W m}^{-2}
\end{aligned}
$$

9.3 SAMPLE PROBLEM 2

Karen measures the sound intensity at a distance of 5.0 m from a lawnmower to be $3.0 \times 10^{-2}\,\text{W m}^{-2}$. Assuming that the lawnmower acts as a point sound source and ignoring the effects of reflection and absorption, what is the total acoustic power of the mower?

SOLUTION:

$$
\begin{aligned}
\boldsymbol{P} &= 4\pi\, \boldsymbol{r}^2 \boldsymbol{I} \\
&= 4\pi\,(5.0\,\text{m})^2 \times 3.0 \times 10^{-2}\,\text{W m}^{-2} \\
&= 9.4\,\text{W}
\end{aligned}
$$

Referring back to the formula for the sound intensity produced by a source, $I = \dfrac{P}{4\pi r^2}$, it can be seen that, for a particular sound source, the sound intensity it produces is inversely proportional to the square of the distance from the source.

$$I \propto \frac{1}{r^2}$$

This is the inverse square law, which can be restated as: *the intensity of sound is inversely proportional to the square of the distance from the source.*

When comparing the sound intensities at two distances r_1 and r_2 from a source, it should be remembered that the power of the source is a constant. Therefore, $P = 4'' r_1^2 I_1 = 4'' r_2^2 I_2$. This relationship then gives the following useful formula:

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$$

9.3 SAMPLE PROBLEM 3

If the sound intensity 3.0 m from a sound source is $4.0 \times 10^{-6}\,\text{W m}^{-2}$, what is the intensity at (a) 1.5 m and (b) 12 m from the source?

SOLUTION:

(a)
$$
\begin{aligned}
r_1 &= 3.0\,\text{m} \\
I_1 &= 4.0 \times 10^{-6}\,\text{W m}^{-2} \\
r_2 &= 1.5\,\text{m} \\
I_2 &= ?
\end{aligned}
$$

$$
\begin{aligned}
\frac{I_2}{I_1} &= \frac{r_1^2}{r_2^2} \\
I_2 &= \frac{I_1 r_1^2}{r_2^2} \\
&= \frac{4.0 \times 10^{-6}\,\text{W m}^{-2} \times (3.0\,\text{m})^2}{(1.5\,\text{m})^2} \\
&= 1.6 \times 10^{-5}\,\text{W m}^{-2}
\end{aligned}
$$

(b) $r_1 = 3.0\,\text{m}$
$I_1 = 4.0 \times 10^{-6}\,\text{W m}^{-2}$
$r_2 = 12\,\text{m}$
$I_2 = ?$

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$$

$$I_2 = \frac{I_1 r_1^2}{r_2^2} = \frac{4.0 \times 10^{-6}\,\text{W m}^{-2} \times (3.0\,\text{m})^2}{(12\,\text{m})^2} = 2.5 \times 10^{-7}\,\text{W m}^{-2}$$

AS A MATTER OF FACT

Loudspeakers for entertainment systems are rated in watts. For example, a system might be fitted with 40 W speakers. In this case, 40 W does not refer to the acoustic power produced by the speakers, but to the maximum electrical power dissipated by the speakers. Under normal operating conditions, the acoustic power produced will be much less than the stated power rating.

9.3.2 Sound intensity level

While the human ear is able to pick up sounds with an intensity as small as $10^{-12}\,\text{W m}^{-2}$, it is less sensitive to larger intensities of sound. Doubling the intensity of a fairly soft sound is a lot more noticeable to our ears than doubling the intensity of a loud one.

As a result, measurement of intensity alone is not a good indication of how loud a noise is. However, this sensitivity to lower intensities can be compensated for by comparing the logarithm of the ratio of sound intensities, called the **sound intensity level** (L).

The sound intensity level, measured in decibels (dB), is a comparison of the intensity of the sound (I) compared to the softest sound audible (I_o), which is $10^{-12}\,\text{W m}^{-2}$. Sound intensity levels can be found using the equation:

$$L = 10 \log\left(\frac{I}{I_o}\right)$$

Using this sound scale, the softest noise able to be heard has a sound intensity level of 0 dB. In order to double sound intensity level, you need to increase sound intensity by a factor of 100.

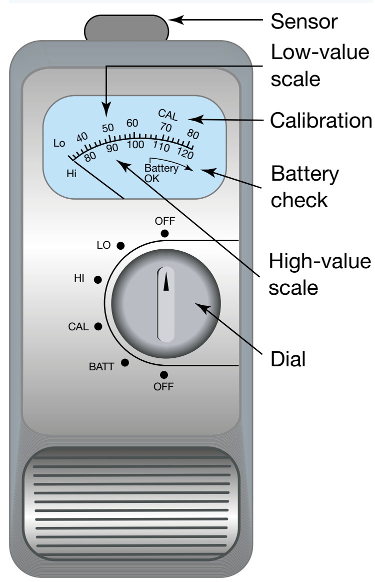
Table 9.2 contains some typical sound levels.

TABLE 9.2 Typical sound intensity levels.

Sound source	Sound intensity level (dB)*
Jet take-off at 30 m	125
Loud indoor rock concert	120
A shout at 1.5 m	100
City traffic, pneumatic drill	80
Interior of a car moving at 90 km h^{-1}	75
Classroom during an experiment	65
Normal conversation at 1 m	60
Quiet bedroom at night	30
Whisper	20

* Unconsciousness can occur at 140 dB, the threshold of pain is 120 dB, and the threshold of hearing is 0 dB.

FIGURE 9.13 A sound level meter measures the sound intensity level.



9.3 SAMPLE PROBLEM 4

What is the sound intensity level of a sound of intensity $2.6 \times 10^{-7}\,\text{W m}^{-2}$?

SOLUTION:

$$
\begin{aligned}
L &= 10 \log_{10} \frac{I}{I_o} \\
&= 10 \log_{10} \frac{2.6 \times 10^{-7}\,\text{W m}^{-2}}{1.0 \times 10^{-12}\,\text{W m}^{-2}} \\
&= 10 \log_{10} 2.6 \times 10^{5} \\
&= 54\,\text{dB}
\end{aligned}
$$

9.3 SAMPLE PROBLEM 5

What is the change in intensity level when a sound intensity is doubled? In this case, $I_2 = 2I_1$.

SOLUTION:

$$
\begin{aligned}
\Delta L &= 10 \log_{10} \frac{I_2}{I_1} \\
&= 10 \log_{10} \frac{2I_1}{I_1} \\
&= 10 \log_{10} 2 \\
&= 3.01\,\text{dB}
\end{aligned}
$$

9.3 Sample problem 5 gives a useful rule of thumb: *if the sound intensity doubles, the sound intensity level increases by 3 dB; if the sound intensity halves, the sound intensity level decreases by 3 dB*. In fact, each 3 dB increase in the sound intensity level requires a factor of 2 increase in the sound intensity.

9.3 SAMPLE PROBLEM 6

The recommended listening distance for music produced by a 40 W speaker is a minimum of 2 m. What is the maximum sound intensity level that a listener at this distance can be exposed to?

SOLUTION:

$$
\begin{aligned}
I &= \frac{40\,\text{W}}{4\,\pi\,(2\,\text{m})^2} \\
&= 0.80\,\text{W m}^{-2} \\
L &= 10 \log \left(\frac{I}{i_o}\right) \\
&= 10 \log \left(\frac{0.80\,\text{W m}^{-2}}{10^{-12}\,\text{W m}^{-2}}\right) \\
&= 119\,\text{dB}
\end{aligned}
$$

9.3.3 Pitch and frequency

Just as loudness is used to qualitatively describe sound intensity, the term **pitch** provides a general indication of a sound's frequency. The higher the frequency of the sound, the more vibrations per second and the higher the pitch; a low-frequency sound is low pitched. If two different frequency (Hz number) tuning forks, producing sounds of equivalent amplitude, are used to produce graphical traces under identical conditions, the amplitudes of the trace waves are equal but the frequency is higher for the higher pitched sound.

FIGURE 9.14

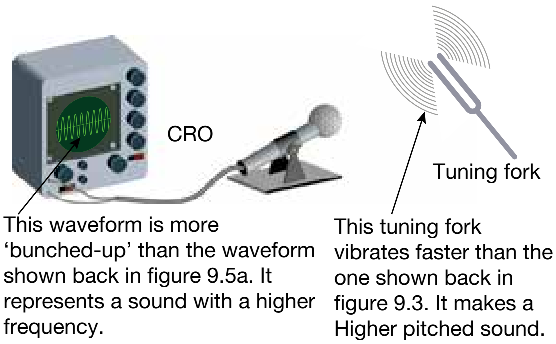
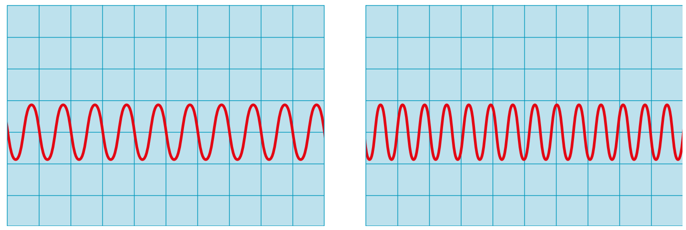


FIGURE 9.15 The traces of two sound waves with identical loudness (amplitude) but different frequencies. The figure on the left shows a low-pitch, lower frequency sound wave trace while the figure on the right shows a high-pitch, higher frequency sound wave trace. The time base represented by horizontal grid divisions on the figures is the same.



AS A MATTER OF FACT

Having 'perfect pitch' is the ability to precisely name the note associated with a heard frequency, such as the hum produced by a ceiling fan or the vibration from a car fanbelt. In years past, a basic job requirement for piano tuners was to have perfect pitch.

At the other end of the scale are people who are tone-deaf. Such people are completely unable to distinguish between sounds differing in frequency; however, very few people are truly tone-deaf. In reality, most people lie between the two extremes of perfect pitch and tone-deafness and can be trained to distinguish pitch more precisely.

WORKING SCIENTIFICALLY 9.1

Use a sound level meter to test the effectiveness of several different devices that are designed to protect hearing. Examine the implications of your results in terms of workplace health and safety practices.

9.3.4 The Doppler effect

If you have ever been passed by a car with its horn blaring, you no doubt would have noticed that the pitch of the sound made by the horn seems to drop as the car approaches, comes level with you and then moves away. Similarly, when people imitate the sound of a fast passing car, they automatically change the pitch of the sound from high to low. This apparent shift in frequency of a sound source is called the Doppler effect. First described by Christian Johann Doppler in 1842, the Doppler effect is the result of the movement of the wave source and/or the observer of the wave relative to the wave medium.

As an example, consider a train whistle that produces a sound with a frequency of 260 Hz. When the train is stopped at a station, the whistle is sounded to alert passengers of its imminent departure. People on the platform ahead of the train, behind the train or even on the train will all hear the whistle at its normal frequency of 260 Hz.

Assuming a speed of sound of $340\ \text{m s}^{-1}$, the wavelength of the whistle can be found using the wave equation:

$$\begin{aligned}\lambda &= \frac{v}{f}\\ &= \frac{340\ \text{m s}^{-1}}{260\ \text{Hz}}\\ &= 1.31\ \text{m}\end{aligned}$$

The period of the whistle (i.e. the time that elapses between successive compressions reaching the observer) can be calculated as the reciprocal of the frequency:

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{260\ \text{Hz}} \\ &= 0.0038\ \text{s or } 3.8\ \text{ms} \end{aligned}$$

Now, let us consider the train in motion along the tracks at a constant speed of 20 m s^{-1} (72 km h^{-1}). As the train approaches a railway crossing, the whistle is sounded. The train driver, who is at rest relative to the train, hears the whistle at its usual 260 Hz. But what pitch is heard by the driver of a car that is stopped at the railway crossing?

Each of the compressions produced by the whistle radiates outwards at the speed of sound (340 m s^{-1}) and there is a time interval of 0.0038 seconds between the production of compressions. However, during this time interval, between the production of one compression and the next, the train (and its whistle) have moved a small distance closer to the crossing at a speed of 20 m s^{-1}:

$$\begin{aligned} d &= v\,t \\ &= 20\ \text{m s}^{-1} \times 0.0038\ \text{s} \\ &= 0.076\ \text{m} \end{aligned}$$

The compressions reaching the driver of the car at the crossing are therefore separated by a distance of 1.31 − 0.076 m = 1.234 m. As the compressions reaching the car driver are 1.234 m apart, the effective wavelength of the whistle as he hears it is also 1.234 metres. As a result, the frequency he hears can be calculated:

$$\begin{aligned} f_{car\ driver} &= \frac{v}{\lambda_{car\ driver}} \\ &= \frac{340\ \text{m s}^{-1}}{1.234\ \text{m}} \\ &= 276\ \text{Hz} \end{aligned}$$

FIGURE 9.16 When the train is stationary relative to the observers, the distance between successive compressions produced by the train whistle is the same for all observers. As a result, all stationary observers hear the same whistle frequency.

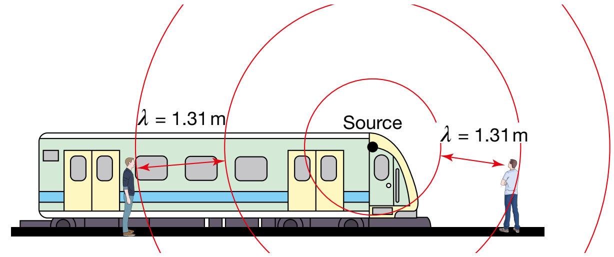
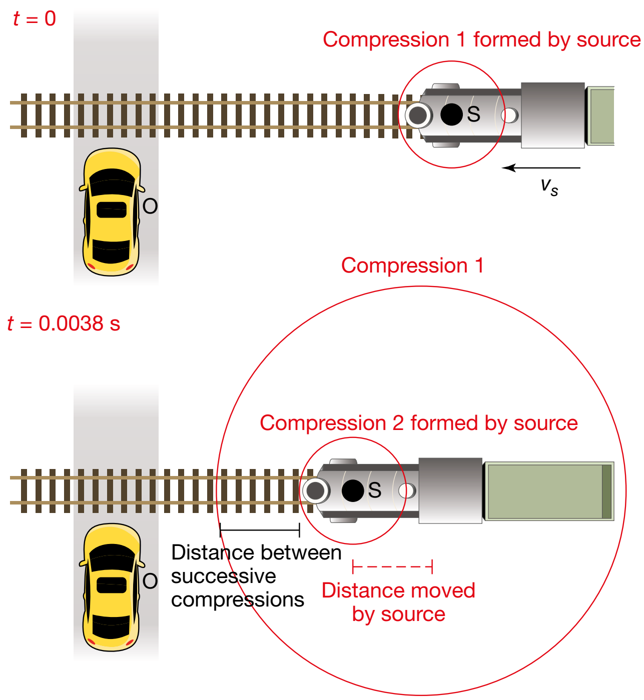


FIGURE 9.17 During the time interval between successive compressions being formed by the train whistle (source S), the train itself has moved closer to the car driver (observer O). This decreases the effective wavelength observed by the car driver, increasing the effective frequency of the train whistle.



This means that the driver of the car perceives the pitch of the approaching train whistle to be higher than its actual frequency of 260 Hz.

When the train moves past the crossing, the whistle moves away from the car driver and he again hears a different pitch than that of the train driver.

Now, during the 0.0038 s between the production of successive compressions by the whistle, the train travels 0.076 m further away from the driver, with the result that the distance between successive compressions reaching the car driver is equal to 1.31 m + 0.076 m = 1.386 m.

With this increase in the effective wavelength of the whistle as experienced by the car driver, there is a corresponding decrease in the whistle frequency:

$$
\begin{aligned}
f_{car\ driver} &= \frac{v}{\lambda_{car\ driver}} \\
&= \frac{340\ \text{m s}^{-1}}{1.386\ \text{m}} \\
&= 245\ \text{Hz}
\end{aligned}
$$

We can use our train whistle example to come up with a general formula for the Doppler effect relating the speed of sound in the medium (v), the speed of the source relative to a stationary observer (v_S), the frequency of the sound produced by the source (f) and the effective frequency of the sound heard by the observer (f').

As the source (whistle) approaches the observer (car driver), the effective wavelength λ' can be determined from the wavelength produced at the source (λ) and the decrease in wavelength produced by the movement of the source towards the observer:

$$
\begin{aligned}
\lambda' &= \lambda - \Delta\lambda \\
&= \frac{v}{f} - \frac{v_S}{f} \\
&= \frac{v - v_S}{f}
\end{aligned}
$$

This means that the effective frequency heard by the observer, f', can be described:

$$
\begin{aligned}
f' &= \frac{v}{\lambda'} \\
&= \frac{v}{\frac{v - v_S}{f}} \\
&= \frac{vf}{v - v_S}
\end{aligned}
$$

Similarly, when the source is moving away from the stationary observer and $\lambda' = \lambda + \Delta\lambda$:

$$
\lambda' = \frac{v + v_S}{f}
$$

and

$$
\begin{aligned}
f' &= \frac{v}{\lambda'} \\
&= \frac{v}{\frac{v + v_S}{f}} \\
&= \frac{vf}{v + v_S}
\end{aligned}
$$

Notice that the effective frequency will always be heard as higher when the sound source approaches the observer, and lower as it moves away.

9.3 SAMPLE PROBLEM 7

A noisy truck approaches a stationary pedestrian operating a frequency meter. The truck motor roars at a frequency of 2000 Hz as it approaches the pedestrian, and 1500 Hz as it moves away. What is the speed of the truck relative to the pedestrian? Take the speed of sound in air to be 340 m s^{-1}.

SOLUTION:

Using the Doppler formulae, as the truck approaches, the effective frequency is:

$$
\begin{aligned}
f' &= \frac{vf}{v - v_S} \\
2000\ \text{Hz} &= \frac{340\ \text{m s}^{-1} f}{340\ \text{m s}^{-1} - v_S}
\end{aligned}
$$

As the truck recedes, the effective frequency is:

$$f' = \frac{vf}{v + v_S}$$

$$1500\,\text{Hz} = \frac{340\text{ m s}^{-1} f}{340\text{ m s}^{-1} + v_S}$$

We now have two equations for f and v_S. We can solve them for v_S by dividing the first equation by the second to eliminate f:

$$\frac{2000}{1500} = \frac{340 + v_S}{340 - v_S}$$

$$\frac{4}{3} = \frac{340 + v_S}{340 - v_S}$$

$$4(340 - v_S) = 3(340 + v_S)$$

$$340 = 7v_S$$

$$v_S = 48.6\text{ m s}^{-1}$$

The Doppler effect is observed whenever the observer, the source or both move relative to the medium. In general, if both the source and the observer are moving towards each other relative to the medium, then the effective frequency heard by the observer is described by:

$$f' = f\frac{(v + v_o)}{(v - v_s)}$$

where v is the speed of sound in the medium, v_o is the speed of the observer, v_s is the speed of the source and f is the frequency of sound produced by the source.

Note that, in the case where a moving source approaches a stationary observer, $v_o = 0$ and so:

$$f' = f\frac{(v + 0)}{(v - v_s)}$$

that is,

$$f' = \frac{fv}{(v - v_s)}$$

as we saw earlier.

If both the source and the observer are moving away from each other relative to the medium, then the effective frequency heard by the observer is described by:

$$f' = f\frac{(v - v_o)}{(v + v_s)}$$

9.3 SAMPLE PROBLEM 8

The horn on a car travelling east at 14 m s^{-1} along a straight road produces a sustained sound of 500 Hz as the driver approaches a jogger travelling west at 3 m s^{-1}. What does the jogger perceive the frequency of the horn to be when the car has driven past her? Assume the speed of sound is 340 m s^{-1}.

SOLUTION:

In this case, neither the observer nor the source are at rest relative to the medium (air) and, when the car has passed the jogger, they are moving away from each other. The effective frequency of the horn as heard by the jogger will be:

$$f' = f\frac{(v - v_o)}{(v + v_s)}$$

$$f' = 500\text{ Hz} \times \frac{(340\text{ m s}^{-1} - 3\text{ m s}^{-1})}{(340\text{ m s}^{-1} + 14\text{ m s}^{-1})}$$

$$= 476\text{ Hz}$$

WORKING SCIENTIFICALLY 9.2

By considering the motion of an observer and a source relative to a stationary medium, derive the equation $f' = f\dfrac{(v - v_o)}{(v + v_s)}$ given that the object and the source are moving away from each other.

WORKING SCIENTIFICALLY 9.3

Standing on a safe place on the kerb, record the sound of a car as it approaches and passes you. Use a sound analysis program such as Audacity® to identify the frequency shift of the car, and thus estimate the car's speed.

9.3 Exercise 1

1 Calculate the sound intensity level of a whisper.
2 A scream at a distance of 1 m has a sound intensity level of 120 dB — twice that of normal conversation levels. How many times greater is the sound intensity of a scream compared to a chat?
3 A particular noise has a sound intensity 1000 times that of the threshold of hearing. What is this sound's intensity level in dB?
4 The intensity of sound experienced 2 m from a sound source is 4 W m^{-2}. How far from the sound source will the intensity be 1 W m^{-2}?
5 How much power is produced by a sound source if an intensity of 2 mW m^{-2} is experienced 3 m away from it?
6 A window has an area of 0.50 m^2, and 4.5×10^{-4} J of energy passes through the window in 30 seconds. Calculate (a) the acoustic power of the sound and (b) the sound intensity at the window.
7 A soccer coach yelling at his team solidly for 20 s produces sound at an intensity of 0.2 mW m^{-2}. How much sound energy per square metre does the coach expend over this time interval?
8 Thuy and her friend Emily both go to a rock concert. Thuy is in the mosh pit located 5 m away from the front of a 450 W speaker while Emily is further back from the same speaker where the intensity of sound that she experiences is half that experienced by Thuy. How far away is Emily from Thuy?
9 The driver of a car travelling east along a road at 60 km h^{-1} hears a police siren although he cannot tell whether the police car is behind him or in front of him. Having perfect pitch, the driver identifies the pitch of the siren to be 528 Hz although he knows that the frequency of a stationary police siren is 500 Hz. At what speed is the police car travelling if it is travelling along the road (a) behind the driver but in the same direction, or (b) in front of the driver but in the opposite direction?

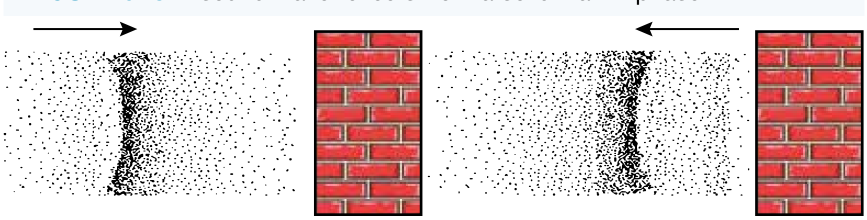
9.4 Reflection of sound waves

9.4.1 Bouncing back!

When waves arrive at a boundary between different media, reflection occurs, causing the return of the wave into the medium from which it was originally travelling. In topic 8, we saw how transverse waves travelling along strings were reflected, depending on the condition the wave met at the string's end. A transverse wave reaching a fixed end is reflected back inverted along the string (and, so, out of phase with the original wave), while a wave reflected from an end that is free to move is not inverted (therefore, no change of phase occurs).

Sound waves, despite being compressional waves, can also be reflected; however, their reflection behaviour differs from that of the transverse waves travelling along strings. When a sound wave encounters a rigid medium such as a cliff wall, it is reflected back from the wall without a change of phase. This means that a compression hitting the wall bounces off as a compression, while rarefactions are reflected as rarefactions.

FIGURE 9.18 A sound wave reflects from a solid wall in phase.



9.4.2 Echoes

An echo occurs when an incident sound wave is reflected repeatedly.

When an echo bounces back from a solid surface, such as a cliff face or a brick wall, you don't hear the full sound, but you do hear the last part of the original sound. If you are a significant distance from the wall, you will hear more of the original sound bounce back. If you are close to the reflecting surface, you probably won't detect an echo. It does still occur, but the original sound drowns it out. There needs to be a time difference between the reflected sound and the original sound so that you can hear the echo. The size of that time difference is a minimum 0.1 seconds. Because sound travels around 340 m s^{-1} in air, both you and the sound must be at least 17 metres from the surface reflecting the sound for you to hear the echo. At this distance, the sound wave takes 0.05 seconds to reach the reflecting surface from the second source and 0.05 seconds to bounce back.

9.4.3 Echolocation

Echoes are used by sonic rangers to determine the distance to objects. In the water, sonar and depth finders on boats are used to determine the distance to objects underwater or to the floor of the ocean (see figure 9.19). Sonic rangers are also used by industry in sonic level controllers to tell how full storage tanks are. In most of these applications it is desirable to use short-wavelength, high-pitch sound waves. These ultrasonic or very high-frequency sound waves are emitted from a source and bounce back from objects. After bouncing back, they are detected by pressure-sensitive detectors. The time they take to return to their source can be determined accurately. Knowing the time for the reflected wave or echo to be received allows the distance to the object to be calculated. When calculating the distance, the speed of sound in a medium is assumed to be constant (even though slight fluctuations in the speed of sound do occur as the density changes).

Because there is a time difference between reflections of the same pulse if the reflecting surface is irregular in shape, it is possible to use ultra-high-frequency sound waves to 'see' objects. The reflections from multiple surfaces are processed by a computer to generate an image of the object's surface. This technology is used extensively in medicine to

FIGURE 9.19 Depth finders on boats use sound echoes to measure the depth to the ocean floor or to other objects or fish.

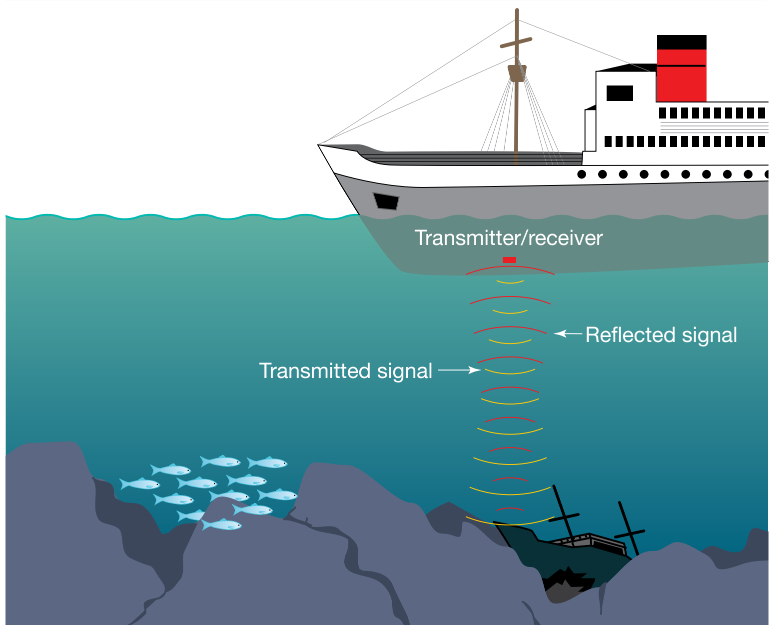
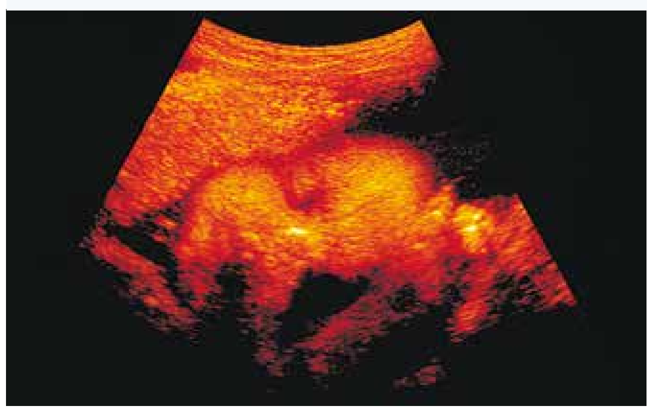


FIGURE 9.20 An ultrasound image of a baby in the womb.



perform non-invasive examinations of soft tissue injuries and diseased organs, or to check unborn children for abnormalities (see figure 9.20).

9.4.4 Reverberation

Acoustics is the scientific study of sound. It has many applications; in architecture and engineering, for example, acoustics can be used to explain how the characteristics of spaces affect sounds within them.

The inside of a concert hall looks quite different from a simple school hall. This is because the surfaces, fittings and even seating of the concert hall have all been designed to make the music played on stage as clear as possible. You will not see flat walls or ceilings or too many hard surfaces, as these have a tendency to cause **reverberation**. This is an effect created when the audience hears a noticeable lag between the played note ending and the dying away of that note.

On the other hand, too many soft surfaces can absorb sound, making it 'acoustically dead'. This can be an advantage in a recording studio where some rooms need to be completely soundproofed, so that the frequency and quality of the sound heard by the performer, the backing musicians and the sound technician are exactly the same. Many of these rooms are lined in heavily textured padding or even the bumpy bottoms of egg cartons, so that there is no spurious reflection or resonance of sound waves.

FIGURE 9.21 This theatre hall in the Sydney Opera House has been designed so that everyone in the audience will be able to hear clearly. The angled surfaces visible in the figure help to achieve this.

The quality of a concert hall can be evaluated in terms of its **reverberation time**, which is the time that elapses between the ending of a note and for the sound level of that note's echoes from around the room to cease being heard. This is usually taken to be when the echoes have a sound level intensity less than 60 dB.

The formula to calculate the reverberation time of a performance space was first derived by Wallace Sabine in 1898 and is still in use today. He proposed that the reverberation time of a space (T_R) was directly related to the volume of the space (V) and inversely proportional to its effective absorbing surface area (A):

$$\boldsymbol{T}_{\mathrm{R}} = \frac{0.161\,\boldsymbol{V}}{\boldsymbol{A}}$$

While it is fairly easy to calculate the volume of a hall, it is not quite so easy to assess how well the surfaces in that hall absorb sound. To calculate the effective absorbing surface area of a performance space, the effective absorbing surface of every fixture in that space must be taken into account by assigning each of the surfaces (curtains, chairs, floors, ceiling, walls and so on) an absorption coefficient, which will be slightly different for sounds of differing frequency range. The absorption coefficient is the proportion of sound that is absorbed by that surface. Glass in a window, for example has an absorption coefficient of 0.18 for 500 Hz; this means that it absorbs 18% of sound at 500 Hz that falls on it. Thick carpet, on the other hand, has an absorption coefficient of up to 0.60 — absorbing 60% of sound.

The total effective absorbing surface area (A) is the sum of each of the individual effective absorption areas for each object and fitting in the hall. For example, a 400 m^2 wooden floor that has an absorption coefficient of 0.10 for 500 Hz sounds would have an effective absorption area equal to $400 \times 0.10 = 40\ m^2$.

Table 9.3 indicates the absorption coefficients for common building materials when exposed to sound with a frequency of 500 Hz (which lies in the middle of the frequency range for the voice and for musical instruments).

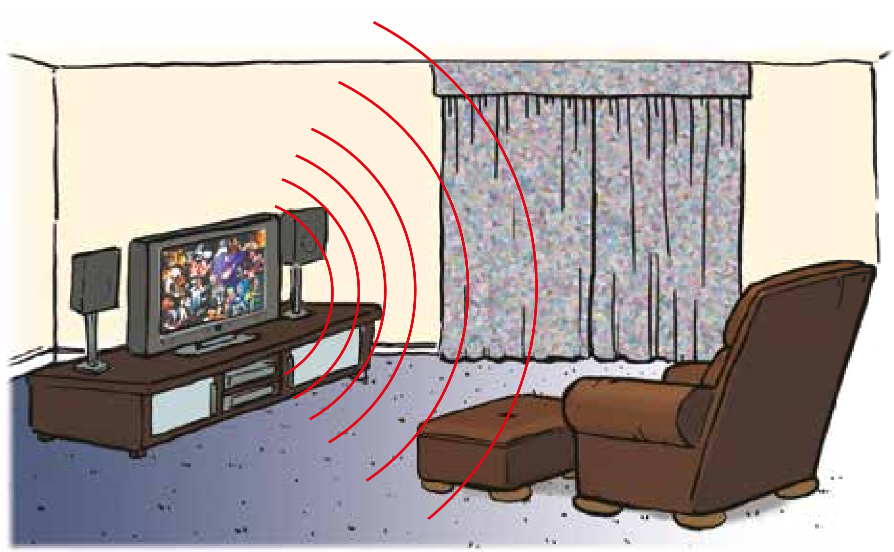
TABLE 9.3 Absorption coefficients for a variety of building materials (500 Hz)

Material	Absorption coefficient
Window glass	0.18
Plaster	0.06
Wood panelling	0.17
Wooden floor	0.10
Carpet laid on concrete	0.14
Carpet laid on underlay	0.57
Lightweight curtains	0.11
Heavy curtains	0.55
Painted concrete blocks	0.06
Unpainted concrete blocks	0.31
Acoustic tile (suspended)	0.83
Acoustic tile (fixed to concrete)	0.76

The best reverberation time for a concert hall depends upon the type of performance being given. Speech sounds best in a hall that has a reverberation time of between 0.4 to 0.8 s, while halls with times of between 1.0 and 2.0 s are generally better for music. A symphony hall for a full orchestra is best with a time of between 1.7 and 2.0 s.

A performance space is considered to have good acoustics if there are no noticeable echoes, the loudness of all sounds is uniform throughout the space, and it does not allow noise from the outside world to be heard inside.

FIGURE 9.22 The contents and structure of a room will affect its acoustic properties.



9.4 SAMPLE PROBLEM 1

Estimate the reverberation time (at 500 Hz) for a small, empty bedroom.

The wooden floor is 5.0 by 5.0 m, three of the walls are plastered and the ceiling (also plastered) is 2.8 m high. The fourth wall is made entirely of glass. (The door, in one of the plastered walls, may be assumed to have the same absorption coefficient as the wall.)

SOLUTION:

First, we will need to calculate the individual effective absorption areas:

wooden floor: $A_1 = (5.0 \times 5.0) \times 0.1 = 2.5\,\text{m}^2$

glass wall: $A_2 = (5.0 \times 2.8) \times 0.18 = 2.52\,\text{m}^2$

3 × plastered walls: $A_3 = 3 \times (5.0 \times 2.8) \times 0.06 = 2.52\,\text{m}^2$

ceiling: $A_4 = (5.0 \times 5.0) \times 0.06 = 1.5\,\text{m}^2$

The total effective absorption surface area (A) can be found:

$$\begin{aligned} A &= A_1 + A_2 + A_3 + A_4 \\ &= 2.5 + 2.52 + 2.52 + 1.5 \\ &= 9.04\,\text{m}^2 \end{aligned}$$

The volume of the bedroom (V) is easily found:

$$\begin{aligned} V &= 5.0 \times 5.0 \times 2.8 \\ &= 70\,\text{m}^3 \end{aligned}$$

Therefore, we can find the reverberation time of the bedroom:

$$\begin{aligned} T_R &= \frac{0.161(70)}{(9.04)} \\ &= 1.25\,\text{s} \end{aligned}$$

WORKING SCIENTIFICALLY 9.4

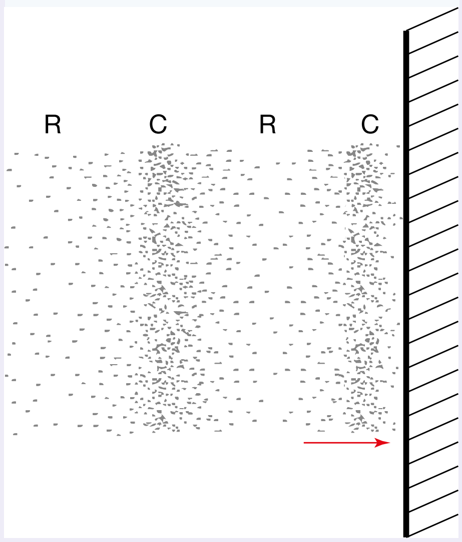
Calculate the reverberation time for several different spaces in your school, such as the auditorium, classrooms and laboratories. Compare their reverberation times and consider whether they are consistent with the function of the space. In cases where they are not suitable, suggest ways of improving their acoustic properties.

9.4 Exercise 1

1 A sound wave approaches a rigid surface as shown in figure 9.23.
Sketch the reflected wave.

2 While standing in a canyon, you hear the echo of your voice 2.8 s later. How far away is the rock wall responsible for this echo if the speed of sound is assumed to be 344 m s^{-1}?

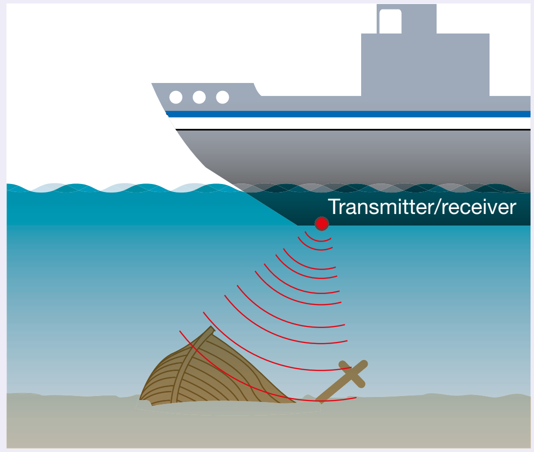
FIGURE 9.23 A compression approaches a rigid surface.



3 Sonar can be used to map the ocean floor or to detect objects within the water. Figure 9.24 shows sonar being used by a salvage ship to find sunken wrecks.

If 0.17 s elapse between the sound waves being emitted and the reflected waves from the wreck being detected, how far below the sonar transmitter/receiver is the wreck? The speed of sound in sea water is $1540\ \text{m s}^{-1}$.

FIGURE 9.24



4 The Cheese Puff Café has a high reverberation rate of 1.2 s when fully booked, which means that the customers find it very noisy and they have difficulty hearing their dining partners. It has been suggested to the owners that replacing the concrete floor with carpet might help reduce this effect. Given that the volume of the café is $90\ \text{m}^3$ and the concrete floor has an area of $30\ \text{m}^2$, by what percentage would the reverberation rate drop if carpet and underlay were put down?

5 A starter pistol is fired into the air at a race meet. The starter is located 200 m from the east wall of the stadium and 500 m from the stadium's western wall. What will be the time interval between the echoes from each wall being heard by the starter?

6 In an experiment to measure the speed of sound in different metals, a sound pulse was sent through identical lengths of aluminium and copper. The experimenter noted that the pulse took 0.3 ms longer to travel through the copper sample than through the aluminium sample. If sound travels through aluminium at $5100\ \text{m s}^{-1}$ and through copper at $3900\ \text{m s}^{-1}$, how long were the samples?

9.5 Superposition of sound

9.5.1 Interference of sound waves

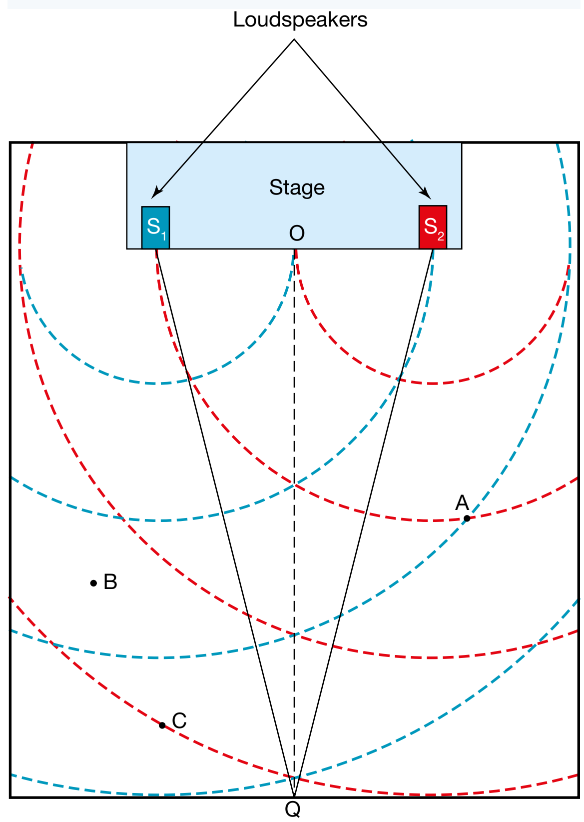
When two sources emit sound at the same frequency and in phase, an interference pattern is produced. The pattern is three-dimensional, but its features are the same as for interference patterns modelled two-dimensionally.

Consider what happens when a sound generator (such as a microphone or signal generator) is connected to two loudspeakers that are positioned at the left and right ends of a stage at the front of a hall, as shown in figure 9.25. Although the loudspeakers are in phase (that is, they both produce a compression at the same time and a rarefaction at the same time), there will be positions in the hall where the combined sound they produce will be very loud and others where the sound will be very soft.

S1 and S2 represent the loudspeakers on the stage, and the concentric circles that are centred on each of the speakers represent the positions of compressions at a particular moment in time. The distance between successive compressions will be equal to the wavelength of the sound produced at the speakers. Points along the central line OQ are equidistant from S_1 and S_2.

Local antinodes, or **maxima**, are points at which constructive interference between the waves produced by the

FIGURE 9.25 Loudspeakers set up in a hall.



two speakers creates a sound of greater intensity than that created by one speaker alone. Points A and B in figure 9.25 are both maxima, with the waves from S_1 and S_2 arriving in phase and interfering constructively. At point A, compressions from S_1 and S_2 coincide, while, at point B, two rarefactions are coincident.

Local nodes, or **minima**, are points where destructive interference produces a sound with a much lower intensity than that produced by one source alone. Point C in figure 9.25 is such a point: compressions from one speaker coincide with rarefactions from the other speaker and vice versa. As the waves pass through these minima, there is very little variation in the air pressure, resulting in a very soft sound.

The same formulas that were used in section 8.2.3 to describe constructive and destructive interference in waves in general can be applied to sound waves and used to predict whether a point is part of a nodal or antinodal region and, therefore, whether the sound heard at that point would be especially softer or louder. For a point, P, to be an antinode, the path difference between the lines connecting each of the two sources to P must be a whole number multiple of the wavelength:

$$d(\mathrm{PS}_1) - d(\mathrm{PS}_2) = n\lambda \quad \text{for } n = 0, 1, 2, 3, 4, \ldots$$

where n is the number of the antinodal region from the centre of the pattern, and S_1 and S_2 are the sound sources.

In figure 9.25, it can be seen that point A is 3 wavelengths away from S_1 and 2 wavelengths from S_2, a path difference of 1 wavelength. Point B is 2.5 wavelengths from S_1 and 3.5 wavelengths from S_2, again providing a path difference of 1 wavelength. Both A and B lie on the first antinodal lines either side of the central antinodal line OQ.

For a point to be a node, the path difference is an odd numbered multiple of half the wavelength:

$$d(\mathrm{PS}_1) - d(\mathrm{PS}_2) = \left(n - \frac{1}{2}\right)\lambda \quad \text{for } n = 1, 2, 3, 4, \ldots$$

where n is the number of the nodal line obtained by counting outwards from the centre line.

The nodal point C is located 4 wavelengths from S_2 and 3.5 wavelengths from S_1, giving a path difference of half a wavelength. It lies on the first nodal line away from the central antinodal line OQ.

Although we have modelled the interference of two sources in two dimensions where a central antinodal line OQ is observed, along which all points are equidistant from S_1 and S_2, we should remember that, in reality, interference between two sound sources occurs in a three-dimensional space. As a result, the points that are equidistant from each sound source lie on a central antinodal plane rather than a line.

AS A MATTER OF FACT

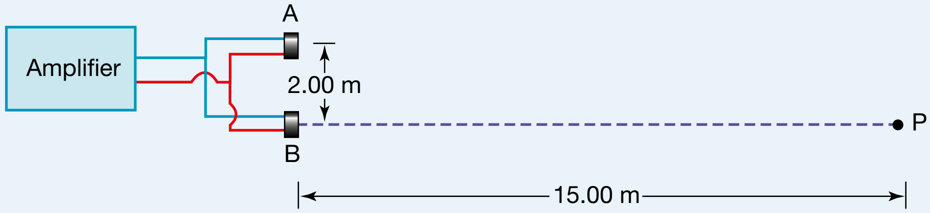
Complete destructive interference rarely occurs as the sounds produced from each source are usually not of equal intensity, due to the different distances travelled by the individual waves and the inverse square law that describes this variation in intensity with distance from the source.

9.5 SAMPLE PROBLEM 1

A student arranges two loudspeakers, A and B, so that they are connected in phase to an audio amplifier. The speakers are then placed 2.00 m apart and they emit sound that has a wavelength of 0.26 m.

Another student stands at a point P, which is 15.00 m directly in front of speaker B. The situation representing this arrangement is shown in figure 9.26. Describe what the student standing at point P will hear from this position.

FIGURE 9.26



SOLUTION:

In this type of question, it is important to determine whether the point is a node or antinode. This is done by determining the path difference and then comparing this to the wavelength.

$\lambda = 0.26\,\text{m},\ d(\text{PB}) = 15.00\,\text{m}$

$d(\text{PA})$ can be found by applying Pythagoras's theorem.

$$d(\text{PA})^2 = 15.00\,\text{m}^2 + 2.00\,\text{m}^2$$
$$= 229\,\text{m}^2$$

So $d(\text{PA}) = 15.13\,\text{m}$

$$d(\text{PA}) - d(\text{PB}) = 15.13\,\text{m} - 15.00\,\text{m}$$
$$= 0.13\,\text{m}.$$
$$0.13\,\text{m} = \frac{1}{2}\lambda$$

Therefore, the student is at a local minimum and will hear only a very soft sound.

WORKING SCIENTIFICALLY 9.5

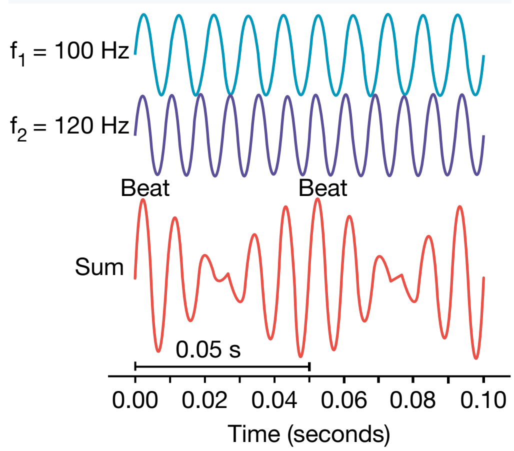
Draw a plan of your school assembly hall and mark the location of the speakers. Draw nodal and antinodal lines on your plan, for sound waves from the speakers having a frequency of 200 Hz (the mid range of human speech) to determine where constructive and destructive interference occurs within your hall.

9.5.2 Beats — a special case of superposition

When two sources of sound of the same amplitude but slightly different frequency are heard together, there will be a rhythmic change to the volume of the sound. When the two sound waves are in phase, the amplitude of the resulting sound wave is the sum of the amplitudes of the two waves, and results in a loud sound. As the waves drift out of phase, the resultant amplitude will become smaller, eventually reaching zero before increasing again as the waves drift back into phase. The term '**beats**' is used to describe the variation in the loudness of the sound.

For example, beats may occur when members of an orchestra are warming up for a performance and are tuning their instruments. As the tuning of the instruments becomes closer, the beat frequency decreases until it disappears. The beat frequency is determined by the difference in the frequency of the notes played by the different instruments when slightly out of tune.

FIGURE 9.27 Two sound waves of differing frequency being played at the same time, and the resulting superposition.



That is, $f_{beat} = |f_2 - f_1|$

In figure 9.27, we can see the traces of two sound waves of differing frequency being played at the same time, and the resulting superposition.

For the two sounds, one at 100 Hz and the other at 120 Hz, the beat frequency is simply found:

$$\begin{aligned} f_{beat} &= 120\,\text{Hz} - 100\,\text{Hz} \\ &= 20\,\text{Hz} \end{aligned}$$

So, every 0.05 s, a louder beat tone will be heard.

9.5.3 Timbre

Timbre: Combining pure tones

The sound produced by a tuning fork is a pure tone. The CRO trace of such a sound is a sine wave, as shown in figure 9.28. Most sounds are not pure tones but are made up of a number of pure tones that have been superimposed in a particular way to produce a sound with a characteristic **timbre** (see figure 9.29).

FIGURE 9.28 A pure tone produced by a tuning fork. The CRO trace is a sine wave shape.

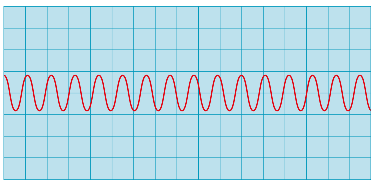


FIGURE 9.29 A complex sound produced by a person singing. Note that in any CRO trace figure, the trace represents a very small 'grab' of time, much like a photograph.

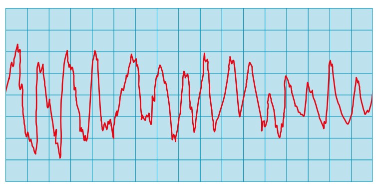
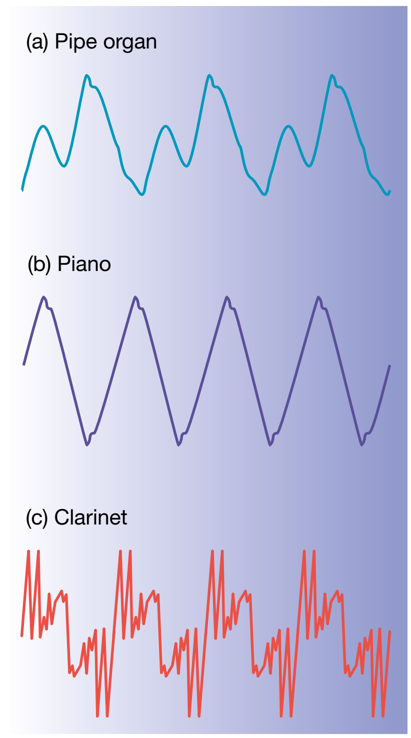


FIGURE 9.30 The CRO traces at the same settings of the same musical note played with approximately the same volume by a variety of different instruments: (a) pipe organ, (b) piano and (c) clarinet. These CRO traces are more complex than the sounds produced by tuning forks and often lack symmetry about the baseline.



Although the shapes of the waves for figures 9.28 and 9.29 are different, the frequencies are approximately the same. The difference is the timbre, or complexity of the note. This is borne out in the difference in shape.

You are probably aware that different musical instruments playing in an orchestra can play the same musical note. However, while the sounds are of the same frequency, they do not appear to be the same. This is because the sounds produced have their own particular timbre. If viewed as a CRO trace, these common notes from the different instruments produce a differently shaped wave trace even though the frequencies are common. Figure 9.30 shows the wave traces for the same note played by a number of different instruments into a microphone.

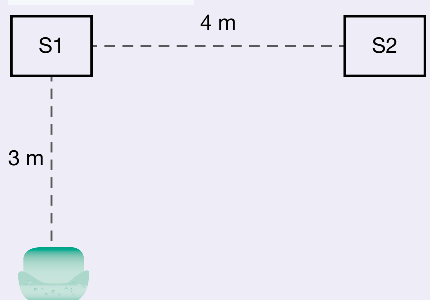
9.5 Exercise 1

1 Two tuning forks with frequencies of 256 Hz and 262 Hz are placed next to each other and struck so that they vibrate. What beat frequency will be heard?

2 While tuning her guitar using a signal generator, a student hears a beat frequency of 4 Hz when the topmost string is plucked at the same time as the generator produces a sound wave with frequency 334 Hz. When she drops the frequency of the signal generator to 323 Hz and plucks the guitar string, she hears a 7 Hz beat frequency. What is the frequency of the guitar string?

3 Two speakers are placed 4 metres apart in Rohith's home theatre and he has placed his chair 3 metres from the front of one of the speakers, as shown in figure 9.31.

FIGURE 9.31



(a) For which of the following frequencies would Rohith be located at an interference maximum? (Assume the speakers produce sound waves in phase and that the speed of sound is 340 m s^{-1}.)
   (i) 170 Hz
   (ii) 255 Hz
   (iii) 510 Hz

(b) Where should Rohith place his chair so that he experiences an interference maximum no matter what frequency the speakers produce?

4 Two cellists sitting 2 metres apart in a practice room are playing the same notes. Will there be places in the room where nothing can be heard because of interference? Explain your answer.

5 Two small portable speakers, S1 and S2, are placed 5.6 m apart. They are in phase and produce sound with a wavelength of 1.4 m. If Astrid stands so that she is 7 metres from S1, what is the minimum distance she must be from S2 if she is to be at (a) a node, or (b) an antinode?

9.6 Sound from strings

9.6.1 Vibrational modes in a string

A string will produce sound if it is held under tension by fixing it at both ends. When the string is made to vibrate by plucking it (such as a guitar string), running a bow across it (a violin string) or striking it with small padded hammers (piano and dulcimer strings), transverse waves are produced, which travel along the string in both directions from the vibration site and are reflected from the fixed ends of the string. When these reflected waves interfere with the waves coming in from the other end of the string, standing waves are produced. As the strings are fixed at each end, nodes (positions at which the string is not displaced) will always form there. Obviously, there are a number of different standing wave combinations that can be formed in the string, having a node at each end. The simplest, shown in figure 9.32, has a single antinode in the middle of the string. This type of standing wave is called the fundamental vibration mode (or the first harmonic). The frequency of the sound waves produced by this vibrational mode is referred to as the **fundamental frequency**.

FIGURE 9.32 The fundamental vibration mode.

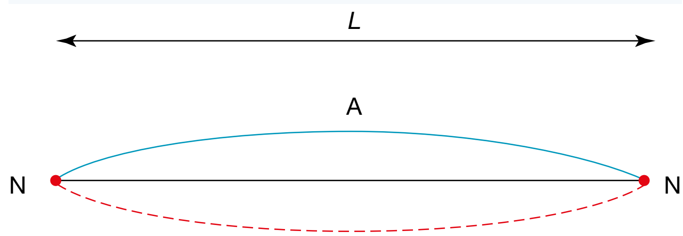
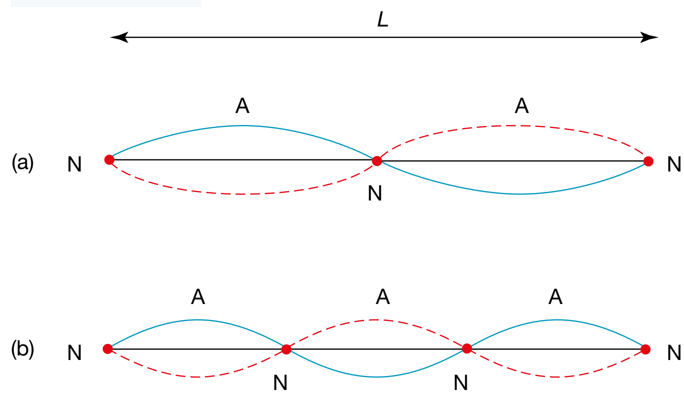


FIGURE 9.33



The wavelength of the fundamental (λ_0) is equal to twice the distance between two successive nodes. In this case, the distance between the nodes is equal to the string length, L. Therefore,

$$\lambda_0 = 2L$$

Substituting this into the wave equation, we derive an expression for the fundamental frequency:

$$f_0 = \frac{v}{2L}$$

where v is the speed of the wave along the string.

A string vibrating in its next vibrational mode, as shown in figure 9.33a, will have two antinodes and three nodes.

The wavelength of such a vibration will equal the length of the string:

$$\lambda_1 = L$$

and the frequency of this second mode of vibration is found:

$$f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{L}$$

Further, by rearranging the equation for the fundamental frequency and substituting for v, we find:

$$f_1 = \frac{2f_0 L}{L}$$

$$f_1 = 2f_0$$

The frequencies produced in a string that are higher than the fundamental frequency are called **overtones**. If the overtone frequency is equal to a whole number multiple of the fundamental frequency, then it is called a **harmonic**. The fundamental frequency, f_0, is therefore the first harmonic while f_1, which is equal to twice the fundamental frequency, is called the second harmonic.

In figure 9.33b, we see the string vibrating in its third vibrational mode, in which it displays three antinodes and four nodes. The wavelength, λ_3, is equal to $\frac{2}{3}L$, and the frequency is found to be:

$$f_2 = \frac{3v}{2L}$$

and

$$f_2 = 3f_0$$

Thus, the third vibrational mode produces the third harmonic.

By now, you can probably see a pattern emerging from all of this. In fact, we can determine that the frequency for any mode of vibration is such that:

$$f_n = \frac{(n+1)v}{2L}$$

where L is the length of the string, v is the speed of the wave along the string and n is the integer number associated with the vibrational mode, where $n = 0, 1, 2$ ...

This means that $n = 0$ for the first harmonic (fundamental), $n = 1$ for the second harmonic, and so on.

A plucked string will vibrate with frequencies such that all harmonics are produced, although the amplitude of the harmonics will decrease as the harmonic number increases. As a result, the frequency heard most loudly is the fundamental of the string.

FIGURE 9.34 The waveform shown is produced by plucking the A string of a guitar. The pattern on the CRO screen results from the superposition of waves with frequencies corresponding to the fundamental and subsequent harmonics.

9.6 SAMPLE PROBLEM 1

What is the frequency of the third harmonic of a string if the fundamental frequency is 250 Hz?

SOLUTION:

The third harmonic, by definition, has a frequency three times the fundamental frequency. Therefore, the answer is 750 Hz.

9.6.2 Wave speed in strings

The speed at which a travelling wave moves along a stretched string depends primarily upon the length of the string and the mass of the string per unit length (also referred to as the string's linear density). The relationship between these factors can be described by the equation:

$$v = \sqrt{\frac{T}{m/L}}$$

where T is the tension (in newtons) in the string and m is the mass in kilograms of the length L (in metres) of the string.

9.6 SAMPLE PROBLEM 2

(a) The lowest E string on a guitar has a fundamental frequency of 82.4 Hz. If the guitar string has a vibrational length of 640 mm, at what speed do waves travel along its length?

(b) The low E string on the average guitar has a mass per unit length of 6.8×10^{-3} kg m^{-1}. What is the tension in the stretched guitar string?

SOLUTION:

(a) In the first vibrational mode,

$$\lambda_0 = 2L = 2(0.64\ \text{m}) = 1.28\ \text{m}$$

Using the wave equation,

$$\begin{aligned} v &= f\lambda \\ &= 82.4\ \text{Hz} \times 1.28\ \text{m} \\ &= 105\ \text{m s}^{-1} \end{aligned}$$

(b) Rearranging the velocity equation for strings, we get:

$$\begin{aligned} T &= v^2\left(\frac{m}{L}\right) \\ &= (105\ \text{m s}^{-1})^2 \times (6.8 \times 10^{-3}\ \text{kg m}^{-1}) \\ &= 75\ \text{N} \end{aligned}$$

WORKING SCIENTIFICALLY 9.6

Simple musical instruments can be made by stretching rubber bands over wooden boxes. Determine if it is possible to make an instrument using a wooden box and rubber bands of a variety of thicknesses with the same acoustical range as (a) a ukulele, (b) a violin, or (c) a guitar.

9.6 Exercise 1

1 A particular guitar string is 60 cm long, and waves move through it at 400 m s^{-1}. Calculate:
(a) the fundamental frequency of the note produced when the string vibrates freely
(b) the fundamental frequency of the note produced when the string is pressed hard against the fingerboard halfway down the string's length
(c) the second harmonic frequency produced by lightly touching the string halfway down the string's length.

2 The A string on a small harp is 0.9 m long and has a fundamental frequency of 440 Hz, while the nearby C string made from the same material has a fundamental frequency of 512 Hz. If the wave speed is the same in both strings, what is the length of the C string?

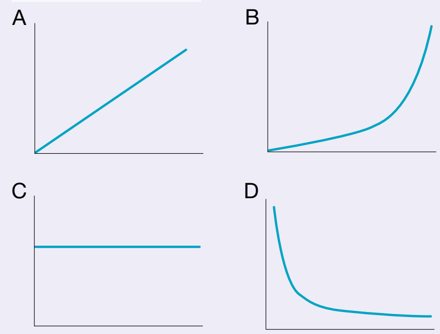
3 What is the frequency of the third harmonic of a string if the fundamental frequency is 250 Hz?

4 A string produces a sound that has a second harmonic of 700 Hz. Calculate:
(a) the fundamental frequency
(b) the frequency of the fourth harmonic of this string.

5 One of the harmonic frequencies of a string fixed at both ends is 375 Hz. The string's next harmonic frequency is 450 Hz. What is the fundamental frequency of the string?

6 Which of the graphs in figure 9.35 best shows the relationship described in each case:
(i) the velocity of a travelling wave in a string with fixed length versus the tension in the wire
(ii) the frequency of the harmonic versus the harmonic number
(iii) frequency versus wavelength for a wave travelling in a string at constant speed?

FIGURE 9.35

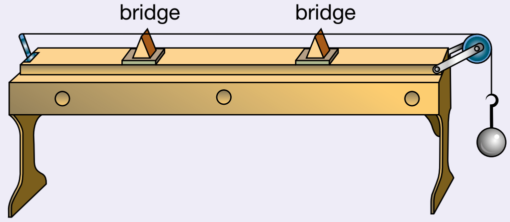


7 The G string on a violin is 30 cm long and vibrates at a fundamental frequency of 196 Hz. How far from the top end of the string must you place your finger to play an A (220 Hz)?

8 The device shown in figure 9.36 is called a monochord, or sonometer.

A string is supported on the two bridges and is tensioned by suspending masses from one end as shown. If the string has a mass per unit length of 2 g m^{-1} and the distance between the bridges is 50 cm, what will be the fundamental frequency of the string if a 3 kg mass is used to tension the string?

FIGURE 9.36 A sonometer.



9.7 Sound from pipes

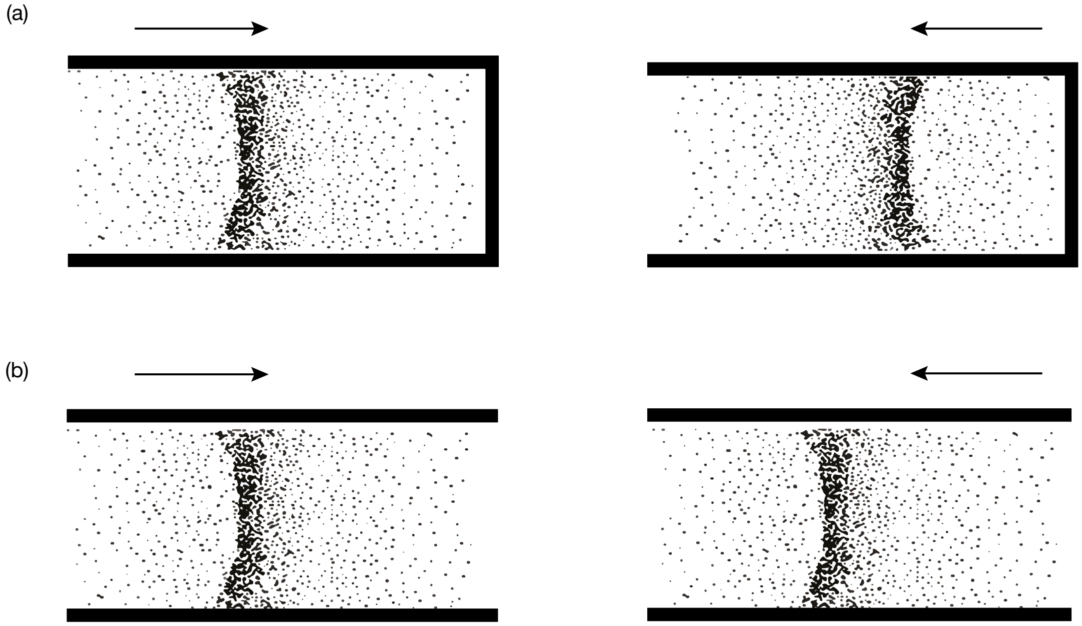
9.7.1 Standing wave formation in pipes

Just as sound can be produced by the vibration of fixed-end strings to form standing waves, it can also be produced by the formation of standing waves in the air cavities of pipes and tubes. Unlike the transverse waves produced in strings, the standing waves in pipes are longitudinal waves produced by reflection that occurs at the ends of the pipes. The standing waves will differ in their geometry according to whether the pipe is open at both ends (referred to as an **open pipe**) or is sealed at one end (**closed pipe**).

A compression travelling from the open end to the closed end of a pipe is reflected back without a phase change, so it returns as a compression. Upon reaching the open end of the pipe, however, the compression is reflected back as a rarefaction. As the waves travel up and down the length of the pipe, the incident and reflected pulses interfere to form a standing wave.

Air particles at the open end of a pipe are able to freely enter and leave, and have the greatest degree of freedom. The air pressure at the open end is the same as the air pressure outside the pipe, and the air

FIGURE 9.37 **(a)** Compressions striking the closed end of a pipe are reflected back as compressions. **(b)** Compressions encountering the open end of a pipe are reflected as rarefactions.



particles maintain a uniform distance between them. As there is no change in air pressure at the open end of a pipe, we say that there is always a **pressure node** located there. At the same time, as the particles are able to move with maximum amplitude, there is always a **displacement antinode** located at the open end.

At the closed end of a pipe, the particles are not able to move as freely, and so it is here that the minimum displacement of air particles occurs. This means that there is a **displacement node** always located at the closed end of a pipe. At the same time, it is at the closed end where incident compressions are reflected as compressions; the waves interfere constructively to create compression waves with twice the amplitude of either wave alone. Similarly, incident rarefactions are reflected as rarefactions and interfere constructively. As the air pressure at the closed end is undergoing maximum change, there is always a **pressure antinode** located there. In general, displacement antinodes occur at the same positions in a pipe as pressure nodes, and displacement nodes occur at the same positions as pressure antinodes.

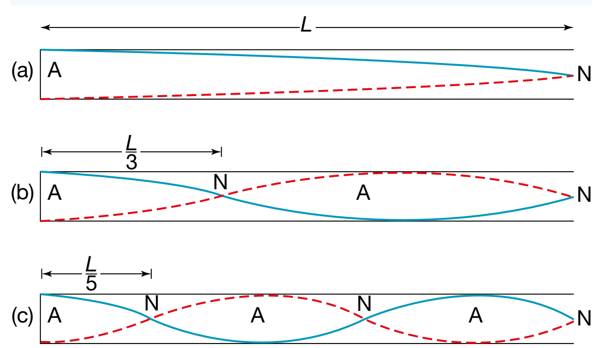
Standing waves in pipes can be represented either in terms of the displacement of particles along the pipe, or as variations in air pressure along the pipe. In our following discussion of resonance in open and closed pipes, we will represent standing waves as variations in air pressure.

9.7.2 Resonance in closed pipes

All the brass instruments in an orchestra as well as instruments such as clarinets, oboes, bagpipes, pipe organs and didgeridoos act as closed pipes, as one end is effectively closed by the player's mouth when the instruments are being played.

As mentioned previously, closed pipes will have a pressure antinode at their closed end and a pressure node at their open end. The fundamental vibrational mode for a closed pipe will have one node and one antinode, as shown in figure 9.38a.

FIGURE 9.38 The first three modes of vibration in a closed pipe: **(a)** the fundamental, **(b)** the second harmonic, and **(c)** the third harmonic.



Remembering that the distance between successive nodes is equal to $\frac{\lambda}{2}$, we can see from our diagram that the fundamental wavelength λ_0 is equal to $4L$, where L is the length of the pipe.

As $v = f\lambda$, we can write an expression for the fundamental frequency f_0 of a closed pipe:

$$f_0 = \frac{v}{\lambda_0} = \frac{v}{4L}$$

The second vibrational mode for a closed pipe has 2 nodes and 2 antinodes (figure 9.38b), and the wavelength is equal to $\frac{4L}{3}$. The frequency f_1 can therefore be derived:

$$f_1 = \frac{3v}{4L}$$

The frequency f_1 is called the first **resonant frequency**, as it is the first frequency above the fundamental frequency at which resonance occurs.

Also, $f_1 = 3f_0$, which can be described as the third harmonic of the closed pipe.

In general, the nth resonant frequency above the fundamental can be expressed as:

$$f_n = \frac{(2n + 1)v}{4L}$$

and

$$f_n = (2n + 1)f_0$$

9.7 SAMPLE PROBLEM 1

(a) What is the fundamental frequency for a pipe closed at one end if it is 0.80 m long and the speed of sound in air is $340\ \text{m s}^{-1}$?

(b) What is the frequency of the third resonant frequency above the fundamental for this pipe?

SOLUTION:

(a) For the fundamental frequency, use $\lambda = 4L$.

$$\lambda = 4 \times 0.80\ \text{m} = 3.2\ \text{m}$$

$$f_0 = \frac{v}{\lambda} = \frac{340\ \text{ms}^{-1}}{3.2\ \text{m}} = 106.25\ \text{Hz}$$

Therefore, the fundamental frequency is 1.1×10^2 Hz.

(b) The third resonant frequency above the fundamental is the seventh harmonic.

$$f_3 = 7f_0 = 7 \times 106.25\ \text{Hz} = 743.75\ \text{Hz}$$

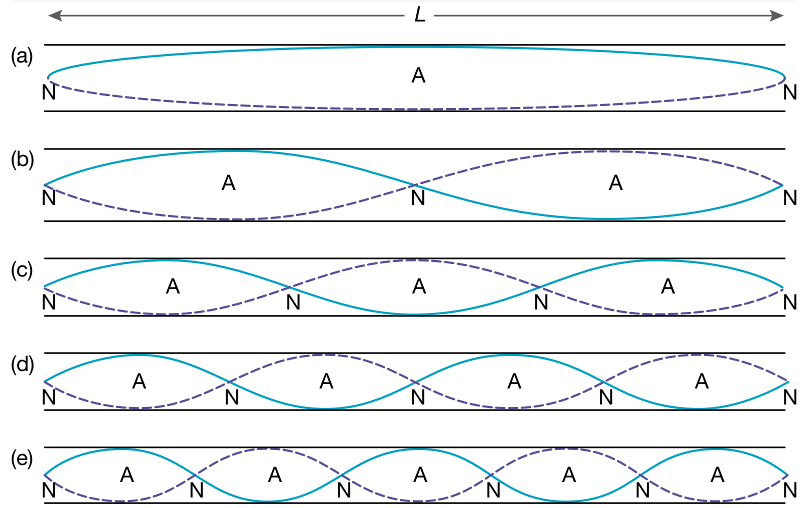
Therefore, the third resonant frequency above the fundamental has a frequency of 7.4×10^2 Hz.

9.7.3 Resonance in open pipes

As both ends of an open pipe allow the free movement of air particles, there is a pressure node at each end. The fundamental mode of vibration for an open pipe will occur when there is an antinode in the centre of the pipe, as shown in Figure 9.39a.

As there are only two nodes in the fundamental mode, we can see that the fundamental wavelength will be equal to twice the length of the pipe: that is, $\lambda_0 = 2L$. Thus, the fundamental frequency $f_0 = \frac{v}{2L}$.

FIGURE 9.39 Standing waves in a pipe open at both ends.



The second vibrational mode occurs when there are two antinodes and three nodes (figure 9.39b); here we can see that the length of the pipe is equal to the wavelength, and so the first resonant frequency above the fundamental can be found:

$$f_1 = \frac{v}{L}$$

As $f_1 = 2f_0$, it is the second harmonic for the open pipe.

Similarly, the second resonant frequency is found to be

$$f_2 = \frac{3v}{2L}$$

and, as $f_2 = 3f_0$, it is also the third harmonic.

In general, the nth resonant frequency above the fundamental for an open pipe is given by:

$$f_n = \frac{n \times v}{2L}$$

and $f_n = (n + 1)f_0$

Note that this is the same relationship as that found for a vibrating string, which, being fixed at each end, also has a node at each end.

9.7 SAMPLE PROBLEM 2

A pipe open at both ends has a length of 40 cm. The speed of sound in air is 340 m s^{-1}. Determine:

(a) the fundamental frequency

(b) the third resonant frequency above the fundamental for this pipe.

SOLUTION:

(a) $L = 40\text{ cm} = 0.40\text{ m}$

$$\begin{aligned}\lambda &= 2L \\ &= 0.80\text{ m}\end{aligned}$$

$$\begin{aligned}f_0 &= \frac{v}{\lambda} \\ &= \frac{340\text{ m s}^{-1}}{0.80\text{ m}} \\ &= 425\text{ Hz}\end{aligned}$$

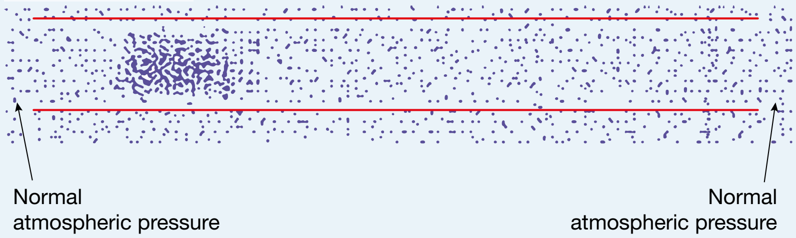
(b) For a pipe open at both ends, the third resonant frequency above the fundamental is the fourth harmonic.

$$\begin{aligned}f_3 &= 4f_0 \\ &= 4 \times 425\text{ Hz} \\ &= 1700\text{ Hz}\end{aligned}$$

9.7 SAMPLE PROBLEM 3

Figure 9.40 shows the pressure variation in a pipe open at both ends. At the instant shown in the figure, the pressure is at its maximum variation from normal pressure. The speed of sound in air is $340\ \text{m s}^{-1}$. The pipe has a length of 0.80 m.

FIGURE 9.40

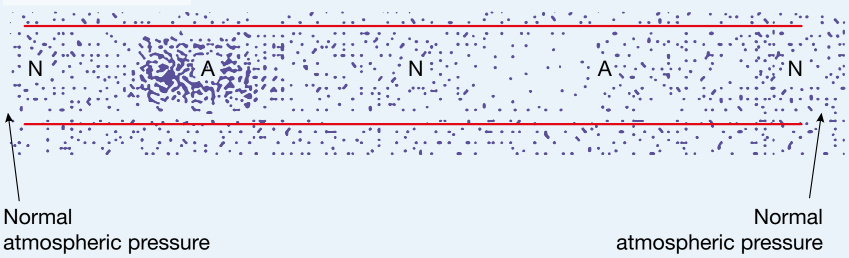


(a) Mark the position of any pressure nodes and antinodes in the pipe.
(b) Sketch a graph showing the variation of air pressure as a function of distance along the pipe:
  (i) at the instant shown in the diagram
  (ii) one-quarter of a period later
  (iii) one-half period later.
(c) What is the wavelength of this standing wave?
(d) What are the frequency and the period of this standing wave?
(e) What harmonic is this standing wave?
(f) What is the fundamental frequency of this pipe?

SOLUTION:

(a) Nodes occur at points where the air pressure is normal, antinodes occur where the air pressure is a maximum or a minimum. These points are shown in figure 9.41.

FIGURE 9.41



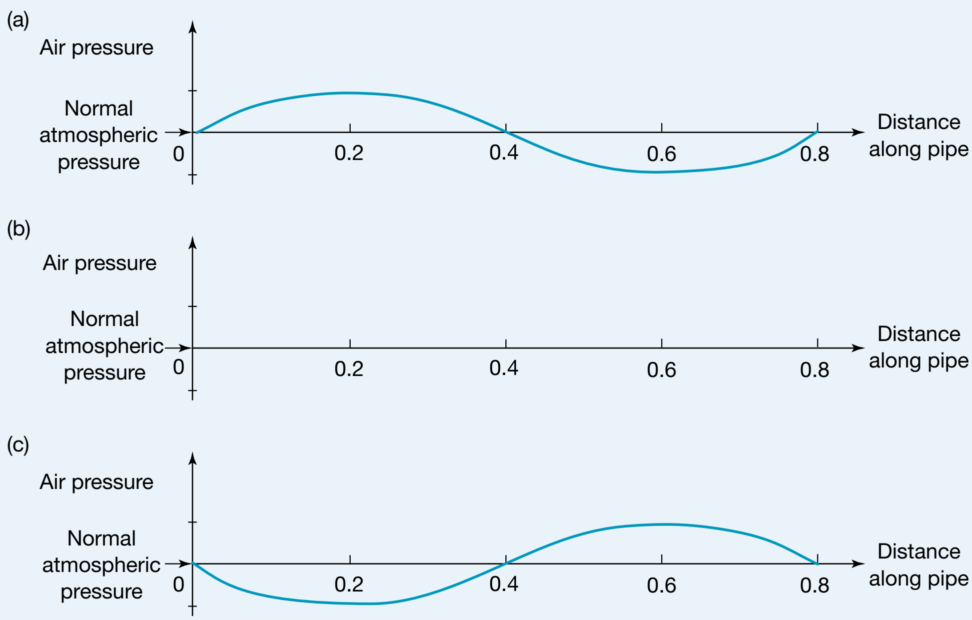
(b) (i) The pressure variations are at a maximum. This situation is shown in figure 9.42 (a).
  (ii) A quarter of a period later, the air pressure will be normal all along the pipe, as shown in figure 9.42 (b).
  (iii) One half a period after the first instant, the pressure variations will again be at a maximum, but they will be the opposite of their original values, as shown in figure 9.42 (c).

(c) The wavelength is twice the distance between adjacent nodes, or the distance between alternate nodes. So $\lambda = 0.80\ \text{m}$.

(d)
$$f = \frac{v}{\lambda} = \frac{340\ \text{m s}^{-1}}{0.80\ \text{m}} = 425\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{425\ \text{Hz}} = 2.35 \times 10^{-3}\ \text{s}$$

FIGURE 9.42



(e) This is the first standing wave above the fundamental, hence it is the second harmonic. (Refer back to figure 9.39e, showing standing waves in a pipe open at both ends.)

(f) The second harmonic is twice the fundamental frequency.

$$f_0 = \frac{425\text{ Hz}}{2}$$
$$= 212.5\text{ Hz}$$

9.7 Exercise 1

1 A pipe that is open at both ends has a length of 60 cm. It produces a sound that has a second harmonic of 550 Hz. Calculate:
(a) the fundamental frequency
(b) the speed of sound in air
(c) the frequency of the third overtone of the pipe.

2 A pipe that is closed at one end has a length of 40 cm. It produces a sound that has a first overtone of 600 Hz. Calculate:
(a) the fundamental frequency
(b) the speed of sound in air
(c) the frequency of the third overtone of the pipe.

3 What is the fundamental frequency of a trumpet that has a tube length of 1.40 m?

4 Calculate the third harmonic wavelength of a 2.75 m long closed pipe.

5 How long does a closed organ pipe need to be in order to have a fundamental frequency of 256 Hz?

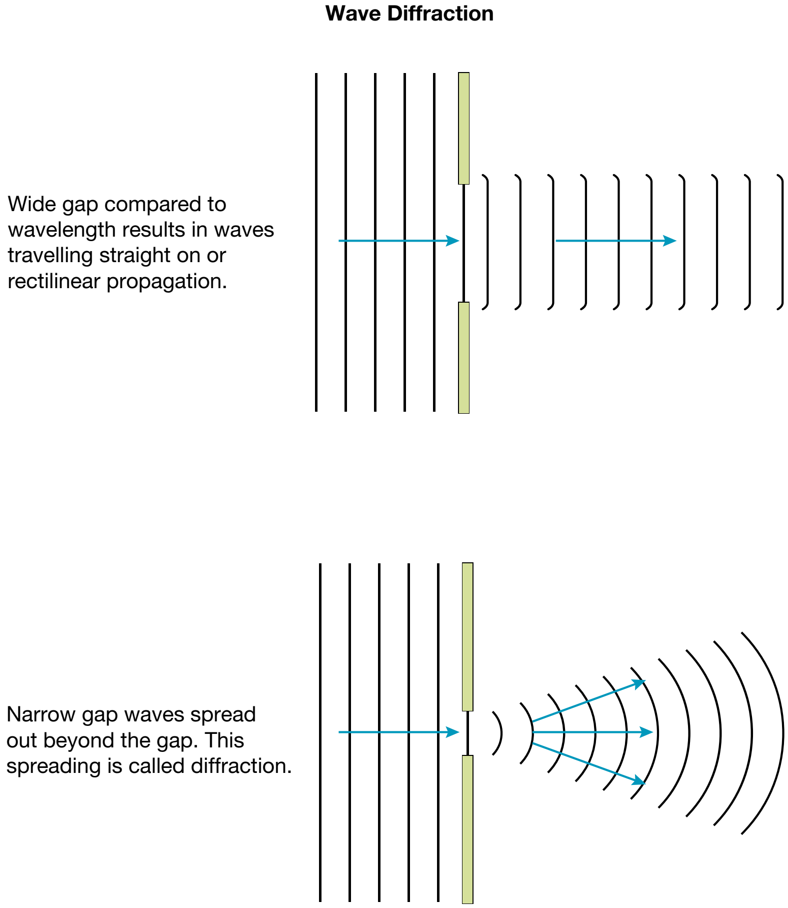
6 Why do pipes that are closed at one end sustain only the odd-numbered harmonics?

9.8 Diffraction of sound waves

9.8.1 Bending sound

Like other types of waves, sound waves demonstrate diffraction when they encounter an edge, barrier or gap, spreading out as they travel beyond the obstacle. The degree of diffraction that occurs depends upon the wavelength of the sound. Sounds with wavelengths that are short compared to the width of the gap or obstacle show very little diffraction, with only slight bending of the waves. Sound waves with wavelengths similar to or longer than the obstacle or gap they encounter are diffracted more significantly.

FIGURE 9.43 Diffraction is most obvious when the gap is approximately the same size as the wavelength of the wave.

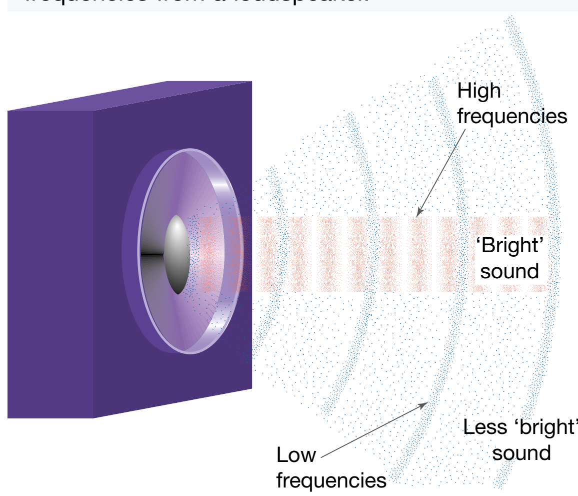


9.8.2 Directional spread of different frequencies

The opening at the end of a wind instrument such as a trumpet, the size of someone's mouth and the size of the loudspeaker opening all affect the amount of diffraction that occurs in the sound produced. High-frequency sounds can best be heard directly in front of these devices.

When a loudspeaker plays music, it is reproducing more than one frequency at a time. Low-frequency soundwaves from a bass have a large wavelength; high-frequency soundwaves from a trumpet have a short wavelength. Short-wavelength, high-frequency sounds do not diffract (spread out) very much when they emerge from the opening of a loudspeaker, but long wavelength sounds do. If a single loudspeaker is used, the best place to hear the sound is directly in front of the speaker.

FIGURE 9.44 The diffraction of high and low frequencies from a loudspeaker.



9.8 SAMPLE PROBLEM 1

Two sirens are used to produce frequencies of 200 Hz and 10 000 Hz. Describe the spread of the two sounds as they pass through a window in a wall. The window has a width of 35 cm. Assume that the speed of sound in air is 330 m s^{-1}.

SOLUTION:

First calculate the wavelengths of the sounds using the formula $v = f\lambda$. These calculate to 165 cm and 3.3 cm respectively. There will be a very small diffraction spread for the sound of wavelength 3.3 cm because the wavelength is small compared with the opening. There will be a large diffraction spread for the sound of wavelength 165 cm because the wavelength is large compared with the opening.

9.8 Exercise 1

1 (a) What is diffraction?
(b) Why is diffraction an important concept to consider when designing loudspeakers?

2 A sound of wavelength λ passes through a gap of width w in a barrier. How will the following changes affect the amount of diffraction that occurs:
(a) λ decreases
(b) λ increases
(c) w decreases
(d) w increases?

9.9 Review

9.9.1 Summary

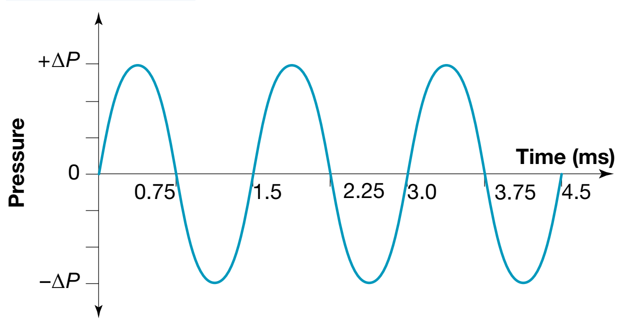
- Sound waves are vibrations of particles in a medium.
- Compressions relate to the crests of a transverse wave and rarefactions relate to the troughs of a transverse wave.
- The pitch of a sound wave increases as the frequency of the sound wave increases.
- The amplitude of a sound wave increases as the sound's volume grows louder.
- An echo is a reflection of a sound wave.
- Waves can interfere when they come into contact. This can result in the amplitude of the waves increasing if the waves are in phase, or decreasing if the waves are out of phase. The addition of waves is called superposition.
- Beats occur when sound waves that are close but not identical in frequency are played at the same time. The beat frequency is equal to the difference in the frequencies of the two sound waves: $f_{beat} = |f_2 - f_1|$.
- Sound waves can be studied with a cathode-ray oscilloscope (CRO) or cathode-ray oscilloscope simulator application. Different musical instruments produce sound waves that produce different shaped traces on a CRO.
- The Doppler effect is the result of a wave source moving through the medium. The waves move at constant speed relative to the medium, resulting in a higher frequency in front of the moving source and a lower frequency behind. For an observer and a source moving towards each other, the effective frequency heard by the observer is $f' = f\dfrac{(v + v_o)}{(v - v_s)}$. For an observer and a sound source moving away from each other, $f' = f\dfrac{(v - v_o)}{(v + v_s)}$ where v is the speed of sound in the medium, v_o is the speed of the observer and v_s is the speed of the source.

- The intensity (I) of a sound is the acoustic power per unit area at a point separated from the sound source by a distance d, and it is measured in W m^{-2}: $I = \dfrac{P}{4\pi d^2}$
- The intensity level (L) of a sound is measured in decibels (dB): $L = 10 \log\left(\dfrac{I}{I_0}\right)$ where I_0 is the intensity of the softest audible sound (10^{-12} W m^{-2}).
- The fundamental frequency, f_0, of a string or pipe is the lowest frequency at which a standing wave occurs.
- Harmonics are whole number multiples of the fundamental frequency.
- Resonant frequencies are frequencies above the fundamental frequency at which resonance occurs.
- Stringed instruments form standing waves that have a node at each end.
- A closed pipe will form a pressure node at its open end and a pressure antinode at its closed end. An open pipe will form pressure nodes at both ends.
- Pitch is a qualitative measurement of frequency.
- Timbre is a qualitative measure of the complexity of sound produced by an instrument. It is dependent upon the number of harmonic frequencies that are produced.

9.9.2 Questions

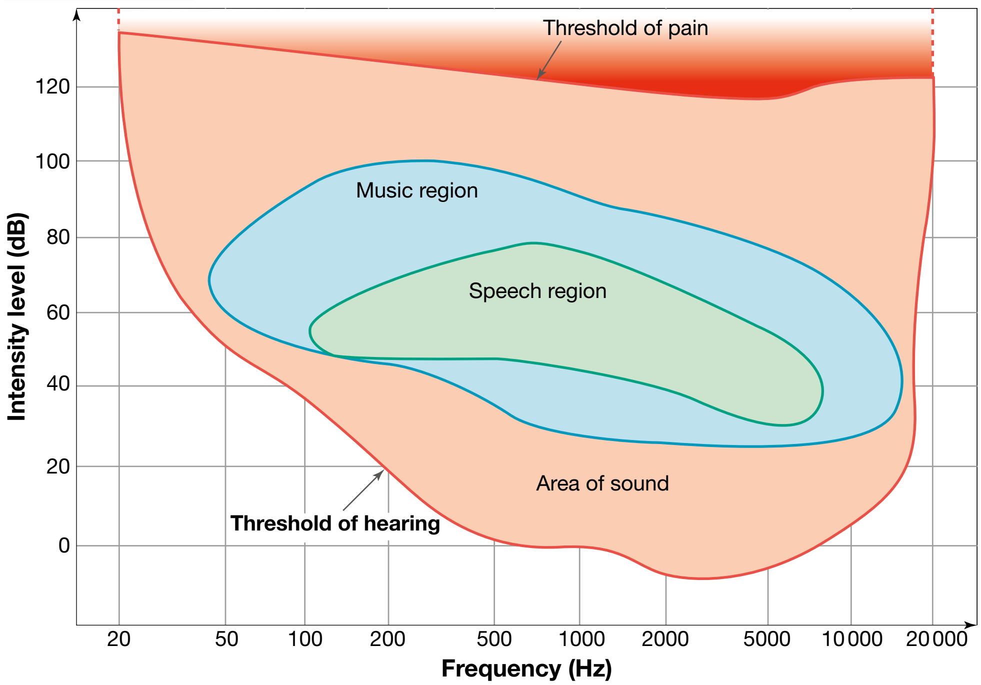
1. What is the intensity level of a sound that has an intensity of 10^{-8} W m^{-2}?
2. Calculate the intensity of a sound that measures 65 dB on a sound level meter.
3. The intensity level of a stereo placed in the open air is 100 dB at a distance of 10.0 m from the speaker. If the speaker has a total power output of 20 W, what percentage of the speaker's power is converted into sound?
4. (a) What is the speed of sound in dry air at a temperature of 30 °C?
   (b) At this temperature, how far away from the observer did lightning strike if the thunder was heard 7 s after the flash was seen?
5. Figure 9.45 shows the variation in air pressure near a sound source producing a single note as a function of time.
   (a) What is the period of this sound?
   (b) What is this sound's frequency?
   (c) What is the wavelength of this sound if the speed of sound is assumed to be 344 m s^{-1}?

FIGURE 9.45



6. An ear trumpet that has a circular opening with a diameter of 15 cm is used by a hearing-impaired person. If we assume that the ear trumpet is 100% efficient, by what factor is sound intensity incident on the eardrum increased by the device if the surface area of the eardrum is 0.5 cm^2?
7. A rifleman fires a shot and hears its echo from a surface 200 m away 1.2 s later. What was the air temperature at this time?
8. Figure 9.46 shows the audibility range of the human ear. Use it to answer the following questions.
   (a) What is the highest frequency noise that can be made with the human voice?
   (b) What is the loudest sound level at that someone can shout?
   (c) Sounds with frequencies that lie below the human hearing threshold are said to be subsonic. Below what frequency are subsonic noises?
   (d) For what frequency range is the pain threshold below 120 dB?
   (e) What frequencies are able to be made by musical instruments but not by human voices?

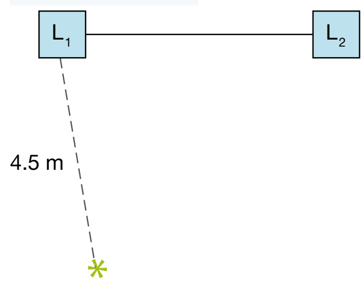
FIGURE 9.46



9. Jan is sitting in a concert hall five rows from the front of the stage on which are mounted two loudspeakers, L_1 and L_2, as shown in figure 9.47.

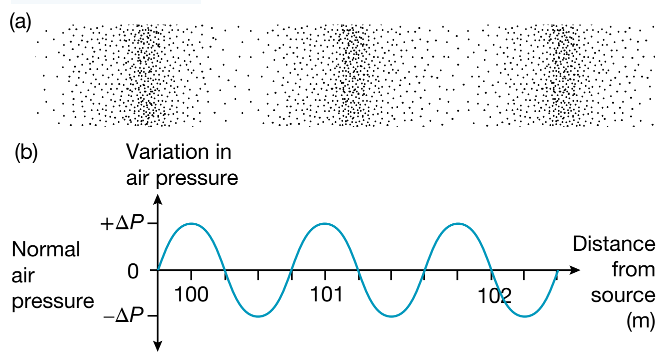
   She notices that the sound from the performance doesn't seem as loud where she's sitting compared to where she has sat in other performances, particularly for notes that have a wavelength of 1.5 m, and she realises (being a Physics student) that she is sitting in a position that is a minimum. Given that she is 4.5 m away from L_1 and at least that from L_2, what is the smallest distance that can be separating her from L_2?

FIGURE 9.47



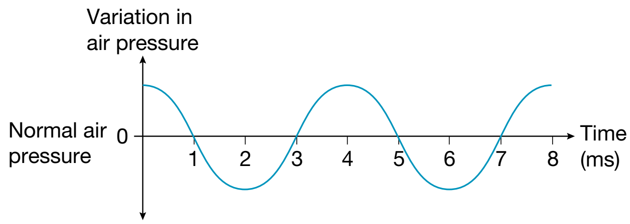
10. Figure 9.48 (a) shows the pressure variation as a function of distance from a sound source at an instant in time. The graph shown in figure 9.48 (b) shows the pressure variation as a function of distance from the sound source. The speed of sound in air is 340 m s^{-1}.
    (a) What is the wavelength of this sound?
    (b) What is the period of this sound?
    (c) Using the same scale as shown in figure 9.48 (b), sketch the distribution of particles, and the pressure variation one-quarter of a period later.
    (d) Using the same axes as shown in figure 9.48 (b), sketch a graph of pressure variation versus distance one-quarter of a period later than shown in the graph.

FIGURE 9.48



11. Figure 9.49 shows the variation of air pressure as a function of time near a sound source.
    (a) What is the period of this sound?
    (b) What is the frequency of this sound?
    (c) What is the wavelength of this sound if the speed of sound is $330\,\text{m s}^{-1}$?

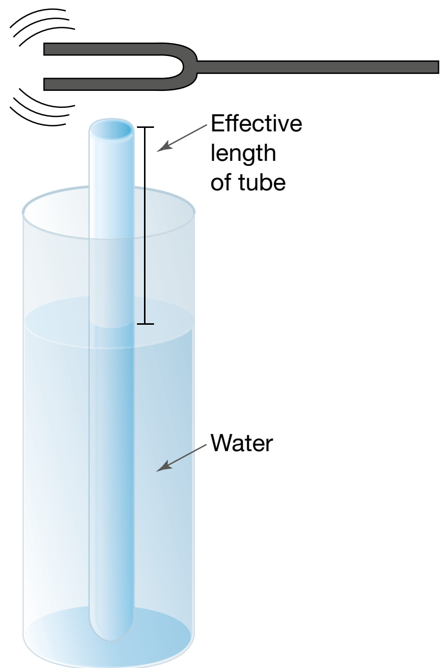
FIGURE 9.49



12. What is the acoustic power of a source if it produces 2.0 J in 100 s?
13. What is the sound intensity if 4.0×10^{-8} W pass through an area of $0.080\,\text{m}^2$?
14. Calculate the power passing through an area of $2.0\,\text{m}^2$ if the sound intensity is $4.5 \times 10^{-5}\,\text{W m}^{-2}$?
15. One siren produces a sound intensity of $3.0 \times 10^{-3}\,\text{W m}^{-2}$ at a point that is 10 m away. What would be the sound intensity produced at that point if five identical sirens sounded simultaneously at the same place as the original?
16. If the sound intensity 4.0 m from a point sound source is $1.0 \times 10^{-6}\,\text{W m}^{-2}$, what will be the sound intensity at each of the following distances from the source?
    (a) 1.0 m
    (b) 2.0 m
    (c) 8.0 m
    (d) 40 m
17. What are the sound intensity levels associated with the following sound intensities?
    (a) $5.0 \times 10^{-10}\,\text{W m}^{-2}$
    (b) $3.2 \times 10^{-7}\,\text{W m}^{-2}$
    (c) $4.9 \times 10^{-3}\,\text{W m}^{-2}$
    (d) $1.8 \times 10^{-9}\,\text{W m}^{-2}$
18. Calculate the sound intensities associated with the following sound intensity levels:
    (a) 7.0 dB
    (b) 25 dB
    (c) 54 dB
    (d) 115 dB
19. The fourth harmonic frequency of a particular string is 880 Hz. What will be its:
    (a) fundamental frequency?
    (b) third harmonic frequency?
20. The speed of waves in a 0.8 m guitar string is $240\,\text{m s}^{-1}$. Find:
    (a) the longest wavelength that can be formed by the string
    (b) the fundamental frequency
    (c) the distance between nodes in the fourth vibrational mode
    (d) the wavelength for the third harmonic.
21. A viola string vibrating in its third harmonic has a wavelength of 0.40 m.
    (a) What is the length of the string?
    (b) How long would a similar string need to be if it vibrates in its second harmonic at the same wavelength as this first string?
    (c) If the frequency of the first string is 400 Hz, calculate the speed of the waves in the string.
22. Calculate the fundamental, second harmonic and third harmonic frequencies for the following pipes on a day when the speed of sound is $340\,\text{m s}^{-1}$:
    (a) 60 cm long open pipe
    (b) 120 cm long closed pipe
    (c) 30 cm long closed pipe
    (d) 2.00 m long open pipe
    (e) 2.00 m long closed pipe.
23. How many octaves apart are 131 Hz and 524 Hz?
24. A closed pipe in air (at 20 °C) resonates at its fundamental frequency of 200 Hz. If the same pipe is submerged completely in a tank of pure water, at what fundamental frequency will it then resonate?

25. A tube is placed with one end in water as shown in figure 9.50. By raising and lowering the tube in the water, we are able to consider it a closed tube of variable length.
    A tuning fork with a frequency of 524 Hz is struck and then held above the mouth of the tube. As the length of the pipe is changed, there are some lengths at which the sound of the tuning fork is very loud and others at which it is very soft.
    (a) Explain why this happens.
    (b) The lengths at which the loud vibrations occur are 0.33 m apart. What was the speed of sound on that day?

FIGURE 9.50

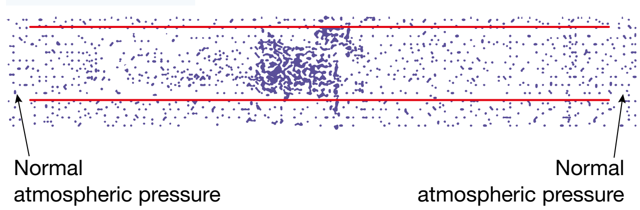


26. A flute is designed to be played in the key of C major; that is, when all of the finger holes are covered and the flute played, middle C (262 Hz) is the fundamental frequency. What is the approximate separation between the mouthpiece and the end of the flute?
27. An open E string is 0.70 m long and vibrates at a frequency of 330 Hz. How far up the length of the string must a finger be pressed onto the string to produce the note A (440 Hz)?
28. (a) What is the resonant frequency of the column of air in the outer ear of a human, which is about 2.5 cm long?
    (b) Is this within our range of hearing? Explain your answer.
29. All the strings of a violin are the same length. How are they able to produce different notes when open?
30. A railway worker strikes a steel rail with a hammer, and the noise is heard both through the air and through the rail by another worker 600 m away. What time interval elapses between the two sounds arriving?
31. Why isn't it possible for a guitar string of length L to form a standing wave pattern that has a wavelength equal to $\frac{3L}{4}$?
32. The node at the open end of a pipe has, in this course, been assumed to be exactly at the end. In fact, the node is located slightly beyond the end of the tube. How far beyond the end of the tube this node is located depends upon the diameter of the pipe itself. We are able to compensate for this effect by using the end correction equation for a pipe open at both ends:

$$L_{\text{effective}} = L_{\text{actual}} + 0.58d$$

    where d is the pipe diameter. The end correction is usually ignored as it is quite small.
    How long would a 1.6 cm diameter pipe instrument need to be in order for the percentage difference between the actual and effective lengths to reach 5%?
33. A maker of organ pipes decides to use aluminium instead of steel to make his pipes. How will the change in material affect the present design of the pipes?
34. Figure 9.51 shows pressure variation in and around a pipe open at both ends as it is resonating at one of its harmonics. Assume that the speed of sound in air is 340 m s^{-1}.
    (a) What harmonic is represented in the diagram?
    (b) If the pipe is 0.85 m, what is the wavelength of the tone that the pipe is producing?

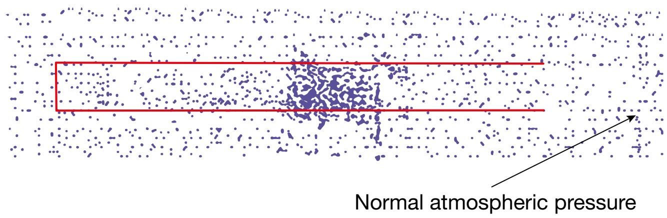
FIGURE 9.51



(c) What is the frequency of the tone being produced?
(d) Make a sketch to show the pressure variation in and around the pipe half a period later than the instant shown.
(e) Sketch the pressure variation in and around the pipe one-quarter of a period later than the instant shown.
(f) What is the period of the sound being produced by the pipe?
(g) What is the frequency of the second resonant frequency above the fundamental for this pipe?

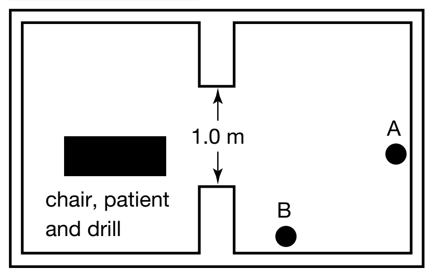
35. Figure 9.52 shows the pressure variation in and around a pipe closed at one end as it is resonating at one of its harmonics. Assume that the speed of sound in air is 340 m s^{-1}.
(a) What harmonic is represented in the diagram?
(b) If the pipe has a length of 50 cm, what is the wavelength of the tone that the pipe is producing?
(c) What is the frequency of the tone being produced?
(d) What is the fundamental frequency for this pipe?
(e) What is the frequency of the third resonant frequency above the fundamental of this pipe? What harmonic does this frequency correspond to?
(f) Make a sketch to show the pressure variation in and around the pipe half a period later than the instant shown.

FIGURE 9.52



36. Figure 9.53 shows the design of a dentist's waiting room and surgery.
There are two people sitting in the waiting room at points A and B. The door to the surgery is open and has a width of 1.0 m. A drill is operating and produces a sound of 5000 Hz frequency. The patient groans at a frequency of 200 Hz. Assume the speed of sound is 340 m s^{-1}.
(a) What is the wavelength of the patient's groan?
(b) What difference, if any, is there between the sound intensity levels produced by the patient's groan at points A and B? Justify your answer.
(c) What difference, if any, is there between the sound intensity levels produced by the dentist's drill at points A and B? Justify your answer.

FIGURE 9.53



37. A 1500 Hz sound and an 8500 Hz sound are emitted from a loudspeaker whose diameter is 0.30 m. Assume the speed of sound in air is 343 m s^{-1}.
(a) Calculate the wavelength of each sound.
(b) Find the angle of the first minimum for each sound for this speaker.
(c) A sound engineer wants to use a different speaker for the 8500 Hz sound so that it has the same angle of dispersion as the 1500 Hz sound has for the 0.30 m diameter speaker. Calculate the diameter of the new speaker if this is to occur.

38. A trumpeter on a moving train first demonstrated the Doppler effect. (Use 340 m s^{-1} as the speed of sound.)
(a) How fast would the train be travelling if the trumpeter played an A (f = 440 Hz) and the observers on the platform heard an A sharp (f = 466 Hz)?
(b) What frequency would the observers hear once the train had passed?
(c) How fast would the train need to be travelling for the pitch of the note to rise a full octave (that is, double its frequency)?

39. Lyn cannot hear sound above 1.5×10^4 Hz. She decided to investigate the Doppler effect by strapping a speaker to the roof of a car. She connects a signal generator to the speaker so that it produces a sound of frequency 1.2×10^4 Hz. She predicts that if the car is driven towards her with sufficient speed she will not be able to hear the sound.
    (a) At what speed can she no longer hear the sound? (Assume there are no other sounds to drown it out.)
    (b) What does she hear as the car accelerates?
40. Shelly is concerned about the speed of traffic in her street. She measures the dominant frequency of the sound of a car as it approaches to be 1100 Hz, and as it moves away to be 919 Hz. What was the speed of the car? (Take the speed of sound in air to be 340 m s^{-1}.)
41. Explain why high-frequency sound waves are preferred for tasks such as echolocation rather than low-frequency sound waves.
42. If astronauts lose radio contact with one another when they are outside their spacecraft, they can converse by touching the face plates of their helmets together. Why do they need to do this, rather than just getting close enough to shout at one another?
43. Does the amplitude of a wave affect its speed through a medium?
44. The voice of a person who has inhaled helium sounds higher than normal. Why does this happen?
45. In the Victorian era, hearing-impaired people used ear trumpets to improve their hearing. An ear trumpet was a cone-shaped device, the smaller end of which was placed in the ear. How would these devices have worked?
46. Timekeepers for races are advised to watch for the smoke from the starter's gun rather than listening for the sound of the gun being fired in order to start their stopwatches at the correct moment. Why is this?

FIGURE 9.54

PRACTICAL INVESTIGATIONS

Investigation 9.1: Analysing sound waves from a tuning fork

Aim

To observe and collect sound traces from a CRO

Apparatus

at least two tuning forks of different frequency
access to a CRO or a CRO simulation program for the computer
a microphone to convert the sound wave into an electrical signal

Theory

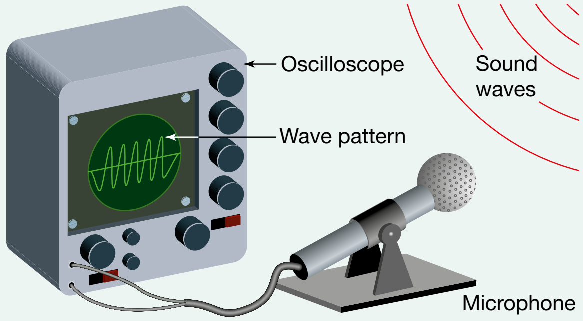
The traces from a CRO can provide you with a snapshot of a number of different sound waves. The waves are a small time-grab of a much larger train of sound waves. These short interval grabs can show you some of the features of a sound.

Method

1. Connect the microphone to the input of the CRO or the microphone input on the computer if using a CRO simulation program.

2. Tune and adjust the CRO so that when a single tuning fork is brought near to the microphone, a sine-wave trace is produced. Observe what happens to the amplitude of the wave as the tuning fork loses its vibrational energy and the sound becomes softer.
3. Check out all the traces of all tuning forks you have. When doing this, keep the same CRO settings to make comparison easier. Note the frequencies and shape of the waves produced. If you are using a CRO simulation computer program, you should be able to freeze the CRO traces, save them and print them out.
4. Try striking two different frequency tuning forks and having the microphone collect the sound from both tuning forks. You will notice the shape of the CRO trace wave becomes more complex.
5. Try adding a third sound from another tuning fork to the input into the CRO. Observe the increasingly complex CRO trace.

FIGURE 9.55 A microphone attached to a CRO



Investigation 9.2: Observing wave interference

Aim

To hear sound waves interfering with each other

Apparatus

tuning fork

Theory

Each of the vibrating tuning fork prongs acts as a coherent source of sound because it has the same frequency, amplitude and phase in relation to the other when producing a sound wave in air. Hence, there are two sound waves generated by the tuning fork prongs. Each one radiates from a slightly different position. As a compression is produced between the prongs, a rarefaction is produced outside each of the prongs and vice versa. The sound waves propagate outwards from each tuning fork prong but on some paths they overlap. This is either because there is a full wavelength difference in the travel path length or because in some directions they meet at a point one half wavelength out of phase. In these directions where the sound waves are exactly one half wavelength out of phase (compression meets rarefaction) the sound waves will add. If the amplitudes are the same, one sound wave's compression is annulled by the rarefaction from the other. This produces a sound minimum.

The sound waves can add to form a maximum if the path difference is equal to a whole number of wavelengths. The result is a higher amplitude sound.

Method

1. Strike a tuning fork so that it produces a note.
2. Hold the tuning fork to your ear and rotate the tuning fork about its long axis.
3. Listen carefully to the sound you hear. Note when the sound waves appear to increase in amplitude and decrease in amplitude.

Investigation 9.3: Analysing sound waves from musical instruments

Aim

To observe the sound from musical instruments on a CRO or CRO computer simulation program

Apparatus

access to a CRO or a CRO simulation computer program
microphone
variety of musical instruments

Theory

The notes produced by a musical instruments have a characteristic timbre. An examination of the CRO traces of the same note played by a number of different musical instruments will highlight the differences in the nature of the sound waves.

Method

1. Connect the microphone to the input of the CRO or the microphone input on the computer if using a CRO simulation program.
2. Tune and adjust the CRO so that, when a single tuning fork is brought near to the microphone, a sine-wave trace is produced.
3. Using a variety of musical instruments, play the same note into the microphone, attached to the CRO and observe the differently shaped wave patterns that are produced. If you are using a CRO simulation computer program, you may be able to freeze and save the CRO traces and then print them out.

Analysis

Compare the shapes of the CRO traces for each instrument. Are there any similarities? Which instruments are most similar?

Investigation 9.4: Observing sound

Connect a signal generator to an audio amplifier that has a built-in speaker. Then connect a CRO across the output of the amplifier. With the signal generator producing a sinusoidal signal, and making any necessary adjustments to the timebase and voltage scales of the CRO, increase the frequency from about 20 Hz to 20 000 Hz. Observe what happens and then answer the following questions.

1. What happened to the period, pitch and loudness as the frequency increased?
2. What were the lowest and highest frequencies you could hear?
3. How is amplitude related to loudness?
4. Replace the signal generator with a microphone and produce sounds on various instruments and voices. Sketch and compare the different waveforms produced. Do they have a definite period?

Investigation 9.5: Sound intensities and intensity levels

This investigation consists of two parts:

1. Use a sound level meter to construct your own table of sound intensities and intensity levels for a number of everyday sounds that you wish to select. Why is it important that you note how far the meter is from the sound sources?
2. The second part of this investigation should be carried out in an outdoor setting where there is little interference from reflection. Measure the sound intensity level at distances of 1.0 m, 2.0 m, 4.0 m, 10.0 m, 20.0 m and 50.0 m from a sound source that radiates sound evenly in all directions (a lawnmower would do). Produce a graph of the log of sound intensity versus the log of distance from the source. Comment on your results.

Investigation 9.6: Finding the speed of sound in air

The aims of this investigation are:

1. To establish that resonance occurs in the tube, at different frequencies.
2. To determine the velocity of sound in air, at room temperature, by means of a resonance tube.

You will need the following equipment:

- length of glass tubing of approx. internal diameter of 40 mm
- retort stand and clamp for the tube
- frequency generator
- audio amp and speaker
- sound level meters
- thermometer.

For all forms of wave motion, $v = f\lambda$, where v is velocity, f is frequency and λ is wavelength.

The air inside the tube will resonate for certain frequencies when standing waves can be set up so that a pressure node occurs at the both open ends of the tube.

FIGURE 9.56

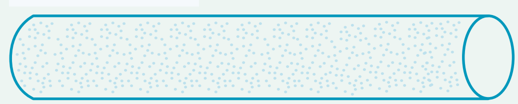


Figure 9.56 shows the simplest standing wave that can be set up in the resonating tube. The frequency that causes such a standing wave is called the FUNDAMENTAL and always occurs for the shortest length of tube that produces resonance.

Note: the standing wave is actually slightly longer than the length of the pipe. The ‘end correction’ at each end is 0.3 × the diameter of the pipe; therefore, the ‘effective length’ is $L + 2 \times 0.3d$.

At resonance, the length of the tube, L, is related to the wavelength of the standing wave set up in the air inside the tube, and, since the resonant frequency, f, and wavelength, λ, are related to the velocity of sound in the air, the speed of sound in air can be determined.

Clamp the tube to the retort stand. Position the audio amp and speaker at one end of the tube. Connect the audio amp to the frequency generator. Select a low frequency and gradually increase the frequency while observing the sound level meter. Look for 'spikes' on the sound level meter and record the frequencies at which these spikes occur. (Resonance causes an increase in the amplitude of the sound.) Repeat until several resonant frequencies are obtained.

Record the pipe length and diameter in both cm and m. Hence, calculate the effective length, L, of the pipe by adding $2 \times 0.3 \times$ diameter of the pipe.

Record the frequencies that caused resonance and their harmonic number, n.

Calculate the speed of sound from $v = \frac{2 \times L \times f}{n}$ where n is the harmonic number.

Analysis

1. Explain what is meant by the term 'resonance'.
2. What is the average value for the speed of sound obtained from this experiment with an error range?
3. What is the theoretical speed of sound based on the air temperature?

Investigation 9.7: Resonant frequencies in a tube

Take the centre and stopper out of a ballpoint pen, then hold the pen vertically. With the bottom uncovered, blow across the top until you produce the fundamental frequency. Blow harder and produce the first resonant frequency above the fundamental and any others if possible.

1. Comment on the pitch of each frequency.

Cover the bottom and produce the fundamental frequency.

2. How does this frequency compare with that produced when the pen has both ends open?
3. Try to produce resonant frequencies above the fundamental and comment on the pitch of each.
4. Explain your observations in terms of the key ideas you have studied so far.

Investigation 9.8: Diffraction and sound intensity levels for sounds of different frequencies

This investigation aims to measure the effect that frequency and gap size have on the diffraction of sound waves. It should be carried out in an outdoor setting where there is little interference from reflection.

You will need the following equipment:

- sine wave signal generator
- audio amplifier
- loudspeaker
- sound level meter
- soundproof box (cardboard lined with pillows with an adjustable gap for sound to pass through).

Place the loudspeaker in the box facing the opening. Start the investigation by using a gap of 0.20 m. Adjust the signal generator to a frequency of 500 Hz and adjust the amplifier to give a sound intensity level of 80 dB at a distance of 5.0 m directly in front of the opening in the box. Maintaining a distance of 5.0 m from the opening, measure the sound intensity at angles of 20, 40, 60 and 80 degrees on either side of the first reading.

Adjust the signal generator to a frequency of 5000 Hz and adjust the amplifier to give a sound intensity level of 80 dB at a distance of 5.0 m directly in front of the opening in the box. Maintaining a distance of 5.0 m from the opening, measure the sound intensity at angles of 20, 40, 60, and 80 degrees on either side of the first reading.

1. Make a table of your results and draw graphs showing how the sound intensity level and the sound intensity carried with the angle for each frequency.
2. Adapt this procedure to investigate how gap size affects the amount of diffraction.

TOPIC 10
Ray model of light

10.1 Overview

10.1.1 Module 3: Waves and Thermodynamics

Ray model of light

Inquiry question: What properties can be demonstrated when using the ray model of light?

Students:

- conduct a practical investigation to analyse the formation of images in mirrors and lenses via reflection and refraction using the ray model of light (ACSPH075)
- conduct investigations to examine qualitatively and quantitatively the refraction and total internal reflection of light (ACSPH075, ACSPH076)
- predict quantitatively, using Snell's Law, the refraction and total internal reflection of light in a variety of situations
- conduct a practical investigation to demonstrate and explain the phenomenon of the dispersion of light
- conduct an investigation to demonstrate the relationship between inverse square law, the intensity of light and the transfer of energy (ACSPH077)
- solve problems or make quantitative predictions in a variety of situations by applying the following relationships to:
  - $n_x = \frac{c}{v_x}$ – for the refractive index of medium x, v_x is the speed of light in the medium
  - $n_1 sin\,(i) = n_2 sin\,(r)$ (Snell's Law)
  - $\sin\,(i_c) = \frac{1}{n_x}$ – for the critical angle i_c of medium x
  - $I_1 r_1^2 = I_2 r_2^2$ – to compare the intensity of light at two points, r_1 and r_2

FIGURE 10.1 Rainbows are the result of refraction, reflection and dispersion of light rays by fine water droplets in the air.

10.2 What is light?

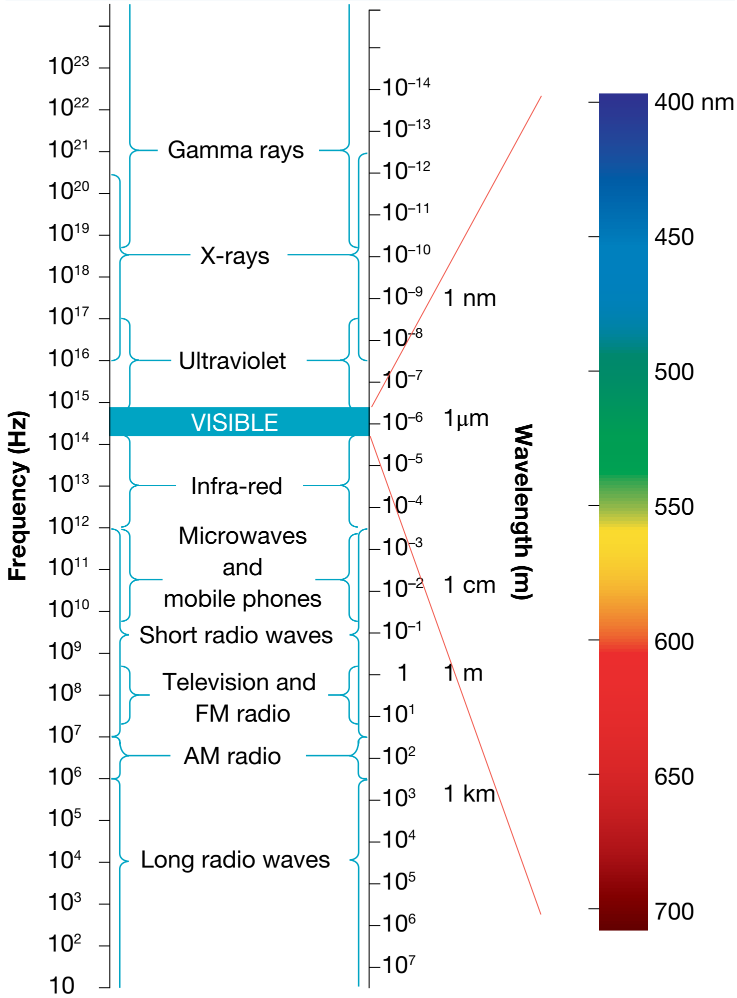
10.2.1 Electromagnetic waves

Light is the term commonly used to describe electromagnetic radiation; more specifically, we tend to use it to describe only a small part of the electromagnetic spectrum known as the visible spectrum.

Electromagnetic radiation travels in the form of transverse waves. However, unlike mechanical waves such as sound, earthquake tremors and pond ripples, electromagnetic waves do not need a medium to travel in; in fact, they slow down when travelling in any physical medium apart from a vacuum. All electromagnetic waves travel at the same speed in a vacuum. This is referred to as the speed of light, c, and it is equal to 299 792 458 m s^{-1}. For most purposes, the speed of light is approximated to 3×10^8 m s^{-1}.

While electromagnetic waves may travel at the same speed, they vary widely in wavelength and frequency. The visible spectrum — the range of electromagnetic radiation to which human eyes respond — makes up only a very small section of the electromagnetic spectrum.

FIGURE 10.2 The electromagnetic spectrum.



10.2.2 Sources of light

Light sources of different kinds can be classified as being either luminous or illuminated. **Luminous bodies** are those that emit electromagnetic radiation in the visible part of the spectrum either as a result of chemical processes or because they are incandescent. **Incandescent** objects glow because they are very hot and, the hotter they are, the more light they produce. The Sun's incandescence is the result of the enormous heat generated by the thermonuclear reactions within its interior. On a much smaller scale, the tiny particles of hot carbon produced in a candle flame provide incandescent light, and the tungsten filament in a light bulb can produce light as a result of becoming white hot when electric charges move through it.

FIGURE 10.3 The Pleiades open cluster in the constellation Taurus. All stars are incandescent sources of light.

AS A MATTER OF FACT

Generally, solid objects start to emit visible light when their temperature reaches about 525 °C (called the Draper point). At temperatures lower than this, electromagnetic radiation is still being released by warm objects but, because the light waves produced are in the infra-red part of the spectrum, we are unable to see the objects.

Non-luminous (or illuminated) objects don't produce their own light; instead, they are reflectors of light produced by a luminous source. A non-luminous body can only act as a light source when there is a luminous body present. The source of the moonlight that allows us to see in the night is actually light from the Sun reflected from the surface of the Moon. When we wish to read a book in a dark room, we turn on a lamp whose light then reflects from the white pages of the book, allowing us to see it.

FIGURE 10.4 When liquid Luminol comes into contact with the iron in blood haemoglobin in the presence of ultraviolet light, the blood is luminous as a result of the chemical reaction rather than because of incandescence.

FIGURE 10.5 The Moon is a non-luminous object; it reflects light from the Sun.

FIGURE 10.6 An image seen through a night-vision device.

10.2.3 The ray model of light

When encountering a surface, light can exhibit a number of different behaviours. It may be reflected from the surface, it may be absorbed by the material, it may pass through the material, or it may exhibit a combination of these behaviours. What light does when it enters a new medium depends upon the nature of the material and the condition of the interface between the media.

All light travels in straight lines. As a result, we can only see an object if the light from that object can travel directly to our eye. You can easily see the book that is right in front of you on the desk, but you won't be able to see the people sitting behind you. You may not even be able to see the people sitting beside you

FIGURE 10.7 Rays are straight lines indicating light propagating from a light source. The further apart they are, the dimmer the light.

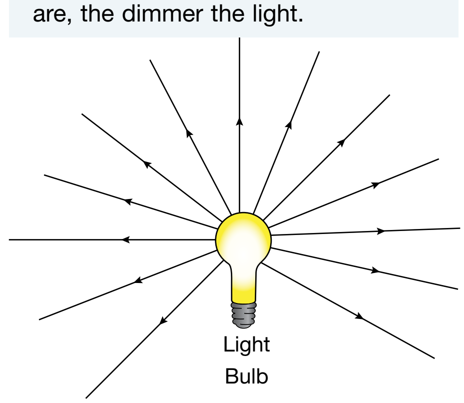


FIGURE 10.8 We can see an object only if light from that object is able to travel to our eyes. This person cannot see the yellow objects.

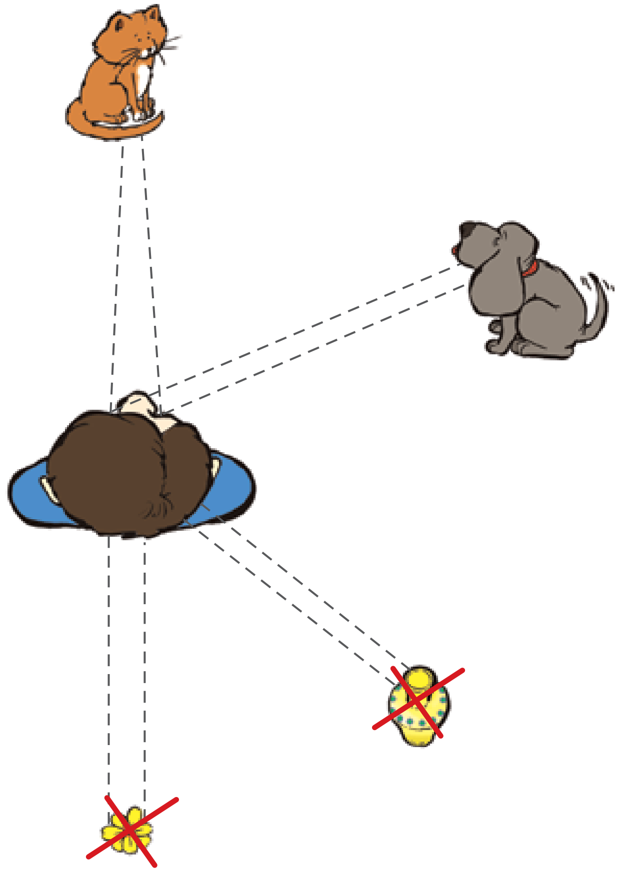


FIGURE 10.9 A ray diagram of a light bulb.

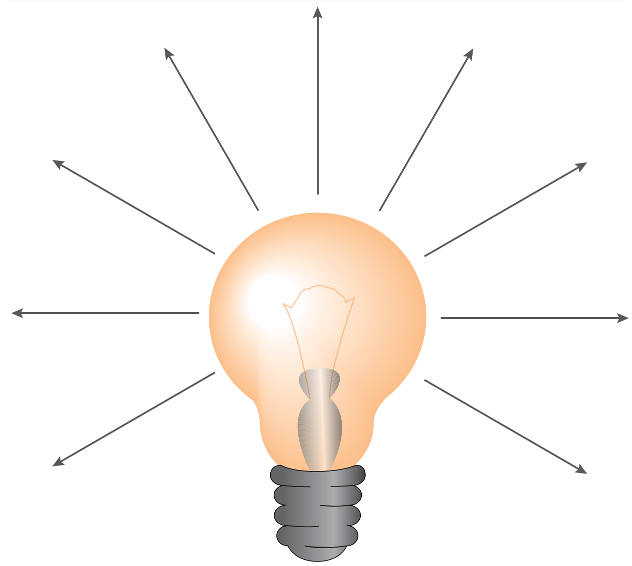
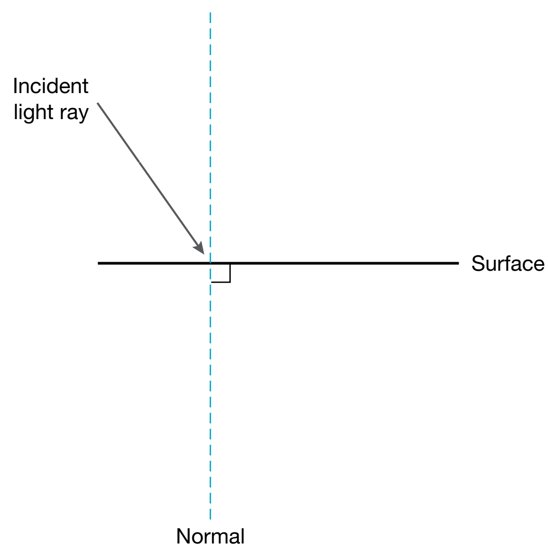


FIGURE 10.10 The normal is an imaginary line drawn perpendicular to a surface where a light ray is incident upon it.



clearly. However, while light may only move in a straight line, we are able to manipulate its path and see what was previously hidden by exploiting light's ability to be reflected and refracted.

As light travels in straight lines, it is often convenient to represent the paths taken by light as rays when drawing diagrams. This will be particularly useful in our next sections. Remember that the ray is only a representation. Although the diagram in figure 10.9 shows nine rays leaving the surface of the light bulb, it should be understood that this is only to imply the spreading nature of the light. We could as easily have drawn ten or a hundred rays in a similar way.

It is also handy at this point to define the normal to a surface. The **normal** is an imaginary line drawn perpendicular to a surface at the point where a light ray is incident upon it. We will be using the concept of the normal a great deal over the rest of the chapter.

10.2.4 Transmission of light through a medium

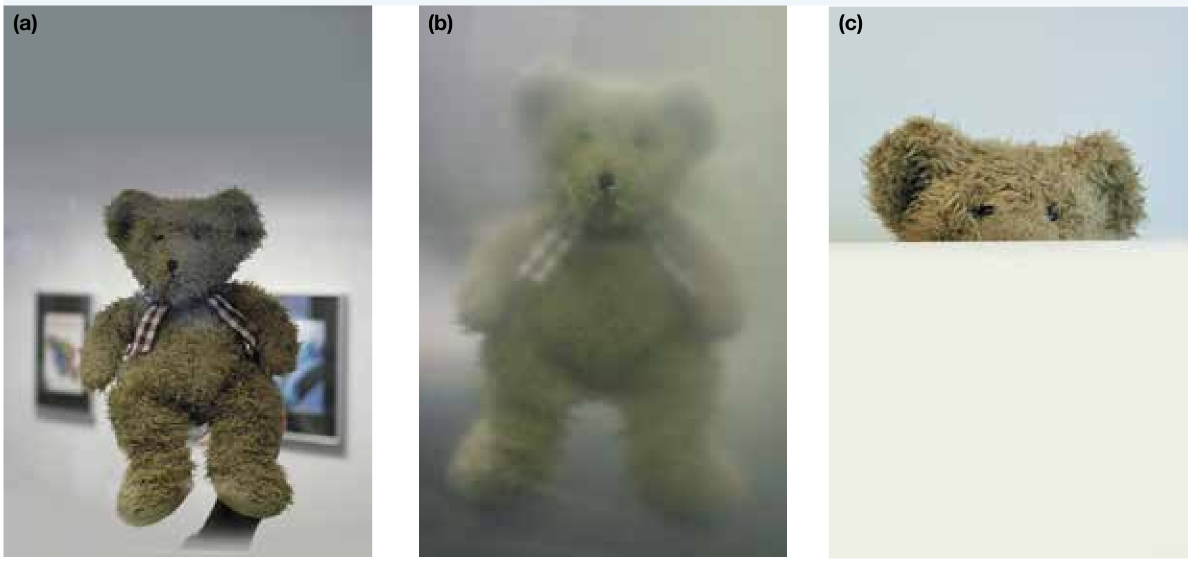
Light that strikes the surface of a material and is not reflected may pass through it. We are able to see through glass windows because light rays travelling from outside objects are able to pass through easily to our eyes. However, we cannot see what lies on the other side of a brick wall because light rays are unable to pass through it. The ability of a material to allow light to pass through it is referred to as its **optical transmissivity**. This is dependent upon the arrangement, type and size of atoms that the material is made from.

A material through which light rays are able to travel without distortion of their relative pathways is said to be **transparent**. Glass, Perspex and cling wrap are all examples of transparent materials. We are able to see and identify objects through them without losing clarity.

Some materials, such as frosted glass and thin rice paper, allow us to see that there is an object on the other side of them, but light rays cannot pass through them easily enough to see a clear image of the object. Such materials are said to be **translucent**. These translucent materials allow light rays to pass through them, but irregularities in their structure cause the rays to be scattered as they do so. As these scattered rays emerge, they are able to give a vague impression of the object but not a clear picture.

Opaque materials are those which light is unable to pass through at all. Wood, brick, concrete and the human body itself are all opaque. (Some parts of the human body, such as skin and nails, are translucent when they are separate from the rest of the body.)

FIGURE 10.11 A teddy bear as seen through materials that are (a) transparent, (b) translucent and (c) opaque



Upon striking an opaque material, light rays may be reflected from it or, in some cases, they may be absorbed completely. A black object with a rough surface will tend to absorb light, allowing almost none of it to be reflected back to the observer's eye. The absorbed light energy is usually converted into heat energy. Light rays striking an opaque material cannot be transmitted through it.

WORKING SCIENTIFICALLY 10.1

Baking paper is a translucent material that allows a limited quantity of light from a light source to pass through it. If enough layers of baking paper are placed over one another, they act as an opaque medium. Use a light meter to measure the amount of light that is transmitted from a light bulb or other consistent light source through a single layer of baking paper. Investigate the relationship between the number of layers and the amount of light transmitted.

10.2.5 Light intensity

The **luminous intensity, *I***, of light is a quantitative measure of the effective brightness of a light source and takes into account the amount of light energy produced by the light source each second and the area over which that light energy is distributed.

The amount of light energy in joules produced by a light source each second is measured by its **luminosity, *L***. The luminosity of a light source (also referred to as its **luminous power**) is measured in joules per second or watts (W).

Like mechanical waves such as sound, light waves travel outwards from their source in three dimensions, forming a spherical wavefront, with the light energy distributed each second over a larger and larger area. The area of this spherical wavefront increases with distance from the source:

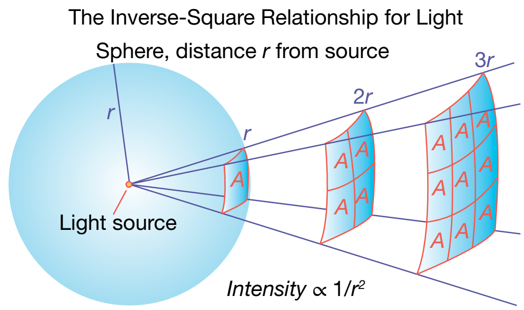
$$A_{sphere} = 4\pi d^2$$

As the intensity of the light can be described as the amount of energy distributed over each square metre of the wavefront each second, we can then describe the intensity in terms of the luminosity of the light source and the observer's distance away from it:

$$I = \frac{L}{4\pi d^2}$$

with intensity having the units W m^{-2}.

FIGURE 10.12 Light obeys an inverse-square relationship with distance.



At a distance 2*r* from the source the radiation is spread over four times the area so is only 1/4 the intensity that it is a distance *r*.

10.2 SAMPLE PROBLEM 1

A white sheet of paper is held 30 cm from a 60 W reading light. By what factor is the intensity of the light incident on the paper reduced if the paper is moved 50 cm further away from the reading light?

SOLUTION:

First, the intensity of the light at the two positions can be described by:

$$I_1 = \frac{L}{4\pi (d_1)^2}$$

and

$$I_2 = \frac{L}{4\pi (d_2)^2}$$

As the luminous power of the source is the same in both cases, the two equations can be combined by substituting for L and cancelling common terms to get the relationship:

$$I_1(d_1)^2 = I_2(d_2)^2$$

and so,

$$\frac{I_1}{I_2} = \left(\frac{d_2}{d_1}\right)^2$$

Substituting in values, we find

$$\frac{I_1}{I_2} = \left(\frac{0.8\text{ m}}{0.3\text{ m}}\right)^2$$

$$\frac{I_1}{I_2} = 7.1$$

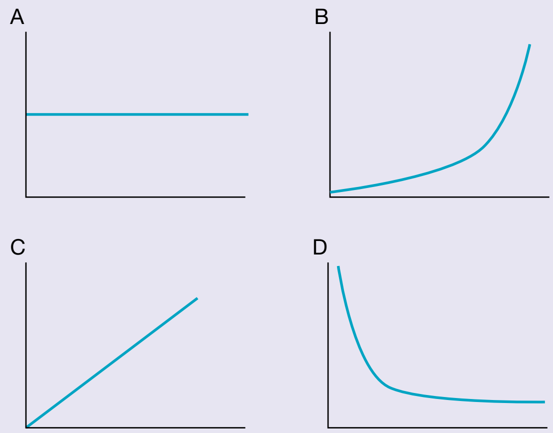
Therefore, by moving the paper 50 cm further away, the intensity of light on the paper has been reduced by a factor of 7.1.

WORKING SCIENTIFICALLY 10.2

Only a fraction of the energy produced by a light bulb is in the form of light energy; most of the energy is lost as heat energy. Devise a method to determine exactly what proportion of light energy is produced by a light bulb and then use this method to investigate the relationship (if any) between the wattage of a light bulb and the proportion of light energy it produces.

10.2 Exercise 1

FIGURE 10.13



1. Which of the graphs in figure 10.13 best describes the relation between luminous intensity and distance from the light source?
2. Which of the graphs in figure 10.13 best describes the relation between luminous intensity and the power of the light source?
3. Which of the following are not luminous bodies in the visible spectrum:
   (a) a star
   (b) the Moon
   (c) a candle
   (d) an incandescent light globe
   (e) an LED light bulb
   (f) coals that are hot but not glowing?
4. Place these forms of electromagnetic radiation in order of increasing wavelength: ultraviolet, infrared, microwaves, gamma rays, visible light, X-rays
5. Which of the following materials can be described as translucent:
   (a) frosted glass
   (b) crystal glass
   (c) cardboard
   (d) steel?
6. The Earth is located 150 million kilometres from the Sun. How long does light from the Sun take to reach the Earth?
7. A light is placed 40 cm from a screen. If the luminous intensity of light falling on the screen is 25 W m^{-2}, what will be the luminous intensity of light on the screen if the light source is placed 70 cm from the screen?
8. A 20 W light bulb illuminates the page of a book with a luminous intensity of 5 W m^{-2}. What will be the luminous intensity incident on the page if the 20 W bulb is replaced with a 50 W bulb?
9. Two light sources, A and B, are placed either side of a white screen. When source A is placed 5 m from the screen, it provides the same luminous intensity as source B does on its side of the screen. If source B is ten times stronger as a light source than source A, how far away from the screen has source B been placed?
10. The Sun has a luminosity of 3.846×10^{26} W and is located 150 million km from the Earth.
    (a) Calculate the intensity of the Sun's light on the Earth.
    (b) Assuming that the Earth is a sphere with a radius of 6370 km, calculate the amount of solar energy that falls on the surface of the Earth each second. (Hint: remember that only half of the Earth's face is illuminated by the Sun at any one time.)
11. Draw light rays to represent light shining from the reading lamp shown in figure 10.14.
12. Draw a diagram showing the normal to the surface at the point where a light ray is incident upon it.

FIGURE 10.14

10.3 Reflection

10.3.1 The Law of Reflection

Reflection is, essentially, the bouncing of light from a surface. Light rays from luminous objects enter our eyes directly but we see non-luminous objects because some of the light rays falling on them from luminous sources are reflected into our eyes. We see the Moon because light from the Sun strikes the surface and bounces (reflects) off it. Some of this reflected light reaches our eye and, so, we see the Moon. Light may bounce from several things before it reaches our eyes. A moonlit river is the result of light from the Sun bouncing off the Moon, and this moonlight then reflecting from the river to us.

The degree to which the light rays falling on a non-luminous object are reflected from it depends upon the nature of the object's surface. In some cases, the surface is such that very little of the light falling upon it is reflected at all — instead, most of the energy from the light rays is absorbed by the surface.

For example, when light from the Sun falls on a black bitumen road, most of the light is absorbed, so very little of the Sun's light is reflected from it to our eyes. As a result, we perceive the bitumen road to be very dark. Conversely, almost all the sunlight falling on packed snow is reflected, so it appears very bright to our eyes.

Under ideal conditions, a surface may be so smooth as to reflect nearly all the light incident upon it, allowing a clear image to be formed.

The geometry of perfect reflection is summarised in the **Law of Reflection**, which states that *the angle between the incident light ray and the normal where it strikes the surface will be equal to the angle between the normal and the reflected ray.*

We can express this mathematically as

$$i = i'$$

where i is the incident angle formed between the incoming ray and the normal, and i' is the angle of reflection.

FIGURE 10.15 Light rays from a luminous object hitting a book and reflecting into an eye.

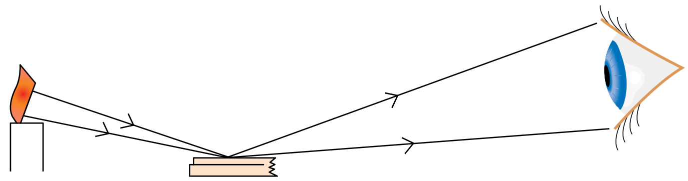
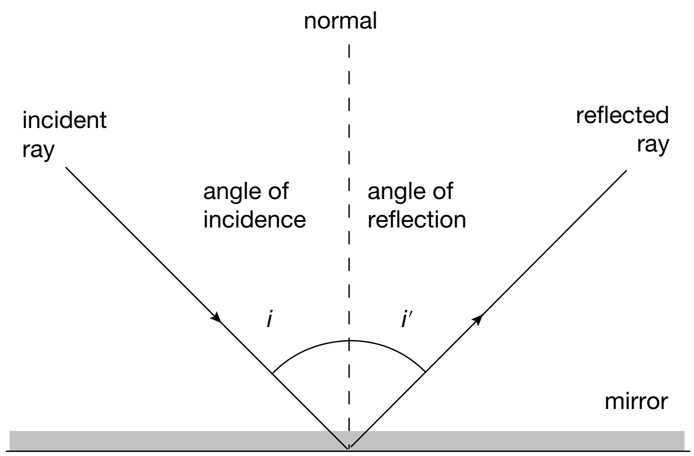
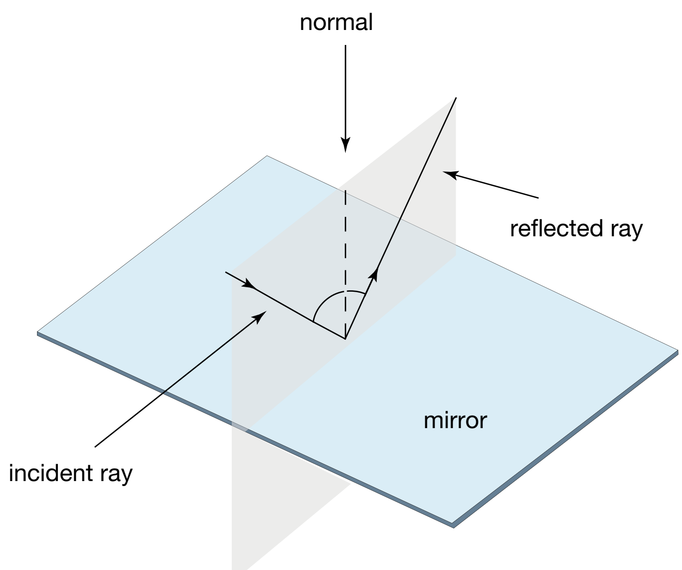


FIGURE 10.16 Diagram showing incident and reflected rays for a plane surface.





10.3.2 Types of reflection

Reflection of light rays from a non-luminous surface can be described as being either specular or diffuse.

Specular reflection (also referred to as **regular** reflection) is observed from surfaces that are very smooth or highly polished, such as still water and good-quality mirrors. When parallel rays strike these surfaces, each ray is reflected evenly from the surface so the reflected rays are also parallel to each other. The more polished and even the surface, the less we see the surface itself and the more we see the reflection of other objects in it instead.

Surfaces that are irregular will not reflect parallel incident rays uniformly but will scatter them. We call this type of reflection **diffuse** (or **irregular**) reflection.

When each incident ray strikes the surface, it obeys the Law of Reflection. However, as the section of surface that each ray strikes is angled differently due to the surface irregularity, the normal for each point of incidence will not be parallel to the normal for the next section. As the angle of incidence for each ray is different, the reflected angles for the rays will also differ, causing the reflected light to be scattered. Only some of these reflected rays will reach our eyes. Thus, while a very white piece of 'smooth' paper will reflect enough diffuse rays to our eyes to appear bright and light, the irregularity of those rays prevents the formation of a coherent reflected image in it.

FIGURE 10.17 Regular reflection occurs when parallel incident rays are reflected from a surface in such a way that the reflected rays are also parallel.

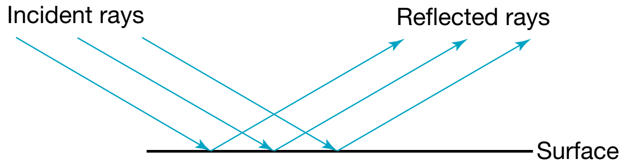


FIGURE 10.18 Aurora Borealis reflected in a perfectly calm fjord on a cold winter night.

10.3.3 Images formed by plane mirrors

When you look in the mirror, what you see is a reflected image of your face. Assuming that the mirror you are looking at is a plane (flat) mirror, you can see that the image is about the same size as you would expect your head to be when viewed at that distance, and it is the right way up. You might also notice that the image appears to be behind the mirror at the same distance behind it as you are in front of it. You know that there is not really an image behind the mirror. That space is probably in another room or outside. The image of your face in the mirror is an optical illusion caused by the reflection of light in the mirror. This image is called a **virtual image**.

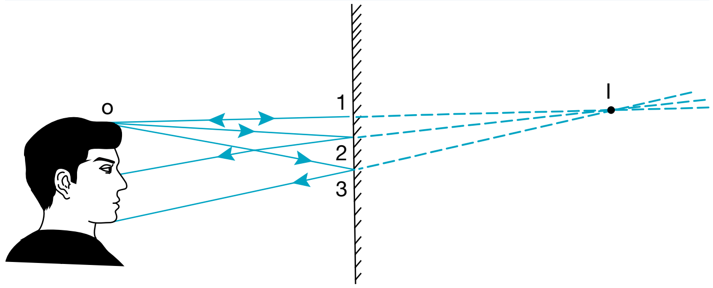
The ray model helps us to understand how this image forms. In figure 10.20 you can see the object (your head) and the mirror from a view to one side. Your head is non-luminous, but because you are in a lit room, light striking your head is diffused in all directions. Consider light striking the top of your head. Some of this light reflects in the direction of the mirror. We can choose to investigate the behaviour of any of the rays that hit the mirror, but let's start with the ray that passes horizontally to the mirror (ray 1). It will reflect with $i = i'$

FIGURE 10.19 The image of your face in the mirror is an optical illusion caused by the reflection of light in the mirror. It is called a virtual image.

so that the reflected ray retraces the path of the incident ray. Since this ray returns to the top of your head, it never actually enters your eye, so it does not contribute to the image formed by your eye.

Now consider what happens to a ray of light that passes from the top of your head to the mirror and reflects back to your eye (ray 2). Again, we know its path because $i = i'$. This ray helps to form the image that your eye sees.

FIGURE 10.20 Locating the image in a plane mirror.



Consider another ray that travels from the top of your head to the mirror and reflects back to touch your chin (ray 3). Again, this ray does not enter your eye.

What we can see is that all three rays can be traced back to a single point behind the mirror. This point, labelled I, is exactly where we see the image of the top of our head in the mirror. There is nothing special about the three rays chosen. Draw any other ray and trace back its reflected ray, and we see that it too appears to come from this point.

Only one ray that we drew enters the eye. How can the eye form an image of the top of the head from a single ray? A ray represents an infinitesimally small beam of light. Many rays of light enter the pupil of the eye, all from slightly different angles, so the eye can interpret them as diverging from a point behind the mirror.

When drawing diagrams such as this, it is much easier to use rays of light that are well spread out, even if they do not enter the eye. It does not make any difference to the result.

We have now located the position of the image of the top of your head using the technique of ray tracing. We can do the same for the chin. See figure 10.22. What we find is that the image is the same size as the object, it is upright and appears to be at the same distance behind the mirror as the object is in front of the mirror. It is also a virtual image, because the light only appears to come from the image. In reality, the light from your head does not pass through the image at all.

FIGURE 10.21 Light diverging from a virtual image to your eye.

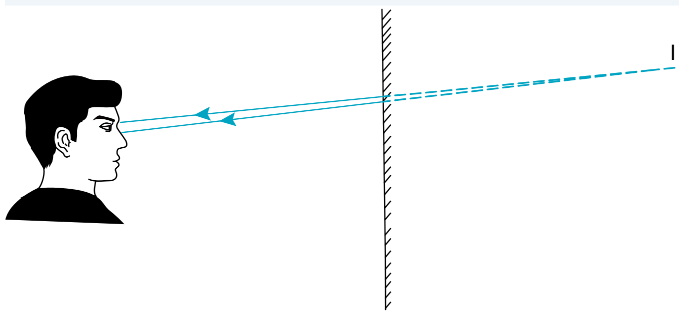
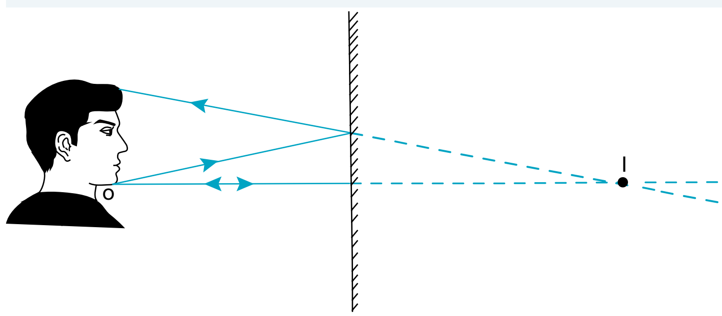


FIGURE 10.22 Locating the image of your chin.



Next we will investigate the formation of other types of images. **Real images** are actually formed by the light rays. These are essential in the eye and in cameras, both of which have sensors that respond to the light of the image. We can only see virtual images because our eyes make real images of the light appearing to come from virtual images.

An interesting fact about plane mirrors is that the image is **laterally inverted**. This means that the left-hand side of the object is the left-hand side of the image, but the image is facing the object. So if you wear a watch on your left hand (the object), the image will have the watch on its right hand. This is simply explained by drawing a ray diagram as seen from above the situation.

Images are not always the same size as their objects. The effect of an optical device on the size of the image is indicated by the magnification:

$$M = \frac{H_1}{H_0}$$

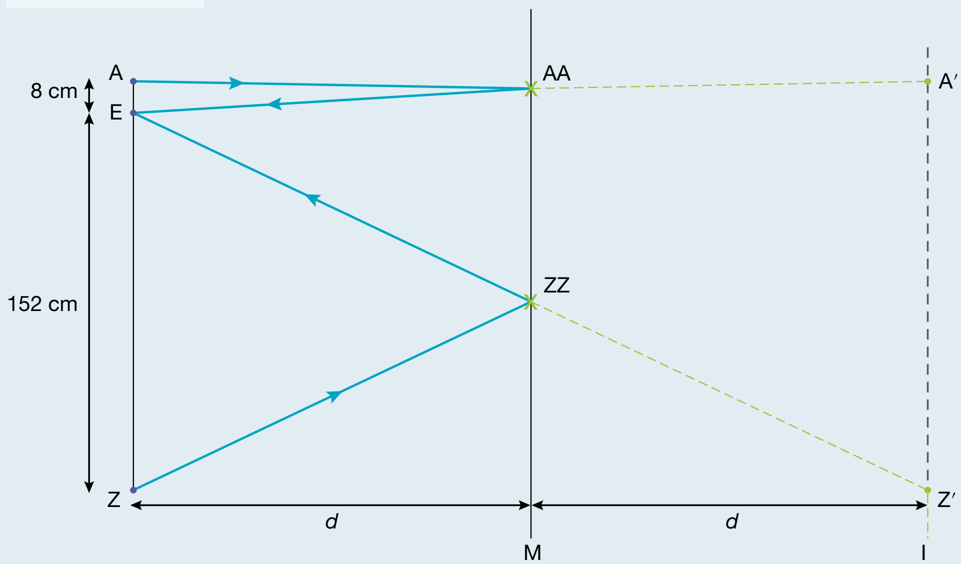
where H_1 is the height of the image and H_0 is the height of the object. As the image in a plane mirror is the same height as the object, the magnification is 1. In a device such as the bottom of a spoon, where the height of the image is smaller than the height of the object, then the magnification is between 0 and 1. This is known as a **diminished** image. When the magnification is greater than 1, the image is said to be **enlarged**. If you look at the reflection of your eye in the **concave** (curved inwards) side of a polished spoon, with your eye very close to the spoon, you may see an enlarged image of your eye.

10.3 SAMPLE PROBLEM 1

Joan is 160 cm tall and her eyes are located a distance 8 cm from the top of her head. What will be the shortest length mirror that she can purchase in order to see a full-length image of herself?

SOLUTION:

FIGURE 10.23



We can represent Joan's length by a line AZ with the point E marking the position of her eyes as shown in the figure above.

We know that her image will be located the same distance from the mirror M as she herself is so. If we say that she is a distance *d* from the mirror when she sees a complete image of herself, then we know her image must be located on the line I.

Light rays leaving A must reflect back to point E, as must those leaving Z if Joan is to see an image of them in the mirror. Joan's eye will see the image of A at A′ and of Z at Z′. The rays reflected from A and *Z* back to E will appear to come from A′ and Z′, and the reflection points will be at AA and ZZ. The distance between AA and *ZZ* is the minimum length of the mirror that Joan needs to buy. Using geometry, we can calculate this distance.

We can see that ZZ will be located at a height halfway between E and Z (which will be 76 cm from the ground), while AA will be located halfway between A and E (a point 4 cm below Joan's head height). This means that AA and ZZ are 80 cm apart.

Hence, Joan will need to buy a mirror that is at least 80 cm long if she is to see a full length-image of herself in it.

10.3 Exercise 1

1 A ray of light strikes the surface of a plane mirror so that the angle between it and the mirror is 35°.
(a) Determine:
(i) the angle of incidence
(ii) the angle of reflection.
(b) What do the incident ray, the normal and the reflected ray all have in common (other than being straight lines)?

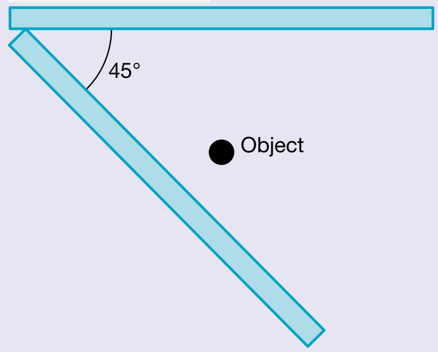
2 A man who is 170 cm tall stands 50 cm in front of a plane mirror mounted on a wall. What is the shortest mirror that could be used if he is to see his entire body reflected? Assume that his eyes are 5 cm down from the top of his head.

3 Explain why the lettering on the front of emergency vehicles is written back-to-front.

4 Explain why you can see your reflection in a highly polished sheet of silver metal but not in a sheet of paper.

5 An object is placed between two mirrors that form a 45° angle between them. Copy the diagram in figure 10.24 and use ray tracing to locate all the images formed in the mirrors.

FIGURE 10.24

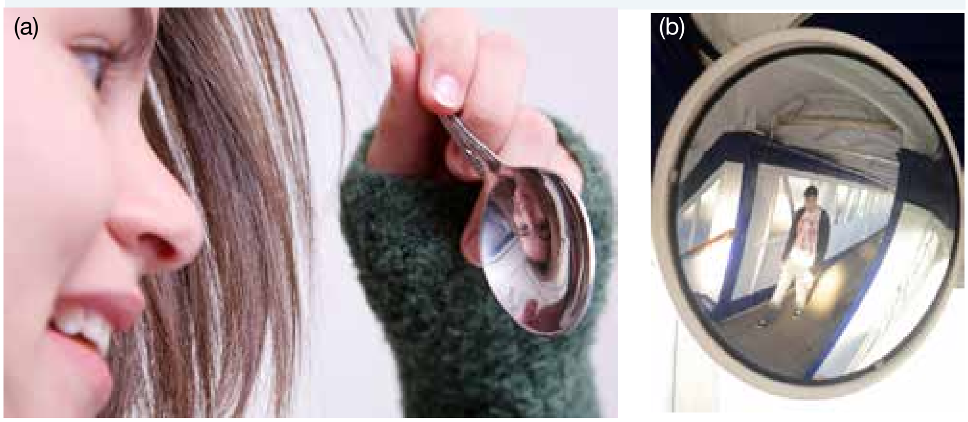


10.4 Curved mirrors

10.4.1 Concave and convex mirrors

Sometimes mirrors are used for purposes that require them to be curved rather than flat plane mirrors. Curved mirrors can be either **concave** (with the polished surface on the interior curve like inside the bowl of a spoon, as shown in figure 10.25a) or **convex** (where the polished surface is on the outside curve, as shown in figure 10.25b).

FIGURE 10.25 Reflections in concave and convex surfaces.



Most curved mirrors are shaped as sections from a sphere or an ovoid; for the moment, we will concentrate on spherical mirrors only.

To better understand how curved mirrors reflect incident rays, consider a concave mirror to perform similarly to a series of plane mirrors arranged in a curve, as shown in figure 10.26.

Parallel incident rays striking these individual plane mirrors will be each reflected according to the angle at which they strike the mirror. For each mirror, the angle at which the reflected ray leaves the mirror surface will be equal to the incident angle. As each mirror is arranged at a different angle to the one beside it, the reflected rays will not be parallel; rather, they meet at a common point referred to as the **focus, F**, of the mirror.

Because concave mirrors cause parallel rays to come together to a point (or converge), they are also referred to as **converging** mirrors (see figure 10.27).

Convex mirrors, on the other hand, cause parallel rays striking the surface to be spread out on reflection, and so they are called **diverging** mirrors (see figure 10.28).

FIGURE 10.26

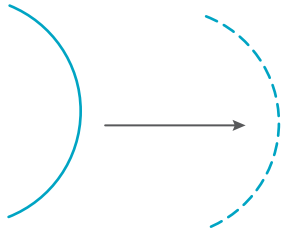
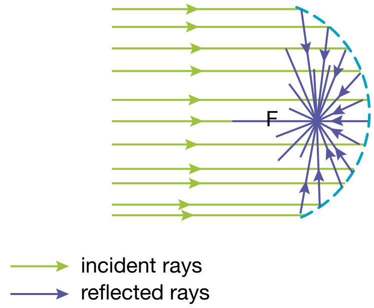
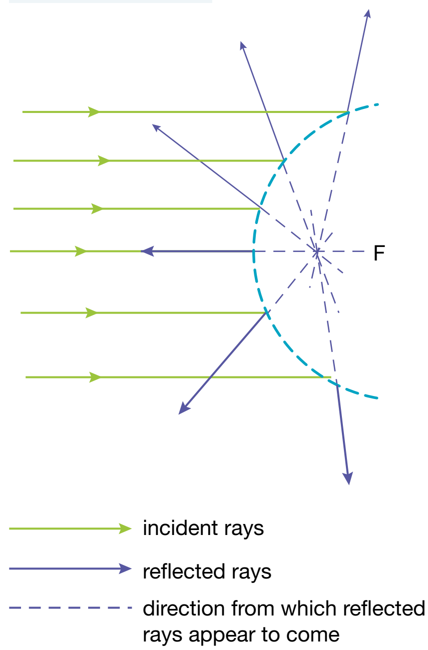


FIGURE 10.27



The spreading rays reflected from a convex mirror appear to originate at a common point inside the convex mirror. As for the concave mirror, this common point is referred to as the focus. The main difference between the foci of the two mirror forms is that reflected light rays actually intersect at the focus of the concave mirror while, for the convex mirror, the focus is virtual and the reflected light rays do not intersect there.

FIGURE 10.28



10.4.2 Mirror terminology

The geometry of concave (and convex) mirrors is critical to the way in which they focus incoming light and form images, so it is useful to describe the main features of a spherical mirror as follows:

- The **centre of curvature (C)** is the geometric centre of the sphere of which the curved mirror is a section.
- The **optical centre (O)** is the centre of the curved mirror's face.
- The **radius of curvature (R)** is the radius of this sphere; this will be the distance between the centre of curvature and the geometric centre of the mirror.
- The **principal focus (F)** is the point at which the reflected rays converge when the incident rays are parallel to the principal axis (concave mirror) or the point from which diverging reflected rays appear to originate (convex mirror).
- The **principal axis** is the line upon which the centre of curvature, the principal focus and the optical centre lie.
- The **focal length (f)** is the distance between the principal focus and the optical centre. For a spherical mirror, the focal length is half the radius of curvature.

FIGURE 10.29 The geometric features of a spherical mirror.

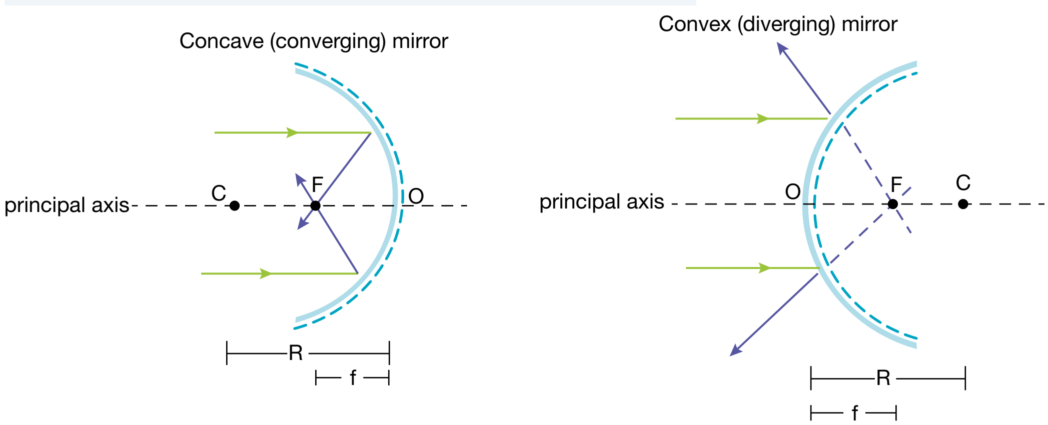
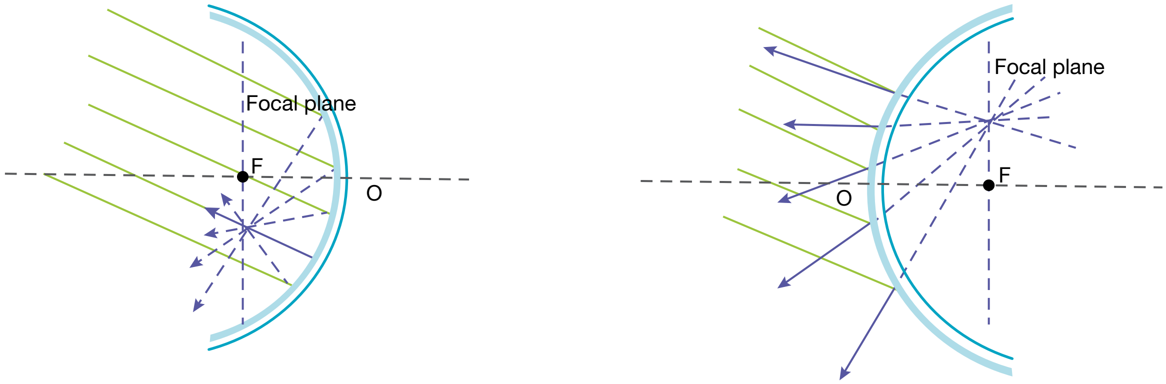


FIGURE 10.30 The focal plane of a spherical mirror.



Parallel rays incident on a spherical mirror but which do not approach parallel to the principal axis will still be focused at a point. However, rather than intersecting at the focal point of the mirror, the reflected rays intersect at a position on the focal plane.

10.4.3 Ray tracing

When an object is placed in front of a curved mirror, an image may be formed. This image may be real or virtual depending upon both the distance the object is placed in front of the mirror and the type of mirror being used.

Scale diagrams incorporating the paths taken by light rays can be used to determine the characteristics of the image formed by each type of mirror at different object distances. This process is referred to as **optical ray tracing**.

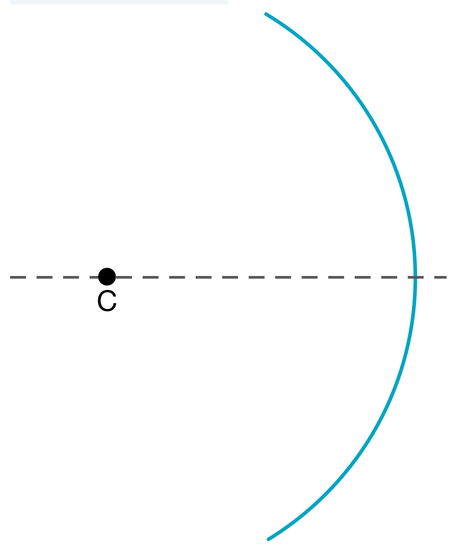
While an infinite number of light rays could be drawn travelling from an object to the mirror, the paths of four rays, in particular, are the most easily traced:

1. Any ray that travels parallel to the principal axis from the object to the mirror will be reflected so that it passes through the mirror's focus.
2. A ray that passes through the focus as it travels from the object to the mirror will be reflected so that it travels parallel to the principal axis.
3. Rays that travel through the centre of curvature as they travel from the object to the mirror are reflected back along their original path.
4. A ray that travels from the object to the optical centre is reflected so that the angle made between the reflected ray and the principal axis is equal to the incident angle.

As an example, let's look at how ray tracing can be used to find the type, height and orientation of an image formed by a converging mirror of an object located between the focus (F) and the centre of curvature (C). In this case, let us assume that the mirror has a focal length of 3 cm.

Step 1. First, a horizontal line is drawn that represents the principal axis. A point is marked on the principal axis to represent the centre of curvature (C) of the mirror. As we know the focal length of the mirror is 3 cm, the radius of curvature of the mirror will be twice that — that is, 6 cm. Using a compass, we draw a part circle with a radius of 6 cm and centred on point C, as shown in figure 10.31.

FIGURE 10.31



Step 2. The focus (F) is marked on the principal axis 3 cm away from the centre of the mirror. The object (represented by a thick arrow, OP) is drawn with its base on the principal axis, as shown in figure 10.32.

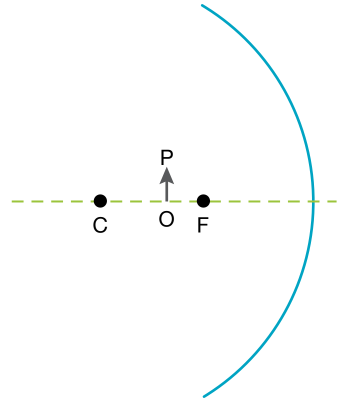
Step 3. Two rays are drawn from the top of the object at P, as shown in figure 10.33:

- Ray 1 leaves the head of the object parallel to the principal axis, is reflected by the mirror and passes back through the focus.
- Ray 2 passes through the focus, is reflected by the mirror, and then travels back parallel to the principal axis.

Where these rays intersect, the image of P (P)′ will be formed. Note that while two, three or even all four of the main ray paths may be employed to locate the image position, the clarity of the diagram drawn can be lost.

Step 4. As OP was placed perpendicularly to the principal axis, the image of O (O)′ will be located on the principal axis such that O′P′ is also perpendicular, as shown in figure 10.34.

FIGURE 10.32



The image O′P′ formed is a real image because light rays actually pass through it. If a screen was placed at that location, the clear image would be seen upon it.

The image is upside-down compared to the object, so we say that it has been inverted.

The image can also be described as **enlarged** or **dilated** because the height of O′P′ is greater than that of OP. The image is located at a distance greater than 2f.

In a similar way, scaled ray diagrams can be drawn to determine the location (d_i), height (h_i) and the nature of the image formed by an object placed at a distance d_o from a converging mirror and having a height of h_o. The formation of these images is summarised in Table 10.1

FIGURE 10.33

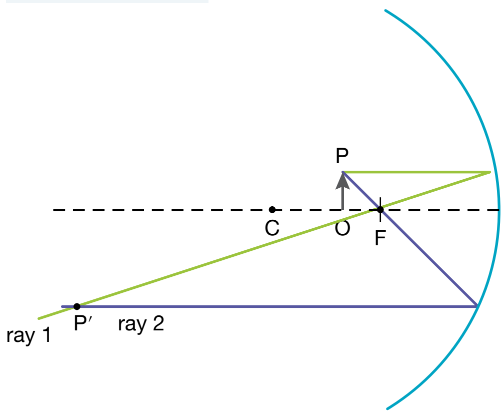


FIGURE 10.34

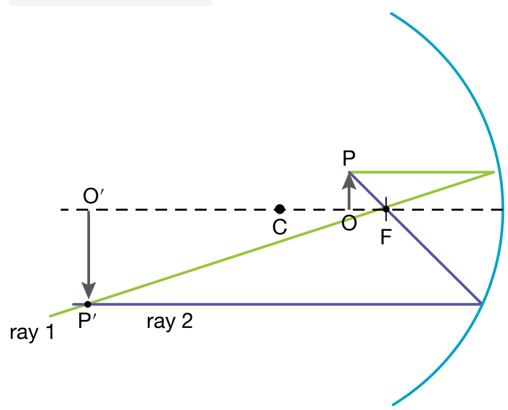
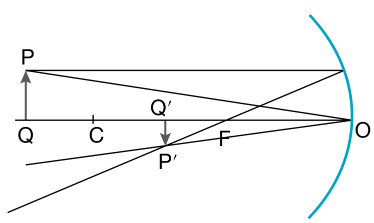
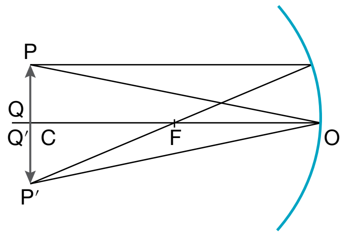
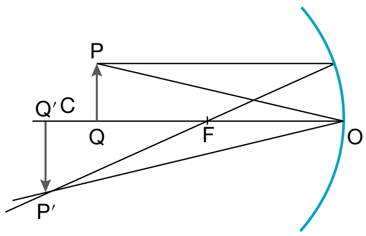
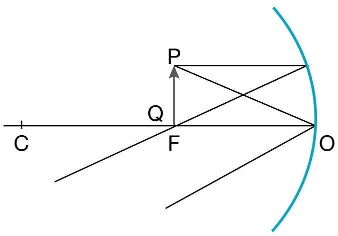
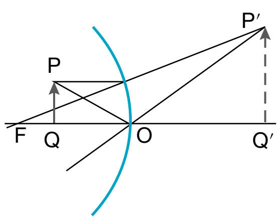


TABLE 10.1 Images formed by a converging (concave) mirror.

Ray diagram	Object position	Image position	Image inverted or upright?	Image size	Image real or virtual
	$d_o > 2f$	$2f > d_i > f$	inverted	$h_i < h_o$	real

(*continued*)

TABLE 10.1 Images formed by a converging (concave) mirror.

Ray diagram	Object position	Image position	Inverted or upright?	Size	Real or virtual
	$d_o = 2f$	$d_i = 2f$	inverted	$h_i = h_o$	real
	$2f > d_o > f$	$d_i > 2f$	inverted	$h_i > h_o$	real
	$d_o = f$	$d_i = \infty$	(no image formed)		
	$d_o < f$	$d_i > -2f$	upright	$h_i > h_o$	virtual

While concave mirrors can form both real and virtual images depending upon how close the object is to the mirror, convex mirrors can only form virtual images.

Ray diagrams for convex mirrors are drawn in much the same way as for concave mirrors. However, as convex mirrors have no true focus, the reflected paths taken by rays travelling from the object to the mirror will be related to the virtual focus, which appears to lie inside the mirror.

When an object PQ is placed in front of the diverging mirror, as shown in figure 10.35, we consider the paths taken by two rays leaving P and travelling to the mirror, as shown in figure 10.36:

- Ray 1 leaves P and travels parallel to the principal axis. On reaching the mirror, the ray is reflected so it appears to travel through the focus.
- Ray 2 travels in a straight line directed from P to the focus; on striking the mirror, the reflected ray travels parallel to the principal axis.

FIGURE 10.35

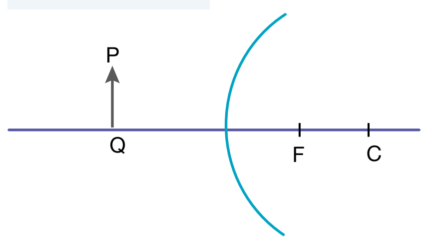
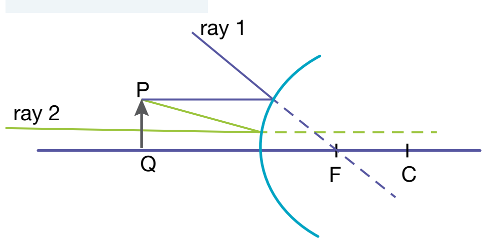


FIGURE 10.36



The intersection of the continuing lines of ray 1 and ray 2 marks the location of the image of P (P′). As PQ is perpendicular to the principal axis, Q′ can be located directly underneath P′, as shown in figure 10.37.

The virtual image P′Q′ formed in this case is smaller than the object PQ and is positioned closer to mirror.

All images formed by convex mirrors are:

- virtual
- upright
- reduced in height.

FIGURE 10.37

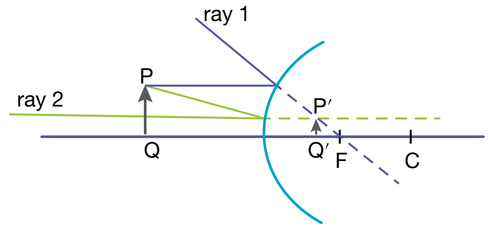


FIGURE 10.38 Convex mirrors are often used to help improve vision around corners.

10.4.4 The mirror equations

While ray tracing provides qualitative information about the position, size and nature of the images formed by concave and convex mirrors, more precise numerical information about the position and size of the images can be obtained by use of the mirror equation and the magnification equation.

The mirror equation relates the distances of the object (d_o) and the image (d_i) from the mirror to the mirror's focal length f:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

In general:

- d_o has a positive value
- for a concave mirror, f is a positive value (as it has a true focus where light rays intersect) while a convex mirror has a negative value for the focal length
- for a virtual image, d_i will have a negative value. This is the case when an object is placed within the focus of a concave mirror, and for all images formed by a convex mirror.

The magnification M describes the height of the image (h_i) relative to the height of the object (h_o):

$$M = \frac{h_\mathrm{i}}{h_\mathrm{o}}$$

The magnification can also be determined from the distances of the object and the image from the mirror:

$$M = -\frac{d_i}{d_o}$$

For a concave mirror, all real images will be inverted and h_i has a negative value, while virtual images are upright and h_i has a positive value.

By combining the two equations, we find a third form of the magnification equation:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

10.4 SAMPLE PROBLEM 1

A 4 cm high object is placed 10 cm in front of a converging mirror having a radius of curvature of 16 cm.

(a) Calculate where the image forms relative to the mirror.
(b) Calculate the height of the image.
(c) Determine whether the image is
  (i) real or virtual,
  (ii) upright or inverted, and
  (iii) enlarged or reduced.

SOLUTION:

(a) As the focal length of a spherical mirror is equal to half the radius of curvature,

$$f = \frac{16\text{ cm}}{2} = 8\text{ cm}$$

As the mirror is a converging mirror, the focal length will have a positive value.
Substituting values into the mirror equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{8\text{ cm}} = \frac{1}{10\text{ cm}} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{8\text{ cm}} - \frac{1}{10\text{ cm}}$$

$$\frac{1}{d_i} = \frac{1}{40\text{ cm}}$$

$$d_i = 40\text{ cm}$$

The image will be located 40 cm from the mirror.

(b) Using the magnification equations:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{4\text{ cm}} = -\frac{40\text{ cm}}{10\text{ cm}}$$

$$h_i = -4 \times 4\text{ cm}$$

$$= -16\text{ cm}$$

The image has a height of 16 cm.

(c) (i) As $d_i > 0$, a real image has formed

(ii) As $h_i < 0$, the image is inverted

(iii) The image is 16 cm high while the object is only 4 cm high, therefore the image is enlarged.

10.4 SAMPLE PROBLEM 2

A 5 cm object is placed 12 cm from a diverging mirror that has a focal length of 10 cm. Determine the location, height and nature of the image formed.

SOLUTION:

As the mirror is convex (diverging), the focal length will have a negative value: $f = -10$ cm.
Substituting values:

$$\frac{1}{-10\text{ cm}} = \frac{1}{12\text{ cm}} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = -\frac{1}{10\text{ cm}} - \frac{1}{12\text{ cm}}$$

$$\frac{1}{d_i} = -\frac{11}{60}\text{ cm}$$

$$d_i = -\frac{60}{11}\text{ cm}$$

$$= -5.4\text{ cm}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{5\text{ cm}} = -\frac{-5.4\text{ cm}}{12\text{ cm}}$$

$$h_i = 2.25\,\text{cm}$$

$$M = \frac{h_i}{h_o} = \frac{2.25\ \text{cm}}{5\ \text{cm}} = 0.45$$

Therefore, the image is 2.25 cm high and it appears to form 5.4 cm inside the mirror. The image is virtual (as $d_i < 0$), upright (as $h_i > 0$) and reduced in size (as $M < 1$).

WORKING SCIENTIFICALLY 10.3

Archimedes is famously credited with using a series of spherical concave mirrors to set enemy ships alight. Some translations of the story give the distance from the mirrors to the ships as being equivalent to 1.5 km. Assess the feasibility of this story by considering the distance over which such a feat might be possible, the diameter of the mirrors, the number of mirrors needed, their radius of curvature, and the temperature achievable by focusing the rays of the Sun.

10.4 Exercise 1

1. Dentists often place a small mirror inside the patient's mouth to examine their teeth. Is the mirror used more likely to be concave, convex or flat?
2. A lit match is placed 5 cm from a converging mirror that has a radius of curvature of 10 cm. Which best describes the image:
   (a) real, reduced and inverted
   (b) real, enlarged and inverted
   (c) virtual, enlarged and upright
   (d) no image is formed?
3. Which of the following statements is true:
   (a) concave mirrors can only form real images
   (b) convex mirrors can only form virtual images
   (c) the image formed by a plane mirror is always the same size as the object
   (d) the image formed in a plane mirror always appears to be the same distance from the mirror as the object
   (e) concave mirrors can be used as magnifying makeup mirrors
   (f) an object placed at the focus of a converging mirror reflects rays parallel to the principal axis
   (g) the Law of Reflection is only true for plane mirrors and does not apply to spherical mirrors?
4. A 3 cm high object is placed 6 cm from a diverging mirror with a focal length of 4 cm. How high is the image formed as a result?
5. An object placed 6 cm from a converging mirror forms a real image 10 cm from the surface of the mirror. What is the mirror's focal length?
6. How far from a concave spherical mirror with a focal length of 12 cm must an object be placed to produce a virtual image that is 3 times larger than the object?
7. Use ray tracing to show the approximate location and nature of the image formed when an object is placed 4 cm in front of a spherical converging mirror with a focal length of 3 cm.
8. Use ray tracing to show the approximate location and nature of the image formed when a 6 cm high object is placed in front of a spherical diverging mirror with a focal length of 4 cm.

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10.5 Refraction

10.5.1 The speed of light

Visible light travels in a vacuum at the same speed as all other electromagnetic radiation — 3×10^8 m s^{-1}. When it encounters any other medium, it will slow down. The degree to which the speed of light is slowed when it moves through a material is described by the **absolute refractive index (n)** of the material. This value is the ratio of the speed of light in a vacuum (c) compared to its speed in the medium (v):

$$n = \frac{c}{v}$$

Table 10.2 shows the absolute refractive indices of some common media.

TABLE 10.2 Absolute refractive indices.

Material	Index of refraction
Vacuum	1.00
Air*	1.00
Water	1.33
Quartz	1.46
Car headlight glass	1.48
Perspex (average)	1.50
Window glass	1.51
Crystal wineglass (24% lead)	1.54
Diamond	2.42

* The slowing of light in air is fairly small and, for most cases, can be assumed to be negligible.

10.5 SAMPLE PROBLEM 1

Light travels at a speed of 2.26×10^8 m s^{-1} in water. Calculate water's absolute refractive index.

SOLUTION:

$$n = \frac{3 \times 10^8 \text{ m s}^{-1}}{2.26 \times 10^8 \text{ m s}^{-1}}$$
$$= 1.33$$

The refractive index of water is 1.33. This means that light travels 1.33 times faster in air than it does in water.

10.5.2 The bending of light

Refraction refers to the bending of light that occurs when light travels through transparent media that have different refractive indices. The reason that the light bends is connected to the fact that light travels at different speeds in different media.

We're going to use an analogy at this point to help us understand how a changing speed leads light to bend when travelling through different media.

You may have noticed that a four-wheel drive travels faster over packed wet sand on the beach than it does over dry loose sand. In this way, the four-wheel drive is much like light in that it will travel more slowly through some media than others. Now, let's say that the four-wheel drive is travelling along a section of wet sand when it comes to a section of dry sand. It is headed towards the demarcation line between the two types of sand at an angle i as shown in Figure 10.39a.

The first tyre to hit the dry sand will be the right front tyre in our diagram. As soon as it enters the dry sand region, it will start turning more slowly than the other wheels (figure 10.39b). This has the effect of causing the front of the car to be dragged off course, and it will veer to the right as it enters the dry sand (figure 10.39c). As a result, the course of the car has been altered.

Light entering a new medium will behave similarly to the four-wheel drive on the beach. If we could look at the light waves as they strike the interface between media, we would see that they too are diverted from their course.

FIGURE 10.39 A four-wheel drive entering an area of dry sand.

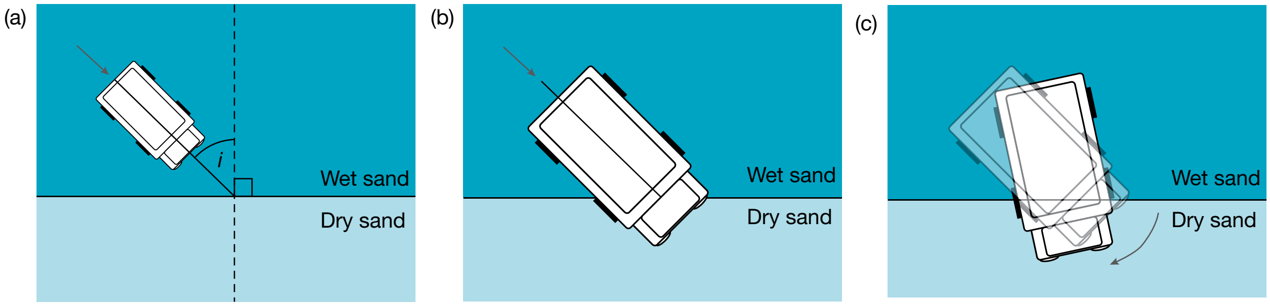
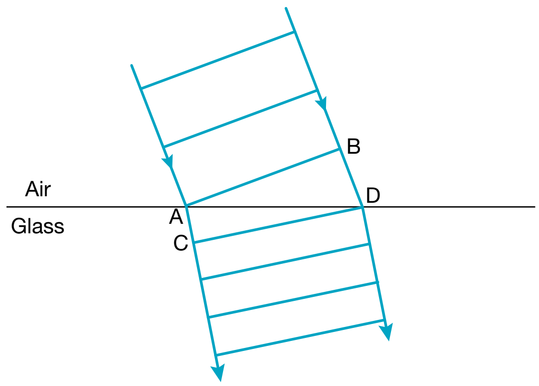


Figure 10.40 shows a light wave entering a medium in which it travels more slowly. The line AB represents a wavefront approaching the interface between air and glass. The section of the wavefront at A strikes the boundary before that — at point B. On entering the glass, the light waves at A will slow down while the rest of each wave continues to travel through air at the original, faster speed. During the time taken for the waves at A to travel to position C in the new medium, the waves at the other end of the wavefront have travelled a larger distance from B to D. As a result, the wavefront changes direction as it crosses the interface.

FIGURE 10.40 The refraction of light at a boundary between different media.



The extent to which light is bent when it enters a second medium depends upon the speed of light in the individual media. As you will recall, the speed of light in a medium can be related by the absolute refractive index (n) of that material.

When light strikes an interface between media at an angle i (which is the angle between the incident ray and the normal), it will be refracted so that the transmitted light will travel at the refracted angle r (the angle between the refracted light and the normal). If light travels from a lower refractive index medium to a medium with a higher refractive index, it will bend towards the normal — that is, if $n_2 > n_1$ then $r < i$. Conversely, if the second medium has a refractive index that is lower than that of the first medium, then the light will be bent away from the normal as it is transmitted — that is, $r > i$ if $n_2 < n_1$.

FIGURE 10.41 Light entering a medium with a higher refractive index will be bent towards the normal as it is transmitted.

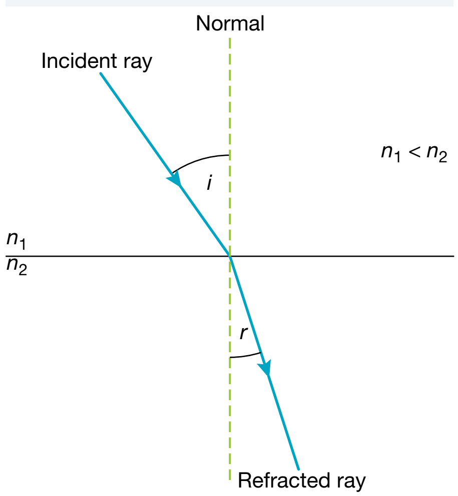
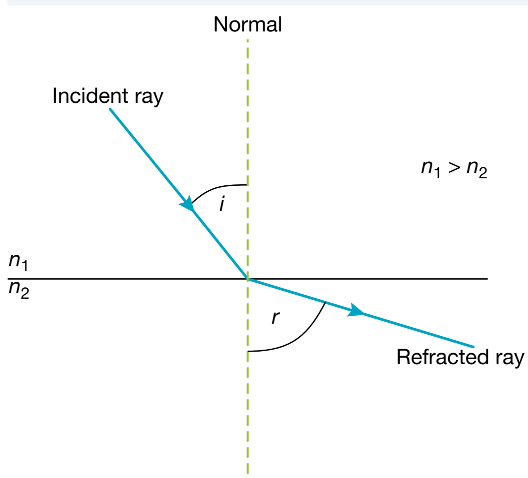


FIGURE 10.42 Light entering a medium with a lower refractive index will be bent away from the normal as it is transmitted.



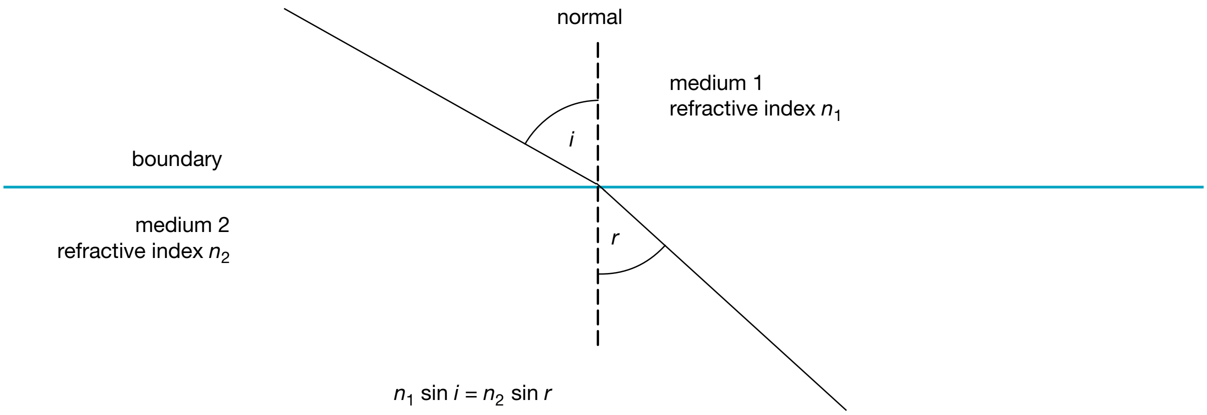
10.5.3 Snell's Law

The extent to which light is bent when it changes medium can be determined using **Snell's Law**, which was first formulated in approximately 1621 by the Dutch scientist Willebrord Snell:

$$n_1 \sin(i) = n_2 \sin(r)$$

where n_1 is the refractive index of the incident medium, i is the incident angle, n_2 is the refractive index of the new medium, and r is the angle of refraction.

FIGURE 10.43 A graphical depiction of Snell's Law for any two substances. Note that the light ray has no arrow, because the relation is true for the ray travelling in either direction.



10.5 SAMPLE PROBLEM 2

A ray of light strikes a glass block of refractive index 1.45 at an angle of incidence of 30°. What is the angle of refraction?

SOLUTION:

$$\begin{aligned}
1.0 \times \sin 30° &= 1.45 \times \sin\theta_{\text{glass}} \text{ (substitute values into Snell's Law)}\\
\sin\theta_{\text{glass}} &= \frac{\sin 30°}{1.45} \text{ (divide both sides by 1.45, the refractive index of glass)}\\
&= 0.3448 \text{ (calculate value of expression)}\\
\theta_{\text{glass}} &= 20.17° \text{ (use inverse sine to find the angle whose sine is 0.3448)}\\
\theta_{\text{glass}} &= 20° \text{ (round off to two significant figures)}
\end{aligned}$$

10.5 Exercise 1

1. Light travels at a speed of 2.1×10^8 m s^{-1} through medium X. What is this medium's refractive index?
2. What will be the frequency of violet light ($\lambda = 420$ nm) as it passes through window glass?
3. Calculate the speed at which light travels through diamond.
4. How many times faster does light travel through glass than it does diamond?
5. Which of these statements is true:
   (a) light rays entering a new medium change frequency
   (b) light rays travel through glass at a lower speed than they do through a vacuum
   (c) light rays entering a medium with a higher refractive index will be bent towards the normal
   (d) light rays directed at right angles to the boundary between two media are not refracted?
6. Light travelling from water into glass ($n_{\text{glass}} = 1.53$) is refracted at an angle of 49°. At what angle was the light incident upon the glass?

7 If a laser light is shone onto a pool of water at an incident angle of 15° to the normal, what will be its angle of refraction?
8 A beam of light shines onto a glass slab (n_{glass} = 1.51) that has a thickness of 4 cm. If the beam makes an angle of 30° with the slab surface, how far horizontally will the beam exit the block from where it entered?
9 A ray of light enters a plastic block at an angle of incidence of 40°. The angle of refraction is 30°. What is the refractive index of the plastic?
10 In a science fiction story, a transparent material called 'slow glass' can slow down light rays entering the material so much that they can take years to emerge from the other side. What would the refractive index of such a material be if light entering a 20 cm thick pane of the glass took one day to emerge from the other side?

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10.6 Lenses

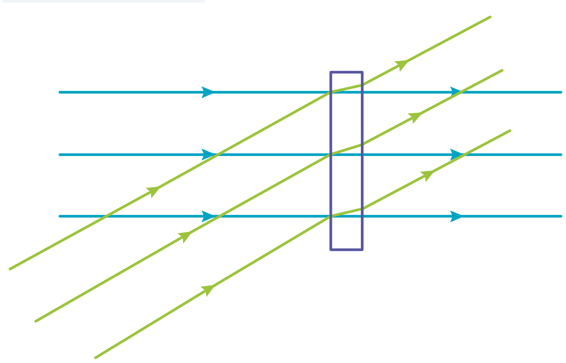
10.6.1 Converging and diverging lenses

The word 'lens' is a familiar one to anyone who wears glasses or has ever used a microscope or telescope. A lens describes any transparent optical object with a curved surface that refracts light as it transmits it, allowing redirection.

To begin to understand a lens, we can start with a rectangular block of glass as in figure 10.44. Parallel rays from the left pass through the block without a change in direction if they are normal to the block (blue lines). Parallel rays that are not normal to the block are refracted when passing through the block, but emerge parallel on the other side (green lines). This is essentially what happens with light passing through a pane of glass in a window.

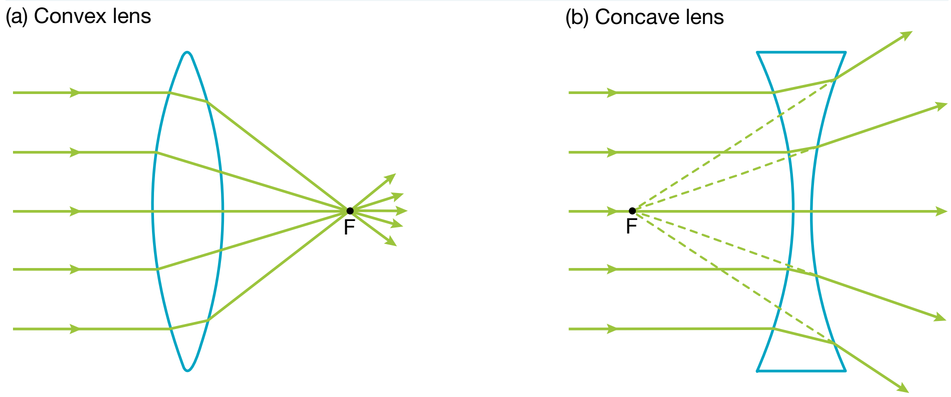
If the block is shaped so that its surface is a continuous curve in the arc of a circle (figure 10.45a), all sets of parallel rays entering the lens converge on the other side.

FIGURE 10.44



Lenses come in a variety of different forms, but can be generally classified as being either converging or diverging.

FIGURE 10.45 Refraction of rays through (a) a convex and (b) a concave lens.



The lens form we have considered so far is referred to as a **converging** lens. A converging lens causes parallel light rays passing through it to be refracted towards a single point. As for a converging mirror, this intersection point is referred to as a focus. The converging lens comes in several forms: the bi-convex, which has convex surfaces on each side; the plano-convex, which has a convex shape on one side but is flat on the other; and the converging meniscus, which is convex on one side but concave on the other. Regardless of their variations in shape, all converging lenses are thicker in the middle than at their edges.

FIGURE 10.46 The converging lens in its different forms: (a) bi-convex, (b) plano-convex, (c) convex meniscus.

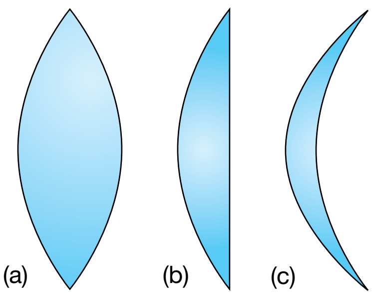
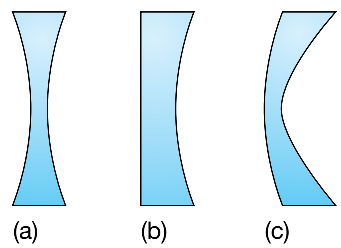


FIGURE 10.47 The diverging lens in its different forms:
(a) bi-concave,
(b) plano-concave,
(c) diverging meniscus.



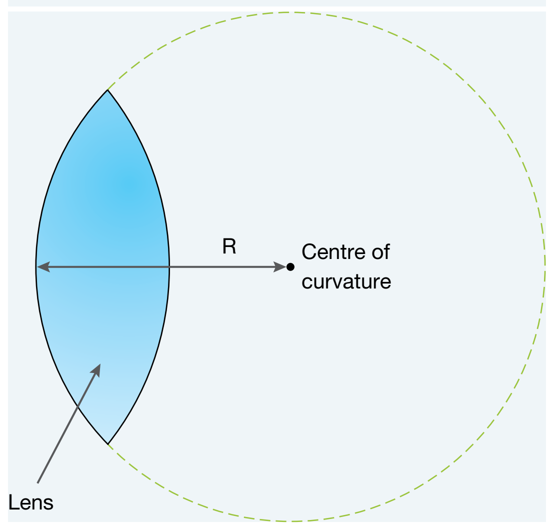
A **diverging** lens causes parallel light rays to be spread further apart after being refracted. The diverging rays appear to come from a focus on the opposite side of the lens. Diverging lenses are thicker at their edges than in their centres and can have a variety of forms: the bi-concave, which has both of its faces concave; the plano-concave, where one of the lens's faces is flat while the other is concave; and the diverging meniscus, which has both concave and convex faces.

10.6.2 Lens terminology

Many of the terms we will encounter in our study of lenses will be familiar from our earlier study of mirrors:

- The **optical centre** (or **pole**) of a lens is the point in the exact centre of the lens itself. Light rays that pass through the optical centre of a lens will not be diverted, but will continue undeflected.
- The **centre of curvature (C)** for the face of a lens is the centre of the circle, an arc of which corresponds to the curve of the lens face. A flat face of a lens has a centre of curvature located at infinity.
- The **radius of curvature (R)** is the distance between the centre of curvature and the surface of the lens.

FIGURE 10.48 The centre of curvature for a lens face.



- The **principal axis** is a line that can be drawn through the centres of curvature for both faces of a lens and the optical centre.
- The **focus (F)** is the point at which light rays entering the lens parallel to the principal axis converge (or, in the case of a diverging lens, appear to originate) on exiting the lens. Because light can pass through either side of a lens, there is one focus on each side.
- The **focal length (f)** is the distance between the optical centre and the focus. The focal length of a lens depends upon the curvature of the lens faces, the thickness of the lens and the material from which it is made. In general, the greater the curvature of the lens face, the shorter the focal length.
- The **focal plane** is a plane through the focus that is perpendicular to the principal axis. When rays that are parallel to one another enter the lens at an angle to the principal axis, they will converge at some point on the focal plane.

FIGURE 10.49 The lens's radius of curvature influences the location of the focus.

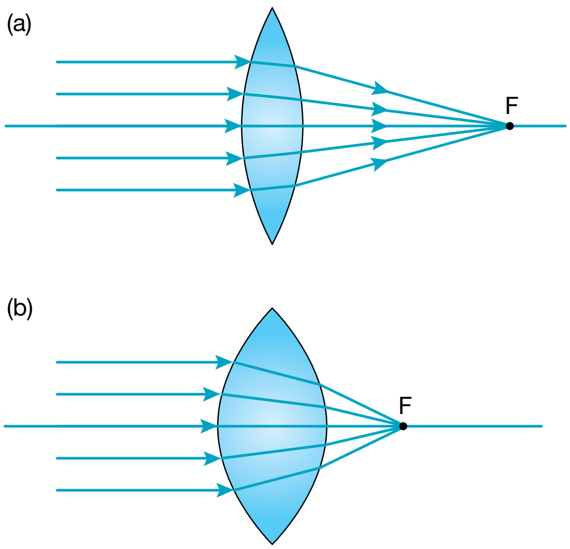
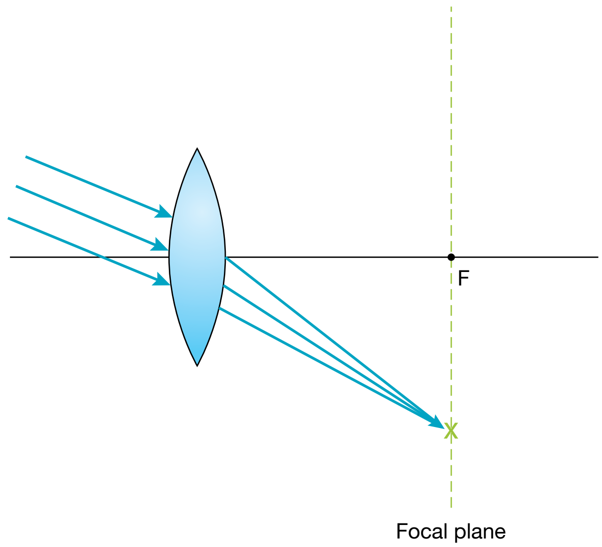


FIGURE 10.50 Parallel rays entering a lens at an angle to the principal axis will converge on the focal plane.



10.6.3 Images formed by converging lenses

Light rays passing from an object through a converging lens can form images of that object. However, the orientation, size and nature of that image depend on how far the object lies from the lens and the focusing ability of the lens itself.

As in the earlier section on images formed by mirrors, ray tracing can be used to give a qualitative impression of the size and location of an image formed by a lens, as well as the nature of the image.

Four main principles are observed when using ray tracing for converging lenses:

1. Incident rays that travel parallel to the principal axis when approaching the lens will be refracted to pass through the focus on the other side.
2. Incident rays that pass directly through the focus on the side nearest to the object as they approach the lens will be refracted to pass parallel to the principal axis on the other side.
3. Incident rays that pass through the optical centre (pole) of a thin lens and that are incident at small angles to the principal axis continue to travel in the same direction.
4. Images form where rays converge.

By considering these principles, a ray diagram can be drawn for an object PQ placed outside the focal length of a converging lens and the image's relative size and location determined. It should be noted that, while refraction of rays occurs at each boundary between the air and the lens, in reality, the lenses in this text are considered to be very thin. As a result, by convention, the bending of the light rays within the lens is represented by a single refraction at the **lens axis** (a line that passes through the pole of the lens that is perpendicular to the principal axis).

From figure 10.51, we see that the image P′Q′ is located at a position greater than 2f and that it is enlarged and inverted. The image is formed on the opposite side of the lens and is described as real. This means that, should a screen be placed at the image position, an image will form on that screen.

The image obtained depends on the placement of the object in relation to the focus. A range of these applications is given in table 10.3.

FIGURE 10.51 The location of the image is determined according to the point where the three rays cross. All the rays that pass through the lens pass through the image.

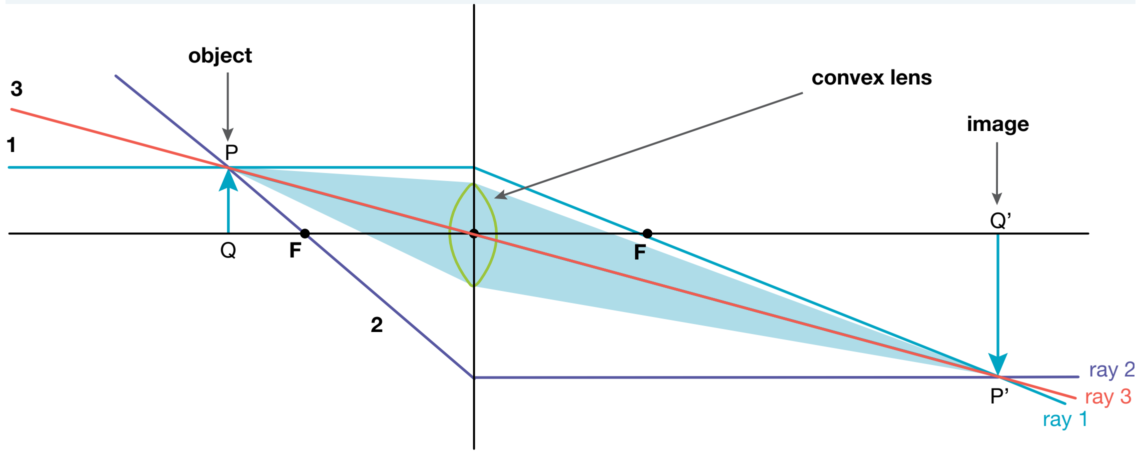


TABLE 10.3 Simple applications of convex lenses

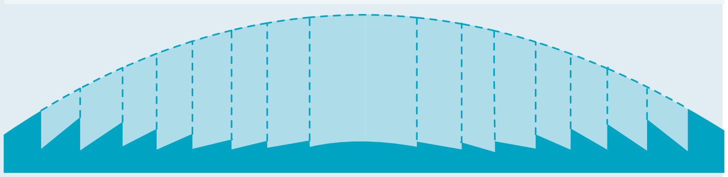
Location of object	Uses	Description of image
Very large distance away from lens	Objective lens of refracting telescope	Real, inverted, diminished and located near the opposite focus
Beyond twice the focal length from lens	Human eye; camera	Real, inverted, diminished and located on other side between one and two focal lengths from lens
At twice the focal length from lens	Correction lens for terrestrial telescope	Real, inverted, same size and located two focal lengths from lens
Between twice the focal length from lens and the focus	Slide projector; objective lens of microscope	Real, inverted, magnified and located on other side of lens beyond two focal lengths
At the focus	Searchlight; eyepiece of refracting telescope	No image. The emerging parallel rays do not meet.
Between focus and lens	Magnifying glass; eyepiece lens of microscope; spectacles for long-sightedness	Virtual, upright, magnified and located on same side of the lens and further away

PHYSICS IN FOCUS

Flat lenses?

A lens works by changing the direction of the light ray at the front surface and then again at the back surface. The glass in the middle is there to keep the two surfaces apart. Augustin-Jean Fresnel devised a way of making a lens without the need for all the glass in the middle.

FIGURE 10.52 A side view of a convex Fresnel lens showing how it is constructed.



The glass surface of the lens is a series of concentric rings. Each ring has the slope of the corresponding section of the full lens, but its base is flat. The slopes of the rings get flatter towards the centre.

This design substantially reduces the weight of the lens, so lenses of this type are used in lighthouses. Their relative thinness means they are also used where space is at a premium, such as in overhead projectors, and as a lens to be used with the ground-glass screens in camera viewfinders.

Flat lenses, or Fresnel lenses as they are called, are now attached to the rear windows of vans and station wagons to assist the driver when reversing or parking.

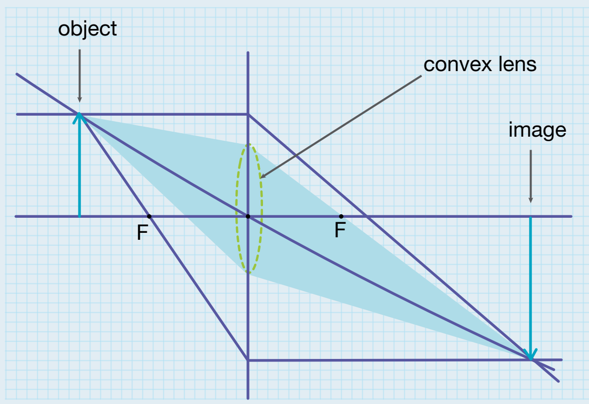
10.6 SAMPLE PROBLEM 1

A convex lens has a focal length of 10 cm. A candle 10 cm tall is located 16 cm in front of the lens. Use ray tracing to determine the location, size, orientation and type of image formed.

SOLUTION:

Draw the principal axis, the focal points, the object and three rays, one passing through the centre of the lens without deviation, one parallel to the principal axis and one passing through the focus.

FIGURE 10.53 The image of the candle is 27 cm on the opposite side of the lens, 15 cm tall, inverted and real.



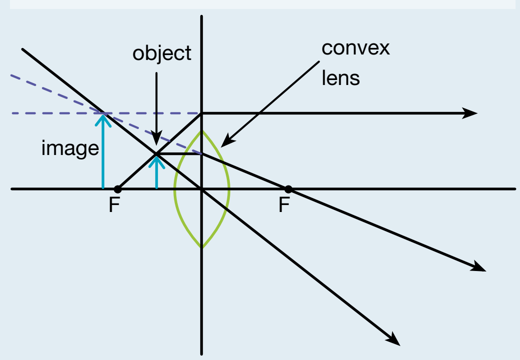
10.6 SAMPLE PROBLEM 2

The candle is moved so that it is now 5 cm in front of the same lens. Use ray tracing to determine the location, size, orientation and type of image formed.

SOLUTION:

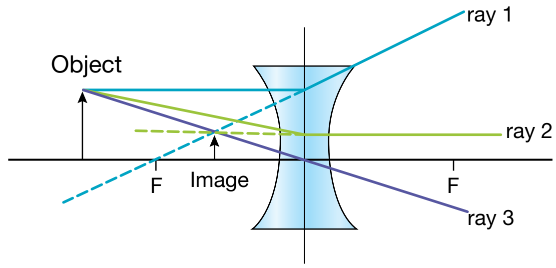
Draw the principal axis, the focal points, the object and the three rays.

FIGURE 10.54 The image of the candle is 10 cm on the same side of the lens, 20 cm tall, upright and virtual.



10.6.4 Images formed by diverging lenses

FIGURE 10.55 The ray diagram for a diverging lens.



The diverging lens shares the same features as the converging lens; however, the diverging lens can only form virtual images, because it can never bring light rays to focus at a point and so form a real image.

Consider an object placed outside the focus of a diverging lens as shown in figure 10.55. By using ray diagrams as we did with converging lenses, we can observe the virtual nature of the image formed by the diverging lens.

In the figure, our first ray approaching the lens parallel to the principal axis is refracted away from the lens axis as it passes through so that the refracted ray appears to come from the focus nearest the object. The second ray is directed towards the focus on the opposite side of the lens. On reaching the lens axis, the emerging ray is directed parallel to the principal axis. The third ray travelling from the top of the object through to the centre of the lens passes through, as before, undiverted.

As the diagram indicates, these three refracted rays will never meet and, so, never form a real image. Instead, they form a virtual image at the location where the three rays seem to have a common origin. The virtual image formed here is smaller than the object, is upright and lies within the focus on the same side of the lens as the object.

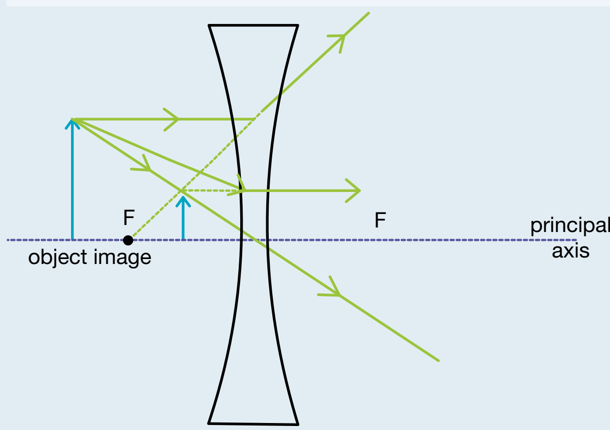
10.6 SAMPLE PROBLEM 3

The lens is switched with a diverging lens with a focal length of -10 cm. What image of the candle is formed when it is placed 15 cm from the lens?

SOLUTION:

Draw the principal axis, focal points, object and the three rays.

FIGURE 10.56 The image of the candle is 6 cm on the same side of the lens, 4 cm tall, upright and virtual.



10.6.5 The thin lens equation

Just as the mirror equations allow more precise evaluations of image size and position than those provided by ray tracing, so too can the thin lens equations allow the calculation of the position and size of the images formed by lenses.

Look at figure 10.57. The two triangles shaded in green are similar triangles as all of their corresponding angles are the same size. This would be true wherever the image is located.

This means that the ratios of equivalent sides are equal, for example:

$$\frac{H_0}{H_1} = \frac{u - f}{f}$$

Also, the triangles shaded in blue are similar:

$$\frac{H_0}{H_1} = \frac{u}{v}$$

The left-hand sides of these equations are equal, so we can say:

$$\frac{u}{v} = \frac{u - f}{f}$$

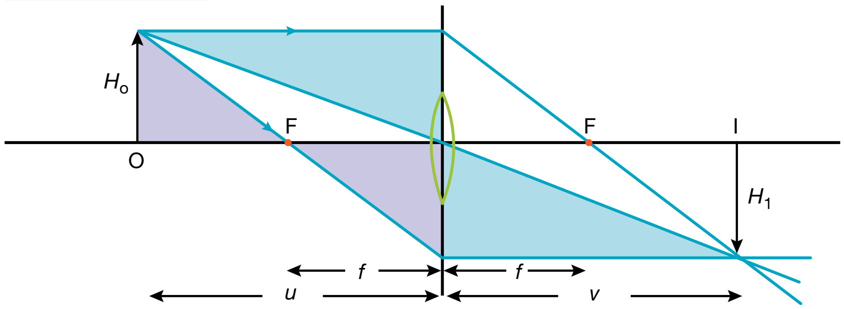
Which is the same as:

$$\frac{u}{v} = \frac{u}{f} - 1$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

FIGURE 10.57



This formula is known as the thin lens formula. It gives a good approximation for thin lenses. When using it, you need to be careful with signs:

- f is positive for converging lenses and negative for diverging lenses
- u is positive
- v is positive when the image is on the opposite side of the lens to the object and negative when on the same side

We can compare the results of this formula with what we determined by ray tracing back in 10.6 Sample problem 1.

10.6 SAMPLE PROBLEM 4

Use the thin lens formula to find the position of the image when $f = 10$ cm and $u = 16$ cm.

SOLUTION:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{16}$$

$$\frac{1}{v} = \frac{8}{80} - \frac{5}{80}$$

$$\frac{1}{v} = \frac{3}{80}$$

$v = \frac{80}{3} = 27$ cm behind the lens.

10.6 SAMPLE PROBLEM 5

Use the thin lens formula to find the position of the image when $f = 10$ cm and $u = 5$ cm.

SOLUTION:

$$\begin{aligned}
\frac{1}{f} &= \frac{1}{u} + \frac{1}{v} \\
\frac{1}{v} &= \frac{1}{f} - \frac{1}{u} \\
\frac{1}{v} &= \frac{1}{10} - \frac{1}{5} \\
\frac{1}{v} &= \frac{1}{10} - \frac{2}{10} \\
\frac{1}{v} &= -\frac{1}{10} \\
v &= -10\,\text{cm}
\end{aligned}$$

The image is 10 cm in front of the lens (on the same side as the object).

We have defined the magnification to be equal to the height of the image divided by the height of the object, $M = \frac{H_1}{H_0}$. We can see from the similar triangles in our derivation of the thin lens formula that $\frac{H_1}{H_0} = \frac{v}{u}$. However, zinverted $\left(\frac{H_1}{H_0}\text{ is negative}\right)$. Conversely, if v is negative, the image is upright $\left(\frac{H_1}{H_0}\text{ is positive}\right)$. To account for this we add a negative to the formula so that we have $M = -\frac{v}{u}$.

WORKING SCIENTIFICALLY 10.4

Design and build a simple telescope, documenting each stage of development and construction.

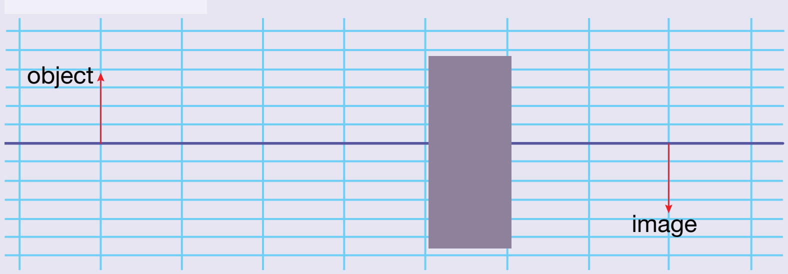
WORKING SCIENTIFICALLY 10.5

Find an old pair of glasses and, through experimentation, determine the focal length of each of the lenses and the possible defects that the glasses were meant to correct.

10.6 Exercise 1

1 The box in the following diagram contains a lens. The scale of the grid is 1 cm per line. Draw rays from the object to the image to determine:
(a) whether the lens is converging or diverging
(b) its focal length.

FIGURE 10.58



2 A convex lens has a focal length of 12 cm and is used as a magnifying glass by placing an object 4 cm from the lens. Determine the magnification achieved by the magnifying glass.

3 A 4 cm high object is placed 10 cm in front of a diverging lens with a focal length of 6 cm.
(a) How far from the lens will the image appear?
(b) Will the image be real or virtual?
(c) How high will the image be?

4 When a 5 cm high object is placed in front of a concave lens of focal length 8 cm, it forms an image 2.5 times smaller than the object. What is the distance between the object and the lens?

5 Optometrists and opticians describe the focusing ability of a lens in terms of its power, P, which is equal to the inverse of the lens's focal length f:

$$P = \frac{1}{f}$$

The unit of measurement for this type of power is the dioptre (D), where f is measured in metres.
Susan has glasses with a power of -4.0 D.
(a) What will be the focal length of her lenses?
(b) Will the lenses be converging or diverging lenses?
(c) Is Susan more likely to be short-sighted or long-sighted?
(d) An object is placed 40 cm in front of one of Susan's lenses. Where will the image formed by the lens appear?

10.7 Tricks of the light

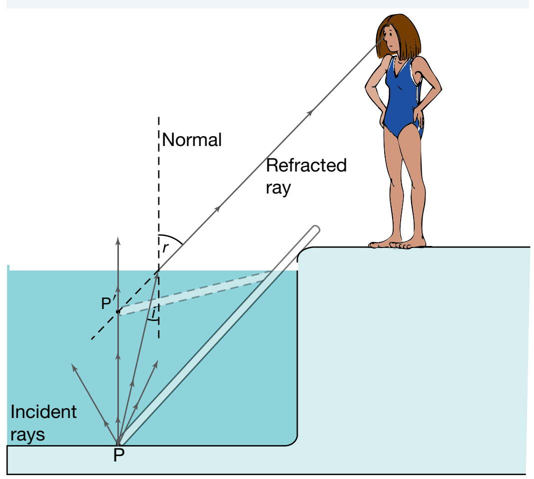
10.7.1 On a bender

Many odd visual effects that you may have noticed can be explained by the refraction of light. One that we see all the time is the way that straight objects such as drinking straws, pencils and poles appear to be bent when placed in water. This phenomenon can be explained using the ray model.

Consider a straight pole placed into the pool in figure 10.60. Light rays from the submerged end of pole (P) can be drawn in all directions and, when they hit the interface between the water and the air, they bend because of refraction. As the refractive index of air is smaller than that of water, the light is bent away from the normal, and $r > i$. Some of these refracted rays originating at P find their way to the observer's eyes. However, as these refracted rays appear to originate from a position P′, the observer sees the image of the end of the pole here rather than in its true position. As a result, P appears to be closer to the surface than it really is. A similar thing happens for every point along the pole. As they all appear closer, the pole appears bent.

FIGURE 10.59 An example of refraction.

FIGURE 10.60 A straight pole appears bent where it enters the water.



PHYSICS IN FOCUS

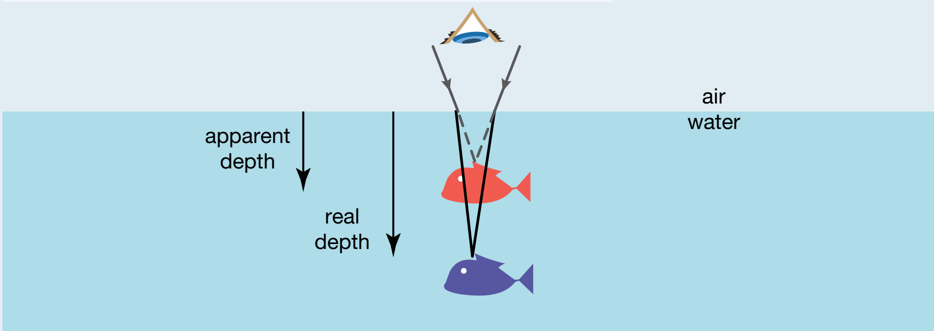
Apparent depth

Spear throwers need to aim below a fish if they are to have a chance of spearing the fish. A similar phenomenon occurs when a spear thrower is directly above a fish. The fish appears to be closer to the surface than it actually is. This observation is known as apparent depth. Swimming pools provide another example of apparent depth: they look shallower than they actually are. The refraction of light combined with our two-eyed vision makes the pool appear shallower.

The relationship is illustrated in figure 10.61 and can be expressed as follows:

$$\frac{\text{real depth}}{\text{apparent depth}} = \text{refractive index}$$

FIGURE 10.61 The phenomenon of apparent depth.



10.7.2 Total internal reflection

As seen earlier in this topic, some of the light incident on a transparent surface will be reflected, while the rest will be transmitted into the next medium, as shown in figure 10.63a. This applies whether the refracted ray is bent towards or away from the normal.

As we know from Snell's Law, an increase in the incident angle results in an increase in the reflected angle. However, a special situation applies when rays travelling from a medium with a higher refractive index into a medium with a lower refractive index meet the interface at certain incident angles. As the incident angle increases in size, it will reach a **critical angle, i_c**, at which the angle of refraction equals its maximum value of 90° with the normal. At this point, the refracted ray travels parallel to the boundary between the two media (figure 10.63b).

FIGURE 10.62 There are no mirrors in a fish tank but strange reflections can be seen.

The value of the critical angle depends upon the refractive indices of the two media. At the critical angle i_c:

$$n_1 \sin(i_c) = n_2 \sin(90^\circ)$$

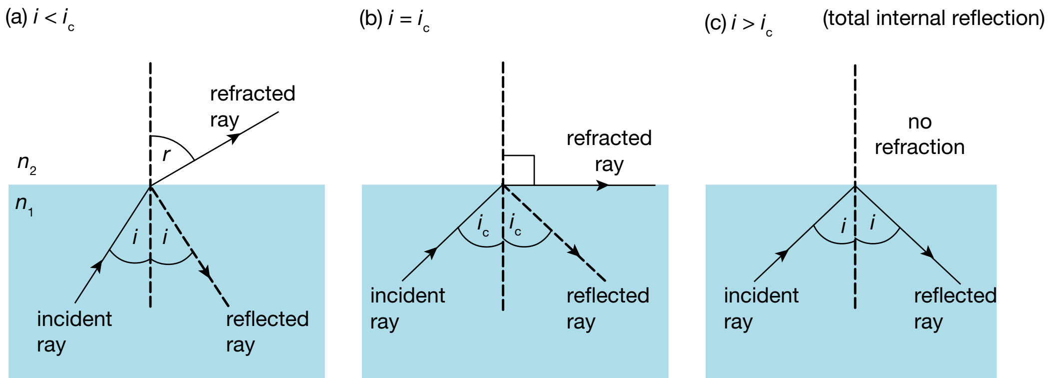
so,

$$n_1 \sin(i_c) = n_2$$

and thus,

$$\sin(i_c) = \frac{n_2}{n_1}$$

FIGURE 10.63 Total internal reflection.



The value n_2/n_1 is referred to as the **relative refractive index** for media 1 and 2. It should be noted that a critical angle can only exist provided that the relative refractive index is less than 1 — that is, $n_1 > n_2$.

At incident angles greater than the critical angle, all the light is reflected back into the original medium and no refracted ray is formed. This circumstance is referred to as **total internal reflection** (figure 10.63c).

Total internal reflection is a relatively common atmospheric phenomenon (as in mirages) and it has technological uses (for example, in optical fibres).

10.7 SAMPLE PROBLEM 1

What is the critical angle for light rays passing from water into air given that the refractive index of water is 1.3?

SOLUTION:

$n_{air} = 1.0;\ \theta_{air} = 90°;\ n_{water} = 1.3;\ \theta_{water} = ?$

$1.3 \times \sin\theta_{water} = 1.0 \times \sin 90°$ (substitute data into Snell's Law)

$\surd \sin\theta_{water} = \dfrac{\sin 90°}{1.3}$ (rearrange formula to get the unknown by itself)

$= 0.7692$ (determine sine values and calculate expression)

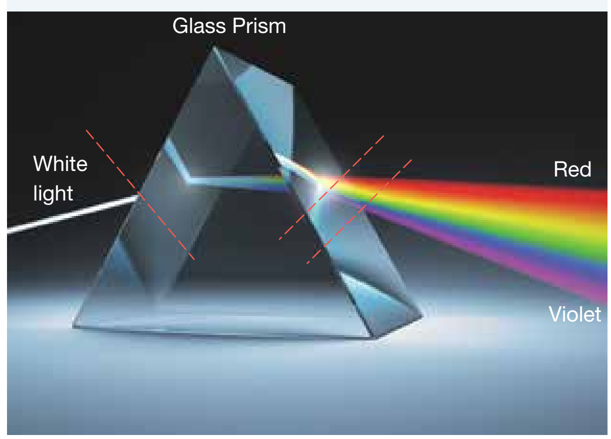
$\theta_{water} = 50.28°$ (use inverse sine to find angle)

$\theta_{water} = 50°$ (round off to two significant figures)

10.7.3 Dispersion of light

Isaac Newton was the first person to discuss the breaking up of white light into the coloured spectrum in a process called **dispersion**. He observed that, as white light passes through a triangular glass prism, the individual spectral colours emerge. This occurs because the different colours of the visible light spectrum travel through the glass at different velocities. This means that light of each colour has a different refractive index and so the different colours are refracted through at different angles. Violet light, which travels the slowest, is refracted the most, while red light, which travels the fastest, is refracted the least.

FIGURE 10.64 Dispersion by a glass prism.

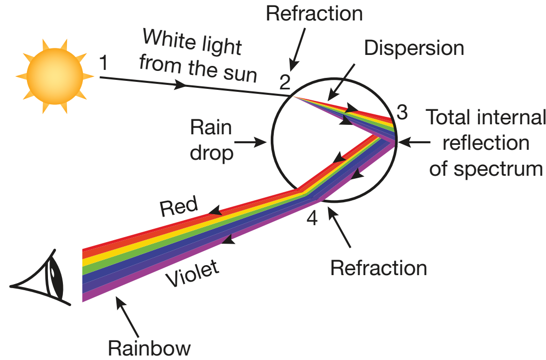


Rainbows are formed when sunlight is incident on water particles suspended in the air, which is why they are most frequently seen after rain showers. When white light from the sun enters the water droplet, it is refracted and dispersion occurs, separating the individual colours. These rays continue to travel until reaching the far surface of the water droplet, where some emerge but the rest are totally internally reflected back into the droplet. When they encounter the front boundary between the water of the droplet and the air, the rays are again refracted, further increasing the angle of dispersion.

The only dispersed rays reaching our eyes from each of those billions of droplets are those that have an angle between 40º (violet light) and 42º (red light) relative to the incident sunlight.

The arc of colour seen by the observer is the section of a circle subtended by these angles at a point called the antisolar point, which lies on the line between the observer's eye and the sun.

FIGURE 10.65 Dispersion in a water droplet.



The higher above the horizon the observer is, the higher above the ground the antisolar point is positioned and so the greater the proportion of the circle that is seen. At a very high altitude, an entire circular rainbow could be observed.

FIGURE 10.66 Angle of dispersion.

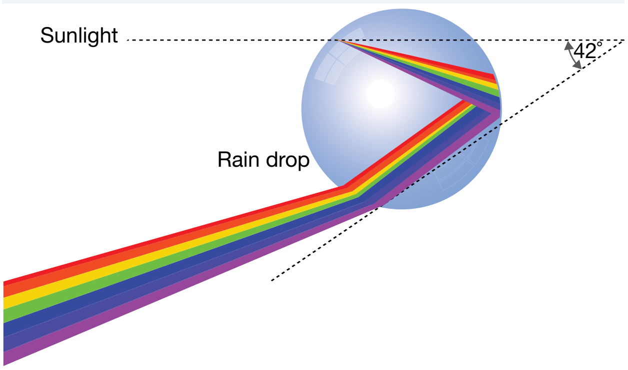
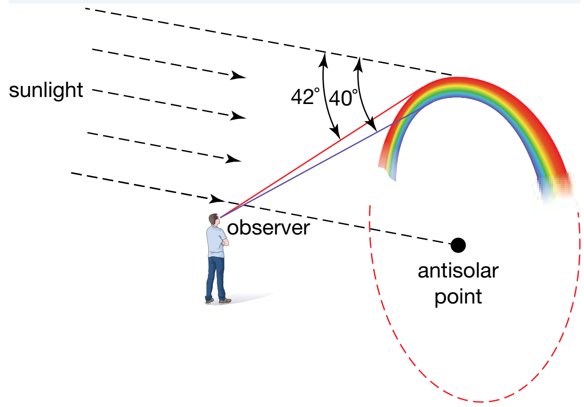


FIGURE 10.67 How the arc of a rainbow forms.



10.7 Exercise 1

1 Which of the following colours of light travels the fastest through glass:
(a) blue (b) green (c) yellow (d) violet?

2 A glass fibre has a refractive index of x and its cladding has a refractive index of y. What is the critical angle in the fibre?

3 What is the critical angle for light passing from diamond into water?

4 The critical angle for light passing from a mystery liquid into air is 43.2º. What is the absolute refractive index of the mystery liquid?

5 A light positioned in the bottom of a 1.5 m pool produces a circle of light on the water's surface. What is the radius of the light circle?

6 Mark stands at the edge of a fish pond and sees a large fish in the water. From where he is standing, the fish is 2 m horizontally from the pond's edge and appears to be 50 cm below the surface. How far below the surface of the pond is the fish actually located? Assume that Mark's eyes are 1.5 m above pond level.

7 Phuong placed a coin in the bottom of an opaque mug. From where she is sitting, she can't quite see the coin. However, when she pours some water into the mug, she finds that she can now see the coin. How is this possible?

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10.8 Review

10.8.1 Summary

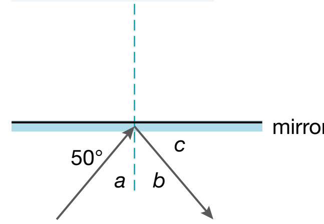
- The ray model depicts light as straight lines in a uniform medium.
- All electromagnetic waves travel at the same speed in a vacuum and are slowed down when they enter any other media.
- The speed of light in a vacuum is 299 792 458 m s^{-1}, usually approximated to 3×10^8 m s^{-1}.
- A luminous body is one that can directly produce light. A body that produces light when heated is said to be incandescent. A non-luminous or illuminated body is one that does not itself produce light, but reflects it from another source of light.
- The incident ray, reflected ray and the normal to the surface all lie in the same plane.
- The absolute refractive index of a transparent medium is the ratio of the speed of light in a vacuum to the speed of light in the medium. The refractive index is always larger than 1.
- A transparent material is one through which an object may be clearly seen. A translucent material allows light through it, but does not allow an object to be seen coherently through it. An opaque material is one through which light cannot pass at all.
- A material may reflect, transmit or absorb light, or a combination of these, depending upon the nature of the material.
- The Law of Reflection: the angle of incidence is equal to the angle of reflection.
- A concave (converging) mirror reflects parallel light rays so that they converge on the focal plane of the mirror. A convex (diverging mirror) reflects parallel light rays so that they spread out.
- Ray tracing and the mirror equations can be used to determine the location, size and nature of images produced by curved mirrors.
- Light is refracted when it passes between different transparent materials. The degree of refraction is described by Snell's Law: $n_1 \sin i = n_2 \sin r$.
- A lens is a device made from a transparent medium that allows the refraction of light to be controlled.
- A converging lens is thicker in the middle than at the edges. Parallel rays passing through a converging lens coincide at the focus of the lens. A diverging lens is thicker at its edges than in its middle. Parallel rays passing through a diverging lens spread out so that they appear to originate at a point on the focal plane nearest the object.
- The focal length of a lens depends upon the curvature of the faces and the refractive index of the medium from which it is made.
- The object distance (u) for lenses is assumed to be positive. The image distance (v) is negative for virtual images and positive for real images.
- A converging lens has a positive focal length, f, while a diverging lens has a negative focal length.
- A real image is one created from converging light rays. It will manifest on a screen placed at the formation position.
- A virtual image is unable to materialise on a screen, and can only be seen when viewed in a mirror or through a lens. Light rays do not converge at a virtual image.
- The position of an image formed by thin lenses can be determined by accurate ray tracing and by using the thin lens equation: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

10.8.2 Questions

1. Which of the following are luminous objects:
   (a) the Sun
   (b) a projector screen
   (c) a star
   (d) a campfire
   (e) a red-hot piece of iron
   (f) a television screen?

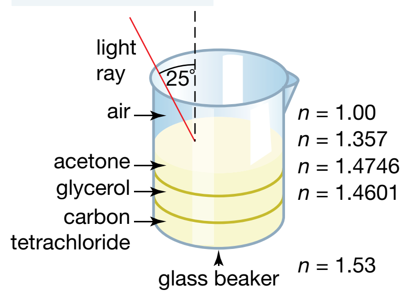
2. True or false? One type of transparent plastic has a refractive index equal to that of water (n = 1.33). If you placed a lump of this plastic into water, you would not see it.
3. Calculate the angles a, b and c in figure 10.68.
4. What is the luminous intensity of a 50 W light bulb at a distance of 2 m if we assume that all of the bulb's energy is converted into light?
5. A light source with a luminous intensity of 36 W m^{-2} that is positioned 1.2 m from a light meter produces the same reading as a second light source that is positioned 2.4 m away from the meter. What is the luminous intensity of the second source?
6. What is the angle of refraction in water (n = 1.33) for an angle of incidence of 40°? If the angle of incidence is increased by 10°, by how much does the angle of refraction increase?

FIGURE 10.68



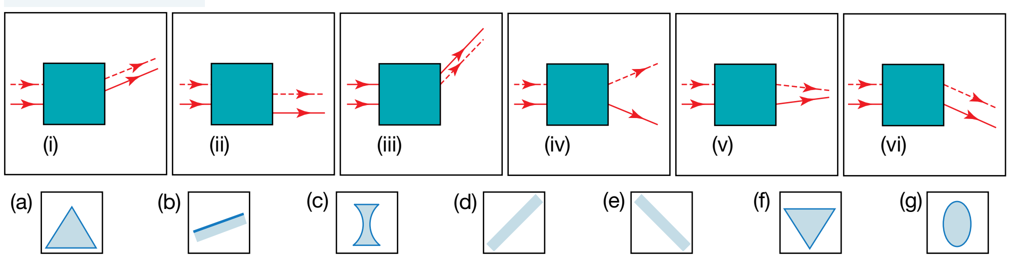
7. A ray of light enters a plastic block at an angle of incidence of 55° with an angle of refraction of 33°. What is the refractive index of the plastic?
8. A ray of light passes through a rectangular glass block with a refractive index of 1.55. The angle of incidence as the ray enters the block is 65°. Calculate the angle of refraction at the first face of the block, then calculate the angle of refraction as the ray emerges on the other side of the block. Com ment on your answers.
9. Immiscible liquids are liquids that do not mix. Immiscible liquids will settle on top of each other, in the order of their density, with the densest liquid at the bottom. Some immiscible liquids are also transparent.
   (a) Calculate the angles of refraction as a ray passes down through immiscible layers as shown in figure 10.69.
   (b) If a plane mirror was placed at the bottom of the beaker, calculate the angles of refraction as the ray reflects back to the surface. Comment on your answers.

FIGURE 10.69



10. Light rays are shown passing through boxes in figure 10.70. Identify the contents of each box from the options (a)–(g) given below. Option (b) is a mirror. All others are solid glass.
    Note: There are more options than boxes.

FIGURE 10.70



11. An object is placed 40 cm in front of a convex mirror that has a focal length of 30 cm.
    (a) Where is the image formed?
    (b) Is the image:
        i real or virtual, ii reduced or enlarged, or iii inverted or upright?
12. A concave mirror has a 40 cm focal length. How far from the mirror must an object be positioned in order for:
    (a) an image to appear 50 cm from the mirror
    (b) a real image to be formed that is twice the height of the object
    (c) a virtual image to be formed that is three times the height of the object?

13. Use ray tracing to determine the full description of the following objects:
   (a) a 4.0 cm high object, 20 cm in front of a convex lens with a focal length of 15 cm
   (b) a 3.0 mm high object, 10 cm in front of a convex lens with a focal length of 12 cm
   (c) a 5.0 cm high object, 200 cm in front of a convex lens with a focal length of 10 cm.
14. What does 'accommodation mechanism' mean? Give an example.
15. (a) You are carrying out a convex lens investigation at a bench near the classroom window and you obtain a sharp image of the window on your screen. A teacher walks past outside the window. What do you see on the screen?
   (b) The trees outside the classroom are unclear on the screen. What can you do to bring the trees into focus?
16. Use ray tracing to determine the magnification of an object placed under the following two-lens microscope. The object is placed 5.2 mm from an objective lens of focal length 5.0 mm. The eyepiece lens has a focal length of 40 mm. The poles of the lenses are 150 mm apart.
17. A convex lens with a focal length of 5.0 cm is used as a magnifying glass. Determine the size and location of the image of text on this page if the centre of the lens was placed:
   (a) 4.0 cm above the page
   (b) 3.0 cm above the page.
18. A 35 mm slide is placed in a slide projector. A sharp image is produced on a screen 4.0 m away. The focal length of the lens system is 5.0 cm.
   (a) How far is the slide from the centre of the lens?
   (b) What is the size of the image?
   (c) Looking from the back of the slide projector, the slide contains a letter 'L'. What shape will appear on the screen?
   (d) The slide projector is moved closer to the screen. The image becomes unclear. Should the lens system be moved closer to or further away from the slide?
19. An object is placed at an equal distance a from two plane mirrors that are placed at right angles as shown in figure 10.71. How many images of the object are formed in the mirrors?
20. (a) What is the angle of refraction in water ($n = 1.33$) of a light ray that has an incident angle of 30°?
   (b) By how much will the angle of refraction increase if the incident angle is increased by 10°?
21. Calculate the distance the photographic film needs to be from the centre of a camera lens of focal length 5.0 cm in order to take a sharp, focused photograph of a family group located 15.0 m away.

FIGURE 10.71

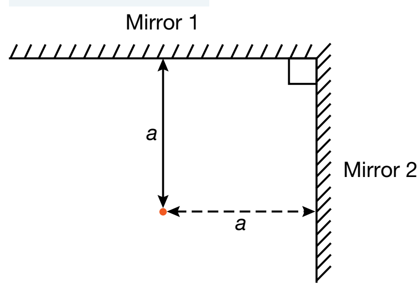
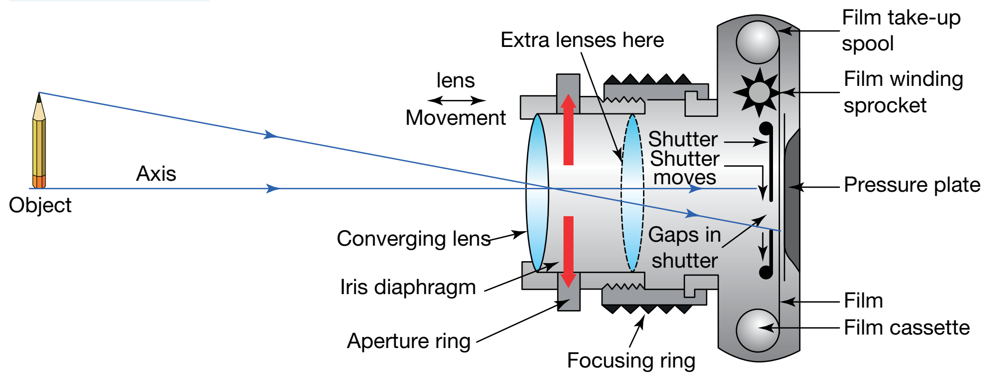
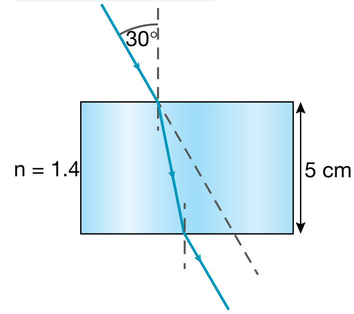


FIGURE 10.72



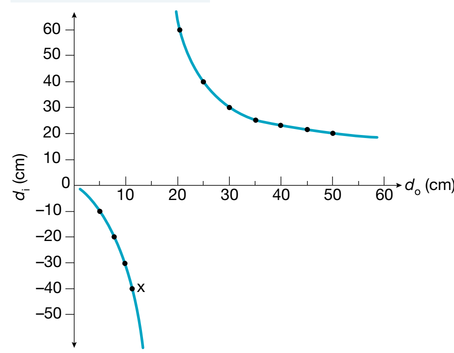
22. Calculate the sideways deflection as a ray of light goes through a parallel-sided plastic block ($n = 1.4$) with sides 5.0 cm apart, as in figure 10.73.

FIGURE 10.73



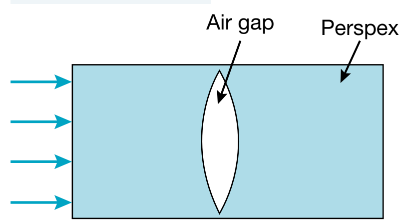
23. During the course of an experiment, a student moves an object into different positions (d_o) in front of a converging lens and measures the resulting positions (d_i) of the image formed. His results are then plotted on a graph as shown in figure 10.74.
    (a) What is the focal length of the lens?
    (b) Describe the image formed for point X. Is it real or virtual? Is it enlarged or reduced?
    (c) If the object is placed at the 50 cm position, where will the image form?
24. A lens held 20 cm from an object produces a real, inverted image of it 30 cm on the other side of the lens. What is the focal length of the lens? Is it converging or diverging?

FIGURE 10.74



25. A convex air pocket is formed inside a block of Perspex as shown in figure 10.75. What effect will this air pocket have on parallel light rays entering the block?
26. A converging lens with a focal length of 40 cm is placed 1 m in front of a diverging lens that has a focal length of 30 cm. A 5 cm birthday candle is lit and placed a distance of 80 cm in front of the converging lens, as shown in figure 10.76. Under the influence of both lenses, where will the final image be formed, and how high will it be?
27. Describe the light path from a light source to your eye in seeing an object.

FIGURE 10.75



28. Use the ray model and the sources of light to rephrase the statements (a) 'I looked at a flower through the window' and (b) 'I watched the TV'.
29. Explain how early astronomers knew the Moon must have a rough surface.
30. Copy figure 10.77 and draw the incident and reflected rays from the two ends of the object to the eye. Locate the image.
31. The two arrowed lines in figure 10.78 represent reflected rays. The line AB represents the plane mirror. Locate the image and the light source in each of the two figures.

FIGURE 10.76

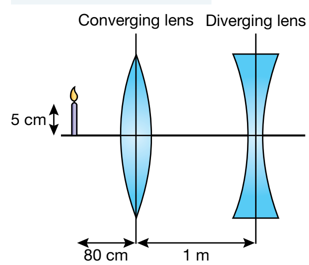


FIGURE 10.78

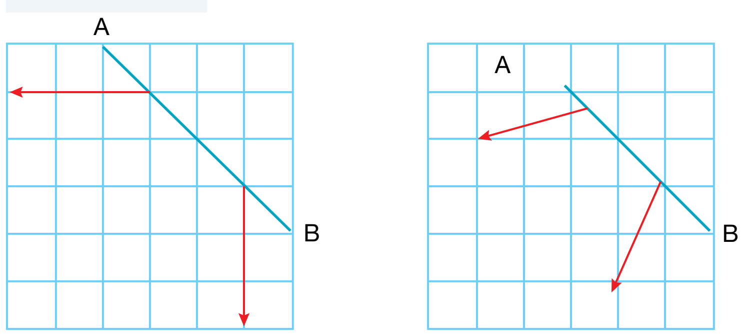
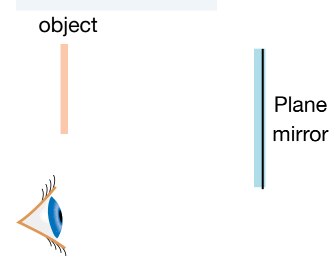


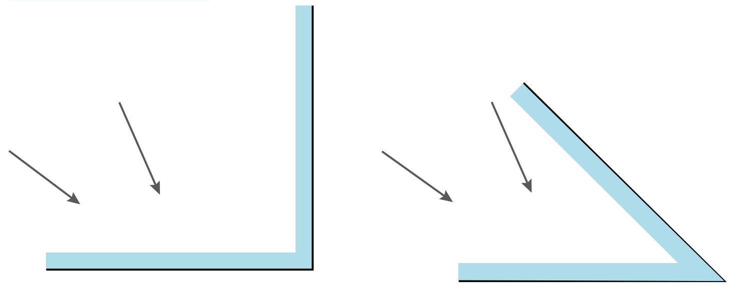
FIGURE 10.77



32. A student argues that you cannot photograph a virtual image because light rays do not pass through the space where the image is formed. How would you argue against this statement?

33. Sketch the path of the rays entering each of the pair of joined mirrors in figure 10.79.
34. When slides are placed in a slide projector, they are put into the cartridge upside-down. Why is this done?
35. Explain how you can use two mirrors to see your view from behind.
36. As part of an experiment on Snell's Law, a student measures the angle of refraction (θ_2) obtained when an incident light ray enters a clear plastic block for a number of different incident angles (θ_1). Her results are shown in the table at right.
    (a) Draw a graph of this data.
    (b) Use your graph to determine the refractive index of the plastic.
37. When you look into a plane mirror, your left and right sides appear reversed. This is called lateral inversion. Draw a diagram showing how you could position a series of plane mirrors so that you see your image upside-down.

FIGURE 10.79



θ_1 (degrees)	θ_2 (degrees)
0	0
10	6
20	12
30	18
40	24
50	29

PRACTICAL INVESTIGATIONS

Investigation 10.1: Snell's Law

Aim

To observe the refraction of light and to use Snell's Law to determine the refractive index of a medium

Materials

Power supply, ray box with single slit card, rectangular Perspex or glass block, ruler, protractor, pencil, blank A4 paper, drawing board, drawing pins

Method

1. Use drawing pins to attach the A4 paper to the drawing board, which should be lying flat on the bench.
2. Place the block in the middle of the page. Use a pencil to draw around the block so that it can always be returned to the same position. Mark a point on the boundary and label it as O.
3. Reduce the amount of light in the room (by drawing curtains etc.). Turn on the ray box and direct a single ray of light so that it enters the block at point O at an angle and emerges on the other side of the block.
4. Without moving the ray or the block, mark 3 points along each of the incident ray and the emerging ray. Place a mark at the block boundary at the point where the light ray emerges from the block and label this R.
5. Turn off the ray box and remove the block from the paper. Using your pencil marks as guides, use a ruler to draw the path of the incident ray into the block, joining points O and R, and to draw the path of the emerging ray. Draw normals to the surface at points O and R.
6. Use your protractor to measure the angle of incidence θ_1 and the angle of refraction θ_2 as shown in figure 10.80. Enter these values into table 10.4A.
7. Repeat steps 1–6 for three other incident angles.

Results

TABLE 10.4A

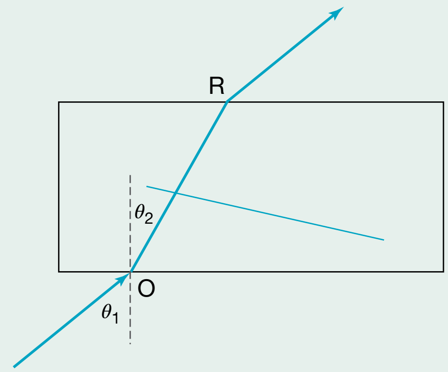
θ_1 (degrees)	θ_2 (degrees)

Analysing the results

1. For each of the angles in table 10.4A, complete table 10.4B at right.
2. Snell's Law states that $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Given that $n_1 = 1.00$ (air), what variable does the ratio $\frac{\sin \theta_2}{\sin \theta_1}$ represent?
3. What is the approximate refractive index of the block?
4. In each case, is the ray refracted towards the normal or away from the normal as it passes into the block?
5. What do you notice about the angles of the incident ray entering the block at O and the ray emerging from the block at R?

TABLE 10.4B

$\sin \theta_1$	$\sin \theta_2$	$\frac{\sin \theta_2}{\sin \theta_1}$



Conclusion

State the relationship between the refractive index of the block, the angle of incidence and the angle of refraction in this investigation.

Investigation 10.2: Concave mirrors — an observation exercise

This investigation involves observing yourself in a concave mirror. You will need the following equipment:

- concave mirror
- tape measure or metre ruler.

Look at yourself in a concave mirror.

1. How does your appearance change as you move towards and away from the mirror?
2. Describe your image (for example, size and orientation) as the distance changes. Note the distance.
3. Was there a distance at which the image changed markedly? If so, where did you notice that this occurred?
4. Do you notice any distortion of the image? If so, how was the image distorted and where did this occur?

Investigation 10.3: Converging Lenses

Aim

To investigate the formation of images by converging lenses

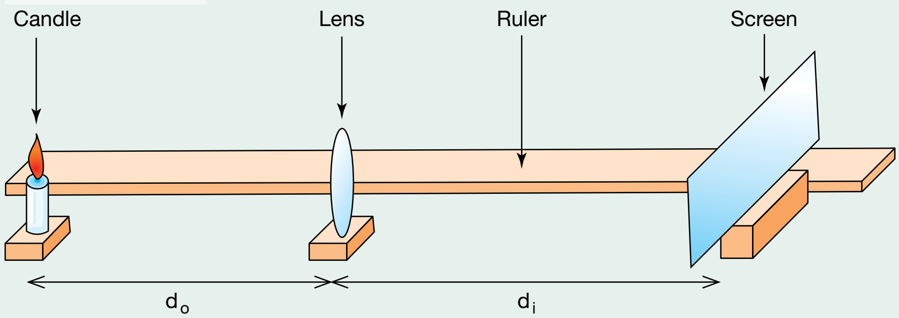
Materials

Biconvex glass lens, lens holder, metre ruler, small birthday candle mounted in a holder, white cardboard screen, masking tape

Method

1. Place the metre ruler flat on the benchtop.
2. Put the biconvex lens in the holder and place it next to the ruler at the 50 cm mark.
3. Light the candle and place it next to the 0 cm mark of the ruler. This location corresponds to $d_o = 50$ cm. Place the screen against the 100 cm mark and move it closer to the lens or further away from the lens until a clear image of the candle flame appears on the screen. Measure the distance d_i between the lens and the screen, and enter this value into table 10.5A. The image formed on the screen is said to be a *real image*.

FIGURE 10.81



4. Continue to move the candle closer to the lens in 5 cm increments and measure the corresponding values of d_i.

5. Eventually, you will reach values of d_o at which no clear image can be formed on the screen. This will occur when the candle is located at the focus of the lens (where no image of the candle can be formed) or closer (where only a virtual image can be formed). In the case of a virtual image, looking directly through the lens itself at the candle will reveal its image. The location of the virtual image can be estimated by using the adjacent ruler. Enter the d_i values of these images as negative values in table 10.5A.

Results

Analysing the results

1. When the candle is at the focus of the lens, F, no image (either real or virtual) can be formed. At what value of d_o did this occur?
2. Using your values for d_o and d_i and the equation $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$, determine the value of f for the lens.
3. How do your values for question 1 and question 2 compare?
4. Converging lenses are used as magnifying glasses. Is this magnified image a real or virtual image? Justify your answer.
5. The power P (in dioptres) of a lens is equal to the inverse of its focal length (in metres). What is the power of the lens you have used here?

TABLE 10.5A

		Image		
(d_o) cm	(d_i) cm	Real or virtual?	Erect or inverted?	Enlarged or reduced?
50				
45				
40				
35				
30				
25				
20				
15				
10				
5				

Conclusion

1. Complete the following table summarising the locations and types of images formed by a converging lens.

TABLE 10.5B

Position of object	Position of image	Description of image
$2f > d_o > f$		
$d_o = f$		
$f > d_o$		

Investigation 10.4: Using apparent depth to determine the refractive index

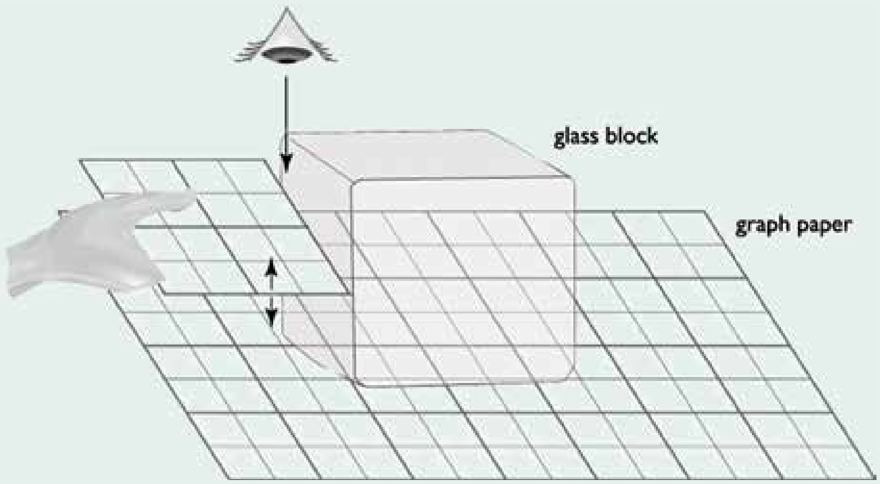
This investigation involves using apparent depth to determine the refractive index.

You will need the following equipment:

- rectangular glass or perspex block
- two pieces of grid graph paper.

Place the smallest face of a rectangular glass block on a sheet of grid graph paper as shown in figure 10.82.

FIGURE 10.82



Look down a vertical face of the block so that you can see the grid of the graph paper through the glass and through the air. The grid seen through the glass will appear larger (closer).

Slowly bring another piece of identical graph paper up that face until the graph pattern seen through the block matches the pattern held beside the block. Take care, this is a difficult task.

Mark the point where the patterns are seen to match and measure its distance from the top of the block.

Repeat this exercise several times and calculate the average of your measurements.

Measure the full length of the glass block and calculate the refractive index of the block, using the equation:

$$\frac{\text{real depth}}{\text{apparent depth}} = \text{refractive index.}$$

Repeat the exercise to calculate the refractive index of water, using a fish tank or a large beaker instead of the glass block.

Investigation 10.5: Floating coins

AIM

To investigate the effect of refraction on the image of a submerged object

You will need:

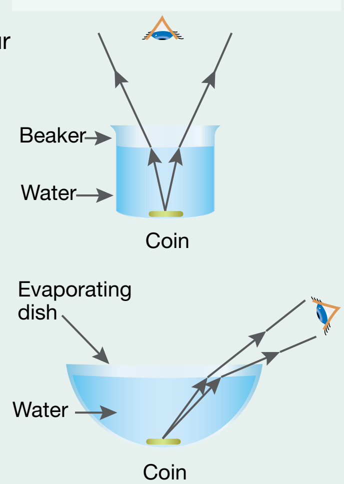
2 beakers
evaporating dish
coin

- Place a coin at the bottom of an empty beaker and look at it from above while your partner slowly adds water from another beaker.
- Place the coin in the centre of an evaporating dish and move back just far enough so you can no longer see the coin. Remain in this position while your partner slowly adds water to the dish.
- Make a copy of the diagrams shown in figure 10.83. Use dotted lines to extend back the rays shown entering the observer's eye to see where they seem to be coming from. This enables you to locate the centre of the image of the coin.

Discussion

1. How does the position of the coin appear to change while the water is being added?
2. Which other feature of the coin appears to change?
3. What appears to happen to the coin as water is added to the evaporating dish?
4. Is the image of the coin above or below the actual coin?

FIGURE 10.83 The image of the coin is not in the same place as the actual coin.



Investigation 10.6: Total Internal reflection

Aim

To observe total internal reflection of light in a Perspex prism

You will need:

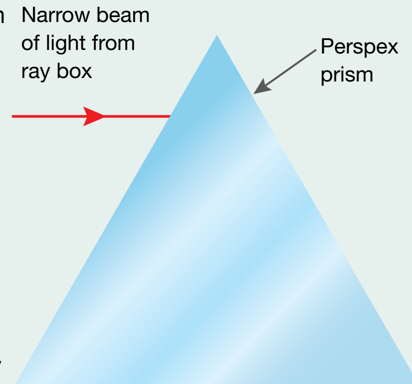
ray box kit

12 V DC power supply

Perspex triangular prism

- Connect the ray box to the power supply. Place the ray box over a page of your notebook. Use one of the black plastic slides in the ray box kit to produce a single thin beam of light that is clearly visible on the white paper.
- Place a Perspex triangular prism on your notebook and direct the thin beam of light towards it as shown in figure 10.84. Observe the beam as it passes through the prism.
- Turn the prism slightly anticlockwise, closely observing the thin light beam as it travels from the Perspex prism back into the air. Continue to turn the prism until the beam no longer emerges from the prism.

FIGURE 10.84 Observe the beam of light as it passes through the prism.



Discussion

1. Describe what happens to the thin light beam as it passes from air into the Perspex prism and back into the air.
2. Outline what happens to the beam of light when it no longer emerges from the prism.
3. Draw a series of two or three diagrams showing how the path taken by the beam of light changed as you turned the prism.

TOPIC 11 Thermodynamics

11.1 Overview

11.1.1 Module 3: Waves and Thermodynamics

Thermodynamics

Inquiry question: How are temperature, thermal energy and particle motion related?

Students:

- explain the relationship between the temperature of an object and the kinetic energy of the particles within it (ACSPH018)
- explain the concept of thermal equilibrium (ACSPH022)
- analyse the relationship between the change in temperature of an object and its specific heat capacity through the equation $\Delta Q = mc\Delta T$ (ACSPH020)
- investigate energy transfer by the process of:
  - conduction
  - convection
  - radiation (ACSPH016)
- conduct an investigation to analyse qualitatively and quantitatively the latent heat involved in a change of state
- model and predict quantitatively energy transfer from hot objects by the process of thermal conductivity
- apply the following relationships to solve problems and make quantitative predictions in a variety of situations:
  - $\Delta Q = mc\Delta T$, where c is the specific heat capacity of a substance
  - $\dfrac{Q}{t} = \dfrac{k A \Delta T}{d}$, where k is the thermal conductivity of a material.

FIGURE 11.1 Incoming radiation is reflected off ice back into space, but is absorbed by the water. How will increasing air and ocean temperatures change this scene?

11.2 Temperature and kinetic energy

11.2.1 Measuring temperature

Our bodies tell us when it is hot or cold. Our fingers warn us when we touch a hot object. However, for all that, our senses are not reliable.

Try this at home: Place three bowls of water in front of you. Put iced water in the bowl on the left, water hot enough for a bath in the bowl on the right, and room temperature water in the one in the middle. Place a hand in each of the two outer bowls, leave them there for a few minutes, then place both hands in the middle bowl. As you would expect, your left hand tells you the water is warmer, while your right hand tells you it is colder.

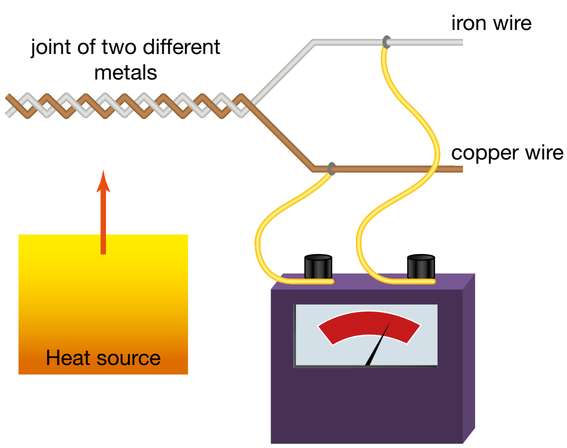
Thermometers were designed as a way to measure temperature accurately. A good thermometer needs a material that changes in a measurable way as its temperature changes. Many materials, including water, expand when heated, so the first thermometer, built in 1630, used water in a narrow tube with a filled bulb at the bottom. The water rose up the tube as the bulb was warmed.

TABLE 11.1 Some temperatures on the Kelvin and Celsius scales.

Event	Temperature K	Temperature °C
Absolute zero	0	−273
Helium gas liquefies	4	−269
Lead becomes a superconductor	7	−266
Nitrogen gas liquefies	63	−210
Lowest recorded air temperature on the Earth's surface (Vostok, Antarctica)	184	−89
Mercury freezes	234	−39
Water freezes	273	0
Normal human body temperature	310	37
Highest recorded air temperature on the Earth's surface (Death Valley, USA)	330	57
Mercury boils	630	357
Iron melts	1535	1262
Surface of the Sun	5778	5505

German physicist Daniel Fahrenheit replaced the water with mercury in 1724. Liquid thermometers now use alcohol with a dye added. Fahrenheit developed a scale to measure the temperature, using the lowest temperature he could reach, an ice and salt mixture, as zero degrees, and the temperature of the human body as 100 degrees. Fahrenheit also showed that a particular liquid will always boil at the same temperature. Swedish astronomer Anders Celsius developed another temperature scale in 1742, which is the one we use today. Celsius used melting ice and steam from boiling water to define 0 °C and 100 °C for his scale.

FIGURE 11.2 This thermocouple is connected to a voltmeter which reads differing voltages as the thermocouple changes temperature.



A third temperature scale was proposed in 1848 by William Thomson, later to be ennobled as Lord Kelvin. He proposed the scale based on the better understanding of heat and temperature that had developed by that time (see figure 11.11). This scale uses the symbol 'K' to stand for 'Kelvin'.

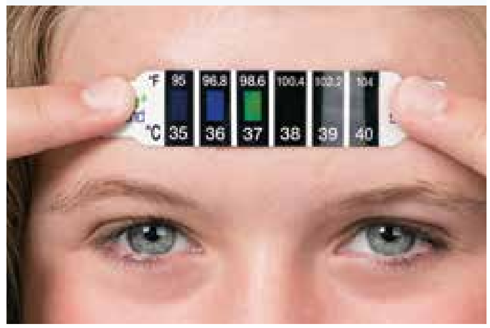
Other materials, including gases and metals, also expand with temperature and are used as thermometers. A bimetallic strip is two lengths of different metals, usually steel and copper, joined together. The two metals expand at different rates, so the strip bends one way as the temperature rises, or the other as it cools. A bimetallic strip can be used as a thermometer, a thermostat or as a compensating mechanism in clocks.

Other properties that change with temperature that can be employed in designing a thermometer are:

- electrical resistance of metals, which increases with temperature
- electrical voltage from a thermocouple, which is two lengths of different metals with their ends joined; if one end is heated, a voltage is produced
- colour change; liquid crystals change colour with temperature
- colour emitted by a hot object; in steelmaking, the temperature of hot steel is measured by its colour.

FIGURE 11.3 This steel is nearly 1000 °C and has recently been poured in a mould to shape it. The steel will continue to glow until it has cooled to about 400 °C.

FIGURE 11.4 This liquid crystal thermometer indicates body temperature when the liquid crystals change colour. The thermometer is registering 37 °C.



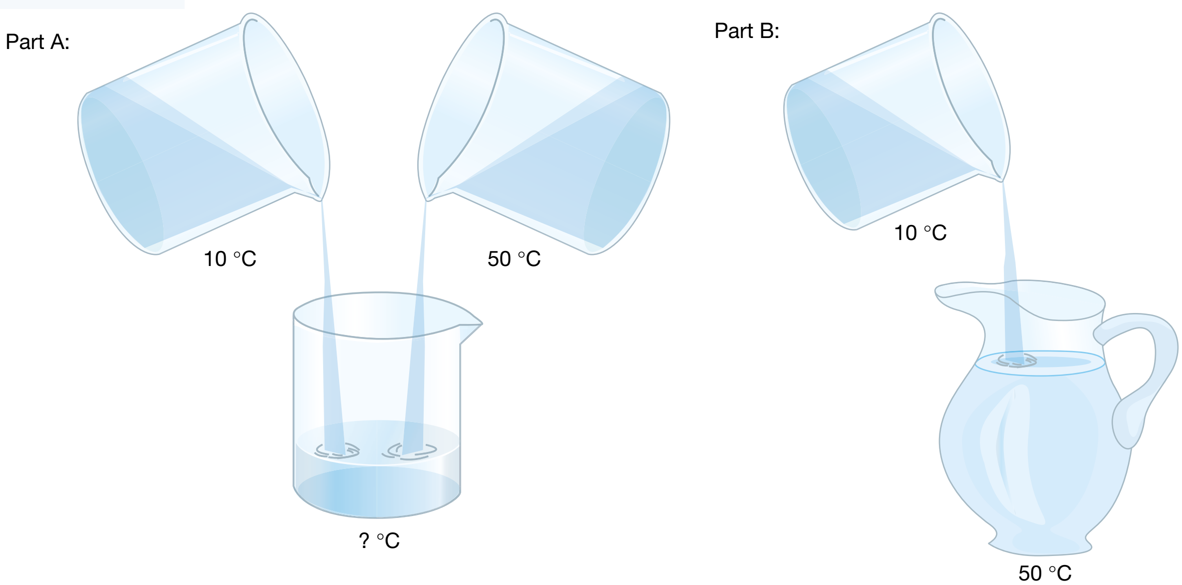
PHYSICS IN FOCUS

The lowest temperature: In 2015, the temperature of a cloud of 100 000 rubidium atoms was reduced to 50×10^{-12} degrees Kelvin (above absolute zero). The average speed of the atoms was less than 70 micrometres per second.

11.2.2 What is the difference between heat and temperature?

If you mix a beaker of cold water at 10 °C with a beaker of hot water at 50 °C, you expect the final temperature to be about 30 °C. But if you mix the beaker of cold water with a jug of hot water, the final temperature will be a lot closer to 50 °C.

FIGURE 11.5



Obviously if an object has more mass, it contains more heat, but how are temperature and heat related? Consider these two analogies to temperature and heat:

(a) If you drop a marble on your big toe from a height of 1 metre, you would notice it, but it would not hurt. However, if you dropped a 1 kg mass from only 10 cm, it would hurt.

The height is to temperature as the impact is to heat.

(b) If you rub your shoes across a carpet, you can generate a voltage as high as 10 000 volts, but the electric shock is no more than a twitch. However, a 6-volt car battery can deliver a charge to start the engine.

The voltage is to temperature as the spark is to heat.

These analogies don't really help to explain what heat is. If an object cools down, there seems to be no physical difference other than the drop in temperature. The object does not weigh less because it has lost heat! What is being transferred when a hot object warms up a cold object?

In 1798 Benjamin Thompson, later to be called Count Rumford, conducted an experiment on the nature of heat. The barrel of a cannon is made by drilling a cylindrical hole in a solid piece of metal. Rumford observed that the metal and the drill became quite hot. He devised an experiment to investigate the source of the heat and how much heat was produced. Rumford put the drill and the end of the cannon in a wooden box filled with water. He measured the mass of water and the rate at which the temperature rose. He showed that the amount of heat produced was not related to the amount of metal that was drilled out. He concluded that the amount of heat produced depended only on the work done against friction. He said that heat was in fact a form of energy, not an invisible substance that is transferred from hot objects to cold objects. Instead a hot object had heat energy, in the same way as a moving object has **kinetic energy** or an object high off the ground has gravitational potential energy.

Kinetic energy is the energy associated with the movement of objects. Like all forms of energy, kinetic energy is a scalar quantity.

AS A MATTER OF FACT

Count Rumford was born Benjamin Thompson in Massachusetts in 1753. By the age of 16 he was conducting experiments on heat. By 1775, when the American War of Independence began, he was already a wealthy man and of some standing in his community. He joined the British side of the war, becoming a senior advisor. While with the army, he also investigated and published a paper on the force of gunpowder.

At the end of the war, he moved to England, where he was known as a research scientist. A few years later he moved to Bavaria, in what is now southern Germany, and spent 11 years there. He moved in royal circles and eventually became Bavaria's Army Minister, tasked with reorganising the army. As part of those duties, he investigated methods of cooking, heating and lighting. He developed a soup, now called Rumford's soup, as a nutritious ration for soldiers. He also used the soup to establish soup kitchens for the poor throughout Bavaria. For his services, he was made a Count of the Holy Roman Empire, taking the name 'Rumford' from his birthplace.

On returning to England, his activities included: (i) redesigning an industrial furnace, which revolutionised the production of quicklime, a component of cement, and also used for lighting ('limelight'); (ii) redesigning the domestic fireplace to narrow the chimney at the hearth to increase the updraught, resulting in greater efficiency and preventing smoke from coming back into the room; and (iii) inventing thermal underwear, a kitchen range and a drip coffeepot.

With Joseph Banks and others, Rumford established the Royal Institution (RI) in London as a scientific research establishment with a strong emphasis on public education. Initial funding came from the 'Society for Bettering the Conditions and Improving the Comforts of the Poor', with which Count Rumford was centrally involved. Famous scientists in its early years included Humphrey Davy and Michael Faraday. Fifteen Nobel prize winners have worked at the RI, and 10 chemical elements were discovered there.

FIGURE 11.6

'Ten chemical elements were discovered at the Royal Institution and 15 Nobel Prize winners have worked there.'

WORKING SCIENTIFICALLY 11.1

Devise an experiment to measure the heat generated when two hands are rubbed together. Use your design to investigate how/if the amount of heat generated is dependent upon (a) the speed with which the hands are rubbed together, (b) the surface area of the hands, or (c) the period of time for which the hands are rubbed together.

Rumford's ideas about heat were not taken up for a few decades until, in 1840, James Prescott Joule conducted a series of experiments to find a quantitative link between mechanical energy and heat. In other words, how much energy is required to increase the temperature of a mass by 1 °C?

Joule used different methods and compared the results:

- Using gravity: A falling mass spins a paddle wheel in an insulated barrel of water, raising the temperature of the water; Measure the temperature of water at the top and bottom of a waterfall.
- Using electricity: Mechanical work is done turning a dynamo to produce an electric current in a wire, which heats the water.
- Compressing a gas: Mechanical work is used to compress a gas, which raises the gas's temperature.
- Using a battery: Chemical reactions at the battery terminals produce a current, which heats the water.

Joule obtained approximately identical answers for all methods. This confirmed heat is a form of energy. To honour his achievement, the SI unit of energy is the **joule** (J). The unit, joule, is used to measure the kinetic energy of a runner, the light energy in a beam, the chemical energy stored in a battery, the electrical energy in a circuit, the potential energy in a lift on the top floor and the heat energy when water boils.

One joule is the energy expended when a force of 1 newton acts through a distance of 1 metre. The usual metric prefixes make the use of the unit more convenient. For example:

1 kJ (kilojoule) = 10^3 J   1 MJ (megajoule) = 10^6 J   1 GJ (gigajoule) = 10^9 J

The chemical energy available from a bowl of breakfast cereal is usually hundreds of thousands of joules and is more likely to be listed on the packet in kilojoules. The amount of energy needed to boil an average kettle full of cold water is about 500 kJ.

Examples of 1 joule include:

- the kinetic energy of a tennis ball moving at about 6 m/s
- the heat energy needed to raise the temperature of 1 gram of dry air by 1 °C
- the heat energy needed to raise the temperature of 1 gram of water by 0.24 °C
- the energy released when an apple falls 1 m to the ground
- the amount of sunlight hitting a square centimetre every 10 seconds when the Sun is directly above
- the amount of sound energy entering your eardrum at a loud concert over 3 hours
- the amount of electrical energy used by a plasma TV screen while on standby every 2.5 seconds
- the energy released by the combustion of 18 micrograms of methane.

11.2.3 Explaining heat: the kinetic theory of matter

The kinetic theory of matter, which considers all objects as assemblies of particles in motion, is an old one. First described by Lucretius in 55 AD, the kinetic view of matter was developed over time by Hooke, Bernoulli, Boltzmann and Maxwell.

The evidence for the existence of particles includes:

- gases and liquids diffuse, that is, a combination of two gases or two liquids quickly becomes a mixture, for example a dye spreading in water. Even solids can diffuse; if a sheet of lead is clamped to a sheet of gold, over time the metals merge to a depth of a few millimetres.
- the mixing of two liquids gives a final volume that is less than the sum of their original volumes
- a solid dissolves in a liquid.

The kinetic theory of matter assumes that:

- all matter is made up of particles in constant, random and rapid motion
- there is space between the particles.

The energy associated with the motion of the particles in an object is called the internal energy of the object. The particles can move and interact in many ways, so there are a number of contributions to the internal energy. For example:

Gases: In a gas made up of single atoms, such as helium, the atoms move around, randomly colliding with each other and the walls of the container: each atom has some translational kinetic energy.

FIGURE 11.7 Iodine crystals sublimate (turn directly into a gas) when heated. (a) This diagram shows a gas jar with iodine crystals. (b) As the crystals warm up, they produce a purple gas that diffuses throughout the jar. (c) After a long period of time, the crystals have completely sublimated.

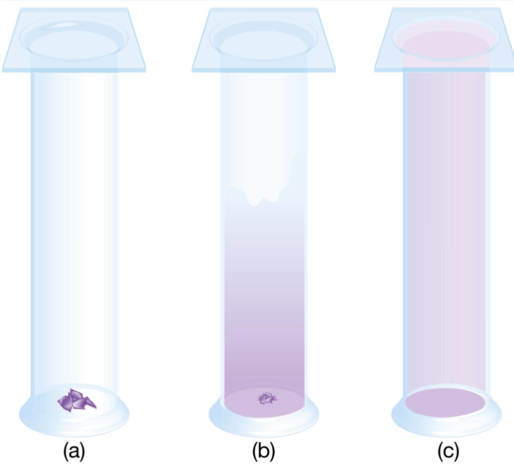
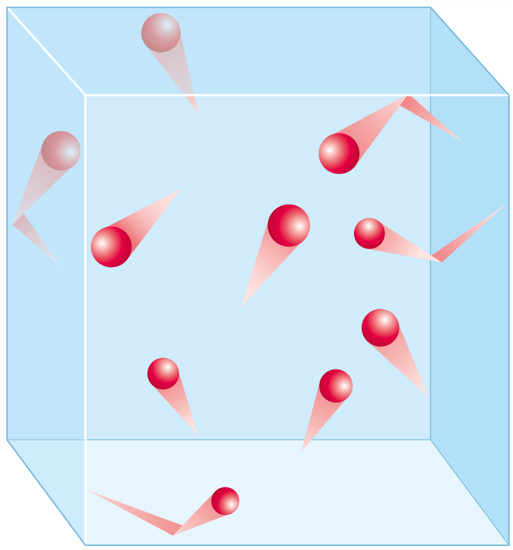
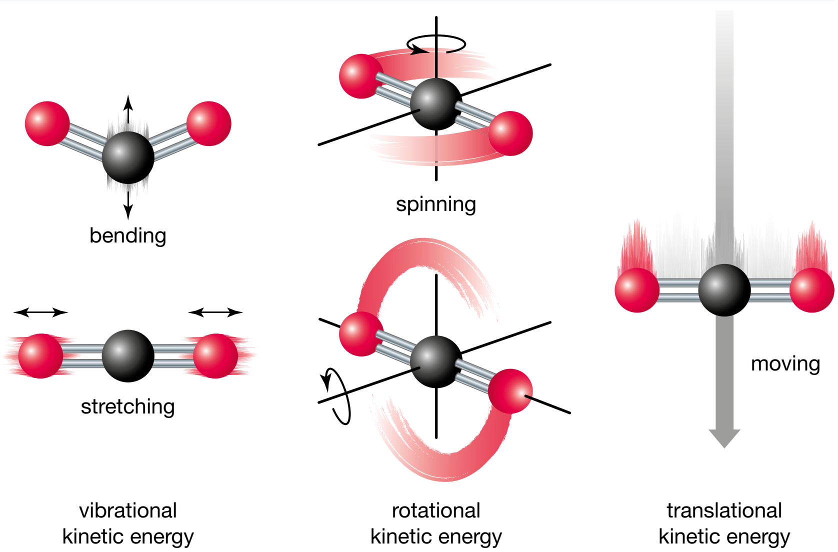


FIGURE 11.8 Moving single atoms have translational kinetic energy.



However, if the gas is made up of molecules with two or more atoms, the molecules can also stretch, contract and spin, so these molecules also have other types of kinetic energy called vibrational and rotational kinetic energy.

FIGURE 11.9 The movements of a molecule.



Liquids: Like a gas, molecules in a liquid are free to move, but only within the confines of the surface of the liquid. There is some attraction between molecules, which means there is some energy stored as molecules approach each other. Stored energy is called potential energy. It is the energy that must be overcome for a liquid to evaporate or boil.

Solids: In a solid, atoms jiggle rather than move around. They have kinetic energy, but they also have a lot of potential energy stored in the strong attractive force that holds the atoms together. This means that a lot of energy is required to melt a solid.

Temperature is a measure of the average translational kinetic energy of particles. The other contributions to the internal energy do not affect the temperature. This becomes important when materials melt or boil because the added heat must go somewhere, but the temperature does not change.

FIGURE 11.10 Movement of atoms in a solid.

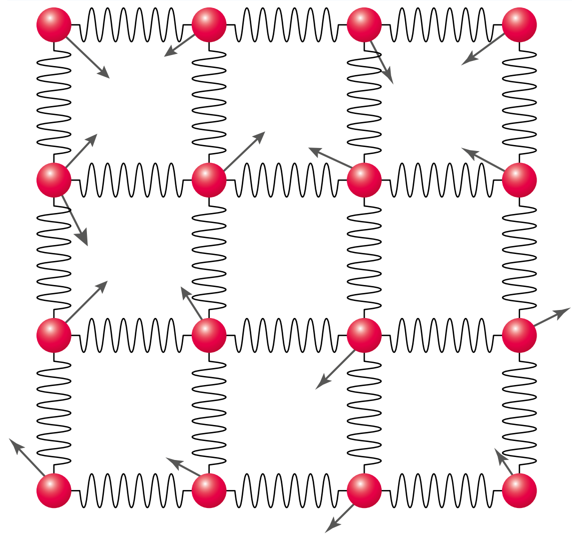
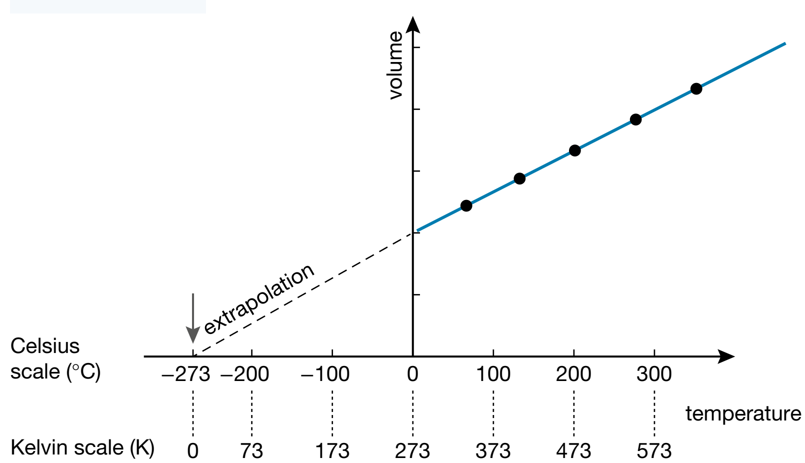


TABLE 11.2

	Internal energy	
	Movement that is NOT related to temperature	Movement that is related to temperature
Atoms in a gas	None	Moving and colliding
Molecules in a gas	Spinning, stretching, compressing and bending	Moving and colliding
Molecules in a liquid	Spinning, stretching, compressing and bending	Moving and colliding
Atoms in a solid	Pulling and pushing	Jiggling
Energy types	Other types of kinetic energy, potential energy	Translational kinetic energy

The kinetic theory of matter is the origin of the Kelvin temperature scale. If temperature depends on the movement of particles, then the slower they move, the lower the temperature. When the particles stop moving, the temperature will be the lowest that is physically possible. This temperature was adopted as **absolute zero**. But how do we measure it and what is its value?

FIGURE 11.11



In the early 1800s, gases were a good material to work with to explore the nature of matter. An amount of gas in a glass vessel could be heated and the variables of temperature, volume and pressure to keep the volume fixed could be easily measured. Joseph Gay-Lussac and Jacques Charles independently investigated how the volume of gases changed with temperature if they were kept under a constant pressure. They found that all gases kept at constant pressure expand or contract by 1/273 of their volume at 0 °C for each Celsius degree rise or fall in temperature.

From that result you can conclude that if you cooled the gas, and it stayed as a gas and did not liquefy, you could cool it to a low enough temperature that its volume reduced to zero. The temperature would be absolute zero. According to their experiments, absolute zero was −273 °C. Nowadays more accurate experiments put the value at −273.15 °C.

In degrees Kelvin, absolute zero is 0 K. The increments in the Kelvin temperature scale are the same size as those in the Celsius scale so, if the temperature increased by 5 °C, it also increased by 5 K. The conversion formula between the two temperature scales is:

$$\text{degrees Kelvin} = \text{degrees Celsius} + 273$$

11.2 SAMPLE PROBLEM 1

What is the Kelvin temperature at which ice melts?

SOLUTION:

Ice melts at 0 °C, so the equivalent Kelvin temperature is $0 + 273 = 273$ K.

Note: In 1968, the international General Conference on Weights and Measures decided that Kelvin temperatures do not use the ° symbol as do Celsius and Fahrenheit temperatures.

11.2.4 Thermal equilibrium

Energy is always transferred from a region of high temperature to a region of lower temperature until both regions reach the same temperature. When the temperature is uniform, a state of **thermal equilibrium** is said to exist.

So, when a hot nail is dropped into a beaker of cold water, energy will be transferred from the hot nail into the water even though the hot nail has less total internal energy than the water. When thermal equilibrium is reached, the temperatures of the water and the nail are the same. The particles of water and the particles in the nail have the same amount of random translational kinetic energy. Figure 11.12 shows how the kinetic particle model can be used to explain the direction of energy transfer in the beaker.

Implicit in the above discussion on thermal equilibrium and internal energy is the subtle but important point made by James Clerk Maxwell that 'All heat is of the same kind'.

FIGURE 11.12 The particles in the nail have more kinetic energy (on average) than those that make up the water. They collide with the particles of water, losing some of their kinetic energy and increasing the kinetic energy of individual particles of water. The temperature of the surrounding water increases.

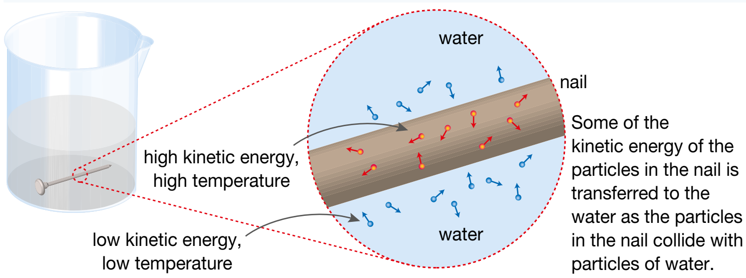
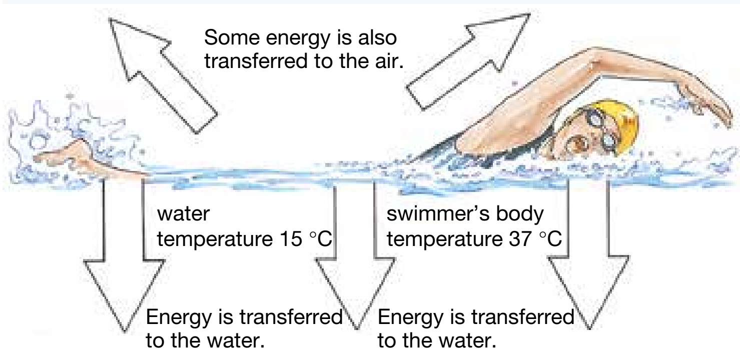


FIGURE 11.13 When you swim in a cold pool, energy is transferred from your body into the water. The water has much more total internal energy than your body because there is so much of it. However, the particles in your body have more random translational kinetic energy that can be transferred to the particles of cold water. Hopefully, you would not remain in the water long enough for thermal equilibrium to be reached.



11.2.5 Laws of thermodynamics

Three laws of thermodynamics were progressively developed during the 19th century, but in the 20th century it became apparent that the principle of thermal equilibrium could be seen as the logical underpinning of these three laws. Consequently, the Zeroth Law of Thermodynamics became accepted.

Zeroth Law of Thermodynamics

Consider three objects: A, B and C. It is the case that A is in thermal equilibrium with B, and C is also in thermal equilibrium with B. Since 'All heat is of the same kind', it follows that A is in thermal equilibrium with C.

In practice this means that all three objects, A, B and C, are at the same temperature, and the law enables the comparison of temperatures.

First Law of Thermodynamics

The **First Law of Thermodynamics** states that energy is conserved and cannot be created or destroyed. If there is an energy change in a system, all the energy must be accounted for. From a thermodynamics point of view, the internal energy of a substance and any change in it are a crucial part of this accounting exercise.

Consider a volume of air inside a balloon that is placed in direct sunlight. The air inside the balloon will get hotter and the balloon will expand slightly.

The First Law of Thermodynamics says:

Change in the internal energy **of** the air	=	Heat energy applied **to** the air	−	Work done **by** the air
Using symbols: ΔU	=	Q	−	W

The energy from the Sun heats the air inside the balloon, increasing the kinetic energy of the air molecules. The air molecules lose some of this energy as they repeatedly collide with the wall of the balloon, forcing it outwards.

The First Law of Thermodynamics applies to many situations: cylinders in a car engine, hot air balloons, food consumption, pumping up a tyre, and the weather. Consequently, the word 'system' is often used as a generic name when discussing thermodynamics.

Note: The words in bold, '**of**', '**to**' and '**by**', and the minus sign are important in the equation as Q and W can be either positive or negative.

Guide:

If a system absorbs heat, e.g. energy from sunlight,	then $Q > 0$.
If a system releases heat, e.g. when you sweat,	then $Q < 0$.
If a system does work on the surroundings, e.g. a hot balloon expands,	then $W > 0$.
If the surroundings do work on the system. e,g, pumping up a tyre,	then $W < 0$.

11.2 SAMPLE PROBLEM 2

(a) A balloon is placed in direct sunlight. The sunlight supplies 200 joules of energy to the balloon. The air inside pushes out the balloon surface, doing 50 joules of work. By how much does the internal energy of the air inside change?

(b) While doing some heavy lifting, you do 2500 joules of work on the weights while releasing 3000 joules of heat. By how much did your internal energy change?

SOLUTION:

(a) $Q = 200\,\text{J},\ W = 50\,\text{J}$, so $\Delta U = 200 - 50 = 150\,\text{J}$. The internal energy of the air in the balloon increased by 150 J.

(b) $Q = -3000\,\text{J},\ W = 2500\,\text{J}$, so $\Delta U = -3000 - 2500 = -5500\,\text{J}$. Your internal energy decreased by 5500 J.

11.2 Exercise 1

1 Estimate each of the following temperatures in Kelvin:
   (a) the maximum temperature in Sydney on a hot summer's day
   (b) the minimum temperature in Sydney on a cold, frosty winter's morning
   (c) the current room temperature
   (d) the temperature of cold tap water
   (e) the boiling point of water.
2 How does the term 'heat' differ from the term 'temperature'?
3 True or false? 'You can tell whether the internal energy of a body is due to an energy transfer as heat or as work.'
4 Explain what is meant by the term thermal equilibrium.
5 Carbon dioxide sublimates, that is, it goes directly from solid to gas, at −78.5 °C. What is this temperature in degrees Kelvin?
6 The temperature of the surface of the planet Mars was measured by the Viking lander: it ranged from 256 K to 166 K. What are the equivalent temperatures in degrees Celsius?
7 A block of ice is melted by 100 joules of energy. What is the size and the sign of W and ΔU?
8 Some countries, such as the United States, use the Fahrenheit scale rather than the Celsius temperature scale. On the Fahrenheit scale, water boils at 212 °F and freezes at 32 °F. The temperature −40 °C is equal to −40 °F.
   (a) Use this information to derive an equation that will convert a Fahrenheit temperature (F) into a Celsius temperature (C).
   (b) What would be the Celsius temperature equivalent of 400 °F?
9 One joule of heat energy is needed to raise the temperature of 1 gram of water by 0.24 °C. How many joules of heat energy would be needed to raise the temperature of 120 grams of water from 20 °C to 60 °C?
10 On an X temperature scale, water freezes at −25 °X and boils at 375 °X. On a Y temperature scale, water freezes at −70 °Y and boils at −30 °Y. What will a temperature of 50 °Y be on the X temperature scale?

11.3 Changing temperature

11.3.1 Specific heat capacity

Once the temperature of materials could be accurately measured, it became apparent that, when heated, some materials increased in temperature more quickly than others. The property of the material that describes this phenomenon is called the specific heat capacity and is defined as the amount of energy required to increase the temperature of 1 kg of the substance by 1 °C (or K).

It takes more energy to increase the temperature of water by 1 °C than any other common substance. Water also needs to lose more energy to decrease in temperature. In simple terms, this means that water maintains its temperature well, cooling down and heating up more slowly than other materials.

Specific heat capacities differ because of two factors:

- the different contributions to the internal energy by the forms of energy other than translational kinetic energy, and
- the varying mass of individual atoms and molecules.

The internal energy of single-atom gases, such as helium, neon and argon, consists of only translational kinetic energy. So, the specific heat capacities should be the same if you account for their difference in mass. Look up the atomic weight for each gas and multiply it by each gas's specific heat capacity and compare your answers.

TABLE 11.3 Specific heat capacity of some common substances.

Substance	Specific heat capacity ($J\ kg^{-1}\ K^{-1}$)
Helium	5193
Water	4200
Human body (average)	3500
Cooking oil	2800
Ethylene glycol (used in car 'antifreeze')	2400
Ice	2100
Steam	2000
Fertile topsoil	1800
Neon	1030
Air	1003
Aluminium	897
Carbon dioxide	839
Desert sand	820
Glass (standard)	670
Argon	520
Iron and steel (average)	450
Zinc	387
Copper	385
Lead	129

The quantity of energy, Q, transferred to or from a substance in order to change its temperature is directly proportional to three factors:

- the mass of the substance (m)
- the change in temperature (ΔT)
- the specific heat capacity of the substance (c).

Thus,

$$Q = mc\Delta T$$

11.3 SAMPLE PROBLEM 1

(a) How much energy is needed to heat 8.0 L (about 8.0 kg) of water from a room temperature of 15 °C to 85 °C (just right for washing dishes)?

(b) A chef pours 200 g of cold water with a temperature of 15 °C into a hot aluminium saucepan with a mass of 250 g and a temperature of 120 °C. What will be the common temperature of the water and saucepan when thermal equilibrium is reached? Assume that no energy is transferred to or from the surroundings.

SOLUTION:

(a) $Q = mc\Delta T$

where

$c = 4200\ \text{J kg}^{-1}\text{K}^{-1}$ (from table 11.3)

$m = 8.0\ \text{kg}$

$\Delta T = 70\ \text{K}$ (same change as 70 °C)

Therefore,

$$\begin{aligned} Q &= 8.0\ \text{kg} \times 4200\ \text{J kg}^{-1}\text{K}^{-1} \times 70\ \text{K} && \text{(substituting data)} \\ &= 2\,352\,000\ \text{J} && \text{(solving)} \\ &= 2352\ \text{kJ} && \text{(using the most appropriate units)} \\ &= 2.4 \times 10^3\ \text{kJ}. \end{aligned}$$

The energy needed is best expressed as 2400 kJ.

(b) The solution to this question relies on the following three factors:

1. Energy is transferred from the saucepan into the water until both the saucepan and the water reach the same temperature (T_f °C).
2. The amount of internal energy (Q_w) gained by the water will be the same as the amount of internal energy lost by the saucepan (Q_s).
3. The internal energy gained or lost can be expressed as $mc\Delta T$. (ΔT can be expressed in K or °C since change in temperature is the same in both units.)

Therefore,

$$Q_w = Q_s$$

$$m_w c_w \Delta T_w = m_s c_s \Delta T_s$$

where

change in temperature of the water, $\Delta T_w = T_f - 15\ °\text{C}$

change in temperature of the saucepan, $\Delta T_s = 120\ °\text{C} - T_f$.

$$0.200\ \text{g} \times 4200\ \text{J kg}^{-1}\,°\text{C}^{-1} \times (T_f - 15\ °\text{C}) = 0.250\ \text{g} \times 900\ \text{J kg}^{-1}\,°\text{C}^{-1} \times (120 - T_f)$$

(substituting data)

$$\begin{aligned} 840T_f - 12\,600 &= 27\,000 - 225T_f && \text{(simplifying and expanding brackets)} \\ 1065T_f &= 39\,600 \\ T_f &= 37\ °\text{C} && \text{(solving)} \end{aligned}$$

The saucepan and water will reach a common temperature of 37 °C.

This example is a good illustration of the implications of a high specific heat capacity. Even though there was a smaller mass of water than aluminium, the final temperature was closer to the original water temperature than the original aluminium temperature.

AS A MATTER OF FACT

Eating a hot pie can be a health hazard! The temperature of the pastry and filling of a hot pie are the same. Thermal equilibrium has been reached. So why can you bite into a pie that seems cool enough to eat and be burnt by the filling?

The reason is that the filling is mostly water, while the pastry is mostly air. When your mouth surrounds that tasty pie, energy is transferred from the pie to your mouth. Each gram of water in the filling releases about 4 J of energy into your mouth for every 1 °C lost (since the specific heat of water is 4200 J kg^{-1}K^{-1}). Each gram of air in the pastry releases only about 1 J of energy into your mouth for every 1 °C lost (since the specific heat of air is 1000 J kg^{-1}K^{-1}). Gram for gram, the filling transfers four times more energy into your mouth than the pastry.

11.3.2 Latent heat and the kinetic particle model of matter

In order for a substance to melt or evaporate, energy must be added. During the process of melting or evaporating, the temperature of the substance does not increase. The energy added while the state is changing is called **latent heat**. The word *latent* is used because it means 'hidden'. The usual evidence of heating — a change in temperature — is not observed.

Similarly, when substances freeze or condense, energy must be released. However, during the process of changing state, there is no decrease in temperature accompanying the loss of internal energy.

In simple terms, the energy transferred to or from a substance during melting, evaporating, freezing or condensing is used to change the state rather than to change the temperature.

During a change of state, internal energy is gained or lost from the substance. Recall, however, that the internal energy includes the random kinetic and potential energy of the particles in the substance. The random translational kinetic energy of particles determines the temperature.

When a substance being heated reaches its melting point, the incoming energy increases the potential energy of the particles rather than the random translational kinetic energy of the particles. After the substance has melted completely, the incoming energy increases the kinetic energy of the particles again. When the substance is being cooled, the internal energy lost on reaching the melting (or freezing) point is potential energy. The temperature does not decrease until the substance has completely solidified.

The same process occurs at the boiling point of a substance. While evaporation or condensation takes place, the temperature of the substance does not change. The energy being gained or lost is latent heat, 'hidden' as changes in internal potential energy take place.

Specific latent heat of fusion

The **specific latent heat of fusion** is the quantity of energy required to change 1 kilogram of a substance from a solid to a liquid without a change in temperature. Note that the same quantity of energy is lost without a change in temperature during the change from a liquid to a solid. The specific latent heat of fusion of water is 334 kJ kg^{-1}.

Specific latent heat of vaporisation

The **specific latent heat of vaporisation** is the quantity of energy required to change 1 kilogram of a substance from a liquid to a gas without a change in temperature. Note that the same quantity of energy is lost without a change in temperature during the change from a gas to a liquid. The specific latent heat of vaporisation of water is 2.3×10^3 kJ kg^{-1}.

FIGURE 11.14 Changes of state and latent heat.

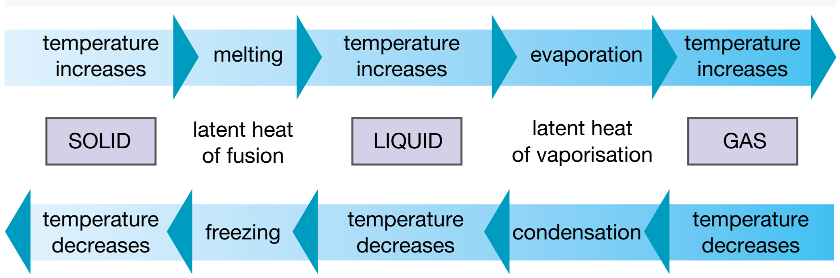


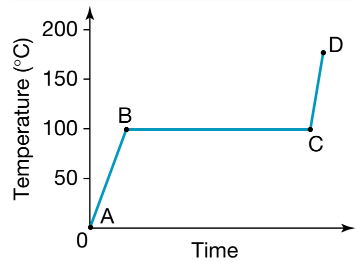
Table 11.4 details both the specific latent heat of fusion and the specific latent heat of vaporisation of a number of common substances.

The graph in figure 11.15 shows how the temperature of water increases as it is heated at a constant rate. During the interval BC, the temperature is not increasing. The water is changing state. The energy transferred to the water is not increasing the random translational kinetic energy of water particles. Note that the gradient of the section AB is considerably less than the gradient of the section CD. What difference in the properties of water and steam does this reflect?

TABLE 11.4 Specific latent heat of some common substances.

Substance	Specific latent heat of fusion (J kg^{-1})	Specific latent heat of vaporisation (J kg^{-1})
Water	3.3×10^5	2.3×10^6
Oxygen	6.9×10^3	1.1×10^5
Sodium chloride	4.9×10^5	2.9×10^6
Aluminium	2.2×10^3	1.7×10^4
Iron	2.8×10^5	6.3×10^6

FIGURE 11.15 A heating curve for water being heated at a constant rate.



Algebraically, the quantity of energy, Q, required to change the state of a substance without a change in temperature can be expressed as:

$$Q = mL$$

where

m = mass of the substance

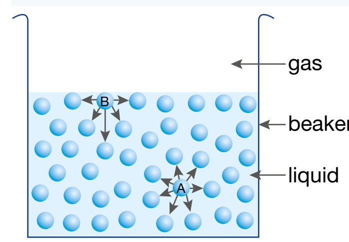
L = specific latent heat of fusion or vaporisation.

11.3.3 Evaporation

Your skin is not completely watertight, which allows water from the skin and tissues beneath it to evaporate. The latent heat of vaporisation required for the water to change state from liquid to gas is obtained from the body, reducing its temperature. Evaporation of water from the mouth and lungs also takes place during the process of breathing. Even without sweating, the energy used to evaporate water in the body accounts for about 17% of the total heat transfer from the body to the environment.

Water evaporates even though its temperature is well below its boiling point. The temperature is dependent on the average translational kinetic energy of the water molecules. Those molecules with a kinetic energy greater than average will be moving faster than the others. Some of them will be moving fast enough to break the bonds holding them to the water and escape from the liquid surface. The escaping molecules obtained their additional energy from the rest of the liquid water, thus reducing its temperature.

FIGURE 11.16 Particle A experiences forces of attraction from the other surrounding particles in all directions. Particle B does not experience as many forces, so it will need less kinetic energy to escape the forces of attraction and evaporate.



AS A MATTER OF FACT

A burn caused by steam at 100 °C is considerably more serious than a burn caused by the same mass of boiling water. Each kilogram of hot steam transfers 2600 kJ of energy to your skin as it condenses to water at 100 °C. Each kilogram of newly condensed steam then transfers 4.2 kJ for each °C drop in temperature as it cools to your body temperature of about 37 °C. That's about 265 kJ. The total quantity of energy transferred by each kilogram of steam is therefore about 2865 kJ. A kilogram of boiling water would transfer 265 kJ of energy as it cooled to your body temperature.

WORKING SCIENTIFICALLY 11.2

Devise and perform an experiment to determine the specific heat constant for a ceramic coffee mug.

11.3 Exercise 1

1 How much energy is needed to increase the temperature of your body by 1°C?
2 True or false? 'When a hot object warms a cool object in a closed system, their temperature changes are equal in magnitude.'
3 Explain why the presence of a large body of water such as an ocean or sea tends to moderate the temperature extremes experienced on adjacent dry land.
4 What temperature increase would be experienced by a 200-gram cube of iron if its thermal energy was increased by 2000 J?
5 A jeweller drops a 10-gram piece of silver that has been annealed at a temperature 520 °C into a 1 litre water bath at a temperature of 20 °C to cool it. What will be the final temperature of both the water bath and the silver when they have reached thermal equilibrium? (Assume that no energy is lost to the surroundings. The specific heat capacity of silver is 233 J kg^{-1} K^{-1}.)
6 An 800 g rubber hot-water bottle that has been stored at a room temperature of 15 °C is filled with 1.5 kg of water at a temperature of 80 °C. Before being placed in a cold bed, thermal equilibrium between the rubber and water is reached. What is the common temperature of the rubber and water at this time? (Assume that no energy is lost to the surroundings. The specific heat capacity of rubber is 1700 J kg^{-1} K^{-1}. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}.)
7 How much energy is required to completely convert 2 kg of ice at –5 °C into steam at 100 °C? Assume no energy loss to the surroundings.
8 A mystery metal is used to make a 3.0 kg container that is then used to hold 12 kg of water. Both the container and the water have an initial temperature of 16 °C. A 2.0 kg piece of the mystery metal is heated to 200 °C and dropped into the water. If the final temperature of the entire system is 20 °C when thermal equilibrium is reached, determine the specific heat of the mystery metal.
9 A 50 kg block of ice at 0 °C slides along the horizontal surface of a thermally insulated material, starting at a speed of 5.4 m s^{-1} and finally coming to rest after travelling 28 m. What mass of the ice melted as a result of friction between the block and the surface? (Assume that all of the energy lost by mechanical energy due to the friction is converted to thermal energy in the block of ice.)

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Try out this Interactivity: Thermal equilibrium
Searchlight ID: int-6390

Try out this Interactivity: Changes of state
Searchlight ID: int-0222

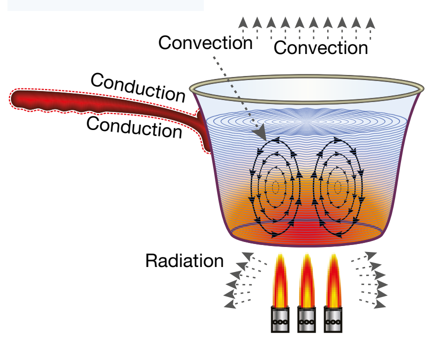
11.4 Transferring heat

11.4.1 Keeping warm, keeping cool

During heating and cooling, energy is always transferred from a region of high temperature to a region of lower temperature. There are many situations in which it is necessary to control the rate at which the energy is transferred.

- Warm-blooded animals, including humans, need to maintain their body temperature in hot and cold conditions. Cooling of the body must be reduced in cold conditions. In hot conditions, it is important that cooling takes place to avoid an increase in body temperature.

FIGURE 11.17



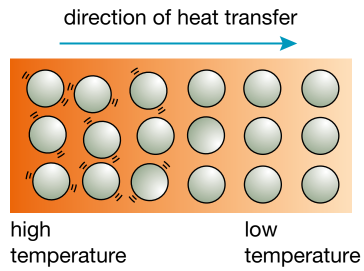
- Keeping your home warm in winter and cool in summer can be a costly exercise, both in terms of energy resources and money. Applying knowledge of how heat is transferred from one place to another can help you to find ways to reduce how much your house cools in winter and heats up in summer, thus reducing your energy bills.
- The storage of many foods in cold temperatures is necessary to keep them from spoiling. In warm climates, most beverages are enjoyed more if they are cold. The transfer of heat from the warmer surroundings needs to be kept to a minimum.

There are three different processes through which energy can be transferred during heating and cooling: **conduction**, **convection** and **radiation**.

11.4.2 Conduction

Conduction is the transfer of heat through a substance as a result of collisions between neighbouring vibrating particles. The particles in the higher temperature region have more random kinetic energy than those in the lower temperature region. As shown in figure 11.18, the more energetic particles collide with the less energetic particles, giving up some of their kinetic energy. This transfer of kinetic energy from particle to particle continues until thermal equilibrium is reached. There is no net movement of particles during the process of conduction.

FIGURE 11.18 Conduction is the transfer of heat due to collisions between neighbouring particles.



Solids are better conductors of heat than liquids and gases. In solids, the particles are more tightly bound and closer together than in liquids and gases. Thus, kinetic energy can be transferred more quickly. Metals are the best conductors of heat because free electrons are able to transfer kinetic energy more readily to other electrons and atoms.

The **thermal conductivity (k)** of a material is a constant that expresses how easily heat is transferred through it. Substances such as metals, which are good conductors of heat, have high thermal conductivity constants, while thermal insulators such as air or polystyrene have very low thermal conductivity constants, as shown in table 11.5.

The rate at which the heat is transferred across an object can be determined by considering the thermal conductivity constant of the material from which the object is made, the difference in temperature across the object (ΔT), its cross-sectional area (A) and its thickness or length (d). The relationship between these variables and the rate of heat transfer $\left(\frac{Q}{t}\right)$ is expressed in the thermal conductivity equation:

$$\frac{Q}{t} = \frac{\mathrm{k}\, A\, \Delta T}{d}$$

where A is in m^2, T is in °C, d is in m, and the rate of heat transfer is in $\mathrm{J\,s^{-1}}$.

FIGURE 11.19 Factors affecting the rate of heat transfer through a material.

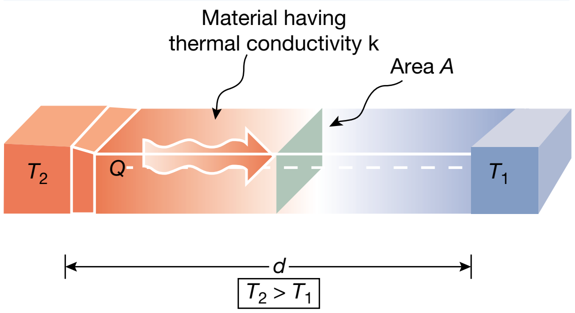


TABLE 11.5 Thermal conductivity constants of some common substances.

Material	Thermal conductivity constant, k ($\mathrm{W\,m^{-1}\,°C^{-1}}$)
Silver	427
Copper	398
Aluminium	237
Cast iron	55
Brick	1.6
Ice	2.18
Water	0.58
Expanded polystyrene	0.3
Air	0.024
Argon	0.016

11.4 SAMPLE PROBLEM 1

The end of a 30 cm long copper rod at 20 ºC is dropped into a campfire at a temperature of 400 ºC. If the rod is circular with a diameter of 1 cm, at what rate will the heat be transferred along the length of the rod?

SOLUTION:

First, the cross-sectional area of the rod is determined:

$$A = \pi(r)^2 = \pi(0.005\ \text{m})^2 = 7.8 \times 10^{-5}\ \text{m}^2$$

Then,

$$\frac{Q}{t} = \frac{\text{k}\,A\,\Delta T}{d}$$

$$\frac{Q}{t} = \frac{(398\ \text{W m}^{-1}\ {}^\circ\text{C}^{-1})\ (7.8 \times 10^{-5}\ \text{m}^2)\ (400\ {}^\circ\text{C} - 20\ {}^\circ\text{C})}{(0.3\ \text{m})}$$

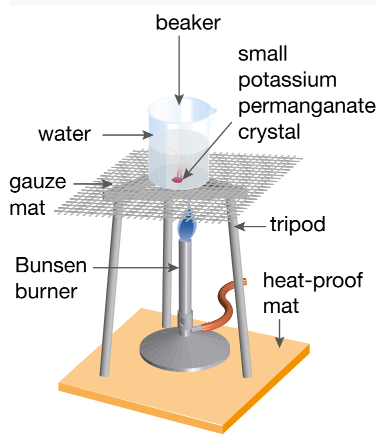
$$\frac{Q}{t} = 39\ \text{J s}^{-1}$$

11.4.3 Convection

Convection is the transfer of heat through a substance as a result of the movement of particles between regions of different temperatures. Convection takes place in liquids and gases where particles are free to move around. In solids, the particles vibrate about a fixed position and convection does not occur.

The movement of particles during convection is called a **convection current**. Faster moving particles in hot regions rise while slower moving particles in cool regions fall. The particles in the warm water near the flame in figure 11.20 are moving faster and are further apart than those in the cooler water further from the flame. The cooler, denser water sinks, forcing the warm, less dense water upwards. This process continues as the warm water rises, gradually cools and eventually sinks again, replacing newly heated water.

FIGURE 11.20 Purple particles from a crystal of potassium permanganate carefully placed at the bottom of the beaker are forced around the beaker by convection currents in the heated water.



Convection currents are apparent in ovens that do not have fans. As the air circulates, the whole oven becomes hot. However, the top part of the oven always contains the hottest, least dense air. As the air cools, it sinks and is replaced by less dense hot air for as long as the energy source at the bottom of the oven remains on. Fans can be used to push air around the oven, providing a more even temperature.

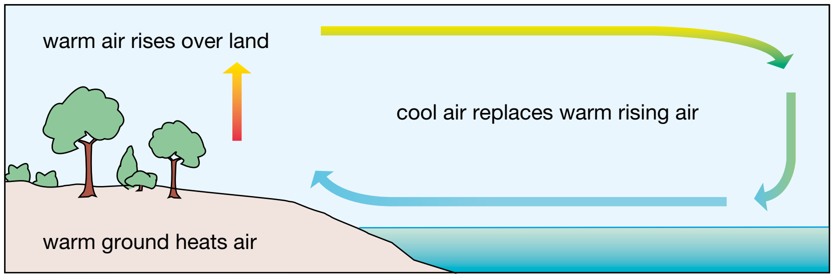
Home heating systems use convection to move warm air around. Ducted heating vents are, where possible, located in the floor. Without the aid of powerful fans, the warm air rises and circulates around the room until it cools and sinks, to be replaced with more warm air. In homes built on concrete slabs, ducted heating vents are in the ceiling. Fans are necessary to push the warm air downwards so that it can circulate more efficiently.

In summer, loose-fitting clothing is more comfortable because it allows air to circulate. Thus, heat can be transferred from your body by convection as the warm air near your skin rises and escapes upwards.

Hot summer days

During hot summer days, radiant energy from the Sun heats the land and sea. The land, however, has a lower specific heat capacity than the sea, and soon has a higher temperature than the water. The air near the ground becomes hot as a result of conduction. As this air gets hot, it expands, becoming less dense than the cooler, denser air over the sea. The air over the sea rushes in towards the land, replacing the rising warm air, causing what is known as a sea breeze. Coastal areas generally experience less extreme maximum temperatures than inland areas as a result of sea breezes.

FIGURE 11.21 A sea breeze is caused by convection currents resulting from temperature differences between the land and the sea.



On the south coast of Australia, strong northerly winds blowing from the land will occasionally prevent convection from causing a sea breeze. When this happens in summer, temperatures can soar — often above 40 °C.

During the night, if the land becomes colder than the sea, convection currents push cool air from the land towards the sea, creating a land breeze.

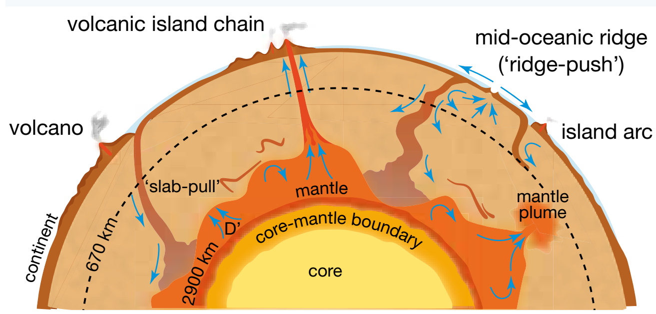
WORKING SCIENTIFICALLY 11.3

Some coffee mugs seem better at keeping the coffee warm for a longer period of time than others. By investigating the effect that the material, wall thickness, shape and surface area of the opening of a coffee mug has on its ability to keep coffee warm, design the most effective mug possible.

Convection inside the Earth

Energy transfer by convection is common in gases and liquids, but it can also occur in solids under the right conditions. The high temperatures, about 2000 °C, and pressures in the Earth's mantle are enough to make solid rock move, only very slowly of course. The speed of the rock movement is a few centimetres per year.

FIGURE 11.22 New crust is formed at a ridge and returns to the mantle at a trench.



The heat energy in the Earth comes from the radioactive decay of elements such as uranium. The heat energy is not evenly distributed and hot spots occur under the mantle. The hot lighter rock at these points slowly rises, while denser rock at colder spots slowly sinks. This sets up a convection cell in the Earth's mantle with the surface crust moving horizontally across the Earth.

The molten rock wells up at mid-ocean ridges and moves out. The rock eventually meets the edge of a continental plate and cools further, becoming denser, then sinks back towards the mantle in a deep ocean trench.

11.4.4 Radiation

Heat can be transferred without the presence of particles by the process of radiation. All objects with a temperature above absolute zero (0 K) emit small amounts of **electromagnetic radiation**. Visible light, microwaves, infra-red radiation, ultraviolet radiation and x-rays are all examples of electromagnetic radiation. All electromagnetic radiation is transmitted through empty space at a speed of $3.0 \times 10^8\ \mathrm{m\,s^{-1}}$, which is most commonly known as the speed of light.

Electromagnetic radiation can be absorbed by, reflected from or transmitted through substances. Scientists have used a wave model to explain much of the behaviour of electromagnetic waves. These electromagnetic waves transfer energy, and reflect and refract in ways that are similar to waves on water.

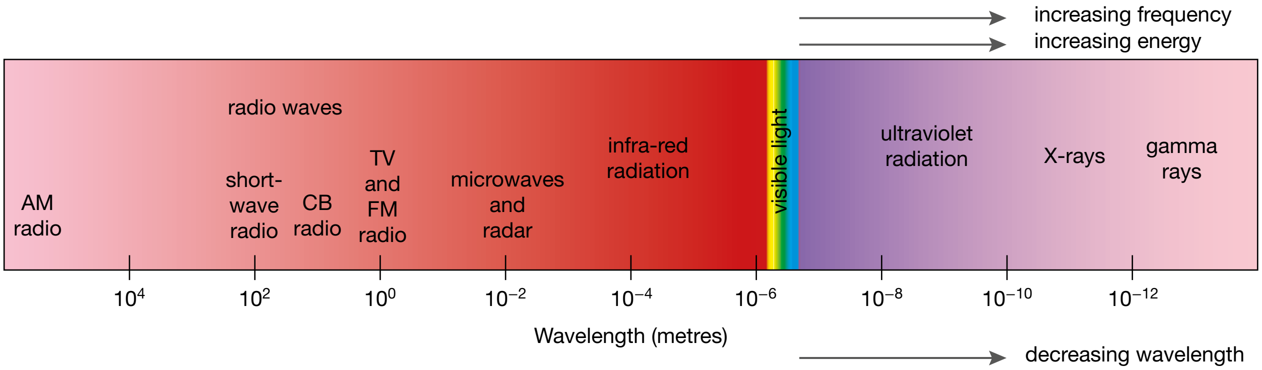
What distinguishes the different types of electromagnetic radiation from one another is:

- their wavelength (the distance the wave takes to repeat itself)
- their frequency (the number of wavelengths passing every second)
- the amount of energy they transfer.

These properties in turn determine their ability to be transmitted through transparent or opaque objects, their heating effect and their effect on living tissue.

Figure 11.23 shows the electromagnetic spectrum and demonstrates that higher energy radiation corresponds to low wavelength.

FIGURE 11.23 The electromagnetic spectrum is the full range of wavelengths of all electromagnetic waves. All objects emit some electromagnetic radiation.

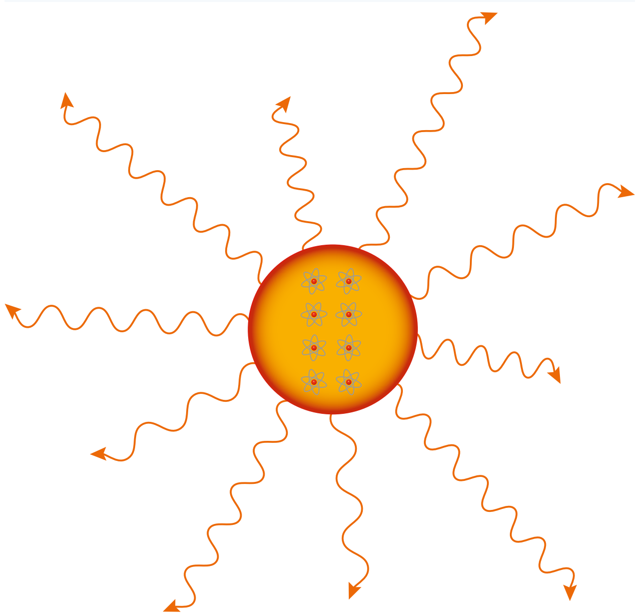


Why do hot objects emit electromagnetic radiation?

All matter is made up of atoms. At any temperature above absolute zero, these atoms are moving and colliding into each other. The atoms contain positive and negative charges. The motion of the atoms and their collisions with other atoms affect the motion of the electrons. Because they are charged and moving around, the electrons produce electromagnetic radiation. Electrons moving in an antenna produce a radio signal, but in a hot object the motion is more random with a range of speeds.

So, a hot object produces radiation across a broad range of wavelengths. If its temperature increases, the atoms move faster and have more frequent and more energetic collisions. These produce more intense radiation with higher frequencies and shorter wavelengths.

FIGURE 11.24 Electromagnetic radiation from a hot body.

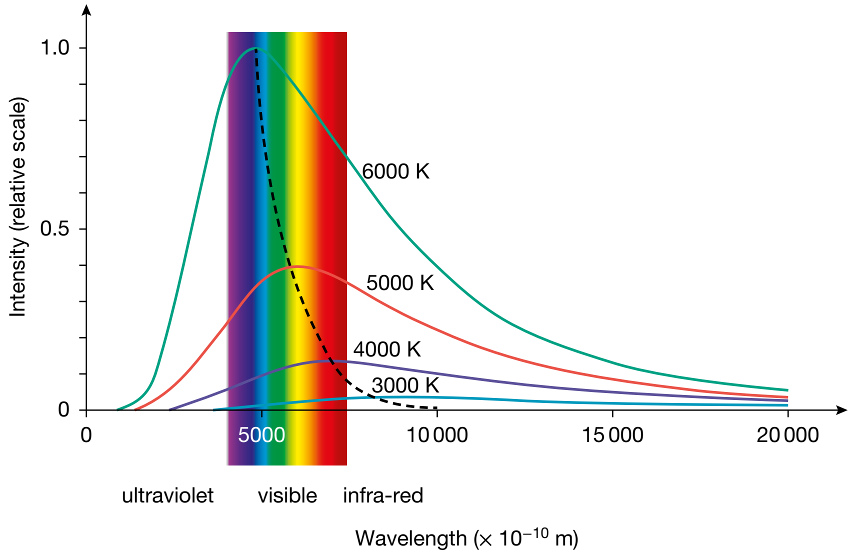


During the late 19th century, scientists conducted investigations into how much radiation was produced across the spectrum and how this distribution changed with temperature. The results are displayed in figure 11.25.

The graphs for different temperatures are roughly the same shape. Starting from the right with long wavelengths, there is very little infra-red radiation emitted. As the wavelength gets shorter, the radiation produced increases to a maximum; finally, as the wavelength shortens even further, the amount of radiation drops away quite quickly. The graphs for higher temperatures have a peak at a shorter wavelength and also have a much larger area under the graph, meaning a lot more energy is emitted.

Early researchers such as Jozef Stefan were keen to find patterns and relationships in the data and to be able to explain their observations. In 1879, Stefan compared the area under the graph for different temperatures. This area is the total energy emitted every second across all wavelengths, in other words, the power.

FIGURE 11.25 Intensity for different wavelengths across the electromagnetic spectrum for four different temperatures: 3000 K, 4000 K, 5000 K and 6000 K.



He found that the power was proportional to absolute temperature to the power of 4, that is,

power $\propto T^4$.

This means that if the absolute temperature of a hot object doubles from 1000 K to 2000 K, the amount of energy emitted every second increases by 2^4 ($2 \times 2 \times 2 \times 2 = 16$ times).

Using this relationship, Stefan was able to estimate the temperature of the surface of the Sun as 5430 °C or 5700 K, which is very close to the value known today of 5778 K.

Ludwig Boltzmann later proved this from a theoretical standpoint, and so the power $\propto T^4$ relationship is called the Stefan–Boltzmann law.

This relationship applies to all objects, but the constant of proportionality depends on the size of the object and other factors.

11.4 SAMPLE PROBLEM 2

(a) When iron reaches about 480 °C, it begins to glow with a red colour. How much more energy is emitted by the iron at this temperature, compared to when it is at a room temperature of 20 °C?

(b) How much hotter than 20 °C would the iron need to be to emit 10 times as much energy?

SOLUTION:

(a) **STEP 1**

Change the temperature to Kelvin.
Temperature of hot iron = 480 °C + 273 = 753 K
Temperature of cold iron = 20 °C + 273 = 293 K

STEP 2

Calculate the ratio.
Ratio of power (hot to cold) = Ratio of temperatures to the power of 4

$$\frac{P_{\text{hot}}}{P_{\text{cold}}} = \left(\frac{T_{\text{hot}}}{T_{\text{cold}}}\right)^4 = \left(\frac{753}{293}\right)^4 = 44$$

The hot iron emits 44 times as much energy every second as it does when it is at room temperature.

(b) **STEP 1**

Change the temperature to Kelvin.

Temperature of cold iron = 20 °C + 273 = 293 K

STEP 2

Calculate the ratio.

Ratio of power (hot to cold) = Ratio of temperatures to the power of 4

$$\frac{P_{hot}}{P_{cold}} = \left(\frac{T_{hot}}{T_{cold}}\right)^4$$
$$10 = \left(\frac{T_{hot}}{293}\right)^4$$

This can be rearranged to give $10^{\frac{1}{4}} = \frac{T_{hot}}{293}$

To calculate $10^{\frac{1}{4}}$, you can use the x^y key on your calculator.

First, enter the number for x, in this case 10, then push the x^y key, then enter the number for y, in this case 0.25, which is $\frac{1}{4}$ as a decimal. Then hit the equals key. You should get the answer 1.778.

$$1.778 = \frac{T_{hot}}{293}$$
$$T_{hot} = 1.778 \times 293 = 521\ \text{K}$$

STEP 3

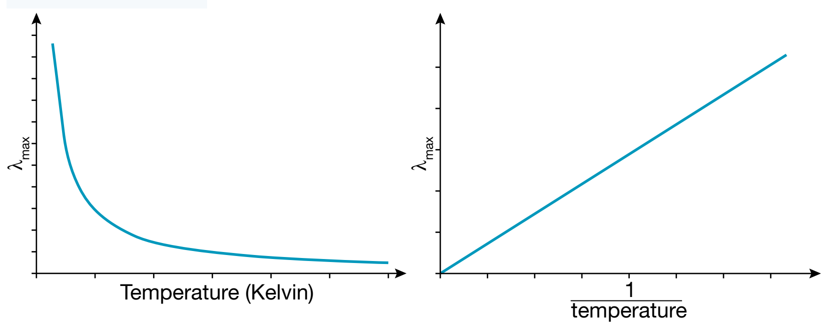
Change the temperature to Celsius.

Temperature of hot iron = 521 − 273 = 248 °C

At 248 °C, the iron will emit 10 times as much energy every second as iron at 20 °C.

Wilhelm Wien (pronounced *Veen*) in 1893 was able to show that as the temperature increased, the wavelength of maximum intensity of energy emitted decreased, and indeed the two quantities were inversely proportional. That is, the wavelength is proportional to the inverse of the temperature. This can be seen in the graph on the right in figure 11.26.

FIGURE 11.26



Wien's law can be written as $\lambda_{max} \times T =$ constant. The value of this constant is 2.90×10^{-3} mK (metre–degree Kelvin).

11.4 SAMPLE PROBLEM 3

(a) At what wavelength is the peak intensity of the light coming from a star whose surface temperature is 11 000 K (about twice as hot as the Sun)?
(b) In what section of the spectrum is this wavelength?

SOLUTION:

(a) $$\lambda_{max} = \frac{2.90 \times 10^{-3}\,\text{mK}}{11\,000\,\text{K}}$$
$$= 2.636 \times 10^{-7}\,\text{m}$$

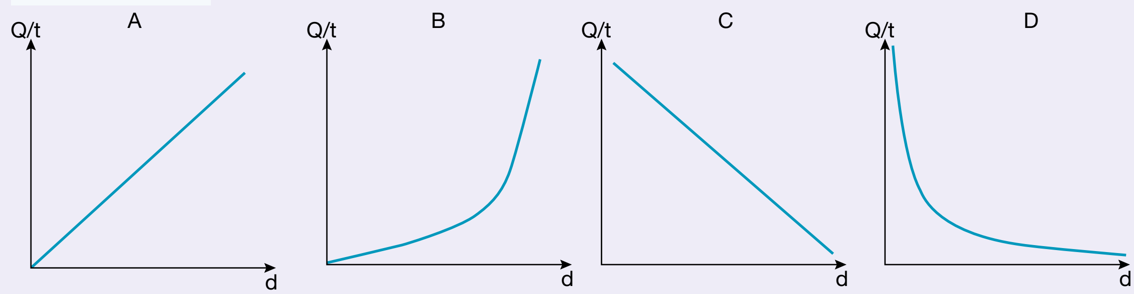
1 nanometre $= 10^{-9}$ m, so $\lambda_{max} = 263.6 \times 10^{-9}\,\text{m} = 264\,\text{nm}$

(b) 264 nm is beyond the violet end of the visible spectrum, so it is in the ultraviolet section of the electromagnetic spectrum.

11.4 Exercise 1

1 Which graph best describes the relationship between the rate of heat transfer through a metal sheet and its thickness?

FIGURE 11.27



2 At what speed does radiant energy move through space?
3 Why is it not practical to drink hot coffee in an aluminium picnic cup?
4 Stainless steel cookware often has a layer of copper or aluminium on the bottom. What is the most likely reason for this?
5 Explain why sweating is the only cooling option for the human body when the surrounding temperature is higher than about 34 °C?
6 Calculate the rate of heat loss through a 90 cm × 90 cm glass window in a house that is heated to 24 °C if the outside temperature is –5 °C and the glass pane is 10 mm thick. ($k_{glass} = 0.27\,\text{W m}^{-1}\,{}^{\circ}\text{C}^{-1}$)
7 The Sun has a surface temperature of 5778 K and radiates energy at a rate of 3.846 × 1026 watts. How much energy would a star of similar size radiate if its surface temperature was 8000 K?
8 Determine the surface temperature of a star that emits light at a maximum intensity of 450 nm.
9 What is the wavelength of the light with the peak intensity from our solar system's closest neighbouring star, Proxima Centauri, which has an average surface temperature of 3042 K?
10 A 5 kg block of ice at 0 °C is placed inside a sealed styrofoam esky that has external dimensions of 40 cm × 40 cm × 60 cm. The walls, floor and cover of the esky are 1.5 cm thick. If it is assumed that the air temperature outside the esky is 25 °C, approximately how long would it take for the ice to melt?

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11.5 Review

11.5.1 Summary

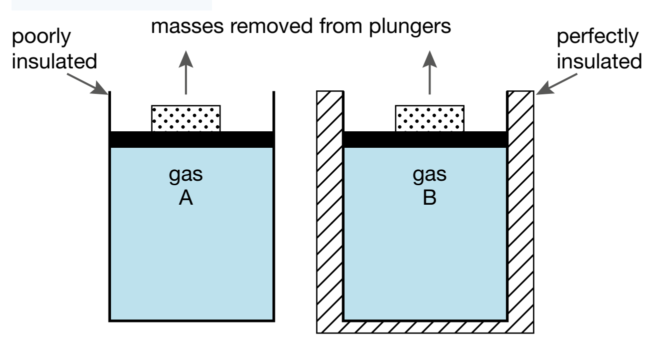
- A thermometer measures temperature, and various properties of materials can be used to make one.
- There are different temperature scales, with Celsius and Kelvin being the common ones. Temperatures in one scale can be converted to any other.
- The kinetic particle model of matter explains heat phenomena.
- Internal energy is the energy associated with the random movement of molecules and it comes in many forms, including translational kinetic energy, rotational and vibrational kinetic energy, and potential energy.
- Temperature is a measure of the average translational kinetic energy of the atoms and molecules in a substance.
- Objects at different temperatures, if placed in contact, will reach a common temperature. This process is called thermal equilibrium and is described as the Zeroth Law of Thermodynamics.
- The First Law of Thermodynamics states that if energy is transferred to or from a system, then the total energy must be conserved, with any changes in the internal energy of the system given by $\Delta U = Q - W$, where ΔU is the change in internal energy, Q is the heat added to the system and W is the work done by the system.
- The specific heat capacity, c, of a substance is the amount of energy required to increase the temperature of 1 kg of the substance by 1 °C.
- When substances of different specific heat capacities and different temperatures are mixed, the final temperature can be determined by using the conservation of energy and the relationship $Q = m\,c\,\Delta T$ for each substance.
- The latent heat, L, of a substance is the amount of energy required to change the state from solid to liquid or liquid to gas of 1 kg of the substance. For a substance of mass m kg, the energy required is given by $Q = m\,L$.
- Evaporation of a liquid occurs because some of the faster particles have sufficient energy and speed to break free of the surface. The removal of these particles lowers the overall average kinetic energy of the remaining particles and consequently the temperature of the liquid.
- Heat energy can be transferred by conduction, convection and radiation.
- Conduction is the transfer of heat energy through a material by collisions between adjacent particles.
- Some materials conduct heat energy well and are called conductors. Others do not and are called insulators.
- The thermal conductivity constant is a measure of how well a material conducts heat energy.
- The rate of heat transfer through a material can be calculated using the equation
  $$\frac{Q}{t} = \frac{\mathrm{k}\,A\Delta T}{d}$$
- Convection is the transfer of heat energy through a substance, usually a liquid or a gas, by the movement of particles between regions of different temperature. Hotter material is less dense because faster moving particles push each other further apart. If free to move, the less dense and hotter material will rise, displacing cooler material.
- The movement of plates in the Earth's crust is caused by convection from heat energy within the Earth.
- Radiation is the transfer of heat energy by the emission of electromagnetic radiation.
- The emitted radiation comes from a range of wavelengths across the electromagnetic spectrum.
- The graph of the energy contribution of different wavelengths of emitted radiation has a characteristic shape.
- For a given temperature, there is a specific wavelength at which the most energy is emitted. Its symbol is λ_{max}.
- The graph of the energy contribution of different wavelengths for a higher temperature has a lower λ_{max} and a larger area under the graph.

- λ_{max} is inversely proportional to the temperature measured in Kelvin ($\lambda_{max}T$ = constant).
- The amount of energy emitted per second is called power.
- The area under the graph of energy contribution against wavelength is a measure of power. The area under the graph is proportional to the Kelvin temperature raised to the power of four. This can be expressed as power $\propto T^4$.

11.5.2 Questions

1. Why is the Celsius scale of temperature commonly used rather than the Kelvin scale?
2. What is the main advantage of an absolute scale of temperature?
3. The temperature of very cold water in a small test tube is measured with a large mercury-in-glass thermometer. The temperature measured is unexpectedly high. Suggest a reason why this might be the case.
4. James Joule showed that mechanical energy could be transformed into the internal energy of a substance or object. The temperature of a nail, for example, can be raised by hitting it with a hammer. List as many examples as you can of the use of mechanical energy to increase the temperature of a substance or object.
5. Explain in terms of the kinetic particle model why a red-hot pin dropped into a cup of water has less effect on the water's temperature than a red-hot nail dropped into the same cup of water.
6. If today's maximum temperature was 14 °C and tomorrow's maximum temperature is expected to be 28 °C, will tomorrow be twice as hot? Explain your answer.
7. Explain why energy is transferred from your body into the cold sea while swimming even though you have less internal energy than the surrounding cold water.
8. Why can't you put your hand on your own forehead to estimate your body temperature?
9. It is said that thermometers indirectly measure the temperature of an object by measuring their own temperature. Explain this statement by referring to the concept of thermal equilibrium.
10. Adam says that 'A thermometer measures the average temperature between itself and the object it is measuring,' while Bob says that 'A thermometer directly measures the temperature of the object.' Explain why each is wrong.
11. For each of the following, calculate the values of Q, W and ΔU and indicate whether the temperature increases, decreases or stays the same.
    (a) A gas in a fixed container is heated by 500 J.
    (b) A gas in a container with a flexible lid is cooled by ice with 250 J of energy extracted.
    (c) A gas in a container with a plunger is squashed by a heavy mass moving down, losing 150 J of gravitational potential energy.
    (d) A stretched rubber band at room temperature with 5 J of stored energy is released.
12. A can filled with a high-pressure gas has a balloon attached over the top. What happens to the temperature of the gas inside the can as you allow the gas to expand into the balloon?
13. In figure 11.28, two beakers are filled with the same gas. A plunger is fitted so that no gas escapes; friction is negligible between the plunger and the beaker walls.
    The block is removed from each plunger, and the plunger moves upward.
    (a) In each case, does the gas do work or is work done on the gas? Explain your reasoning in a few sentences.
    (b) Is there a larger transfer of thermal energy as heat between Gas A and the surroundings or between Gas B and the surroundings?
    Explain your reasoning in a few sentences. Draw an arrow on each figure indicating the direction of thermal energy flow.

FIGURE 11.28



(c) For the expansion of Gas A, how do the work and heat involved in this process affect the internal energy of the gas? Explain your reasoning in a few sentences.
(d) For the expansion of Gas B, how do the work and heat involved in this process affect the internal energy of the gas? Explain your reasoning in a few sentences.

14. A barbecue uses gas from a gas bottle as its energy source. After the BBQ has been running a while, ice is noticed around the top of the gas bottle. Explain the physics principles behind this observation.
15. Two insulated containers are connected by a valve. The valve is closed. One container is filled with gas, the other is a vacuum. The valve is opened. Is there any change in temperature? Is there any change in the internal energy? What are the values of Q and W in this situation?
16. Consider these three scenarios, then complete the table at the right, using either 0, − or +.
(a) An insulated container, such as a thermos flask, has a piston that can be moved up and down without letting the air out. The piston is pushed down.
(b) A metal tin with a lid is heated.
(c) A metal tin with a sliding lid and a mass on top is heated.

	A	B	C
Heat (+ is in)			
Work (+ is out)			
ΔU			

Use the table of specific heat capacities at the right to answer questions 17, 18 and 19.

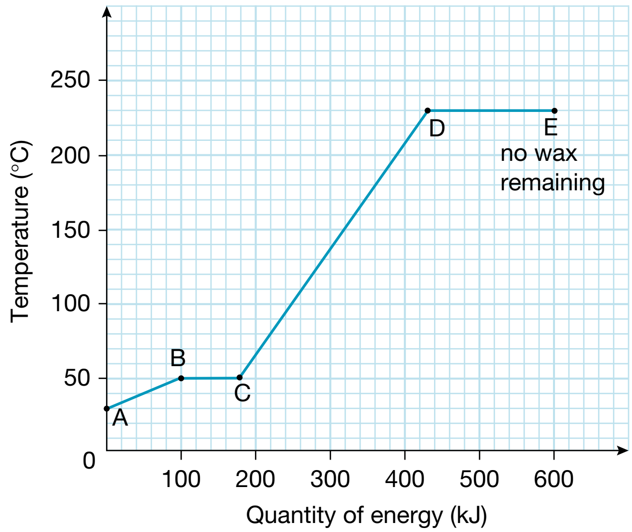
Substance	Specific heat capacity ($J\,kg^{-1}K^{-1}$)
Helium	5193
Water	4200
Human body (average)	3500
Cooking oil	2800
Ethylene glycol (used in car ‘antifreeze’)	2400
Ice	2100
Steam	2000
Fertile topsoil	1800
Neon	1030
Air	1003
Aluminium	897
Carbon dioxide	839
Desert sand	820
Glass (standard)	670
Argon	520
Iron and steel (average)	450
Zinc	387
Copper	385
Lead	129

17. The same hotplate is used to heat 50 g of ethylene glycol (used in car antifreeze) and 50 g of cooking oil. Both substances are heated for 2 minutes. Use the data in the table to determine:
(a) which liquid needs more energy to raise its temperature by 1 °C
(b) which liquid will experience the greater increase in temperature.
18. The quantity of energy needed to increase the temperature of a substance is directly proportional to the mass, specific heat capacity and the change in temperature of the substance. If 200 kJ is used to increase the temperature of a particular quantity of a substance, how much energy would be needed to bring:
(a) twice as much of the substance through the same change in temperature?
(b) three times as much of the substance through a temperature change twice as great?
19. Use the table to answer the following questions.
(a) Why is the specific heat capacity of the human body so high?
(b) Why is the specific heat capacity of desert sand so much lower than that of fertile topsoil?
(c) When heating water to boiling point in a saucepan, some of the energy transferred from the hotplate is used to increase the temperature of the saucepan. Which would you expect to gain the most energy from the hotplate: an aluminium, copper or steel saucepan?
(d) Make some general comments about the order of substances listed in the table of specific heat capacities.
20. Use the data in the short table at the right to determine the quantity of energy needed to evaporate 500 g of water without a change in temperature.

Substance	Specific latent heat of fusion ($J\,kg^{-1}$)	Specific latent heat of vaporisation ($J\,kg^{-1}$)
Water	3.3×10^5	2.3×10^6

21. The graph in figure 11.29 shows the heating curve obtained when 500 g of candle wax in solid form was heated from room temperature in a beaker of boiling water.
   (a) What is the boiling point of the candle wax?
   (b) During the interval BC, there is no increase in temperature even though heating continued. What was the energy transferred to the candle wax being used for during this interval?
   (c) In which state of matter was the candle wax during the interval CD?
   (d) Use the heating curve to determine the latent heat of fusion of candle wax.
   (e) Which is higher: the specific heat capacity of solid candle wax or the specific heat capacity of liquid candle wax? Explain your answer.
   (f) Explain in terms of the kinetic particle model what is happening during the interval DE.

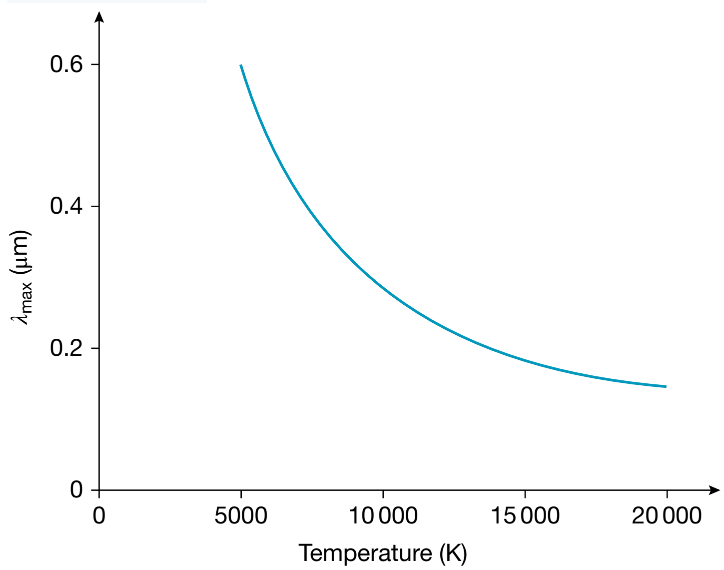
FIGURE 11.29



22. How much ice at 0°C could be melted with 1 kg of steam at 100 °C, assuming no loss of energy to the surroundings? Use the specific latent heat values quoted in table 11.4. The specific heat capacity of water is $4200\ \text{J kg}^{-1}\text{K}^{-1}$.
23. Explain why vegetables cook faster by being steamed than boiled.
24. Why are burns caused by steam more serious than those caused by boiling water?
25. In hot weather, sweat evaporates from the skin. Where does the energy required to evaporate the sweat come from?
26. Explain the importance of keeping a lid on a simmering saucepan of water in terms of latent heat of vaporisation.
27. Explain in terms of the kinetic particle model why you can put your hand safely in a 300 °C oven for a few seconds, while if you touch a metal tray in the same oven your hand will be burned.
28. How does the evaporation of water cause a reduction in the temperature of the surrounding air?
29. Give two reasons why you feel cooler when the wind is blowing than you would in still air at the same temperature.
30. In humid weather, evaporation of perspiration takes place as it does in dry weather. However, the cooling effect is greatly reduced. Why?
31. Explain with the aid of a well-labelled diagram how heat is transferred through a substance by conduction.
32. Why are liquids and gases generally poorer conductors of heat than solids?
33. Explain in terms of conduction and convection why you don't heat a test tube of water with the Bunsen burner flame near the top of the test tube.
34. Explain with the aid of a well-labelled diagram how convection occurs in a liquid that is being heated from below.
35. Why is it not possible for heat to be transferred through solids by convection?
36. When you swim in a still body of water on a hot afternoon there is a noticeable temperature difference between the water at the surface and the deeper water.
   (a) Explain why this difference occurs.
   (b) If the water is rough, the difference is less noticeable. Why?
37. The daytime temperature of an area can decrease for several days after a major bushfire. Why does this happen?

38. The microwave cooking instructions for frozen pies state that pies should be left to stand for two minutes after heating. What happens to the pie while it stands?
39. Standing near the concrete wall of a city building after a hot day, you can instantly feel its warmth from a few metres away.
   (a) How is the energy transferred to you?
   (b) What caused the building to get hot during the day?
40. Why do ducts in the ceiling need more powerful fans than those in the floor?
41. Why do conventional ovens without fans have heating elements at the bottom. What is the advantage of having an oven with a fan?
42. A 100 W light globe has a tungsten filament, which has a temperature of 2775 K when switched on.
   (a) How much radiation does the filament emit at 20 °C?
   (b) The voltage on the light globe is reduced to increase the lifetime of the filament. The temperature of the filament is now 2000 K. What is the power saving?
   (c) The voltage is now increased so that the power output is 200 W. What is the new filament temperature in Kelvin?
43. (a) A piece of iron has a yellow glow when it reaches 1150 °C. How much more energy is emitted every second at this temperature compared to when the iron glows red at 480 °C?
   (b) At what temperature in degrees Celsius would the iron give off 10 times as much energy as it does at 480 °C?
44. Our Sun gives off most of its light in the 'yellow' portion of the electromagnetic spectrum. Its λ_{max} is 510 nm. Calculate the average surface temperature of the Sun.
45. The Earth's surface has an average temperature of 288 K. What is the wavelength of maximum emission from the Earth's surface?
46. The human body has a surface temperature of about 37 °C.
   (a) What is the wavelength at which the human body emits the most radiation?
   (b) In what part of the spectrum is this wavelength ?
47. (a) A violet star has a spectrum with a peak intensity at a wavelength of 4×10^{-7} m. Determine the temperature at the surface of this star.
   (b) A red star has a spectrum with a peak intensity at a wavelength of 7×10^{-7} m. Determine the temperature at the surface of this star.
48. Figure 11.30 shows how λ_{max} (the wavelength of the peak of the radiation spectrum) for a range of stars varies with their surface temperatures.
   (a) Use values from the graph to confirm Wien's Law.
   (b) Use the graph to estimate the surface temperature of a star whose intensity peaks at a wavelength of:
      (i) 0.4 μm
      (ii) 0.27 μm.
   (c) Use the graph to estimate the peak wavelength for a star with a surface temperature of:
      (i) 15 000 K
      (ii) 5550 K.

FIGURE 11.30



49. Suppose the surface temperature of the Sun was about 12 000 K, rather than about 6000 K.
   (a) How much more thermal radiation would the Sun emit?
   (b) What would happen to the Sun's wavelength of peak emission?
50. Two stars have identical diameters. One has a temperature of 5800 K; the other has a temperature of 2900 K. What are the colours of these stars? Which is brighter and by how much?

PRACTICAL INVESTIGATIONS

Investigation 11.1: The good oil on heating

Aim

This investigation aims to show that different substances require different quantities of energy to change their temperatures by the same amount.

It also aims to show that the quantity of energy required to change the temperature of a given substance is directly proportional to the mass of the substance.

Apparatus

hotplate
two 100 mL beakers
50 mL of water
cooking oil
electronic balance or other equipment to measure mass
2 thermometers
Stopwatch or clock with a second hand.

Method

Switch on a hotplate to about half its maximum setting. While waiting for the temperature of the hotplate to stabilise, pour 50 mL of water at room temperature into a 100 mL beaker and measure its mass. Add cooking oil to an identical beaker so that the beaker and oil have the same total mass as the beaker of water. Record the temperature of each liquid.

Place the beaker of water on the hotplate and, while gently stirring, use a stopwatch to record the time taken for the temperature of the water to increase by 10 °C.

Place the beaker of cooking oil on the hotplate and, while gently stirring, record the time taken for the temperature of the cooking oil to increase by 10 °C.

Results and analysis

1. Which liquid required more energy to increase its temperature by 10 °C?
2. Which liquid has the greater resistance to change in temperature?
   Repeat the procedure above using 100 mL of water at room temperature.
   Before commencing, however, predict how long it will take the water to increase its temperature by 10 °C.
3. By what factor did the amount of energy required to increase the temperature by 10 °C change when the amount of water was doubled?
4. Was the result consistent with your prediction? If not, suggest some reasons for the inconsistency.
5. If the cooking oil and water were supplied with the same amount of energy by heating for the same amount of time on the same flame, which would experience the greater increase in temperature?

Investigation 11.2: Cooling

Aim

This investigation aims to show that the internal energy of a substance can change without a subsequent change in temperature.

It also aims to produce a cooling curve that illustrates the concept of latent heat.

Apparatus

large test tube containing about 2–3 cm depth of paraffin wax
250 mL beaker
heat-proof mat, Bunsen burner and matches
tripod and gauze mat
retort stand and clamp
thermometer.

Method

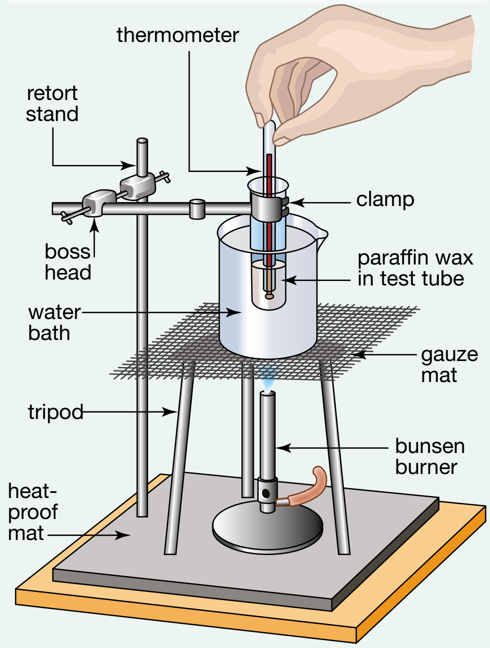
Place some solid paraffin wax into a large test tube. Heat the test tube in a water bath until the temperature of the paraffin wax is about 80 °C.

Remove the test tube from the water bath and record the temperature of the paraffin every minute until the temperature has fallen to about 30°C. Gently and carefully stir with the thermometer while the liquid paraffin is cooling.

Results and analysis

1. Construct a graph of temperature versus time to display your data.
2. What causes the decrease in temperature of the liquid paraffin?
3. How does the rate of cooling change as the liquid paraffin solidifies?
4. During the process of solidification, what form of internal energy is being lost from the paraffin? Where is it going?
5. What is the meaning of the term 'latent heat of fusion' and how does it relate to this investigation?

FIGURE 11.31 A water bath is used to heat the paraffin wax.



Investigation 11.3: Relating colour to temperature

Aim

To relate the colour of a hot object to its temperature

Apparatus

12 V mounted lamp
voltmeter
variable 12 V power supply
ammeter
hand spectroscope

Theory

Wien's Law for black-body radiation tells us that as an object gets hotter, the dominant wavelength of the electromagnetic radiation it emits shifts towards the blue end of the spectrum. The object in this experiment is a lamp filament. Voltage and current readings will be taken during the experiment so that the resistance of the filament can be calculated. This will be used as an indication of temperature, as the resistance of a filament is approximately proportional to its temperature.

Method

1. Set up a circuit with the power supply, ammeter and mounted lamp in series; place the voltmeter in parallel with the lamp.
2. Darken the room, set the power supply to its lowest setting and turn it on. Record the readings on the voltmeter and ammeter, then calculate the resistance of the filament using Ohm's Law.
3. Use the spectroscope to examine the dim light from the glowing filament, noting which colours are present and their relative intensities. Record this information by shading in the section of spectrum observed.
4. Repeat this process for each successively higher setting on the power supply.

Results

Your results table should resemble that shown below.

Questions

1. Does each successively higher setting on the power supply produce a higher resistance of the filament?
2. What colours are present in the spectrum on the lowest setting? Describe your impression of the filament colour, looking directly at it without the spectroscope.
3. What colours are present in the spectrum on the highest setting? Describe your impression of the filament colour this time, once again without the spectroscope.
4. Looking at your series of diagrams representing the spectra, what general change can be seen as temperature increases (aside from becoming brighter)?
5. Do your results agree with Wien's Law?

Power supply setting	Voltmeter reading, v(v)	Ammerter reading I(A)	Resistance $R = \frac{V}{I}(\Omega)$	Spectrum						
				V	I	B	G	Y	O	R
A										
B etc										

TOPIC 12 Electrostatics

12.1 Overview

12.1.1 Module 4: Electricity and Magnetism

Electrostatics

Inquiry question: How do charged objects interact with other charged objects and with neutral objects?

Students:

- conduct investigations to describe and analyse qualitatively and quantitatively:
  - processes by which objects become electrically charged (ACSPH002)
  - the forces produced by other objects as a result of their interactions with charged objects (ACSPH103)
  - variables that affect electrostatic forces between those objects (ACSPH103)
- using the electric field lines representation, model qualitatively the direction and strength of electric fields produced by:
  - simple point charges
  - pairs of charges
  - dipoles
  - parallel charged plates
- apply the electric field model to account for and quantitatively analyse interactions between charged objects using:
  - $\boldsymbol{E} = \dfrac{\boldsymbol{F}}{q}$ (ACSPH103, ACSPH104)
  - $\boldsymbol{E} = -\dfrac{v}{\boldsymbol{d}}$
  - $\boldsymbol{F} = \dfrac{1}{4\pi\epsilon_0} \times \dfrac{q_1 q_2}{r^2}$ (ACSPH102)
- analyse the effects of a moving charge in an electric field, in order to relate potential energy, work and equipotential lines, by applying: (ACSPH105)
  - $V = \dfrac{\Delta U}{q}$, where U is potential energy and q is the charge

FIGURE 12.1 Lightning is a naturally occurring example of electrical phenomena.

12.2 Electric Charge

12.2.1 Electric charge and the structure of atoms

The words *electric* and *electricity* are derived from the Greek word for amber: *electron*. Amber is a naturally occurring substance exuded as a resin from certain trees. As long ago as 500 BC, the Greeks had observed that if amber was rubbed, it would attract small pieces of material. Today we can observe this phenomenon more conveniently using certain man-made materials such as perspex. When a perspex rod is rubbed with silk, the rod acquires the ability to attract small pieces of materials such as paper. The rod is said to have become electrically charged.

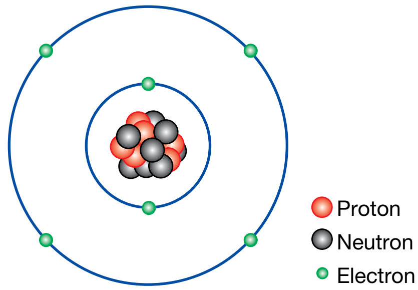
Some other common observations of bodies becoming electrically charged are:

- when you walk on a carpet on a dry day, your body becomes electrically charged. If you touch a metal door handle, you feel a slight shock as your body is discharged.
- on a dry day a car becomes electrically charged as it moves through the air. If you touch the car you feel a slight shock as the car discharges through your body.

We now understand electric charge in terms of the basic structure of matter. All matter is made of atoms that are themselves made of electrons, protons and neutrons, as shown in figure 12.2.

Electric charge is a property of electrons and protons. Because of their electric charge, these particles exert electric forces on each other. Protons carry a **positive charge**; electrons carry a **negative charge**. The positive charge on a proton is equal in magnitude to the negative charge on an electron.

FIGURE 12.2 The structure of an atom.



The directions of the forces between electric charges act such that:

- two positive charges repel one another
- two negative charges repel one another
- a positive charge and a negative charge attract one another.

This is summarised as: *like charges repel; unlike charges attract*.

Neutrons, the third type of particle in atoms, have no electric charge and do not experience electric forces. Neutrons are uncharged or **neutral**.

The SI unit of electric charge is the **coulomb** (C). The name, is derived from Charles Augustin Coulomb (1736–1806), a French physicist who studied the forces between electric charges. The coulomb is defined in terms of electric current, but, for our purposes, it is sufficient to state that a charge of one coulomb is approximately equal to the total charge on 6.25×10^{18} electrons (or 6.25×10^{18} protons). That is, the charge on one electron is approximately -1.60×10^{-19} C, and the charge on one proton is approximately $+1.60 \times 10^{-19}$ C.

FIGURE 12.3 Charles Augustin Coulomb, French physicist.

A coulomb is a very large charge; two charges of 1 C placed 1 metre apart would exert forces on each other of approximately 10^{10} N. A smaller unit of charge, the microcoulomb (μC), is often used ($1\,\mu C = 10^{-6}$ C).

The symbols Q and q are usually used to represent electric charge. For example: $Q = 1.4 \times 10^{-5}$ C.

12.2.2 Neutral and charged bodies

A body that has equal numbers of protons and electrons will be **neutral**. As each atom in a body normally has equal numbers of protons and electrons, most bodies are neutral. However, it is possible for a body to lose some of its electrons or to gain extra electrons.

If a body has *gained* electrons it will have more electrons than protons. The body has an **excess of electrons** and is **negatively charged**. If a body has lost electrons it will have fewer electrons than protons. The body has a **deficiency of electrons** and is **positively charged**. A charge on a body due to an excess or deficiency of electrons is called an **electrostatic charge**.

It is always electrons that are gained or lost by a body, as the protons are strongly bound in the nuclei at the centres of the atoms. For this reason, we talk about excess and deficiency of electrons rather than deficiency and excess of protons.

The deficiency or excess of electrons in a charged body is only a minute fraction (typically no more than 1 in 10^{12}) of the total number of electrons in the body. When we refer to the charge on a body, it is always the net charge that is meant.

12.2 SAMPLE PROBLEM 1

A body has a charge of $+4.60\,\mu C$.
(a) Does it have an excess or a deficiency of electrons?
(b) Calculate how many excess or deficient electrons the body has.

SOLUTION:
(a) Since the body is positively charged, it has a deficiency of electrons.
(b) As the charge on one electron is 1.60×10^{-19} C, the number of deficient electrons, n, that have a charge equal to $4.60\,\mu C$ is given by:

$$n = \frac{(4.60 \times 10^{-6})}{(1.60 \times 10^{-19})}$$
$$= 2.88 \times 10^{15}$$

12.2.3 Conductors and insulators

A **conductor** is a material that contains **charge carriers**; that is, charged particles that are free to move through the material. Examples of conductors and their charge carriers are:
- salt solutions — the charge carriers are positive and negative ions that are free to move through the solution
- metals — the charge carriers are electrons.

In this preliminary course, metals will be the only conductors studied.

An **insulator** is a material that contains no charge carriers. Common insulating materials are dry air, glass, plastics, rubber and ceramics. If an insulator is given an electrostatic charge at a particular area on the insulator, the charge will remain at that area.

If a conductor is given an electrostatic charge, there are two possibilities:
- If the conductor is **insulated** (not earthed), there will be a movement of electrons within the conductor so that the electrostatic charge is as widely spread as possible. The electrostatic charge will be distributed on the surface of the conductor (see figure 12.4).
- If the conductor is **earthed**, there is a conducting path between the conductor and the Earth. Electrons will move to or from the Earth to neutralise the conductor. (See figure 12.5 — note the symbol for an earth connection.) The Earth is so big that the negative charges going to or leaving the Earth produce no detectable charge on it.

FIGURE 12.4 Charge on insulated conductors: (a) Charge on a positively charged conductor and (b) charge on a negatively charged conductor.

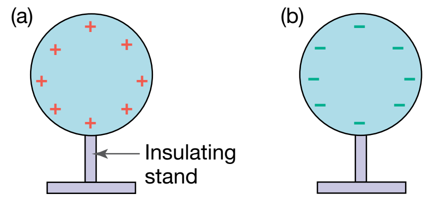
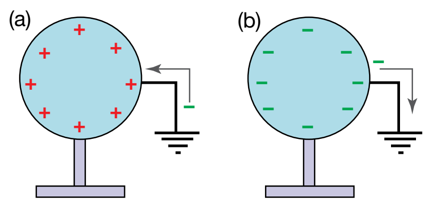


FIGURE 12.5 (a) Earthing a positively charged conductor and (b) earthing a negatively charged conductor.



12.2.4 Methods of charging

Charging by friction

If two bodies made of different materials are rubbed together, a small number of electrons will be transferred from one body to the other. The body that has lost electrons will have a deficiency of electrons and will be positively charged. The body that has gained electrons will have an excess of electrons and will be negatively charged.

The direction in which electrons are transferred depends on what two materials are rubbed together. It is possible to list materials so that when two of them are rubbed together, the first-listed material becomes positively charged and the later-listed material becomes negatively charged. A partial list of this type is shown here: *Air, Rabbit fur, Glass, Human hair, Nylon, Wool, Silk, Steel, Wood, Perspex.*

For example:

- If glass is rubbed with silk, electrons are transferred from the glass to the silk (see figure 12.6).
- If perspex is rubbed with wool, electrons are transferred from the wool to the perspex (see figure 12.7).

FIGURE 12.6 Positively charging glass by friction.

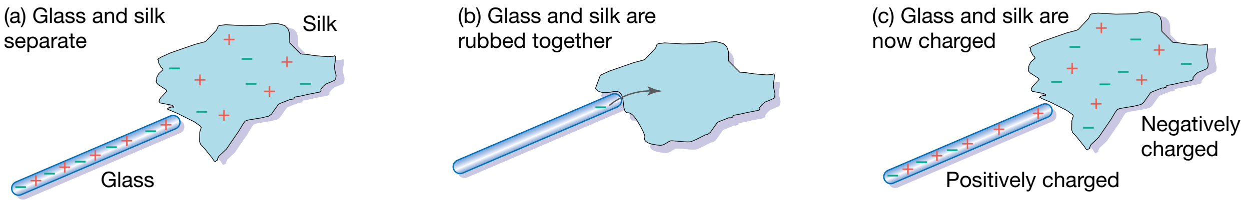
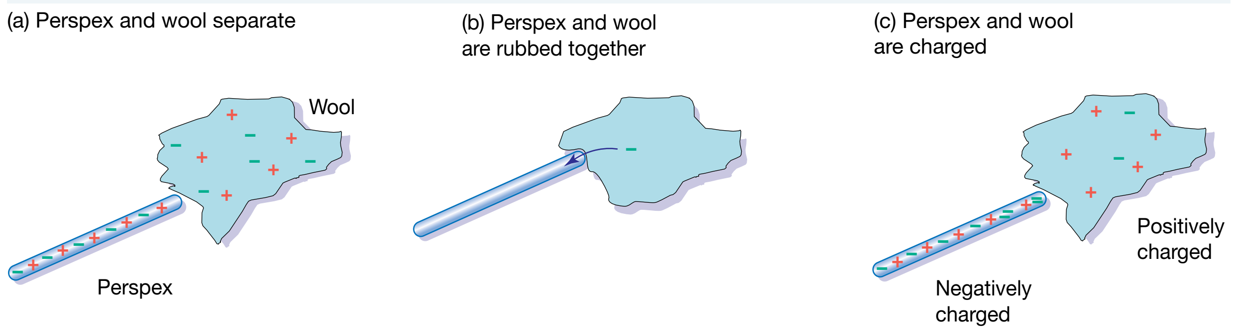


FIGURE 12.7 Negatively charging perspex by friction.



Charging by contact

If a charged conductor is brought into contact with an uncharged conductor, the charge will be shared between the two conductors. The uncharged conductor will be charged by contact.

Figure 12.8 shows a neutral conductor being charged by contact with a positively charged conductor. Figure 12.9 shows a neutral conductor being charged by contact with a negatively charged conductor.

FIGURE 12.8 Charging by contact with a positively charged body.

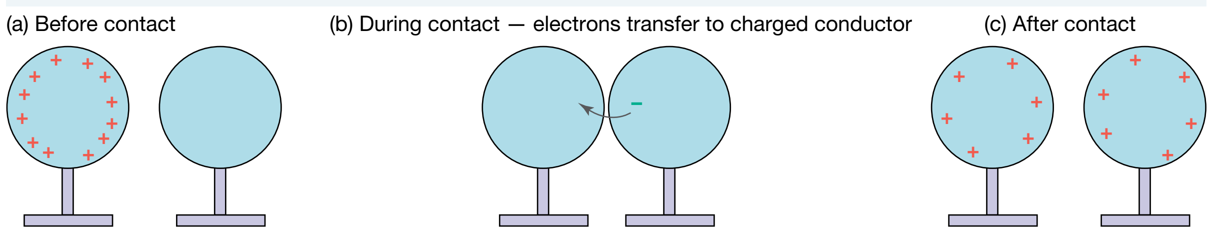
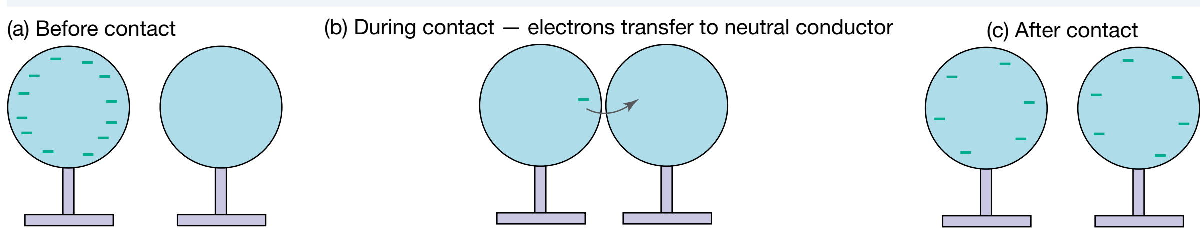


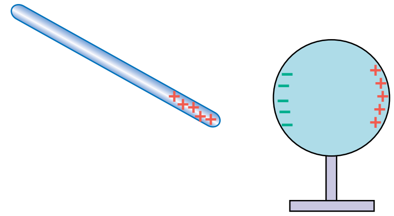
FIGURE 12.9 Charging by contact with a negatively charged body.



Charging by Induction

As an example of induced charges, consider what happens when a positively charged body is brought near an insulated, uncharged conductor. The positively charged body will attract electrons in the conductor. Some of these electrons will move to the area of the conductor closest to the positively charged body. As a result, that end of the conductor will have an excess of electrons (be negatively charged) and the opposite end of the conductor will have a deficiency of electrons (be positively charged). The charges on the conductor are called **induced charges** and the process is called **induction**. This is illustrated in figure 12.10.

FIGURE 12.10 Induced charges in an insulated conductor



Because they are closer, the positively charged body attracts the negative induced charges more strongly than it repels the positive induced charges. There will be a net force of attraction between the positively charged body and the conductor. This is illustrated in figure 12.11.

Induced charges are also produced if a charged body is brought near an insulator. For example, a positively charged body will attract the electrons and repel the nuclei in each atom of the insulator. These forces of attraction and repulsion result in a slight separation of positive and negative charges within each atom. As a result, one end of the insulator will be negatively charged and the other end will be positively charged, as illustrated in figure 12.12.

Induction explains why a charged body attracts small uncharged bodies such as pieces of paper. This is illustrated in figure 12.13.

FIGURE 12.11 Attraction between a charged body and a neutral conductor.

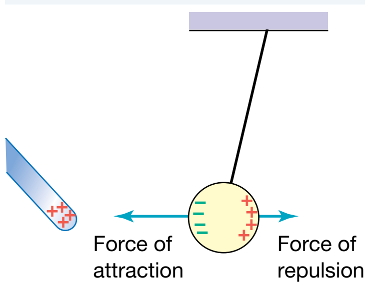


FIGURE 12.12 Induced charges in an insulator.

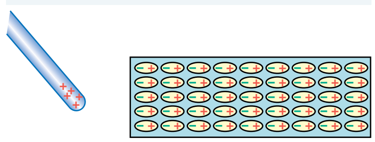
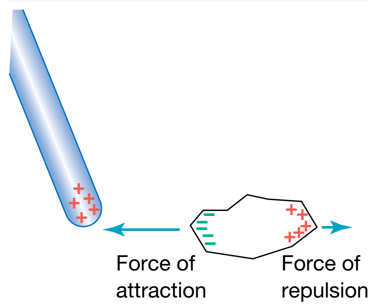
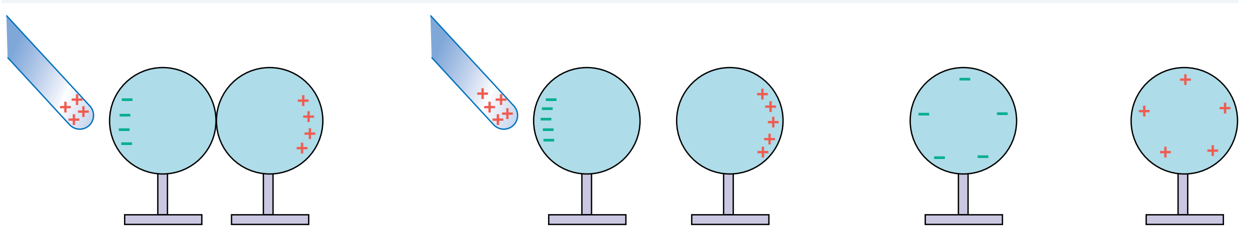


FIGURE 12.13 Attraction between a charged body and a neutral insulator.



Induced charges are usually not permanent; when the charging body is removed, the induced charges disappear. However, it is possible to charge an insulated conductor permanently by induction. One method of doing this is shown in figure 12.14.

FIGURE 12.14 Charging a conductor permanently by induction.



(a) Bring the charged body near two touching, insulated conductors.

(b) Move the insulated conductors apart.

(c) Remove the charged body.

12.2.5 Conservation of charge

When two previously neutral bodies are charged by friction, the amount of positive charge produced on one body is equal to the amount of negative charge produced on the other body.

When two charged conductors are brought into contact, there is a redistribution of charge between the bodies but the total amount of charge remains the same.

Observations such as these lead to the conclusion that the total amount of electric charge never changes; that is, electric charge is conserved.

12.2 SAMPLE PROBLEM 2

Two identical, insulated metal spheres carry charges of $+3.0\,\mu$C and $-7.0\,\mu$C. The spheres are brought into contact and then separated. Calculate the new charge on each sphere.

SOLUTION

The total charge is $-4.0\,\mu$C. As the spheres are identical, this charge will be shared equally by the two spheres. Therefore, the charge on each sphere will be $-2.0\,\mu$C.

PHYSICS IN FOCUS

Lightning

Lightning is a natural phenomenon that illustrates some of the properties of electric charge. Processes inside a storm cloud cause the bottom of the cloud to become negatively charged and the top to become positively charged. The mechanism responsible for this separation of charge is not known for certain, but many scientists think that the following happens. The inside of the cloud contains minute particles of ice. When two ice particles of different sizes collide, there is a transfer of electrons, so that the smaller particle becomes positively charged and the larger particle becomes negatively charged. Under the influence of gravity and updraughts within the cloud, the larger, negatively charged ice particles move towards the bottom of the cloud and the smaller, positively charged ice particles move towards the top of the cloud. Charges in the order of a coulomb can accumulate in this way.

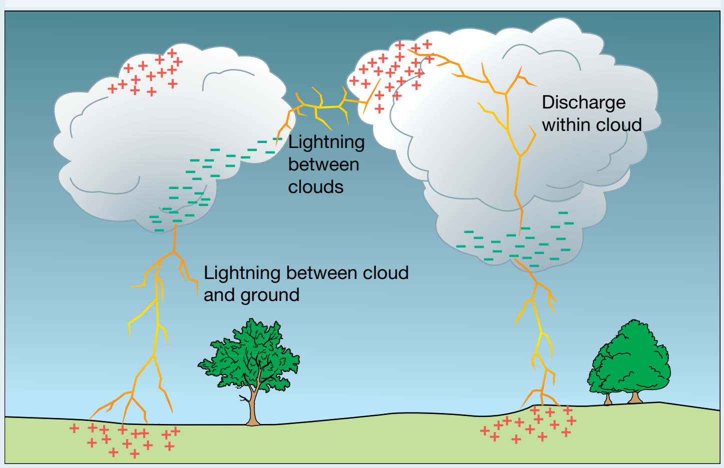
The negative charge at the bottom of the cloud repels electrons from the Earth's surface. The ground under the cloud therefore becomes positively charged by induction.

Air is normally an insulator. Before lightning can occur, the air must become conducting. The negative charges at the bottom of the cloud and the positive charges on the ground under the cloud exert strong forces on the electrons and nuclei of atoms of air between the cloud and the Earth. The electrons experience forces towards the Earth; the nuclei experience forces towards the cloud. These forces can be so great that some electrons are removed from their atoms. An atom that has lost one or more electrons is positively charged. The liberated electrons and the positively charged atoms act as charge carriers, so the air becomes conducting. Once this process begins, a complicated chain of events follows, which leads to the establishment of a conducting path between the cloud and the Earth. Immediately after the conducting path is complete, there is a flow of negative charge from the cloud to the ground. This is a lightning flash and is shown in figure 12.1. A lightning flash usually consists of about four separate strokes, each stroke lasting about 30 microseconds.

The air along the path of a lightning stroke is heated to a temperature of about 20 000°C, producing light. The heated air expands, producing a shock wave in the surrounding air that is heard as thunder.

As well as occurring between a cloud and the Earth, lightning also occurs within a cloud and between two clouds. Figure 12.15 illustrates different paths that lightning may travel.

FIGURE 12.15 Different paths for lightning.



WORKING SCIENTIFICALLY 12.1

A charged rod can be used to pick up small piece of paper or confetti. Devise an experiment that would allow you to find the relationship between the amount of charge given to the rod and the maximum mass of paper that can be picked up.

12.2.6 The development of Coulomb's Law

For thousands of years, philosophers and scientists tried to explain the various manifestations of electricity, but an understanding of the phenomenon was elusive. Both attraction and repulsion were observed, but initially repulsion was considered less important. In 1551 Girolamo Cardano realised that this electrical attraction was different from magnetic attraction. In 1600 William Gilbert, the physician to Elizabeth I, found that other substances such as glass and wax could be 'electrified', but he concluded that metals could not. In 1729 Stephen Gray discovered that electric charge could pass through materials such as the human body and metals. He concluded that some objects are conductors and others insulators. In 1734 Charles du Fay showed that Gilbert was wrong about metals: they could be charged as long as the metal was in a handle of glass. However, du Fay thought there were two fluids, to explain the two types of charge, whereas Benjamin Franklin in 1746 suggested there was only one fluid. Objects with an excess of this fluid were designated positively charged, while negatively charged objects were deficient in the fluid.

Experiments continued, not only to identify what electricity was but also to determine how strong the electric force was and what affected its strength.

In 1766 Franklin tried an experiment involving a hollow metal sphere with a small hole. He charged up the sphere and then lowered a small cork carrying an electric charge inside the sphere. Nothing happened to it — it was not pushed around, no matter where he placed the test charge. He wrote about this to his friend Joseph Priestley in England. Priestley was aware of Newton's Law of Universal Gravitation, which is an inverse square law ($F \propto \frac{1}{r^2}$). He also knew that Newton had proved mathematically that, because of the inverse square law, no net gravitational force exists inside a hollow sphere. That is, at every point inside the sphere, the gravitational force from the mass on one side is balanced by the force from the mass on the other side.

Priestley confirmed Franklin's results and realised that this was strong evidence that the inverse square law applied to electricity. In 1767 he published his finding that electric force was an inverse square law. Unfortunately, his paper went unnoticed by other scientists of his time.

If the force between two charges was an inverse square law (that is $F \propto \frac{1}{r^n}$ where $n = 2$) could the value of n be experimentally confirmed?

In 1769 John Robison investigated how the force between charges changed with separation. He determined the value of the power, n, to be 2.06, very close to 2. In the 1770s Henry Cavendish measured the value as between 1.96 and 2.04, but he never published his results.

In 1788 and 1789, Charles-Augustin de Coulomb published a series of eight papers on different aspects of his electrical experiments, showing that the electric force satisfied the inverse square law.

TABLE 12.1 The results of some of Coulomb's experiments.

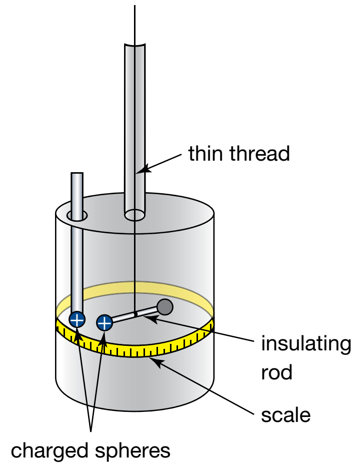
	Distance	
Observed force	Observed	Calculated from the inverse square law
36 units	36 units	36 units
144 units	18 units	18 units
576 units	8.5 units	9 units

These results are no better than the earlier ones, so why was Coulomb's Law named after him?

Coulomb's papers were excellent examples of scientific writing. They were well organised and thorough. He described his apparatus in detail, and he discussed possible sources of error in his measurements. He also used two different methods to determine the value of n, obtaining the same result with each.

To investigate the force between two charges, Coulomb designed a torsion balance. His torsion balance had a long silk thread hanging vertically with a horizontal rod attached at the end. On one end of the rod was a small metal-coated sphere. On the other end was a sphere of identical mass to keep the rod level. The metal sphere was given a quantity of charge and a second metal sphere, charged with the same type of charge, was lowered to be in line with the first sphere. The electrical repulsion caused the silk thread to twist slightly. The angle of twist or deflection of the rod was a measure of the strength of the repulsive force.

FIGURE 12.16 Coulomb's torsion balance.



Coulomb was able to measure the force to an accuracy of less than a millionth of a newton.

Coulomb's Law: *The force between two charges at rest is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.*

$$F \propto \frac{q_1 q_2}{r^2}$$

This expression has no equals sign; it is not an equation or formula. Coulomb was able to measure the force and separation very accurately, but charge was such a new concept that there were no units to measure it. Coulomb was only able to show that halving the size of each charge reduced the size of the force by a quarter.

It was not until the unit for current, the ampere, was defined and precisely measured that a unit for charge could be defined and calculated using following the relationship: charge = current × time ($Q = I \times t$). This unit was called the coulomb after Charles-Augustin de Coulomb. One coulomb of charge equals the amount of charge that is transferred by one ampere of current in one second.

A coulomb of charge is a large quantity of charge. For example, the amount of charge transferred when fur is rubbed against a glass rod is a few millionths of a coulomb. In a typical lightning strike, about 20 coulombs of charge is transferred, whereas, in the lifetime of an AA battery, about 5000 coulombs passes through the battery.

When the electron was discovered, its charge was determined as 1.602×10^{-19} coulombs, which means that the total charge of 6.241×10^{18} electrons would equal one coulomb.

Once a unit to measure charge was available, the above relationship for the force between charges could be written as an equation with a proportionality constant, k:

$$F = \frac{\mathrm{k} q_1 q_2}{r^2}$$

The value of the constant k depends upon the medium in which the charges are placed. Some media are more resistant to the establishment of an electric field, a quality that is reflected in the material's permittivity constant, ε. The value of k is related to the permittivity by the equation

$$\mathrm{k} = \frac{1}{4\pi\varepsilon}$$

In a perfect vacuum, where there are no molecules, the permittivity is equal to $8.854 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$. This value is referred to as the **permittivity of free space**, ε_o. As a result, for a vacuum,

$$\begin{aligned}\mathrm{k} &= \frac{1}{4\pi\varepsilon_o} \\ &= \frac{1}{4\pi 8.854 \times 10^{-12}} \\ &= 8.9988 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}\end{aligned}$$

The value of k for air is similar to that for a vacuum. For ease of calculation and remembering, the value of k is usually approximated to $9.0 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$. While this constant has no special name, it is usually referred to as 'the electric force constant' or 'Coulomb's constant'.

WORKING SCIENTIFICALLY 12.2

Build an apparatus that would allow you to replicate Coulomb's experiment.

12.2 Exercise 1

1. When glass is rubbed with silk, the glass becomes positively charged. Explain, using a diagram, how this happens.
2. When referring to charged bodies, explain what is meant by:
   (a) excess of electrons
   (b) deficiency of electrons.
3. When each of these pairs of materials are rubbed together, identify which of the pair ends up with excess negative charge:
   (a) nylon and perspex
   (b) rabbit fur and steel
   (c) glass and wool.
4. A body has a positive charge of 2.00×10^{-6} coulombs. Calculate the number of electrons it has lost.
5. How far apart would two charges, each of 1.0 coulomb, need to be to each experience an electric force of 10 N?
6. How many electrons would need to be removed from a coin to give it a charge of $+10\,\mu$C?
7. The radius of a hydrogen atom is 5.3×10^{-11} m. What is the strength of the electric force between the nucleus and the electron?
8. If the force between two charges was 400 mN, how far apart would they need to be moved for the force to reduce by one-eighth?
9. Two small positively charged spheres have a combined charge of 5.0×10^{-5} C. If each sphere is repelled from the other by an electrostatic force of 1.0 N when the spheres are 2.0 m apart, what is the charge on each sphere?
10. Two point charges of $+6\,\mu$C and $-4\,\mu$C are placed on the x-axis at $x = 0$ and $x = 20$ cm respectively. What will be the magnitude and direction of the net electrostatic force acting on a $+8\,\mu$C point charge positioned at $x = 30$ cm?

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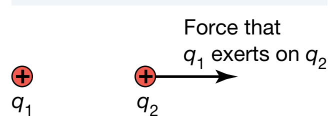
12.3 Electric fields

12.3.1 Field model of electric forces

Up to now we have considered a model where two electric charges exert forces directly on one another as shown in figure 12.17.

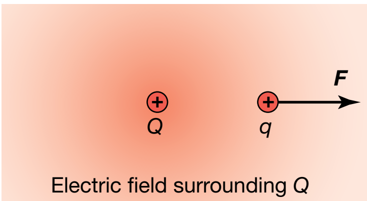
A different way of looking at the interaction between electric charges is in terms of **electric fields**. An electric field is a region where an electric charge experiences a force. On the field picture, every electric charge is surrounded by an electric field. If another charge, q, is placed in this electric field, the field exerts a force, F, on it. The field picture of electric force is illustrated in figure 12.18.

FIGURE 12.17 An electric charge exerting a force directly on another charge.



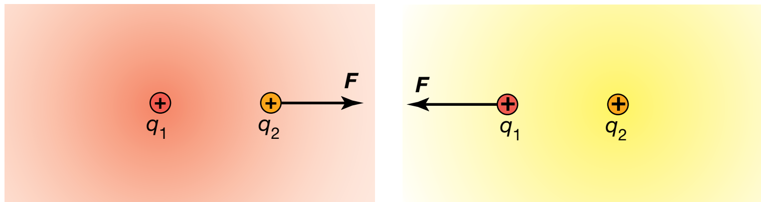
When two charges are brought close together, each charge is in the field of the other and experiences a force, as illustrated in figure 12.19. Note that the field surrounding a charge does not exert any force on the charge itself, only on other charges placed in the field.

FIGURE 12.18 Field picture of electric force.



Field surrounding Q exerts a force on q.

FIGURE 12.19 Field picture of forces between two charges.



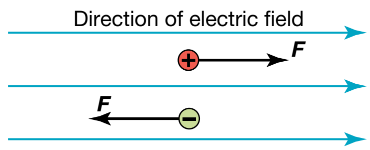
Field surrounding q_1 exerts a force on q_2.

Field surrounding q_2 exerts a force on q_1.

12.3.2 Electric field strength

An electric field exists at a point if an electric charge placed at the point experiences a force. *The direction of the electric field at a point is defined as the direction of the force that acts on a positive electric charge placed at the point*. A negative charge placed in an electric field experiences a force in the opposite direction to the field. This is illustrated in figure 12.20.

FIGURE 12.20 Direction of force on a charge place in an electric field.

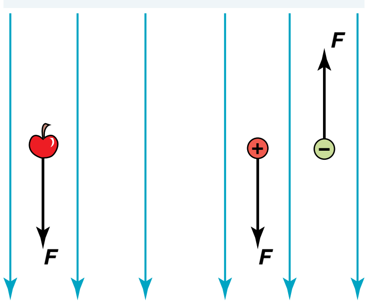


Compare an electric field with a gravitational field. An apple placed in the Earth's gravitational field experiences a force in the direction of the field, that is, downwards. The electrical situation is complicated by the fact that there are two different types of charge: positive and negative. A positive charge experiences a force in the direction of the field; a negative charge experiences a force in the opposite direction to the field. This is illustrated in figure 12.21.

The magnitude of the **electric field strength**, E, at a point is found by putting a charge, q, at the point and measuring the force, F, which the field exerts on it.

This is illustrated in figure 12.22. The magnitude of the electric field strength is defined by the formula $E = \frac{F}{q}$.

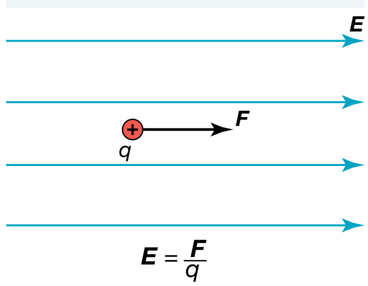
FIGURE 12.21 Comparison of electric and gravitational fields.



(a) Gravitational field

(b) Electric field

FIGURE 12.22 Finding the magnitude of the electric field strength.



12.3 SAMPLE PROBLEM 1

A charge of $+3.0\,\mu C$, placed at a point in an electric field, experiences a force of 2.0×10^{-4} N east. Calculate the electric field strength at the point.

SOLUTION:

The magnitude of the electric field strength is given by the formula:

$$\begin{aligned} E &= \frac{F}{q} \\ &= \frac{(2.0 \times 10^{-4})}{(3.0 \times 10^{-6})} \\ &= 6.7 \times 10^{1}\,\mathrm{N\,C^{-1}}. \end{aligned}$$

The positive charge experiences a force to the east, therefore the direction of the electric field is east.

12.3 SAMPLE PROBLEM 2

A charge of $-4.0\,\mu C$ is placed at a point where the electric field strength is $6.0 \times 10^{3}\,\mathrm{N\,C^{-1}}$ north. Calculate the force that will act on the charge.

SOLUTION

$$\begin{aligned} E &= \frac{F}{q} \\ 6.0 \times 10^{3} &= \frac{F}{(4.0 \times 10^{-6})} \\ F &= 2.4 \times 10^{-2}\,\mathrm{N} \end{aligned}$$

As the charge is negative, it will experience a force in the opposite direction to the field. Therefore, the direction of the force is south. Note that the sign of q is not used in this calculation.

This can be expressed as: *The magnitude of the electric field strength at a point is the magnitude of the force per unit charge at the point*.

The SI unit of electric field strength is **newton coulomb**$^{-1}$ ($\mathrm{N\,C^{-1}}$).

Force and electric field strength are vector quantities — they have magnitude and direction. We have defined the magnitude and the direction of the electric field strength separately, but it is possible to use a single definition that covers both.

If a force, $\boldsymbol{F}$, acts on a positive charge, $+q$, placed at a point in an electric field, then the electric field strength, $\boldsymbol{E}$, at the point is given by the equation: $\boldsymbol{E} = \frac{\boldsymbol{F}}{+q}$. That is, the magnitude and direction of the electric field strength at a point is equal to the magnitude and direction of the force per unit positive charge placed at the point.

Note the use of bold to indicate vector quantities.

By combining the equation for electric field strength with Coulomb's Law, an equation can be derived that can determine the electric field strength at a set distance from a point charge.

Let q be a test charge separated from a point charge Q by a distance r. According to Coulomb's Law, the force exerted by the point charge Q on the test charge q in free space is described by:

$$\boldsymbol{F} = \frac{1}{4\pi\varepsilon_o} \times \frac{Q\,q}{\boldsymbol{r}^2} \qquad (1)$$

The electric field strength experienced by the test charge q is related to the force exerted on it by Q:

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q} \qquad (2)$$

This can be rearranged to give F in terms of E and q:

$$\boldsymbol{F} = \boldsymbol{E}\,q \qquad (3)$$

As (1) and (3) describe the same force, they can be equated:

$$\frac{1}{4\pi\varepsilon_o} \times \frac{Q\,q}{\boldsymbol{r}^2} = \boldsymbol{E}\,q$$

This can be reduced to:

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_o} \times \frac{Q}{\boldsymbol{r}^2}$$

or,

$$\boldsymbol{E} = \frac{\mathrm{k}\,Q}{\boldsymbol{r}^2}$$

12.3 SAMPLE PROBLEM 3

What is the magnitude and direction of the electric field at a point 30 cm left of a point charge of $+2.0 \times 10^{-5}$ C?

SOLUTION

Using $E = \dfrac{\mathrm{k}Q}{r^2}$,

$$E = \frac{9.0 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}} \times 2.0 \times 10^{-5}\,\mathrm{C}}{(30 \times 10^{-2}\,\mathrm{m})^2}$$
$$= 2.0 \times 10^{6}\,\mathrm{N\,C^{-1}}.$$

Because the point charge is positive, the direction of the electric field is to the left.

12.3.3 Examples of electric fields

Electric fields are represented in a diagram by **lines of electric field**. These lines have no physical reality but are very useful for picturing the direction and the magnitude of the electric field.

The *direction* of the electric field lines indicates the *direction* of the electric field.

The *spacing* of the electric field lines indicates the *magnitude* of the electric field. The closer together the lines, the stronger the field.

Electric field surrounding a positive point charge

Consider a positive point charge, $+Q$. To determine the direction of the electric field surrounding $+Q$, a small positive test charge, $+q$, is placed near $+Q$. The direction of the force on $+q$ will be away from $+Q$, as shown in figure 12.23. The electric field surrounding $+Q$ will therefore point away from $+Q$.

Figure 12.24 shows the electric field surrounding a positive point charge, $+Q$. As you go further from the charge, the electric field lines are further apart indicating that the field is becoming weaker.

Figure 12.25 shows the forces that act on positive and negative charges placed in the field surrounding a positive point charge. A positive charge, $+q$, placed in the field will experience a force in the direction of the field, that is, away from the positive charge $+Q$. A negative charge, $-q$, placed in the field will experience a force in the opposite direction to the electric field, that is, towards the positive charge $+Q$.

FIGURE 12.23 Direction of the force on a positive test charge placed in the field surrounding a positive point charge.

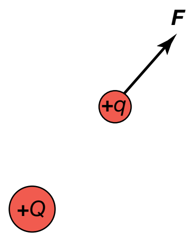


FIGURE 12.24 Electric field surrounding a positive point charge.

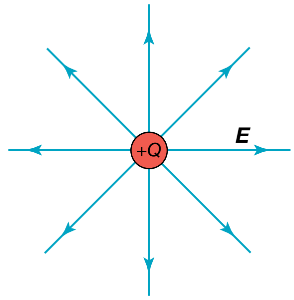
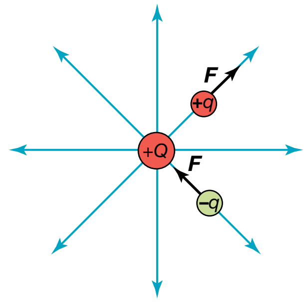


FIGURE 12.25 Forces on charges placed in the field surrounding a positive point charge.



In diagrams, electric fields are represented in two dimensions. It should be noted that electric fields are three dimensional.

Electric field surrounding a negative point charge

Consider a negative point charge, $-Q$. To determine the direction of the electric field surrounding $-Q$, a small positive test charge, $+q$, is placed in the field. The force on $+q$ will be towards $-Q$. Therefore, the electric field surrounding $-Q$ will be towards $-Q$. This is illustrated in figure 12.26.

Figure 12.27 shows the electric field surrounding a negative point charge, $-Q$.

Figure 12.28 shows the forces that act on charges placed in the field surrounding a negative point charge. A positive charge, $+q$, placed in the field will experience a force in the direction of the field, that is, towards the negative charge $-Q$. A negative charge, $-q$, placed in the field will experience a force in the opposite direction to the electric field, that is, away from the negative charge $-Q$.

FIGURE 12.26 Force on a positive test charge placed in the field surrounding a negative point charge.

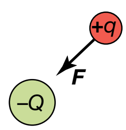


FIGURE 12.27 Electric field surrounding a negative point charge.

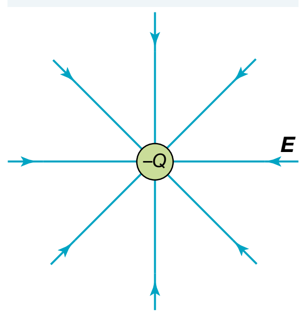
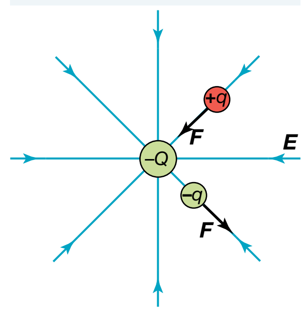


FIGURE 12.28 Forces on charges placed in a field surrounding a negative point charge.



Electric fields surrounding pairs of point charges

When there is more than one point charge producing an electric field, the fields from the individual charges combine to produce a single resultant field. Figures 12.29 to 12.33 show examples of electric fields produced by two point charges a small distance apart. In some of these examples, there are points where the fields from the two charges cancel one another. At these points, called *null points*, the electric field strength is zero. Null points are marked 'n' in the diagrams.

In drawing lines of electric field the following points should be noted:

- lines start on positive charges and end on negative charges

- lines never cross (the field cannot have two directions at a point)
- the greater the charge, the greater the number of lines starting or ending on it
- equal charges have equal numbers of lines starting or ending on them.

FIGURE 12.29 Electric field due to two equal positive point charges.

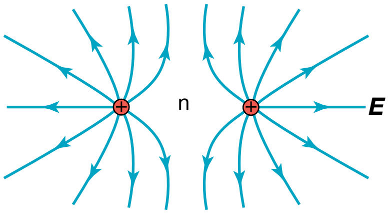


FIGURE 12.30 Electric field due to two equal negative point charges.

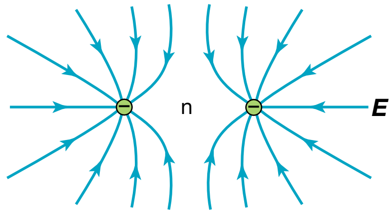


FIGURE 12.31 Electric field due to equal positive and negative point charges.

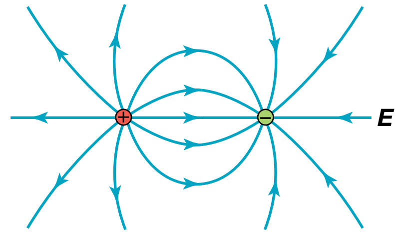


FIGURE 12.32 Electric field surrounding unequal positive and negative point charges.

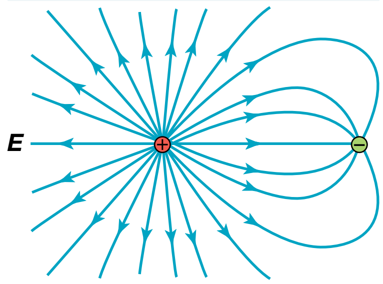
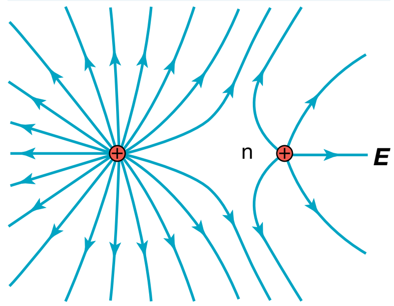


FIGURE 12.33 Electric field surrounding two unequal positive point charges.



Dipole fields

When a positive charge and a negative charge are separated by a short distance, the electric field around them is called a **dipole field**. This concept is more relevant to magnetic fields, where the ends of a bar magnet have different polarities (north and south). However, electric dipoles do occur in nature.

Electric dipoles mainly occur with the shared electrons in the bonds between atoms in molecules. For example in a molecule of water, H_2O, the oxygen atom more strongly attracts the shared electrons than do each of the hydrogen atoms. This makes the oxygen end of the molecule more negatively charged and the hydrogen end more positively charged. Because of this, the water molecule is called a polar molecule. It is this polarity that makes water so good at dissolving substances.

An antenna can be described as a varying electric dipole. To produce a radio or a TV signal, electrons are accelerated up and down the antenna. At one moment the top may be negative and the bottom positive, then a moment later the reverse is the case.

FIGURE 12.34 A water molecule (H_2O) displays polarity because the shared electrons are attracted more strongly to the oxygen atom than to the hydrogen atoms.

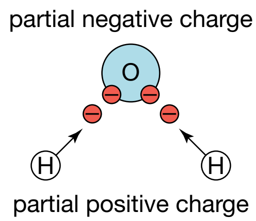
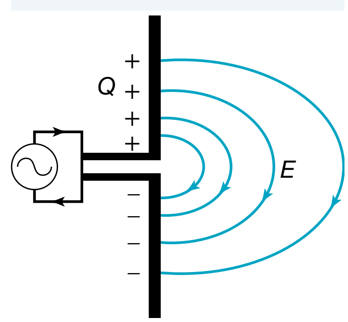


FIGURE 12.35 Partial circuit diagram of an antenna.



AS A MATTER OF FACT

The structure of DNA and electrical attraction

A DNA molecule is a long chain molecule built from four small molecules: adenine (A), cytosine (C), guanine (G) and thymine (T). These are arranged along the DNA molecule according to a code called the genetic code. Different sequences of A, C, G and T code for different amino acids, which are combined one after the other to produce different protein molecules. Two DNA molecules wrap around each other in a spiral to produce a double-helix chromosome.

The two DNA molecules in the helix are held together by electrical attraction between the polar ends of the four small molecules, A, C, G, and T. The chromosome is able to replicate itself because A and T can only pair up with each other, and likewise C and G can only pair up with each other. If there is an A on one strand, there must be a T immediately opposite on the other strand, and so on.

Figure 12.37 shows that one of the oxygen atoms in the thymine molecule is slightly negative, and one of the hydrogen atoms in the adenine molecule is slightly positive. Similarly, a hydrogen atom in the thymine molecule is slightly positive, and a nitrogen atom in the adenine molecule is slightly negative. These two slight electrical attractions are enough to hold these two molecules together, and the separations across these weak bonds are comparable in length.

Guanine and cytosine have a similar arrangement, except that there are three pairs of electrical attraction. Most importantly, the separations of the weak bonds between guanine and cytosine are comparable to each other and also to those of adenine and thymine. Without this matchup of separations, a chromosome could not hold together, nor could it form a double helix.

FIGURE 12.36 Electrical attraction in a DNA molecule.

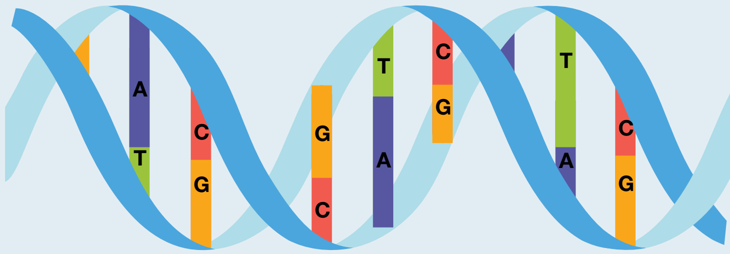


FIGURE 12.37 Electrical attraction between thymine and adenine molecules.

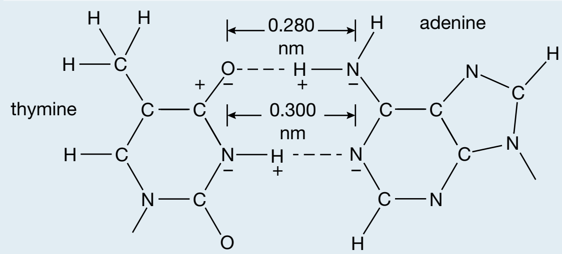
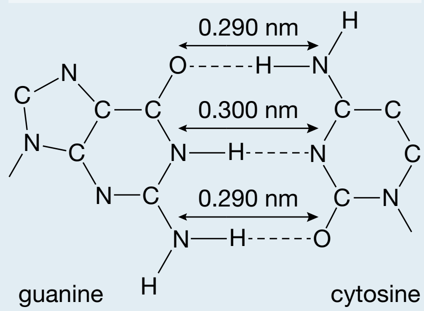


FIGURE 12.38 Electrical attraction between guanine and cytosine molecules.



12.3 Exercise 1

1. Draw the electrical fields around the following configurations:
   (a) two separated negative charges
   (b) two positive charges and two negative charges at the corners of a square with like charges diagonally opposite each other.
2. Sketch the electric field around two positive charges, A and B, where the charge on A is twice that on B.
3. Two charged objects, A and B, are held a short distance apart. Which object is the source of the electric field that acts on B?
4. What is the magnitude and direction of the electric field at a point 50 cm to the right of a point charge of -3.0×10^{-6} C?
5. An electric force of 3.0 N acts downwards on a charge of $-1.5\,\mu$C. What is the strength and direction of the electric field?
6. Determine the strength of the electric field 30 cm from a charge of 120 μC.
7. What is the strength of the electric field 1.0 mm from a proton?

8 At a point in an electric field, a positive charge of 3.00×10^{-6} C experiences a force of 6.50×10^{-4} N east. Calculate the electric field strength at the point in magnitude and direction.
9 A $+60\,\mu$C charge and a $+80\,\mu$C charge are placed 20 cm apart. At what point is the net electric field equal to zero?
10 A proton and an electron form two corners of an equilateral triangle that has a side length of 2.0×10^{-6} m. What is the magnitude and direction of their net electric field at the third corner?

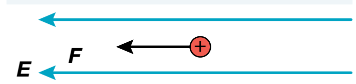
12.4 Electric Potential Energy

12.4.1 Electric potential of charges in fields

Recall that potential energy is stored energy. An electric charge placed in an electric field has **electric potential energy**, which is similar to the gravitational potential energy of a mass in a gravitational field. The SI unit of electric potential energy is the joule (J).

A positive charge, $+q$, in an electric field will experience a force, $\boldsymbol{F}$, in the direction of the field. If the charge is free to move, it will move in the direction of the field, increasing in speed and therefore gaining kinetic energy. The gain in kinetic energy has come from a loss in electric potential energy. This is illustrated in figure 12.39. The free movement of a charge in an electric field is similar to a mass falling in the Earth's gravitational field.

FIGURE 12.39 Positive charge moving freely in an electric field.



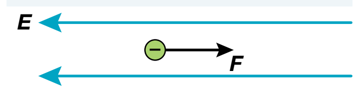
Charge accelerates in direction of field.

To move the positive charge in the opposite direction to the field, energy must be expended to increase the electric potential energy of the charge.

When a positive charge moves in the direction of an electric field, its electric potential energy decreases. When it moves in the opposite direction to an electric field, its electric potential energy increases. Moving a positive charge in the opposite direction to an electric field is similar to raising a mass in a gravitational field.

A negative charge, $-q$, in an electric field will experience a force, $\boldsymbol{F}$, in the opposite direction to the field. If the negative charge is free to move, it will move in the opposite direction to the field, increasing in speed and therefore gaining kinetic energy. The gain in kinetic energy has come from a loss in electric potential energy. This is illustrated in figure 12.40.

FIGURE 12.40 Negative charge moving freely in an electric field.



Charge accelerates in opposite direction to field.

To move a negative charge in the direction of the field, energy must be expended to increase the electric potential energy of the charge.

When a negative charge moves in the opposite direction to an electric field, its electric potential energy decreases. When a negative charge moves in the direction of an electric field, its electric potential energy increases.

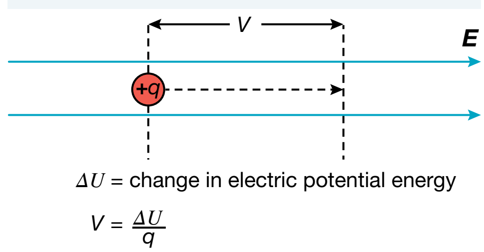
12.4.2 Potential difference

The **potential difference** between two points in an electric field is the change in electric potential energy per coulomb of charge that moves between the points.

Consider a charge, q, that moves between two points in an electric field. If the change in the electric potential energy of the charge is ΔU, then the potential difference, V, between the points is given by the formula: $V = \dfrac{\Delta U}{q}$.

Figure 12.41 shows a charge moving between two points in an electric field.

FIGURE 12.41 Potential difference between two points in an electric fields.



The SI unit of potential difference is the **volt** (V). A volt is equivalent to a joule coulomb^{-1}. Potential difference is also referred to as **voltage**.

Note that in this book the symbol 'V' is used for potential difference and the symbol 'V' for its unit, the volt. For example $V = 10\text{V}$. This distinction is not made in ordinary handwriting.

12.4 SAMPLE PROBLEM 1

When a charge of 2.50×10^{-4}C moves between two points in an electric field, the electric potential energy of the charge changes by 5.00×10^{-2}J.

(a) Calculate the potential difference between the two points.

(b) Calculate the change in potential energy if a charge of 7.60×10^{-2}C were moved between the two points.

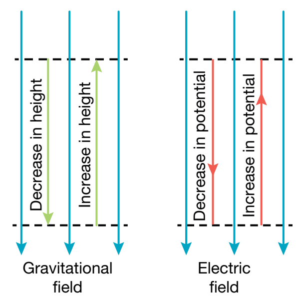
SOLUTION

(a)
$$\begin{aligned} V &= \frac{\Delta U}{q} \\ &= \frac{(5.00 \times 10^{-2})}{(2.50 \times 10^{-4})} \\ &= 2.00 \times 10^{2}\,\text{V} \end{aligned}$$

(b)
$$\begin{aligned} V &= \frac{\Delta U}{q} \\ 2.00 \times 10^{2} &= \frac{\Delta U}{(7.60 \times 10^{-2})} \\ \Delta U &= (2.00 \times 10^{2}) \times (7.60 \times 10^{-2}) \\ &= 1.52 \times 10^{1}\,\text{J} \end{aligned}$$

A more complete treatment of potential difference distinguishes between a potential rise and a potential drop. This is similar to a rise in height and a drop in height in a gravitational field. Figure 12.42 illustrates the comparison between an electric field and a gravitational field.

FIGURE 12.42 Comparison between electric potential difference and change in height in a gravitational field.



12.4.3 Changes in potential energy and kinetic energy in an electric field

A small positive charge is placed at point Q, some distance from a central positive charge. To move the charge to point P, you will need to push inwards against the repulsive electrical force. At point P the small charge will have electrical potential energy, like a compressed spring. The amount of potential energy it has will be equal to the area under the field–distance graph times its charge. If the small charge was released, all this potential energy would be converted into kinetic energy by the time the charge reached Q.

If instead a small negative charge was placed at Q, it would experience an attractive electrical force, and when the charge reached P, the shaded area would represent its gain in kinetic energy.

FIGURE 12.43 Diagrams and field–distance graphs for the electric field around (a) a positive charge and (b) a negative charge.

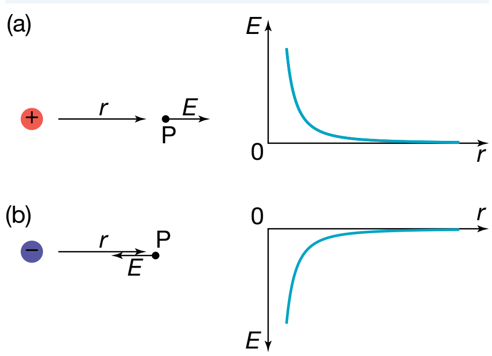
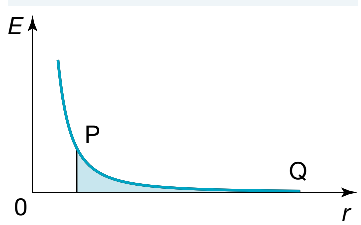


FIGURE 12.44 A field–distance graph for a positive charge at P near a central positive charge at Q.



12.4 Exercise 1

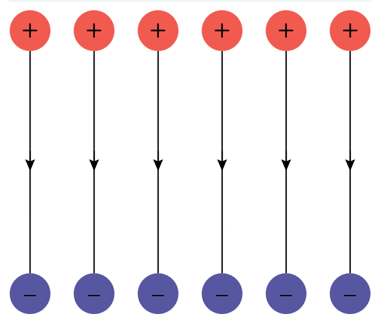
1 True or false? "Electrons tend to move towards regions of high electric potential."
2 In which of the following cases is positive work done:
  (a) moving a negative charge from a potential of 10 V to a potential of 5 V
  (b) moving a positive charge closer to another positive charge
  (c) moving a negative charge in the same direction as the electric field
  (d) moving a positive charge in the same direction as the electric field?
3 It takes 5 J of energy to move a charge of $+5 \times 10^{-4}$ C from point A to point B. What is the potential difference between A and B?
4 What is the potential difference between two points if it takes 10 J of work to move −0.04 C from one point to another?
5 What is the potential difference between two points that are 50 cm and 80 cm respectively from a point charge of $+2\,\mu$C?
6 A proton is placed 2 cm from a point charge of $+4\,\mu$C. When released it begins to move.
  (a) What is the magnitude and direction of the electric field strength experienced by the proton at its starting position?
  (b) What is the potential difference between the proton's starting position and its position when it has travelled 8 cm?
  (c) At what speed is the proton travelling when it is 10 cm away from the point charge?

12.5 Uniform electric fields

12.5.1 Electric field between parallel plates

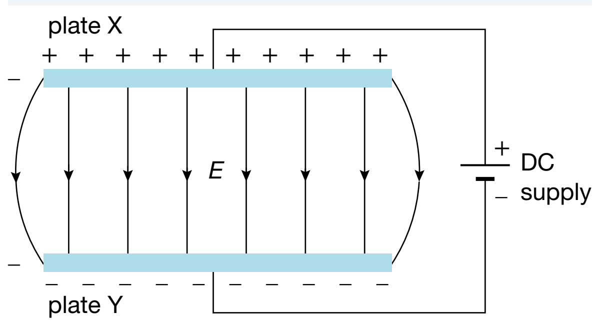
If a set of positive and negative charges were lined up in two rows facing each other, the lines of electric field in the space between the rows would be evenly spaced, that is, the value of the strength of the field would be constant. This is called a uniform electrical field.

FIGURE 12.45 A uniform electric field.



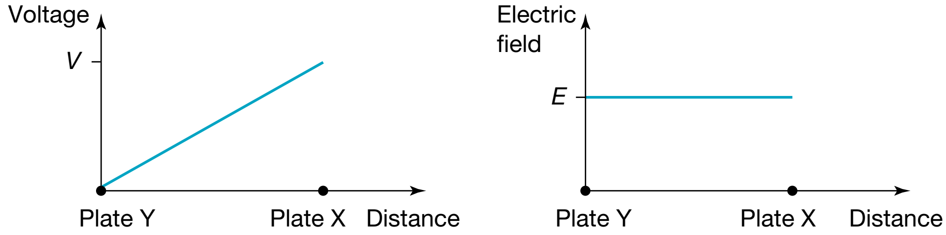
It is also very easy to set up. Just set two metal plates a few centimetres apart, then connect one plate to the positive terminal of a battery and connect the other plate to the negative terminal of the battery. The battery will transfer electrons from one plate, making it positive, and put them on the other, making that one negative. The battery will keep on doing this until the positive plate is so positive that the battery's voltage, or the energy it gives to each coulomb of electrons, is insufficient to overcome the attraction of the positively charged plate. Similarly, the negatively charged plate will become so negative that the repulsion from this plate prevents further electrons being added.

FIGURE 12.46 An electric field between two plates.



If a space contains a uniform field, that means that if a charge was placed in that space it would experience a constant electric force, $F = Eq$. The direction of the force on a positive charge will be in the direction of the field, and the force on a negative charge will be opposite to the field direction. Also, because the force is constant, the acceleration will be constant. As seen earlier, the situation with a charged particle in the space between the plates in the figure above is similar to the vertical motion under gravity. Indeed, if a charged particle is injected with speed into the field from one side, its subsequent motion is similar to projectile motion.

FIGURE 12.47 Electric field strength equals the gradient of the voltage–distance graph.



12.5.2 What is the strength of a uniform electric field?

In the situation of an electric field between two plates, it is not easy to apply Coulomb's Law, as there are many charges on each plate interacting with each other. An alternative approach is needed — one that uses the concept of energy.

The emf of a battery, or its voltage, is the amount of energy that the battery gives to each coulomb of charge. A battery of V volts would use up V joules of energy transferring one coulomb of electrons from the top plate through the wires to the bottom plate. Once on the negative plate, this coulomb of electrons would have V joules of electrical potential energy.

If this coulomb of electrons could be released from the negative plate, it would be accelerated by the constant force of the electric field between the plates, gaining kinetic energy like a stone falling in a gravitational field. And as in a gravitational field, the gain in kinetic energy equals the loss in electrical potential energy.

The gain in kinetic energy of one coulomb of charge = V joules.
The gain in kinetic energy for q coulombs of charge = qV joules.
This is the relationship $\Delta U = qV$.

$$\begin{array}{c} \text{Work done on} \\ q \text{ coulombs of charge } (\Delta U) \end{array} = \begin{array}{l} \text{quantity of} \\ \text{charge } (q) \end{array} \times \begin{array}{l} \text{voltage drop or} \\ \text{potential difference } (V) \end{array}$$

However, work done (ΔU) also has a definition of motion:

$$\begin{aligned} \text{Work done } (\Delta U) &= \text{force } (F) \times \text{displacement } (d) \\ \Delta U &= Fd \end{aligned}$$

But the force, if it is an electrical force, is given by $F = qE$, so $\Delta U = qE \times d$, where d in this instance is the separation of the plates.

Equating the two expressions for work done,

$$qE \times d = q \times V$$

Cancelling the charge, q, gives

$$E = \frac{V}{d}$$

This provides an alternative unit for electric field of volts per metre or Vm^{-1}. So, like gravitational field strength, electric field strength has two equivalent units: either newtons per coulomb or volts per metre. Using volts per metre makes it very easy to determine the strength of a uniform electric field.

12.5 SAMPLE PROBLEM 1

What is the strength of the electric field between two plates 5.0 cm apart connected to a 100 V DC supply?

SOLUTION

$V = 100\text{ V},\ d = 5.0\text{ cm} = 5.0 \times 10^{-2}\text{ m},\ E = ?$

$$
\begin{aligned}
E &= \frac{V}{d} \\
&= \frac{100\text{ V}}{5.0 \times 10^{-2}\text{ m}} \\
&= 2000\text{ V m}^{-1}
\end{aligned}
$$

12.5 Exercise 1

1 A 6 V battery is connected to two parallel plates as shown. X, Y and Z are positions between the plates.

(a) Which of these statements is true:

A. X and Y are at the same electric potential

B. X and Z are at the same electric potential

C. The electric field strength at Z is greater than at X

D. The electric field strength at X is greater than at Y?

(b) If the plates are 4 cm apart and position X is located 1 cm from the top plate, what is the electric field strength at X?

(c) What is the direction of the electric field at Y?

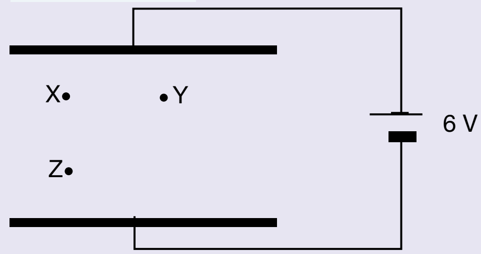
A. Left to right

B. Right to left

C. Top plate to bottom plate

D. Bottom plate to top plate.

FIGURE 12.48



2 Two parallel metal plates are separated by 1 cm and are connected to a 12 V battery. How much work must be done to move an electron from the positive plate to the negative plate?

3 (a) Calculate the strength of the electric field between a storm cloud 1.5 km above ground and the ground itself if the voltage drop or potential difference is 30 000 000 V. Assume a uniform field.

(b) How would the strength of the electric field change if the storm cloud was higher?

4 Two metal plates, X and Y, are set up 10 cm apart. The X plate is connected to the positive terminal of a 60 V battery and the Y plate is connected to the negative terminal. A small positively charged sphere is suspended midway between the plates and it experiences a force of 4.0×10^{-3} newtons.

(a) What would be the size of the force on the sphere if it was placed 7.5 cm from plate X?

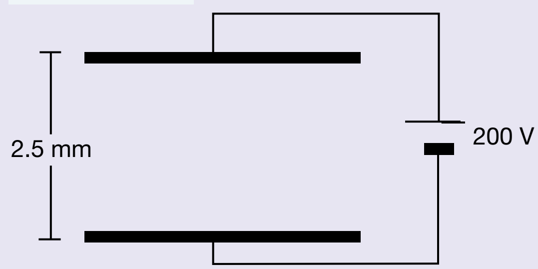
(b) The sphere is placed back in the middle and the plates are moved apart to a separation of 15 cm. What is the size of the force now?

(c) The plates are returned to a separation of 10 cm but the battery is changed. The force is now 6.0×10^{-3} newtons. What is the voltage of the new battery?

5 Two parallel plates are placed 2.5 mm apart vertically and connected to a 200 V power supply as shown in figure 12.49.

A charged oil drop is suspended halfway between the plates and remains stationary. If the oil drop holds a charge of $+5.0\,\mu$C, what is the mass of the oil drop?

FIGURE 12.49



12.6 Review

12.6.1 Summary

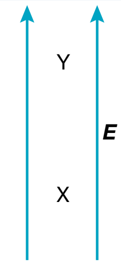
- Protons have a positive electric charge; electrons have a negative electric charge.
- Like charges repel; unlike charges attract.
- The SI unit of charge is the coulomb (C).
- Positively charged bodies have a deficiency of electrons; negatively charged bodies have an excess of electrons.
- Bodies can be given electrostatic charges by friction, contact and induction.
- An electric field is a region where an electric charge experiences a force.
- The electric force between two objects with charges q_1 and q_2 separated by a distance of r metres is given by $F = k\frac{q_1 q_2}{r^2}$ where $\mathrm{k} = \frac{1}{4\pi\varepsilon_0}$. This equation is called Coulomb's Law, and $\mathrm{k} = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ in air.
- Every electric charge is surrounded by an electric field E. The electric field strength at a point is defined as the force per unit charge on a positive charge placed at that point: $E = \frac{F}{q}$
- The direction of the electric field strength at a point is the direction of the force on a positive charge placed at the point.
- The direction of the electric field surrounding a positive charge is away from the charge; the direction of the electric field surrounding a negative charge is towards the charge.
- A charge in an electric field has electric potential energy. When a positive charge moves in the direction of an electric field, its electric potential energy decreases. When a negative charge moves in the opposite direction to an electric field its electric potential energy decreases.
- The potential difference between two points in an electric field is the change in electric potential energy per coulomb when a charge moves between the two points: $V = \frac{\Delta U}{q}$
- The SI unit of potential difference is the volt. One volt is equivalent to one joule per coulomb.
- A uniform electric field exists between two metal plates connected to a DC supply. The strength of the electric field, E, is given by the voltage drop or potential difference across the plates, V, over the plate separation, d: $E = \frac{V}{d}$.

12.6.2 Questions

1. If one coulomb is equal to the charge on 6.25×10^{18} electrons, calculate the charge in coulombs on one electron.
2. When a piece of perspex is rubbed with a piece of silk, 3.40×10^5 electrons are transferred from the perspex to the silk. Calculate the charge in coulombs on:
   (a) the perspex
   (b) the silk.
3. (a) Define the direction of an electric field.
   (b) Using this definition, explain why the field surrounding a positive charge points away from the charge.
4. At a point in an electric field, a negative charge of $2.50 \times 10^{-5}\,\mathrm{C}$ experiences a force of $7.50 \times 10^{-6}\,\mathrm{N}$ south. Calculate the electric field strength, in magnitude and direction, at the point.
5. At a certain point, the electric field strength is $4.30 \times 10^2\,\mathrm{N\,C^{-1}}$ east. Calculate the force, in magnitude and direction, on each of the following placed at the point:
   (a) an electron
   (b) a proton
   (c) a charge of $+2.30 \times 10^{-4}\,\mathrm{C}$
   (d) a charge of $-6.50 \times 10^{-4}\,\mathrm{C}$.

6. Figure 12.50 shows two points, X and Y, in an electric field.
   (a) Explain why a positive charge would have more electric potential energy at X than at Y.
   (b) Explain why a negative charge would have more electric potential energy at Y than at X.
7. A positive charge, $+q$, is brought nearer to a positive charge, $+Q$. As $+q$ gets closer to $+Q$, discuss whether its electric potential energy increases or decreases.
8. A positive charge, $+q$, is brought closer to a negative charge, $-Q$. As the charge $+q$ gets closer to the charge $-Q$, discuss whether its electric potential energy increases or decreases.

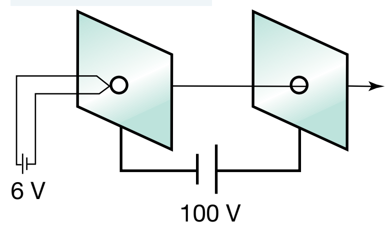
FIGURE 12.50



9. When a charge of 3.75×10^{-4} C moves between two points in an electric field, the electric potential energy of the charge changes by 7.50×10^{-2} J.
   (a) Calculate the potential difference between the two points.
   (b) Calculate the change in potential energy if a charge of 2.60×10^{-3} C moved between the two points.
10. What is the experimental evidence for there being two types of charge?
11. A and B are metal spheres x metres apart. Each has a charge of $+q$ coulombs. The force they exert on each other is 5.0×10^{-4} newtons. Determine the magnitude of the force in each of the following situations. (Consider the situations separately.)
    (a) The separation of A and B is increased to $2x$ metres.
    (b) A charge of $+2q$ coulombs is added to B. Are the forces on A by B and on B by A still equal in magnitude?
    (c) A charge of $-3q$ coulomb is added to A.
    (d) The distance is halved and the charges are changed to $+0.5q$ on A and $4q$ on B.
12. Find the force of repulsion between two point charges with charges of 5.0 microcoulombs (μC) and 7.0 microcoulombs (μC) if they are 20 cm apart.
13. Two charged spheres are 5.0 cm apart, with one holding twice the amount of charge of the other. If the force between is 1.5×10^{-4} newtons, how much charge does each sphere have?
14. Two small spheres are placed with their centres 20 cm apart. The charges on each are $+4.0 \times 10^{-8}$ C and $+9.0 \times 10^{-8}$ C. Where between the two spheres would a test charge experience zero net force?
15. Coulomb's Law is very similar to Newton's Law of Universal Gravitation. How do these two laws differ? Compare electric charge and gravitational mass.
16. The nucleus of an iron atom has 26 protons, and the innermost electron is 1.0×10^{-12} m away from the nucleus. What is the strength of the electric force between the nucleus and the electron?
17. The nucleus of a uranium atom has 92 protons, and the innermost electron is about 5.0×10^{-13} m away from the nucleus. What is the strength of the electric force between the nucleus and the electron?
18. A proton is made up of two 'up' quarks of charge $+\frac{2e}{3}$ and one 'down' quark of charge $-\frac{1e}{3}$. The diameter of a proton is about 8.8×10^{-16} m. Using the diameter as the maximum value for the separation of the two 'up' quarks, calculate the size of the electrical repulsion force between them.
19. What equal positive charge would the Earth and the Moon need to have for the electrical repulsion to balance the gravitational attraction? Why don't you need to know the separation of the two objects?
20. What is the charge in coulombs of 10 kg of electrons?
21. One example of alpha decay is uranium-238 decaying to thorium-234. The thorium nucleus has 90 protons and the alpha particle has two protons. At a moment just after the ejection of the alpha particle, their separation is about 9.0×10^{-15} m. What is the size of the electrical repulsion force between them, and what is the acceleration of the alpha particle at this point?
22. What is the size of the electric force between a positive sodium ion (Na^+) and a negative chloride ion (Cl^-) in a NaCl crystal if their spacing is 2.82×10^{-10} m?

23. An electric force of 1.5 N acts upwards on a charge of $+3.0\,\mu$C. What is the strength and direction of the electric field?
24. A proton is suspended so that it is stationary in an electric field. Using the value of $g = 10\,\mathrm{m\,s^{-2}}$, determine the strength of the electric field.
25. Use the statement 'the electric force exerted by a charged object A on a charged object B is proportional to the charge on B' and Newton's Third Law to show that the electric force between the two charges is proportional to the **product** of the charges.
26. Electric field lines can never cross. Why?
27. If a charged particle is free to move, will it move along an electric field line?
28. One of the units for gravitational field is that of acceleration. Is that also true for electric field? If not, why not?
29. Sketch the electric field around a positively charged straight plastic rod. Assume the charge is distributed evenly. Sketch the electric field as if the rod had a curve in it. If the plastic rod was bent into a closed circle, what would be the strength of the electric field in the middle?
30. A negative test charge is placed at a point in an electric field. It experiences a force in an easterly direction. What is the direction of the electric field at that point?
31. Two small spheres, A and B, are placed with their centres 10 cm apart. P is 2.5 cm from A. What is the direction of the electric field at P in the following situations?
    (a) A and B have the same positive charge.
    (b) A has a positive charge, B has a negative charge and the magnitudes are the same.
32. Determine the strength of the electric field 30 cm from a charge of $120\,\mu$C.
33. What is the strength of the electric field 1.0 mm from a proton?
34. Electrons from a hot filament are emitted into the space between two parallel plates and are accelerated across the space between them.
    (a) Which battery supplies the field to accelerate the electrons?
    (b) How much energy would be gained by an electron in crossing the space between the plates?
    (c) How would your answer to (b) change if the plate separation was halved?
    (d) How would your answer to (b) change if the terminals of the 6 V battery were reversed?
    (e) How would your answer to (b) change if the terminals of the 100 V battery were reversed?
    (f) How would the size of the electric field between the plates, and thus the electric force on the electron, change if the plate separation was halved?
    (g) Explain how your answers to (c) and (f) are connected.

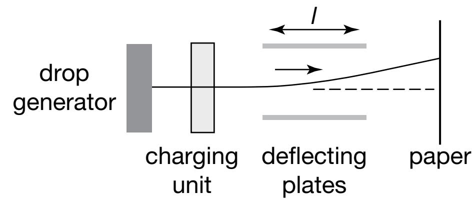
FIGURE 12.51



35. (a) Calculate the acceleration of an electron in a uniform electric field of strength $1.0 \times 10^6\,\mathrm{N\,C^{-1}}$.
    (b) Starting from rest, how long would it take for the speed of the electron to reach 10% of the speed of light? (Ignore relativistic effects.)
    (c) What distance would the electron travel in that time?
    (d) If the answer to (c) was the actual spacing of the plates producing the electric field, what was the voltage drop or potential difference across the plates?
36. In an inkjet printer, small drops of ink are given a controlled charge and fired between two charged plates. The electric field deflects each drop and thus controls where the drop lands on the page.

    Let m = the mass of the drop, q = the charge of the drop, v = the speed of the drop, l = the horizontal length of the plate crossed by the drop, and E = electric field strength.

FIGURE 12.52



(a) Develop an expression for the deflection of the drop. *Hint:* This is like a projectile motion question.
(b) With the values $m = 1.0 \times 10^{-10}$ kg, $v = 20\,\text{m s}^{-1}$, $l = 1.0$ cm and $E = 1.2 \times 10^{6}\,\text{N C}^{-1}$, calculate the charge required on the drop to produce a deflection of 1.2 mm.

PRACTICAL INVESTIGATIONS

Investigation 12.1: The Van de Graaff generator

Aim
To investigate electrostatic charge

CAUTION
Your teacher will carry out this activity. Do not touch the charged dome of a Van de Graaff generator unless instructed to by your teacher. Always use an earthed rod to discharge. Carry out the demonstration while standing on a plastic tray.

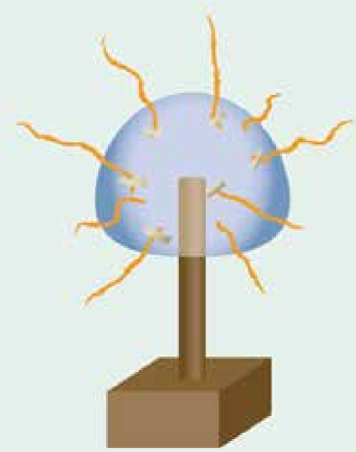
Apparatus:
Van de Graaff generator
several strands of wool
plastic tray

Method
Part A
- Turn the Van de Graaff generator on and let it charge up. Bring the **earthed** metal rod near it.
- Turn the generator off and discharge it using the earthed metal rod.

Part B
- Tape several strands of wool onto the dome. Make sure they are spread out over the surface of the dome. Turn the generator on and let it charge up once more.

Discussion
1. What do you observe occurring between the rod and the dome when it is turned on?
2. Explain your observation. Use words like charging and discharging in your explanation.
3. What happens to the wool?
4. Explain why this happens in terms of the charges on the dome and on the wool.
5. The wool forms a pattern around the dome. Explain why this pattern forms.

FIGURE 12.53



TOPIC 13
Electric circuits

13.1 Overview

13.1.1 Module 4: Electricity and Magnetism

Electric circuits

Inquiry question: How do the processes of the transfer and the transformation of energy occur in electric circuits?

Students:

- investigate the flow of electric current in metals and apply models to represent current, including:
  - $I = \frac{q}{t}$ (ACSPH038)
- investigate quantitatively the current–voltage relationships in ohmic and non-ohmic resistors to explore the usefulness and limitations of Ohm's Law using:
  - $V = \frac{W}{q}$
  - $R = \frac{V}{I}$ (ACSPH003, ACSPH041, ACSPH043)
- investigate quantitatively and analyse the rate of conversion of electrical energy in components of electric circuits, including the production of heat and light, by applying $P = VI$ and $W = Pt$ and variations that involve Ohm's Law (ACSPH042)
- investigate qualitatively and quantitatively series and parallel circuits to relate the flow of current through the individual components, the potential differences across those components and the rate of energy conversion by the components to the laws of conservation of charge and energy, by deriving the following relationships: (ACSPH038, ACSPH039, ACSPH044)
  - $\Sigma I = 0$ (Kirchoff's current law — conservation of charge)
  - $\Sigma V = 0$ (Kirchoff's voltage law — conservation of energy)
  - $R_{series} = R_1 + R_2 + \cdots + R_n$
  - $\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$
- investigate quantitatively the application of the law of conservation of energy to the heating effects of electric currents, including the application of $P = VI$ and variations of this involving Ohm's Law (ACSPH043).

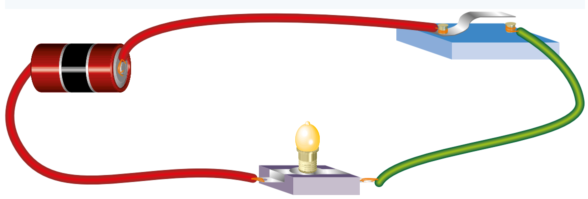
FIGURE 13.1 Electricity is an integral part of modern life. How do the processes of the transfer and the transformation of energy occur in electric circuits?

13.2 Electric currents

13.2.1 A simple electric circuit

Before studying electric currents in more detail, we will look at a simple example to recall earlier work you have done on this subject. Figure 13.2 shows a familiar situation involving an electric current.

FIGURE 13.2 A simple electric circuit.



Note the following:

- For the globe to light up there must be a complete conducting path between the terminals of the battery. The switch must be closed.
- The battery is necessary for a current to flow around the circuit. The ability of the battery to cause a current to flow is often referred to as its voltage.
- The battery has two terminals, marked positive and negative.
- The light globe resists the flow of the current. As a result of this resistance, the current causes the light globe to heat up to such an extent that it gives off light.
- The wires connecting the light globe to the battery do not heat up.

As an aid in representing electric circuits in diagrams, the symbols shown in figure 13.3 are used. Therefore, the circuit shown in figure 13.2 can be represented more simply as shown in figure 13.4.

FIGURE 13.3 Symbols for circuit components.

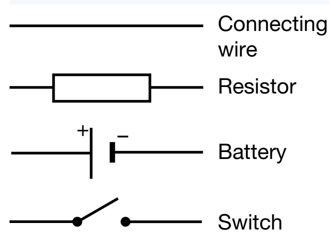
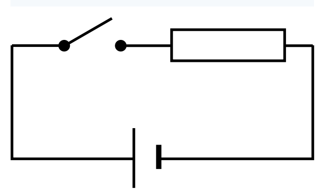


FIGURE 13.4 Circuit diagram for circuit shown in figure 13.2.

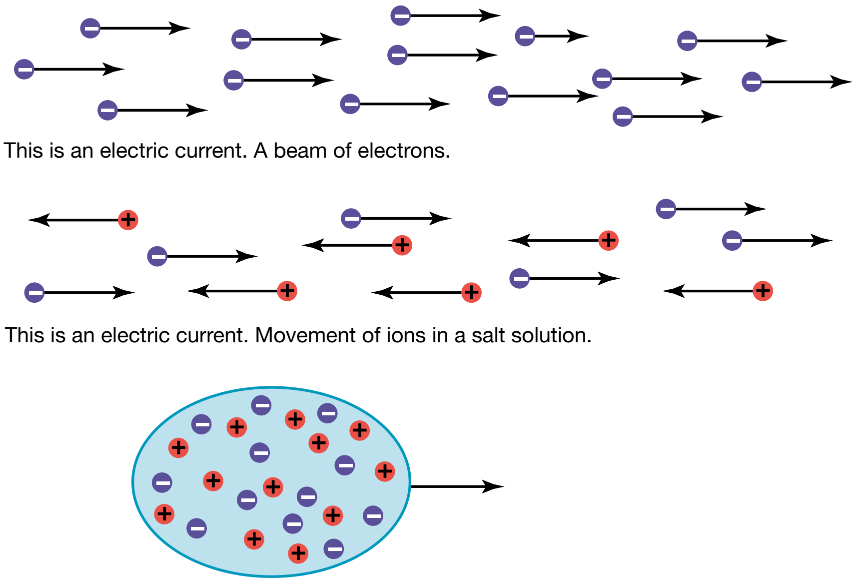


13.2.2 Defining current

Electric current is the movement of charged particles from one place to another. The charged particles may be electrons in a metal conductor or ions in a salt solution. Charged particles that move in a conductor can also be referred to as '**charge carriers**'.

There are many examples of electric currents. Lightning strikes are large currents. Nerve impulses that control muscle movement are examples of small currents. Charge flows in household and automotive electrical devices such as light globes and heaters. Both positive and negative charges flow in cells, in batteries and in the ionised gases of fluorescent lights. The solar wind is an enormous flow of protons, electrons and ions being blasted away from the Sun.

FIGURE 13.5 Electric currents.



(a) This is an electric current. A beam of electrons.

(b) This is an electric current. Movement of ions in a salt solution.

(c) This is not an electric current. Movement of a neutral body.

Not all moving charges constitute a current. There must be a net movement of charge in one direction for a current to exist. In a piece of metal conductor, electrons are constantly moving in random directions, but there is no net movement in one direction and no current. A stream of water represents a movement of millions of coulombs of charge as the protons and electrons of the water molecules move. There is no electrical current in this case, because equal numbers of positive and negative charge are moving in the same direction.

For there to be a current in a circuit there must be a complete conducting pathway around the circuit and a device to make the charged particles move. When the switch in the circuit is open, the pathway is broken and the current stops almost immediately.

Electric current is a measure of the rate of flow of charge around a circuit. It can be expressed as:

$$I = \frac{Q}{t}$$

where I is the current and Q is the quantity of charge flowing past a point in the circuit in a time interval t.

FIGURE 13.6 André-Marie Ampère

The unit of current is the **ampere** (A). It is named in honour of the French physicist André-Marie Ampère (1775–1836).

The unit for charge is the **coulomb** (C), named after the French physicist Charles-Augustin de Coulomb (1736–1806).

One coulomb of charge is equal to the amount of charge carried by 6.24×10^{18} electrons. The charge carried by a single electron is equal to -1.602×10^{-19} C.

One ampere is the current in a conductor when 1 coulomb of charge passes a point in the conductor every second.

The charge possessed by an electron is the smallest free charge possible. All other charges are whole-number multiples of this value. This so-called elementary charge is equal in magnitude to the charge of a proton. The charge of an electron is negative, whereas the charge of a proton is positive.

AS A MATTER OF FACT

Charges smaller than that carried by the electron are understood to exist, but they are not free to move as a current. Particles such as neutrons and protons are composed of quarks, with one-third of the charge of an electron, but these are never found alone.

13.2 SAMPLE PROBLEM 1

What is the current in a conductor if 10 coulombs of charge pass a point in 5.0 seconds?

SOLUTION:

$$\begin{aligned} Q &= 10\,\text{C} \\ t &= 5.0\,\text{s} \\ I &= \frac{Q}{t} \\ &= \frac{10\,\text{C}}{5.0\,\text{s}} \\ &= 2.0\,\text{C s}^{-1} \\ &= 2.0\,\text{A} \end{aligned}$$

13.2 SAMPLE PROBLEM 2

How much charge passes through a load if a current of 3.0 A flows for 5 minutes and 20 seconds?

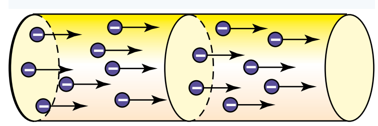
SOLUTION:

$$\begin{aligned} I &= \frac{Q}{t} \\ \text{or } Q &= It \\ I &= 3.0\,\text{A} \\ t &= 5 \text{ minutes and 20 seconds} = 320\,\text{s} \\ Q &= 3.0\,\text{A} \times 320\,\text{s} \\ &= 960\,\text{C} \\ &= 9.6 \times 10^2\,\text{C} \end{aligned}$$

In real circuits, currents of the order of 10^{-3} A are common. To describe these currents, the milliampere (mA) is used. One milliampere is equal to 1×10^{-3} ampere.

To convert from amperes to milliamperes, multiply by 1000 or by 10^3. To convert from milliamperes to amperes, divide by 1000 or multiply by 10^{-3}.

FIGURE 13.7 Meaning of ampere in terms of electrons.



When the current is 1 ampere, 6.25×10^{18} electrons pass through a cross-section of the conductor in one second.

13.2 SAMPLE PROBLEM 3

Convert 450 mA to amperes.

SOLUTION:

$$\frac{450\,\text{mA}}{1000} = 0.450\,\text{A}$$

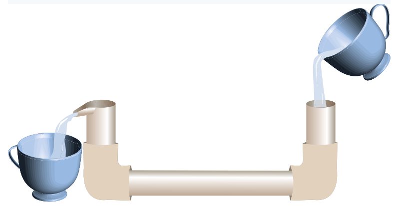
So 450 mA is equal to 0.450 A.

13.2.3 The hydraulic model of current

Most circuits have metal conductors, which means that the charge carriers will be electrons.

Metal conductors can be considered to be a three-dimensional arrangement of atoms that have one or more of their electrons loosely bound. These electrons are so loosely bound that they tend to drift easily among the atom. Metals are good conductors of both heat and electricity because of the ease with which these electrons are able to move, transferring energy as they go. Diagrammatically, the atoms are represented as positive ions (atoms that have lost an electron and have a net positive charge) in a 'sea' of free electrons.

FIGURE 13.8 The hydraulic model for current flow. One cupful of water in one end of the pipe means one cupful out the other end.



When the ends of a conductor are connected to a battery, the free electrons drift towards the positive terminal. The electrons are attracted by the positive terminal and indeed accelerate, but constantly bump into atoms, so on average they just drift along.

The flow of electrons through a metallic conductor can be modelled by the flow of water through a pipe.

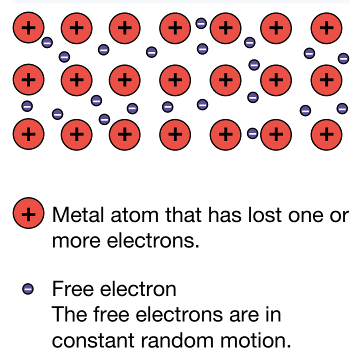
Electrons cannot be destroyed, nor, in a closed circuit, can they build up at a point. Therefore, if electrons are forced into one end of a conductor, an equal number will be forced out the other end. This is rather like pouring a cupful of water into one end of a full pipe. It forces a cupful of water to come out the other end, as shown in figure 13.8.

Note that when water is put in one end it is not the same water that comes out the other end, because the pipe was already full of water.

13.2.4 Charge movement through a metal

In a metal, some electrons become detached from their atoms and are able to move freely within the metal. These are called **free electrons**. The atoms that have lost electrons become positively charged ions. The positively charged atoms form a lattice through which the free electrons move freely. The free electrons are the charge carriers that allow the metal to conduct an electric current.

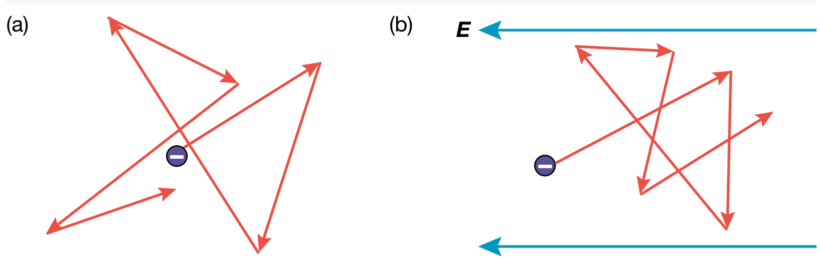
FIGURE 13.9 Structure of a metal.



The free electrons in a metal are in constant random motion. Each free electron makes frequent collisions with positive ions of the lattice. At each collision, the electron changes direction. Although the average speed of free electrons between collisions is of the order of $10^6\,\mathrm{m\,s^{-1}}$, there is no net movement of the electrons, so there is no electric current. The random movement of a free electron through a metal lattice is shown in figure 13.10a.

If there is an electric field in a metal, the free electrons, being negatively charged, will experience a force in the opposite direction to the field. As a result of this force, there will be a net movement in the direction of the force superimposed on the random movement of the free electrons. This net movement is called **electron drift** and constitutes an electric current. Electron drift is illustrated in figure 13.10b. The drift velocity of an electron is of the order of $10^{-4}\,\mathrm{m\,s^{-1}}$, much smaller than the average speed of its random motion.

FIGURE 13.10 Motion of free electron (a) with no electric field (b) with an electric field.



13.2 SAMPLE PROBLEM 4

A current of 5.00 A passes through a wire for 6.00 s. Calculate the number of electrons passing through a cross-section of the wire in this time.

SOLUTION:

As the current is 5.00 A, a charge of 5.00 C will pass through a cross-section of the wire in 1 s. Therefore, in 6.00 s a charge of 6.00 × 5.00 C will pass through. 1 C of charge is equivalent to 6.25×10^{18} electrons. If n is the number of electrons, then:

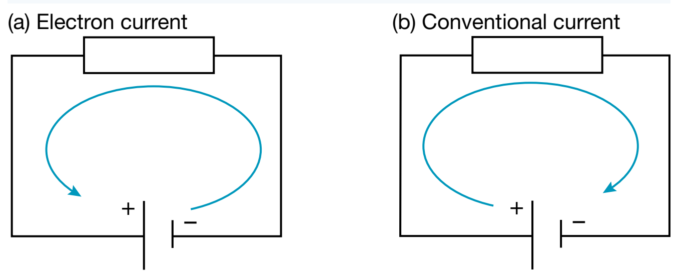
$$\begin{aligned} n &= 6.00 \times 5.00 \times 6.25 \times 10^{18} \\ &= 1.88 \times 10^{20}. \end{aligned}$$

As the free electrons drift in the opposite direction to the field, they lose electric potential energy and gain kinetic energy. The free electrons continually collide with the positive ions in the metal lattice. In these collisions, the kinetic energy gained by the electrons is transferred to the positive ions of the lattice, causing them to vibrate with greater energy. The energy of vibration of the atoms of a body is heat energy. Thus, when an electric current flows through a metal, electric potential energy is transformed into heat energy.

13.2.5 Describing current direction

By the time the battery was invented by Alessandro Volta in 1800, it was accepted that electric current was the movement of positive charge. It was assumed that positive charges left the positive terminal of the battery and travelled through a conductor to the negative terminal. This is called **conventional current**.

FIGURE 13.11 (a) Electron current direction. (b) Conventional current direction.



In reality, the charge carriers in a metal conductor are electrons moving from the negative terminal towards the positive terminal of the battery. The effect is essentially the same as positive charges moving in the opposite direction.

When dealing with the mechanisms for the movement of electrons, the term '**electron current**' is used.

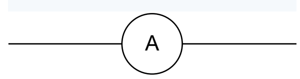
Direct current (DC) refers to circuits where the net flow of charge is in one direction only. The current provided by a battery is direct current, which usually flows at a steady rate.

Alternating current (AC) refers to circuits where the charge carriers move backwards and forwards periodically. The electricity obtained from household power points is alternating current.

13.2.6 Measuring electric current

Electric current is measured with a device called an **ammeter**. This must be placed directly in the circuit so that all the charges being measured pass through it. This is known as placing the ammeter in series with the circuit.

FIGURE 13.12 The circuit diagram symbol for an ammeter.



Ammeters are designed so that they do not significantly affect the size of the current by their presence. Their resistance to the flow of current must be negligible.

The circuit diagram symbol for an ammeter is shown in figure 13.12.

Most school laboratories now use digital multimeters. These can measure voltage drop and resistance as well as current. Each quantity has a few settings to allow measurement of a large range of values. Labels on multimeters may vary, but those given below are most common.

The black or common socket, labelled 'COM', is connected to the part of the circuit that is closer to the black or negative terminal of the power supply. The red socket, labelled 'VΩmA', is used for measuring small currents and is connected to the part of the circuit that is closer to the red or positive terminal of the power supply. The red socket, labelled ‘10A MAX’ or similar, is used for measuring large currents (see warning below). The dial has a few settings. First choose the setting for current, labelled ‘A’, with the largest value. If you want more accuracy in your measurement, turn the dial to a smaller setting. If the display shows just the digit ‘1’, the current you are trying to measure exceeds the range of that setting and you need to go back to a higher setting.

WARNING: While for most quantities, multimeters are quite tolerant of values beyond a chosen setting, care must be taken when measuring current. Multimeters have a fuse that can blow if the current exceeds the rated value. For this reason, they have two red sockets. One socket is for exclusive use when measuring currents in the range 200 mA to 10A. This is labelled ‘10A MAX’. (Some multimeters may be able to measure up to 20 A.) The other red socket is for currents less than 200 mA as well as the other quantities of voltage and resistance.

If you are using a needle type ammeter, the instructions above generally apply.

FIGURE 13.13 A digital multimeter, which can measure current, voltage drop and resistance.

FIGURE 13.14 A needle-deflection ammeter.

13.2 Exercise 1

1. State the difference between conventional current and electron current.
2. (a) Identify the charge carriers in a metal.
   (b) Describe how charge carriers move in a conductor under the influence of a power supply.
3. True or False? ‘It is possible to have a charge of 6.0×10^{-19} C.’
4. Convert 45 mA to amperes.
5. What is the current passing through a conductor if 15 coulombs of charge pass a point in 3.0 seconds?
6. For how long must a current of 2.5 amperes flow to make 7.5 coulombs of charge pass a point in a circuit?
7. How long will it take an electron to travel from a car’s battery to a rear light globe if it has a drift velocity of 1.0×10^{-4} m s^{-1} and there is 2.5 m of metal to pass through? (Electrons travel from the negative terminal of the battery through the car body towards the circuit elements.)
8. What is the current flowing through a section of wire if 2.0 C of charge passes through it in 5 seconds?
9. How many electrons move through a conductor every second if they produce a 3 A current?
10. How many electrons will move through the cross-section of a wire in an 8-second period of time if the wire has a current of 1 A?

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13.3 Supplying energy

13.3.1 Voltage

A power supply is a source of electric potential energy that enables electrons to move around a circuit. It separates positive and negative charges to produce a positively charged terminal and a negatively charged terminal. If a conductor joins the positive and negative terminals of a power supply, an electric field is established through the conductor from the positive terminal to the negative terminal of the power supply. If the conductor is a metal, this field will exert forces on the free electrons in the opposite direction to the field, causing them to move towards the positive terminal.

Most power supplies convert another form of energy into electric potential energy. For example, a battery uses a chemical reaction to separate electrons, leaving one terminal short of electrons and, therefore, with an excess of positive charge. The other battery terminal has an excess of electrons and so is the negative terminal.

In the school laboratory, the power supply is usually a power pack that converts electric potential energy from the mains supply into a form of electric potential energy that is more suitable for school use.

The **potential difference** V across a power supply is a measure of the number of joules of electric potential energy given to each coulomb of charge that passes through it. This potential difference is often referred to as the **voltage** across the power supply.

For example, a 5-volt battery gives 5 joules of electric potential energy to each coulomb of charge that passes through the battery. If electric potential energy, W, is given to a charge, q, that passes through the power supply, then the potential difference, V, across the power supply is given by the formula:

$$V = \frac{W}{q}$$

FIGURE 13.15 A power pack is used in school laboratories to supply electrical potential energy to a circuit.

13.3 SAMPLE PROBLEM 1

Calculate the amount of electric potential energy given to 2.00 coulomb of charge that passes through a 1.50×10^2 V power supply.

SOLUTION:

$$\begin{aligned} V &= \frac{W}{q} \\ 1.50 \times 10^2 &= \frac{W}{2.00} \\ W &= 3.00 \times 10^2 \text{ J} \end{aligned}$$

13.3.2 The conventional point of view

Looking from the perspective of conventional current, that is, positive charge carriers, the current would go in the opposite direction. In the circuit that follows, positive charges at A, the positive terminal, would leave with energy and arrive at F with no energy. The graph below shows the energy held by one coulomb of charge, that is, the voltage, as the charge moves around the circuit from A to F.

At the positive terminal, A, the coulomb of charge has 9 joules of energy; its voltage is 9 V. The wire, AB, from the battery to the globe is a good conductor, so no energy is lost and the voltage is still at 9 V. In the globe, as the current goes from B to C, the coulomb of charge loses 3 joules of energy and now has a voltage

FIGURE 13.16 The circuit symbol for a battery showing direction of conventional current.

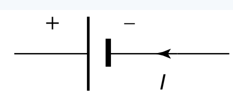
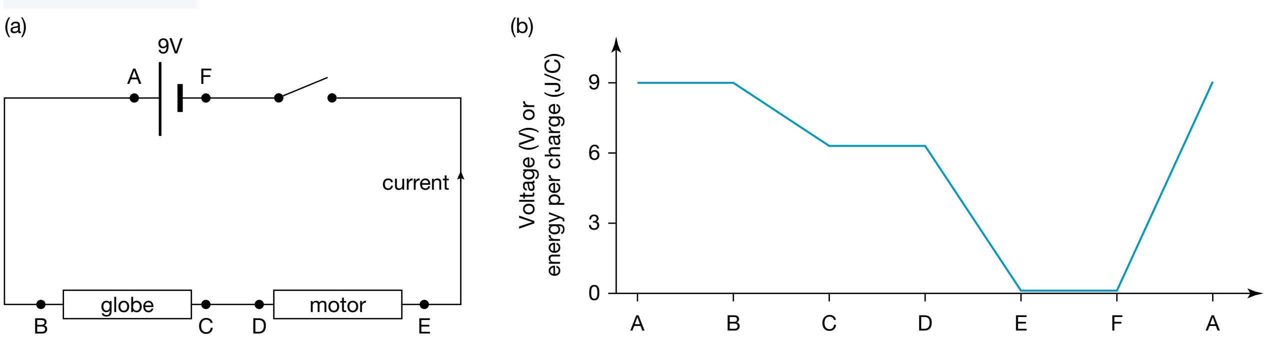


FIGURE 13.17



of 6 V at C. The conducting wire from C to D has no effect, so the coulomb of charge arrives at the motor, DE, with 6 joules of energy. This energy is used up in the motor so that at E the voltage is 0 V. The charge then moves on to F, the negative terminal, where the battery re-energises the charge to go around again.

Voltage is also called the electric potential. Using the hydraulic model, at A the charge is like water in a high dam that has gravitational potential energy that can be released when the dam opens. The charge at A has an electric potential of 9 V or 9 joules for every coulomb, which can be released when a switch is closed.

13.3.3 Measuring potential difference or voltage drop

The potential difference or voltage drop between any two points in a circuit can be measured with a **voltmeter**. The voltmeter must be connected across a part of the circuit. If the voltmeter was connected to points A and B in the circuit shown in figure 13.17a, it would display zero, as there is no difference in the potential or voltage between those two points. If instead it was connected across the globe, at BC, it would show a voltage drop of 3 V ($9 - 6 = 3$ V). This means that in the globe, 3 joules of electrical energy are lost by each coulomb of charge and transformed by the globe into light and heat.

FIGURE 13.18 The circuit diagram symbol for a voltmeter.

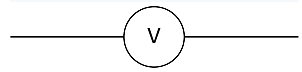
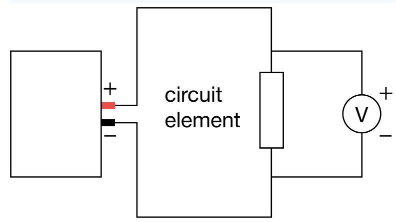


FIGURE 13.19 Connecting a voltmeter into a circuit.



Voltmeters are designed so that they do not significantly affect the size of the current passing through the circuit element. For this reason, the resistance of the voltmeter must be much higher than the resistance of the circuit elements involved. Resistance will be discussed later in this chapter. The circuit diagram symbol for a voltmeter is shown in figure 13.18.

As discussed in section 13.2.6, most school laboratories now use digital multimeters, which can generally measure both AC and DC voltages. To measure DC voltages, use one of the settings near the 'V' with a bar beside it.

The black or common socket, labelled 'COM', is connected to the part of the circuit that is closer to the black or negative terminal of the power supply. The red socket, labelled 'VΩmA', is used for measuring voltages and is connected to the part of the circuit closer to the red or positive terminal of the power supply. The other red socket, labelled '10A MAX' is only for large currents. The dial has a range of settings; when first connecting the multimeter, choose the setting with the largest value. If you want more accuracy in your measurement, turn the dial to a smaller setting. If the display shows just the digit '1', the voltage you are trying to measure exceeds the range of that setting and you need to go back to a higher setting.

13.3.4 Energy transformed by a circuit

Charges experience a potential difference as they go from the negative terminal of a power supply to its positive terminal; this potential difference is equal in magnitude to the potential difference across the power supply. A charge travelling from the negative to positive terminal of a 12-volt power supply goes from having an electrical potential of 0 volts up to an electrical potential of 12 volts. With this increase in electrical potential comes an increase in each charge's electrical potential energy. This increase in energy is the result of work being done by the power supply on the charge. As a result, we have until now used W (the symbol for work) to represent the increase in electrical potential energy that a charge q experiences when it moves through a potential difference V:

that is, $W = Vq$.

When the charges travel from the positive terminal of a power supply through an electric circuit and then back to the negative terminal, the charges experience a drop in electrical potential. This is because the electrical potential energy carried by the charges decreases at every point in the circuit where some of their electrical energy is converted into some other form of energy. For example, when electric charges travel through a light globe, some of the electric potential energy of the charges is converted into heat energy, causing the bulb filament to glow. With the decrease in each charge's electrical energy from one side of the globe to the other, there is a corresponding drop in each charge's electric potential. A circuit component over which such a potential drop occurs is called a **load**.

Since the potential difference is a measure of the loss in electrical potential energy by each coulomb of charge, the amount of energy (W) transformed by a charge (Q) passing through a load can be expressed as:

$$W = Vq$$

since $V = \dfrac{W}{q}$

where V is the potential difference across the load.

The amount of charge passing through a load in a time interval t can be expressed as:

$$q = It$$

Thus, $W = VIt$

where I is the current.

13.3 SAMPLE PROBLEM 2

What is the potential difference across a heater element if 3.6×10^4 J of heat energy is produced when a current of 5.0 A flows for 30 s?

SOLUTION:

As $W = VIt$,

$$V = \frac{W}{It}$$

$$= \frac{3.6 \times 10^4 \text{ J}}{5.0\,A \times 30 \text{ s}}$$

$$= 240\,V$$

13.3.5 Power delivered by a circuit

In practice, it is the rate at which energy is transformed in an electrical load that determines its effect. The brightness of an incandescent light globe is determined by the rate at which electrical potential energy is transformed into the internal energy of the filament.

Power is the rate of doing work, or the rate at which energy is transformed from one form to another. Power is equal to the amount of energy transformed per second, or the amount of energy transformed divided by the time it took to do it. Power can therefore be expressed as:

$$P = \frac{W}{t}$$

where P is the power delivered when an amount of energy is transformed (i.e. an amount of work W is done) in a time interval t.

The SI unit of power is the watt (W).

1 watt = 1 joule per second = $1\,\text{J}\,\text{s}^{-1}$

Since $W = VIt$ and $P = \dfrac{W}{t}$

then $P = \dfrac{VIt}{t}$

or $P = VI$

And since $V = \dfrac{W}{q}$ and $q = It$,

therefore, $V = \dfrac{W}{It}$

or $W = VIt$

This is a particularly useful formula because the potential difference V and electric current I can be easily measured in a circuit.

13.3 SAMPLE PROBLEM 3

What is the power rating of an electric heater if a current of 5.0 A flows through it when there is a voltage drop of 240 V across the heating element?

SOLUTION:

$$\begin{aligned} I &= 5.0\ \text{A} \\ V &= 240\ \text{V} \\ P &= VI \\ &= 240\ \text{V} \times 5.0\ \text{A} \\ &= 1200\ \text{W} \end{aligned}$$

or 1.2 kW

13.3.6 Transposing formulae

If you have trouble transposing formulae to solve a question, you could use the triangle shown in figure 13.20.

Cover the pronumeral you want to be the subject, for example I. What is visible in the triangle shows what that pronumeral equals. In this example,

$$I = \frac{P}{V}.$$

This method can also be used for any formula of the form $x = yz$. For example, $q = It$ and $W = Vq$.

FIGURE 13.20 Power formula triangle.

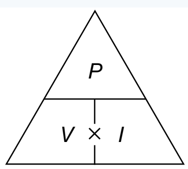
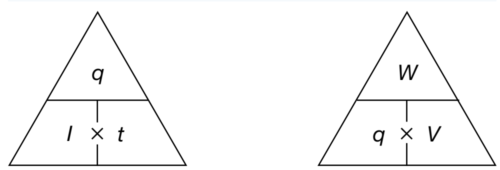


FIGURE 13.21 Variants of the power formula triangle.



13.3 SAMPLE PROBLEM 4

How much energy is supplied by a mobile phone battery rated at 3.7 V and 1200 mAh? 'mAh' stands for milliamp hours, which means that the battery would last for one hour supplying a current of 1200 mA or two hours at 600 mA.

SOLUTION:

$V = 3.7\ \text{V}$

$I = 1200\ \text{mA} = 12\ \text{A},$

$t = 60 \times 60 = 3600\ \text{s}.$

The energy produced is given by $E = VIt$, so

$$\begin{aligned} E &= 3.7 \times 1.2 \times 3600 \\ &= 16\ \text{KJ} \end{aligned}$$

WORKING SCIENTIFICALLY 13.1

The power packs used in most school labs have a knob that is turned to change the size of the voltage supplied. While the positions are marked 2V, 4V, etc., do you think this actually indicates the potential difference supplied to the circuit? Investigate whether the percentage difference between the marked voltage value and the voltage supplied to the circuit varies between packs and if it changes depending upon the resistances in the circuit that it is supplying.

13.3 Exercise 1

1. If electrons are the carriers of charge around an electric circuit, why is current taken to travel from the positive terminal to the negative terminal of a power supply?
2. A mobile phone battery has a voltage of 3.7 V. During its lifetime, 4000 coulombs of charge leave the battery. How much energy did the battery originally hold?
3. What is the potential difference across a light globe if 1.44×10^3 J of heat is produced when a current of 2.0 A flows for 1.0 minute?
4. How much energy does a 1.5 V battery give to 0.5 coulombs of charge?
5. The charge on an electron is 1.6×10^{-19} coulombs. How much energy does each electron have as it leaves a 1.5 V battery?
6. How much electrical potential energy will 5.7 μC of charge transfer if it passes through a voltage drop of 6.0 V?
7. A 6.0 V source supplies 3.6×10^{-4} J of energy to a quantity of charge. Determine the quantity of charge in coulombs and microcoulombs.
8. Copy and complete the following table:

Potential difference	Energy	Charge
? V	32 J	9.6 C
? V	4.0 J	670 mC
9.0 V	? J	3.5 C
12 V	? J	85 mC
4.5 V	12 J	? C
240 V	7.5 kJ	? C

9. What is the energy loss when a current of 5 mA flows for 10 minutes through a conductor across which the potential difference is 2000 V?
10. An electron-Volt (eV) is a unit of energy representing the work done in moving an electron through a potential difference of 1 volt. Approximately how many joules is equal to one electron-Volt?

13.4 Resistance

13.4.1 Defining Resistance

The **resistance**, R, of a substance is defined as the ratio of the voltage drop, V, across it to the current, I, flowing through it.

$$R = \frac{V}{I}$$

The resistance of a device is a measure of how difficult it is for a current to pass through it. The higher the value of resistance, the harder it is for the current to pass through the device.

The SI unit of resistance is the ohm (symbol Ω). It is the resistance of a conductor in which a current of one ampere results from the application of a constant voltage drop of one volt across its ends.

$$1\,\Omega = 1\,\text{VA}^{-1}$$

The ohm is named in honour of Georg Simon Ohm (1787–1854), a German phyiscist who investigated the effects of different materials in electric circuits.

FIGURE 13.22 Georg Simon Ohm.

PHYSICS IN FOCUS

The lie detector

The lie detector, or polygraph, is a meter that measures the resistance of skin. The resistance of skin is greatly reduced by the presence of moisture. When people are under stress, as they may be when telling lies, they sweat more. The subsequent change in resistance is detected by the polygraph and is regarded as an indication that the person *may* be telling a lie.

13.4.2 Factors that determine resistance

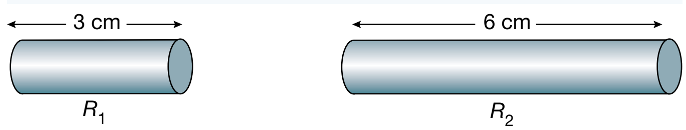
The resistance of a conductor is a result of collisions between the free electrons and the lattice of positive ions. The greater the number of collisions, the greater the resistance. The resistance of a particular conductor is determined by four factors:

- length
- material
- area of cross-section
- temperature.

Resistance and length

Consider free electrons drifting through a metal wire. The longer the wire, the greater the chance of a collision between a free electron and an ion in the lattice. Therefore, the longer the wire, the greater its resistance. If two conductors differ only in length, the longer conductor will have the greater resistance. The resistance, R, is proportional to the length, I: $R \propto I$.

FIGURE 13.23 Dependence of resistance on length.



R_2 will have greater resistance than R_1.

If two conductors, differing only in length, have lengths l_1 and l_2 and resistances R_1 and R_2, then: $\frac{R_1}{R_2} = \frac{l_1}{l_2}$. If the length of a conductor is doubled, keeping all the other factors the same, the resistance will be doubled.

A useful comparison is with the flow of water through a pipe. It is more difficult for water to flow through a long pipe than to flow through a short pipe.

13.4 SAMPLE PROBLEM 1

A 2.0 cm length of wire has a resistance of 1.6 Ω. What would be the resistance of 1.0×10^2 cm of the wire?

SOLUTION:

The resistance of 1.0 cm of the wire is $\frac{1.6\,\Omega}{2.0\,\text{cm}}$.

Therefore, the resistance, R, of 1.0×10^2 cm of the wire will be: $\left(\frac{1.6}{2.0}\right) \times (1.0 \times 10^2)\,\Omega$.

Therefore, $R = 8.0 \times 10^1\,\Omega$.

Resistance and area of cross-section

Consider free electrons drifting through a metal wire. The smaller the area of cross-section of the wire, the greater the chance of a collision of a free electron with an ion in the lattice. Therefore, the smaller the area of cross-section of the wire, the greater its resistance. (Note that doubling the area of cross-section is not the same as doubling the diameter of a wire. If the diameter is multiplied by two, the area of cross-section is multiplied by four.)

If two conductors differ only in area of cross-section, the conductor with the greater area of cross-section will have the lesser resistance. The resistance, R, of a wire is inversely proportional to the area of cross-section, A: $R \propto \frac{1}{A}$.

FIGURE 13.24 Dependence of resistance on area of cross-section.

R_1 will have greater resistance than R_2.

If two conductors, differing only in area of cross-section, have areas of cross-section A_1 and A_2 and resistances R_1 and R_2, then: $\frac{R_1}{R_2} = \frac{A_2}{A_1}$.

If the area of cross-section of a conductor is doubled, keeping all the other factors the same, the resistance will be halved. This is illustrated in figure 13.24.

The flow of water comparison applies here also. It is more difficult for water to flow through a narrow pipe than to flow through a wide pipe.

13.4 SAMPLE PROBLEM 2

Two pieces of resistance wire, X and Y, have the same length. Wire X has a cross-sectional area of 1.00 mm^2, and a resistance of 5.00 Ω. Wire Y has a cross-sectional area of 4.00 mm^2. What will be the resistance of wire Y?

SOLUTION:

$$\frac{\text{Area of cross-section of X}}{\text{Area of cross-section of Y}} = \frac{1.00}{4.00}.$$

$$\frac{\text{Resistance of X}}{\text{Resistance of Y}} = \frac{4.00}{1.00}.$$

$$\frac{5.00}{\text{Resistance of Y}} = \frac{4.00}{1.00}.$$

$$\text{Resistance of Y} = \frac{5.00 \times 1.00}{4.00}$$

$$= 1.25\,\Omega.$$

Resistance and material

When a free electron is drifting through a wire, the chance of a collision with an ion in the lattice depends, in a complex way, on what metal the wire is made of. The size of the postive ions, the geometry of the lattice and the distance between the ions will all have an effect on how many collisions are made between ions and electrons. These factors are reflected in the conductor's resistivity (ρ) measured in ohm metres (Ω m). Resistance is directly proportional to the resistivity:

$R \propto \rho$

The larger the resistivity, the greater the resistance of the material. Two conductors made of different materials but having the same length, area of cross-section and temperature will have different resistivities and, so, will have differing resistances. Table 13.1 shows the resistivities of some common materials at 20 °C.

TABLE 13.1 Comparative resistivities of materials

Material	Resistivity (Ω m)	Material	Resistivity (Ω m)
Silver	1.5×10^{-8}	Tungsten	5.5×10^{-8}
Copper	1.7×10^{-8}	Carbon	3.5×10^{-5}
Aluminium	2.6×10^{-8}	Rubber	approx. 10^{13}–10^{16}
Iron	12×10^{-8}	Glass	approx. 10^{10}–10^{14}
Nichrome	1×10^{-6}	Wood (maple)	4×10^{11}

The resistivity of a material influences its use. When a conductor with negligible resistance is required, copper is commonly used. When a conductor is required to have some resistance, for example, in a heating coil, a material such as nichrome is used. Materials such as glass and rubber are used to make insulators. For household circuits, copper wiring is used. Aluminium and steel (iron) are usually used for transmission lines as copper is too expensive and is not mechanically strong enough.

Resistance and temperature

When the temperature of a conductor is increased, the ions in the lattice vibrate with greater amplitude. This increases the chance of a collision between a free electron and an ion in the lattice. Therefore, increasing the temperature of a conductor increases its resistance.

As an example, consider a conductor made of copper with a resistance of $1.000 \times 10^{-3}\,\Omega$ at 0° C. If its temperature is raised to 100°C, its resistance will be $1.393 \times 10^{-3}\,\Omega$.

If the material experiences very little change in its temperature, as is assumed for the majority of the circuits we will consider, then the combined effects of length L, cross-sectional area A and resistivity ρ on the overall resistance R of a conductor can be related by the formula:

$$R = \frac{\rho L}{A}$$

where A is in m^2, L is in m and ρ is in Ω m.

13.4 SAMPLE PROBLEM 3

Calculate the resistance of a copper wire that is 40 cm long and has a diameter of 2 mm.

SOLUTION:

Given: $D = 2$ mm $= 2 \times 10^{-3}$ m; $L = 40$ cm $= 0.40$ m; $\rho = 1.7 \times 10^{-8}$ (from table 13.1)

To find: R

First, we will need to calculate the cross-sectional area of the wire:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{D}{2}\right)^2 \\ &= \pi \times (1 \times 10^{-3})^2 \\ &= 3.1 \times 10^{-6}\,\text{m}^2 \end{aligned}$$

Now, we can substitute in values to find R:

$$\begin{aligned} R &= \frac{\rho L}{A} \\ &= \frac{(1.7 \times 10^{-8}) \times 0.40}{3.1 \times 10^{-6}} \\ &= 2.2 \times 10^{-3}\,\Omega \end{aligned}$$

So, the wire has a fairly small value of resistance, 2.2 mΩ.

WORKING SCIENTIFICALLY 13.2

Investigate the resistance of a light globe at different temperatures. Does the degree of non-ohmic behaviour depend upon the wattage of the bulb?

13.4.3 Resistors

In many electrical devices, **resistors** are used to control the current flowing through, and the voltage drop across, parts of the circuits. Resistors have constant resistances ranging from less than one ohm to millions of ohms. There are three main types of resistors. ‘Composition’ resistors are usually made of the semiconductor carbon. The wire wound resistor consists of a coil of fine wire made of a resistance alloy such as nichrome. The third type is the metal film resistor, which consists of a glass or pottery tube coated with a thin film of metal. A laser trims the resistor to its correct value.

FIGURE 13.25 A 240-volt, 60-watt globe.

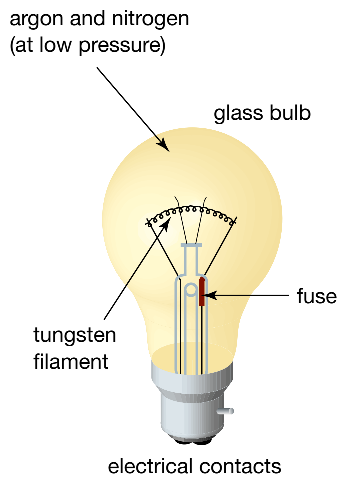
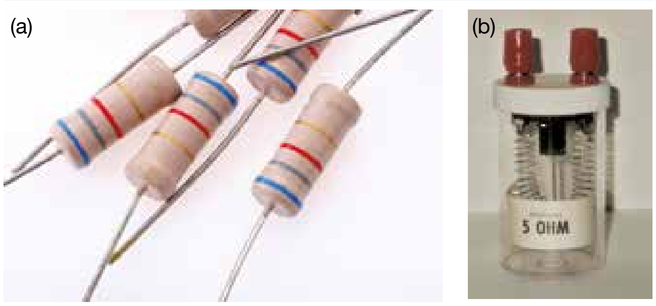


FIGURE 13.26 Examples of (a) carbon resistors and (b) a coiled wire resistor.



Some large resistors have their resistance printed on them. Others have a colour code to indicate their resistance, as shown in figures 13.27 and 13.28, and in table 13.2. The resistor has four coloured bands on it. The first two bands represent the first two digits in the value of resistance. The next band represents the power of ten by which the two digits are multiplied. The fourth band is the manufacturing tolerance.

FIGURE 13.27 Example of a metal film resistor.

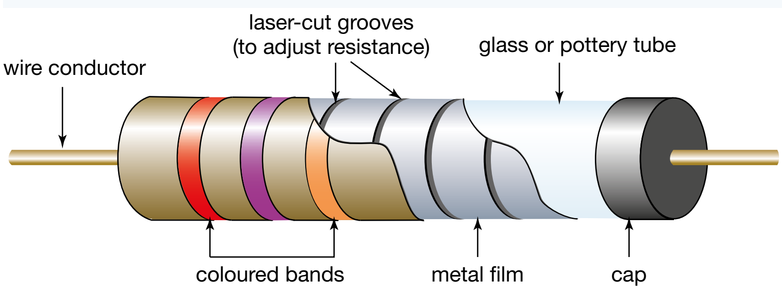


FIGURE 13.28 A resistor, showing the coloured brands.

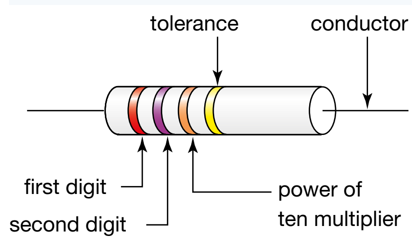


TABLE 13.2 The resistor colour code.

Colour	Digit	Multiplier	Tolerance
Black	0	10^0 or 1	
Brown	1	10^1	
Red	2	10^2	±2%
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	±5%
Silver		10^{-2}	±10%
No colour			±20%

13.4 SAMPLE PROBLEM 4

What is the resistance of the following resistors if their coloured bands are:
(a) red, violet, orange and gold
(b) brown, black, red and silver?

SOLUTION:

(a) Red = 2, violet = 7, so the first two digits are 27.
The third band is orange, which means multiply the first two digits by 10^3. So the resistance is $27\,000\,\Omega$, or $27\,\text{k}\Omega$.
The fourth band is gold, which means there is a tolerance of 5%. This means that the true value is $27\,000\,\Omega \pm 1350\,\Omega$ (5% of $27\,000\,\Omega$).

(b) Brown and black give 10. Red means multiply by 10^2, so the resistance is $1.0 \times 10^3\,\Omega$, and the tolerance is 10%.
When holding a resistor to read its value, keep the gold or silver band on the right and read the colours from the left.

13.4.4 Ohm's Law

Georg Ohm established experimentally that the current I in a metal wire is proportional to the voltage drop V applied to its ends.

$I \propto V$

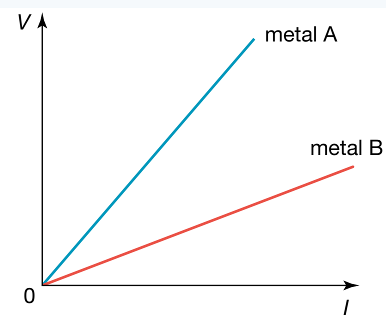
When he plotted his results on a graph of V versus I, he obtained a straight line.

The equation of the line is known as Ohm's Law and can be written:

$V = IR$

where R is numerically equal to the constant gradient of the line. This is known as the resistance of the metal conductor to the flow of current through it. Remember that the SI unit of resistance is the ohm.

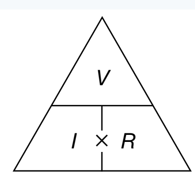
FIGURE 13.29 Graphs of V versus I for two different metal wires.



If you have trouble transposing equations, you can use the triangle method for Ohm's Law: simply cover the quantity you wish to calculate, and the triangle indicates what to do.

For example, if you are given the voltage drop and the current and you wish to find the resistance, cover R, and the triangle shows that $R = \dfrac{V}{I}$.

FIGURE 13.30 Triangle for Ohm's Law.



13.4 SAMPLE PROBLEM 5

A transistor radio uses a 6 V battery and draws a current of 300 mA. What is the resistance of the radio?

SOLUTION:

From Ohm's Law: $R = \dfrac{V}{I}$

$$\text{so } R = \frac{6\ V}{0.3\ \text{A}} \text{ (since 300 mA} = 0.3 \text{ A)}$$
$$= 20\,\Omega.$$

Remember, the voltage drop must be expressed in volts and current must be expressed in amperes.

Note that the equation used for defining resistance is $R = \dfrac{V}{I}$. This is not Ohm's Law unless R is constant. The V versus I graph is not a straight line for a metallic conductor unless the temperature is constant.

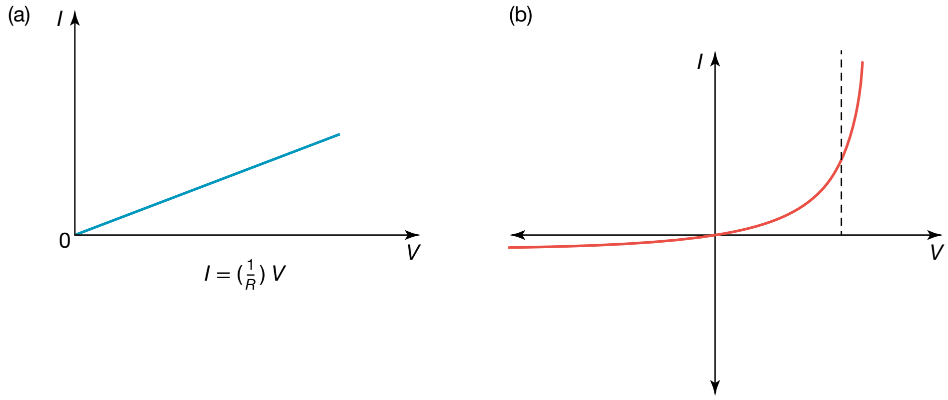
13.4.5 Ohmic and non-ohmic devices

An **ohmic device** is one for which, under constant physical conditions such as temperature, the resistance is constant for all currents that pass through it.

A **non-ohmic device** is one for which the resistance is different for different currents passing through it.

The graph in figure 13.29 has voltage on the y-axis and current on the x-axis. The graph is drawn this way so that the gradient of lines for the metals A and B gave the resistance of each. However, the size of the current in a circuit depends on the magnitude of the voltage; that is, the voltage is the independent variable and the current is the dependent variable. The accepted convention graphs the independent variable on the x-axis and the dependent variable on the y-axis. So in figure 13.31a, the gradient equals $\dfrac{1}{R}$.

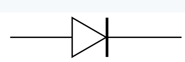
FIGURE 13.31 The I versus V graphs for (a) an ohmic resistor and (b) a diode, which is a non-ohmic device.



Non-ohmic devices

Many non-ohmic devices are made from elements that are semiconductors. They are not insulators as they conduct electricity, but not as well as metals. Common semiconductor elements are silicon and germanium, which are in Group 14 of the periodic table. Many new semiconductor devices are compounds of Group 13 and Group 15 elements such as gallium arsenide.

FIGURE 13.32 Circuit symbol for a diode.



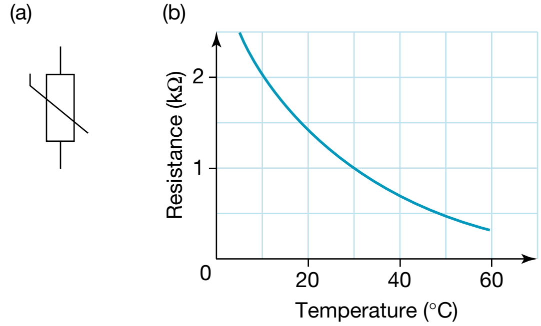
The conducting properties of silicon and germanium can be substantially changed by adding a very small quantity of either a Group 13 element or a Group 15 element. This is called **doping** and affects the movement of electrons in the material.

A **diode** is formed by joining two differently doped materials together. A diode allows current to flow through it in only one direction. This effect can be seen in the current–voltage graph for a diode in figure 13.31b, where a small positive voltage produces a current, while a large negative or reverse voltage produces negligible current.

Light-emitting diodes (LEDs) are diodes that give off light when they conduct. They are usually made from gallium arsenide. Gallium nitride is used in blue LEDs.

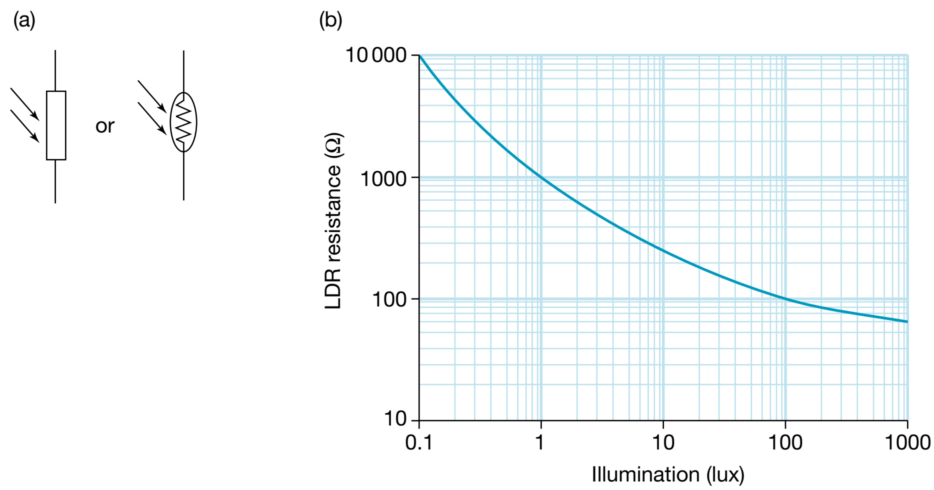
Thermistors are made from a mixture of semiconductors so they can conduct electricity in both directions. They differ from metal conductors, whose resistance increases with temperature, as in thermistors an increase in temperature increases the number of electrons available to move and the resistance decreases.

FIGURE 13.33 (a) Circuit symbol for a thermistor. (b) resistance-versus-temperature graph for a thermistor.



Light-dependent resistors (LDRs) are like thermistors, except they respond to light. The resistance of an LDR decreases as the intensity of light shining on it increases. The axes in figure 13.34b for an LDR have different scales to the other graphs. As you move from the origin, each number is 10 times the previous one. This enables more data to fit in a small space.

FIGURE 13.34 (a) Circuit symbols for an LDR; (b) resistance-versus-light intensity graph for an LDR



13.4.6 Heating effects of currents

Whenever a current passes through a conductor, thermal energy is produced. This is due to the fact that the mobile charged particles, for example electrons, repeatedly collide with the atoms of the conductor, causing them to vibrate more and producing an increase in the temperature of the material.

This temperature increase is not related to the direction of the current. A current in a conductor always generates thermal energy, regardless of which direction the current flows. Examples of devices that make use of this energy include radiators, electric kettles, toasters, stoves, incandescent lamps and fuses.

WORKING SCIENTIFICALLY 13.3

When the transformers/rectifiers for charging mobile phones are connected to the power point, they will often heat up. Investigate the relationship between the period of time the transformer/rectifier is plugged into the power point and the temperature of the casing when (a) the phone is connected and charging, (b) the phone is connected and already fully charged and (c) no phone is connected.

AS A MATTER OF FACT

Nichrome is a heat-resistant alloy used in electrical heating elements. Its composition is variable, but is usually around 62% nickel, 15% chromium and 23% iron.

13.4.7 Power and resistance

The rate at which energy is dissipated by any part of an electric circuit can be expressed as:

$P = VI$

where

P = power

I = current

V = voltage drop.

This relationship can be used, along with the definition of resistance, $R = \dfrac{V}{I}$, to deduce two different formulae describing the relationship between power and resistance:

$$P = VI = (IR)I$$

Thus $P = I^2R$. [1]

$$P = VI = V\left(\frac{V}{R}\right)$$

Thus $P = \dfrac{V^2}{R}$. [2]

You now have three different ways of determining the rate at which energy is transferred as charge flows through a voltage drop in an electric circuit:

$P = VI$ $\qquad$ $P = I^2R$ $\qquad$ $P = \dfrac{V^2}{R}$.

In addition, the quantity of energy transferred, ΔE, can be determined:

$$\begin{aligned}\Delta E &= W \\ &= VIt \\ &= I^2Rt \\ &= \frac{V^2t}{R}\end{aligned}$$

These formulae indicate that in conducting wires with low resistance, very little energy is dissipated. If the resistance, R, is small and the voltage drop, V, is small, the rate of energy transfer is also small.

13.4 SAMPLE PROBLEM 6

A portable radio has a total resistance of 18 Ω and uses a 6.0 V battery consisting of four 1.5 V cells in series. At what rate does the radio transform electrical energy?

SOLUTION:

$$\begin{aligned}P &= \frac{V^2}{R} \\ &= \frac{(6.0\ V)^2}{18\ \Omega} \\ &= 2.0\ W\end{aligned}$$

13.4 SAMPLE PROBLEM 7

A pop-up toaster is labelled '240 V, 800 W'.
(a) What is the normal operating current of the toaster?
(b) What is the total resistance of the toaster while it is operating?

SOLUTION:

(a)
$$P = VI$$
$$\Rightarrow I = \frac{P}{V}$$
$$= \frac{800\text{ W}}{240\text{ V}}$$
$$= 3.3\text{ A}$$

(b)
$$P = \frac{V^2}{R}$$
$$\Rightarrow R = \frac{V^2}{P}$$
$$= \frac{(240\text{ V})^2}{800\text{ W}}$$
$$= 72\,\Omega$$

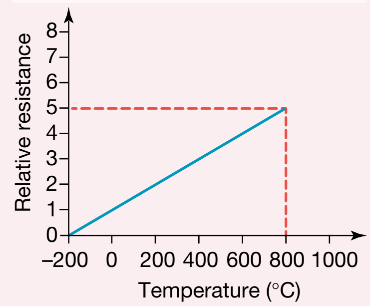
PHYSICS IN FOCUS

Resistance thermometers

The change of resistance of a conductor with temperature change can be used to make a thermometer. Such a thermometer can be used over a much greater range of temperatures than a liquid-in-glass thermometer.

The metal element of such a thermometer consists of a fine wire (approx. 10^{-4} m diameter). As this element is fragile, it is wound around a support made of mica, a mineral that is an insulator with a high melting point. The element is connected to an electrical circuit so that its resistance can be measured. To use the thermometer, the element is inserted into the place where the temperature is to be measured. The resistance of the element is measured using the electric circuit connected to it, and the temperature is calculated from the known temperature–resistance characteristics of the element.

FIGURE 13.35 Relative resistance against temperature for copper.



The most common metals used to make resistance thermometers are platinum, nickel and copper. Figure 13.35 shows how the relative resistance of copper varies with temperature. The relative resistance of a metal at a particular temperature is the ratio of the resistance of the metal at that temperature to its resistance at 0°C.

Copper wire elements are used between 120°C and 200°C; nickel elements between between 150°C and 300°C; platinum elements between 258°C and 900°C. A resistance thermometer can measure temperature to an accuracy of $\pm$ 0.01°C.

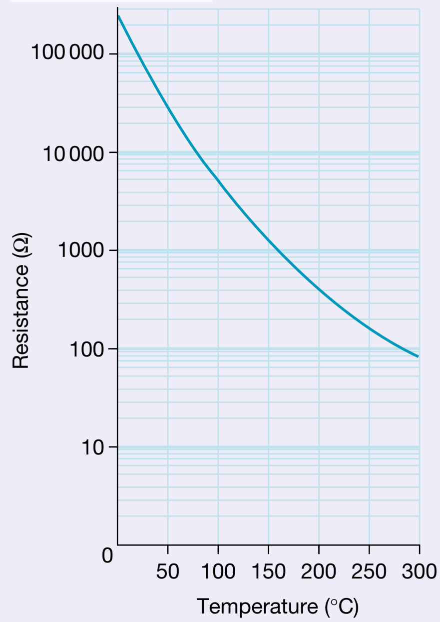
13.4 Exercise 1

1. You are given four pieces of wire made of the same material. The lengths and diameters of the wires are given in the following table. List these in order of increasing resistance.

	Length (cm)	Diameter (mm)
(a)	10	1
(b)	10	2
(c)	20	0.5
(d)	20	11

2. A 30 cm length of wire has a resistance of 1.6 Ω. How much resistance will a 90 cm length of wire have that is cut from the same roll of wire?
3. What will be the resistance of a 2 m length of nichrome wire if it has a diameter of 1 mm? ($\rho_{nichrome} = 1 \times 10^{-6}\,\Omega\,m$)
4. A 4 m length of wire has a cross-sectional area of $1 \times 10^{-3}\,m^2$ and a resistance of 60.0 m Ω.
   (a) Calculate the resistivity of the metal the wire is made from.
   (b) What metal is the wire made from? (Use table 13.1.)
5. A microwave oven is labelled '240 V, 600 W'.
   (a) What is the normal operating current of the microwave oven?
   (b) What is the total resistance of the microwave oven when it is operating?
6. A temperature-sensing system in an oven uses a thermistor with the characteristics shown in figure 13.36.
   (a) What is the resistance of the thermistor when the temperature in the oven is 100 °C?
   (b) What is the temperature in the oven when the resistance of the thermistor is 400 Ω?

FIGURE 13.36



7. Calculate the current drawn by:
   (a) a 60 W light globe connected to a 240 V source
   (b) a 40 W globe with a voltage drop of 12 V across it
   (c) a 6.0 V, 6.3 W globe when operating normally
   (d) a 1200 W, 240 V toaster when operating normally.
8. How much energy is provided by a 6.0 V battery if a current of 3.0 A passes through it for 1.0 minute?
9. Copy and complete the following table.

Potential difference	Current	Resistance
?	8.0 A	4.0 Ω
?	22 mA	2.2 kΩ
12 V	?	6.0 Ω
240 V	?	$8.0 \times 10^4\,\Omega$
9.0 V	6.0 A	?
1.5 V	45 mA	?

10. What are the resistances and tolerances of resistors with the following colour codes:
   (a) blue, green, red, gold
   (b) orange, black, brown, silver
   (c) black, brown, black, red?

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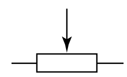
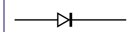
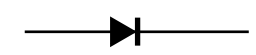
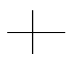
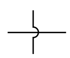
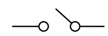
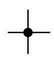
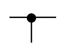
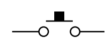
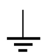
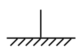
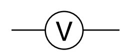
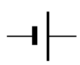
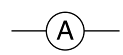
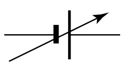
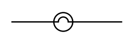
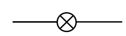
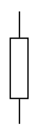
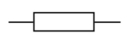
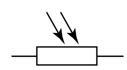
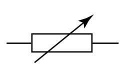
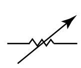
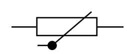
13.5 Series and parallel circuits

13.5.1 Circuit diagrams

A circuit diagram shows schematically the devices used in constructing an electrical circuit. Table 13.3 shows the symbols commonly used in drawing circuits.

TABLE 13.3 Symbols used in circuit diagrams.

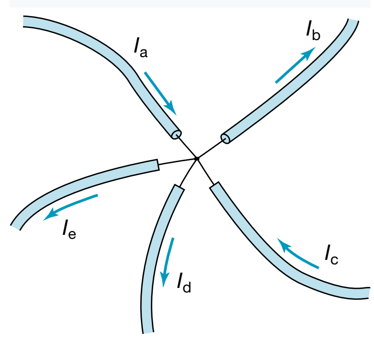
Circuit component	Symbol	Circuit component	Symbol
Connection between conductors		Resistor with sliding contact to give a variable resistance	
Terminal		Semiconductor diode*	 or 
Conductors not connected*	 or 	Single pole switch (open)	
Conductors connected*	 or 	Button switch (open)	
Earth*	 or 	Voltmeter	
Battery		Ammeter	
Variable power supply*	 or 	Incandescent lamp*	 or 
Resistor	 or 	Light-dependent resistor (LDR)	
Variable resistor*	 or 	Heat-dependent resistor (thermistor)	

*The first of the two alternative symbols is used in this book.

13.5.2 Kirchoff's Laws

We owe much of our understanding of electrical circuits to the German physicist Gustav Kirchoff. Building upon the work of Georg Ohm, Kirchoff developed two essential laws that allow us to calculate the voltages, currents or resistances for even very complex electrical networks.

FIGURE 13.37 Five wires soldered at a junction.



Kirchoff's Current Law — the conservation of charge

Electric charge is conserved. At any point in a conductor, the amount of charge (usually electrons) flowing into that point must equal the amount of charge flowing out of that point. Electrons do not build up at a point in a conductor, nor will they magically disappear. You don't get traffic jams in electric circuits.

The sum of the currents flowing into a junction is equal in magnitude to the sum of the currents flowing out of that junction: $I_{in} = I_{out}$ or, $\Sigma I = 0$.

In figure 13.37, $I_a + I_c = I_b + I_d + I_e$.

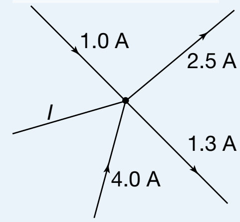
13.5 SAMPLE PROBLEM 1

Calculate the magnitude and direction of the unknown current in figure 13.38, showing currents meeting at a junction.

SOLUTION:

Currents flowing into the junction equal
$1.0 + 4.0 = 5.0$ A.
Currents flowing out of the junction equal
$2.5 + 1.3 = 3.8$ A.
Therefore the unknown current must be
$5.0 - 3.8 = 1.2$ A out of the junction.

FIGURE 13.38



13.5 SAMPLE PROBLEM 2

Find the values of currents a, b, c, d, e and f as marked in figure 13.39.

SOLUTION:

At junction A,

$$I_{in} = I_{out}$$
$$\Rightarrow 15.3 = 7.9 + a$$
$$\Rightarrow a = 7.4 \text{ mA.}$$

At junction B,

$$I_{in} = I_{out}$$
$$\Rightarrow 7.9 + 7.4 = b$$
$$\Rightarrow b = 15.3 \text{ mA.}$$

At junction C,

$$I_{in} = I_{out}$$
$$\Rightarrow 15.3 = c + 2.1$$
$$\Rightarrow c = 13.2 \text{ mA.}$$

At junction D,

$$I_{in} = I_{out}$$
$$\Rightarrow c = d + 6.5$$
$$\Rightarrow 13.2 = d + 6.5$$
$$\Rightarrow d = 6.7 \text{ mA.}$$

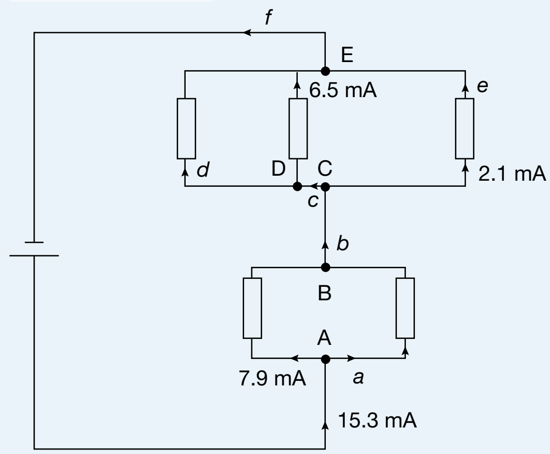
At the resistor,

$e = 2.1$ mA, as $I_{in} = I_{out}$ for the resistor.

At junction E,

$$I_{in} = I_{out}$$
$$\Rightarrow d + 6.5 + e = f$$
$$\Rightarrow 6.7 + 6.5 + 2.1 = f$$
$$\Rightarrow f = 15.3 \text{ mA.}$$

FIGURE 13.39



Kirchoff's Voltage Law — the conservation of energy

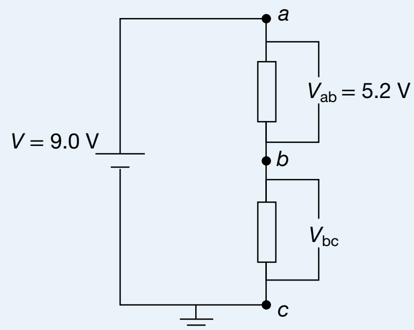
Around a circuit, electrical energy must be conserved. This can be stated as:

In any closed loop of a circuit, the sum of the voltage drops must equal the sum of the emfs (increases in voltage) in that loop, that is $\Sigma V = 0$ for any closed-circuit loop.

13.5 SAMPLE PROBLEM 3

Calculate the unknown voltage drop V_{bc} in figure 13.40.

FIGURE 13.40



SOLUTION:

$$
\begin{aligned}
V &= \text{the sum of the voltage drops} \\
V &= 9.0\text{ V} \\
\Rightarrow 9.0\text{ V} &= 5.2\text{ V} + V_{bc} \\
\Rightarrow V_{bc} &= 3.8\text{ V}
\end{aligned}
$$

13.5.3 Resistors in series

There are two ways in which circuit elements can be connected: in **series** and in **parallel**.

When devices are connected in series, they are joined together one after the other. There is only one path for the current to take.

When devices are connected in parallel, they are joined together so that there is more than one path for the current to flow through.

Many devices can be connected in series and parallel. These include resistors and cells.

When a number of resistors are placed in series, some basic rules can be derived.

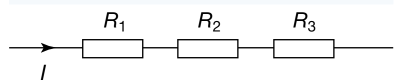
There is only one path for the current to flow through. Therefore in figure 13.41, the current in R_1 equals the current in R_2 and in R_3. I_1 refers to the current in R_1; I_2 to the current in R_2, etc. Similarly V_1 is the voltage drop across R_1; V_2 is the voltage drop across R_2, etc.

$I = I_1 = I_2 = I_3$

Since $V = IR$,

$$
\begin{aligned}
V_1 &= IR_1 \\
V_2 &= IR_2 \\
V_3 &= IR_3
\end{aligned}
$$

FIGURE 13.41 Resistors connected in series.



The total voltage drop, V_T, across resistors in series is equal to the sum of the voltage drops across each individual resistor.

$$
\begin{aligned}
V_T &= V_1 + V_2 + V_3 \\
\Rightarrow V_T &= IR_1 + IR_2 + IR_3 \\
\Rightarrow V_T &= I(R_1 + R_2 + R_3)
\end{aligned}
$$

Since $V_T = IR_T$ (where R_T is the effective resistance of all three resistors), the effective resistance offered by resistors in series is found by obtaining the sum of the individual resistances:

$R_T = R_1 + R_2 + R_3$.

This means that the effective resistance of a circuit is increased by adding an extra resistor in series with the others. The resistance of a series circuit is greater than that for any individual resistor.

13.5 SAMPLE PROBLEM 4

Find the effective resistance of a circuit comprising three resistors, having resistance values of 15 Ω, 25 Ω and 34 Ω, connected in series.

SOLUTION:

$$
\begin{aligned}
R_T &= R_1 + R_2 + R_3 \\
&= 15\,\Omega + 25\,\Omega + 34\,\Omega \\
&= 74\,\Omega
\end{aligned}
$$

13.5 SAMPLE PROBLEM 5

In the series circuit shown in figure 13.42 the emf of the power supply is 100 V, the current at point a, I_a, equals 1.0 A, and the value of R_2 is 60 Ω. Find the following quantities:

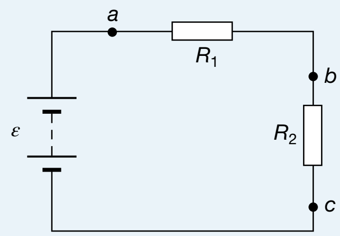
(a) the current at point b

(b) the voltage drop across R_2

(c) the voltage drop across R_1

(d) the value of R_1.

FIGURE 13.42



SOLUTION:

(a) The current is the same at all points of a series circuit. Therefore the current at point b, I_b, is 1.0 A.

(b)
$$\begin{aligned} V_2 &= IR_2 \\ &= 1.0\,\text{A} \times 60\,\Omega \\ &= 60\,\text{V} \end{aligned}$$

(c)
$$\begin{aligned} \varepsilon &= V_1 + V_2 \\ \Rightarrow 100\,\text{V} &= V_1 + 60\,\text{V} \\ V_1 &= 40\text{V} \end{aligned}$$

(d)
$$\begin{aligned} V_1 &= IR_1 \\ 40\,\text{V} &= 1.0\,\text{A} \times R_1 \\ R_1 &= 40\,\Omega \end{aligned}$$

13.5.4 Resistors in parallel

In a parallel branch of a circuit, there is more than one path for the current to flow through.

The total current flowing into the parallel section of a circuit equals the sum of the individual currents flowing through each resistor.

$I_T = I_1 + I_2 + I_3$

As can be seen in figure 13.43, the left-hand sides of all the resistors are connected to point A, so they are all at the same voltage. This means that all charges on that side of the resistors have the same amount of electrical potential energy. Similarly, the right-hand sides of the resistors are connected to point B, therefore they also are at the same voltage. This means that each resistor in a parallel section of a circuit has the same voltage drop across it.

$V_T = V_1 = V_2 = V_3$

In a parallel section of a circuit, the total current equals the sum of the individual currents, and the voltage drops across each resistor are the same. It is possible to derive an expression for the effective resistance, R_T, of a parallel section of a circuit.

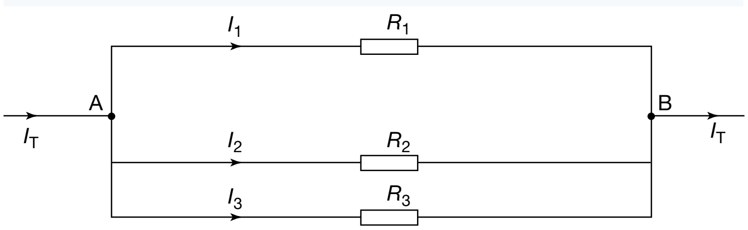
$$I_T = I_1 + I_2 + I_3$$

$$\Rightarrow \frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

(since $I = \frac{V}{R}$ for each resistor and the whole section of the circuit)

$$\Rightarrow \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \text{ (dividing both sides by } V\text{)}$$

FIGURE 13.43 A parallel branch of a circuit.



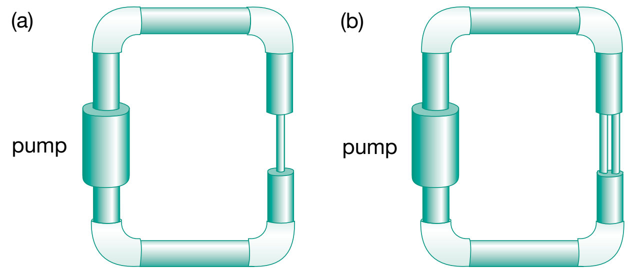
This means that the reciprocal of the effective resistance is equal to the sum of the reciprocals of the individual resistances. The effective resistance is less than the smallest individual resistance. The more resistors there are added in parallel, the more paths there are for the current to flow through, and the easier it is for the current to flow through the parallel section.

Modelling resistors in parallel

One way to help understand this concept is to use the hydraulic model. Current is represented by water flowing in a pipe. Resistors are represented as thin pipes. The thinner the pipe, the greater the resistance and therefore less water can flow in the circuit. A conductor is represented by a large pipe through which water flows easily. The source of emf is represented by a pump that supplies energy to the circuit.

If there is only one thin pipe, it limits the flow of water. Adding another thin pipe beside the first allows more water to flow. The total resistance offered by the two thin pipes in parallel is less than that offered by an individual thin pipe.

FIGURE 13.44 The hydraulic model for resistors in parallel showing (a) a circuit with one 'resistor' and (b) a second 'resistor' added in parallel, which allows more current to flow and reduces the effective resistance.



13.5 SAMPLE PROBLEM 6

What is the effective resistance of three resistors connected in parallel if they have resistance values of 5.0 Ω, 10 Ω and 20 Ω respectively?

SOLUTION:

$$\frac{1}{R_\mathrm{T}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_\mathrm{T}} = \frac{1}{5.0\ \Omega} + \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega}$$

$$\Rightarrow \frac{1}{R_\mathrm{T}} = \frac{4}{20\ \Omega} + \frac{2}{20\ \Omega} + \frac{1}{20\ \Omega}$$

$$\Rightarrow \frac{1}{R_\mathrm{T}} = \frac{7}{20\ \Omega}$$

$$\Rightarrow R_T = \frac{20\ \Omega}{7} = 2.9\ \Omega$$

Note that the effective resistance of a set of resistors connected in parallel is always less than the value of the smallest resistor used. Adding resistors in parallel increases the number of paths for current to flow through, so more current can flow and the resistance is reduced.

If there are n resistors of equal value, R, the effective resistance will be $\frac{R}{n}$.

$$R_T = \frac{R}{n}$$

13.5 SAMPLE PROBLEM 7

Consider the parallel circuit shown in figure 13.45.

The emf of the power supply is 9.0 V, R_2 has a resistance of 10 Ω, and the current flowing through the power supply is 1.35 A. Find the following quantities:

(a) the voltage drop across R_1 and R_2
(b) I_2, the current flowing through R_2
(c) I_1, the current flowing through R_1

(d) the resistance of R_1
(e) the effective resistance of the circuit.

SOLUTION:

(a) For a parallel circuit, $V_1 = V_2$, which in this case is 9.0 V.

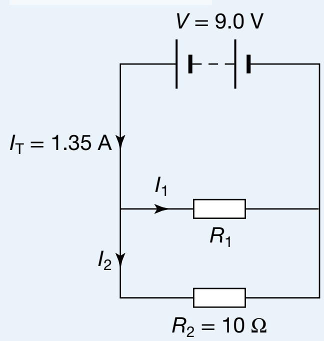
(b)
$$I_2 = \frac{V}{R_2}$$
$$\Rightarrow I_2 = \frac{9.0\ \text{V}}{10\ \Omega} = 0.90\ \text{A}$$

(c)
$$\begin{aligned} I_T &= I_1 + I_2 \\ \Rightarrow I_1 &= I_T - I_2 \\ &= 1.35\ \text{A} - 0.9\ \text{A} \\ &= 0.45\ \text{A} \end{aligned}$$

(d)
$$\begin{aligned} R_1 &= \frac{V}{I_1} \\ \Rightarrow R_1 &= \frac{9.0\ \text{V}}{0.45\text{A}} \\ \Rightarrow R_1 &= 20\ \Omega \end{aligned}$$

(e)
$$\begin{aligned} \frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} \\ \Rightarrow \frac{1}{R_T} &= \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} \\ \Rightarrow \frac{1}{R_T} &= \frac{3}{20\ \Omega} \\ \Rightarrow R_T &= \frac{20\ \Omega}{3} \\ &= 6.7\ \Omega \end{aligned}$$

FIGURE 13.45



13.5 SAMPLE PROBLEM 8

Find the effective resistance when a $10\ \Omega$ resistor is placed in parallel with a $10\ \text{k}\Omega$ resistor.

SOLUTION:

$$\begin{aligned} \frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{10\ \Omega} + \frac{1}{10\,000\ \Omega} \\ &= \frac{1000}{10\,000\ \Omega} + \frac{1}{10\,000\ \Omega} \\ &= \frac{1001}{10\,000\ \Omega} \\ \Rightarrow R_T &= \frac{10\,000\ \Omega}{1001} \\ &= 9.99\ \Omega \end{aligned}$$

Note: Adding a large resistance in parallel with a small resistance slightly reduces the effective resistance of that part of a circuit.

Parallel circuits are used extensively. Australian households are wired in parallel with an AC voltage of 230 V. This is equivalent to a DC voltage of 230 V, and all the formulae that have been presented so far can be used for analysing AC circuits.

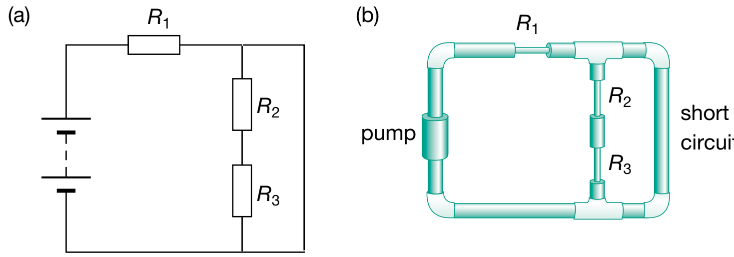
The advantage of having parallel circuits is that all appliances have the same voltage across them and the appliances can be switched on independently. If appliances were connected in series, they would all be on or off at the same time; and they would share the voltage between them, so no appliance would receive the full voltage. This would present problems when designing the devices, as it would not be known what voltage to allow for.

Car lights, front and rear, are wired in parallel for the same reason. If one lamp 'blows', the other lamps will continue functioning normally.

Short circuits

A short circuit occurs in a circuit when a conductor of negligible resistance is placed in parallel with a circuit element. This element may be a resistor or a globe. The result of a short circuit is that virtually all the current flows through the conductor and practically none flows through the circuit element. Because there is effectively no voltage drop across the wire, there is also no voltage drop across the circuit element, and no current flows through it. Think of what would happen in the hydraulic model if a conducting pipe were placed beside a thin pipe. This situation is represented in both ways in figure 13.46.

FIGURE 13.46 (a) Circuit diagram showing a short circuit. (b) Hydraulic model of a short circuit.

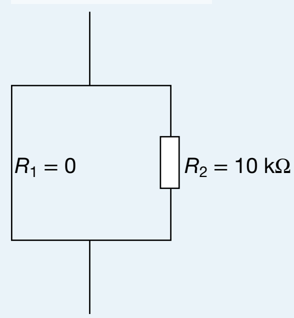


In this case, the current through the power supply passes through R_1, but then flows through the short circuit, effectively avoiding R_2 and R_3.

13.5 SAMPLE PROBLEM 9

Figure 13.47 shows a 10 kΩ resistor that has been short circuited with a conductor of 0 Ω resistance. Calculate the effective resistance of this arrangement.

FIGURE 13.47



SOLUTION:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$= \frac{1}{0\,\Omega} + \frac{1}{10\,000\,\Omega}$$
$$= \infty$$
$$\Rightarrow R_T = 0\,\Omega$$

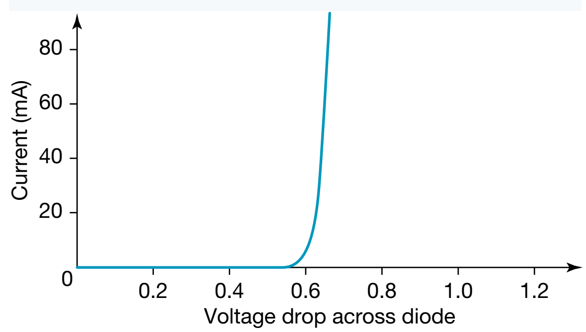
13.5.5 Non-ohmic devices in series and parallel

Non-ohmic devices do not obey Ohm's Law. Their current-versus-voltage characteristics can be presented graphically.

The value of $\frac{V}{I}$ is not constant for non-ohmic devices.

The rules for series and parallel circuits still apply when analysing circuits containing non-ohmic devices. Devices in series have the same current and share the voltage. Devices in parallel have the same voltage and the current is shared between them. The actual values of the voltage or current are obtained from the V–I graphs for the devices.

FIGURE 13.48 V–I characteristic for a silicon diode.



13.5 SAMPLE PROBLEM 10

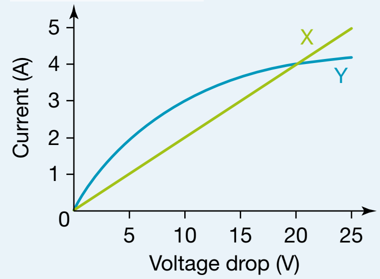
Figure 13.49 shows the current-versus-voltage graph for two electrical devices.

If X and Y are in parallel, and the current through X is 2.0 A, calculate:

(a) the voltage across Y

(b) the current through Y.

FIGURE 13.49



SOLUTION:

(a) As X and Y are in parallel, the voltage across X equals the voltage across Y. From the graph, when the current through X is 2.0 A, the voltage is 10 V, so the voltage across Y is 10 V also.

(b) When the voltage across Y is 10 V, the current through Y is seen to be 3.0 A.

13.5.6 Power in circuits

Recall that the power being used in a circuit element is the product of the voltage drop across it and the current through it: $P = VI$. The total power being provided to a circuit is the sum of the power being used in, or 'dissipated by', the individual elements in that circuit. It does not matter if the elements are connected in series or in parallel.

$$P_T = P_1 + P_2 + P_3 = \ldots$$

13.5 SAMPLE PROBLEM 11

A household electrical circuit is wired in parallel. Find the total current flowing in the circuit if the following appliances are being used: a 600 W microwave oven, a 450 W toaster and a 1000 W electric kettle. Household circuits provide a voltage drop of 230 V across each appliance.

SOLUTION:

The total power being used in the circuit is 600 + 450 + 1000 = 2050 W.

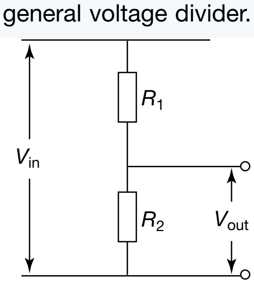
$$\begin{aligned} I_T &= \frac{P_T}{V} \\ &= \frac{2050\text{ W}}{230\text{ V}} \\ &= 8.91\text{ A} \end{aligned}$$

The voltage divider

The **voltage divider** is an example of resistors in series. It is used to divide or reduce a voltage to a value needed for a part of the circuit. A voltage divider is used in many control circuits, for example, turning on the heating in a house when the temperature drops.

The voltage divider has an input voltage, V_{in}, and an output voltage, V_{out}. A general voltage divider is shown in figure 13.50.

FIGURE 13.50 A general voltage divider.



The current I flowing through R_1 and R_2 is the same, since R_1 and R_2 are in series.

$$\begin{aligned} V_{in} &= I(R_1 + R_2) \\ \Rightarrow I &= \frac{V_{in}}{R_1 + R_2} \\ V_{out} &= IR_2 \\ \Rightarrow V_{out} &= \frac{V_{in}}{R_1 + R_2} \times R_2 \end{aligned}$$

This can be rewritten as:

$$V_{\text{out}} = \left[\frac{R_2}{R_1 + R_2}\right] V_{\text{in}}, \text{ or more generally,}$$

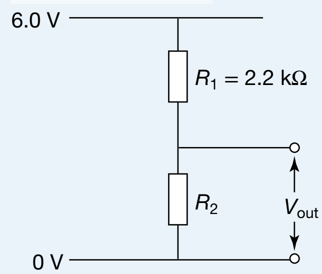
$$V_{\text{out}} = \left[\frac{\text{resistance across which } V_{\text{out}} \text{ is taken}}{\text{sum of all resistances}}\right] V_{\text{in}} = \frac{R_{\text{out}}}{R_{\text{total}}} V_{\text{in}}.$$

If R_1 and R_2 are equal in value, the voltage will be divided equally across both resistors. If R_1 is much greater than R_2, then most of the voltage drop will be across R_1.

13.5 SAMPLE PROBLEM 12

Calculate the value of the unknown resistor in the voltage divider shown in figure 13.51, if the output voltage is required to be 4.0 V.

FIGURE 13.51



SOLUTION:

$$V_{\text{out}} = \frac{R_2 V_{\text{in}}}{R_1 + R_2}$$

$$\Rightarrow 4.0\text{ V} = \frac{6.0\text{ V} \times R_2}{2.2\text{ k}\Omega + R_2}$$

$$\Rightarrow 8.8\text{ k}\Omega\text{ V} + 4.0\text{ V}\, R_2 = 6.0\text{ V}\, R_2$$

$$\Rightarrow 2.0\text{ V}R_2 = 8.8\text{ k}\Omega\text{ V}$$

$$\Rightarrow R_2 = 4.4\text{ k}\Omega$$

13.5 Exercise 1

1. Find the effective resistance of three resistors in series if they have the following values: 1.2 kΩ, 5.6 kΩ and 7.1 kΩ.
2. Find the effective resistance of two resistors in parallel if they have resistance values of 1.2 kW and 4.8 kW.
3. Find the total current owing through a household circuit when the following devices are being used: a 400 W computer, a 200 W DVD player, a 500 W television and a 60 W lamp.
4. Calculate the value of the unknown resistor in the voltage divider in figure 13.51 from sample problem 12 if the output voltage is to be 1.5 V.
5. Two resistors of resistance 12.0 Ω and 4.00 Ω are connected in parallel to a power supply of 30.0 V. Calculate what the current will be through the power supply.
6. Calculate how many 20.0 W resistors, connected in parallel to a 2.00 V power supply, would result in a current of 1.00 A through the power supply.
7. Two resistors of 12.0 Ω and 18.0 Ω are connected in series to a 60.0 V power supply. Calculate the current flowing through the circuit.
8. Consider the circuit shown in figure 13.52. (a) Calculate the current through the 2.50 Ω resistor. (b) Calculate the current through the 6.00 Ω resistor. (c) Calculate the voltage drop across the 6.00 Ω resistor. (d) Calculate the voltage gain across the power supply.
9. (a) Find the output voltage for the voltage divider shown in the circuit in figure 13.53.
   (b) What is the output voltage of the circuit in figure 13.54 if a load of resistance 4.4 kΩ is connected across the output terminals of the voltage divider?

FIGURE 13.52

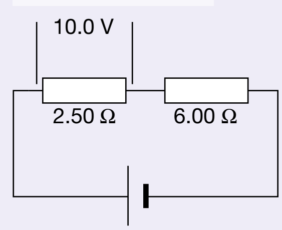


FIGURE 13.53

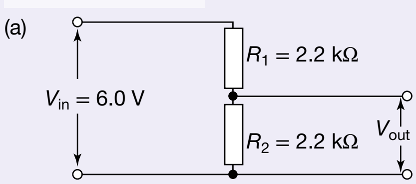
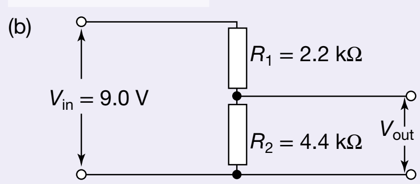
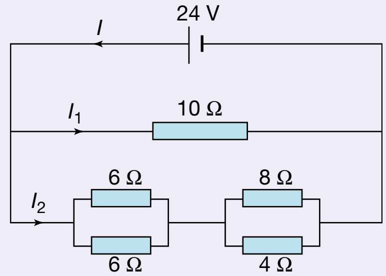


FIGURE 13.54



10. Consider the circuit shown in figure 13.55. Determine:
   (a) the total equivalent resistance R that would replace all of the resistors in the circuit without changing the value of I
   (b) the value of I
   (c) the value of I_2
   (d) the potential drop across the 8 Ω resistor.

FIGURE 13.55



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13.6 Review

13.6.1 Summary

- An electric current is a net movement of electric charge. The SI unit of current is the ampere (A). One ampere is equivalent to one coulomb per second: $I = \frac{q}{t}$.
- Metals are electrical conductors because they have free electrons that act as charge carriers.
- Insulators are materials that have no charge carriers and, therefore, cannot carry an electric current.
- The potential difference (voltage) between the terminals of a power supply is the number of joules of electric potential energy given to each coulomb of electric charge: $\mathrm{V} = \frac{W}{q}$.
- If a conductor connects the terminals of a power supply, a current will flow through the conductor. The movement of electrons is from the negative to the positive terminal of the power supply. The conventional current direction is from the positive to the negative terminal of the power supply.
- A resistor is a conductor that resists the movement of the current through it.
- When current flows through a resistor, electric potential energy is dissipated as heat energy: $W = I^2R$.

- The potential difference (voltage) between the ends of a resistor is the number of joules of electric potential energy dissipated for each coulomb of charge that passes through the resistor.
- The resistance of a resistor is equal to the potential difference across the resistor divided by the current passing through the resistor: $R = \frac{V}{I}$. The SI unit of resistance is the ohm (Ω).
- The resistance of a resistor depends on length, cross-section area, material and temperature: $R = \frac{\rho L}{A}$ at constant temperature.
- An ammeter is used to measure current and is connected into a circuit in series. A voltmeter is used to measure electric potential difference and is connected into a circuit in parallel.
- When resistors are connected in series to a power supply, the same current passes through each resistor and through the power supply.
- When resistors are connected in series to a power supply, the sum of the voltage drops across the resistors equals the voltage rise across the power supply.
- When resistors are connected in parallel to a power supply, the sum of the currents through the resistors equals the current through the power supply.
- When resistors are connected in parallel to a power supply, the voltage drop across each resistor equals the voltage rise across the power supply.
- Non-ohmic devices such as LDRs, LEDs, diodes and thermistors do not obey Ohm's Law.
- Circuits containing non-ohmic devices can be analysed using the rules for series and parallel circuits with their voltage–current characteristic graphs.
- The total power used in a circuit equals the sum of the powers used in individual devices.
- A voltage divider is used to reduce an input voltage to some required value.
- A voltage divider consists of two or more resistors arranged in series to produce a smaller voltage at its output.
- The output of a voltage divider can be calculated using the equation: $V_{out} = \left[\frac{R_2}{R_1 + R_2}\right]V_{in}$

13.6.2 Questions

1. Define an electric current.
2. (a) Identify the charge carriers in a metal.
   (b) Describe how these charge carriers move under the influence of an electric field in the metal.
   (c) Describe how an electric field can be produced in the metal.
3. A current of 2.00 A is flowing through a wire.
   (a) Calculate how many coulombs pass through a cross-section of the wire in 3.00 s.
   (b) Calculate how many electrons pass through a cross-section of the wire in this time.
4. An electric current is flowing through a wire. A charge of 4.20 C passes through a cross-section of the wire in 3.00 s. Calculate the current in amps.
5. A current flows through a wire for 2.50 s. During this time 5.60×10^{18} electrons pass through a cross-section of the wire.
   (a) Calculate the charge passing through a cross-section of the wire in this time.
   (b) Calculate the current in amps.
6. A current of 5.00 amps is passing through a wire.
   (a) Calculate the charge passing through a cross-section of the wire in:
      (i) 1.00 second
      (ii) 1.00 minute.
   (b) Calculate the number of electrons passing through a cross-section of the wire in each of these periods of time.
7. Explain how heat energy is produced when an electric current passes through a metal.

8. Explain what is meant by the potential difference across the ends of a resistor.
9. When 2.50 C pass through a certain resistor, 50.0 J of heat energy is generated. Calculate the potential difference across the resistor.
10. The potential difference across a certain resistor is 32.0 V. Calculate how much heat energy is produced when 12.0 C of electric charge pass through the resistor.
11. A battery is marked 25 V. Explain what this means.
12. Calculate the change in electric potential energy as 2.00 C of charge passes through a 2.50 V battery.
13. Explain what is meant by:
    (a) a conductor
    (b) a resistor
    (c) an insulator.
14. When a current of 2.00 A passes through a certain resistor, there is a potential difference of 16.0 volts across it. Calculate the resistance of the resistor.
15. A coil of wire has a resistance of 3.20 Ω. Calculate the potential difference across the coil when there is a current of 2.00 A passing through it.
16. The following table refers to the potential difference, V, across a resistor of resistance R when a current, I, passes through it. Calculate the values of the missing quantities and complete the table.

	V(v)	*I* (A)	*R*(Ω)
(a)	10.0	2.00	
(b)		6.00	12.0
(c)	16.0		32.0
(d)	3.00	18.0	
(e)	24.0		8.00
(f)		5.00	2.00

17. The table shown at right refers to the change in potential energy, W, when an electric charge, q, passes through an electrical potential difference, V. Calculate the values of the missing quantities and complete the table.

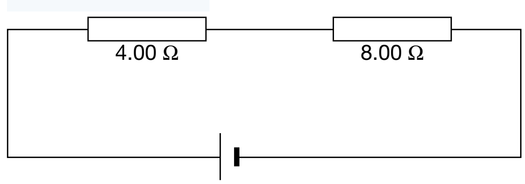
	V(v)	*q*(C)	*W*(J)
(a)	12.0	2.00	
(b)		25.0	50.0
(c)	6.00		42.0
(d)		8.00	32.0
(e)	4.00		0.50
(f)	2.00	16.0	

18. You are given four pieces of wire made of the same material. The lengths and diameters of the wires are given in the following table. List these in order of increasing resistance. Justify your answer.

	Length (cm)	Diameter (mm)
(a)	10	1
(b)	10	2
(c)	20	0.5
(d)	20	1

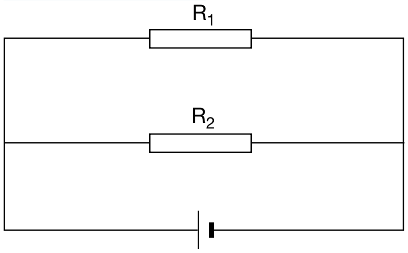
19. Demonstrate by means of diagrams the correct way of connecting into a circuit:
    (a) an ammeter
    (b) a voltmeter.
20. Draw circuit diagrams showing resistors of 2.00 Ω, 4.00 Ω and 6.00 Ω (a) in series and (b) in parallel with a 12.0 V battery.
21. In the circuit shown in figure 13.56, the current through the 4.00 Ω resistor is 5.00 A. Calculate:
    (a) the current through the 8.00 Ω resistor
    (b) the current through the power supply.

FIGURE 13.56



22. In the circuit shown in figure 13.57, the current through the power supply is 6.00 A and the current through R_1 is 4.00 A.

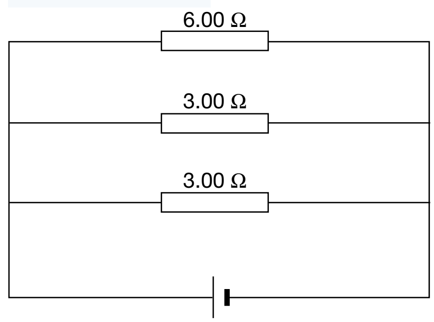
FIGURE 13.57



(a) Calculate the current through R_2.
(b) Deduce which resistor has the greater resistance.

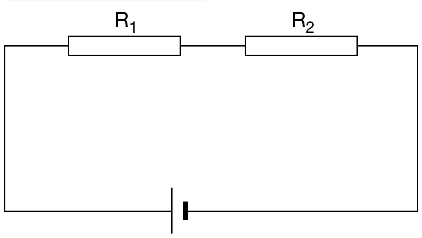
23. In the circuit shown in figure 13.58, the current through the power supply is 20.00 A and the current through the 6.00 Ω resistor is 4.00 A. Calculate the current through each of the 3.00 Ω resistors.

FIGURE 13.58



24. In the circuit shown in figure 13.59, the power supply has a voltage of 7.00 V and the voltage drop across R_1 is 4.00 V.
(a) Calculate the voltage drop across R_2.

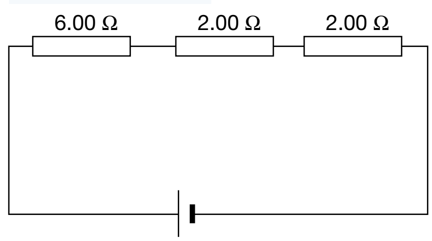
FIGURE 13.59



(b) Deduce which resistor has the greater resistance.

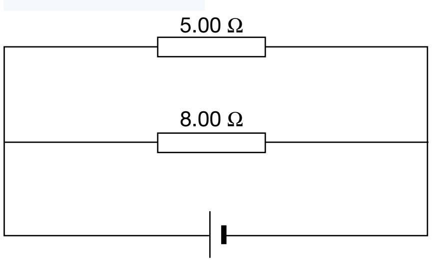
25. In the circuit shown in figure 13.60, the power supply has a voltage of 20.0 V and the voltage drop across the 6.00 Ω resistor is 12.0 V. Calculate the voltage drop across each of the 2.00 Ω resistors.

FIGURE 13.60



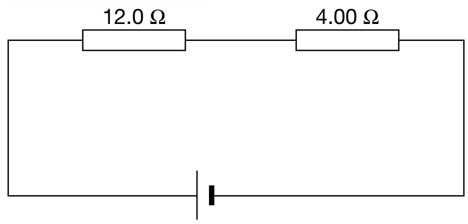
26. In the circuit shown in figure 13.61, the voltage drop across the 5.00 Ω resistor is 20.0 V.
Calculate:
(a) the voltage drop across the 8.00 Ω resistor
(b) the voltage of the battery.

FIGURE 13.61



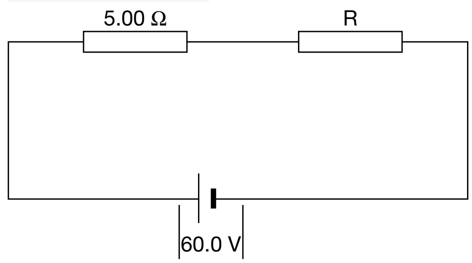
27. In the circuit shown in figure 13.62, the voltage drop across the 12.0 Ω resistor is 36.0 V.
Calculate:
(a) the current through the 12.0 Ω resistor
(b) the current through the 4.00 Ω resistor
(c) the voltage drop across the 4.00 Ω resistor
(d) the voltage of the power supply.

FIGURE 13.62



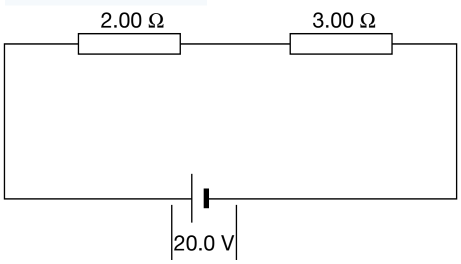
28. In the circuit shown in figure 13.63, the current through the 5.00 Ω resistor is 4.00 A. Calculate:
(a) the voltage drop across the 5.00 Ω resistor
(b) the voltage drop across the resistor R
(c) the current through the resistor R
(d) the resistance of the resistor R.

FIGURE 13.63



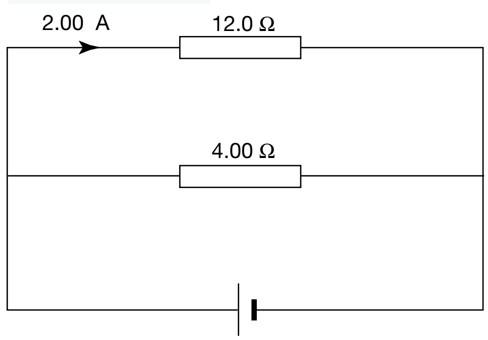
29. For the circuit shown in figure 13.64, calculate the current through the circuit.

FIGURE 13.64



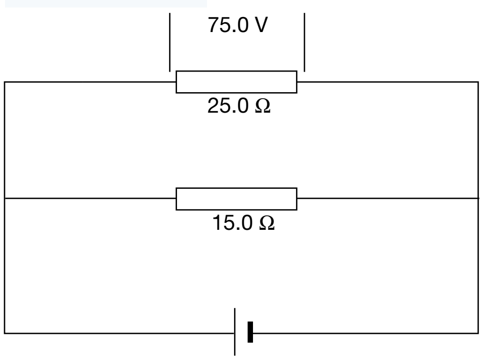
30. For the circuit shown in figure 13.65, calculate:
(a) the potential drop across the 12.0 Ω resistor
(b) the potential drop across the 4.00 Ω resistor
(c) the current through the 4.00 Ω resistor
(d) the potential gain across the power supply
(e) the current through the power supply.

FIGURE 13.65



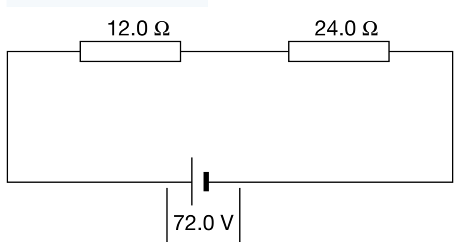
31. For the circuit shown in figure 13.66, calculate:
(a) the current through the 25.0 Ω resistor

FIGURE 13.66



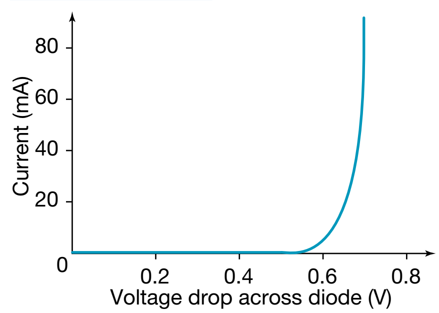
(b) the current through the 15.0 Ω resistor
(c) the current through the power supply.
32. For the circuit shown in figure 13.67, calculate:
(a) the total resistance
(b) the current through the power supply
(c) the potential drop across the 12.0 Ω resistor.

FIGURE 13.67



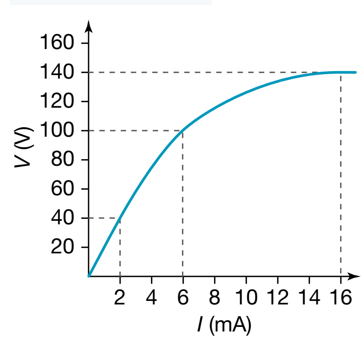
33. When a certain resistor is connected to a 25.0 V power supply, the current through the resistor is 12.5 A. Calculate the resistance of a second resistor, connected in series, that reduces the current to 5.00 A.
34. In each of the following cases, calculate the total resistance:
    (a) resistors of $12.0\,\Omega$ and $16.0\,\Omega$ in series
    (b) resistors of $12.0\,\Omega$ and $4.00\,\Omega$ in parallel
    (c) resistors of $12.0\,\Omega$, $10.0\,\Omega$ and $17.0\,\Omega$ in series
    (d) resistors of $12.0\,\Omega$, $10.0\,\Omega$ and $17.0\,\Omega$ in parallel
    (e) six $15.0\,\Omega$ resistors in series
    (f) five $2.00\,\Omega$ resistors in parallel.
35. In each of the following cases, calculate:
    (i) the current through each resistor
    (ii) the voltage drop across each resistor.
    (a) $5.00\,\Omega$ and $7.00\,\Omega$ resistors connected in series to a 6.00 V power supply
    (b) $10.0\,\Omega$, $20.0\,\Omega$ and $40.0\,\Omega$ resistors connected in series to a 140.0 V power supply
    (c) $2.4\,\Omega$ and $3.7\,\Omega$ resistors connected in series to a 15 V supply
    (d) $11.2\,\Omega$, $20.4\,\Omega$ and $31.5\,\Omega$ resistors connected in series to a 128 V power supply
    (e) $2.00\,\Omega$ and $3.00\,\Omega$ resistors connected in parallel to a 12.0 V power supply
    (f) $12.0\,\Omega$, $24.0\,\Omega$ and $60.0\,\Omega$ resistors connected in parallel to a 48.0 V power supply
    (g) $17.3\,\Omega$ and $25.6\,\Omega$ resistors connected in parallel to a 125 V power supply
    (h) $2.53\,\Omega$, $7.12\,\Omega$ and $4.28\,\Omega$ resistors connected in parallel to a 30.5 V power supply.
36. What are the resistances and tolerances of resistors with the following colour codes:
    (a) orange, white, black, gold
    (b) green, blue, orange, silver
    (c) violet, green, yellow, gold?
37. Figure 13.68 shows the current versus voltage characteristic for an electronic device.
    (a) Is this device ohmic or non-ohmic? Justify your answer.
    (b) What is the current through the device when the voltage drop across it is 0.5 V?
    (c) What is the resistance of the device when the voltage drop across it is 0.5 V?
    (d) Estimate the voltage drop across the device, and its resistance, when it draws a current of 20 mA.

FIGURE 13.68



38. At what rate is thermal energy being transferred to a wire if it has a resistance of $5.0\,\Omega$ and carries a current of 0.30 A?
39. Calculate the resistance of the following globes if their ratings are:
    (a) 240 V, 60 W
    (b) 6.0 V, 6.3 W
    (c) 12 V, 40 W.
40. What is the power rating of an electric jug if it has a resistance of $48\,\Omega$ when hot and is connected to a 240 V supply?
41. The voltage-versus-current characteristic graph for a non-ohmic device is shown in figure 13.69.
    (a) What is the device's current when the voltage drop across it is 100 V?
    (b) What is the voltage drop across the device when the current through it is 16 mA?
    (c) What is the resistance of the device when it carries a current of 16 mA?

FIGURE 13.69



42. Two students are discussing how to produce more power in a circuit.

'$P = VI$,' says Tom. 'Since $V = IR$, $P = I^2R$. This shows that power is proportional to resistance. Therefore, if I want more power from a supply I should use a bigger resistance.'

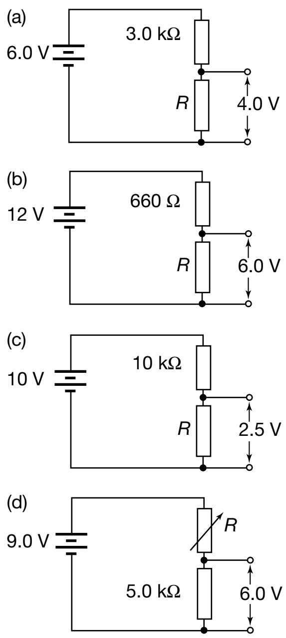
'I don't agree', says Henrietta. 'It is true that $P = VI$, but Ohm's Law gives $I = \frac{V}{R}$. This means that $P = \frac{V^2}{R}$, in which case power is proportional to the inverse of resistance. If I want to draw more power from a supply, I would use a smaller resistance.'

Which student is correct, and under what circumstances is he/she correct?

43. What happens to the voltage drop across a variable resistor in a two-element voltage divider when its resistance decreases and the other resistance is unchanged?

44. Find the value of the unknown resistor in the voltage dividers shown in figure 13.70.

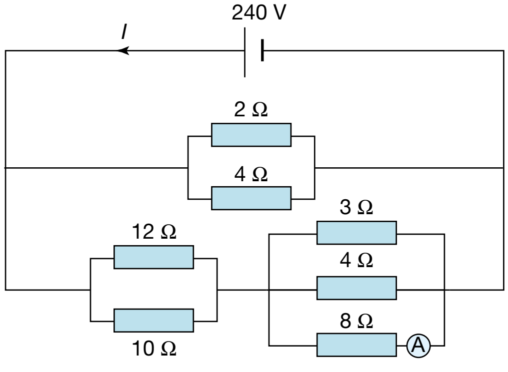
FIGURE 13.70



45. (a) Given four $10\,\Omega$ resistors, how many different total resistances can be obtained by placing them in varying combinations?
    (b) What will be the highest resistance possible?
    (c) What will be the lowest?

46. Determine the value on the ammeter in the circuit shown in figure 13.71.

FIGURE 13.71



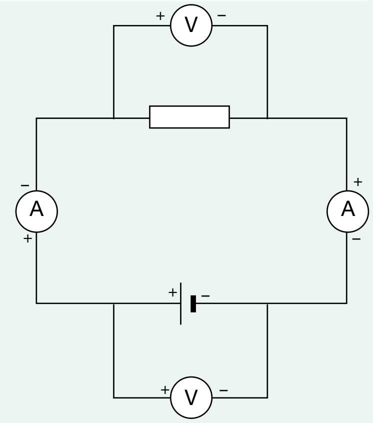
PRACTICAL INVESTIGATIONS

Note on practical activities

Instructions for activities involving electric currents:

- To measure the current through a component, connect an ammeter in series with the component.
- To measure the voltage across a component, connect a voltmeter in parallel with the component.
- Ammeters and voltmeters should be connected so that the connection from the positive terminal of the power supply goes to the positive terminal of the ammeter or voltmeter (see figure 13.72).
- If there is more than one range on a meter, start with the largest range (for example, if there is a 0–1 A range and a 0–5 A range on an ammeter, start with the 0–5 A range).
- Start with the lowest voltage setting on the power supply.
- If using a variable resistor to vary the current, start with its highest resistance.
- Tap the switch closed to check that the ammeters and voltmeters are correctly connected.
- When making a measurement, close the switch just long enough to make the measurement.
- Ask your teacher to check your circuit before switching on the power supply.
- Your teacher will advise you about what maximum current or maximum voltage you should use in a particular experiment.

FIGURE 13.72 Connecting ammeters and voltmeters to a power supply



Investigation 13.1: Potential difference in a simple circuit

Aim

To investigate the potential difference between different points around a simple circuit

Apparatus

power supply
voltmeter
two resistors, R_1 and R_2, of different resistance
switch
connecting wires

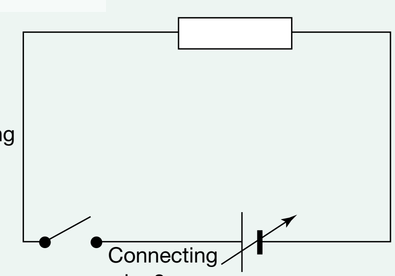
Theory

Potential difference is the change in electric potential energy per coulomb of charge. When a current flows through a circuit, several changes in potential occur. In the power supply there is an increase in electric potential energy and therefore, a potential rise. In a resistor, there is a decrease of electric potential energy and therefore, a potential drop. In a connecting wire, there is no change of electric potential energy and therefore, no potential difference.

Method

1. Connect the apparatus as shown in figure 13.73. Use R_1.
2. Set the voltage of the power supply to a low voltage.
3. Connect the voltmeter, in turn, across each of the following components shown in figure 13.73:
   - power supply
   - resistor
   - connecting wire 1
   - connecting wire 2
   - connecting wire 3 (including switch).
4. For each component, measure the voltage with the switch closed.
5. Repeat 3 using a higher voltage power supply.
6. Repeat 2, 3, and 4 using R_2.

FIGURE 13.73



Results

Record the results in a table as shown below.

R_1, low power supply voltage

Component	Potential difference (V)
Power supply	
Resistor	
Connecting wire 1	
Connecting wire 2	
Connecting wire 3	

Make similar tables for:

(i) R_1, high power supply voltage
(ii) R_2, low power supply voltage
(iii) R_2, high power supply voltage.

Analysis

1. What are the voltages across the connecting wires?
2. What is the relation between the voltage across the power supply and the voltage across the resistor?

Question

Explain, in terms of energy, why the potential rise across the power supply equals the potential drop across the resistor.

Investigation 13.2: Measuring resistance

Aim

To show the relationship between voltage and current, and measure the resistance of a resistor

Apparatus

power supply
ammeter
voltmeter
variable resistor
two resistors, R_1 and R_2, of different resistance
connecting wires
switch

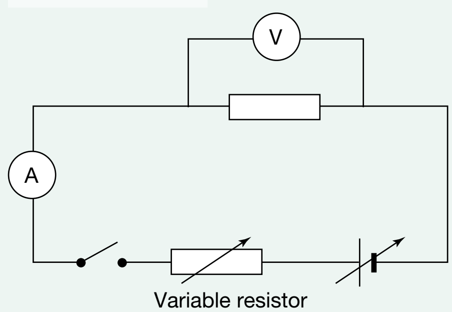
Theory

The voltage drop, V, across a resistor is proportional to the current, I, through the resistor. The graph of V against I will be a straight line. The slope of the line will equal the resistance.

Method

1. Connect the apparatus as in the circuit diagram in figure 13.74, using R_1.
2. Pass at least five different currents through the resistor by varying both the voltage of the power supply and the resistance of the variable resistor.
3. For each value of the current, record the current through R_1 and the voltage across R_1.
4. Repeat steps 1, 2 and 3 using resistor R_2.

FIGURE 13.74



Results

Record your results in a table as shown below.

Measurements for R_1

v(v)	I(A)

Analysis
1. For each resistor, draw a graph of V against I using the same sheet of graph paper.
2. Measure the slope of each graph.
3. Use the slopes of the graphs to find the values of the resistance.

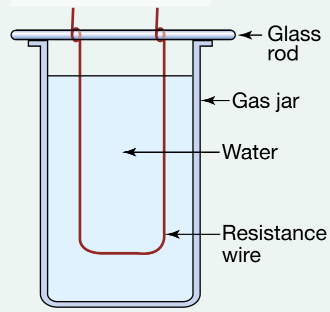
Questions
1. What are the shapes of the graphs?
2. Which graph has the steeper slope?
3. How do the values you found for the resistances compare with the values marked on the resistors?
4. Why would you expect your graphs to pass through the origin?
5. If your graphs do not pass through the origin suggest a reason.

Investigation 13.3: Dependence of resistance on length of resistance wire

Aim
To investigate how the resistance of a resistance wire varies with length

Apparatus
power supply
voltmeter
ammeter
variable resistor
switch
two different lengths of resistance wire, l_1 and l_2
gas jar
metre rule

FIGURE 13.75

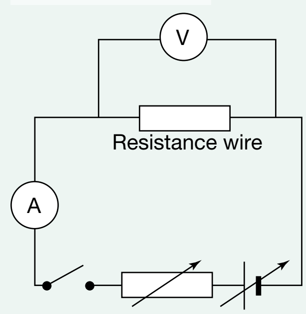


Theory
The resistance of a conductor of constant thickness increases as its length increases. Resistance can be calculated by the formula: $R = \frac{V}{I}$.

Figure 13.75 Setup for resistance wire

Method
As the resistance of a wire varies with temperature, it is necessary to keep the resistance wire at constant temperature. This is done by immersing the resistance wire in water as shown in figure 13.75.
1. Measure the lengths of resistance wire.
2. Connect the apparatus as shown in figure 13.76 using one of the resistance wires.
3. Vary the voltage of the power supply and the resistance of the variable resistor until the current has the maximum value given by your teacher.
4. Measure the voltage and the current with the switch closed.
5. Repeat using the other length of resistance wire.

FIGURE 13.76



Results
Record your results in a table as shown below and calculate the resistances.

Length of wire (cm)	Current (A)	Voltage (V)	Resistance (Ω) $R = \frac{V}{I}$

Analysis
1. What is the ratio of the lengths of the wires?
2. What is the ratio of the resistances of the wires?
3. What conclusion can you draw from these values?

Questions
1. What would be the resistance of a 30 cm length of this wire?
2. What would be the resistance of a 2 cm length of this wire?
3. What is the resistance per unit length of this wire?

Investigation 13.4 (a) Current in a series circuit

Aim

To investigate current in a series circuit

Apparatus

power supply
ammeter
two resistors, R_1 and R_2, with different resistances
switch
connecting wires

Theory

In a series circuit the same current flows through each resistor.

Method

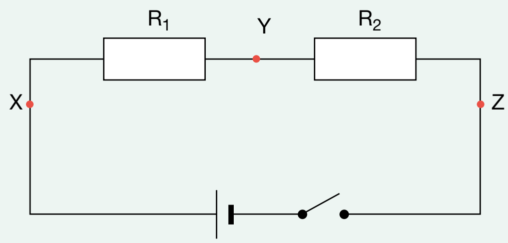
1. Connect the apparatus as shown in figure 13.77.
2. Connect the ammeter in turn to measure the current at each of the points X, Y and Z, when the switch is closed.

Results

Record the current measurements as in the table below.

Current at X	
Current at Y	
Current at Z	

FIGURE 13.77



Analysis

1. What is the current passing through the power supply?
2. What is the current passing through R_1?
3. What is the current passing through R_2?
4. Write a sentence to sum up what you have observed about the current in a series circuit.

Questions

1. Does current decrease as it passes through a resistor?
2. What does decrease as a current passes through a resistor?
3. Does current increase as it passes through a power supply?
4. What does increase as a current passes through a power supply?

Investigation 13.4 (b) Current in a parallel circuit

Aim

To investigate current in a parallel circuit

Apparatus

power supply
ammeter
two resistors, R_1 and R_2, with different resistances
switch
connecting wires

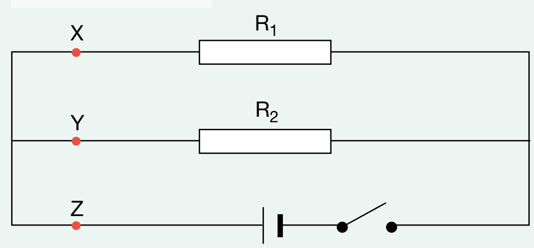
Theory

When current passes through resistors in parallel, the current passes partly through each of the resistors. The current through the power supply is equal to the sum of the currents through the resistors.

Method

1. Connect the apparatus as shown in figure 13.78.
2. Connect an ammeter in turn to measure the current at each of the points X, Y, and Z, when the switch is closed.

FIGURE 13.78



Results
Record your results as in the table below.

Current at X	
Current at Y	
Current at Z	

Analysis
1. What is the sum of the currents through the two resistors in parallel?
2. Compare this with the current through the power supply.
3. Write a sentence to sum up what you have observed about currents in a parallel circuit.

Questions
1. Which resistor had the higher current passing through it?
2. What is the ratio of current through R_1 to the current through R_2?
3. What is the ratio of R_1 to R_2?
4. Comment on your answers to questions 2 and 3.

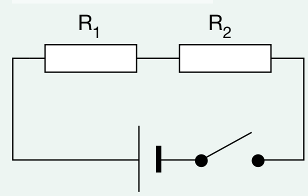
Investigation 13.4 (c) Voltage in a series circuit

Aim
To investigate voltage in a series circuit

Apparatus
power supply
voltmeter
two resistors, R_1 and R_2, with different resistances
switch
connecting wires

Theory
In a series circuit, the voltage rise across the power supply equals the sum of the voltage drops across the resistors.

FIGURE 13.79



Method
1. Connect the circuit as in figure 13.79.
2. Connect the voltmeter in turn across the power supply and each of the resistors and record the readings when the switch is closed.

Results
Record the measurements as in the table below.

V across power supply	
V across R_1	
V across R_2	

Analysis
1. What is the sum of the voltage drops across the resistors?
2. Compare this with the voltage gain across the power supply.
3. Write a sentence to state the relationship between the voltage across the power supply and the voltage drops across the resistors.

Questions
1. Which resistor had the higher voltage drop across it?
2. What is the ratio of the voltage drop across R_1 to the voltage drop across R_2?
3. What is the ratio of R_1 to R_2?
4. Compare your answers to questions 2 and 3 and make a comment.

Investigation 13.4 (d) Voltage in a parallel circuit

Aim

To investigate the voltage in a parallel circuit

Apparatus

power supply
voltmeter
two resistors, R_1 and R_2, with different resistances
switch
connecting wires

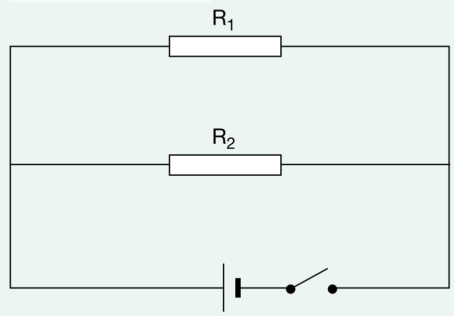
Theory

In a parallel circuit, the voltage drop across each resistor is equal to the voltage gain across the power supply.

Method

1. Connect the apparatus according to the circuit diagram in figure 13.80.
2. Connect the voltmeter in turn to measure the voltage (with the switch closed) across the power supply and across each resistor.

FIGURE 13.80



Results

Record your results as in the table below.

V across power supply	
V across R_1	
V across R_2	

Analysis

Write a sentence to sum up these results.

Questions

1. Does any current pass through *both* R_1 and R_2?
2. Consider *only* that part of the current that passes through R_1.
   (a) What is its voltage gain in the power supply?
   (b) What is its voltage loss in R_1?
3. Consider *only* that part of the current that passes through R_2.
   (a) What is its voltage gain in the power supply?
   (b) What is its voltage loss in R_2?
4. Consider the *total* current in the circuit.
   (a) What is its voltage gain in the power supply?
   (b) What is its voltage loss in the resistors?

Investigation 13.5: Addition of resistances

Aim

To use the laws of addition of resistances to predict the resistance of two resistors connected (a) in series and (b) in parallel and to test the predictions experimentally

Apparatus

power supply
variable resistor
ammeter
voltmeter
two resistors, R_1 and R_2, with different resistances
switch
connecting wires

Theory

The total resistance, R_{series}, of two resistors, R_1 and R_2, connected in series, is given by the rule:

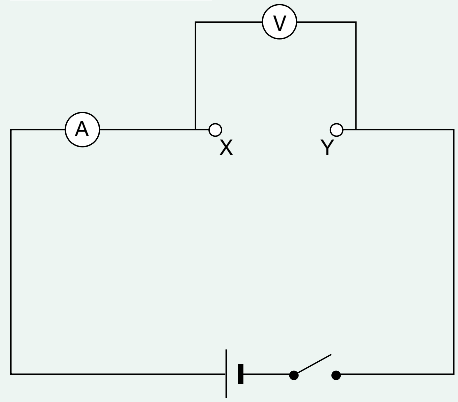
$$R_{series} = R_1 + R_2$$

The total resistance, $R_{parallel}$, of two resistors, R_1 and R_2, connected in parallel, is given by the rule:

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

To measure the total resistance of a combination of resistors, measure the voltage drop, V, across the combination, and the current, I, passing through the combination. The total resistance is given by the formula: $R = \frac{V}{I}$.

FIGURE 13.81



Method

1. Connect the circuit as shown in figure 13.81. Between the points X and Y, connect in turn:
   (a) R_1
   (b) R_2
   (c) R_1 and R_2 in series
   (d) R_1 and R_2 in parallel.
2. In each case adjust the voltage of the power supply and the resistance of the variable resistor so that the current has the value suggested by your teacher.
3. With the switch closed, measure the total current passing from X to Y and the voltage across XY.

Results

Record your results as in the table below.

	V(V)	I(A)	$R = \frac{V}{I}(\Omega)$
R_1			
R_2			
R_1 and R_2 in series			
R_1 and R_2 in parallel			

The third column is calculated from the measurements of voltage and current. This column gives the *experimental* values of the resistances and their series and parallel combinations.

Analysis

1. Using the laws of addition of resistances, calculate the *theoretical* values R_{series} and $R_{parallel}$. (Use the values of R_1 and R_2 found experimentally, not the nominal values written on the resistors.)
2. Compare the theoretical and experimental values by completing the following table.

	Experimental value of resistance (Ω)	Theoretical value of resistance (Ω)
R_1 and R_2 in series		
R_1 and R_2 in parallel		

3. Comment on any differences between the experimental and theoretical values.

TOPIC 14 Magnetism

14.1 Overview

14.1.1 Module 4: Electricity and Magnetism

Magnetism

Inquiry question: How do magnetised and magnetic objects interact?

Students:

- investigate and describe qualitatively the force produced between magnetised and magnetic materials in the context of ferromagnetic materials (ACSPH079)
- use magnetic field lines to model qualitatively the direction and strength of magnetic fields produced by magnets, current-carrying wires and solenoids and relate these fields to their effect on magnetic materials that are placed within them (ACSPH083)
- conduct investigations into and describe quantitatively the magnetic fields produced by wires and solenoids, including: (ACSPH106, ACSPH107)
  - $B = \frac{\mu_o I}{2\pi r}$
  - $B = \frac{\mu_o NI}{L}$
- investigate and explain the process by which ferromagnetic materials become magnetised (ACSPH083)
- apply models to represent qualitatively and describe quantitatively the features of magnetic fields.

FIGURE 14.1 This strong magnet sitting on top of a glass shelf creates a magnetic field that is able to attract small pieces of metal.

14.2 Properties of magnets

14.2.1 Magnetic poles

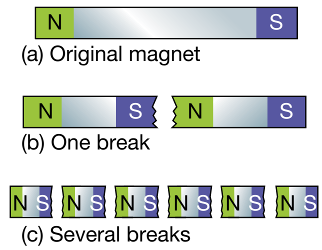
The words *magnet*, *magnetism* and *magnetic* are derived from the name of a district in Greece called Magnesia. By 600 BC, the Greeks had discovered a mineral there, now called magnetite, with the property of attracting iron.

In any sample of the mineral, the property of attracting iron is concentrated in two regions called the **poles**. If the sample of mineral is suspended freely, it will align itself so that one pole points roughly north and the other pole points roughly south. The pole that points north is called the north-seeking pole; the pole that points south is called the south-seeking pole. These names are now abbreviated to **north pole** and **south pole**.

A **natural magnet** is made by shaping a piece of magnetite so that the poles are at the ends. Early compasses were made using natural magnets. Today, natural magnets are not used because better magnets can be made artificially. The method of making these **artificial magnets** will be described later in this chapter.

North and south poles always occur together in equal pairs. Such a pair of equal and opposite magnetic poles is called a *magnetic dipole*. An isolated north or south pole has never been observed. If a magnet is broken in two in an attempt to separate the north and south poles, new south and north poles appear, as shown in figure 14.2.

FIGURE 14.2 Breaking a bar magnet.

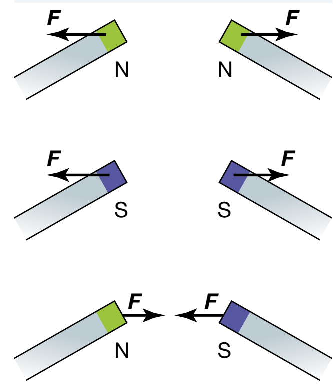


If two magnetic poles are brought close together they exert forces on one another as shown in figure 14.3. The directions of the forces between magnetic poles are:

- two north poles repel each other
- two south poles repel each other
- a north pole and a south pole attract each other.

That is, *like poles repel; unlike poles attract*.

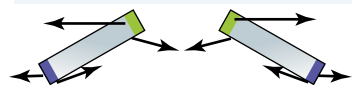
FIGURE 14.3 Forces between magnetic poles.



The closer two magnetic poles are to one another, the stronger the force of attraction or repulsion between them.

When two magnets are brought close to one another, there will be four pairs of forces between the poles, as shown in figure 14.4. This will result in an overall force of attraction or repulsion (depending on the positions of the two magnets).

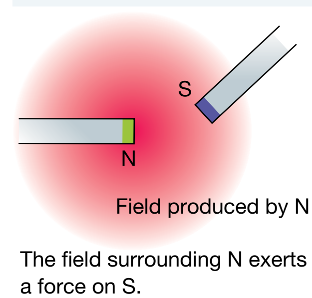
FIGURE 14.4 Forces between two magnets.



14.2.2 Magnetic fields

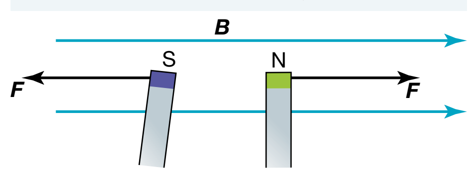
Interactions between magnetic poles can be described by magnetic fields in the same way that interactions between electric charges were described by electric fields. In the field picture of magnetic interactions, each magnetic pole is surrounded by a **magnetic field** that exerts forces on other magnetic poles placed in the field, as shown in figure 14.5.

FIGURE 14.5 Magnetic field picture of magnetic interactions.



When a magnet is placed in a magnetic field, the north and south poles experience forces in opposite directions. The **direction of the magnetic field** at a point is defined as the direction of the force on a very small north pole placed at the point. The directions of the forces on the north and south poles of a magnet placed in a magnetic field are shown in figure 14.6. (In diagrams, a magnetic field is labelled ***B***.)

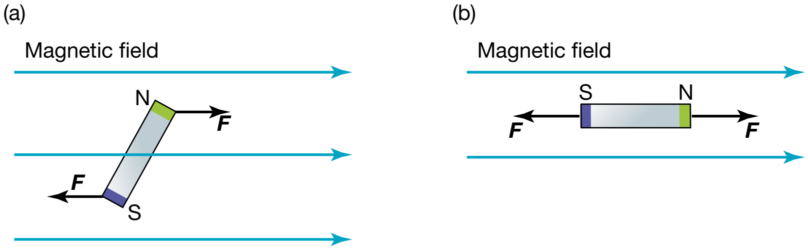
FIGURE 14.6 Forces on the north and south poles in a magnetic field.



A compass consists of a magnet suspended so that it is free to rotate. When a compass is placed in a magnetic field, the forces on the north and south poles cause the compass to rotate until the north pole of the compass points in the direction of the magnetic field.

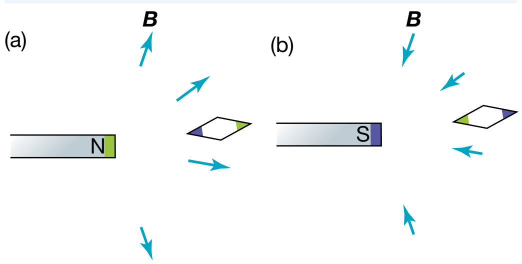
In practice, the direction of a magnetic field at a point is found by placing a small compass at that point. The direction in which the north pole of a compass points shows the direction of the magnetic field.

FIGURE 14.7 Forces on a compass in a magnetic field. (a) Forces causing compass needle to rotate and (b) compass needle aligned with magnetic field.



If a compass is placed at a point near a north pole, N, the north pole of the compass will experience a force away from N, and the south pole of the compass will experience a force towards N. The compass will point in the direction shown in figure 14.8a. The magnetic field therefore points away from N. Similarly, the magnetic field surrounding a south pole points towards the south pole. This is illustrated in figure 14.8b.

FIGURE 14.8 (a) Magnetic field near a north pole. (b) Magnetic field near a south pole.



14.2.3 Representing Magnetic Fields

- Magnetic fields are represented by magnetic field lines.
- Magnetic field lines start at north poles and end at south poles.
- The direction of the magnetic field lines, shows the direction of the magnetic field.
- The spacing of the magnetic field lines shows the strength of the magnetic field: the closer the lines, the stronger the field.

A magnet has equal north and south poles at the ends. The magnetic field surrounding a magnet is shown in figure 14.9.

Consider two identical magnets with their north poles placed close to one another. The magnetic field in the region near the two equal north poles is shown in figure 14.10a. Similarly, the magnetic field near the two equal south poles is shown in figure 14.10b.

FIGURE 14.9 Magnetic field surrounding a magnet.

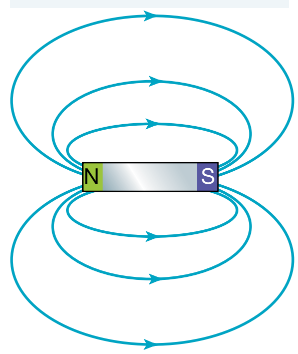
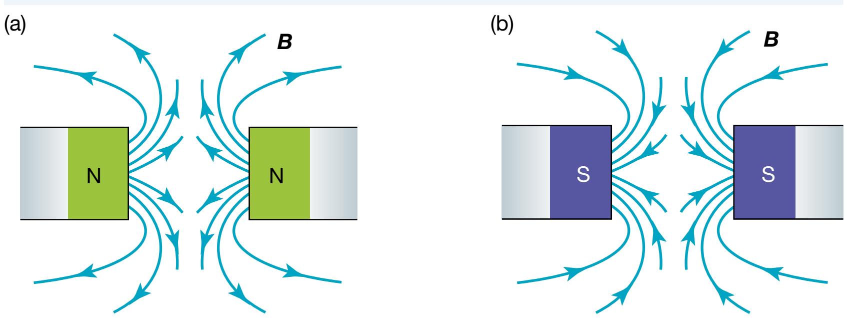
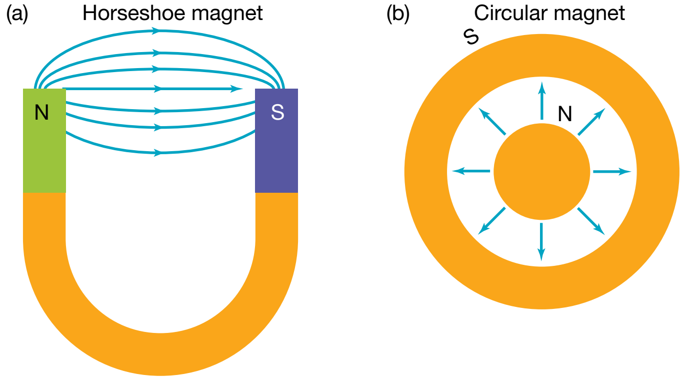


FIGURE 14.10 (a) Magnetic field near two equal north poles. (b) Magnetic field near two equal south poles.



Magnets can be designed to produce fields of different shapes. A horseshoe magnet with the ends adjacent produces a strong and even field between the ends. A circular magnet with a north end in the middle produces a radial field that points outward all the way around. This design is used in loudspeakers.

FIGURE 14.11 Differently shaped magnetic fields can be created by arranging the north and south ends of the magnet, as shown by (a) a horseshoe magnet and (b) a circular magnet.



14.2.4 Measuring the magnetic field

The strength of the magnetic field around a magnetic material varies according to where it is measured. The magnetic field can be qualitatively represented by the separation of the field lines. Where field lines are shown close together — such as at the poles of a magnet — the magnetic field is stronger than at positions where the field lines are shown spread further apart. Generally, the influence of a magnetic material decreases with distance. This is readily seen with a magnet and iron filings, where the attraction of the filings to the magnet increases as the magnet gets closer to them.

The magnetic field strength (B) at a position is measured quantitatively in terms of the amount of twisting force exerted on a compass needle when it is positioned at an angle to the magnetic field. The greater the force twisting the compass needle so that it aligns with the magnetic field, the greater the magnetic field strength. Magnetic field strength is a vector quantity and it is measured in Tesla (T). The Earth's magnetic field strength is fairly small: only 10^{-4} T or 0.1 mT. In comparison, the average fridge magnet has a magnetic field strength of about 30 mT, while a typical bar magnet used in the school lab is around 0.1 T.

AS A MATTER OF FACT

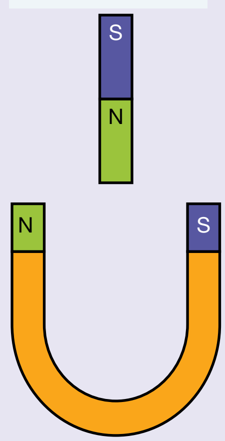
The strength of a magnetic field 1.0 cm from a wire carrying 100 A is about 2.0 m T. The small currents in the nerves of the human body produce magnetic fields of about 10^{-11} T. Electromagnets used in research have a short-term strength of about 70 T, which requires a momentary current of 15 000 A.

The magnetic field around the human heart is about 5×10^{-11} T, about one millionth of Earth's magnetic field. To measure fields of this size, it is necessary to use a magnetically shielded room and a very sensitive detector called a SQUID (a Superconducting Quantum Interference Device) that can measure fields down to 10^{-14} T. The magnetocardiogram produced is a useful diagnostic tool.

14.2 Exercise 1

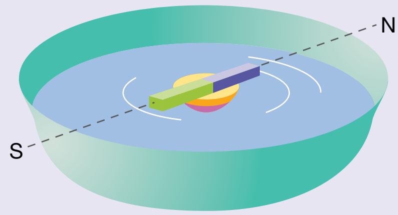
1 If north poles repel, why does the north pole of a compass point to the Earth's north pole?
2 Draw field lines to represent the magnetic field around
   (a) a single bar magnet
   (b) two bar magnets with their south poles facing each other.
3 Draw field lines to represent the magnetic field for the diagram in figure 14.12.
4 Early Chinese explorers used a system of floating a piece of magnetite in a bowl of water, as shown in figure 14.13, for navigation purposes.
   (a) Why was the magnetite placed in water rather than left on a flat surface?
   (b) Give one disadvantage of this compass system when used at sea.
5 When a compass is used, the needle may be seen to tilt downwards rather than staying parallel. Explain why this happens.

FIGURE 14.12



6 How would you use a magnet to test whether a piece of metal is magnetic?
7 Why do both ends of a magnet attract an iron nail?
8 What is the polarity of Earth's magnetic field at the magnetic pole in the southern hemisphere?

FIGURE 14.13



14.3 Magnetic fields and electric currents

14.3.1 Magnetic fields produced by a wire

Because the forces between magnetic poles are similar to the forces between electric charges, many early scientists suspected that there was a connection between magnetism and electricity.

In 1821, a Danish scientist, Hans Christian Oersted, while demonstrating to friends the flow of an electric current in a wire, noticed that the current caused a nearby compass needle to change direction. Oersted's observation showed that there was a magnetic field surrounding the electric current. Further investigation showed that all electric currents are surrounded by magnetic fields.

The magnetic field surrounding a wire carrying an electric current depends, in a complex way, on the shape of the wire. In this course, two important cases will be studied where the field produced by the current is comparatively simple.

The magnetic field lines surrounding a long, straight wire carrying a current are concentric circles around the conductor.

The direction of the magnetic field is given by the **right-hand grip rule**. This states:

Grip the wire with the right hand, with the thumb pointing in the direction of the conventional current and the fingers will curl around the wire in the direction of the magnetic field.

In drawing a diagram to represent the magnetic field surrounding a wire carrying a current, it is often convenient to imagine the wire being perpendicular to the page. In such a diagram, the wire is represented by a small circle at the point where the wire passes through the page. The direction of the current will be *into the page* or *out of the page*. Figure 14.18 shows the magnetic fields surrounding electric currents passing into and out of the page.

FIGURE 14.14 Oersted's experiment.

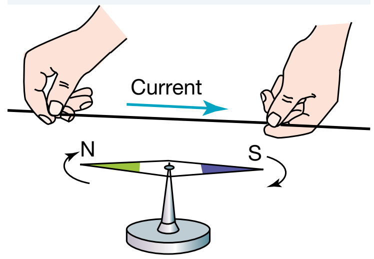


FIGURE 14.15 Hans Christian Oersted (1851).

FIGURE 14.16 Magnetic field surrounding a long, straight wire carrying a current.

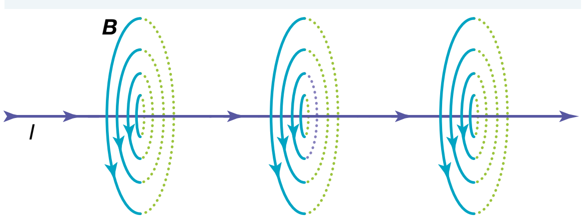
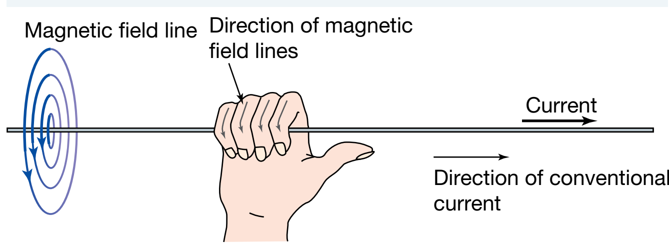


FIGURE 14.17 Right-hand grip rule.



Magnetic fields directed into and out of the page are represented by the symbols 'x' and '•'. Figure 14.19 shows the magnetic field at the sides of a wire carrying a conventional current towards the right of the page. By the right-hand grip rule, the magnetic field comes out of the page above the wire and goes into the page below the wire.

FIGURE 14.18 Magnetic fields surrounding currents passing into and out of the page.

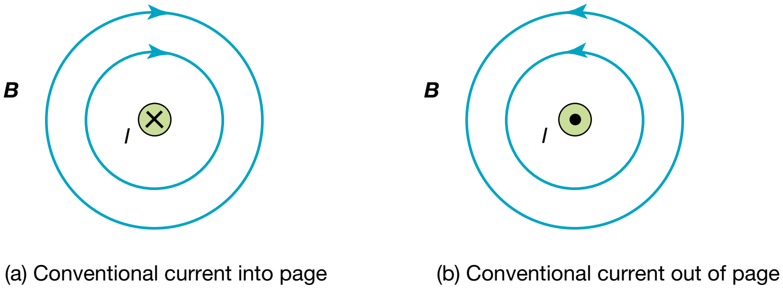
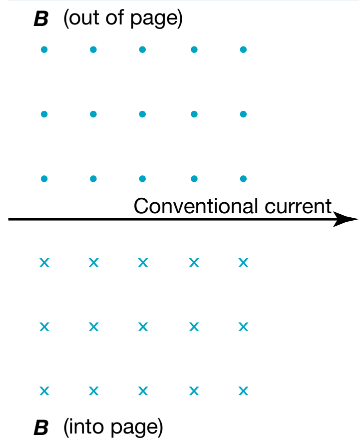


FIGURE 14.19 Magnetic fields into and out of the page.



14.3.2 Magnetic field strength around a current-carrying wire

The strength of the magnetic field produced by a long straight wire through which a current flows depends upon the size of the current and the distance from the wire at which the field is being measured. This relationship can be described by the equation:

$$B = \frac{\mu_o}{2\pi}\frac{I}{r}$$

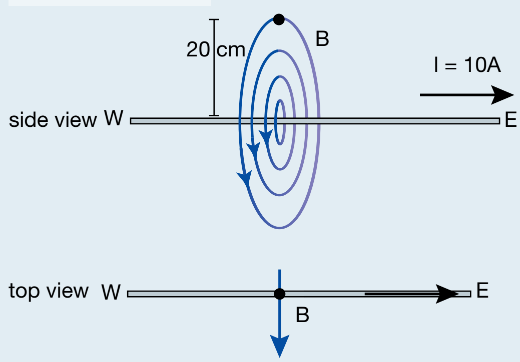
where I is the current through the wire, r is the perpendicular distance from the wire and μ_o is a constant referred to as the **permeability of free space**. The value of μ_o is taken to be equal to $4\pi \times 10^{-7}$ T m A^{-1}.

It should be noted that this equation assumes that r is much smaller than the distance to the ends of the wire.

14.3 SAMPLE PROBLEM 1

A long straight wire carries a current of 10 A from west to east. What will be the magnetic field strength at a point 20 cm above the wire?

FIGURE 14.20



SOLUTION

By using the right-hand grip rule, we can see that, if the current is travelling west to east, the direction of the magnetic field above the wire will be directed from north to south.

To calculate the magnitude of the field, we substitute values into the wire equation:

$$\begin{aligned} B &= \frac{\mu_o}{2\pi}\frac{I}{r} \\ &= \frac{4\pi \times 10^{-7}\ \text{T m}\ A^{-1} \times (10\ \text{A})}{2\pi \times (0.2\ \text{m})} \\ &= 1 \times 10^{-5}\ \text{T} \end{aligned}$$

Therefore, the magnetic field strength is 1×10^{-5} T, south.

14.3.3 Magnetic field produced by a solenoid carrying a current

If a straight wire carrying a current is bent into a loop, the magnetic field is as shown in figure 14.21. The magnetic field lines come out at one side of the loop, which is therefore like the north pole of a magnet. The magnetic field lines go in to the other side of the loop, which is therefore like the south pole of a magnet. The right-hand grip rule, applied to a section of the loop, gives the direction of the magnetic field.

Magnetic fields produced by coils have many more practical applications than magnetic fields produced by straight wires.

The strength of the magnetic field can be increased by using a coil with many turns.

A **solenoid** is a wire that has been wound into a closely packed helix (corkscrew shape). When a current passes through a solenoid, magnetic fields are produced both inside and outside the solenoid. The magnetic field outside the solenoid is similar to the magnetic field surrounding a bar magnet. For the solenoid, however, the lines of magnetic field do not stop at the ends of the solenoid but pass through the inside as parallel lines. The lines of magnetic field form closed loops. The end where the lines of magnetic field emerge from the solenoid is the north pole. The end where the lines of magnetic field enter the solenoid is the south pole. There are two methods of determining which end of a solenoid is the north pole.

1. Grip the solenoid with the right hand, with the fingers pointing in the direction of the conventional current around the solenoid. The thumb will point in the direction of the north pole of the solenoid as shown in figure 14.23.
2. Observe the solenoid end on. If the direction of the conventional current is anti-clockwise, the end is a north pole; if the conventional current is clockwise, the end is a south pole. This can be remembered by writing an N or an S with arrows as shown in figure 14.24.

FIGURE 14.21 Magnetic field produced by a current loop.

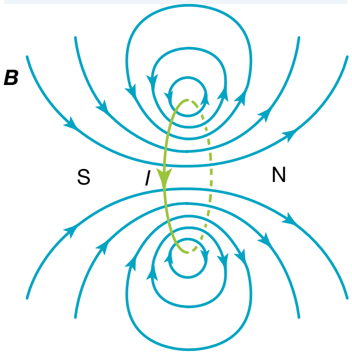


FIGURE 14.22 Magnetic fields produced by a current in a solenoid.

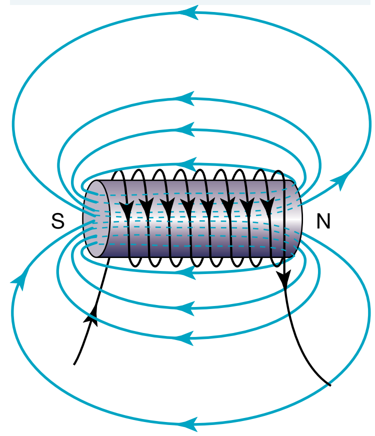


FIGURE 14.23 Poles of a solenoid — first method.

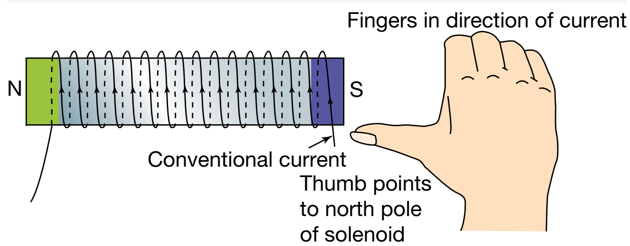
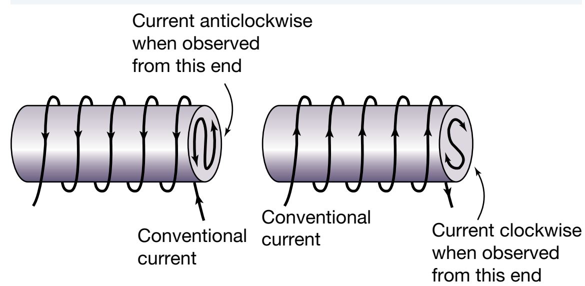


FIGURE 14.24 Poles of a solenoid — second method.



14.3.4 Magnetic field inside a solenoid

The strength of the magnetic field inside a solenoid is directly proportional to the number of coils the solenoid has and the size of the current flowing through them, and is inversely proportional to the solenoid's length. The relationship between these variables is described by the equation:

$$B = \frac{\mu_o N I}{L}$$

where N is the number of loops or coils in the solenoid, I is the current, L is the solenoid length and μ_o is the permeability of free space.

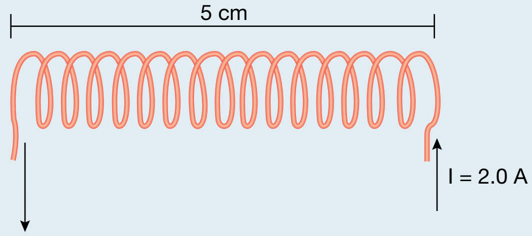
As B is uniform inside the solenoid, the field strength is independent of the position.

14.3 SAMPLE PROBLEM 2

A 2.0 A current flows through a 5 cm long thin solenoid as shown.

What is the magnetic field strength in the solenoid's interior if it has 500 turns of wire?

FIGURE 14.25



SOLUTION

Using the right-hand rule for coils, it can be determined that the north pole of the solenoid will be on the right side of the coil. This means that the magnetic field inside the coil will be directed from left to right.

Substituting values into the coil equation:

$$\begin{aligned} B &= \frac{\mu_0 N I}{L} \\ &= \frac{(4\pi \times 10^{-7}\,\text{T m A}^{-1})\ (500)\ (2.0\ \text{A})}{(0.05\ \text{m})} \\ &= 0.025\ \text{T} \end{aligned}$$

Therefore, the magnetic field strength inside the coil is 25 mT, directed from left to right.

14.3 Exercise 1

1 A magnetic field is set up in a solenoid. What would happen to the magnetic field strength inside the solenoid if:

(a) the current was doubled

(b) the solenoid was stretched so that its coils were further apart

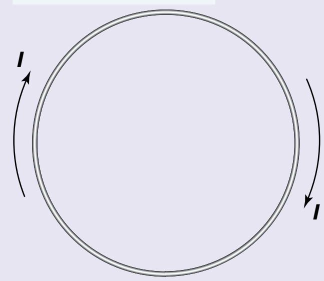
(c) the direction of the current through the coil was reversed?

2 The equation for the magnetic field strength around a current-carrying wire may also be encountered in the form $B = k\frac{I}{r}$.

Calculate the value of k.

3 A loop of wire has a current flowing through it as shown in figure 14.26. Use the symbols ⊗ and • to represent the direction of the magnetic field inside and outside the loop.

FIGURE 14.26



4 A vertical wire attached to a wall carries a current of 4.0 A upwards. What is the magnitude and direction of the magnetic field at a point 50 cm in front of the wire?

5 A magnetic field of 0.3 mT is measured inside a solenoid that has a current of 5.0 A passing through it. How many coils per metre does the solenoid have?

6 Two 3 m long, current-carrying wires are positioned vertically as shown in figure 14.27.

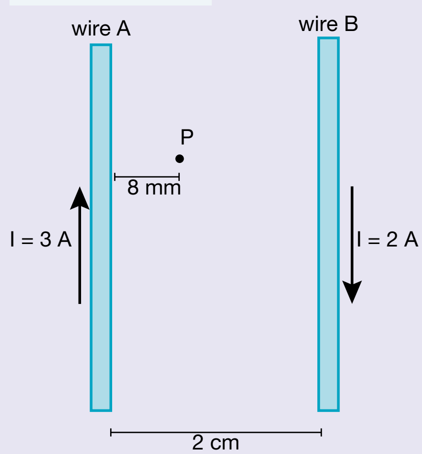
(a) What is the magnetic field strength due to wire A at point P?

(b) What is the magnetic field strength due to wire B at P?

(c) What is the net magnetic field strength experienced at point P?

(d) The direction of the current in wire A is reversed. What is the magnetic field strength at P now?

FIGURE 14.27



7 A 40 cm long, thin solenoid has 2000 loops of wire along its length. What is the magnitude of the magnetic field inside the solenoid if a current of 10 A passes through the coils?

8 A 30 cm long solenoid with a 2 cm diameter is to produce a 0.2 T magnetic field in its centre. If the maximum current is 8 A, how many turns must the solenoid have? (Assume that each coil is circular rather than helical.)
9 A solenoid with a radius of 1 cm has a magnetic field of 0.1 mT inside it directed from east to west. A long, straight wire is placed inside the solenoid along the solenoid's central axis. If a current of 10 A is passed through the wire from west to east, what will be the magnitude of the magnetic field experienced at a distance of 4 mm from the wire?
10 If a maximum magnetic field strength of 1 mT is allowed at a distance of 30 cm from an electrical wire, what is the maximum current that the wire can carry?

14.4 Magnets and electromagnets

14.4.1 Ferromagnetism

It is now thought that *all* magnetic fields (including the magnetic fields of magnets) are produced by electric currents. This raises the question: 'Where is the electric current that produces the magnetic field of a magnet?'.

Within every atom there are electric currents due to the movement of the electrons. Each electron moves in an orbit around the nucleus and spins on its axis. (This picture of the motion of an electron in an atom is much simpler than what physicists believe to be the actual situation.)

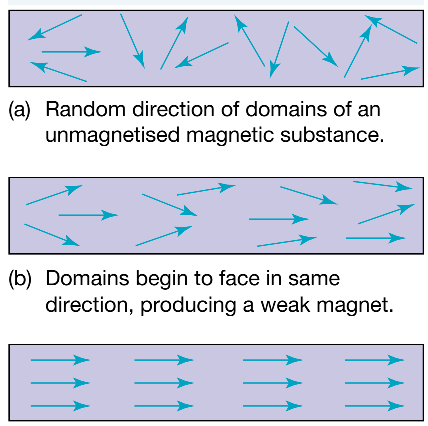
As a result of the movement of its electrons, each atom behaves like a current loop with a north and south pole. The magnetic properties of a material are due to the magnetic properties of its atoms.

As all materials consist of atoms, it follows that all materials will have magnetic properties. For most materials, the magnetism is very weak and cannot be detected without using very strong magnetic fields. In a few materials, the magnetism is strong. In these materials, the spins of the electrons producing the atomic magnetism tend to line up in neighbouring atoms. As a result, the north and south poles of the atomic magnets tend to line up and point in the same direction producing a strong magnetic field. These materials are called ferromagnetic materials. Examples of ferromagnetic materials are iron, cobalt and nickel. Magnets are made of ferromagnetic materials.

A ferromagnetic material is made up of many magnetic domains. Within each domain nearly all the atomic magnets are lined up in the same direction. Each domain contains 10^9–10^{15} atoms. When a ferromagnetic material is unmagnetised, the domains point in random directions so that there is no overall magnetism.

When the ferromagnetic material is placed in a magnetic field, the directions of magnetism of the domains tend to line up in the direction of the magnetic field. When all the domains are lined up with the magnetic field, the material is fully magnetised, or saturated. Figure 14.28 shows the magnetic domains in an unmagnetised, a partially magnetised and a fully magnetised sample of ferromagnetic material.

FIGURE 14.28 Magnetic domains.



(a) Random direction of domains of an unmagnetised magnetic substance.

(b) Domains begin to face in same direction, producing a weak magnet.

(c) All domains face the same direction, resulting in a strong magnet.

14.4.2 Temporary and permanent magnets

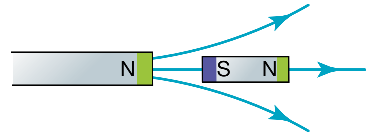
When a magnetic material, such as iron, is placed in a magnetic field, it becomes magnetised. 'Soft iron' is a type of iron that becomes magnetised very quickly when placed in a magnetic field, and loses its magnetism very quickly when removed from the field. When soft iron is magnetised by being placed in a magnetic field, it is said to be a **temporary magnet**.

A piece of iron is attracted to a magnetic pole because it becomes magnetised by the magnetic field surrounding the pole.

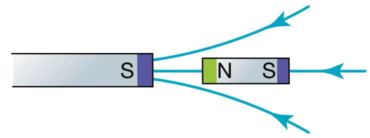
'Hard iron' is used to refer to any alloy of iron that becomes magnetised slowly when placed in a magnetic field but retains its magnetism for a long time after it is removed from the field. Such an alloy is used to make a **permanent magnet**. To make a permanent magnet, a bar of hard iron is placed inside a solenoid, and a current is passed through the solenoid for a sufficient time to magnetise the iron.

A solenoid with a soft iron core is called an **electromagnet**. A current through the coil produces a magnetic field that magnetises the soft iron core almost instantaneously. This produces a much stronger magnet than would be produced by the solenoid without the soft iron core. When the current is switched off, the soft iron core loses its magnetism almost instantaneously.

FIGURE 14.29 Attraction of iron to (a) a north pole and (b) a south pole.

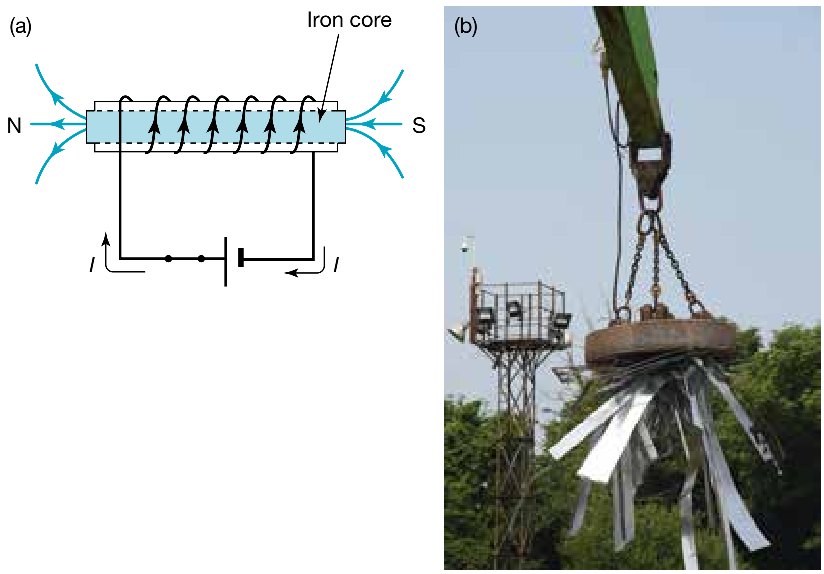


(a) Iron becomes magnetised and is attracted to the north pole.



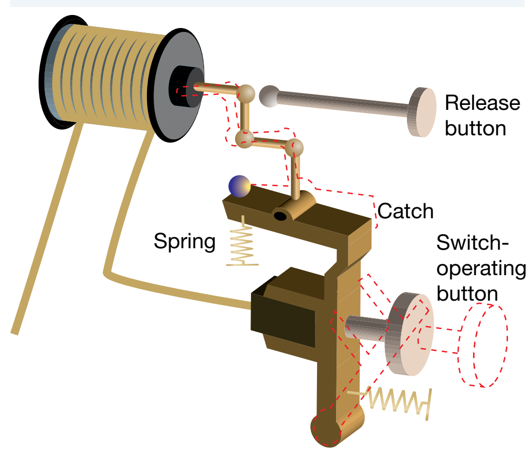
(b) Iron becomes magnetised and is attracted to the south pole.

FIGURE 14.30 (a) An electromagnet (b) An electromagnet in action.



Electromagnets have many practical applications. Figure 14.31 shows a circuit breaker that uses an electromagnet. If a sufficiently high current passes through the solenoid, the electromagnet will trip the mechanism and break the circuit.

FIGURE 14.31 A circuit breaker.



14.4 Exercise 1

1 Explain in terms of domains how a magnetic material can be 'demagnetised' when it is heated.
2 Which of the following materials can be described as ferromagnetic:
   (a) aluminium
   (b) stainless steel
   (c) copper
   (d) nickel?
3 What is the function of the coil in an electromagnet?
4 Explain why iron is used as the core of an electromagnet.
5 How could naturally occurring magnets have been formed?

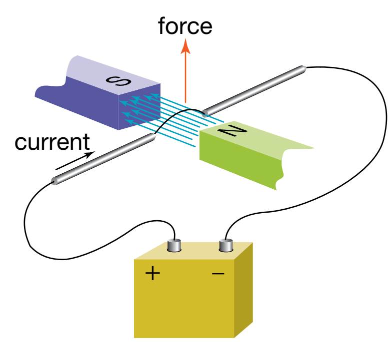
14.5 Magnetic force

14.5.1 Magnetic force on an electric current

Once the technology of electromagnets was developed, very strong magnetic fields could be achieved. This enabled the reverse of Oersted's discovery to be investigated: what is the effect of a magnetic field on a current in a wire?

In Oersted's experiment the magnetic field due to the current exerts a force on the magnetic field of the compass. So, according to Newton's Third Law of Motion, the compass exerts an equal and opposite force on the current. What is the size of this force and in what direction does it act?

FIGURE 14.32 The magnetic field exerts a magnetic force on a current-carrying wire.



Observations of the magnetic force applied to the current-carrying wire show that:

- if the strength of the magnetic field increases, there is a larger force on the wire
- if the magnetic field acts on a larger current in the wire, there is a larger force
- if the magnetic field acts on a longer wire, there is a larger force
- it is only the component of the magnetic field that is perpendicular to the current that causes the force
- if there are more wires in the magnetic field, there is a larger force.

Combined, these findings can be expressed as:

magnetic force on a current (***F***) = number of wires (n) × current in each wire (I) × length of wire (***l***) × strength of the magnetic field (***B***),

or

$$\boldsymbol{F} = n \times I \times \boldsymbol{l} \times \boldsymbol{B}.$$

The units are expressed as:

1 newton = 1 × 1 ampere × 1 metre × 1 tesla.

When the magnetic field is perpendicular to the direction of the current (and hence the length vector) in a single wire, the magnitude of the force is given by:

$$F = IlB.$$

When the magnetic field is not perpendicular to the direction of the current, it is important to remember that the force on the wire is less. In fact, if the magnetic field is parallel to the direction of the current, the force on the wire is zero. That is because the component of magnetic field perpendicular to the current is zero.

14.5.1 SAMPLE PROBLEM 1

If a straight wire of length 8.0 cm carries a current of 300 mA, calculate the magnitude of the force acting on it when it is in a magnetic field of strength 0.25 T if:
(a) the wire is at right angles to the field
(b) the wire is parallel with the field.

SOLUTION:

(a) The magnetic field is perpendicular to the direction of current.

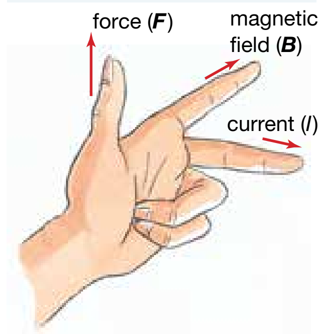
$$\begin{aligned} F &= IlB \\ &= 3.00 \times 10^{-1}\,\text{A} \times 8.0 \times 10^{-2}\,\text{m} \times 0.25\,\text{T} \\ &= 6.0 \times 10^{-3}\,\text{N} \end{aligned}$$

(b) The magnetic field is parallel to the direction of current. Therefore the component of magnetic field that is perpendicular to the current is zero.

$$\begin{aligned} F &= IlB \\ &= 3.00 \times 10^{-1}\,\text{A} \times 0\,\text{m} \times 0.25\,\text{T} \\ &= 0 \end{aligned}$$

If the magnetic field is pointing to the right across the page, and the current is going down the page, the direction of the magnetic force is up, out of the page. The direction of this force will be important in applications such as meters and motors, so it is necessary to have a rule to determine the direction of the force in a variety of situations. There are two alternative hand rules commonly used. These are described below.

FIGURE 14.33 Left-hand rule for determining the direction of the magnetic force of a magnetic field on a current.



Left-hand rule

The left-hand rule applies as follows:
- the index finger, pointing straight ahead, represents the magnetic field (***B***)
- the middle finger, at right angles to the index finger, represents the current (***I***)
- the thumb, upright at right angles to both fingers, represents the force (***F***).

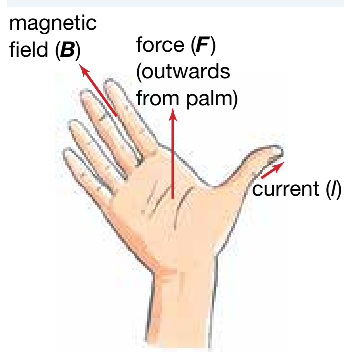
Lock the three fingers in place so they are at right angles to each other. Now rotate your hand so that the field and current line up with the directions in your problem. The thumb will now point in the direction of the force.

Right-hand-slap rule

The right-hand-slap rule applies as follows:
- the fingers (out straight) represent the magnetic field (***B***)
- the thumb (out to the side of the hand) represents the current (***I***)
- the palm of the hand represents the force (***F***).

Hold your hand flat with the fingers outstretched and the thumb out to the side, at right angles to your fingers. Now rotate your hand so that the field and current line up with the direction in your problem. The palm of your hand now gives the direction of the force, hence the name.

FIGURE 14.34 Right-hand-slap rule for determining the direction of the magnetic force of a magnetic field on a current.



14.5.2 Magnetic force on charges

Electric current consists of electrons moving in a wire. A magnetic field acts on the electrons and pushes them sideways. This force then pushes the nuclei in the wire, and the wire moves. If the moving electrons were in a vacuum, free of the wire, the magnetic field would still exert a force at right angles to their velocity. What would be the effect of this force on a freely moving electron?

When an electron is moving across a magnetic field, it experiences a sideways force, which deflects the movement of the electron. The electron now moves in another direction given by the hand rule; however, it is still moving at right angles to the magnetic field, so the strength of the force is unchanged. The direction of the force will again be at right angles to the electron's motion, and deflecting it again. The deflecting force on the moving electron will be constant in size and will always be at right angles to its velocity. This results in the electron travelling in a circle.

The magnetic force is always at right angles to the direction of the charge's motion. So the magnetic force cannot increase the speed on the charge; it can only change its direction at a constant rate.

The mass spectrometer, the electron microscope and the synchrotron are instruments that use a magnetic force in this manner.

So what is the radius of the circle? How does it depend on the strength of the magnetic field, the speed of the charge and size of the charge?

The magnitude of the magnetic force on a current-carrying wire is given by:

$$F = IlB \qquad (1)$$

Imagine a single charge, q, travelling along at speed, v. The charge travels through a distance, or length, in a time of t seconds given by:

$$l = vt \qquad (2)$$

The electric current is given by:

$$I = \frac{q}{t} \qquad (3)$$

Substituting equations (2) and (3) into (1):

$$F = \frac{q}{t} \times vt \times B$$

$$\Rightarrow F = qvB$$

Does this relationship make sense?

What do we observe?	What does the formula predict?	Match
If the charge is stationary, the current is zero, so no force.	If $v = 0$, then $F = 0$.	Yes
A stronger magnetic field will deflect the charge more.	Force is proportional to the field.	Yes

The magnitude of the net force on the charged particle as it moves in the magnetic field is:

$$F_{net} = ma.$$

In this case the only significant force is the magnetic force, $F = qvB$.

$$\Rightarrow qvB = ma$$

Because the acceleration is centripetal and constant in magnitude, its magnitude can be expressed as $a = \frac{v^2}{r}$, where r is the radius of the circular motion.

$$\Rightarrow qvB = \frac{mv^2}{r}$$

The expression for the radius is therefore:

$$r = \frac{mv}{Bq}.$$

Does this relationship make sense?

What do we observe?	What does the formula predict?	Match
Hard to turn heavy objects	The heavier the mass, the larger the radius	Yes
Hard to turn fast objects	The faster the object, the larger the radius	Yes
The larger the force, the smaller the radius	The stronger the field, the smaller the radius; the larger the charge, the smaller the radius	Yes

Note that because the direction of the magnetic field is always at right angles to the direction in which the charged particles are moving, the magnetic field cannot make the particles go faster — it can only change their direction. In this context, magnetic fields are not 'particle accelerators'.

14.5.1 SAMPLE PROBLEM 2

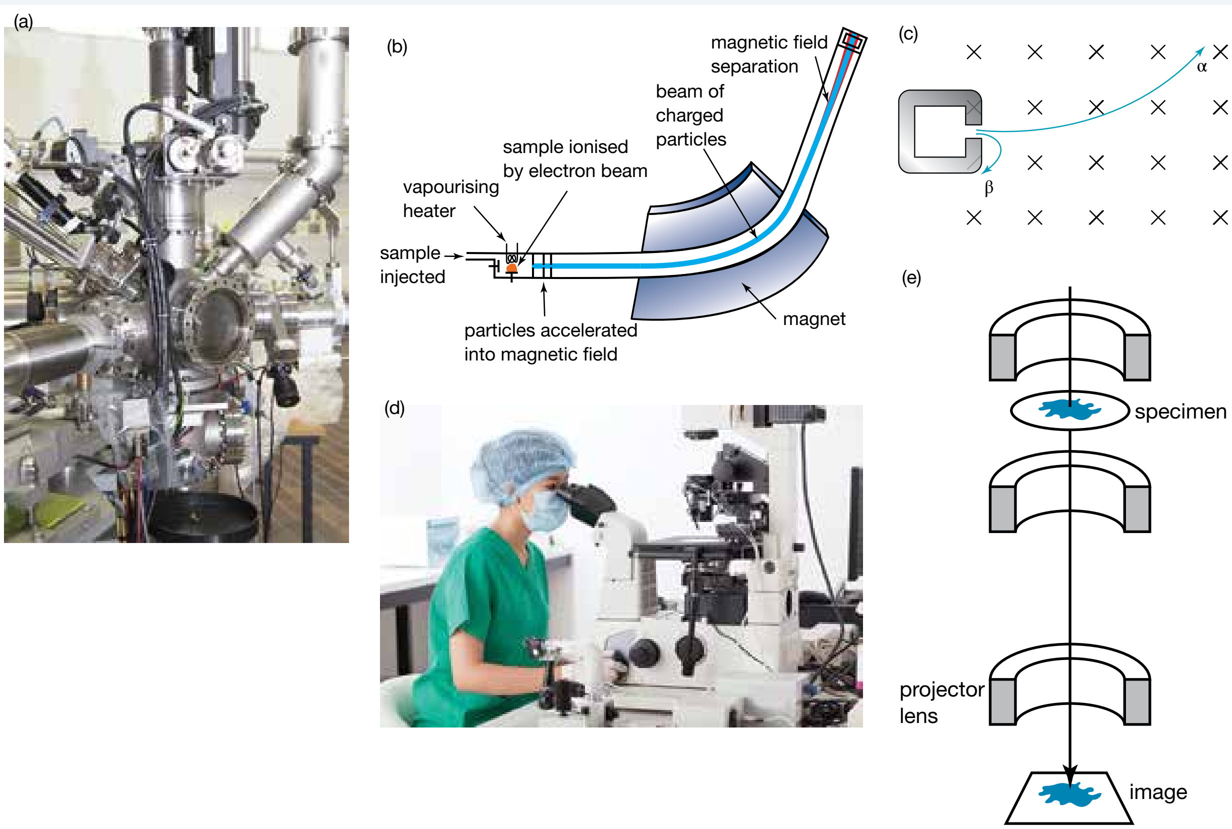
An electron travelling at $5.9 \times 10^6\,\mathrm{m\,s^{-1}}$ enters a magnetic field of 6.0 mT. What is the radius of its path?

SOLUTION:

$m = 9.1 \times 10^{-31}\,\mathrm{kg}$, $q = 1.6 \times 10^{-19}\,\mathrm{C}$, $v = 5.9 \times 10^6\,\mathrm{m\,s^{-1}}$, $B = 6.0\,\mathrm{mT}$

$$\begin{aligned} r &= \frac{mv}{Bq} \\ &= \frac{9.1 \times 10^{-31}\,\mathrm{kg} \times 5.9 \times 10^6\,\mathrm{m\,s^{-1}}}{6.0 \times 10^{-3}\,\mathrm{T} \times 1.6 \times 10^{-19}\,\mathrm{C}} \\ &= 5.6 \times 10^{-3}\,\mathrm{m} = 5.6\,\mathrm{mm} \end{aligned}$$

FIGURE 14.35 (a) and (b) A mass spectrometer. (c) Positive alpha particles are deflected up and beta particles are deflected down. (d) and (e) An electron microscope.



AS A MATTER OF FACT

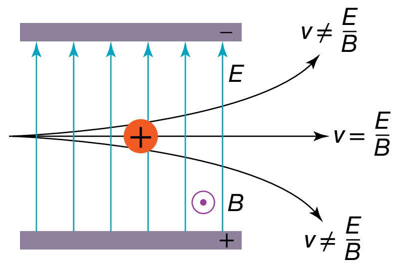
What happens to a stationary electron in a magnetic field? Surprisingly, there is no force! The electron is not moving, so there is, in effect, no current, and therefore no magnetic force. Similarly, the faster the electron moves, the stronger the force. This is a strange situation — that the size of a force on an object is determined by how fast that object is travelling. This raises an interesting conundrum: if you were sitting on an electron moving through a magnetic field, what would you observe? This question can only be resolved by Einstein's Special Theory of Relativity.

14.5.3 Crossed electric and magnetic fields

For mass spectrometers and electron microscopes to work, the charged particles all need to be travelling at the same speed. This is because the radius of the path in a magnetic field for a particle with a given charge and mass depends on the particle's speed.

In 1898, Wilhelm Wien (after whom Wien's Law in thermodynamics is named) was investigating the charged particles that are produced when electricity is passed through gases. To investigate their speed and their charge, he set up a magnetic field to deflect the beam of charged particles in one direction, and an electric field to deflect the beam in the opposite direction. For the charged particles that were undeflected, the magnetic force must have been balanced by the electric force.

FIGURE 14.36 A Wien filter (also known as a velocity selector).



The electric force on a charge in an electric field is $\boldsymbol{F} = q\boldsymbol{E}$, and the magnetic force on a moving charge is $\boldsymbol{F} = qv\boldsymbol{B}$. Equating these formulae gives

$$q\boldsymbol{E} = qv\boldsymbol{B}$$

and cancelling q gives

$$v = \frac{\boldsymbol{E}}{\boldsymbol{B}}$$

This configuration is now called a Wien filter.

AS A MATTER OF FACT

The aurorae at the North Pole and South Pole are glorious displays of waves of coloured light high in the atmosphere. They are produced when charged particles ejected by the Sun enter Earth's magnetic field. The particles spiral down to the pole, producing an amazing display of light as they move in smaller and smaller circles from the increasing magnetic field.

FIGURE 14.37 Charged particles entering Earth's magnetic field.

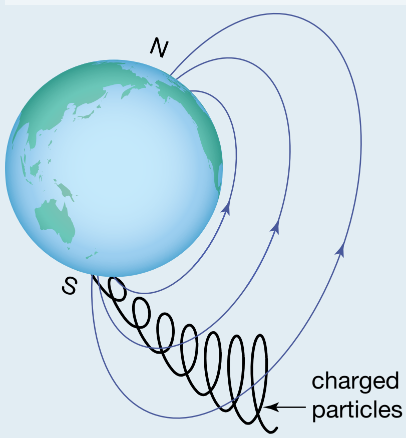


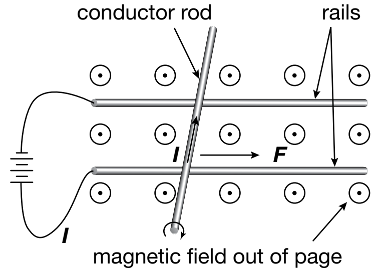
FIGURE 14.38 Aurora Australis, seen from the International Space Station.

14.5.4 Harnessing magnetic forces

Magnetic propulsion

When a current flows along the closest rail (the lower of the two rails in figure 14.39), through the conductor rod and back to the power supply, the conductor will experience a force to the right due to the magnetic field. This force will make the conductor accelerate. If there is little friction, it can move at high speeds.

FIGURE 14.39 A metal conductor rod rolling along two rails.

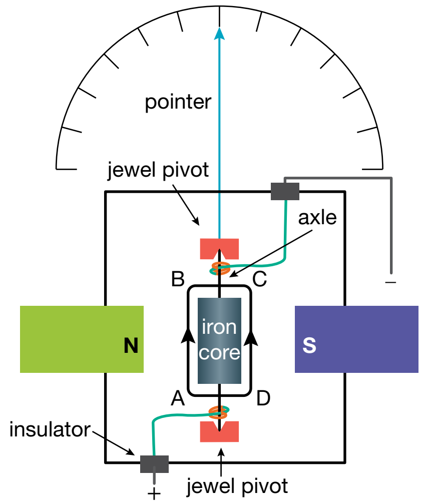


Meters

In the electrical meter illustrated in figure 14.40, the force on the wire BA is out of the page. The current travels around to D and then to C, so the force on wire DC is into the page. The two forces are the same size because the strength of the magnetic field is the same on both sides of the coil, the current through the coil is the same at all points and the lengths BA and DC are the same. However, the forces are in opposite directions. The net force is therefore zero. However, the forces do not act through the centre of the coil, so the combined forces have a turning effect. The turning effect of the forces is called a **torque**. The magnitude of the torque on a coil is the product of the force applied perpendicular to the plane of the coil and the distance between the line of action of the force and the shaft or axle.

If a spring is attached to the axle, the turning effect of the forces unwinds the spring until the spring pushes with an equal torque. A pointer attached to the axle measures the size of the torque, which depends on the size of the current. The larger the current through the meter, the larger the magnetic force and torque on the coil and the further the spring and the pointer are pushed back to achieve balance. Spiral springs have the fortunate property that the deflection of the pointer is proportional to the torque. This means that the scale on the meter can be linear, or evenly spaced.

FIGURE 14.40 An electrical meter.



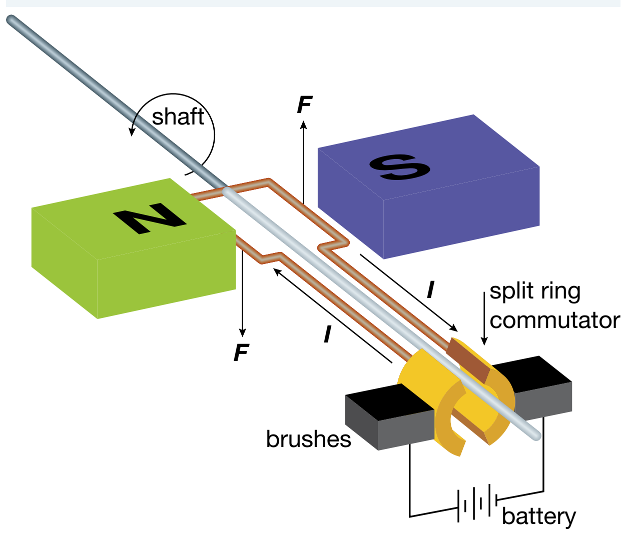
DC motors

A DC motor (a simplified example of which is given in figure 14.41) uses the current from a battery flowing through a coil in a magnetic field to produce continuous rotation of a shaft. How is this done?

A first attempt at a design might be to remove the restoring spring that is used in a meter.

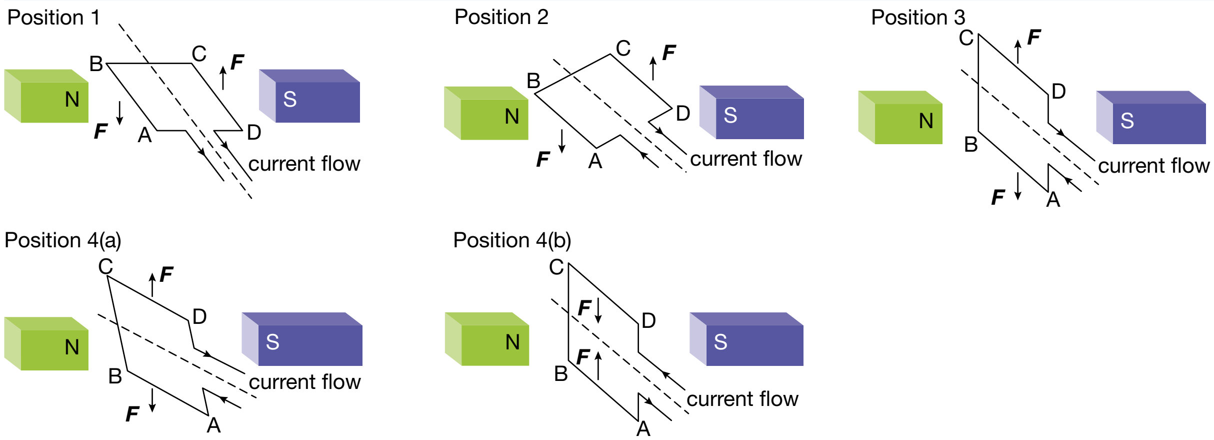
When a coil is in position 1 (as shown in the top left figure in figure 14.42), the forces will make it rotate. As the coil rotates (position 2), the forces remain unchanged in size and direction. This is because the magnetic field and the current in the wire are still the same size and in the same direction. However, their lines of action are closer to the axle, so they have less turning effect. When

FIGURE 14.41 A simplified DC motor.



the coil reaches position 3, at right angles to the magnetic field, the forces are still unchanged in size and direction, but in this case the lines of action of the forces pass through the axle and have no turning effect. Since the coil was already moving before it got to position 3, the momentum of its rotation will carry it beyond position 3 to position 4(a). In position 4(a), the current is still travelling in the same direction, so in this position the forces will act to bring the coil back to position 3.

FIGURE 14.42 Force on a coil in a DC motor.



If this was the design of a DC motor, the coil would turn 90° and then stop! If the coil was in position 3 when the battery was first connected, the coil would not even move.

So, if the motor is to continue to turn, it needs to be modified when the coil reaches position 3. If the direction of the forces can be reversed at this point, as shown in position 4(b), the forces will make the coil continue to turn for another 180°. The coil will then be in the opposite position to that shown for position 3. The current is again reversed to complete the rotation.

The current needs to be reversed twice every rotation when the coil is at right angles to the magnetic field.

This reversal is done with a **commutator**. The commutator consists of two semicircular metal pieces attached to the axle, with a small insulating space between their ends. The ends of the coil are soldered to these metal pieces.

Wires from the battery rest against the commutator pieces. As the axle turns, these pieces turn under the battery contacts, called brushes. This enables the current through the coil to change direction every time the insulating spaces pass the contacts.

Brushes are often small carbon blocks that allow charge to flow and the axle to turn smoothly.

A DC motor is a device used to turn electrical energy into kinetic energy, usually rotational kinetic energy. As an energy transfer device of some industrial significance, there are some important questions to be asked about the design for a DC motor. Are there some starting positions of the coil that won't produce rotation? How can this be overcome? Can it run backwards and forwards? Can it run at different speeds?

FIGURE 14.43 Commutator and coil from a hair dryer.

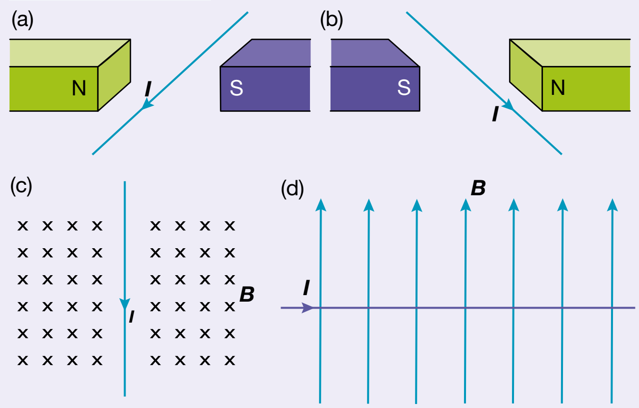
AS A MATTER OF FACT

The principle of the electric motor was proposed by Michael Faraday in 1821, but a useful commercial motor was not designed until 1873. Direct current (DC) motors were installed in trains in Europe in the 1880s.

14.5 Exercise 1

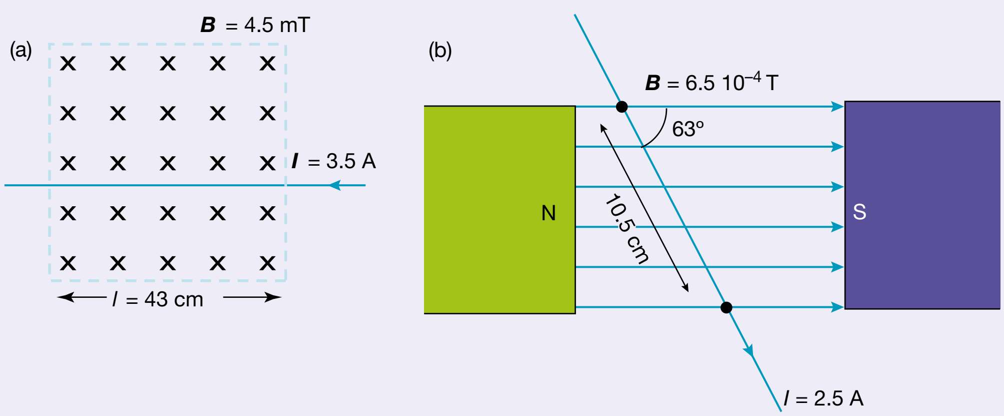
1 Explain how a moving electron could remain undeflected by a magnetic field.
2 Can a magnetic field move a stationary electron? Explain.
3 An electron and a proton are fired vertically upwards through a magnetic field that runs parallel to the earth from north to south. Describe the paths of the two particles.
4 Identify the direction of the force acting on each of the current-carrying conductors shown in figure 14.44. Use the terms ‘up the page’, ‘down the page’, ‘into the page’, ‘out of the page’, ‘left’ and ‘right’.
5 A wire of length 25 cm lies at right angles to a magnetic field of strength 4.0×10^{-2} T. A current of 1.8 A flows in the wire. Calculate the magnitude of the force that acts on the wire.
6 A long straight wire carries a current of 50 A. An electron, travelling at 10^{7} m s^{-1}, is 5 cm from the wire. What force acts on the electron if the electron’s velocity is directed
(a) perpendicular to the wire
(b) parallel to the wire?

FIGURE 14.44



7 Calculate the force on a 100 m length of wire carrying a current of 250 A when the strength of Earth’s magnetic field at right angles to the wire is 5.00×10^{-5} T.
8 The force on a 10 cm wire carrying a current of 15 A when placed in a magnetic field perpendicular to B has a maximum value of 3.5 N. What is the strength of the magnetic field?
9 Calculate the speed of an electron that would move in an arc of radius 1.00 mm in a magnetic field of 6.0 mT.
10 What magnetic field strength would cause an electron travelling at 10% of the speed of light to move in a circle of 10 cm?
11 Calculate the size of the force of a magnetic field of strength 0.25 T on a wire of length 0.30 m carrying a current of 2.4 A at right angles to the field.
12 A wire of length 25 cm lies at an angle of 30° to a magnetic field of strength 4.0×10^{-2} T. A current of 1.8 A flows in the wire. Calculate the magnitude of the force that acts on the wire. (Hint: you will need to consider the components of the magnetic field that lie perpendicular or parallel to the wire.)
13 Deduce both the magnitude and direction of the force acting on the lengths of conductors shown in figure 14.45.

FIGURE 14.45



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14.6 Review

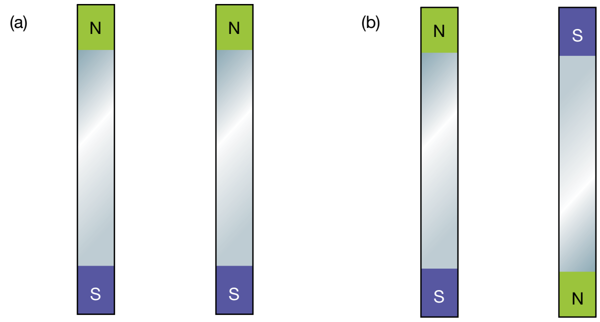
14.6.1 Summary

- Like magnetic poles repel; unlike magnetic poles attract.
- The direction of a magnetic field is the direction of the force that acts on a very small north pole placed in the field.
- Magnetic fields point away from north poles and towards south poles.
- Magnetic field lines are used to represent magnetic fields.
- An electric current in a long, straight wire produces a magnetic field represented by field lines in the form of concentric circles around the wire.
- The right-hand grip rule relates the direction of the current in a wire to the direction of the magnetic field.
- The magnetic field produced by a current in a solenoid is similar to that produced by a magnet.
- An electromagnet consists of a solenoid with a soft iron core. When a current flows through the solenoid, the soft iron core becomes strongly magnetised.
- A magnetic field exerts a force on a wire carrying an electric current. When the magnetic field and electric current are perpendicular to each other, the magnitude of the force can be calculated using the formula $F = IlB$.
- The magnetic field affects moving charge as if it were an electric current in a wire.
- The force by a magnetic field on a moving charged particle is always at right angles to the direction the particle is heading. The force constantly changes the direction of travel, producing a circular path.
- The size of the magnetic force on a moving charged particle is equal to qvB, where q and v are the charge and speed of the particle respectively, and B is the strength of the magnetic field.

14.6.2 Questions

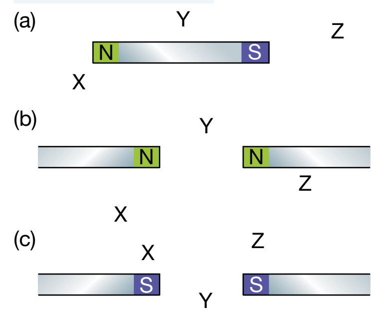
1. Two identical magnets are placed in each of the positions shown in figure 14.46. For each case draw a diagram showing the forces acting between the poles of one magnet and the poles of the other magnet (four pairs of forces in each case). Indicate the strengths of the forces by the lengths of the lines representing the forces. For each case, state whether the magnets will attract or repel one another.

FIGURE 14.46



2. In each of the cases shown in figure 14.47, sketch the magnetic field and mark the direction of the magnetic field at the points X, Y and Z.

FIGURE 14.47



3. Sketch the magnetic fields due to the solenoids shown in figure 14.48. Mark the north and south poles in each case. Show the direction in which a compass needle will point at points X, Y and Z in each case. In each case, state the direction in which a compass needle inside the solenoid would point.
4. The diagrams in figure 14.49 represent currents in wires perpendicular to the page (⊗ represents a current into the page and ⊙ represents a current out of the page). For each case, draw lines of magnetic field strength to represent the magnetic field surrounding the wire. Mark the directions in which the north pole of a compass would point at X, Y and Z in each of the cases shown.

FIGURE 14.48

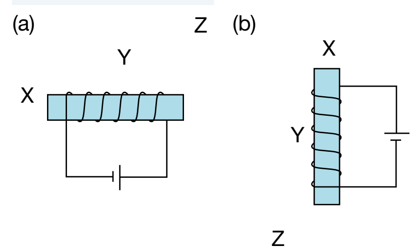


FIGURE 14.49

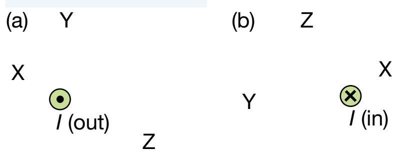
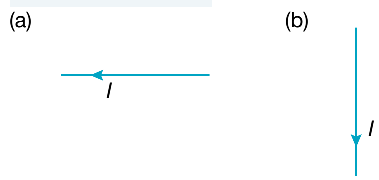
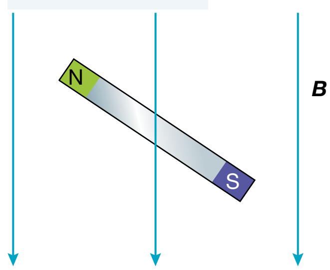


FIGURE 14.50



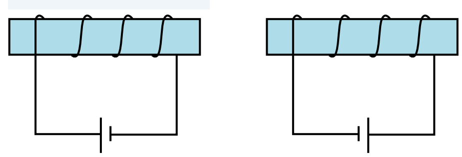
5. In each of the cases shown in figure 14.50, show the magnetic field using the symbol ‘x’ to represent a field into the page and the symbol ‘•’ to represent a field out of the page.
6. Figure 14.51 represents a magnet placed in a magnetic field. Draw a diagram showing the forces that act on the poles of the magnet. If the magnet is free to move, how will it move? Justify your answer.

FIGURE 14.51



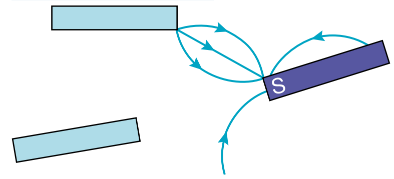
7. Figure 14.52 shows two electromagnets. Will they attract or repel one another? Justify your answer.
8. State the law of magnetic poles.
9. Draw a bar magnet and the magnetic field around it. Label the diagram to show that you understand the characteristics of magnetic field lines.
10. Are the north magnetic pole of the Earth and the north pole of a bar magnet of the same polarity? Explain your reasoning.

FIGURE 14.52



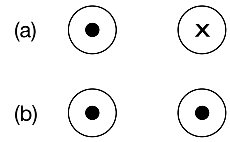
11. Figure 14.53 shows three bar magnets and some of the field lines of the resulting magnetic field.
    (a) Copy and complete the diagram to show the remaining field lines.
    (b) Label the polarities of the magnets.

FIGURE 14.53



12. Draw a diagram to show the direction of the magnetic field lines around a conductor when the current is (a) travelling towards you and (b) away from you.
13. Each diagram in figure 14.54 represents two parallel current-carrying conductors. In each case, determine whether the conductors attract or repel each other. Explain your reasoning.

FIGURE 14.54



14. Each empty circle in figure 14.55 represents a plotting compass near a coiled conductor. Copy the diagram and label the N and S poles of each coil, and indicate the direction of the needle of each compass.
15. The diagrams in figure 14.56 show electromagnets. Identify which poles are N and which are S.
16. In figure 14.57 a current-carrying conductor is in the field of a U-shaped magnet. Identify the direction in which the conductor is forced.

FIGURE 14.55

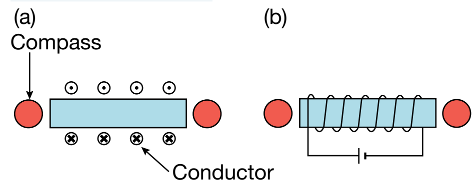


FIGURE 14.56

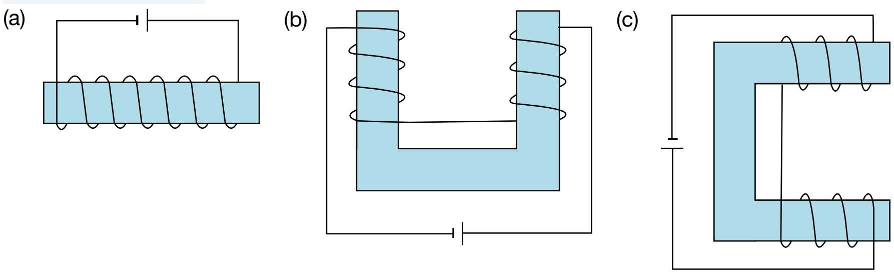
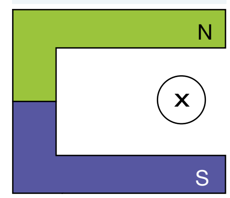


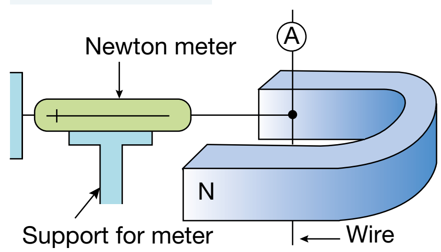
FIGURE 14.57



17. A student wishes to demonstrate the strength of a magnetic field in the region between the poles of a horseshoe magnet. He sets up the apparatus shown in figure 14.58.

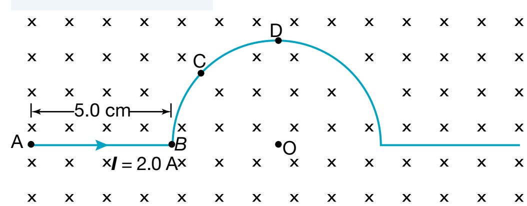
    The length of wire in the magnetic field is 2.0 cm. When the ammeter reads 1.0 A, the force measured on the newton meter is 0.25 N.
    (a) What is the strength of the magnetic field?
    (b) In this experiment the wire moves to the right. In what direction is the current flowing, up or down the page?

FIGURE 14.58



18. A wire with the shape shown in figure 14.59 carries a current of 2.0 A. It lies in a uniform magnetic field of strength 0.60 T.
    (a) Calculate the magnitude of the force acting on the section of wire, AB.
    (b) Which of the following gives the direction of the force acting on the wire at the point, C?
        i. Into the page
        ii. Out of the page
        iii. In the direction OC
        iv. In the direction CO
        v. In the direction OD
        vi. In the direction DO

FIGURE 14.59



(c) Which of the following gives the direction of the net force acting on the semicircular section of wire?

i. Into the page
ii. Out of the page
iii. In the direction OC
iv. In the direction CO
v. In the direction OD
vi. In the direction DO

19. Draw the magnetic field lines for the following items (shown in figure 14.60):
    (a) a loudspeaker magnet
    (b) a horseshoe magnet.
20. In Oersted's experiment, the compass needle initially points north–south. What would happen if the current in the wire above the needle ran:
    (a) west–east
    (b) east–west?
21. Use the right-hand-grip rule to determine the direction of the magnetic field at point X in the diagrams in figure 14.61.
22. Copy the diagrams in figure 14.62 and use the right-hand-grip rule and the direction of the magnetic field at X to determine the direction of the current in the wire in each case.
23. Use the right-hand-grip rule to determine the direction of the magnetic field at W, X, Y, Z in the diagrams in figure 14.63. Figure (a) represents a circular loop of wire with a current and figure (b) represents a solenoid.

FIGURE 14.60

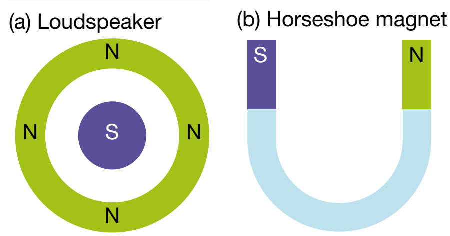


FIGURE 14.61

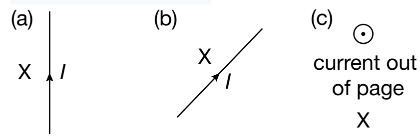


FIGURE 14.62

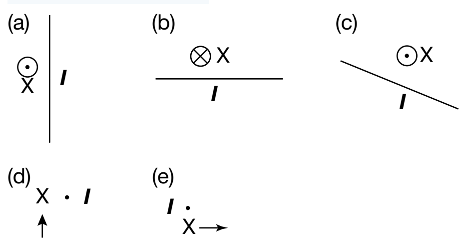
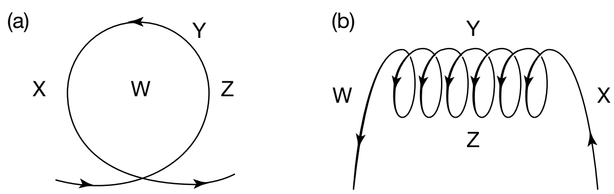
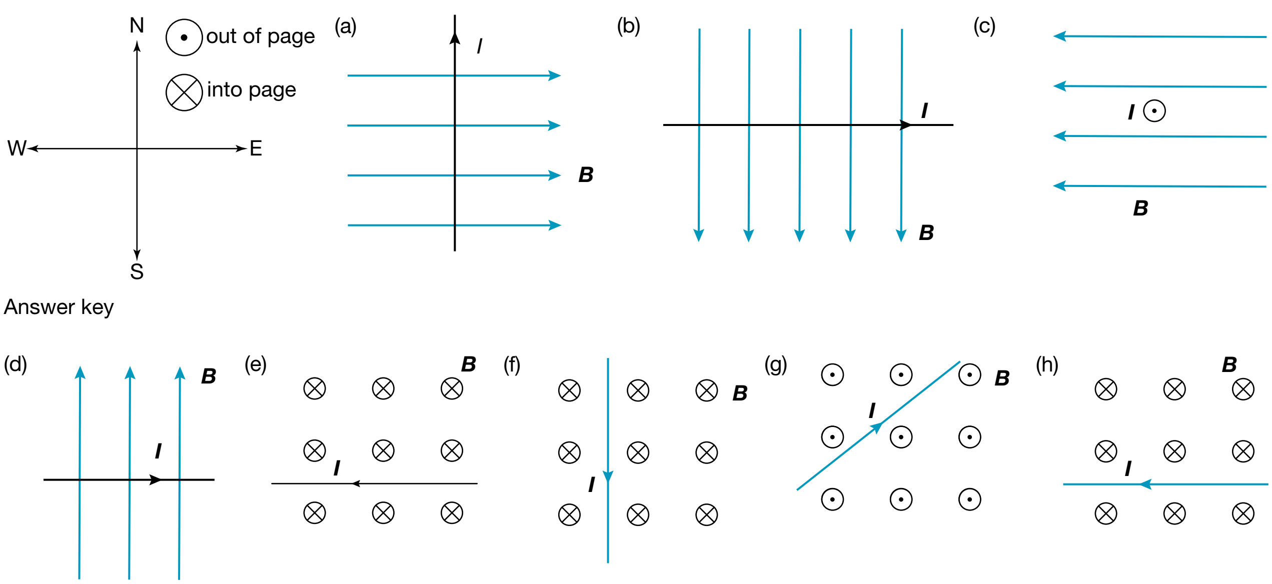


FIGURE 14.63



24. Use the answer key provided to indicate the direction of the force of the magnetic field on the current-carrying wire in diagrams (a) to (h) in figure 14.64.

FIGURE 14.64



25. Wires A and B are parallel to each other and carry current in the same direction.
    (a) Draw a diagram to represent this situation, and determine the direction of the magnetic field at B due to wire A.
    (b) This magnetic force will act on the current in wire B. What is the direction of the force by wire A on wire B?
    (c) Now determine the direction of the magnetic field at A due to wire B and the direction of the force by wire B on wire A.
    (d) Is the answer to (c) what you expected? Why? (*Hint:* Consider Newton's laws of motion.)
26. Calculate the size of the force on a wire of length 0.05 m in a magnetic field of strength 0.30 T if the wire is at right angles to the field and it carries a current of 4.5 A.
27. Calculate the size of the force exerted on a loudspeaker coil of radius 1.5 cm and 500 turns that carries a current of 15 mA in a radial magnetic field of 2.0 T. (*Hint:* Consider what aspect of the circle takes the place of l in this question.)
28. Calculate the size of the force on a wire carrying a current of 1.8 A at right angles to a magnetic field of strength 40 mT, if the length of the wire is 8.0 cm.
29. Design a compass without a permanent magnet.
30. Describe a method to use a moving charge to determine the direction of a magnetic field.
31. Describe and discuss the force of Earth's magnetic field on a horizontal section of a power line that runs in an east–west direction.
32. (a) A beam of electrons is directed at right angles to a wire carrying a conventional current from left to right. What happens to the electrons?
    (b) A beam of electrons is directed parallel to the same wire with the conventional current travelling in the same direction. What happens to the electrons?
33. An electron moving north enters a magnetic field that is directed vertically upwards.
    (a) What happens to the electron?
    (b) If the electron's motion was inclined upwards at an angle, as well as travelling north, what would be the path of the electron?
34. An electron travelling east at $1.2 \times 10^5\,\mathrm{m\,s^{-1}}$ enters a region of uniform magnetic field of strength 2.4 T.
    (a) Calculate the size of the magnetic force acting on the electron.
    (b) Describe the path taken by the electron, giving a reason for your answer.
    (c) Calculate the magnitude of the acceleration of the electron.
35. (a) What is the size of the magnetic force on an electron entering a magnetic field of 250 mT at a speed of $5.0 \times 10^6\,\mathrm{m\,s^{-1}}$?
    (b) Use the mass of the electron to determine its centripetal acceleration.
    (c) If a proton entered the same field with the same speed, what would be its centripetal acceleration?
36. Determine the direction of the magnetic force in the diagrams in figures 14.65, 14.66 and 14.67, using your preferred hand rule. Use the following terminology in your answers: up the page, down the page, left, right, into the page, out of the page.
    (a) Magnetic field into the page, electron entering from left

FIGURE 14.65

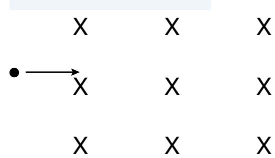


FIGURE 14.66

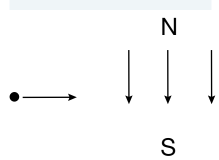
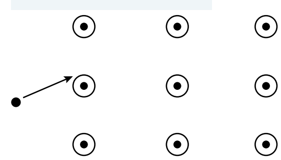


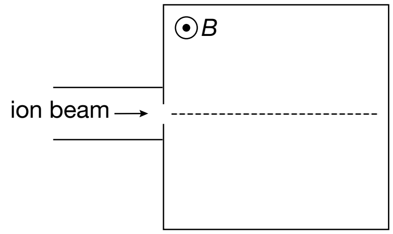
FIGURE 14.67



    (b) Magnetic field down the page, electron entering from left
    (c) Magnetic field out of the page, proton entering obliquely from left

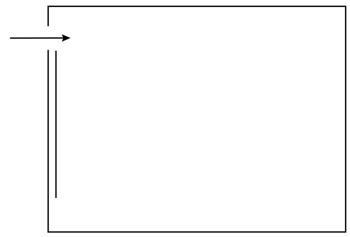
37. An ion beam consisting of three different types of charged particle is directed eastwards into a region having a uniform magnetic field, B, directed out of the page. The particles making up the beam are (i) an electron, (ii) a proton and (iii) a helium nucleus or alpha particle. Copy figure 14.68 and draw the paths that the electron, proton and helium nucleus could take.

FIGURE 14.68



38. In a mass spectrometer, positively charged ions are curved in a semicircle by a magnetic field to hit a detector at different points depending on the radius and mass. The ions enter the chamber at the top left corner, and curve around to hit the detector (see figure 14.69). What should be the direction of the magnetic field for the spectrometer to work properly? Use the terminology from question 36 for your answers.

FIGURE 14.69



39. Calculate the radius of curvature of the following particles travelling at 10% of the speed of light in a magnetic field of 4.0 T.
    (a) An electron
    (b) A proton
    (c) A helium nucleus
40. What strength of magnetic field would be needed to obtain a radius of 1000 m if an electron has momentum of 1.0×10^{-18} kg m s^{-1}? (Assume the direction of the momentum of the electrons is perpendicular to the direction of the magnetic field.)
41. The storage ring of the Australian Synchrotron has a radius of 34.4 m and the strength of the magnetic field is 2.0 T. What is the momentum of an electron in the storage ring?
42. Design a velocity selector with a magnetic field down the page, assuming the charged particles are coming from the left.
43. (a) Calculate the speed acquired by an electron accelerated by a voltage drop of 100 V.
    (b) The electron from part (a) enters a velocity selector with a magnetic field of strength 6.0 mT. For what electric field strength would the electron be undeflected?
    (c) If the plate separation for the electric field was 5.0 cm, what is the voltage across the plates?

PRACTICAL INVESTIGATIONS

Investigation 14.1: Magnetic field surrounding a magnet

Aim

To use a compass to map the magnetic field surrounding a bar magnet

Apparatus

bar magnet
compass
large sheet of paper

Theory

The direction of a magnetic field at a point can be found by placing a small compass at the point. The north pole of the compass points in the direction of the magnetic field at the point.

Method

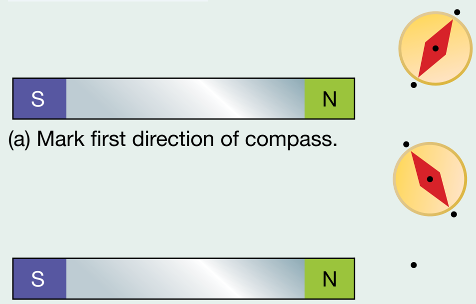
1. Place the sheet of paper on a horizontal surface.
2. Use the compass to find the N–S direction and mark this direction at the centre of the paper.
3. Place the bar magnet on the paper along the N–S line marked on the paper with the north pole of the magnet pointing north.
4. Mark on the paper the outline of the magnet and label the poles N and S.
5. Place the compass at a point near the north pole of the magnet. Mark with two points the position taken up by the compass needle.

6. Move the compass to a new position so that the position of the compass needle follows from the previous position. This is illustrated in figure 14.70.
7. Continue in this way until you reach a position near the south pole of the magnet.
8. Draw a continuous curve through the points you have marked on the paper.
9. Mark with arrows the direction of the magnetic field at several points along your line.
10. Repeat five times starting from different positions of the compass.

Analysis

1. At which pole do the magnetic field lines begin?
2. At which pole do the magnetic field lines end?
3. Where is the magnetic field strongest? How is this shown by the magnetic field lines?

FIGURE 14.70



(a) Mark first direction of compass.

(b) Place compass so that the direction follows on from the previous direction.

Investigation 14.2: Magnetic field produced by a current in a long, straight wire

Aim

To map the magnetic field surrounding a long, straight wire carrying an electric current

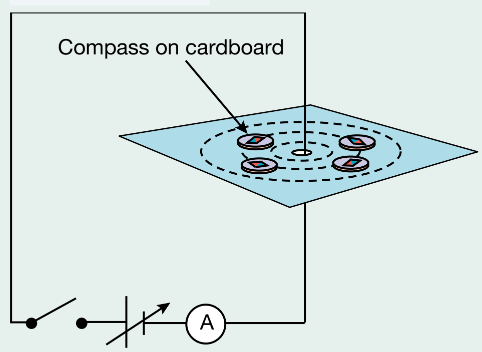
Apparatus

50 cm length of straight wire
sheet of cardboard approximately 20 cm × 20 cm
power supply
variable resistor
connecting wire
switch
compass
some means of supporting the wire
some means of supporting the cardboard

Method

1. Set up the apparatus as shown in figure 14.71. To increase the strength of the magnetic field, a number of loops of wire can be used.
2. Connect the power supply so that the conventional current flows downwards through the wire.
3. Adjust the voltage of the power supply and the variable resistance so that the current has the value given by your teacher.
4. Place the compass about 5 cm from the wire.
5. Switch on the current and mark the positions of the ends of the compass on the cardboard.
6. Proceed as in Investigation 14.1, tracing out the magnetic field line. (Ideally this should return to the starting point to form a closed loop.)
7. Mark the direction in which the north pole of the compass pointed at several places on the magnetic field line.
8. Repeat this a number of times with the initial position of the compass at different distances from the wire.
9. Draw smooth lines of magnetic field through each set of points.
10. Reverse the direction of the current and observe what happens to the compass needle.

FIGURE 14.71



Analysis

Show that your result is compatible with the right-hand grip rule.

Questions

1. When the current was coming upwards out of the cardboard, was the direction of the magnetic field lines around the wire clockwise or anticlockwise?
2. Can you use this to formulate an alternative rule for determining the direction of the magnetic field surrounding a current-carrying wire?

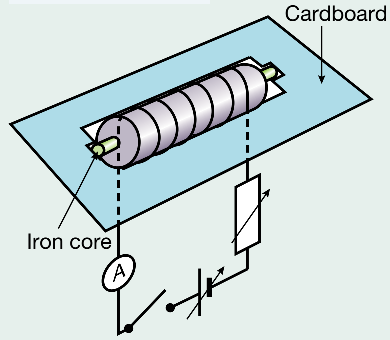
Investigation 14.3: Magnetic field of a solenoid carrying a current

Aim
To map the magnetic field surrounding a solenoid

Apparatus
solenoid
sheet of cardboard approximately 20 cm × 20 cm
scissors
connecting wires
power supply
variable resistor
switch
compass

Method
1. Connect the apparatus as shown in figure 14.72.
2. Note the direction of the conventional current around the solenoid.
3. Map the magnetic field around the solenoid using the same method as was used in the previous two practical activities.
4. Reverse the direction of the current through the solenoid. Note what happens to the direction of the magnetic field.

FIGURE 14.72



Analysis
1. With the first direction of the conventional current, which end of the solenoid was the north pole? Explain.
2. Is this result compatible with the right-hand grip rule for solenoids?
3. Draw a sketch showing how the right-hand grip rule for solenoids applies to your result.
4. What happened to the magnetic field when the direction of the current was reversed?
5. How does the magnetic field produced by a current in a solenoid compare with the magnetic field surrounding a magnet?

Investigation 14.4: Building an electromagnet

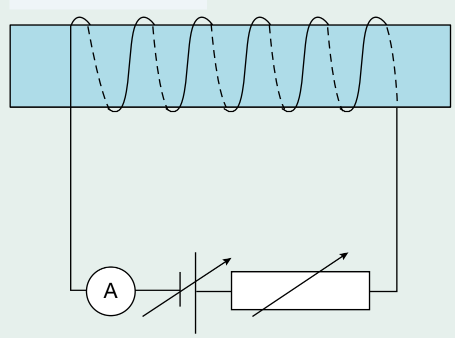
Aim
To build an electromagnet and observe its properties

Apparatus
iron rod for core of electromagnet
insulated conducting wire for coil
power pack
connecting wire
variable resistor
ammeter
small iron nails

Theory
A soft iron core is placed in a solenoid carrying a current and becomes magnetised. When the current is switched off the soft iron core loses its magnetism.

Method
1. Build the electromagnet by winding the conducting wire closely from one end of the iron core to the other. To make the electromagnet stronger one or more layers of coils can be wound on top of the first. It is essential that all layers of coils are wound in the same direction around the core.
2. Connect the electromagnet to the power supply as shown in figure 14.73.
3. Test the magnetism of the electromagnet by observing the attraction of small iron nails to the end of the soft iron rod. The greater the number of iron nails attracted to the rod, the greater is the magnetism.

FIGURE 14.73



4. Observe the magnetism of the electromagnet when there is no current.
5. Observe the magnetism of the electromagnet for a range of currents.
6. Observe how much time is taken for the electromagnet to gain and lose its magnetism when the current is switched on and off.

Analysis

1. How did the magnetism change as the current was increased?
2. Was there any delay observed in the gain or loss of magnetism when the current was switched on or off?
3. Was there any magnetism left when the current was turned off?

GLOSSARY

absolute refractive index (n): the ratio of the speed of light in a vacuum to the speed of light in that material

absolute zero: the lowest temperature that is physically possible. At this temperature, particles cease to vibrate. It is equal to 0 K or approximately – 273 ^{o}C.

acceleration: the rate at which an object changes its velocity

accuracy: a measure of how well a measurement agrees with a set or standard value

acoustic power: amount of sound energy in joules being produced by a source every second

air resistance or **drag:** the force applied to an object, opposite to its direction of motion, by the air through which it is moving

alternating current: (AC), refers to circuits where the charge carriers move backwards and forwards periodically

ammeter: an instrument used to measure the electric current in an electric circuit. An ammeter is connected into a circuit in series.

ampere: (A), the SI unit of electric current. An ampere is equivalent to a coulomb second^{-1}.

amplitude: an object's maximum displacement from its rest position while undergoing periodic motion

antinodal lines: lines where constructive interference occurs on a surface

antinodes: the points in a medium that are disturbed the most when standing waves form

beats: refers to the change in volume of a sound that occurs when two sounds of slightly different frequencies occur together

centre of curvature: the theoretical centre of the circle of which an arc corresponds to the curvature of a particular surface, such as a concave mirror. The centre of curvature of a mirror is located at a distance equal to twice the mirror's focal length.

centre of curvature (C) for a lens: The centre of curvature (C) for the face of a lens is the centre of the circle, an arc of which corresponds to the curve of the lens face. A flat face of a lens has a centre of curvature located at infinity.

change of phase: the inversion of a wave when it is reflected

charge carrier: a charged particle that is free to move through a material

coefficient of friction (μ): the ratio of the frictional force to the normal reaction force acting on an object moving across a surface. It is a measure of how easily the object moves across the surface, and depends upon characteristics of both the object and the surface.

commutator: a device that reverses the direction of the current flowing through an electric circuit

component: a part. Any vector can be resolved into a number of components. When all of the components are added together, the result is the original vector.

compression: a zone where the particles of the medium are pushed closer together. It is a zone of higher pressure.

concave: the profile of a lens or mirror that is shaped like the interior curve of a sphere or circle section (see **diverging**)

conduction: the transfer of heat through a substance as a result of collisions between neighbouring vibrating particles

conductor: a material that contains charge carriers

constructive interference: the disturbance caused when two waves reach a position at the same time and give rise to an amplitude that is greater than that due to each of the waves alone

contact forces: forces that arise between objects that are in physical contact with each other

convection: the transfer of heat in a fluid (a liquid or gas) as a result of the movement of particles within the fluid

convection current: a movement of particles during the transfer of heat through a substance

conventional current: the movement of positive charges from the positive terminal of a cell through the conductor to the negative terminal

converging: term describing a lens or mirror that bends or reflects incident light rays such that they meet at a common point (focus)

convex: the profile of a lens or mirror that is shaped like the exterior curve of a sphere or circle section

coulomb: (C), the SI unit of electric charge

crest: the highest part of a transverse wave

critical angle: the angle where total internal reflection prevents the ray from escaping from a higher optical-density medium to a lower optical-density medium

deficiency of electrons: exists when a body has fewer electrons than protons

destructive interference: the disturbance that occurs when the sum of two superimposed waves is zero

diffraction: a phenomenon exhibited by waves when waves either bend behind a barrier or the wavefront is broken up into many small sources

diffuse reflection: occurs when parallel light rays striking a surface are scattered when reflected

dilated: same as **enlarged**

diminished: description of an image that is smaller than the original object

diode: a device that allows current to pass through it in one direction

dipole field: the field produced around two proximate objects with opposite polarities, for example: a positive charge and a negative charge; a north magnetic pole and a south magnetic pole

direct current: (DC), refers to circuits where the net flow of charge is in one direction only

direction of a magnetic field: the direction of the force on a very small magnetic north pole placed in the field

dispersion: the separation of light into a spectrum of colours as the result of refraction

displacement: a vector quantity representing the location of the destination relative to the origin of motion only, irrespective of the path actually negotiated between the two points

displacement antinode: position in an air column at which the particles are able to move with maximum amplitude

displacement node: position in an air column at which minimum displacement of air particles occurs

distance: the total length of the pathway taken between the origin and the destination point

diverging: term describing a lens or mirror that bends or reflects incident light rays such that they are spread out, appearing to come from a focal point behind the lens or mirror

doping: the process in which small quantities of either a Group 13 element or a Group 15 element are introduced into a semiconducting material to affect the way in which electrons move through it

driving frequency: the frequency of a forced vibration

earthed: when a body is connected to the Earth by a conducting path

elastic collision: a collision in which both momentum and kinetic energy are conserved

electric charge: a property of electrons and protons by which they exert electric forces on one another

electric current: the rate at which charge flows under the influence of an electric field

electric field: a field of force with a field strength equal to the force per unit charge at that point

electric field strength: $\boldsymbol{E}$, given by the formula $\boldsymbol{E} = \frac{\mathbf{F}}{q}$. The direction of the electric field strength is the direction of the force that acts on a positive charge placed in the field.

electric potential energy: the potential energy of an electric charge in an electric field

electromagnet: a temporary magnet produced when a solenoid wound around an iron core carries an electric current

electromagnetic wave: a wave that propagates as a perpendicular electric and magnetic field. Electromagnetic waves do not require a medium for propagation.

electron current: the term used when dealing with the mechanisms for the movement of electrons
electron drift: the slow movement of electrons through a conductor in the opposite direction to the electric field. This movement is superimposed on the much faster, random motion of the electrons.
electrostatic charge: a charge due to an excess or deficiency of electrons
energy: the capacity to do work. It is a scalar quantity.
enlarged or **dilated:** describes an image that is larger than the original object
equilibrant: the force that, when added to an unbalanced system of forces, brings the system into equilibrium
excess of electrons: exists when a body has more electrons than protons
First Law of Thermodynamics: $\Delta U = Q - W$ or $Q = U + W$, where Q is the heat energy in joules, W is the work done in joules and U is the internal energy in joules
focal length: (of a lens or mirror) the distance between the geometric centre and the principal focus
focal plane: (of a lens or mirror) the plane set at right angles to the principal axis and passing through the focus
focus (F): the point where all rays from a converging lens or mirror are concentrated. It is also the point where the rays appear to originate after passing through a diverging lens or being reflected by a diverging mirror.
force: an external influence that is able to alter the state of motion of an object. It is a 'push' or a 'pull' that has both magnitude and a direction.
forced vibration: occurs when an object, surface or medium is made to vibrate at the same rate as an adjacent vibrating object; also known as forced resonance
free electrons: electrons in a metal that are detached from their atoms and are free to move through the metal. A metal conducts an electric current by the movement of the free electrons.
frequency (f): (of a wave) equal to the number of waves that move past a given point in 1 second
fundamental frequency: the lowest frequency at which a standing wave is produced
gravitational field strength: (g), the force of gravity on a unit of mass
gravitational potential energy: the energy stored in an object as a result of its position relative to another object to which it is attracted by the force of gravity
harmonic: overtone frequency that is equal to a whole number multiple of the fundamental frequency
idealisation: an idealisation makes modelling a phenomenon or event easier by assuming ideal conditions that don't exactly match the real situation
illuminated bodies: bodies that reflect light from another source and are not able to produce light by themselves
impulse: vector quantity representing the change in momentum of an object. It is the product of the force acting on the object and the time interval over which the force acts. It has SI units of N s or kg m s^{-1}.
incandescent: refers to materials and objects that give off light when they reach a high enough temperature
induced charge: a charge produced in a body when another charged body is near it
induction: the production of induced charges
inelastic: (collision) a collision in which kinetic energy is not conserved
inertia: the tendency of an object to resist a change in its motion
instantaneous speed: the speed at a particular instant of time
instantaneous velocity: the velocity at a particular instant of time
insulated: refers to an object that is electrically isolated from its surroundings but not earthed
insulator: a material that does not contain charge carriers
intensity: an objective measure of how much energy a sound is able to transfer to a 1 m^2 area of surface at a specific distance from the source
interference: the disturbance caused by the interaction of two or more waves at the same location
irregular reflection: occurs when parallel light rays striking a surface are scattered when reflected [note: same as diffuse reflection]

joule: (J), the SI unit of work or energy. One joule is the energy expended when a force of 1 newton acts through a distance of 1 metre.
kinetic energy: the energy associated with the movement of an object
latent heat: the heat added to a substance undergoing a change of state that does not increase the temperature
Law of Conservation of Momentum: Law describing the momentum of a system: *For any closed isolated system, the sum of the momenta of all objects in that system is a constant.*
Law of Reflection: states that the angle between an incident light ray and the normal where it strikes the surface will be equal to the angle between the normal and the reflected ray
lens axis: a line that passes through the pole of the lens that is perpendicular to the principal axis
light-dependent resistor (LDR): a device that has a resistance which varies with the amount of light falling on it
light-emitting diode (LED): a small semiconductor diode that emits light when a current passes through it
limiting friction: the maximum amount of force that can be applied to a stationary object at a particular time before it commences movement
lines of electric field: the lines drawn on a diagram to represent the direction and magnitude of an electric field
load: the force acting on a structure or building component
local antinodes or **maxima:** points at which constructive interference between the waves produced by two sources in phase occurs. At these points, the amplitude of the combined wave is greater than that produced by one source alone.
local nodes or **minima:** points at which destructive interference between the waves produced by two sources in phase occurs. At these points, the amplitude of the combined wave is less than that produced by either source alone.
longitudinal wave or **compression wave:** a wave in which the disturbance moves in the same direction as the wave
luminosity (L): of a star; the total energy radiated by a star per second
luminous bodies: bodies that give off light directly
luminous intensity, (I): a quantitative measure of the effective brightness of a light source. It is dependent upon the amount of light energy produced by the light source each second and the area over which that light energy is distributed: $I = \frac{L}{4\pi d^2}$. It is measured in W m^{-2} .
luminous power (see **luminosity**)**:** another term for luminosity. it is a measure of the amount of light energy in joules produced by a light source each second. Its units are J s^{-1} or W.
magnetic domain: region in a material in which the magnetic fields of the material's atoms are aligned in the same direction
magnetic field: a force field surrounding a magnetic pole that exerts forces on other magnetic poles placed in the field
mechanical energy transfer: the transfer of energy by the action of a force
mechanical interaction: an interaction in which energy is transferred from one object to another by the action of a force
mechanical wave: a wave that requires the movement of particles to propagate forward
momentum: the product of the mass of an object and its velocity. It is a vector quantity.
natural frequency: the frequency at which an object will vibrate when stimulated. It is independent of the size of the stimulus, depending solely upon the object's size, shape and composition. It is also the rate at which resonance occurs.
natural magnet: naturally occurring iron ores or materials such as magnetite that have magnetic properties
negative charge: the type of charge on an electron
negatively charged: a body that has an excess of electrons

negligible: a quantity that is negligible is so small that it can be ignored when modelling a phenomenon or an event

net force: the vector sum of the forces acting on an object

neutral: refers to an object that carries an equal amount of positive and negative charge

newton: the derived unit of force; 1 newton (N) = 1 kilogram-metre per second squared (kg m s^{-2})

newton coulom^{-1}: (N C^{-1}), is the unit of electric field strength

nodal lines: lines where destructive interference occurs on a surface, resulting in no displacement of the surface

nodes: the points in a medium that remain undisturbed when two waves cancel each other out due to destructive interference

non-contact forces: forces that arise between objects that are not in direct physical contact with each other

non-ohmic device: a device for which the resistance is different for different currents passing through it

normal: an imaginary line drawn perpendicular to the interface between two media at the point where a light ray enters the interface

normal reaction: a force that acts perpendicular to a surface as a result of an object applying a force to the surface

ohmic device: a device for which, under constant physical conditions such as temperature, the resistance is constant for all currents that pass through it

opaque: refers to material that does not allow light to pass through it

optical centre (O) for a mirror: The optical centre (O) is the centre of the curved mirror's face.

optical centre (or **pole**) **for a lens:** The optical centre (or pole) of a lens is the point in the exact centre of the lens itself. Light rays that pass through the optical centre of a lens will not be diverted, but will continue undeflected.

optical ray tracing: process by which the position, nature and size of an image produced by a lens or a mirror is determined by use of a scale diagram

optical transmissivity: the ability of a material to allow light to pass through it

overtones: frequencies produced in a string or air column that are higher than the fundamental frequency

parallel: devices connected in parallel are joined together so that one end of each device is joined at a common point and the other end of each device is joined at another common point

period: (of a cycle or series of events) is the amount of time one cycle or one event takes to occur

periodic waves: disturbances that repeat themselves at regular intervals

permanent magnet: a material that keeps its iron-attracting properties regardless of whether an external electromagnetic field is present

permeability of free space (μ_0): measure of a vacuum's resistance to the establishment of a magnetic field within it. In a perfect vacuum, permeability is equal to $4\pi \times 10^{-7}$ T m A^{-1}.

permittivity of free space (ε_0): measure of a vacuum's resistance to the establishment of an electric field within it. In a perfect vacuum, permittivity is equal to 8.854×10^{-12} C^2 N^{-1} m^{-2}.

pitch: directly related to the frequency of a sound. The higher the frequency of the sound, the more vibrations per second and the higher the pitch. A low-frequency sound is a low-pitched sound.

poles: regions of a magnetic material where the property of attracting iron is concentrated

positive charge: the type of charge on a proton

positively charged: a body that has a deficiency of electrons

potential difference: the change in potential energy per unit charge moving between two points

potential difference across a power supply: the number of joules of electric potential energy given to each coulomb of charge that passes through the power supply

potential drop (or **voltage drop**)**:** the amount of electrical potential energy lost by each coulomb of charge in a given part of a circuit

power: the rate at which work is done, or the rate at which energy is transferred or transformed. It is measured in watts (W)

precision: a measure of how well a set of measurements agree with each other

pressure antinode: position in an air column at which air pressure is undergoing maximum change over time
pressure node: position in an air column at which there is no change in air pressure
principal axis: (for a mirror) The principal axis is the line upon which the centre of curvature, the principal focus and the optical centre lie.
principal axis: (of a lens) the line that passes through the centre of the lens and is perpendicular to the plane of the lens
principal focus: (of a lens or mirror) the point on the principal axis at which incident parallel rays are converged
progressive (or **travelling**) **waves:** waves that move freely through a medium until a boundary is met.
radiation: heat transfer without the presence of particles
radius of curvature (R) for a lens: The radius of curvature (R) is the distance between the centre of curvature and the surface of the lens.
radius of curvature (R) for a mirror: The radius of curvature (R) is the radius of this sphere; this will be the distance between the centre of curvature and the geometric centre of the mirror.
rarefaction: a zone where the particles of the medium are spread further apart. It is a zone of lower pressure.
real image: an image through which light passes. A real image can be seen on a screen placed at the location of the image.
refraction: the change in direction that a light ray experiences when passing into a new medium
regular reflection (or **specular reflection**)**:** occurs when parallel light rays striking a reflective surface are reflected in parallel
relative refractive index: a measure of how much light bends when it travels from any one substance into any other substance
resistance: a measure of how easily charge carriers are able to move through a conductor. It is equal to the potential difference across the resistor divided by the current passing through the resistor. The unit of resistance is the ohm (Ω).
resistor: a device used in circuits to control the current flowing through, and the potential drop across, a section of circuit. It is a conductor in which the electric potential energy of a current is converted into heat energy.
resolution: the fineness to which an instrument can be read
resonance: the condition where a medium responds to a periodic external force by vibrating with the same frequency as the force
reverberation: an effect created when the audience hears a noticeable time delay between the played note ending and the dying away of that note.
reverberation time: the period of time that elapses between the incidence of a sound and the noise level of that sound's echo dropping below 60 dB. The reverberation time of a space depends upon its size and shape, and the nature of the surfaces and objects within it.
right-hand grip rule: a rule for finding the direction of the magnetic field surrounding an electric current
scalar: a quantity that specifies size (magnitude) but not direction
series: devices connected in series are joined together one after the other
Snell's Law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$
solenoid: a coil of wire wound into a cylindrical shape
sound intensity level: a comparison of the intensity of a sound compared to the softest sound audible
specific latent heat of fusion: the quantity of energy required to change 1 kilogram of a substance from a solid to a liquid without a change in temperature
specific latent heat of vaporisation: the quantity of energy required to change 1 kilogram of a substance from a liquid to a gas without a change in temperature
specular reflection: see **regular reflection**
speed: a measure of the time rate at which an object moves over a distance. It is a scalar quantity.
speed of light: represented by the symbol, c. It is the speed at which all electromagnetic waves travel in a vacuum; $c = 299\ 792\ 458$ m s^{-1} (usually approximated to 3×10^8 m s^{-1}).

standing wave (stationary wave): occurs at wave frequencies when there is interference between the initially generated waves and the reflected waves
strain potential energy: the energy stored in an object as a result of a reversible change in shape. It is also known as elastic potential energy.
superposition: the adding of two or more waves
temperature: a measure of the average translational kinetic energy of particles
temporary magnet: an object that acts like a permanent magnet only while it is exposed to a strong electromagnetic field
thermal conductivity (k): a constant that expresses how easily heat is transferred through a material
thermal equilibrium: occurs when the temperature of two regions is uniform
thermistor: a device having a resistance that changes with a change in temperature
timbre: used to describe the richness of sound produced by a musical instrument. Good timbre depends upon the ability of the instrument to produce different harmonic frequencies at once.
torque: also referred to as moment; the turning effect of a force about a pivot or reference point
total internal reflection: the total reflection of light from a boundary between two substances. It occurs when the angle of incidence is greater than the critical angle.
translucent: refers to materials that allow light rays to pass through them, although the rays will be dispersed
transparent: refers to material that allows light to pass through it without dispersion or distortion
transverse wave: a wave in which the disturbance caused by the transfer of energy acts perpendicularly to the direction of motion of the wave itself
trough: the lowest part of a transverse wave
vector: a quantity that specifies size (magnitude) and direction
velocity: a measure of the time rate of displacement, or the time rate of change in position. It is a vector quantity.
virtual image: an image that is seen because light appears to be coming from it. It is unable to be 'captured' on a screen.
volt: (V), the SI unit of potential difference
voltage: another name for potential difference. It is equal to the amount of electrical energy available for transformation at a point in a circuit by each coulomb of charge passing through that point.
voltage divider: a device used to reduce, or divide, a voltage to a value needed for a part of the circuit
voltage drop: see **potential drop**
voltmeter: an instrument used to measure the potential difference across a component in an electric circuit. A voltmeter is connected into a circuit in parallel.
wave: a disturbance that transfers energy through a medium or across a distance
wave equation: equations describing the relationship between the speed, frequency and wavelength of a periodic wave: $v = f\lambda$
wave number: equal to the number of waves per unit distance for a periodic wave series. It is represented by the symbol k, and is equal to the reciprocal of the wavelength: $k = 1/\lambda$.
wavelength (λ): the distance between corresponding points on successive waves
weight: the force applied to an object due to gravitational attraction
work: a scalar quantity that is equal to the amount of energy transferred to or from an object by the action of a force

APPENDIX 1 Formulae sheet

$\boldsymbol{v}_{av} = \frac{\boldsymbol{s}}{t}$

$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

$\boldsymbol{a}_{av} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$

$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$\boldsymbol{s} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t$

$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$

$\boldsymbol{W} = m\boldsymbol{g}$

$\boldsymbol{F}_f = \mu \boldsymbol{R}$

$\boldsymbol{F}_{net} = \mathrm{m}\boldsymbol{a}$

$\boldsymbol{W} = \boldsymbol{F}_{net}\boldsymbol{s}$

$\Delta U = m\boldsymbol{g}\Delta \boldsymbol{h}$

$P = \frac{\Delta E}{t}$

$P = \boldsymbol{F}\boldsymbol{v}$

$E_k = \frac{1}{2} m\boldsymbol{v}^2$

$\Sigma\, \mathrm{m}\boldsymbol{v}_{before} = \Sigma\, \mathrm{m}\boldsymbol{v}_{after}$

$\Sigma \frac{1}{2}\mathrm{m}\boldsymbol{v}^2_{before} = \Sigma \frac{1}{2}\mathrm{m}\boldsymbol{v}^2_{after}$

$\Delta \boldsymbol{p} = \boldsymbol{F}\Delta t$

$v = f\lambda$

$f = \frac{1}{T}$

$k = \frac{2\pi}{\lambda}$

$f_{beat} = |f_2 - f_1|$

$f' = f\frac{(v_{wave} + v_{observer})}{(v_{wave} - v_{source})}$

$I = \frac{P}{4\pi d^2}$

$L = 10 \log\left(\frac{I}{I_o}\right)$

$n_x = \frac{c}{v_x}$

$n_1 sin\,(i) = n_2 sin\,(r)$

$\sin\,(i_c) = \frac{1}{n_x}$

$I_1 r_1^2 = I_2 r_2^2$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\Delta Q = mc\Delta T$

$\frac{Q}{t} = \frac{kA\Delta T}{d}$

$\boldsymbol{E} = \frac{\boldsymbol{F}}{q}$

$\boldsymbol{E} = -\frac{v}{\boldsymbol{d}}$

$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$

$V = \frac{\Delta U}{q}$

$I = \frac{q}{t}$

$V = \frac{W}{q}$

$R = \frac{V}{I}$

$R_{series} = R_1 + R_2 + \cdots + R_n$

$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

$W = I^2 R$

$B = \frac{\mu_o I}{2\pi r}$

$B = \frac{\mu_o NI}{L}$

APPENDIX 2
Periodic table

Key: Atomic number / Symbol / Atomic weight / Name

Period	Group 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H 1.008 Hydrogen																	2 He 4.003 Helium
2	3 Li 6.941 Lithium	4 Be 9.012 Beryllium											5 B 10.81 Boron	6 C 12.01 Carbon	7 N 14.01 Nitrogen	8 O 16.00 Oxygen	9 F 19.00 Fluorine	10 Ne 20.18 Neon
3	11 Na 22.99 Sodium	12 Mg 24.31 Magnesium											13 Al 26.98 Aluminium	14 Si 28.09 Silicon	15 P 30.97 Phosphorus	16 S 32.07 Sulfur	17 Cl 35.45 Chlorine	18 Ar 39.95 Argon
4	19 K 39.10 Potassium	20 Ca 40.08 Calcium	21 Sc 44.96 Scandium	22 Ti 47.87 Titanium	23 V 50.94 Vanadium	24 Cr 52.00 Chromium	25 Mn 54.94 Manganese	26 Fe 55.85 Iron	27 Co 58.93 Cobalt	28 Ni 58.69 Nickel	29 Cu 63.55 Copper	30 Zn 65.38 Zinc	31 Ga 69.72 Gallium	32 Ge 72.64 Germanium	33 As 74.92 Arsenic	34 Se 78.96 Selenium	35 Br 79.90 Bromine	36 Kr 83.80 Krypton
5	37 Rb 85.47 Rubidium	38 Sr 87.61 Strontium	39 Y 88.91 Yttrium	40 Zr 91.22 Zirconium	41 Nb 92.91 Niobium	42 Mo 95.96 Molybdenum	43 Tc [98] Technetium	44 Ru 101.1 Ruthenium	45 Rh 102.9 Rhodium	46 Pd 106.4 Palladium	47 Ag 107.9 Silver	48 Cd 112.4 Cadmium	49 In 114.8 Indium	50 Sn 118.7 Tin	51 Sb 121.8 Antimony	52 Te 127.6 Tellurium	53 I 126.9 Iodine	54 Xe 131.3 Xenon
6	55 Cs 132.9 Caesium	56 Ba 137.3 Barium	57–71 * Lanthanides	72 Hf 178.5 Hafnium	73 Ta 180.9 Tantalum	74 W 183.8 Tungsten	75 Re 186.2 Rhenium	76 Os 190.2 Osmium	77 Ir 192.2 Iridium	78 Pt 195.1 Platinum	79 Au 197.0 Gold	80 Hg 200.6 Mercury	81 Tl 204.4 Thallium	82 Pb 207.2 Lead	83 Bi 209.0 Bismuth	84 Po [209] Polonium	85 At [210] Astatine	86 Rn [222] Radon
7	87 Fr [223] Francium	88 Ra [226] Radium	89–103 ** Actinides	104 Rf [261] Rutherfordium	105 Db [262] Dubnium	106 Sg [266] Seaborgium	107 Bh [264] Bohrium	108 Hs [265] Hassium	109 Mt [268] Meitnerium	110 Ds [271] Darmstadtium	111 Rg [280] Roentgenium	112 Cn Copernicium	113 Nh Nihonium	114 Fl Flerovium	115 Mc Moscovium	116 Lv Livermorium	117 Ts Tennessine	118 Og Oganesson

*Lanthanide series

57 La 138.9 Lanthanum	58 Ce 140.1 Cerium	59 Pr 140.9 Praseodymium	60 Nd 144.2 Neodymium	61 Pm [145] Promethium	62 Sm 150.4 Samarium	63 Eu 152.0 Europium	64 Gd 157.3 Gadolinium	65 Tb 158.9 Terbium	66 Dy 162.5 Dysprosium	67 Ho 164.9 Holmium	68 Er 167.3 Erbium	69 Tm 168.9 Thulium	70 Yb 173.1 Ytterbium	71 Lu 175.0 Lutetium

**Actinide series

89 Ac [227] Actinium	90 Th 232.0 Thorium	91 Pa 231.0 Protactinium	92 U 238.0 Uranium	93 Np [237] Neptunium	94 Pu [244] Plutonium	95 Am [243] Americium	96 Cm [247] Curium	97 Bk [247] Berkelium	98 Cf [251] Californium	99 Es [252] Einsteinium	100 Fm [257] Fermium	101 Md [258] Mendelevium	102 No [259] Nobelium	103 Lr [262] Lawrencium

- For elements with no stable nuclides, the mass of the longest living isotope is given in square brackets.
- The atomic weights of Np and Tc are given for the isotopes ^{237}Np and ^{99}Tc.

ANSWERS

Topic 1 Learning to think like a physicist

1.2 Exercise 1

1. (a) 1.5 A; (b) 0.750 kg; (c) 250 000 000 000 W; (d) 520 m; (e) 0.000 000 6 m; (f) 0.000 15 s; (g) 0.05 m; (h) 50 000 000 V; (i) 1.2 m

2. $m\,s^{-2}$

3. (a) $N\,kg^{-1}$; (b) $m\,s^{-2}$

1.3 Exercise 1

(a) 6.37×10^5 m; (b) $3.0 \times 10^8\ m\,s^{-1}$; (c) 3×10^{-10} m

1.4 Exercise 1

1. (a) 4; (b) 2; (c) 4; (d) 6; (e) 2; (f) 3

2. (a) 465 m^2; (b) 337.4 m

3. 8.6 $m\,s^{-1}$

1.5 Exercise 1

(a) (i) 9.6 ± 0.25; (ii) 8.5 ± 0.125; (iii) 11.85 ± 0.05

(b) 63.9 ± 0.05

Topic 2 Motion in a straight line

2.2 Exercise 1

1. (a) 100 m, N; (b) 220 m; (c) 100 m; (d) 60 m, S

2. (a) 800 m; (b) 400 m, N; (c) 0

3. 400 m, W

2.3 Exercise 1

1. 720 km

2. (i) 10 $km\,h^{-1}$; (ii) 2.8 $m\,s^{-1}$

3. 12 s

2.4 Exercise 1

1. approximately 16 m

2. approximately 30 m

2.5 Exercise 1

1. (a) 6.0 s; (b) 36 m

2. (a) 18 m; (b) $-16\ m\,s^{-2}$

3. $-0.8\ m\,s^{-2}$

4. (a) 8 $m\,s^{-2}$; (b) 0.5 s

2.5 Exercise 2

(a) 1 $m\,s^{-2}$; (b) $t = 0$, $t = 20$ s, $35\text{ s} \leq t \leq 40$ s; (c) 25 m

2.6.2 Questions

1. (b), (d)

2. (a) 14.67 $m\,s^{-1}$; (b) 2 h 56 min 6 s; (c) 1.9 h; (d) (i) 78 $km\,h^{-1}$ (ii) 0 $km\,h^{-1}$

4. (b) 89 $km\,h^{-1}$

6. 50 s

7. (a) 1.8 s; (b) 5.0 s

8. (a) (i) $-40\ km\,h^{-1}$ (ii) 40 $km\,h^{-1}$, south or $-40\ km\,h^{-1}$, north;
(b) (i) $-20\ m\,s^{-1}$ (ii) $-20\ m\,s^{-1}$ in original direction;
(c) (i) $+5\ m\,s^{-1}$ (ii) $-55\ m\,s^{-1}$ in original direction

9. 4.0 $m\,s^{-2}$, north

10. (a) 2.7 s; (b) 27 m s^{-1}

11. (a) 12 m s^{-1}; (b) −6.0 m s^{-2}

12. (a) 3.45 s; (b) −8.05 m s^{-2}

13. (a) 3.2 s; (b) 3.8 m s^{-2} down the slope; (c) 1.6 s

14. (a) B, C; (b) B, D; (c) A, E; (d) A, E; (e) D

15. (a) 10 m, south; (b) 5.0 m s^{-2}, north; (c) 3.0 m s^{-2}, south; (d) 6.1 m s^{-1}, south

16. (a) B; (b) A, D, E; (c) 40 s; (d) 20 m, north; (e) 260 m; (f) D; (g) E; (h) 3.3 m s^{-1}; (i) 6.0 m s^{-1}, south; (j) approx. 3 m s^{-1}, north

18. (a) B, D, F; (b) +20 m; (c) 0.25 m s^{-1}; (d) 30 s; (e) It didn't; (f) C, G; (g) 20–30 s, 50–55 s, 70–80 s; (i) 0.20 m s^{-2}; (j) 0.050 m s^{-2}

20. (a) The jet skier after 8.0 s; (b) (i) 21 m s^{-1} (ii) 33 m s^{-1}

21. 3.7 h or 3 h 41 min

22. 995 m

23. (a) 3.0 s; (b) 2.5 m s^{-2}; (c) 10 s; (d) 80 m

Topic 3 Motion in a plane

3.2 Exercise 1

2. (a) 30 m s^{-1}, N 31° W; (b) 203 km h^{-1}, S 2° E

3.3 Exercise 1

1. (a) 12.8 km, S 51° W; (b) 18 km

2. 12 m s^{-1}, S 57° E

3. 6.4 m s^{-1}, S 39° E

4. (a) X = 78 km, Y = 290 km; (b) X = –5.7 m s^{-1}, Y = –8.2 m s^{-1}; (c) X = 3.8 m, Y = –4.6 m; (d) X = 2 m s^{-2}, Y = –3.5 m s^{-2}

5. 1.1 km h^{-1}, S 28° W

3.4 Exercise 1

1. –1 m s^{-2}, S (or +1 m s^{-2}, N)

2. 56 m s^{-2}, 63° relative to the original direction of motion

3. 2.4 m s^{-2}, N 73° W

4. (a) 21 kn, N 73° E; (b) 21 kn, S 73° W

5. (a) N 60° E; (b) 35 s

3.5.2 Questions

1. no; yes

2. 19 N; 1 N

3. 45° and 225°

4. (c)

7. (a) 50 m; (b) 10 m

8. (a) 25 m, 90°; (b) 2 km, 90°; (c) 1.0 km, 81°; (d) 180 km, 68°

9. (b) (i) 9.2 m, 130°; (ii) 57 km, 234°; (iii) 85 km, 95°; (iv) 2200 m, 24°

11. (a) 4.0 m s^{-2}, N; (b) 10 m s^{-2}, N 37° E; (c) 2.8 m s^{-2}, SE

12. 11 m s^{-2}, S 45° W

13. 645 km, S 36° E

14. (b) 62 km; (c) 33 km, S 12° W; (d) (i) 6.2 km h^{-1} (ii) 3.3 km h^{-1}, S 12° W

15. (a) 216 s; (b) 360 m

16. 3.5 km h^{-1}, 27° to the vertical

17. 1147 m

18. 460 km h^{-1}, S 46° E

19. 11 m s^{-2}

20. 154 km h^{-1}, N 25° W

21. 7 km h^{-1}, N 48° E

22. 91 m, 57° downward angle with the vertical

Topic 4 Forces

4.3 Exercise 1

1. 481 N

4. 39.2 N

5. (a) 31.8 N; (b) 7.8 N

6. 33.4 N

7. 11.8 N

8. 0.18

9. 0.13

4.4 Exercise 1

1. 188 N, S 37° W

3. S 35° E

4. 2.0×10^3 N

5. (a) 1.7×10^4 N; (b) 4.7×10^4 N

6. (a) 0; (b) 1.5×10^4 N

7. (a) 1.7 m s^{-2}; (b) 0.35 m s^{-2}

4.5 Exercise 1

1. 700 N

2. 0.08 kg

3. (a) 2.0 m s^{-2}; (b) 320 N

4. (a) 42 N; (b) 0.42 m s^{-2}; (c) 5.0 m s^{-1}

5. (a) 400 N; (b) 1800 N

4.6 Exercise 1

1. (a) 2.0 m s^{-2}; (b) 200 N; (c) 300 N

4.7.2 Questions

2. (b), (c), (e)

4. (a) 1.4×10^4 N; (b) 5.0×10^3 N; (c) 1400 kg

5. If your mass is m kg, then (a) 9.8 m N; (b) 3.6 m N; (c) m kg

6. The beam balance

7. (a) 3 N, east; (b) 1.4×10^2 N, east

8. (a) 346 N, east; (b) 53.6 N, east

9. (a) (i) equal to (ii) equal to (iii) greater than (iv) less than

10. (b) zero

16. (a) 1.8×10^2 N; (b) 100 N; (c) zero

17. (b) down the hill; (c) weight

18. (a) zero; (b) 3.9×10^2 N (392 N); (c) 4.0×10^2 N (402 N); (d) 6.8×10^2 N

19. (a) down the slope; (c) 2.9×10^2 N (294 N); (d) 2.9×10^2 N (286 N)

20. (a) zero; (b) 9.8 m s^{-2}; (c) 4.9 N down

21. (a) 6.4×10^3 N; (b) zero; (c) 6.6×10^6 N

22. (a) 7500 N; (b) 6.3 m s^{-2}; (c) 31 m s^{-1}; (d) 78 m

23. (a) 9.6×10^6 N; (b) 3.8×10^2 m (375 m)

24. (a) 823 N; (b) 6.6×10^2 N (655 N); (c) 9.9×10^2 N

25. 5.8 m s^{-2}

26. (a) 42 N; (b) 19°

31. (a) 2.0 m s^{-2} to the right; (b) 6.0 N; (c) 8.0 N to the right; (d) 3.5 m s^{-2} to the right

32. (a) 4.0 m s^{-2} to the right; (b) 160 N to the right; (c) 240 N to the left; (d) 240 N to the right

33. $0.86\ m\ s^{-2}$ to the right

34. (a) 20 N; (b) 2 kg; (c) 15 N

35. $T = 36\ N$, $\theta = 34°$

Topic 5 Energy and work

5.2 Exercise 1

1. (a) 1500 J; (b) 392 J; (c) 0 J

2. (a) 940 J; (b) –940 J

3. (a) 600 J; (b) 120 J

5.3 Exercise 1

1. (a) 6.4×10^4 J; (b) (i) approximately 900 J (ii) approximately 2×10^{-8} J

2. $2.0\ m\ s^{-1}$

5.4 Exercise 1

1. (a) 2.4×10^4 J; (b) $28\ m\ s^{-1}$

2. (a) 82 m; (b) 1200 J

3. $1.9\ m\ s^{-1}$

5.5 Exercise 1

1. (a) 0.16 N; (b) $6.8\ m\ s^{-1}$

2. (a) $6.8\ m\ s^{-1}$; (b) $4.8\ m\ s^{-1}$

3. (a) 0 J; (b) $7\ m\ s^{-1}$; (c) 882 J; (d) $5.4\ m\ s^{-1}$

5.6 Exercise 1

1. (a) 1.4×10^6 J; (b) 114 s

2. (a) 1.4 m; (b) 600 W

5.7.2 Questions

3. 59 J

4. zero

5. (a) approx. 1×10^5 J; (b) approx. 40 J; (c) approx. 3×10^3 J; (d) approx. 5×10^{-9} J

6. (a) 1.8×10^5 J; (b) 1.8×10^5 J; (c) $17\ m\ s^{-1}$

10. (a) 6.9 J; (b) 1.7×10^2 W

11. (a) 40%

12. (a) 0.46 N; (b) $7.2\ m\ s^{-1}$

13. (a) zero; (b) zero

15. 90 J

16. (a) 2 kJ; (b) 600 J

17. (a) 200 J; (b) 200 J

19. (a) 3.2 J; (b) 1.0 J; (c) 0.64 m

20. (a) 240 J; (b) $20\ m\ s^{-1}$

21. (a) 1.9×10^3 J (1920 J); (b) 10 N; (c) 3600 J; (d) 5000 J

22. (a) 3.2×10^4 J; (b) At B, $23\ m\ s^{-1}$; at C, $20\ m\ s^{-1}$; (c) 27 m

23. (a) 8.9×10^5 J; (b) 3.6×10^5 J; (c) $30\ m\ s^{-1}$

25. 51%

27. 399 kN

28. 2.2×10^4 W

29. 50 W

30. (a) 3.2×10^3 J; (b) 1.1 kW; (c) none

31. 14 kW

32. 36 W

33. (a) $4.6\ m\ s^{-1}$; (b) 1.7×10^3 W

36. 5.8 kW

Topic 6 Momentum, energy and simple systems

6.2 Exercise 1

1. 2000 N

2. 4.7 m s^{-1}

3. (a) 530 N s; (b) 8.8 m s^{-1}; (c) 4.0 m

4. (a) 15 kg m s^{-1}, E; (b) 1.5 m s^{-1}, E; (c) 2.5 kg m s^{-1}, W; (d) 2.5 kg m s^{-1}, E; (e) 2.5 N s; (f) opposite in direction

6.3 Exercise 1

1. (a) 2.5 m s^{-1}; (b) N 53° E

6.4 Exercise 1

1. 1.3 m s^{-1}

2. 0.12 m s^{-1}

6.5 Exercise 1

1. (a) 0.7 m s^{-1} in its original direction of motion; (b) not elastic

6.6.2 Questions

7. (a) 8×10^2 kg m s^{-1}; (b) 1×10^4 kg m s^{-1}; (c) 6×10^2 N s; (d) 4 N s; (e) 10 kg m s^{-1}

8. (a) 1500 N s due east; (b) 1500 kg m s^{-1} due east; (c) 750 N

9. (a) 60 N s; (b) 90 kg m s^{-1}

10. (a) 2.3×10^4 N s opposite to the initial direction of the car;
(b) 2.9×10^5 N opposite to the initial direction of the car;
(c) 2.1×10^2 m s^{-2}

11. (a) approx. 160 N s; (b) 2.7 m s^{-1}

12. (a) 240 N s upwards; (b) 3.1×10^3 N upwards

16. (a) 3.0 kg m s^{-1} opposite to the initial direction of the toy car;
(b) 3.0 N s in the initial direction of the car

17. (a) 0.30 m s^{-1}; (b) 0.60 m s^{-1}

18. (a) 40 kg; (b) 60 N s; (c) 60 N s; (d) zero

19. (a) 1.7 m s^{-1}; (b) 120 N s; (c) 120 kg m s^{-1}; (d) 120 kg m s^{-2}; (f) 0.92 m s^{-1}

20. (a) 15 m s^{-1}; (b) 1.1×10^4 N s in the initial direction of the car;
(c) 420 N s opposite to the initial direction of the car; (d) 1.1×10^5 N

21. (a) 2.9 m s^{-1}, east; (b) the car;
(c) they experience the same; (d) they experience the same

22. 2.9 m s^{-1}

27. 0.47 m

28. 10 m s^{-1} and 5 m s^{-1} in opposite directions

29. 2.4 m s^{-1}

Topic 7 Wave properties

7.2 Exercise 1

1. 0.15 m

2. 2 m s^{-1}

3. 254 Hz

4. 335 m s^{-1}

7.3 Exercise 1

1. (a) C, D; (b) A, F, G; (c) B, E

2. (a) 40 cm; (b) 15 cm; (c) 10 Hz

7.4.2 Questions

7. $f = 7.317 \times 10^{14}$ Hz

8. 0.8 Hz

11. $\lambda = 1.29$ m

21. 1.02 m

22. 330 m s^{-1}

23. (a) 1.33 m; (b) 5.86 m

24.

v (m s^{-1})	f (Hz)	λ (m)
335	500	0.67
300	12	25
1500	**5000**	0.30
60	24	2.5
340	1000	**0.34**
260	440	**0.59**

Topic 8 Wave behaviour

8.2 Exercise 1

4. 4

5. minimum

8.3 Exercise 1

1. 60 cm

2. (a) 120 cm; (b) 240 m s^{-1} (c) 100 Hz

3. (i) (iv) (v) (vii) (ix)

8.4 Exercise 1

1. 6 cm

2. A

8.6.2 Questions

2. (a) C; (b) A and B

8. 1.50 m

10. (b) 1.0 m (c) 330 m s^{-1}

11. (a) 4.8 m s^{-1}; (b) 0.60 m; (c) 20 cm; (d) 1.2 m; (e) 8

Topic 9 Sound waves

9.2 Exercise 1

1. 335 m s^{-1}

2. 125 Hz

3. 650 Hz

4. (a) 512 Hz; (b) 10.1 m

5. (a) 328 m s^{-1}; (b) 32° C

6. approximately 3 km away

9.3 Exercise 1

1. 10^{-10} W m^{-2}

2. 10^6

3. 30 dB

4. 4 m

5. 0.23 W

6. (a) 1.5×10^{-5} W; (b) 3×10^{-5} W m^{-2}

7. 4×10^{-3} J m^{-2}

8. 2.1 m

9. (a) 123 km h^{-1} (b) 3 km h^{-1}

9.4 Exercise 1

2. 482 m

3. 131 m

4. 58%

5. 1.8 s

6. 5 m

9.5 Exercise 1

1. 6 Hz

2. 330 Hz

3. (a) (i) and (iii)

5. (a) 2.1 m; (b) 1.4 m

9.6 Exercise 1

1. (a) 333 Hz; (b) 667 Hz; (c) 667 Hz

2. 0.78 m

3. 750 Hz

4. (a) 350 Hz; (b) 1400 Hz

5. 75 Hz

6. (i) B; (ii) A; (iii) D

7. 3.3 cm

8. 121 Hz

9.7 Exercise 1

1. (a) 275 Hz; (b) 330 m s^{-1}; (c) 1100 Hz

2. (a) 200 Hz; (b) 320 m s^{-1}; (c) 1400 Hz

3. 59 Hz

4. 3.7 m

5. 0.33 m

9.8 Exercise 1

2. (a) amount of diffraction decreases; (b) amount of diffraction increases; (c) amount of diffraction increases; (d) amount of diffraction decreases

9.9.2 Questions

1. 40 dB

2. 3.2×10^{-6} W m^{-2}

3. 63%

4. (a) 349 m s^{-1}; (b) 2.4 km

5. (a) 1.5 ms; (b) 667 Hz; (c) 0.52 m

6. 90

7. 3.9° C

8. (a) 8000 Hz; (b) 78 dB; (c) 20 Hz; (d) 1400–7500 Hz; (e) 45–100 Hz and 8000–15 000 Hz

9. 5.25 m

10. (a) 0.8 m; (b) 2.35 ms

11. (a) 4 ms; (b) 250 Hz; (c) 1.32 m

12. 2×10^{-2} W

13. 5.0×10^{-7} W m^{-2}

14. 9.0×10^{-5} W

15. 1.5×10^{-2} W m^{-2}

16. (a) 1.6×10^{-5} W m^{-2}; (b) 4.0×10^{-6} W m^{-2}; (c) 2.5×10^{-7} W m^{-2}; (d) 1.0×10^{18} W m^{-2}

17. (a) 27 dB; (b) 55 dB; (c) 97 dB; (d) 33 dB

18. (a) 5×10^{-12} W m^{-2}; (b) 3.2×10^{-10} W m^{-2}; (c) 2.5×10^{-7} W m^{-2}; (d) 0.32 W m^{-2}

19. (a) 220 Hz; (b) 660 Hz

20. (a) 1.6 m; (b) 150 Hz; (c) 0.2 m; (d) 0.53 m

21. (a) 0.6 m; (b) 0.4 m; (c) 160 m s^{-1}

22. (a) 283 Hz, 567 Hz, 850 Hz; (b) 71 Hz, (no second harmonic), 213 Hz;
(c) 283 Hz, (no second harmonic), 850 Hz; (d) 85 Hz, 170 Hz, 255 Hz;
(e) 43 Hz, (no second harmonic), 129 Hz

23. two octaves

24. 871 Hz

25. (b) 346 m s^{-1}

26. 65 cm

27. 17.5 cm

28. (a) 3400 Hz

30. 1.63 s

32. 0.19 m

34. (a) third harmonic; (b) 0.57 m; (c) 600 Hz; (f) 1.7×10^{-3} s; (g) 600 Hz

35. (a) third harmonic; (b) 0.67 m; (c) 510 Hz; (d) 170 Hz;
(e) 1200 Hz, seventh harmonic

36. (a) 1.7 m

37. (a) 0.229 m, 0.040 m; (b) 69°, 9.4°; (c) 5.2 cm

38. (a) 20 m s^{-1}; (b) 416 Hz; (c) 170 m s^{-1}

39. (a) 68 m s^{-1}

40. 110 km h^{-1}

Topic 10 Ray model of light

10.2 Exercise 1

1. D

2. C

3. (b) and (f)

4. gamma rays, X-rays, UV, visible, IR, microwaves

5. (a)

6. 500 s

7. 8.2 W m^{-2}

8. 12.5 W m^{-2}

9. 15.8 m

10. (a) 1360 W m^{-2}; (b) 3.5×10^{17} J

10.3 Exercise 1

1. (a) (i) 55°; (ii) 55°

2. 85 cm

10.4 Exercise 1

1. convex

2. (d)

3. (b), (c) and (d)

4. 1.2 cm

5. 3.75 cm

6. 8 cm

10.5 Exercise 1

1. 1.43

2. 7.14×10^{14} Hz

3. 1.24×10^8 m s^{-1}
4. 1.6 times faster
5. (b), (c) and (d)
6. 60°
7. 11.2°
8. 2.8 cm
9. 1.29
10. 1.3×10^{14}

10.6 Exercise 1

1. (a) converging; (b) 1.5 cm
2. 1.5
3. (a) 3.75 cm; (b) virtual; (c) 3.75 cm high
4. 12 cm
5. (a) –25 cm; (b) diverging; (c) short-sighted; (d) 15.4 cm in front of the lens

10.7 Exercise 1

1. (c)
2. $\theta = \sin^{-1}$ (Y/X)
3. 33.3°
4. 1.46
5. 1.7 m
6. 0.8 m

10.8.2 Questions

1. (a), (c), (d), (e), (f)
2. True
3. a = 40°; b = 40°; c = 50°
4. 1 W m^{-2}
5. 144 W
6. increases by 6°
7. 1.5
9. (a) acetone = 18.1°; glycerol = 16.7°; carbon tetrachloride = 16.8°
10. (a) vi; (b) iii; (c) iv; (d) ii;
(e) none; (f) i; (g) v
11. (a) 17 cm within the mirror;
(b) (i) virtual, (ii) reduced, (iii) upright
12. (a) 200 cm; (b) 60 cm; (c) 26.7 cm
13. (a) The image is real and inverted and located 60 cm from the lens on the other side from the object. It is about 12 cm high;
(b) The image is virtual and upright and located 60 cm from the lens on the same side as the object. It is 18 mm high;
(c) The image is real and inverted and located 10.5 cm from the lens on the other side from the object. It is about 2.5 mm high.
16. M = 50
17. (a) $d_i = -20$ cm, M = 5; (b) $d_i = -7.5$ cm, M = 2.5
18. (a) 5.06 cm; (b) 2.76 m wide; (d) further away
20. (a) 22°; (b) 7°
21. 5 cm
22. 0.85 cm
23. (a) 15 cm; (b) virtual, enlarged; (c) 20 cm from the lens
24. 12 cm, converging
26. 88 cm behind converging lens and 12 cm in front of diverging lens, 3 cm high (inverted)
36. (b) 1.6

Topic 11 Thermodynamics

11.2 Exercise 1

5. 194.5 K

6. –17° C, –107° C

7. W = –100 J, ΔU = 100 J

8. (b) 204° C

9. 2×10^4 J

10. 1175° X

11.3 Exercise 1

1. your mass × 3500 J

4. 22.2° C

5. 20.3° C

6. 68.5° C

7. 6.12×10^6 J

8. 579 J kg^{-1} K^{-1}

9. 2.2 g

11.4 Exercise 1

1. D

2. 3×10^8 m s^{-1}

6. 634 J s^{-1}

7. 1.4×10^{27} W

8. 6400 K

9. 9.53×10^{-7} m

11.5.2 Questions

11. (a) Q = 500 J, $\Delta U =$ 500 J, increase;
(b) Q = −250 J, W = −250 J, $\Delta U = 0$, no change;
(c) Q = 0 J, W = −150 J, $\Delta U = 150$ J, increase;
(d) Q = 0 J, W = −5 J, $\Delta U = 5$ J, increase

16.

	A	B	C
Heat (+ is in)	0	+	+
Work (+ is out)	–	0	+
ΔU	+	+	+

17. (a) cooking oil; (b) ethylene glycol

18. (a) 400 kJ; (b) 1200 kJ

20. 1.15×10^6 J

21. (a) 230° C; (c) liquid; (d) 160 kJ kg^{-1}

22. 8.2 kg

42. (a) 0.012 W; (b) 100 − 27 = 73 W; (c) 3300 K

43. (a) 12.8 times; (b) 1066° C

44. 5690 K

45. 1.0×10^{-5} m

46. (a) 9.4×10^{-6} m; (b) far infra-red

47. (a) 7250 K; (b) 4140 K

48. (b) (i) 7500 K (ii) 10 500 K; (c) (i) 0.4 μm (ii) 0.52 μm

49. (a) 16 times; (b) The wavelength would be halved.

50. The colours of the stars are blue and infra-red; the blue star is 16 times brighter.

Topic 12 Electrostatics

12.2 Exercise 1

3. (a) perspex; (b) steel; (c) wool

4. 1.25×10^{13}

5. 30 km

6. 6.15×10^{13}

7. 8.2×10^{-8} N

8. They would need to be moved 2.8 times further apart.

9. 3.86×10^{-5} C, 1.14×10^{-5} C

10. 24 N, towards $x = 0$

12.3 Exercise 1

3. A

4. 1.08×10^{5} N C^{-1}, to the left

5. 2×10^{6} N C^{-1}, upwards

6. 1.2×10^{7} N C^{-1}

7. 1.44×10^{-3} N C^{-1}

8. 217 N C^{-1}, east

9. Between the two charges, at a distance of 9.28 cm from the 60 μC charge

10. 360 N C^{-1}, directed at an external angle of 60° to the line connecting the corner with the electron

12.4 Exercise 1

1. true

2. (a), (b) and (c)

3. 10 000 V

4. 250 V

5. 4.4×10^{4} V

6. (a) 9×10^{7} N C^{-1}, away from the point charge;
(b) 1.44×10^{6} V;
(c) 1.7×10^{7} m s^{-1}

12.5 Exercise 1

1. (a) A; (b) 150 N C^{-1}; (c) C

2. 1.92×10^{-18} J (or 12 eV)

3. (a) 20 000 V m^{-1} (b) decrease in strength

4. (a) 4×10^{-3} N; (b) 2.61×10^{-3} N; (c) 90 V

5. 41 g

12.6.2 Questions

1. 1.60×10^{-19} C

2. (a) $+5.44 \times 10^{-14}$ C; (b) -5.44×10^{-14} C

4. 3.00×10^{-1} N C^{-1}, north

5. (a) 6.88×10^{-17} N, west; (b) 6.88×10^{-17} N, east;
(c) 9.89×10^{-2} N, east; (d) 2.79×10^{-1} N, west

9. (a) 2.00×10^{2} V; (b) 5.20×10^{-1} J

11. (a) 1.3×10^{-4} N;
(b) 1.5×10^{-3} N; the forces remain equal;
(c) 1.0×10^{-3} N, but the force is now an attractive force;
(d) 4.0×10^{-3} N

12. 7.9 N

13. 2.04×10^{-8} C

14. 8 cm from the 4×10^{-6} C charge or 1.2 cm from the 9×10^{-6} C charge

16. 6.0×10^{-3} N

17. 8.5×10^{-2} N

18. 1.3×10^{2} N

19. 5.7×10^{13} C

20. 1.76×10^{12} C

21. $F = 512$ N, $a = 7.7 \times 10^{28}$ m s^{-2}

22. 2.9×10^{-9} N

23. 5.0×10^{5} N C^{-1}, up

24. 1.02×10^{-7} N C^{-1}

30. west

31. (a) right; (b) right (and stronger than in (a))

32. 1.2×10^{7} N C^{-1}

33. 1.44×10^{-3} N C^{-1}

34. (a) The 100 V battery;

(b) 1.6×10^{-17} J;

(c) The answer does not change;

(d) The answer does not change;

(e) The electrons would not be accelerated, so would not gain any energy;

(f) The field strength is doubled.

35. (a) 1.75×10^{17} m s^{-1}; (b) 1.7×10^{-11} s; (c) 2.57×10^{-3} m; (d) 2.57×10^{3} V

36. (b) 8×10^{-13} C

Topic 13 Electric circuits

13.2 Exercise 1

3. false

4. 4.5×10^{-2} A

5. 5 A

6. 3 s

7. 6.94 h

8. 0.4 A

9. 1.875×10^{19}

10. 5×10^{19}

13.3 Exercise 1

2. 14 800 J

3. 12 V

4. 0.75 J

5. 2.4×10^{-19} J

6. 3.42×10^{-5} J

7. 6×10^{-5} C, 60 μC

8. (from top to bottom): 3.3 V, 6.0 V, 31.5 J, 1.02 J, 2.7 C, 31.3 C

9. 6000 J

10. 1.6×10^{-19} J

13.4 Exercise 1

1. (b), (a), (d), (c)

2. 4.8 Ω

3. 2.6 Ω

4. (a) 1.5×10^{-8} Ω m; (b) silver

5. (a) 2.5 A; (b) 96 Ω

6. (a) 5000 Ω; (b) 200° C

7. (a) 0.25 A; (b) 3.3 A; (c) 1.05 A; (d) 5 A

8. 1080 J
9. (from top to bottom): 32 V, 48.4 V, 2.0 A, 3.0×10^{-3} A or 3.0 mA, 1.5 W, 33.3 W
10. (a) $6500\,\Omega \pm 5\%$; (b) $300\,\Omega \pm 10\%$; (c) $1\,\Omega \pm 2\%$

13.5 Exercise 1

1. 13.9 kΩ
2. 960 Ω
3. 4.8 A
4. 710 Ω
5. 10 A
6. 10
7. 2 A
8. (a) 4 A; (b) 4 A; (c) 24 V; (d) 34 V
9. (a) 3 V; (b) 6 V
10. (a) 3.6 Ω; (b) 6.63 A; (c) 4.23 A; (d) 11.3 V

13.6.2 Questions

3. (a) 6.00 C; (b) 3.75×10^{19}
4. 1.40 A
5. (a) 8.96×10^{-1} C; (b) 3.58×10^{-1} A
6. (a) (i) 5.00 C (ii) 3.00×10^{2} C; (b) 3.13×10^{19}, 1.88×10^{21}
9. 2.00×10^{1} V
10. 3.84×10^{2} J
12. 5.00 J
14. 8.00 Ω
15. 6.40 V
16. (a) 5.00 Ω; (b) 72.0 V; (c) 0.500 A; (d) 0.167 Ω; (e) 3.00 A; (f) 10.0 V
17. (a) 24.0 J; (b) 2.00 V; (c) 7.00 C; (d) 4.00 V; (e) 0.125 C; (f) 32.0 J
18. (b), (a), (d), (c)
21. (a) 5.00 A; (b) 5.00 A
22. (a) 2.00 A; (b) R_2
23. 8.00 A
24. (a) 3.00 V; (b) R_1
25. 4.0 V
26. (a) 20.0 V; (b) 20.0 V
27. (a) 3.00 A; (b) 3.00 A; (c) 12.0 V; (d) 48.0 V
28. (a) 20.0 V; (b) 40.0 V; (c) 4.00 A; (d) 10.0 Ω
29. 4.00 A
30. (a) 24.0 V; (b) 24.0 V; (c) 6.00 A; (d) 24.0 V; (e) 8.00 A
31. (a) 3.00 A; (b) 5.00 A; (c) 8.00 A
32. (a) 36.0 Ω; (b) 2.00 A; (c) 24.0 V
33. 3.00 Ω
34. (a) 28.0 Ω; (b) 3.00 Ω; (c) 39.0 Ω; (d) 4.13 Ω; (e) 90.0 Ω; (f) 0.400 Ω
35. (a) (i) 0.500 A, 0.500 A (ii) 2.50 V, 3.50 V;
    (b) (i) 2.00 A, 2.00 A, 2.00 A (ii) 20.0 V, 40.0 V, 80.0 V
    (c) (i) 2.5 A, 2.5 A (ii) 5.9 V, 9.1 V;
    (d) (i) 2.03 A, 2.03 A, 2.03 A (ii) 22.7 V, 41.4 V, 63.9 V;
    (e) (i) 6.00 A, 4.00 A (ii) 12.0 V, 12.0 V;
    (f) (i) 4.00 A, 2.00 A, 0.800 A (ii) 48.0 V, 48.0 V, 48.0 V;
    (g) (i) 7.23 A, 4.88 A (ii) 125 V, 125 V;
    (h) (i) 12.1 A, 4.28 A, 7.13 A (ii) 30.5 V, 30.5 V, 30.5 V

36. (a) $39 \pm 2\,\Omega$; (b) $56\,000 \pm 5600\,\Omega$; (c) $750 \pm 37.5\,\mathrm{k}\,\Omega$

37. (b) 0; (c) ∞ (infinity); (d) $32.5\,\Omega$

38. 0.45 W

39. (a) $960\,\Omega$; (b) $5.7\,\Omega$; (c) $3.6\,\Omega$

40. 1200 W

41. (a) 6 mA; (b) 140 V; (c) $8.75\,\mathrm{k}\Omega$

44. (a) $6.0\,\mathrm{k}\Omega$; (b) $660\,\Omega$; (c) $3.3\,\mathrm{k}\Omega$; (d) $2.5\,\mathrm{k}\Omega$

45. (a) 10; (b) $40\,\Omega$; (c) $2.5\,\Omega$

46. 6.2 A

Topic 14 Magnetism

14.2 Exercise 1

8. North

14.3 Exercise 1

1. (a) field strength would be doubled;
(b) field strength would decrease;
(c) field strength would remain the same but would act in opposite direction

2. $2.0 \times 10^{-7}\,\mathrm{N\,A^{-2}}$

4. 1.6×10^{-6} T to the right

5. 48

6. (a) 7.5×10^{-5} T into the page; (b) 3.3×10^{-5} T into the page;
(c) 1.08×10^{-4} T into the page; (d) 4.2×10^{-5} T into the page

7. 6.3×10^{-2} T

8. 6000

9. 5.1×10^{-4} T

10. 1500 A

14.4 Exercise 1

2. (b)

14.5 Exercise 1

4. (a) up the page; (b) down the page; (c) to the right; (d) out of the page

5. 1.8×10^{-2} N

6. (a) 0; (b) 2×10^{-4} N

7. 1.25 N

8. 2.3 T

9. $1.1 \times 10^{6}\,\mathrm{m\,s^{-1}}$

10. 1.7×10^{-3} T

11. 0.18 N

12. 9×10^{-3} N

13. (a) 6.8×10^{-3} N down the page; (b) 1.5×10^{-4} N out of the page

14.6.2 Questions

17. (a) 12.5 T

18. (a) 6.0×10^{-2} N; (b) iii; (c) v

24. (a) into page; (b) into page; (c) S; (d) out of page;
(e) N; (f) E; (g) SE; (h) S

25. (b) force up the page; (c) force down the page

26. 0.07 N

27. 1.4 N

28. 0.0058 N

34. (a) 4.6×10^{-14} N; (b) $5.1 \times 10^{16}\,\mathrm{m\,s^{-2}}$

35. (a) 2.0×10^{-13} N; (b) 2.2×10^{17} m s^{-2}; (c) 1.2×10^{14} m s^{-2}

36. (a) down the page; (b) out of the page; (c) obliquely right and down

38. out of the page

39. (a) 0.043 mm; (b) 78 mm; (c) 16 mm

40. 6.3 mT

41. 1.1×10^{-17} kg m s^{-1}

43. (a) 5.9×10^{6} m s^{-1}; (b) 3.56×10^{4} V m^{-1} or N C^{-1}; (c) 1.78×10^{3} V

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