Heisenberg Uncertainty

Quantum Physics

Student Notes

Never Stand Still | Science | School of Physics

Skills Gained

- Fraunhofer diffraction techniques
- Data capture and storage
- Error analysis
- Limits of measurements
- Laser safety (Class II He-Ne laser)

Assumed Knowledge

- PHYS 1121/1131
- PHYS 1221/1231 or PHYS1241
- Optional: MATH2121/2221

1 Experimental aim

You will study the intensity of the diffraction pattern from single slits using monochromatic, coherent light and compare results with the predictions of quantum mechanics to demonstrate the Heisenberg Uncertainty Principle (HUP).

2 Background

The HUP places a fundamental limit on the precision with which we can know – and measure – simultaneously, the values of certain pairs of variables. The HUP is quantified in the famous expressions:

$$\Delta x \Delta p_x \geq \hbar/2$$

for position x and momentum p_x and,

$$\Delta E \Delta t \geq \hbar/2$$

for energy *E* and time *t* where $\hbar = h/2\pi$ is known as the reduced Planck constant. For more information see the PHYS2111 course lecture notes or course texts.

In this experiment you will study the Fraunhofer diffraction pattern of single slits using monochromatic, coherent light from a He-Ne laser.

SAFETY: never look directly into a laser beam

Fraunhofer refers to the condition where the diffraction pattern is viewed at a long distance from the diffracting object – the single slit. This is in contrast to the Fresnel or near-field condition where the diffraction pattern is created near to the diffracting object.

In this experiment the laser light we observe may be considered from both the wave (producing a diffraction pattern) and particle (a stream of photons having momentum which we can calculate) points of view.

Careful measurement and judicious estimation of experimental uncertainties allows a comparison between the measured diffraction pattern and the predictions of quantum theory showing agreement within the limits of your measurements.

2.1 Theoretical background

There are two major topics required here:

1. Recall the theory for diffraction from a single slit (covered in first year physics lectures)
2. Relate the uncertainty in momentum and position of photons passing through a single slit to the HUP.

2.1.1 Diffraction from a single slit

The intensity of fringes in the diffraction pattern of monochromatic, coherent light is given by:

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2$$

where

$$\beta = \frac{\pi d}{\lambda} \sin \alpha$$

where the n^th minimum is at angular position

$$\alpha_n = \frac{\pi d}{\lambda} \sin^{-1} \left(\frac{n \lambda}{d} \right) \qquad n = 1, 2, 3 \ldots$$

For derivations, animations and more information see for example the Physclips pages on diffraction at

http://www.animations.physics.unsw.edu.au/jw/light/single-slit-diffraction.html#1

or

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinint.html#c1

at the HyperPhysics website of Georgia State University, USA.

2.1.2 Photon passing through a single slit: momentum and position considerations

The Heisenberg Uncertainty Principle (HUP) provides the following relationship for position x and momentum p_x:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} = \frac{h}{4\pi}$$

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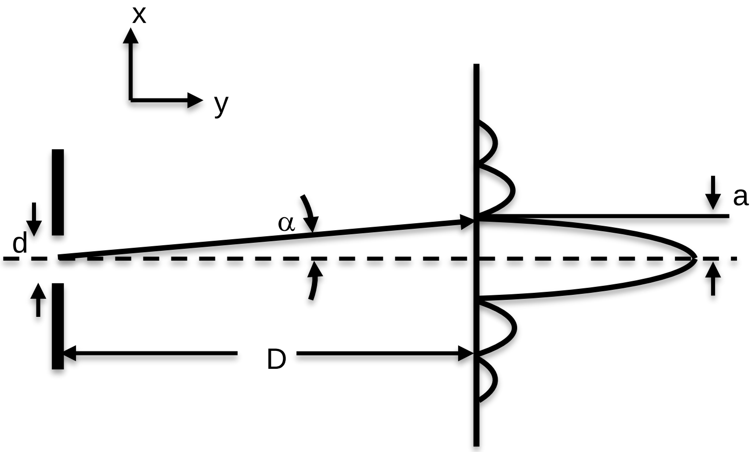
Consider the geometry of the diffracting slit of width d arranged as in the figure below.



Photons passing through the slit have a velocity component in the x-direction so that

$$\Delta x = d$$

$$\Delta v_x = c \sin \alpha$$

where α is the angular position on the screen of the first diffraction minimum subtended at the slit (see diagram above). This gives a momentum uncertainty in the x-direction

$$\Delta p_x = mc \sin \alpha$$

Recalling the de Broglie relation,

$$\frac{h}{\lambda} = p = mc$$

we find the uncertainty in the momentum is

$$\Delta p_x = \frac{h}{\lambda} \sin \alpha$$

and noting that for the first diffraction minimum

$$\sin \alpha = \frac{\lambda}{d}$$

we obtain

$$\Delta x \Delta p_x = h$$

The angular position of the first diffraction minimum is given by (see diagram above)

$$\tan \alpha = \frac{a}{D}$$

and by appropriate substitutions we find

$$\Delta p_x = \frac{h}{\lambda} \sin\left(\tan^{-1} \frac{a}{D}\right)$$

and

$$\frac{d}{\lambda} \sin\left(\tan^{-1} \frac{a}{D}\right) = 1$$

3 Preparation for lab

1. Given the experimental conditions listed in the notes use the Fraunhofer diffraction condition to determine if you will be observing far-field diffraction. (assume the distance between the diffracting slit and detector is 1 m)

2. Calculate the expected diffraction angles for the 0.1 mm slits assuming far-field conditions.

3. Explain how the size of the slit in front of the detector will affect the resolution and uncertainty of the measurements.

4. Consider a similar experiment where photons are replaced with (a) electrons moving with an average speed of 0.15c and (b) buckyballs (C_{60}, m.w. 12g/mol) moving at 220m/s. Calculate the required slit size to the achieve the same first order diffraction angles as in Q2.

4 Experimental plan

Begin by inspecting and identifying the components of the apparatus. Refer to the Operating Instructions provided.

Consider which parameters can be varied and which are fixed or should be fixed.

4.1 What to measure and how?

- Consider which quantities you want to measure and the best ways of doing this.
- How can you optimise the quality of data for the measureable quantities?
- What precision do you require and,
- How many data sets are required to achieve the precision you want?
- How will you estimate the errors on measured quantities such that you will have sufficient confidence that agreement between theory and experiment is met?

4.1.1 Tabulation of data

- Draw up tables for recording of your data, include columns for logging of error estimates and comments on any relevant observations which you'll want to record at the time of the measurements.
- There is usually an obvious procedure to obtain data but pause here and think – is there a superior way of carrying out the measurements that will provide better data?

4.1.2 Pilot (trial run) measurements

- Make a series of test measurements as a 'pilot' to see that what you plan to do is sensible, before you proceed to record extensive data sets.
- Is your planned procedure leading to sensible results? If not identify the problem and rectify.
- The trial runs may quickly reveal better procedures or issues with your experimental plan which needs modification. It's obviously better to identify potential issues at the start.

4.1.3 Analysis and errors

Consider the best way to represent the data in analysis.

- Is a tabulated format best or can graphical representation provide a more convincing analysis – graphs are often the superior analysis if a graphical representation fits the method.
- How will you make the comparison between the HUP theory and the diffraction measurements?
- Errors (experimental uncertainties) are tiresome but they are absolutely key and critical to experiment. Review your error estimates: are they reasonable? Does the analysis indicate that you may have under or over-estimated the uncertainties?
- Make sure that any procedure that you have used to combine uncertainties is correct. There are many good sources of information on error analysis on the web and books on errors are available in the lab. Many top research-intensive universities have information on errors for example see below those from Cornell, Columbia and Oxford (the latter recommended by Berkeley Physics):

  https://courses.cit.cornell.edu/virtual_lab/LabZero/Propagation_of_Error.shtml

  https://phys.columbia.edu/~tutorial/
- http://physics111.lib.berkeley.edu/Physics111/Reprints/Data%20Analysis%20Book%20PDF/Error%20Analysis%20Book-Louis%20Lyons.pdf
- Significant figures: record only the number of digits that is sensible, i.e. that corresponds to the precision of the measurements. Writing down all 8 or 10 digits from your calculator display looks poor – in most undergraduate lab experiments precision is often at the level of a few percent and rarely better than 1 part in 10^4.

5 Planning and writing the report

Plan the report by thinking carefully about what is to be included. A good way to prepare a plan is to write down a series of bullet points or section headings that essentially reflect the structure and content of the report.

Bear in mind the following:

- The most important parts of the report are the data, the experimental uncertainties and the interpretation and discussion of the results.
- Reproducing multiple diagrams and text from the experimental guide or operating instructions is not useful, it is better to refer to these with a statement (e.g. “the set-up is shown in overview in…” or “a close up of the diffraction slits is given in…”, and so on.
- However, *do* include simple sketches, figures or pictures (perhaps obtained elsewhere and so properly acknowledged) where these illustrate an observation, method or some feature that is not given elsewhere in the materials provided.
- Conclusion: always provide a clear Conclusion (as a separate section with it’s own heading, not a few hasty lines buried in amongst calculations as an afterthought). The Conclusion is usually a summary of your findings and results and comment on the outcome overall. A suggestion for further work or something that would be done differently had time allowed may be added.

6 References and Additional Information

To follow in v1.01.