# Hysteresis

PHYS2114 Experimental Physics

Student Notes

## 1 Experimental Aim

The aim of this lab is to gain some practical experience with traditional ferromagnetic materials (i.e. iron alloys) and understand how they interact with an applied magnetic field. To do this you will measure the magnetising properties of the iron core inside a toroidal transformer. Specifically, you are asked to:

- Calculate the energy dissipated by the iron sample over one hysteresis cycle;
- Plot the value of the remanent induction as a function of maximum applied field intensity, and;
- Determine the value at which the remanent induction saturates.

Finally you should develop a physical interpretation of how the microscopic physical model relates to the macroscopic fields of the magnetic field, $\mathbf{B}$, auxiliary field $\mathbf{H}$, and magnetisation, $\mathbf{M}$.

## Contents

# 2 Introduction

## 2.1 Hysteresis

If a sample of magnetic material is subjected to an alternating auxiliary field, $H$, and the total magnetic field, $B$, in the sample is plotted against $H$, we obtain a hysteresis loop as shown in Fig.1.

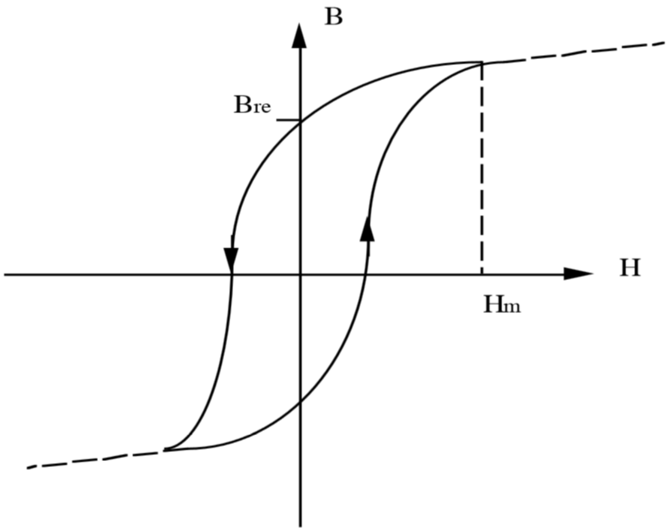


Figure 1: Schematic of a B-H curve showing a hysteresis loop. $B_{re}$ is the remanent magnetisation, and $H_m$ is the maximum field intensity required to saturate the system.

The arrows on the B-H curve show the trace obtained when $H$ is swept from negative to positive (up) and from positive to negative (down). When the auxiliary field imposed on the sample is removed the magnetised material will not relax back to zero, it will remain magnetised. The hysteresis loop is a signature that the system has a time-dependent magnetisation and is highly dependent on the past and current values of the external magnetic field. This time dependence is due to the existence of magnetic domains in the material. Once the magnetic domains are orientated to an applied field, it takes some energy to change their orientation.

The maximum magnetisation occurs when the system is saturated. In Fig. 1 we see if an external field $H > H_m$ is applied the magnetisation does not increase appreciably. The minimum applied field required to saturate the system is called saturation field, $H_m$.

To reverse the magnetisation of the system an opposing magnetic field needs to be applied to drive the system back to zero. In Fig.1 this is the trace with the “down" arrow. The remanent magnetisation, $B_{re}$, is the residual magnetism in the system after the external magnetic field is removed and is a measure of the systems magnetisation. If an alternating magnetic field is applied to a magnetic system, the applied magnetic field quickly sweeps from negative to positive and back, thus the magnetisation of the system will trace out a hysteresis loop. In this experiment we will display such a curve on an oscilloscope screen by varying the magnetising field at 50 Hz and using voltage signals for the X and Y inputs of the oscilloscope, which are proportional to applied field H and total field B, respectively.

## 2.2 Power dissipation

The hysteresis loop can be used to calculate the energy dissipated by the system in one cycle.

The work done on unit volume of the specimen when the applied field changes, is given by $HdB$. Thus in going through one cycle, the energy dissipated per unit volume of the specimen (as heat) is equal to the area of the hysteresis loop:

$$W = \oint HdB \tag{1}$$

# 3 Theory

The magnetic behaviour of materials may be characterised by the functional relationship between the magnetisation **M** (magnetic dipole moment per unit volume), and the magnetic field strength **H** [1]. In all substances there is a so-called diamagnetic effect by which **H** causes a small magnetisation proportional but anti-parallel to it. Thus $\mathbf{M} = \chi\mathbf{H}$, with $\chi$ (the susceptibility) a negative constant.

In many materials there are permanent magnetic dipole moments, which in a magnetic field tend to align parallel to the field, creating a magnetisation parallel to **H**. The corresponding positive contribution to $\chi$ is much larger than the negative diamagnetic contribution. In the so-called paramagnetic materials $\mathbf{M} = \chi\mathbf{H}$ with $\chi$ a positive constant.

[1] In some texts you will find that the B-field or the H-field are alternately ascribed the name magnetic field and the other field is either, magnetic induction (B) or the auxiliary field (H). Try to remember them for what they do and not how they are named.

This constancy refers only to a fixed temperature. $\chi$ varies with temperature, down to extremely low temperatures, according to the Curie law:

$$\chi = \frac{A}{T} \tag{2}$$

where $A$ is a constant. Some materials are paramagnetic only above a certain temperature $T_C$, called the Curie point, with $\chi$ varying according to the Curie-Weiss law:

$$\chi = \frac{A}{T - T_C} \tag{3}$$

Below the Curie point these materials, e.g. iron, become ferromagnetic: their magnetisation does not change linearly with **H** (thus susceptibility is no longer a constant). In fact **M** now depends not only on **H** and the temperature but also on the previous "history" of the sample, i.e. on the way in which the value of **H** and temperature was arrived at.

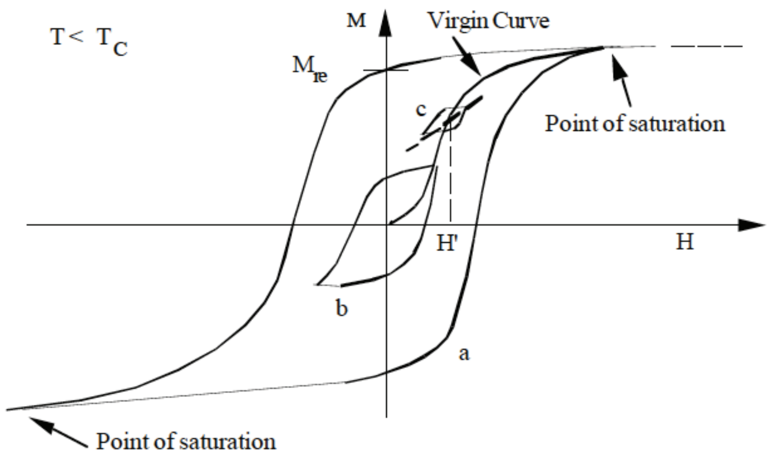


Figure 2: Diagram showing the salient features of the hysteresis curve.

Fig. 2 qualitatively shows the main features of the ferromagnetic behaviour. Starting from **M** = 0, **H** = 0, and maintaining a constant temperature but increasing **H**, **M** will increase along the so-called virgin curve, up to the "point of saturation" above which **M** remains virtually constant.

If we subsequently decrease **H**, the magnetisation will go back, not along the virgin curve, but along curve (a). Reaching **H** = 0, **M** will not be zero but will have a so-called remanent value $\mathbf{M_{re}}$. It needs a certain value of **H** in the opposite direction (negative in Fig. 2) to bring the sample back to its unmagnetised state (**M** = 0). Changing **H** back and forth between the two saturation points produces the closed curve (a) in Fig. 2, called the hysteresis curve. If we change **H** back and forth symmetrically through a smaller range then we obtain the hysteresis curve (b) of Fig. 2. Going through a range of **H** not centred at **H** = 0 but at **H**', we obtain curve (c).

There are further types of magnetic behaviour: anti-ferromagnetism, ferrimagnetism, etc., which are out of the scope of our laboratory program. In single crystals, $\chi$ may not even be a scalar but a tensor quantity.

For convenience, we give here the relationship between the magnetic field **B** , **H** and **M**:

$$\mathbf{B} = \mu_0 (1 + \chi) \mathbf{H} = \mu \mathbf{H} \tag{4}$$

$\mu$ is called permeability and $\mu_0$ is the permeability of free space (vacuum). In vacuum,

$$\mathbf{B} = \mu_0 \mathbf{H} \tag{5}$$

where $\mu_0 = 4\pi \times 10^{-7}$ in Henry / metre ($\mathrm{Hm}^{-1}$)

# 4 Experimental Apparatus

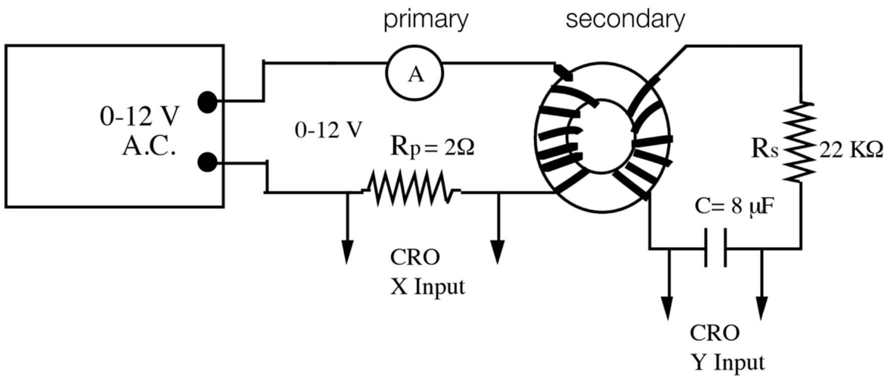


Figure 3: Schematic of the experimental set up used to obtain the B- H hysteresis loop.

The circuit used to display the hysteresis loops is shown in Fig. 3. The primary AC current is obtained from a 12 V transformer and can be varied by means of the variable auto-transformer (“Variac"). A resistor $R_p$ is inserted into the primary circuit to provide a voltage proportional to the primary current $I$. This signal is thus proportional to the magnetic $H$-field (see Eqn. 6) and is used as the X input to the oscilloscope. The secondary voltage is integrated by means of the circuit consisting of $R_s$ and $C$ to obtain a signal proportional to the magnetic B-field (see Eqn. 8) for use as the Y input to the oscilloscope.

## 4.1 Transformer

To display a hysteresis loop as in Fig. 1 we use a transformer consisting of a toroidal specimen of iron wound with a primary coil of $n_p$ turns, by means of which the applied magnetising $H$-field is produced, and a secondary coil of $n_s$ turns which is used to detect the magnetic $B$-field. Fig. 4 is a schematic of a cross section of the toroidal transformer (a demonstration model of the toroidal transformer you will use has been cut open so you can see the windings - ask your demonstrator if you can’t find it).

The magnetising $H$-field is produced by the current $I$ in the primary coil:

$$H = \frac{n_p I}{2\pi r} \tag{6}$$

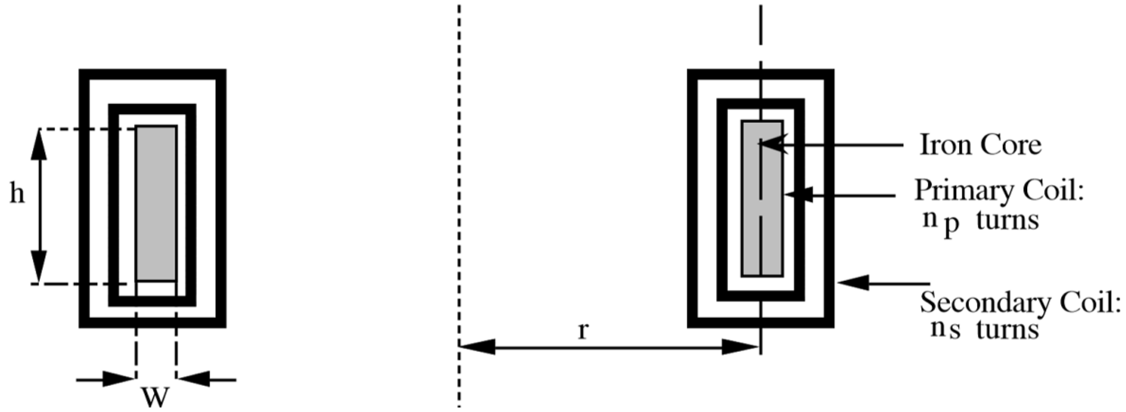


Figure 4: Schematic of the cross section of the toroid transformer you will use. The iron core has a rectangular cross section with dimensions $h \times w$ ($h = 30$ mm, $w = 2$ mm), and curves around to form a toroid (doughnut).The toroid has a diameter $2\pi r$ and is shown above ($r = 50$ mm). A coil, the primary coil, is wrapped around the iron core. When a current is passed through the primary coil it produces am auxiliary field, $H$, which magnetises the iron core. The secondary coil is used to sense the total magnetic field of the iron core and applied field. When the secondary coil is exposed to a magnetic field a current will be induced in the wires of the secondary coil which can be measured.

where $r = 50$ mm and the value $n_p$ is given on the day. The varying magnetic B-field in the core of the toroid induces a voltage in the secondary coil given by:

$$V_s = n_s A \frac{dB}{dt} \tag{7}$$

where $A$ is the area of the core cross section (see Fig. 4). Thus:

$$B = \frac{1}{n_s A} \int V_s dt. \tag{8}$$

To obtain a signal proportional to $B$ it is therefore necessary to integrate the secondary voltage and a means of achieving this is discussed in the next section.

## 4.2 Integrating circuit

Consider a varying voltage $V_{in}$ applied to a resistor and capacitor in series as shown in Fig. 5. It follows that:

$$V_C = \frac{Q}{C} = \frac{1}{C} \int I dt \tag{9}$$

Where $I$ is the current flowing through the resistor and into the capacitor. If the voltage across the capacitor $V_C$, is always kept much less than $V_{in}$, then the voltage across the resistor is very nearly equal to $V_{in}$ in and we have:

$$I = \frac{V_{in}}{R} \tag{10}$$

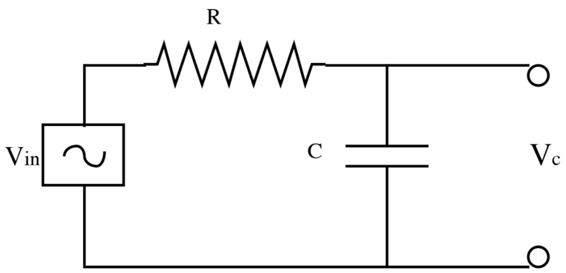


Figure 5: Schematic of an integrating circuit.

and hence:

$$V_C = \frac{1}{RC}\int V_{in}dt \tag{11}$$

Thus the circuit provides an output signal $V_C$, which is proportional to the integral of the input signal $V_{in}$. For a periodic signal $V_C$ of angular frequency $\omega$ the condition for $V_C \ll V_{in}$ is that the impedance of the capacitor must be much smaller than $R$, i.e. $1/\omega C \ll R$. Provided that this condition holds for the lowest frequency component of $V_{in}$ the circuit will integrate correctly.

### 4.3 Oscilloscope output

Thus the relation between B and H can be displayed on the oscilloscope. From Eqn. 6 and the relation $V_X = IR_p$:

while from Eqn. 8 and Eqn. 9 we get:

$$H = \frac{n_p}{2\pi r R_p}V_X \tag{12}$$

and,

$$B = \frac{R_s C}{n_s A}V_Y \tag{13}$$

## 5 Pre-Work

Please complete the preparatory work on Moodle and submit prior to the start of the laboratory session. This will be a graded exercise and will contribute to the final mark for the lab as indicated on the marking rubric.

### 5.1 Pre-Work: Theory

1. What hysteresis loop shape would you require for a magnetic material to be used in (a) a permanent magnet (b) a transformer core? Give reasons for your choice.

2. Using Eqn. 11 show if $V_{in} = V\cos(\omega t)$ for the circuit in Fig. 5 then $V_C = V_0 \sin(\omega t)$ and $V_0 = V/(RC\omega)$.

3. Show that the component values shown in Fig. 3 are suitable for the integration of a 50 Hz signal.

4. Derive Eqn. 14, specifically deriving expressions for the scaling factors $k_1$ and $k_2$. Ensure your scaling factors give a final value for $E$ with the correct units: energy dissipated per cycle, and per unit volume of the specimen. You will need to show a dimensional analysis or use units throughout your calculation.

5. Using the results from the previous section, how would you calculate the total energy dissipated by the iron sample over one hysteresis cycle, $E_{\text{Toroid}}$?

# 6 Experiment Plan

This section is designed to help you plan your experiments.

## 6.1 Energy dissipation

You are going to calculate the energy dissipated by the toroid in one cycle. In this section we will discuss the experimental set up used. The analysis section covers the calculations you will use to extract the dissipated energy.

The first step is to set up the circuit in Fig. 3. Next you will display the hysteresis curve on the oscilloscope as described in the Operating Instructions. This curve will be shown on a computer and printed out after which you will perform your analysis steps.

## 6.2 Remanent magnetisation

Part of the experimental aim is to determine the value at which the remanent induction $B_{re}$ saturates using a plot of $B_{re}$ versus the applied field, $H$. In this measurement you will increase the primary current from zero in appropriate steps and measure the variation of $B_{re}$ as a function of $H$. Look at Fig. 3 to remind yourself which feature is $B_{re}$ on the hysteresis curve. Calculate and plot the value of the remanent induction $B_{re}$ as a function of the field intensity $H$. Estimate the saturation value for $B_{re}$. Estimate the minimum applied field intensity required to produce the strongest magnet from the core material of the toroid by momentary immersion in this field $H_m$.

# 7 Analysis hints and tips

One of the aims in this experiment is to calculate the energy dissipated in the whole volume of the core of the toroid per second.

You're going to calculate the energy dissipated by the iron core by displaying the hysteresis curve on an oscilloscope. You will use the area of the hysteresis loop to calculate the dissipated energy.

From Eqn. 1 you know the area of the hysteresis loop $S$ is equal to the energy $E$ dissipated per cycle, and per unit volume of the specimen (as heat). Thus $E$ can be written as a function of S, where the scaling factors $k_1$ and $k_2$ are used to covert the area of the loop into an energy.

$$E = k_1 k_2 S \tag{14}$$

## 7.1 The "modern" method

You should have access to a .csv (comma delimited) version of the data files. These can be read into Excel or your favourite high-level programming language such as Matlab, Python, etc. An example script for Matlab has been provided. Once you have imported the two date you can plot the channels to reproduce the hysteresis curve. Appropriate scaling should be applied so that any area calculation will have the correct units.

You can use any standard numerical integration method you like to calculate the area (e.g. the trapezoidal rule or Simpson's rule), but keep in mind the following:

1. You should be subtracting the lower branch from the upper branch.
2. The oscilloscope trace has more than one full cycle of the hysteresis curve, so you will need to select the appropriate sections for the area calculation.
3. The data points on the abscissa are not equally spaced.
4. You will still need to make some error estimate of your curve.

## 7.2 The "old school" method

To get $E$ you will print out an image of the hysteresis loop on your oscilloscope and convert this physical area into an energy using the steps below.

As $S$ is a curve it's easier to obtain the area by weighing the curve. You will cut out and weigh the whole image from the oscilloscope, which will contain a certain number of divisions with total weight $w_t$. This will enable you to calculate the weight of each square division $w_{sq}$. Thus when you weigh the curve you can calculate the area (in number of square divisions D) from its weight $w_S$.

The area of the hysteresis curve in square divisions can be expressed as $S = nD$, where the hysteresis curve has n square divisions. From Equations (8) and (9) the amplification factors of the oscilloscope can be converted from volts per division into A/m per division and Tesla (T) per division. Note, a square division D can be thought of as a square with one "edge" for the X scaling factor and one "edge" for the Y scaling factor.

These results enable you to calculate $H$ and $B$ in units of A/m and T, respectively, on the printout.

Remember that Eqn. 14 gives you energy rate with respect to time per unit volume. Remember also that the current frequency is 50 Hz.

Finally you will need to calculate an error for the dissipated energy. One way to estimate the error in cutting the paper is to cut the square and the hysteresis area on the outside of the line and weigh it; then cut the line off and weigh again.

Write down your result and the error with appropriate units.