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Determination of the Klein–Cook parameter in ultrasound light diffraction

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Abstract

For weak and strong acousto-optic interaction analytical solutions are available which describe the diffraction problem. In the intermediate range between these limiting cases, however, the light–sound interaction is far more difficult to analyze for two reasons: (i) Only numerical approaches are available and (ii) the diffraction problem depends on two parameters: the Raman–Nath parameter and the Klein–Cook parameter. This paper describes a technique which makes possible the determination of the Klein–Cook parameter with an adequate accuracy for many fields of application. Once the Klein–Cook parameter is known, the Raman–Nath parameter can easily be obtained from numerical approaches.

Keywords: Acousto-optics; Light diffraction; Ultrasound field investigation

1. Introduction

In ultrasound light diffraction, two limiting cases are usually considered. The diffraction of light by a thin phase grating is characterized by many diffraction orders with distinct propagation directions and Doppler shifts. In this so-called Raman–Nath regime, the light intensity in the individual diffraction orders is given by the square of Bessel functions [1]. The diffraction of light by a thick phase grating is distinguished by only one diffraction order provided that the light is incident at a specific angle, the Bragg angle [2]. This limiting case is referred to as the Bragg regime.

Both limiting cases of light–sound interaction can be characterized by the Klein–Cook parameter, Q; this parameter was originally introduced by Klein, Cook and Mayer in 1965 [3]. The condition $Q \ll 1$ applies to the Raman–Nath regime and $Q \gg 1$ applies to the Bragg regime. The permissible upper value of Q for the Raman–Nath regime has been the subject of various investigations [4–7]. Similarly, the lower value for pure Bragg reflection has been investigated [5,6]. Independent of the respective definitions, in the intermediate range between the limiting cases, knowledge of the actual Klein–Cook parameter is, however, indispensable in many fields of application. Although there are various techniques for the determination of the Klein–Cook parameter, a convincing method suitable for practical use is not yet available. The main reason for this lack is the vagueness of the boundaries of the sound field and with this of the light–sound interaction length.

In this paper a method is presented which allows Q to be measured with an accuracy which is adequate in many fields. It is assumed that the sound field consists of purely sinusoidal progressive waves with plane or almost plane phase surfaces. For this purpose, measurements are to be made very close to the sound source in order to avoid finite amplitude effects. The organization of the paper is as follows: In Section 2 the basic theoretical formulas and the experimental arrangement are briefly described. The measurement method and the results are dealt with in detail in Section 3, with special regard to the experimental requirements. Finally, in the discussion in Section 4 a bridge is thrown to applications which are of practical interest, and the consequences are discussed.

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2. Basic theoretical formulas and experimental arrangement

In the intermediate range between the Raman–Nath regime and the Bragg regime the amplitude of the electric field of the diffracted light becomes complex-valued. As a consequence, additional optical phase shifts occur. Moreover, the light intensity depends not only on the sound pressure but also on the sound frequency. Since only numerical approaches have so far been made for the intermediate range, the discussion is confined to a general formulation. Let the normalized electric field amplitude in the nth diffraction order for normal light incidence be represented by:

$$E_n(v, Q) = \{\alpha_n(v, Q) + i\beta_n(v, Q)\} \exp(in(\Omega t - Kx)), \tag{1}$$

where

$$v = k \frac{\partial n}{\partial p} pL \tag{2}$$

is the Raman–Nath parameter and

$$Q = 2\pi\lambda L/\mu_0 \Lambda^2 \tag{3}$$

is the Klein–Cook parameter. Here, Ω is the angular frequency of the sound, k, K and λ, Λ the wave numbers and the wavelengths of the light and the sound, $\partial n/\partial p$ the adiabatic piezo-optic coefficient, L the sound field depth, p the sound pressure amplitude, μ_0 the refractive index of the undisturbed medium and x the direction of sound propagation. The normalized light intensity in the 0th and ± 1st orders and the combination of both the 0th and ± 1st orders then read as follows:

$$I_0(v, Q) = \alpha_0^2(v, Q) + \beta_0^2(v, Q), \tag{4}$$

$$I_{\pm 1}(v, Q) = \alpha_1^2(v, Q) + \beta_1^2(v, Q), \tag{5}$$

$$I_{0,\pm 1}(v, Q) = C_0 - C_1 \sin(\Omega t - Kx), \tag{6}$$

where

$$C_0 = \alpha_0^2 + \beta_0^2 + \alpha_1^2 + \beta_1^2 = I_{0,\pm 1}^{\mathrm{dc}}(v, Q) \tag{7}$$

is the dc component and

$$C_1 = 2\sqrt{(\alpha_0\alpha_1 + \beta_0\beta_1)^2 + (\alpha_0\beta_1 - \alpha_1\beta_0)^2} = I_{0,\pm 1}^{\mathrm{ac}}(v, Q) \tag{8}$$

represents the interference term. The acoustical phase and all additional optical phase shifts [7] are disregarded in Eq. (6) since they are of no interest any longer.

Various methods are available for the numerical solution of these equations. As early as 1967, Klein and Cook [6] solved the diffraction problem by converting the well-known Raman–Nath system of equations [1] to a set of difference equations. A further approach is the N order approximation method (NOA method) which was originally introduced by Nagendra Nath [9] for $N = 1$ and later on extended by Mertens et al. [10] and by Blomme [11,12]. Similar to the Klein–Cook approach, the NOA method, too, is based on the assumption that the number of orders of any diffraction pattern containing any significant amount of light is limited. This assumption is in agreement with experimental findings. Mertens et al. [10] solved the diffraction problem using both an eigenvalue method and an operator method, while Blomme [11,12] applied the Laplace transform. All these methods are based on plane-wave assumptions for both the acoustic and the optic field. A more general theory was developed by Venske et al. [13] who used the split-step Fourier method. In the experiments presented here, the plane-wave assumptions are appropriate. In this paper, the NOA-method will be used for the numerical calculations.

The experimental arrangement for the determination of the Klein–Cook parameter is shown in Fig. 1. The collimated beam of an He-Ne-laser source passes the sound field and is brought to focus in the plane of the spatial filter by the lens. Furthermore, the lens is used to image the sound beam to the image plane on a magnified scale. The transducers can be adjusted so that normal incidence of the light beam upon the sound field can be achieved. The width and vertical position of the spatial filter can be varied to verify the experimental situations specified in Eqs. (4) to (6).

3. Measurement method and results

The subject under discussion can be explained best by means of Fig. 2. This figure shows the light intensity in the zero diffraction order I_0 (Fig. 2a), the dc component of the zero and first diffraction orders $I_{0,\pm 1}^{\mathrm{dc}}$ (Fig. 2b) and the amplitude of the interference term between the zero and first diffraction orders $I_{0,\pm 1}^{\mathrm{ac}}$ (Fig. 2c) as a function of both the Raman–Nath parameter and the Klein–Cook parameter for normal light incidence. The

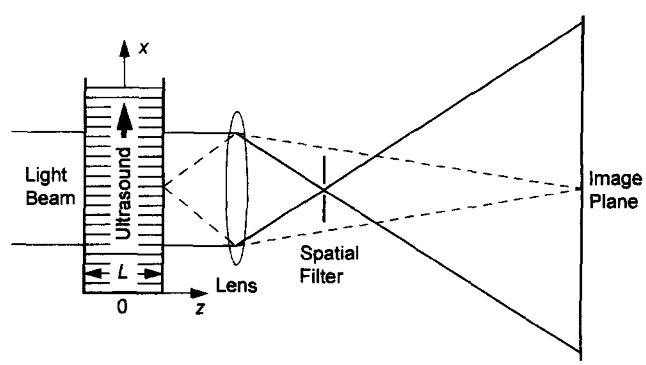
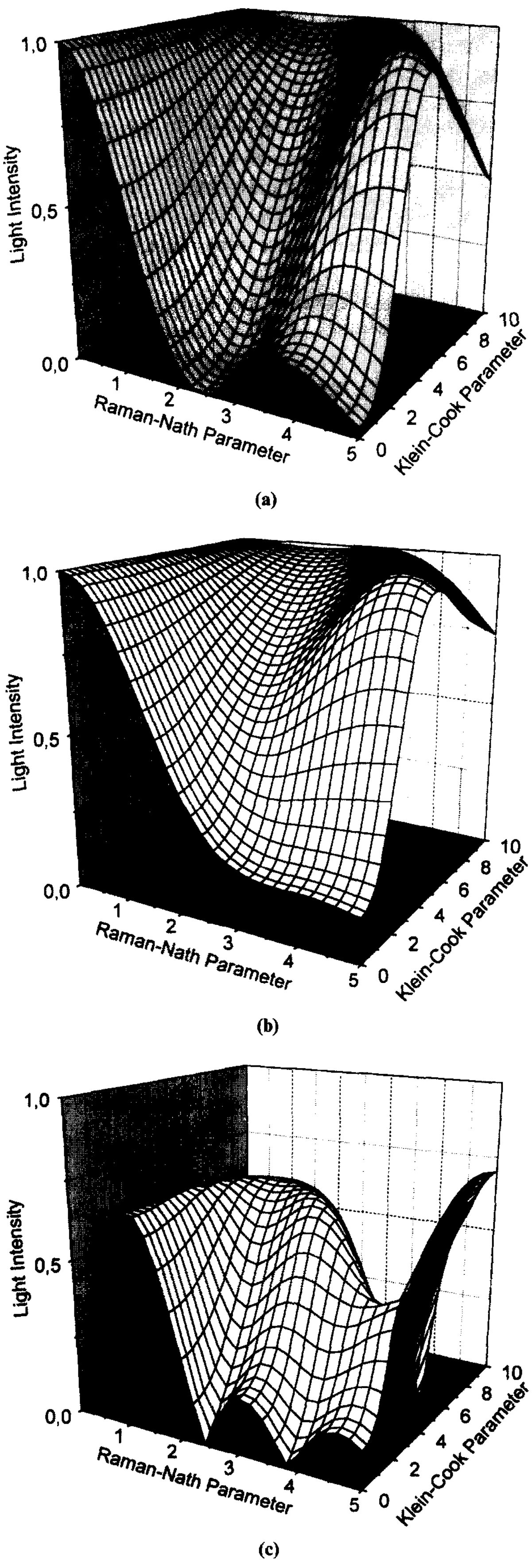


Fig. 1. Schematic representation of the experimental arrangement. The measurement plane $z \geq L/2$ is imaged onto the observation plane on an enlarged scale. Spatial filtering is performed in the focal plane of the lens.



profiles were calculated using the N order approximation method (NOA-method). It can be seen that for $Q \approx 0$ the Raman–Nath approach or Bessel function solution applies, i.e. $I_0 = J_0^2(v)$, $I_{0,\pm1}^{dc} = J_0^2(v) + J_1^2(v)$, $I_{0,\pm1}^{ac} = 2J_0^2(v)J_1^2(v)$. For $Q \approx 10$, the diffraction effect approaches the lower limit of pure Bragg diffraction ($Q = 4\pi$) [5,6]. It should be mentioned that, in the Raman–Nath regime and for pure Bragg diffraction, the light intensity is only a function of the Raman–Nath parameter. It is obvious from the figure that in the range between the Raman–Nath regime and the lower limit of Bragg diffraction, the light intensity strongly depends on both the Raman–Nath parameter and the Klein–Cook parameter.

From the point of view of experimental practice, only the sound pressure amplitude and with this the Raman–Nath parameter can be varied without the structure of the sound field being affected, provided that non-linear distortion of the sound propagation can be neglected. A variation of the Klein–Cook parameter by a change of the frequency is inadmissible since the sound field will undergo diffraction as it travels through the medium, and hence the light–sound interaction length will change as well. From Fig. 2 it follows that, for the specified physical conditions, the light intensity can be measured by varying the voltage U applied to the transducer ($U \propto v$); Q will be kept constant under this condition. The best fit of the measured intensity shape with one of the curves depicted in Fig. 2 will then yield the scaling factor for the v-axis and at the same time the Klein–Cook parameter. This procedure has proved to be an excellent one. It is, however, too time-consuming for the sampling of the data of an ultrasound beam.

The investigation has, therefore, been concentrated on the determination of the Klein–Cook parameter from the intensity minimum which occurs in a range around $v \approx 2.4$, which means around the first zero of the Bessel function $J_0(v)$. For I_0, the minimum light intensity shifts from $v = 2.4$ to $v = 1.53$ in the range $0 \leq Q \leq 10$, for $I_{0,\pm1}^{dc}$ from $v = 2.85$ to $v = 2.24$ in the range $2 \leq Q \leq 10$ and for $I_{0,\pm1}^{ac}$ from $v = 2.4$ to $v = 2.24$ in the range $0 \leq Q \leq 5$. Fig. 3 shows the minimum light intensity as a function of the Klein–Cook parameter for the physical conditions represented in Fig. 2. The minimum light intensity in the zero order increases monotonically with the Klein–Cook parameter. It reaches the value $I_0 =$ 1for $Q = 4\pi$, which means that the diffraction effect is completely cancelled under this condition [5,6]. The

Fig. 2. Normalized light intensity as a function of the Raman–Nath parameter and the Klein–Cook parameter for normal light incidence calculated by the NOA method [11,12]. (a) Zero diffraction order $I_0(v, Q)$, (b) dc component of the zero and first diffraction orders $I_{0,\pm1}^{dc}(v, Q)$, (c) ac component of the zero and first diffraction order $I_{0,\pm1}^{ac}(v, Q)$.

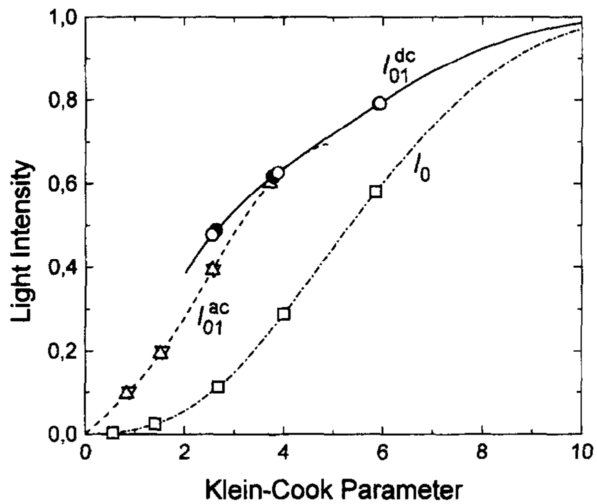


Fig. 3. Minimum of the normalized light intensities I_0, $I^{dc}_{0,\pm 1}$, $I^{ac}_{0,\pm 1}$ in the range $0 \leq v \leq 3$ as a function of the Klein–Cook parameter. The experimental data points are represented by symbols. Similar symbols are used for the combinations of the 0th and +1st and the 0th and −1st diffraction orders.

minimum light intensity for the zero and first diffraction orders $I^{dc}_{0,\pm 1}$ also increases monotonically with increasing Klein–Cook parameter. Below $Q \approx 2$, no intensity minimum occurs in the range $0 \leq v \leq 3$ (see also Fig. 2b). Finally, the minimum light intensity of the interference term $I^{ac}_{0,\pm 1}$ is distinguished by a steeper slope compared with I_0 and $I^{dc}_{0,\pm 1}$ and a maximum for $Q \approx 5$. For completeness, it should be mentioned that the maximum light intensity of $I^{ac}_{0,\pm 1}$ does not provide appropriate experimental conditions because of its insignificant variation with Q (see also Fig. 2c).

The numerical simulations shown in Fig. 3 were experimentally verified using a lithium niobate transducer 25.4 mm in diameter, with a nominal fundamental frequency of 1 MHz. This transducer was excited at harmonic frequencies to cover the range from 5 MHz to 13 MHz. The experimental arrangement used was in accordance with that shown in Fig. 1. The sound-transmitting medium is degassed and deionized water. This is a useful precondition to ensure stable experimental conditions.

To avoid a non-linear distortion of the ultrasound waves, samples were taken very close to the linearly responding transducer. By varying the distance it was made sure that the production of higher harmonics in the sound wave was negligible. In the adjustment of optical arrangement and transducer, special care was taken to guarantee a symmetrical diffraction pattern. The requirements were threefold: First, the laser beam had to be accurately collimated. Secondly, care had to be taken that light incidence upon the sound field was normal. Thirdly, the lens in Fig. 1 had to be correctly adjusted, in particular as regards the interference term $I^{ac}_{0,\pm 1}$. Data acquisition was computer-controlled. In order to measure the minimum light intensity for the different experimental conditions, the voltage applied to the transducer was swept automatically over a small range around the (first) minimum. It should be stressed that, for this measurement technique, only the minimum light intensity must be known and not the Raman–Nath parameter. The frequencies used were 5.29 MHz, 7.43 MHz, 9.56 MHz, 11.69 MHz and 13.81 MHz.

The results obtained have been added to the theoretical shapes in Fig. 3. This figure allows both the symmetry properties of the diffraction patterns and the uncertainties in the determination of the Klein–Cook parameter to be judged. Independent of the specific experimental technique, there is rather good agreement for the frequency settings used. Only the two lowest Q-values for 5.25 MHz and 7.43 MHz, which have been determined from the light intensity in the zero diffraction order, turned out to be less accurate because of the small amount of light available. When these two measured values are disregarded, the agreement of the Q-values is within $\pm 4\%$ of the respective mean values.

Some useful criteria for an adequate measurement technique can be deduced from the experimental results:

(1) In the range $0 \leq Q \leq 4$, the technique characterized by the interference term of the 0th and 1st diffraction orders is the adequate method. Compared with the other techniques, in this range the shape of $I^{ac}_{0,\pm 1}$ is distinguished by the steepest slope and thus the highest sensitivity. For a rough estimation of the range, Q can be calculated from Eq. (3) using the geometrical dimensions of the transducer. It should be mentioned, however, that, with increasing frequency, the adjustment of the experimental arrangement becomes more and more critical.

(2) In the range $3 \leq Q \leq 8$, both dc techniques can be applied. The measurement of the minimum light intensity of the 0th diffraction order should be preferred in this case because the sensitivity is higher and the requirements as regards symmetry lower.

(3) Although this aspect has not been the subject of the present investigation, it is to be expected that the accuracy in the determination of Q will substantially decrease for $Q \geq 8$. The reason for this assumption lies mainly in the imponderable behaviour of the acoustic field, especially as regards non-linear effects, acoustic streaming and heat production. These effects are, of course, not encountered in the theoretical model used.

4. Conclusions

In this paper, a technique has been described which allows the Klein–Cook parameter to be determined. This parameter is an essential quantity in ultrasound light diffraction, especially in the intermediate range between Raman–Nath diffraction and pure Bragg diffraction. When the Klein–Cook parameter is known, the value of the effective sound field depth is known as well (Eq. (3)). It should be stressed that the technique which is characterized by the interference term (Eq. (8)) establishes a consistent link to the Raman–Nath regime.

These measurement techniques may be of particular interest because of their relevance to current technology. They can contribute to an optimization of acousto-optic devices such as modulators, scanners and deflectors. Furthermore, the method of quantitative sound field investigations by light-diffraction tomography [14–19] is no longer restricted to the Raman–Nath regime or a small range beyond this regime but can be substantially extended to higher frequencies.

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