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Analysis on diffraction properties of the transmission phase grating

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Abstract

Diffraction properties of the transmission phase grating are analyzed in terms of the Fourier optics theory. The analysis results show that the diffraction intensity distribution of the transmission phase grating is closely related to the optical thickness of the grating and the wavelength of the incident light, and not only determined by the grating's period and slit width. For monochromatic incident light, more than 80% of the diffraction energy will concentrate to the first diffraction order under certain conditions. For white incident light, the energy of the first order of diffraction spectrum may be much higher than other orders. Based on these results, some possible applications of the transmission phase grating are discussed.

Keywords: Phase grating; Fourier transform; Diffraction

1. Introduction

Phase grating is an important optical element which has been used widely. Gratings used as multiple imaging devices or beam splitters divide one wave front into many wave fronts [1,2]. Gratings used in the spectrometer transfer most diffraction energy to nonzero diffraction order. The diffraction behavior of phase gratings is usually described by the rigorous diffraction theory [3] or the Kogelnik theory [4]. To calculate the efficiency of gratings, one has to solve the differential equations for the diffracted electromagnetic fields with the grating surface as boundary. So, the solution of this problem used to be rather difficult. Many works in this field had been reported. Schmahl and Rudolph [5] studied holographic diffraction gratings in detail. Goodman [6] analyzed the diffraction properties of the sinusoidal phase grating. In this paper, the diffraction property of phase gratings is analyzed using the Fourier transform [7], and the transmission phase gratings is modelled in a simple way. So the solution becomes quite easy. Our calculation results show that the diffraction field of phase grating is closely related to the optical thickness of grating and the wavelength of incident light. Gratings can be designed according to the practical requirements. For example, under certain conditions, the zero and the second order of diffraction will vanish and the diffraction energy will mainly be concentrated to the first diffraction order for monochromatic incident wave. This phenomenon is discussed in details.

2. The analysis on the diffraction of the phase grating

The structure of the phase grating is schematically shown in Fig. 1. The grating is consisted by two different transparent materials, the refraction indexes of the two

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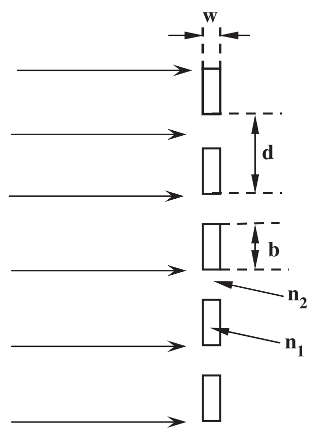

Fig. 1. The structure of the rectangle phase grating.

kinds of materials are noted by n_1 and n_2, respectively (see Fig. 1). The period of the grating is d, the thickness of the grating is w, and the width of the portion with the refractive index n_1 is b. The total width of the grating is L. A plane wave is normally incident on the grating.

2.1. Incidence with monochromatic wave

Suppose that the wavelength of the incident plane wave is λ. The transmission function of a phase grating, $t(x)$, can be considered as the addition of two functions, $t_1(x)$ and $t_2(x)$, as shown in Fig. 2. So,

$$t(x) = [t_1(x) + t_2(x)]\mathrm{rect}\left(\frac{x}{L}\right), \tag{1}$$

where

$$t_1(x) = \frac{1}{d}\,\mathrm{rect}(x/b)\mathrm{comb}(x/d)\exp(j2\pi w n_1/\lambda), \tag{2}$$

$$t_2(x) = \frac{1}{d}\,\mathrm{rect}\left(\frac{x-d/2}{d-b}\right)\mathrm{comb}(x/d)\exp(j2\pi w n_2/\lambda). \tag{3}$$

It is known that the field distribution of Fraunhofer diffraction of an aperture is the Fourier transform of the amplitude distribution across the aperture. As the plane wave is normally incident on the grating, the amplitude distribution across the grating can be described by the transmission function $t(x)$ of the grating. Making Fourier transformation of Eq. (1), we have

$$\begin{aligned} T(f_x) = \{&b\exp(j2\pi w n_1/\lambda)\sin c(bf_x)\sum_{j=-\infty}^{+\infty}\delta(df_x - m) \\ &+ (d-b)\exp[j2\pi(wn_2/\lambda - f_x d/2)] \\ &\times \sin c[(d-b)f_x]\sum_{j=-\infty}^{\infty}\delta(df_x - m)\} \\ &\times L\sin c(Lf_x), \end{aligned} \tag{4}$$

where $m = \pm 1,\ \pm 2, \ldots,$ and $T(f_x)$ is the Fourier transform of $t(x)$, f_x is related to the diffraction angle θ and the wavelength λ, and it is giving by $f_x = \sin\theta/\lambda$.

If $d = 2b$, Eq. (4) can be reduced as

$$\begin{aligned} T(f_x) = 2b\Bigg\{&\sum_{n=-\infty}^{+\infty}\sin c(m/2)\delta(\mathrm{d}f_x - m) \\ &\times \exp[j\pi(w(n_1+n_2)/\lambda - m/2)] \\ &\times \cos[\pi w(n_1-n_2)/\lambda + m\pi/2]\Bigg\} L\sin c(Lf_x). \end{aligned} \tag{5}$$

So, the intensity distribution of the Fraunhofer diffraction of the phase grating can be expressed as

$$\begin{aligned} I(f_x) = L^2&\sum_{n=-\infty}^{+\infty}\sin c^2\left(\frac{m}{2}\right)\cos^2\left(\frac{\pi w(n_1-n_2)}{\lambda} + \frac{m\pi}{2}\right) \\ &\times \sin c^2\left(L\left(f_x - \frac{m}{d}\right)\right). \end{aligned} \tag{6}$$

When $m = 0$, Eq. (6) becomes

$$I_0(f_x) = L^2\cos^2\left(\frac{\pi(n_1-n_2)w}{\lambda}\right)\sin c^2(Lf_x). \tag{7}$$

This equation represents the intensity distribution of the zero diffraction order. Then the peak intensity of the zero diffraction order is

$$I_{0\mathrm{peak}} = L^2\cos^2\left(\frac{\pi w(n_1-n_2)}{\lambda}\right). \tag{8}$$

Similarly, if $m = 1$, we have

$$\begin{aligned} I_1(f_x) = L^2&\sin c^2\left(\frac{1}{2}\right)\sin^2\left[\frac{\pi w(n_1-n_2)}{\lambda}\right] \\ &\times \sin c^2\left[L\left(f_x - \frac{1}{d}\right)\right], \end{aligned} \tag{9}$$

$$I_{1\mathrm{peak}} = L^2\sin c^2\left(\frac{1}{2}\right)\sin^2\left[\frac{\pi w(n_1-n_2)}{\lambda}\right]. \tag{10}$$

In the same way, the peak intensities of higher diffraction orders are: $I_{2\mathrm{peak}} = 0$, $I_{3\mathrm{peak}} = 4L^2/9\pi^2\sin^2(\pi w(n_1-n_2)/\lambda), \ldots$ by making $m = 2, 3, \ldots,$ etc.

We can see that the most diffraction energy concentrate on the zero order and the first order. It is found that, the value of the peak intensities of both zero order and first order, $I_{0\mathrm{peak}}$ and $I_{1\mathrm{peak}}$, vary with the change of the grating thickness w, and they reach their maximums alternately in the case of

$$\frac{\pi w(n_1-n_2)}{\lambda} = \left(k + \frac{1}{2}\right)\pi, \quad k = 0, \pm 1, \pm 2, \ldots. \tag{11}$$

The value of $I_{0\mathrm{peak}}$ is zero and the value of $I_{1\mathrm{peak}}$ reaches its maximum. This means that the zero diffraction order vanishes and the first diffraction order becomes strongest under this condition. More than 80% of the diffraction energy concentrates on this order.

As an example, we calculated the intensity distributions of the diffraction field of a grating with different values of w, and keep all the other parameters unchanged: $n_1 = 1.5$, $n_2 = 1$, $\lambda = 0.5\,\mu\mathrm{m}$, $d = 4\,\mu\mathrm{m}$, $b =$

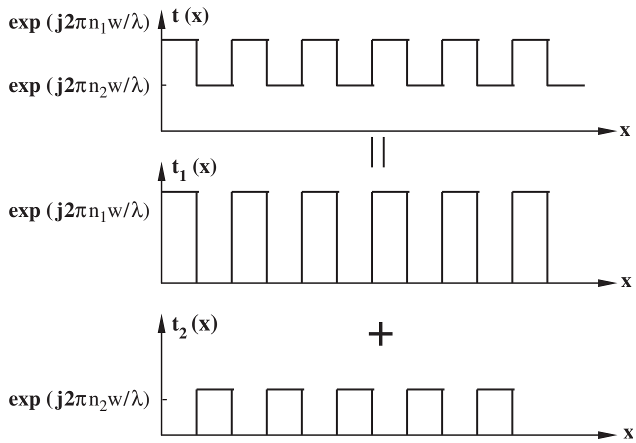

Fig. 2. Structure of a phase grating consist of two separate parts.

$2\,\mu m$, $L = 50\,\mu m$. These parameters are chosen based on our experiments. The calculated results are shown in Fig. 3. Fig. 3(a) is the case for $w = 1.8\,\mu m$, Fig. 3(b) is the case for $w = 1.5\,\mu m$, It is found that the intensity distribution of the two cases is obviously different. For showing the thickness dependence of the intensity distributions of gratings, Fig. 4 gives a 3D plot of the case for $w = 0.1-4\,\mu m$.

From Fig. 4 it can be seen that, when $w = 0.5$, 1.5, 2.5, $3.5\,\mu m$, etc., which satisfy Eq. (11), the zero diffraction orders vanish and the first diffraction orders become strongest. All other higher diffraction orders are very small.

2.2. Incidence with white light

When the grating is normally illuminated by white plane wave, the diffraction fields for various wavelengths are different. Both the diffraction angle and the peak intensity of each diffraction order are wavelength dependent. Comparing Eqs. (7) and (9), it can be found that the first diffraction order is stronger than the zero order i.e. $I_1(f_x)/I_0(f_x) > 1$ when

$$k\pi + \frac{\pi}{4} < \frac{\pi w(n_1 - n_2)}{\lambda} < k\pi + \frac{3\pi}{4}. \tag{12}$$

It means that for all wavelength of the incident beam the first diffraction orders would be stronger than the zero orders by properly choosing the value of w. For example, if the wavelength range of the incident beam is 0.4–0.7 μm, supposing $n_1 = 1.5$, $n_2 = 1$ and $k = 0$, Eq. (12) gives $0.35\,\mu m < w < 0.6\,\mu m$. Thus, the first diffraction order will be stronger than the zero order in this range of w. The diffraction spectra for various wavelengths are calculated by choosing grating thickness $w = 0.5\,\mu m$, Fig. 5(a) and (b) show the results in 2D and 3D plots, respectively. It can be seen that the first order of diffraction spectrum is much stronger than the zero order.

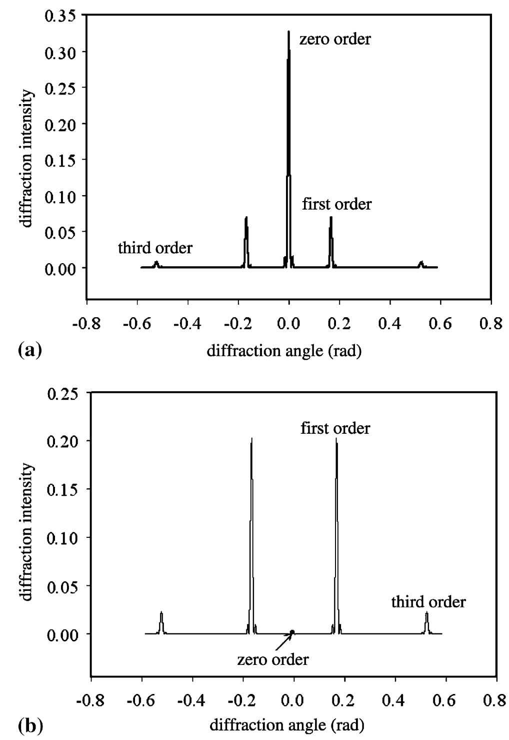

Fig. 3. The intensity distributions of the diffraction field of the phase gratings with various thicknesses. $n_1 = 1.5$, $n_2 = 1$, $\lambda = 0.5\,\mu m$, $d = 4\,\mu m$, $b = 2\,\mu m$, $L = 50\,\mu m$: (a) $w = 1.8\,\mu m$ and (b) $w = 1.5\,\mu m$.

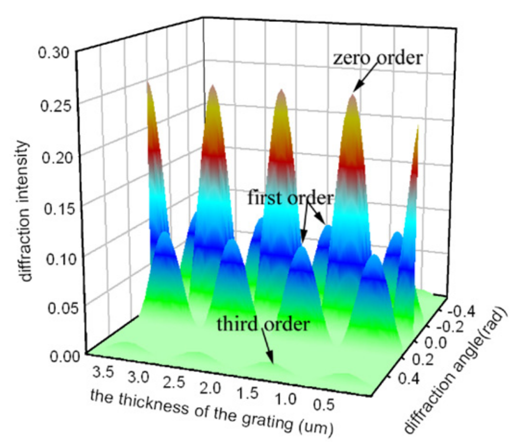

Fig. 4. The intensity distributions of the diffraction field of the phase gratings with various thicknesses, $n_1 = 1.5$, $n_2 = 1$, $\lambda = 0.5\,\mu m$, $d = 4\,\mu m$, $b = 2\,\mu m$, $L = 50\,\mu m$.

2.3. The experimental verifications

The layout for measuring diffraction spectra of phase grating is shown schematically in Fig. 6. The gratings used were made with photo-resist. The CCD used is Toshiba model with the pixel size of $8\,\mu m$.

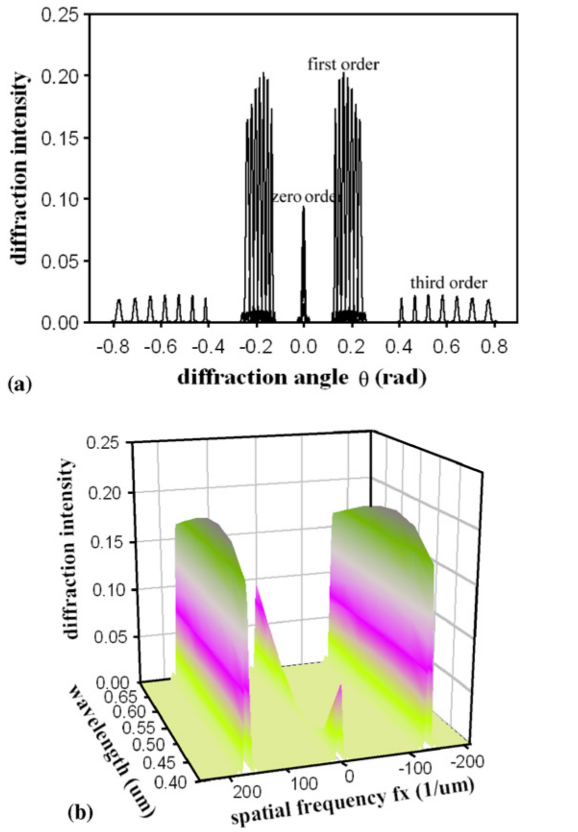

Fig. 5. Calculated diffraction spectra of grating with white light incidence. $d = 3\,\mu\text{m}$, $L = 50\,\mu\text{m}$, $n_1 = 1$, $n_2 = 1.5$, $W = 0.5\,\mu\text{m}$, $\lambda = 0.4-0.7\,\mu\text{m}$: (a) 2D plot, (b) 3D pot.

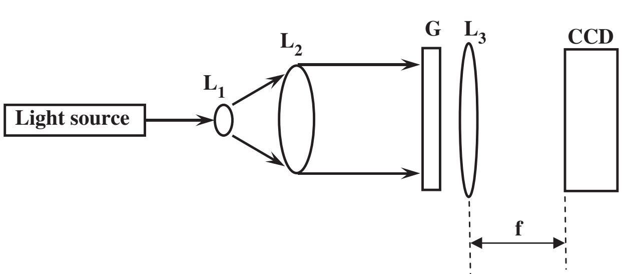

Fig. 6. Measuring layout of diffraction spectra of phase grating.

In our experiments, the white light source is a bromine-tungsten lamp, the monochromatic light source is a frequency stable He–Ne laser. The focal length of the lens L_3 is 28 mm. The diffraction angle corresponding to each diffraction order can be calculated by the focal length of L_3 and the pixel size of the CCD. The period of the grating is $3\,\mu\text{m}$ and the thickness is $0.7\,\mu\text{m}$. Fig. 7 gives the measured results. It can be seen obviously that, in the case of monochromatic light incidence, the measured spectrum (see Fig. 7(a)) is in good agreement with calculated result shown in Fig. 3(b). In the case of white light incidence, the diffraction energy concentrates to the first diffraction order, and all characteristics of the measured spectra are well consistent with the calculated result shown in Fig. 5(a).

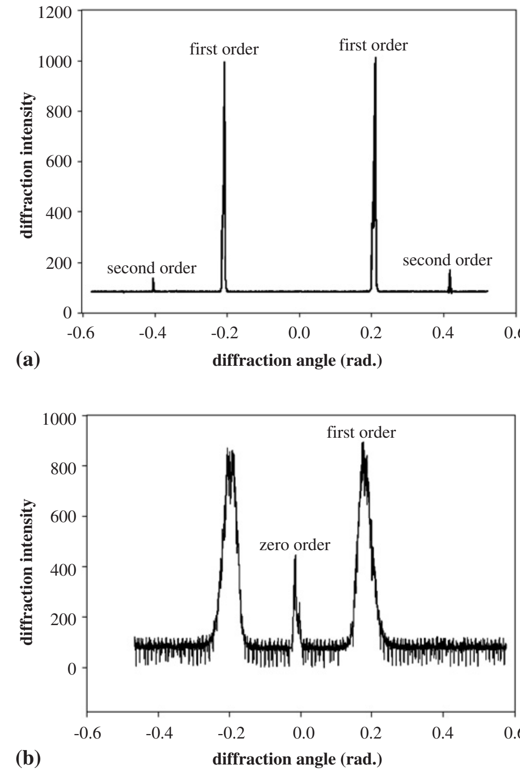

Fig. 7. Measured results of a phase grating: (a) monochromatic incident light, $\lambda = 0.6328\,\mu\text{m}$ and (b) white incident light, $\lambda = 0.4-0.7\,\mu\text{m}$.

3. Discussion

From all mentioned above, it can be seen that the diffraction energy distribution of a phase grating is closely related to the optical thickness of the grating, and not only determined by the grating's period and slit width. If the grating's thickness satisfies Eq. (11), the zero diffraction order will vanish and most diffraction energy will concentrate to the first diffraction order. So, a monochromatic beam can be split into two individual beams with same intensity, their propagating directions can be controlled by carefully choosing the grating's period and the orientation of the grooves. This property can be used in the shearing interferometers and optical interconnections.

Assuming $n_1 = 1.5,\ n_2 = 1,\ w = 0.5\,\mu\text{m},\ L = 50\,\mu\text{m}$, $\lambda = 0.5\,\mu\text{m}$, diffraction energy distributions of gratings with the periods changed from 1 to $4\,\mu\text{m}$ are calculated. Fig. 8 shows the calculated result. It can be seen that the zero diffraction orders for all the periods vanish, and the diffraction angle of the first diffraction order becomes larger with the decrease of the period.

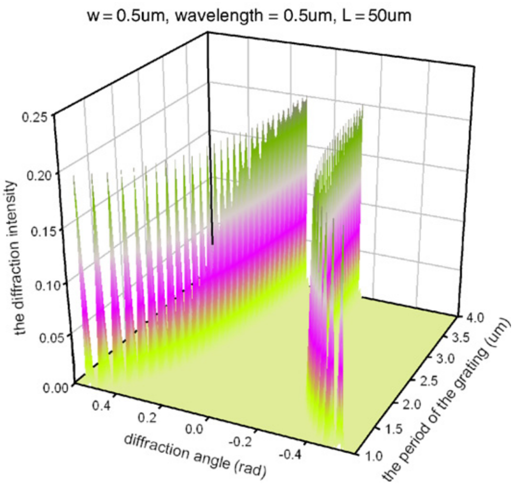

Fig. 8. Diffraction energy distribution of gratings with different periods.

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