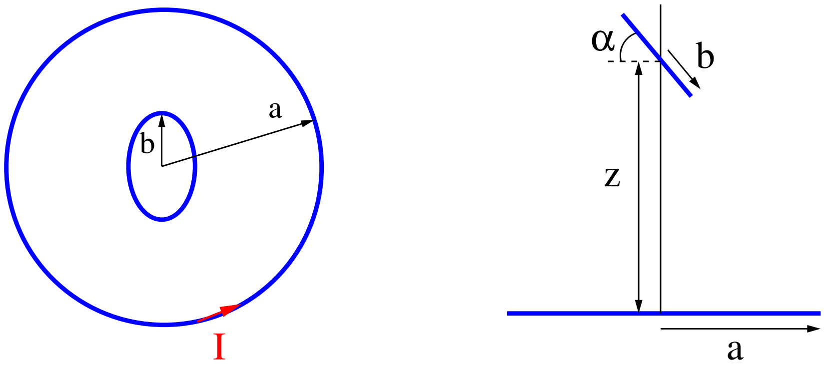
PHYS2114, 2nd year Electromag Assignment, 2024, Due: 11:00 pm, Friday 2 August 2024

Question 1.
(13 marks)

Consider two concentric metallic rings of radii a and b, $b \ll a$. The figure shows the top view



(left) and the side view (right) of the setup. The small ring is shifted up by distance z from the large ring and also it is tilted by an angle α.

a. Derive an approximate expression for the mutual inductance between the two coils.
Note that the condition $b \ll a$ greatly simplifies the calculation.

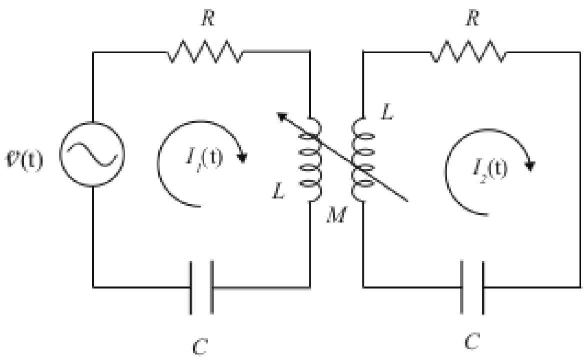
b. The electric current linearly increasing with time

$$I = \beta t$$

is generated in the larger coil. Here β is a constant. The current flows in the anticlockwise direction as it is shown in the left panel of the figure. The resistance of the smaller coil is R.
(i) Find the induced current in the smaller coil.
(ii) Show the direction of the current in the smaller coil and justify your answer.

Question 2.
(13 marks)

Consider the circuit given in the figure that is driven by an AC voltage source and where the two branches are connected by mutual inductance, M.



a. Use Kirchhoff's Laws to determine the relationship between the applied voltage in the first branch and the current flowing in each branch.

b. By setting the losses in the circuit to zero (i.e. $R = 0$), find an expression for the resonant frequencies in terms of M, L, and C

Question 3.
(13 marks)

Maxwell equations in the standard form are shown below in the black font.

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} - cg\vec{B}\cdot\vec{\nabla}a$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} + \frac{g}{c}\left(\vec{B}\frac{\partial a}{\partial t} - \vec{E}\times\vec{\nabla}a\right)$$

It is well established that there is dark matter in the Universe. One of candidates for the dark matter is a particle called axion. In presence of dark matter axions Maxwell equations are modified by terms shown in red. Here a is the axion field, g is a coupling constant, and c is speed of light. In a good approximation the dark matter field is independent of position,

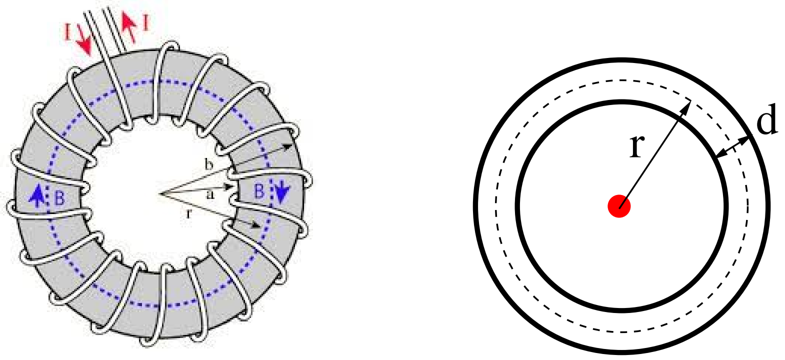
$$a = a_0\cos(\omega t)$$

Assume the following parameters for the dark matter

$$\omega = 10^5 rad/sec\ , \quad a_0 = 2.7\times 10^{-3}GeV\ , \quad g = 10^{-10}GeV^{-1}, \quad \to \quad ga_0 = 2.7\times 10^{-13}\ .$$

Note that values of g and a_0 are given in units different from SI because nobody uses SI in cosmology. However for your purposes you need only the product ga_0 that is dimensionless.

A setup for the dark matter detection is a toroidal coil shown in the left panel of the figure,



$r = 10cm$, $d = b - a = 2cm$. The coil carries a dc current I that creates a dc magnetic field $B_0 = 1Tesla$ inside the coil.

a. Show that according to modified Maxwell Eqs. axions generate the ac magnetic field at a point **R**

$$B(t,\mathbf{R}) = b_0(\mathbf{R})\cos(\omega t + \varphi)$$

b. What is the value of the phase φ?

c. Calculate the ac field amplitude b_0 at the centre of the toroid indicated by the red dot in the right panel of the figure. Present a formula for b_0 and calculate the value of b_0 in Teslas. *Use the approximation $d \ll r$, this approximation greatly simplifies the calculation.*