Stirling's Engine

PHYS3113 - Thermodynamics and Statistical Mechanics

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1 Introduction

The Stirling Engine is a heat engine that uses a fluid such as air or other gas to produce mechanical work. By exposing the fluid to different temperatures, the fluid will expand and contract, producing that mechanical work, effectively converting heat energy to mechanical work. The thermodynamic cycle can be found in the Theoretical pre work below.

Stirling Engines can also be used as a refrigerator by reversing the thermodynamic cycle. This allows it for a wide range of applications such as engine-powered aircraft or autmotive engines as well as solar power generation and cryocooling.

2 Method

2.1 Measuring Mechanical Power

2.1.1 Theory

From classical mechanics, the mechanical power of a rotating object is given by

$$P = \tau\omega. \tag{1}$$

Since, the torque of the object is dependent on the angular velocity of the object we have to account for this dependence,

$$P = \tau(\omega)\omega. \tag{2}$$

Then we find that the mechanical power of a system has quadratic dependence on its angular velocity and so we can model the mechanical power of the system based on the following relationship,

$$P = a\omega^2 + b\omega + c \tag{3}$$

where a, b, c are fitted parameters.

2.1.2 Experiment setup

A torque meter was attached to the rotating axle of the Stirling Engine and the counter weight would rotate the needle to measure the applied torque of the engine. The smallest increments on the torque reading was 1×10^{-3} Nm^{-1}, leading to an absolute uncertainty of $\pm 0.5 \times 10^{-3}$ Nm^{-1}. The major source of uncertainty for this experiment and for all other experiments was the frequency reading. This is because the frequency of the spinning wheel was dependent on the temperature of the engine which was constantly flunctuating. To obtain an uncertainty value, we can use the percentage uncertainty from the temperature readings. Since there are two temperature readings, we will take a percentage uncertainty of both temperature measurements and then add them in quadrature.

From the table in Figure 1, we calculate the standard deviation and average of T_h and T_c. Taking the quotient of the two values and then summing them in quadrature gives us an percentage uncertainty of 15.7%. The percentage uncertainty in the torque meter readings was found by taking the average of the quotient of 0.5 and the torque readings which came out to be 5.2%.

2.2 Measuring Electrical Power

2.2.1 Theory

The torque done by the Stirling Engine is used to spin the coil of a generator. The electromotive force (emf) produced by the generator is given by Faraday's law,

$$emf = -n\frac{\Delta\Phi}{\Delta t}. \tag{4}$$

As the magnetic flux is a function of angular displacement, we can say that the emf is a function of angular velocity. The power produced in the circuit is given by

$$\begin{aligned} P &= V(\omega)I \qquad (5)\\ &= \frac{V(\omega)^2}{R}. \qquad (6) \end{aligned}$$

Again, we note the quadratic dependence of angular velocity and so we can model electrical power as a function of angular velocity as a quadratic,

$$P = a\omega^2 + b\omega + c \tag{7}$$

where a, b, c are fitted parameters.

2.2.2 Experimental setup

The torque meter and needle and mass apparatus were detached from the engine. A elastic rubber band was placed along the circumference of the Stirling Engine wheel and connected to a large pulley. The pulley would then spin a coil in a magnetic field to induce an emf. The circuit is set up as shown in Figure 1.

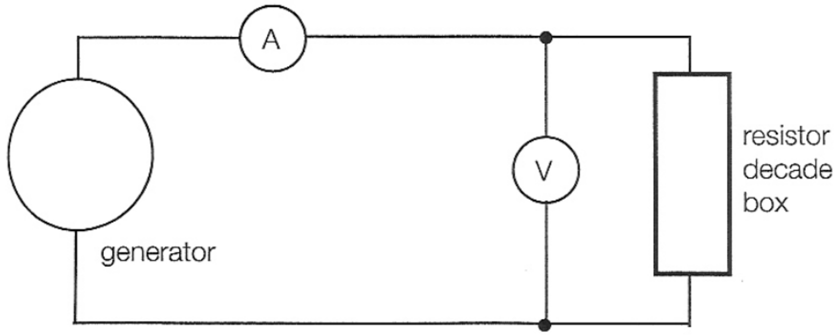

Figure 1: Circuit diagram displaying the circuit used in the electrical power experiment.

The induced emf would give rise to a current in the circuit. Increasing the electrical load by increasing resistance in the decade box would decrease the current in the coil which would lower the emf produced, allowing for the Stirling Engine to spin faster. Conversely, decreasing the load would increase the current in the circuit, increasing the emf induced which then increases the opposition to the torque produced by the Stirling Engine.

The major source of uncertainty in this experiment, like the one previous, would be the temperature and frequency readings. The method of uncertainty calculation was repeated for this experiment, achieving a value of 8.9% .This experiment introduces two new sources of uncertainty being the ammeter and voltmeter readings. According to the Sanwa multimeter manual, the voltmeter has an accuracy of $\pm 0.9\% rdg + 2dgt$ and the ammeter has an accuracy of $\pm 1.8\% rdg + 6dgt$ in their respective ranges. The decade box used as the resistance load has an uncertainty of $\pm 1\%$. A lengthy script doing these calculations can be found in the Appendix, nevertheless we obtain an uncertainty of 8.7%, mainly due to major contributions from the voltmeter readings.

2.3 Cooling

2.3.1 Theory

After shutdown, both chambers of the engine will cool and the relationship between the temperature of the system over time can be modelled using Newton's Law of Cooling.

$$\frac{dT_h}{dt} = -k(T_h - T_c). \tag{8}$$

Since the temperature function of the two chambers are simply exponentials, we can take the temperature difference and model it over time.

$$T_{\text{diff}}(t) = T_{\text{ambient}} + (T_0 - T_{\text{ambient}})e^{-kt}. \tag{9}$$

2.3.2 Experimental setup

The ammeter, voltmeter and the decade box was disconnected. The flame was removed and the engine was allowed to cool. The uncertainty in this experiment was calculated by using the temperature uncertainty values found in the mechanical power experiment.

3 Results

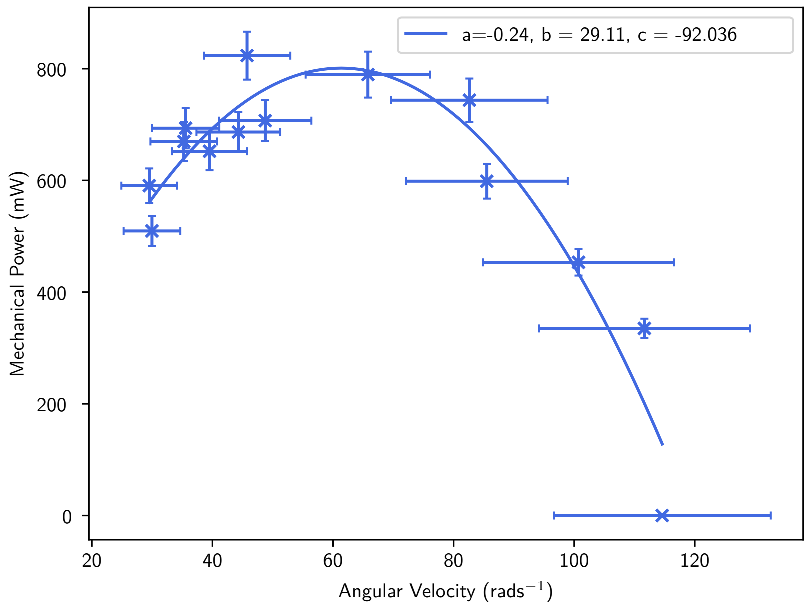

Figure 2: Scatter plot displaying the results of increasing the applied counter torque on the engine over the range of 0 to 20 $\times 10^{-3}$ Nm^{-1}.

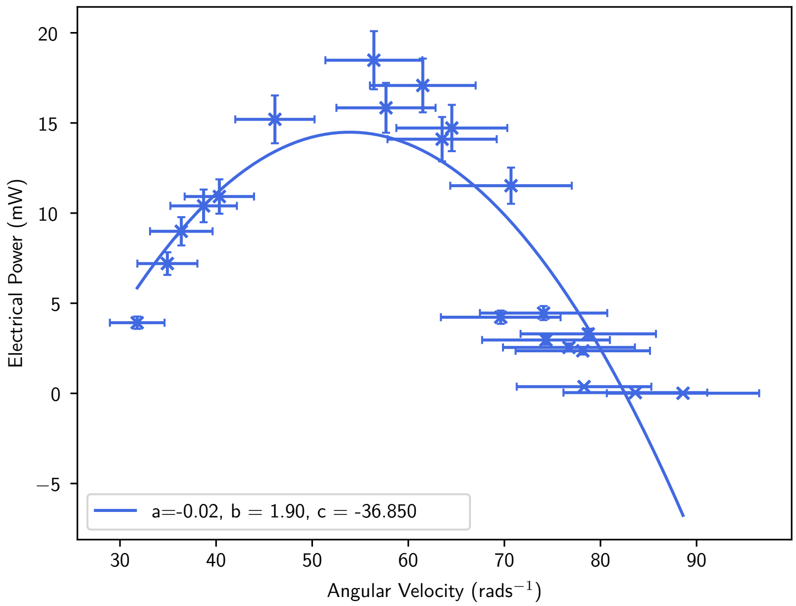

Figure 3: Scatter plot displaying the results of decreasing resistance in the load in the range of magnitudes from 10^7 to 1.

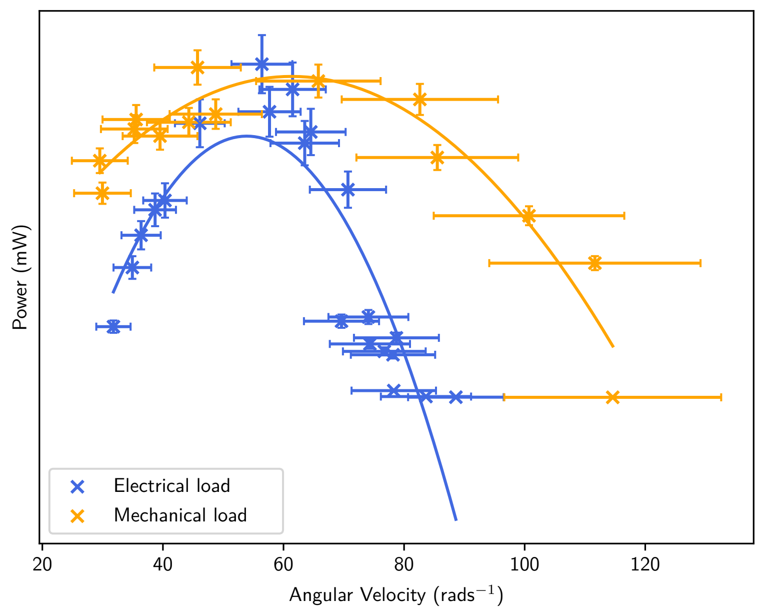

Figure 4: Plot comparing the peaks and shape of the two parabolas

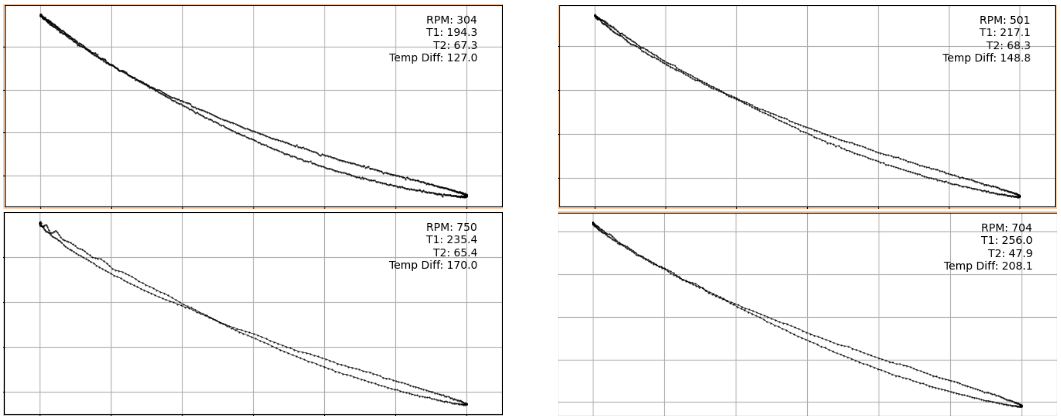

Figure 5: PV diagrams of (a) Top left the engine as a generator providing current to a circuit with a load of 2 Ω (b) Same as befeore but with 30 Ω (c) Max resistance of 11111110 Ω (d) The engine providing mechanical power against a counter torque of 12×10^{-3} Nm^{-1}.

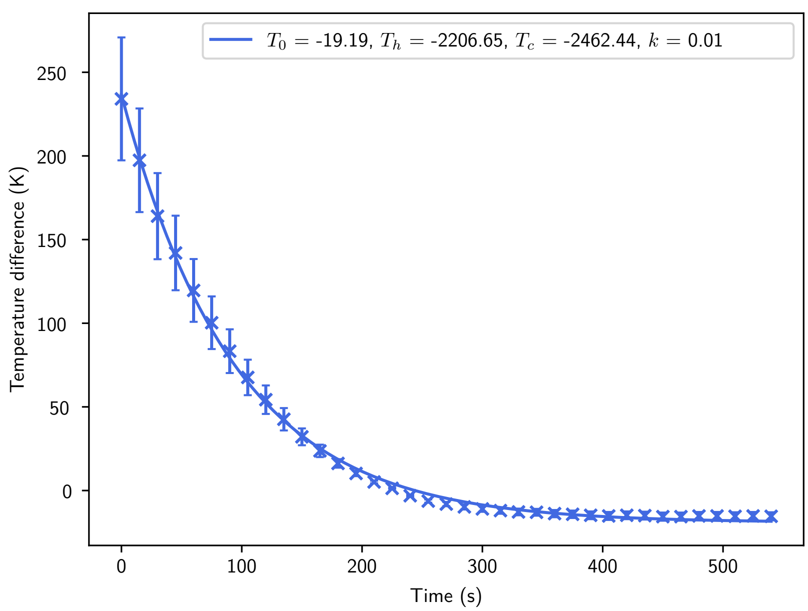

Figure 6: Exponential decay of temperature difference between the two chambers.

4 Analysis

4.1 Optimal angular velocity

Comparing the two parabolas produced by different types of loads in Figure 4, we can see that the optimal angular velocity is substantially different between the two. We find that for the mechanical load experiment, the optimal angular velocity was found to be 45.7 ± 7 rads^{-1} and for the electrical load experiment, the optimal angular velocity was found to be 56.4 ± 5 rads^{-1}. We can also compare the peaks of the two parabolas, using the formula

$$\omega_{peak} = -\frac{b}{2a}. \tag{10}$$

The experimental peak as derived from the parabola of best fit for the mechanical load is 60.6 and for the electrical load, the optimal angular velocity is predicted to be 47.5. We find that neither peak value agree with the predicted value for the same experiment.

4.2 Work done by system

By finding the areas found in the PV diagrams in Figure 5, we can calculate the work done by the system per cycle. One rectangle in the grid weighs 0.0941 g. The half rectangle on the bottom part of the diagrams weighs 0.0468 g. For the PV diagram with a load of 2 Ω, the percentage of the average area of the curve compared to the weight of the paper is then,

$$\begin{align} S &= \frac{0.1003 + 0.0599}{2 \times (18 \times 0.0941 + 6 \times 0.0468)} \tag{11} \\ &= 0.041 \text{ (mV)}^2 \tag{12} \end{align}$$

To convert this reading into a work done per cycle reading we use the conversion factors, $V_{\text{factor}} = 417$ mV/mL and $P_{\text{factor}} = 3.75 \times 10^{-5}$ V/Pa. Therefore, the work done per cycle is then

$$\begin{align} W_{\text{per cycle}} &= S \times V_{\text{factor}} \times P_{\text{factor}} \tag{13} \\ &= 0.041 \times 417 \times 3.75 \times 10^{-5} \tag{14} \\ &= 6.4 \times 10^{-4} \text{ J/cycle} \tag{15} \end{align}$$

To convert this into a power reading, we can multiple by frequency, so for the 2Ω load, the power produced by the engine is

$$\begin{align} P_{2\Omega} &= 6.4 \times 10^{-4} \times 5.06 \tag{16} \\ &= 3.2 \text{ mW}. \tag{17} \end{align}$$

The measured electrical power for this load was 3.9 mW, representing a 18% error.

The uncertainty in cut weigh is quite difficult to measure as there are many random uncertainties such as cutting and screenshotting of the PV cycle which influences the size of the grid. There are also measuring uncertainties such as the mass which can be measured but is negligible in comparison to these other random uncertainties.

The same calculations as above are repeated for different loads shown in the table below in Figure 4.2

Load	Power (mW)	Measured Power (mW)	Error (%)
30 Ω	15.8	18.4	14.1
Max Ω	0.0028	0.0085	67.0
12 Nm^{-1}	748.2	789.4	5.2

5 Discussion

The main problem with this experiment is the random uncertainty brought about by the flame. We were able to quantify this uncertainty by taking temperature measurements for every torque/power measurement we took. To improve we need to use an adiabatic wall or material to surround the flame to ensure there is minimal heat loss to environment.

The results of this experiment are tentative due to the high variance in engine temperature. This is reflected in the high error values found in Figure 4.2 and the non agreeance between values in the electrical and mechanical load experiments.

The method of finding the work per cycle could be drastically improved. The PV cycles found in Figure 5 are quite thin so changing the scales would have greatly improved accuracy of cutting out the curve. The scale factors were given from the instructions which could potentially be outdated. More focus on deriving the scale factors would improve the work per cycle measurement drastically.

6 Theoretical Prework

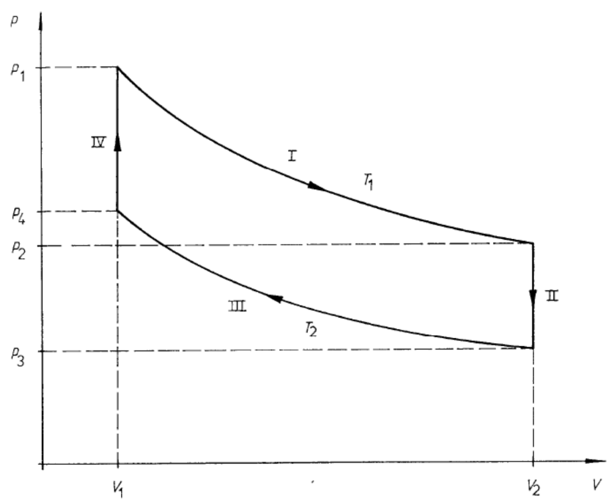

Figure 7: PV diagram of Stirling's Engine

6.1 Derive expressions for the heat flow in (or out) and the work done by each step in the Stirling cycle

The internal energy, heat and work done by the gas of a ideal gas follows the relationship,

$$\Delta U = Q - W. \tag{18}$$

From Figure 7,

I: This is an isothermal expansion, so there is no change in internal energy,

$$\Delta U = 0. \tag{19}$$

Therefore, to find the heat flow of this isothermal process, we can exploit the fact that the heat absorbed by the expansion of the gas is equal to the work done by the gas,

$$Q = W. \tag{20}$$

The work done by the gas can be found using the formula,

$$W = \int_{V_i}^{V_f} PdV. \tag{21}$$

If we use the ideal gas law and substitute P with an equation involving V,

$$PV = nRT \tag{22}$$

$$P = \frac{nrT}{V}. \tag{23}$$

We obtain

$$W = \int_{V_i}^{V_f} \frac{nRT}{V} dV. \tag{24}$$

Since temperature is constant in an isothermal process,

$$W = nRT \int_{V_i}^{V_f} \frac{1}{V} dV. \tag{25}$$

Completing the integral,

$$W = nRT \log \frac{V_f}{V_i}. \tag{26}$$

And so the heat absorbed by the gas as it expands is

$$Q = nRT \log \frac{V_f}{V_i}. \tag{27}$$

III: Since this is also an isothermal process it will follow the same process as in I, however the signage of the work done by the gas and the heat it absorbs changes. This is representative of the fact that now the gas is being compressed and so work is done on it and to ensure the temperature is constant, it must release heat.

II: This is an isochoric process and so the work done is 0. This means that the heat transferred is equal to the internal energy change of the gas,

$$Q = \Delta U. \tag{28}$$

We can then use the formula,

$$Q = nC_v \Delta T. \tag{29}$$

In this specific process, II, there is a depressurisation meaning heat and energy are both released.

IV: This is a isochoric pressurisation meaning there is an inflow of heat and thus an increase in internal energy.

6.2 Calculate the heat flow in each step

We are given the following values for a monoatomic gas undergoing the Stirling Cycle,

$$T_{upper} = 400 \text{ K} \tag{30}$$
$$T_{lower} = 300 \text{ K} \tag{31}$$
$$V_{max} = 10 \text{ m}^3 \tag{32}$$
$$V_{min} = 2 \text{ m}^3. \tag{33}$$

Since it is a monoatomic gas,

$$C_V = \frac{3}{2}R. \tag{34}$$

I: The heat absorbed is

$$Q = nRT \log \frac{V_f}{V_i}. \tag{35}$$

Substituting in values,

$$Q = n(8.314)(400) \log \frac{10}{2} \tag{36}$$
$$= 5352.0\text{J per mole} \tag{37}$$

II: The heat released is

$$Q = nC_v \Delta T. \tag{38}$$

Substituting in values,

$$Q = n(3/2)(8.314)(300 - 400) \tag{39}$$
$$= -1247.1 \text{ J per mole} \tag{40}$$

III: The heat released is

$$Q = nRT \log \frac{V_f}{V_i}. \tag{41}$$

Substituting in values,

$$\begin{aligned} Q &= n(8.314)(300) \log \frac{2}{10} && (42) \\ &= -4014.3 \text{ J per mole} && (43) \end{aligned}$$

IV: The heat absorbed is

$$Q = nC_v \Delta T. \tag{44}$$

Substituting in values,

$$\begin{aligned} Q &= n(3/2)(8.314)(400 - 300) && (45) \\ &= 4014.3 \text{ J per mole} && (46) \end{aligned}$$

6.3 Calculate the work done by the gas in the cycle

Aforementioned, work done is 0 in processes II and IV. The work done by the gas in I is simply equal to the heat absorbed whilst the work done on the gas in III is equal to the heat released.

6.4 Calculate the efficiency of the engine

The efficiency of the engine is calculated by taking the quotient of the work done by the gas and the work done on the gas and the heat absorbed by the gas,

$$\begin{aligned} \text{Efficiency} &= \frac{\text{output}}{\text{input}} && (47) \\ &= \frac{\text{work done by the gas}}{\text{work done on the gas} + \text{ heat absorbed}} && (48) \\ &= \frac{1247.1}{4014.3 + 1247.1} && (49) \\ &= 23.7\%. && (50) \end{aligned}$$

7 Experimental Prework

7.1 Explain how the torque meter works

The torque from the Stirling engine will be countered by the torque created by the mass due to gravity, the counter torque increases as the mass is lifted up as the angle between the force of gravity and the radius from the centre of the torque meter increases. The two torques will be equal when the contraption stops and since the torque due to gravity is easily calculated, the scale can be used to indciate the torque due to the engine.

8 Appendix

8.1 Raw Data

τ (10^{-3}Nm^{-1})	T_h (K)	T_c (K)	ΔT (K)	f (s^{-1})	ω rads^{-1}	P (Js^{-1})
0	171.5	60.2	111.3	18.24	114.6053	0
3	164.5	58.6	105.9	17.77	111.6522	334.9566
4.5	203.2	65.6	137.6	16.03	100.7195	453.2376
9	190.5	63.6	126.9	13.15	82.62389	743.615
7	182.2	62.4	119.8	13.61	85.51415	598.5991
12	177.6	61	116.6	10.47	65.78495	789.4194
14.5	172.9	59.4	113.5	7.76	48.75752	706.984
15.5	175.8	58.4	117.4	7.05	44.29646	686.5951
16.5	186	55.7	130.3	6.29	39.52124	652.1004
17	192.9	54.7	138.2	4.77	29.97079	509.5035
18	220.7	54.1	166.6	7.28	45.74159	823.3486
19	242.5	57	185.5	5.61	35.24867	669.7247
19.5	253.4	58.6	194.8	5.66	35.56283	693.4752
20	249.1	58.2	190.9	4.7	29.53097	590.6194

Table 1: Raw data obtained from mechanical power experiment

R (Ω)	T_h (K)	T_c (K)	ΔT (K)	f (s^{-1})	ω rads^{-1}	$\mathcal{E}$ (V)	I (A)	P (Js^{-1})
11111110	236.2	70.5	165.7	14.1	88.59291	6.5	0.0000013	8.45E-06
1111110	201.1	73.5	127.6	13.31	83.6292	6.56	0.0000065	4.26E-05
111110	187.4	73.1	114.3	12.5E+0	78.28849	6.4	0.0000581	0.000372
11110	186.4	73.1	113.3	12.53	78.72831	6	0.00055	0.0033
1110	199.5	75.1	124.4	12.44	78.16283	5.1	0.000463	0.002361
110	189.7	74.3	115.4	11.25	70.68583	3.6	0.0032	0.01152
70	186	72.5	113.5	10.27	64.52831	3.2	0.0046	0.01472
60	177	68.5	108.5	10.11	63.523	3	0.0047	0.0141
50	178	67.5	110.5	9.79	61.51238	2.8	0.0061	0.01708
40	198.3	69.1	129.2	9.18	57.67964	2.4	0.0066	0.01584
30	225.6	69.9	155.7	8.98	56.423	2.2	0.0084	0.01848
20	204	68.9	135.1	7.34	46.11858	1.6	0.0095	0.0152
10	210	69.3	140.7	6.16	38.70442	0.8	0.013	0.0104
8	200.5	66.7	133.8	6.42	40.33805	0.78	0.014	0.01092
6	203.4	65.8	137.6	5.79	36.37964	0.58	0.0155	0.00899
4	235.4	66	169.4	5.56	34.93451	0.4	0.018	0.0072
2	214.4	64.4	150	5.06	31.79292	0.2	0.0196	0.00392
700	205.8	67.3	138.5	12.21	76.71769	5.2	0.00049	0.002548
400	197.1	70.7	126.4	11.08	69.61769	4.4	0.00096	0.004224
600	190.7	70.3	120.4	11.83	74.33008	4.7	0.00063	0.002961
500	198.5	71.5	127	11.79	74.07875	4.5	0.00099	0.004455

Table 2: Raw data obtained from electrical power experiment

8.2 Code snippets

```
#Theoretical Model
def curve(x,a,b,c):
    return a*x**2 + b*x + c
def curve2(t,T_0,T_h,T_c,k):
    return T_0 + (T_h - T_c)*np.exp(-k*t)
```

```
#Error for Mech Power
t1std = np.std(df1['T1'])
t2std = np.std(df1['T2'])

t1mean = np.mean(df1['T1'])
t2mean = np.mean(df1['T2'])

t1err = t1std/t1mean
t2err = t2std/t2mean

print(t1err,t2err)

xerr1 = np.sqrt(t1err**2+t2err**2)
print(xerr1)

yerr1 = np.mean([0.5/x for x in df1['Torque Meter'] if x > 0])
print(yerr1)
```

```
#Error for Elec Power
t1std = np.std(df2['T1'])
t2std = np.std(df2['T2'])

t1mean = np.mean(df2['T1'])
t2mean = np.mean(df2['T2'])

t1err = t1std/t1mean
```

```
t2err = t2std/t2mean

print(t1err,t2err)

xerr2 = np.sqrt(t1err**2+t2err**2)
print(xerr2)

def digit(V):
    if V == 0 or not isinstance(V,(int,float)) or math.isnan(V):
        return 0
    number = '0.'
    for i in range(1,len(str(V))-2):
        number += '0'
        i += 1
    return float(number + str(abs(V))[-1])

verrlst = [0.009 * V + 2*digit(V) if isinstance(V,(int,float)) and not math.isnan(V) else
                                              None for V in df2s['Voltage (V)']]
aerrlst = [0.018 * A + 2*digit(A) if isinstance(A,(int,float)) and not math.isnan(A) else
                                              None for A in df2s['Current (A)']]

def average(lst):
    clean_lst = [x for x in lst if x is not None]
    return sum(clean_lst)/len(clean_lst) if clean_lst else None

verr = average(verrlst)
aerr = average(aerrlst)

yerr2 = np.sqrt(verr**2+aerr**2+0.01**2)
print(yerr2)
```

9 Lab Book Pages

<u>Experiment notes</u>

No. of stirling engine: #4

Need to measure: frequency (angular velocity), torque, temperature difference

Uncertanties

Torque is 10^{-3}

Temperature: ± 0.05

Frequency: ± 0.005

Torque: ± 1

Voltmeter: ± 0.05

Ammeter: $\pm 0.05\ \mu A$ or $\pm 0.05\ mA$

Large temp diff flunctuations $\Rightarrow$ frequency flunctuation

Time(s)	ΔT (°C)
0	3.6°

Cut and weigh

1 square = 0.0941g

Max Ω smaller square = 0.0468g

Max Ω without black = 0.0606g

30Ω outer = 0.0657g

30Ω w/out outer = 0.043g

2Ω withboth border : 0.1003g

2 w/out border : 0.0599g

12Nm^{-1} with border = 0.0925g

12 Nm^{-1} w/out border = 0.0491g

Cooling

Nathan
19/02/25

Time(s)	ΔT
0	132.5°C
30	107.3°C
60	84.1°C
90	65.8°C
120	51.9°C
150	41.6°C
180	33.6°C
210	27.5°C
240	21.7°C
270	17.1°C
300	14.8°C
330	13.8°C
360	12.6°C
390	11.4°C
420	10°C
450	10°C