# Magnetic Hysteresis

PHYS2114 - Electromagnetism

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# Contents

# 1 Introduction

Magnetic Hysteresis is a phenomenon arising from the make up of ferromagnets. Ferromagnetism involves the magnetic dipoles associated with the spins of unpaired electrons like in the case of paramagnetism. However, due to quantum mechanical effects such as the Pauli exclusion principle which restricts the occupancy of electrons' spin states in atomic orbitals, the magnetic dipoles in ferromagnets tend to 'like' to point in the same direction as its neighbours, making it very distinct from paramagnetism.

Alignment of these dipoles occurs in relatively small patches, called domains. The domains contain billions of magnetic dipoles all aligned in the same direction with each other. Each domain is randomly orientated but can be changed by the influence of external magnetic fields. When an external magnetic field is deployed over a ferromagnet, the torque $N = m \times B$ will align the dipoles parallel to the field however aforementioned, these dipoles will resist in order to be aligned with their neighbours. On the boundaries, there are competing neighbours and so the torque will be most effective here, aligning the dipoles that are closest in parallel to the field.

Once the domain wins some converts from its neighbouring domains, the boundary line is pushed and the domain is expanded and in consequence, the other domains in the ferromagnet will begin to shrink. When one domain expands enough and takes over entirely, the ferromagnetic material is said to be saturated. It should be noted that the magnetic field strength will have diminishing effect on the material as once more and more domains are converted, the difficulty or required magnetic field strength to convert the remaining domains increase up to a saturation point.

This process, however, is not reversible. Once the magnetic field is turned off, not all the domains will return to their original randomly orientated alignments. Some will but there is enough that remain aligned with the now gone magnetic field to produce a permanent magnet. Running a magnetic field in the opposite direction of the first one will re-orientate some of the dipoles to remove this permanent magnet, i.e demagnetise. Some materials will withstand this field more than others, with the property denoted as magnetic coercivity. Magnetically hard materials, which tend to be used as permanent magnets have high coercivity, retaining their permanent magnet status despite the existence of external magnetic fields whilst magnetically soft materials have low coercivity and are such more easily demagnetised and magnetised.

This entire process is visualised in Figure 1 below.

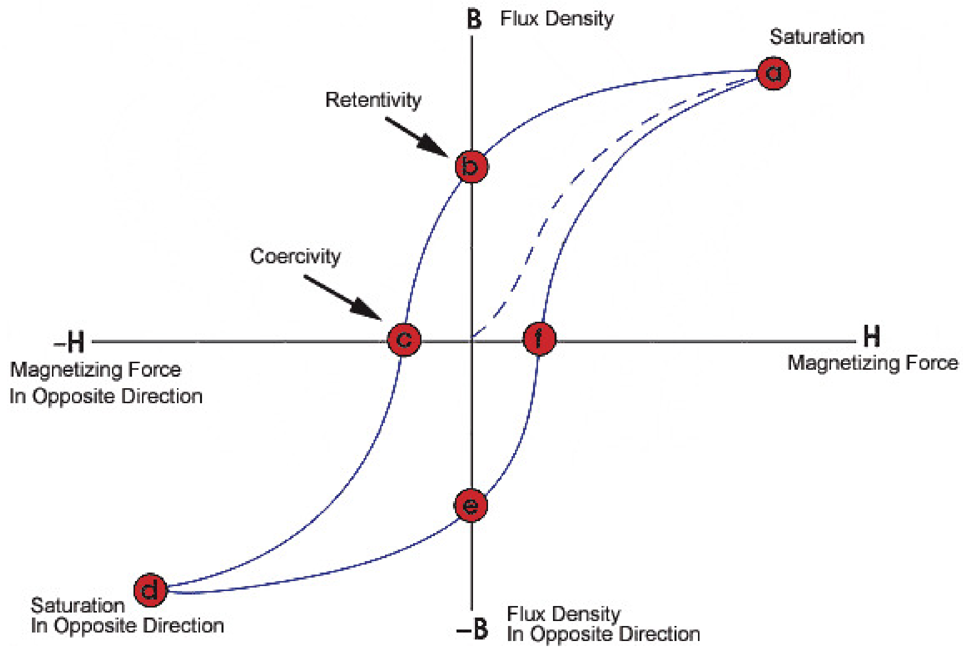



Figure 1: Hysteresis loop

We can see that points a and d represent the saturation points, where the one domain takes over the material completely only under an external magnetising force. Once H goes to zero, i.e the external field is turned off, the material will be at points b or e, retaining some magnetic field. Turning on a magnetic field in the opposite direction will bring the material to points c and f, a state of complete demagnetisation as all the domains cancel out each other before repeating the process once the field is turned on to be stronger. Note the dotted line, the virgin curve, will represents the process in which the material is first put in a magnetic field. At the origin, the material's domains are randomly orientated and so once magnetised, it cannot go back into that state again without undergoing a phase transition. This is why ferromagnets have magnetic history, and its magnetisation is dependent on its state in the hysteresis loop.

The most common way to demagnetise a permanent magnet is to place it in a region of zero magnetic field and then heat it up above the Curie point. This will turn the ferromagnet into a paramagnet destroying the previous alignments of the domains. There are other ways such as just simply dropping the magnet onto a hard surface or placing it between the poles of an electromagnet and then magnetise it back and forth whilst reducing the current to reduce the field slightly but none of these methods will return the magnet to the exact state in which it was before.

## 2 Aim

The aim of this experiment is to validate the model of the hysteresis loop shown in Figure 1 and thus the interpretation of the microscopic interactions of dipoles explored in the previous section. By studying the interaction of traditional ferromagnetic materials with an applied magnetic field, we can gain some insights into the how the microscopic physical model of magnetism relates to macroscopic fields.

# 3 Method

## 3.1 Theory

The auxiliary field is denoted with $H$ and it represents the magnetic field produced by free current,

$$H = \frac{1}{\mu_0}B - M \tag{1}$$

where B is total magnetic field strength and M is the magnetic dipole moment per unit volume in a region.

In linear media, the magnetisation of materials is proportional to the field $H$, not $B$, so

$$M = \chi_m H \tag{2}$$

where $\chi_m$ is the magnetic susceptibility of the material.

Thus,

$$B = \mu_0(H + M) = \mu_0(1 + \chi_m)H. \tag{3}$$

So,

$$B = \mu H. \tag{4}$$

## 3.2 Experimental Setup

To explore the relationship between $H$ and $B$ for a ferromagnet, the following circuit is set up in Figure 2 below.

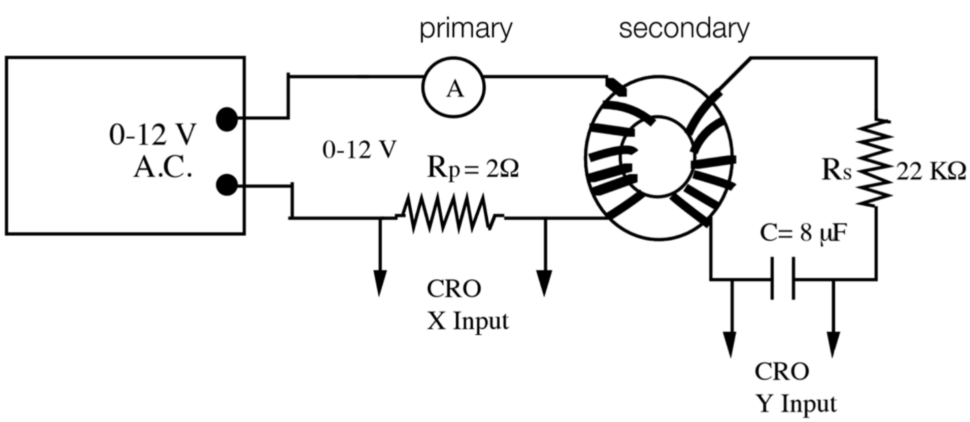



Figure 2: Circuit diagram of the setup

There is a primary circuit measuring $H$ and a secondary circuit measuring $B$ which are both connected to the same oscilloscope with two different channels. In the case of a toroidal transformer, the magnetising $H$ field is produced by the current $I$ in the primary coil,

$$H = \frac{n_p I}{2\pi r}. \tag{5}$$

The varying $B$ field in the core of the toroid induces a voltage in the secondary coil,

$$V_s = n_s A \frac{dB}{dt}. \tag{6}$$

Rearranging for $B$ gives,

$$B = \frac{1}{n_s A} \int V_s dt. \tag{7}$$

To obtain the signal proportional to B, we must integrate the secondary voltage.

For a varying voltage $V_{in}$ applied to a resistor and capacitor in series, the potential difference at the capacitor is

$$V_C = \frac{Q}{C} = \frac{1}{C} \int I dt \tag{8}$$

where $I$ is the current flowing through the resistor and into the capacitor.

If $V_C \ll V_{in}$ then the voltage across the resistor is almost equal to $V_{in}$ and so,

$$I = \frac{V_{in}}{R}. \tag{9}$$

Thus,

$$V_C = \frac{1}{RC} \int V_{in} dt. \tag{10}$$

In the case above, the input voltage $V_{in}$ is $V_s$ and the voltage across the capacitor can be measured by the oscilloscope so we have,

$$B = \frac{R_s C}{n_s A} V_Y \tag{11}$$

and

$$H = \frac{n_p}{2\pi r R_p} V_X \tag{12}$$

where $V_X$ and $V_Y$ are the voltage readings across the resistor and capacitor respectively displayed in Figure 2.

## 3.3 Energy Dissipation

To find the energy dissipated by the toroid in one cycle, we will use the following equation,

$$E = \frac{n_p R_s C}{2\pi n_s R_p r A} S \tag{13}$$

where S is the area of the hysteresis loop. To obtain S we will be using two methods. Numerical integration involves the use of scipy.integrate package, where we can use Trapezoidal and Simpson rule to obtain two values of the area of the curves. The second method is to cut and weigh the graph. The top left hysteresis curve in Figure 4 is cut out and weighed to find the area S using the formula,

$$S = (2)(0.1)\frac{m_{outer} + m_{inner}}{2m_{total}} \tag{14}$$

where the 2 and 0.1 are the adjustment factors for the scaling of the graph.

## 3.4 Remanent Magnetisation

The remanent magnetisation of a magnet will occur at point b in the hysteresis loop in Figure 1. To find the relationship between $B_{re}$ and $H$, a plot of the remanent magnetisation against $H$ will be produced. The limiting value of $B_{re}$ can be calculated as $H \to \infty$.

## 3.5 Systematic Uncertainties

The main drivers of systematic uncertainty would be the input voltage from the VARIAC and the oscilloscope used to measure $V_X$ and $V_Y$. Upon looking at their manuals, no uncertainty values were given so we're going to assume the combined Uncertainties from these two instruments will be 5% which will be the total systematic uncertainty.

# 4 Results

## 4.1 Hysteresis Loop

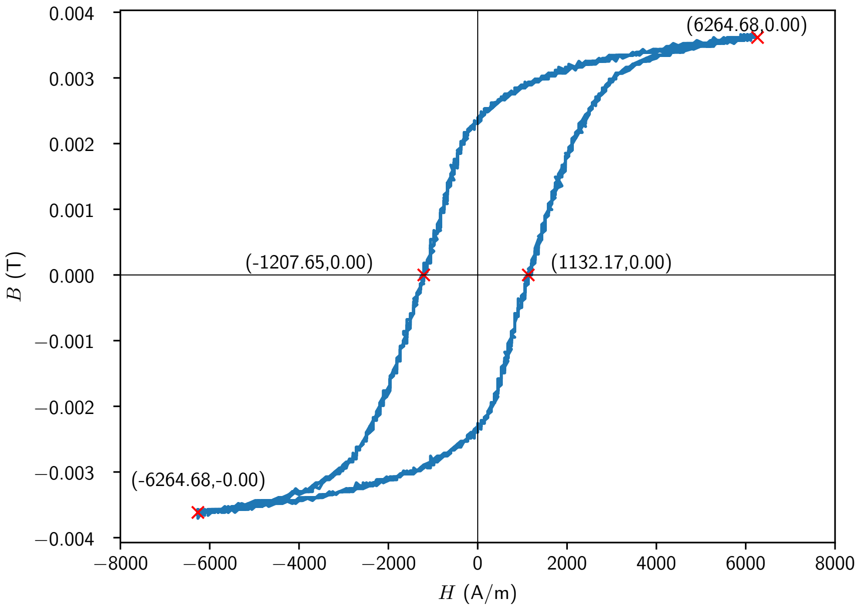



Figure 3: Hysteresis Loop found experimentally for I = 1.6A.

## 4.2 Numerical Integration curves

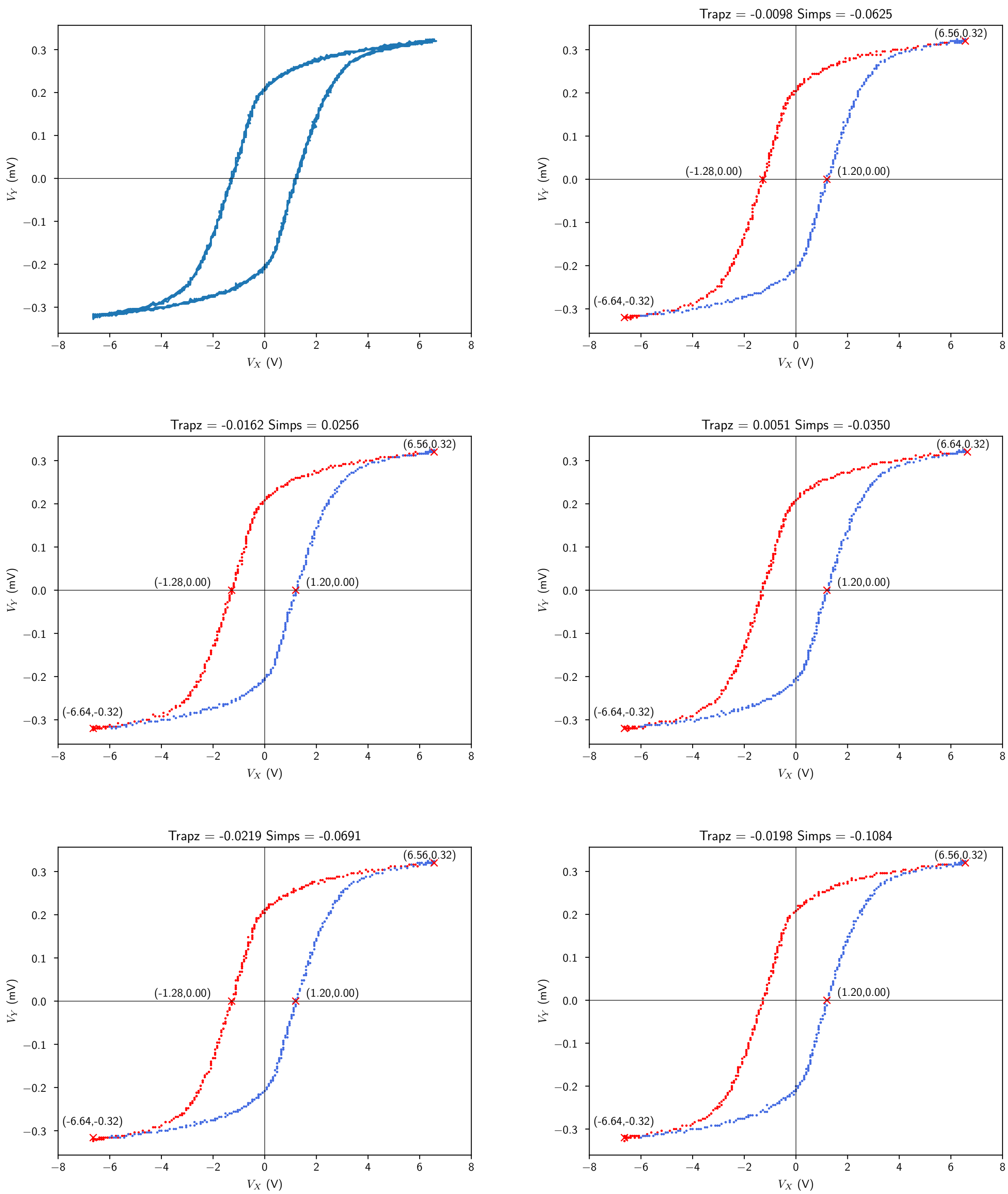



Figure 4: Top left curve is the sum of the 5 loops found experimentally.

## 4.3 Plot of Remanent Magnetisation

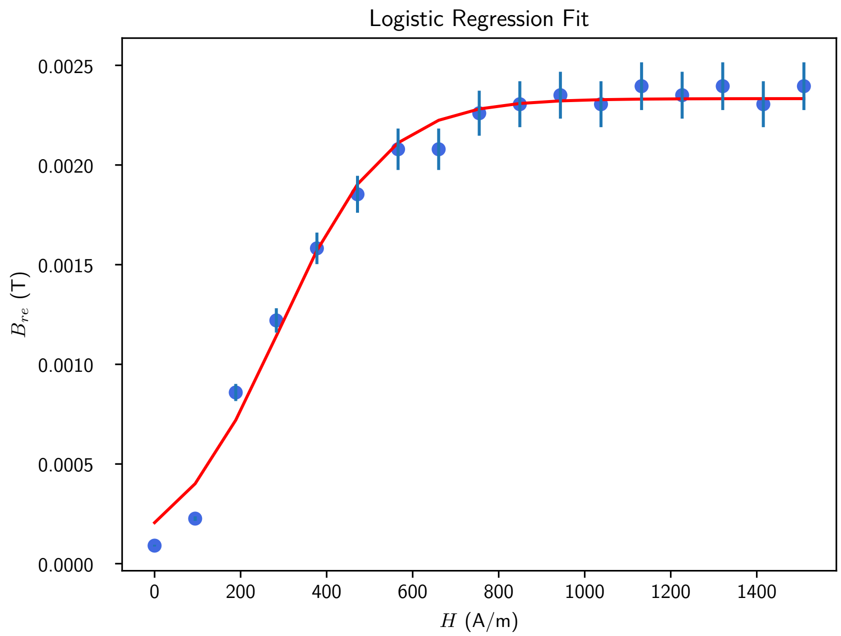



Figure 5: Logistic curve fitted onto the graph.

# 5 Analysis

## 5.1 Energy Dissipation

### 5.1.1 Numerical Integration

For the numerical integration using the trapezoidal rule the area value obtained over 1 cycle is the average of the absolute value of the areas found in the 5 cycles above. So,

$$\begin{aligned} S_{avg} &= \frac{0.0098 + 0.0162 + 0.0051 + 0.0219 + 0.0198}{5} \\ S_{avg} &= 0.01456V^2. \end{aligned} \tag{15}$$

The uncertainty of this value is found by taking the difference between the max value and min value and dividing by two,

$$\begin{aligned} \sigma &= \frac{max - min}{2} \\ &= \frac{0.0219 - 0.0051}{2} \\ &= 0.0084. \end{aligned} \tag{16}$$

Combining the statistical uncertainty with the systematic uncertainty found in Section 3.5 will give,

$$\begin{aligned} sigma_{total} &= \sqrt{(\frac{0.0084}{0.01456})^2 + 0.05^2} \\ &= 57.9\%. \end{aligned} \tag{17}$$

So the area value obtained by the trapezoidal rule is given as $0.01328 \pm 0.0084V^2$, representing a 58% uncertainty value.

Similarly, for Simpson's rule, the value obtained is $0.06012 \pm 0.041V^2$, representing a 69% uncertainty value.

#### 5.1.2 Cut and Weigh

The total mass of the paper was 3.00g whilst the mass of the outer curve was 0.47g and the inner curve was 0.34g. The grid was made up of 64 2V by 0.2V squares, making the area of the curve,

$$\begin{aligned} S &= (0.2)(2)\frac{0.47+0.34}{2*3} \\ &= 0.054\text{V}^2. \end{aligned} \tag{18}$$

The uncertainty of this measurement is driven mainly by the aforementioned systematic uncertainties but also the uncertainty of the cutting, adding another 1% error to the measurement.

Thus,

$$S = 0.054 \pm 0.0027\text{V}^2. \tag{19}$$

#### 5.1.3 Energy Dissipation

From the value of S, the energy dissipated per volume per cycle is given by,

$$E = \frac{n_p R_s C}{2\pi r R_p A} S. \tag{20}$$

We find the energy dissipated per volume per cycle of the S value found using the trapezoidal rule to be,

$$\begin{aligned} E &= \frac{(584)(21892)(9.9741 \times 10^{-6})}{(2\pi)(0.05)(1.9703)(0.00006)}(0.01456) \\ &= (4441.8)(0.01456) \\ &= 64.67 \text{ Jm}^{-3}/\text{cycle}. \end{aligned} \tag{21}$$

Using the value of S from the Simpson's rule we find that,

$$\begin{aligned} E &= (4441.8)(0.06012) \\ &= 267.04 \text{ Jm}^{-3}/\text{cycle}. \end{aligned} \tag{22}$$

The value of S found from the cut and weigh came out to be 0.054, using this to find E,

$$E = 239.86 \text{ Jm}^{-3}/\text{cycle}. \tag{23}$$

The corresponding uncertainties are then,

$$\begin{aligned} E_{trapz} &= 64.67 \ \pm \ 37.5 \text{ Jm}^{-3}/\text{cycle}, \\ E_{simps} &= 267.04 \ \pm \ 184.26 \text{ Jm}^{-3}/\text{cycle}. \\ E_{cw} &= 239.86 \ \pm \ 12 \text{ Jm}^{-3}/\text{cycle} \end{aligned} \tag{24}$$

The volume of the toroid is given by the formula,

$$V = (\pi r^2)(2\pi R). \tag{25}$$

Given that $A = \pi r^2 = h \times w = (0.03)(0.0002) = 0.00006\text{m}^2$, the volume is then,

$$\begin{aligned} V &= (0.00006)(2\pi)(0.05) \\ &= 1.88 \times 10^{-5}\text{m}^3. \end{aligned} \tag{26}$$

Thus the total power loss with a 50Hz frequency as calculated for each $E$ will be, $P_{trapz} = 0.061\,W$, $P_{simps} = 0.251\,W$ and $P_{cw} = 0.225\,W$.

## 5.2 Remanent Magnetisation

The logistic curve fit onto Figure 4.3 is given by the equation,

$$f(x) = \frac{L}{1 + e^{-k(x - x_0)}}. \tag{27}$$

The fitted parameters are then, $L = 0.023$, $k = 0.081$ and $x_0 = 294.2307$.

As $x \to \infty$, we find that $f(x) \to L$ so the limiting value of the remanent magnetisation is then 0.023T. The minimum value of the applied field intensity required to produce the strongest magnet from the core material of the toroid by momentary immersion in this field $H_m$ can be found by taking the regression line and solving for $x$ when $f(x)$ is 95% of the limiting value of $B_{re}$. It is found by,

$$f(x) > 0.95 * 0.023 = 0.02185. \tag{28}$$

This occurs for $x > 657.74$ or a value of $H_m$ greater than 658 A/m.

# 6 Discussion

We find that the results for energy dissipation come with high uncertainty with the trapezoidal rule producing a value of 0.01328 $\pm$ 58% and the Simpson's rule producing an area value of 0.06012 $\pm$ 69%. To reduce uncertainties in future experiments we can take more cycles of the hysteresis loops and use a standard deviation measure for our uncertainty. We can also use a less volatile VARIAC to reduce randomness in the voltage input and thus the current. The integration method itself is less effective for less smooth curves so obtaining more data points to smoothen out the curve would improve the accuracy of the reading.

The cut and weigh method provided a much lower uncertainty value of 5%. Out of the three methods, this would be the most accurate. We notice that the area value found by using cut and weigh method being $S = 0.054 \pm 0.0027\mathrm{V}^2$, agrees with the Simpson's rule integration method, mainly due to the high uncertainty of the numerical integration. Taking the cut and weigh's value of area, turning it into energy dissipated per volume per cycle gives $E = 239.86 \pm 12\mathrm{Jm}^{-3}$/cycle. Finally the power loss is calculated to be $P = 0.225 \pm 0.01\,W$. The maximum power loss of the circuit is given as $P = VI$ where V = 12V input voltage and I = 1.6A, combining to be 19.2 W.

In the remanent magnetisation experiment, the residuals of each data point is quite high indicating higher variance in the data collected. To obtain a more accurate measure of error, more trials should have been taken to obtain a statistical uncertainty as well as reducing residuals as the mean for each data point could have been used. To reduce systematic uncertainty we can use less volatile VARIAC and more accurate oscilloscope.

# 7 Conclusion

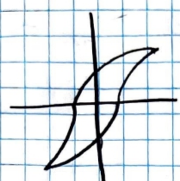
Although the experiment can be majorly improved with greater uncertainty analysis, the overall aim was tested and the relationship between the auxiliary field, H and its effect on the magnetic field strength of the ferromagnet was seen and verified, indicating validity in the interaction between the microscopic model of magnetism and how it relates to macroscopic fields.

# A Prework

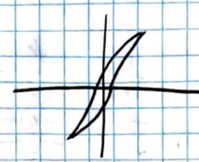
Pre-work: Theory

1.

a)



b)



Permanent magnets are ferromagnets.

We want least amount of energy dissipated hence low area.

2. $V_{in} = V\cos(\omega t)$

Using

$$V_C = \frac{1}{RC}\int V_{in}\,dt$$
$$= \frac{1}{RC}\int V\cos(\omega t)\,dt$$
$$= \frac{1}{RC}\left[\frac{V}{\omega}\sin(\omega t)\right]$$

At $t=0$, $V_C = 0$ so

$$V_C = \frac{V}{RC\omega}\sin(\omega t) \quad \cancel{\text{where}}$$
$$= V_0 \sin(\omega t) \quad \text{where} \quad V_0 = \frac{V}{R\omega C}$$

3. For suitable values for an integrating circuit,

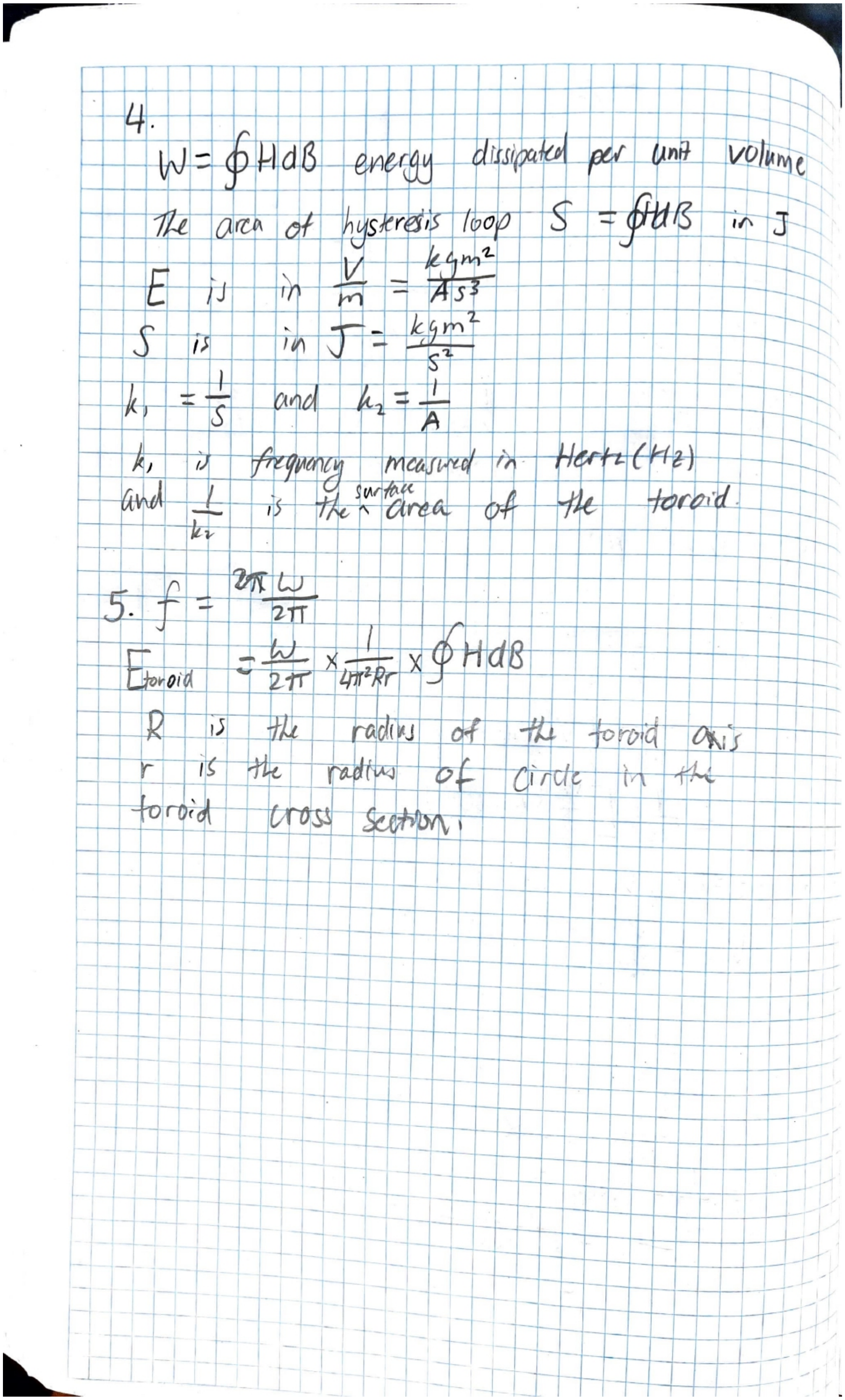
$$\frac{1}{\omega C} << R$$

$$R_{total} = 22000 + 2 = 22002$$

$$\frac{1}{\omega C} = \frac{1}{50 \times 8 \times 10^{-6}} = 2500$$

We find $\frac{1}{\omega C} << R$.

Figure 6: Prework Theory.



4.

$W = \oint H\,dB$ energy dissipated per unit volume

The area of hysteresis loop $S = \oint H dB$ in J

$E$ is in $\frac{V}{m} = \frac{kg\,m^2}{A s^3}$

$S$ is in $J = \frac{kg\,m^2}{s^2}$

$k_1 = \frac{1}{S}$ and $k_2 = \frac{1}{A}$

$k_1$ is frequency measured in Hertz (Hz)

and $\frac{1}{k_2}$ is the surface area of the toroid.

5. $f = \frac{2\pi\,\omega}{2\pi}$

$E_{toroid} = \frac{\omega}{2\pi} \times \frac{1}{4\pi^2 R r} \times \oint H\,dB$

$R$ is the radius of the toroid axis

$r$ is the radius of circle in the toroid cross section.

Figure 7: Prework Theory.

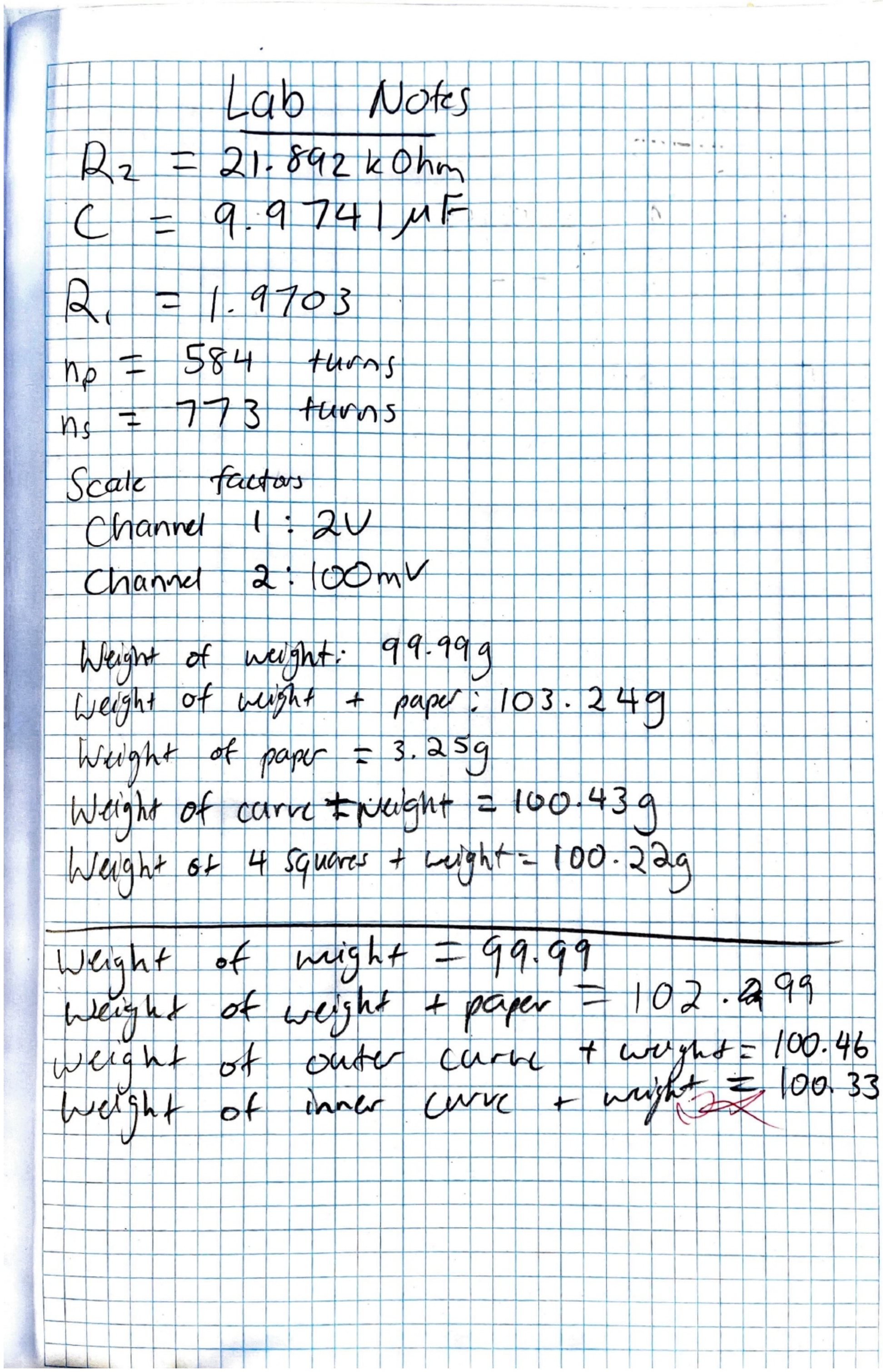
## A.1 Corrections

For Question 1a, permanent magnets have thicker hysteresis loops as to better retain their remanent magnetisation.

For Question 3, the resistance of the integrating circuit should just be 22000 Ohms, although this will not affect the final conclusion.

For Question 4, $k_1 = \frac{n_p}{2\pi r R_p}$ and $k_2 = \frac{R_s C}{n_s A}$. Energy, $E$, is measured in Joules (J). The area of the hysteresis curve for $V_X$ and $V_Y$ will be in $V^2$. $k_2$ contains Farads which are equivalent to $\frac{J}{V^2}$. The remaining $A$ in $k_2$ and $2\pi r$ in $k_1$ will represent the volume of the toroid.

# B Lab Notes



Lab Notes

$R_2$ = 21.892 kOhm
C = 9.9741 μF

$R_1$ = 1.9703
$n_p$ = 584 turns
$n_s$ = 773 turns

Scale factors
Channel 1 : 2V
Channel 2 : 100mV

Weight of weight: 99.99g
Weight of weight + paper: 103.24g
Weight of paper = 3.25g
Weight of curve + weight = 100.43g
Weight of 4 squares + weight = 100.22g

Weight of weight = 99.99
Weight of weight + paper = 102.~~2~~99
Weight of outer curve + weight = 100.46
Weight of inner curve + weight = 100.33

Figure 8: Signed Lab book.