ISSN 1062-8738, Bulletin of the Russian Academy of Sciences. Physics, 2014, Vol. 78, No. 12, pp. 1250–1253. © Allerton Press, Inc., 2014.
Original Russian Text © A.V. Zakharov, N.V. Polikarpova, V.B. Voloshinov, 2014, published in Izvestiya Rossiiskoi Akademii Nauk. Seriya Fizicheskaya, 2014, Vol. 78, No. 12, pp. 1534–1538.

The Klein–Cook Parameter in Analyzing Acousto-Optic Interaction in Acoustically Anisotropic Media

A. V. Zakharov, N. V. Polikarpova, and V. B. Voloshinov

Faculty of Physics, Lomonosov Moscow State University, Moscow, 119991 Russia
e-mail: azakharov@physics.msu.ru

Abstract—A theoretical study of acousto-optic interaction in acoustically anisotropic media is performed. A new criterion for estimating the Raman–Nath and Bragg regimes of light diffraction by means of ultrasound in acoustic crystals is proposed. The modified Klein–Cook parameter is found to consider the walkoff of ultrasonic wave energy as a result of the elastic anisotropy of a medium.

DOI: 10.3103/S1062873814120302

INTRODUCTION

In the physics and engineering of recent years, there has been great interest in creating materials with strong anisotropy of different physical properties: artificial periodical crystal structures with negative refractive indices, metamaterials, photonic crystals, and so on [1, 2]. Nevertheless, many natural materials are also characterized by significant anisotropy of their physical properties; the crystals that are widely used in modern acousto-optics and acoustoelectronics [3, 4] are one example of media with the strong anisotropy of acoustic and optic properties. Recent studies revealed that light diffraction by ultrasound in these crystals differs from diffraction in isotropic media [5–7]. Practical needs are thus driving the development of acoustic interaction theory in the application of acoustic anisotropic media. This work is therefore dedicated to investigating acousto-optic interaction in acoustically anisotropic materials; i.e. acoustic crystals.

As is well known, the acoustic anisotropy of a crystal is obvious in the strong deviation of the energy flux vector from the direction of the wave vector and the phase velocity vector of the acoustic wave [8]. The angle between these vectors, referred to as acoustic walkoff angle Ψ, can be as high as $\Psi = 74°$ in modern acousto-optic materials [2]. Paratellurite (TeO_2) crystal, widely used in devices for control of optical radiation, is an example of materials with strong anisotropy of their elastic properties. Considerable acoustic anisotropy has also been detected in a number of other crystals that are promising in terms of fabricating new acousto-optic devices, particularly tellurium (Te) [9], double lead molybdate (Pb_2MoO_5) [10] and crystalline mercury halides (Hg_2Cl_2, Hg_2Br_2 and Hg_2I_2) [3].

Diffraction of light on a phase grating excited by travelling acoustic wave in a crystal [11–14] lies at the heart of acousto-optic devices. In this work, we consider light diffraction in media with acoustic anisotropy only, while optical anisotropy is ignored. Since the acoustic walkoff angle is as high as 30°–40° in most acousto-optic materials (for many crystals, it exceeds 60°), acoustic anisotropy must therefore be considered when analyzing acousto-optic interaction [5].

Our analysis of acousto-optic interaction in a medium with strong acoustic anisotropy is based on parametric acousto-optic interaction theory [11, 12], which is in turn built upon phase matching and allows us to calculate the phase mismatch parameters. This model can be applied to diffraction analysis for both the orthogonal and oblique incidence of light on ultrasound over a wide range of Raman–Nath (υ) and Klein–Cook (Q) parameters [5]. It can therefore be used in analyzing all possible regimes of acousto-optic interaction. However, this work focuses on the Klein–Cook parameter, which is traditionally used in acousto-optics to determine the boundaries of interaction between the Raman–Nath and the Bragg regimes, and to describe conditions of phase matching in acoustically isotropic media [11, 12]. Here, we generalize the Klein–Cook parameter for interactions in acoustically anisotropic media.

THEORETICAL

In this work, we consider the diffraction of an incident plane light wave with wave vector $\vec{k}$ by an acoustic wave with wave vector $\vec{K}$, which spreads in the crystal. The acoustic energy is assumed to be concentrated in space in the form of a plane-parallel acoustic column that can be considered continuous in the direction of acoustic wave energy propagation and in one of the orthogonal directions (i.e., along the z and y axes, as is shown in Fig. 1). At the same time, the acoustic field is basically limited in another orthogonal direction: along the x axis, as is also shown in Fig. 1. The width of the acoustic column l is calculated along the x axis perpendicular to its boundaries.

As is well known, acousto-optic interaction can be described by a set of coupled wave equations [11, 12]:

$$\frac{dC_p}{dx} = \frac{\upsilon_p}{2l}[C_{p-1}\exp(j\Delta k_{p-1}x) - C_{p+1}\exp(-j\Delta k_p x)]. \quad (1)$$

The intensity of light in the pth diffraction maximum at the end of the column when $x = l$ can be found as the product of the complex conjugated amplitudes of the diffracted light: $I_p = C_p(l)C_p^*(l)$. The boundary conditions are written in the forms $C_0(0) = 1$ and $C_{p\neq 0}(0) = 0$. As follows from coupled equations (1), amplitude C_p of the pth diffraction maximum rises with coordinate x and depends on the light field amplitudes of adjacent maxima C_{p-1} and C_{p+1}. This shows that light energy is exchanged only between adjacent orders of diffraction upon acousto-optic interaction. As is also seen from Eqs. (1), the efficiency of energy exchange is determined by two principal parameters: Raman–Nath (υ_p) and phase mismatch (Δk_p). The Raman–Nath parameter is known to depend on the acoustic wave amplitude and the photoelastic properties of a medium. The mismatch parameter is determined by the difference between the light and sound wave vectors projected onto the x axis [11]:

$$\Delta k_p = k_{p+1,x} - k_{p,x} - K_x. \quad (2)$$

The system of coupled wave equations can be used to describe acousto-optic interaction in both acoustically isotropic and anisotropic media [5]. In latter, acoustic anisotropy is taken into account as the dependence of υ_p and Δk_p parameters on acoustic walkoff angle Ψ. Since the lengths of the light and sound wave vectors are usually related to each other via the ratio $k_p \gg K$ and the diffraction angles are quite narrow, we may therefore ignore the change in light wavelength due to the Doppler effect. The Raman–Nath parameter for all diffraction maxima in an acoustically anisotropic medium is determined by the relationship

$$\upsilon_p \approx \upsilon_0 = 2\pi l\delta n/\lambda\cos(\Psi + \theta_0). \quad (3)$$

The phase mismatch parameters in an acoustically anisotropic medium are determined as

$$\Delta k_p = \sqrt{k^2 - [(p+1)K\cos\Psi + k\sin(\Psi+\theta_0)]^2} - \sqrt{k^2 - [pK\cos\Psi + k\sin(\Psi+\theta_0)]^2} + K\sin\Psi. \quad (4)$$

In ratios (3) and (4), δn is the change in the refractive index caused by an ultrasonic wave in the crystal; λ is the wavelength of light; θ_0 is the light's angle of incidence, defined in acousto-optics as the angle between the direction of an incident light wave's vector and the ultrasound's wave front; k and K are the wave numbers of light and sound (we assume that $k_p \approx k_0 = k$). Combined

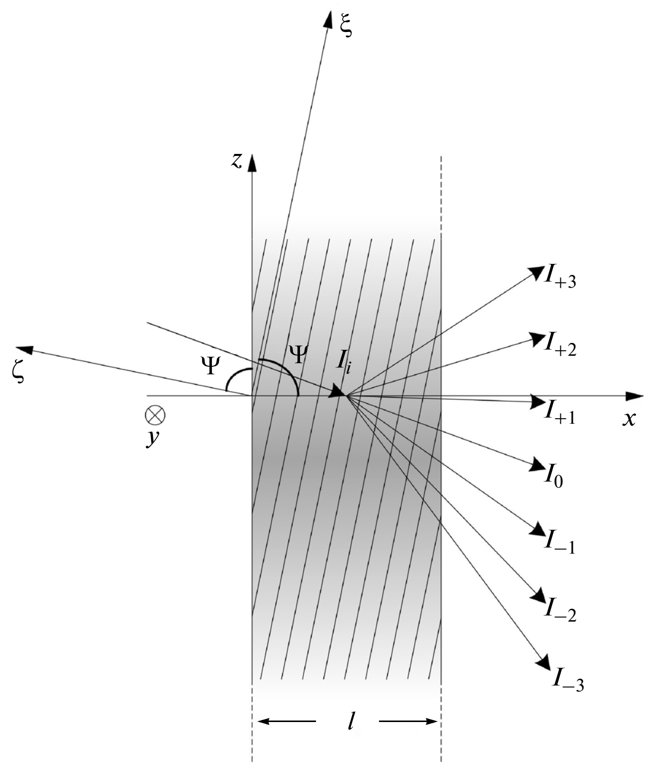

Fig. 1. Scheme of acousto-optic interaction in an acoustically anisotropic medium.

with the Raman–Nath (3) and phase mismatch (4) parameters, coupled wave equations (1) thus allow us to calculate the intensity of light for all diffraction maxima [5].

There are two known acousto-optic limits: the Raman–Nath and Bragg diffraction regimes [11, 12]. The Raman–Nath regime is observed when the diffraction grating induced by an ultrasonic wave is considered to be thin. The diffraction pattern consists of a considerable number of diffraction maxima arranged symmetrically relative the incident light wave, while diffraction is observed at any angle of light incidence.

The Bragg regime is observed when the grating has enough width to be three-dimensional in principle. The pattern then contains only two diffraction maxima (including the zero maximum), and diffraction is observed only at the Bragg angle of light incidence (θ_B) determined by the ratio

$$|\sin\theta_B| = K/2k. \quad (5)$$

The Klein–Cook parameter [11, 12, 15] is traditionally used to determine the acousto-optic diffraction regime in acoustically isotropic media:

$$Q = K^2 l/k. \quad (6)$$

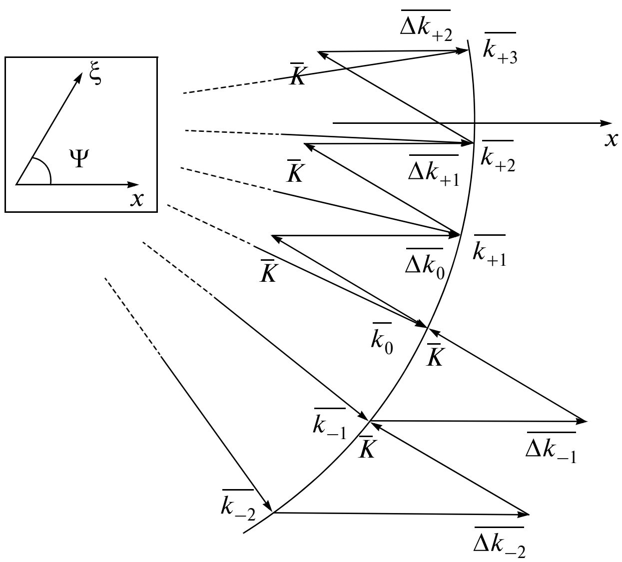

Fig. 2. Vector diagram of acousto-optic interaction in an acoustically anisotropic medium.

The Raman–Nath regime is observed when $Q \ll 1$; $Q \gg 1$ is seen in the Bragg regime; and there is an intermediate regime at $Q \sim 1$. The last is characterized by several maxima in the diffraction pattern, and diffraction is observed when the light is incident over a certain range of angles. Nevertheless, the Klein–Cook parameter determined by formula (6) cannot be used to estimate the diffraction regimes in acoustically anisotropic media, as was shown in [5].

In order to generalize the Klein–Cook parameter for an acoustically anisotropic medium, we must consider a number of factors. As has already been mentioned, the efficiency of light energy exchange between the adjacent diffraction maxima is largely due to the phase mismatch parameters. As can be seen from the vector diagram in Fig. 2, these parameters are generally greater the farther the corresponding diffraction orders from the zero maximum. The Raman–Nath regime thus requires that the mismatch parameters do not grow along with the order of diffraction. In other words, the maximum values of $\Delta k_p x = \Delta k_p l$ products in the exponents of coupled wave equations (1) must not depend on number p of the diffraction maximum. Since the wave numbers of light and sound are usually related to each other by the ratio $k \gg K$ and the diffraction angles are narrow, we can expand the expression for phase mismatch parameters (4) in a Taylor series, relative to the ratios

$$[(p+1)K\cos\Psi/k + \sin(\Psi+\theta_0)]^2,$$
$$[pK\cos\Psi/k + \sin(\Psi+\theta_0)]^2. \tag{7}$$

Ignoring the highest-order terms of a series, we obtain the following approximate expression for the phase mismatch parameters:

$$\Delta k_p = -\left(p+\frac{1}{2}\right)\frac{k^2\cos^2\Psi}{k\cos^3(\Psi+\theta_0)} - K[\cos\Psi\,\mathrm{tg}(\Psi+\theta_0) - \sin\Psi]. \tag{8}$$

It obviously follows from this ratio that product $\Delta k_p l$ does not depend on number p when $[K^2\cos^2\Psi/k\cos^3(\Psi+\theta_0)]l \ll 1$. This condition coincides with the Raman–Nath regime. In other situations, the mismatch parameters grow along with p. We therefore obtain a parameter that allows us to determine the diffraction regime:

$$Q_A = \frac{K^2 l\cos^2\Psi}{k\cos^3(\Psi+\theta_0)} = Q\frac{\cos^2\Psi}{\cos^3(\Psi+\theta_0)}. \tag{9}$$

This magnitude can be considered a modified Klein–Cook parameter for an acoustically anisotropic medium. Assuming that $\Psi = 0$ in ratio (9), we obtain a stricter interaction regime parameter for an isotropic medium than traditional Klein–Cook parameter (6), which does not consider the light's angle of incidence θ_0.

CALCULATIONS

To prove the validity of the above parameter, we calculated the dependences of the light intensity of the diffraction maxima on the light's angle of incidence θ_0, and on modified Klein–Cook parameter Q_A for acousto-optic interaction in the paratellurite crystal (001) plane. In this plane, the maximum angle of acoustic walkoff is 74° for a slow shear acoustic wave propagating in the crystal with a velocity of 0.89×10^5 cm s^{-1}. The calculations were performed for red-range light with $\lambda = 633$ nm. Analysis showed that the polarization of light does not change during interaction.

Figure 3 shows the dependence of light intensity at the end of the acousto-optic cell on the angle of incidence (in degrees) in the +1st- (curves *1*), +2nd- (curves *2*) and +3rd- (curves *3*) order diffraction maxima for three different modified Klein–Cook parameter values and at the same Raman–Nath parameter $\upsilon = \pi$. When $Q_A \approx 0.06$ (solid curves), the light intensity reaches its highest values in the range of 15–35%, which is typical of Raman–Nath diffraction. When $Q_A \approx 1$ (dashed curves), the predominance of the 1st order, which reaches 97% diffraction efficiency at Bragg angle of incidence $\theta_0 = -0.62°$, is obvious. The 2nd-order diffraction maximum reaches 8% at angle of incidence $\theta_0 = -0.74°$. This corresponds to the intermediate diffraction regime. Finally, when $Q_A \approx 5$ there is only a single 1st-order maximum among those listed above (shown by the dotted-and-dashed curve in Fig. 2). Its

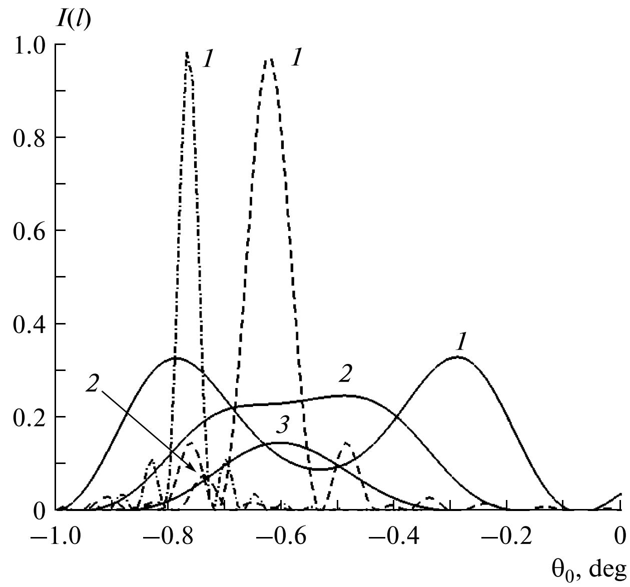

Fig. 3. Dependence of light intensity in diffraction maxima on the light's angle of incidence at different values of the modified Klein–Cook parameter.

intensity reaches 98% at Bragg angle of incidence $\theta_0 = -0.77°$, which corresponds to Bragg diffraction. The proposed Klein–Cook parameter can thus be used to study the regimes of acousto-optic interaction in an acoustically anisotropic medium.

CONCLUSIONS

Acousto-optic interaction was considered in an acoustically anisotropic medium using intermediate diffraction regimes and the examples of Raman–Nath and Bragg. It was shown that our modified Klein–Cook parameter can be used to determine the regimes of diffraction in such media. The difference between the traditional and modified Klein–Cook parameters was particularly noticeable at acoustic walkoff angles higher than 50°. It is thus recommended that this feature be considered when analyzing diffraction regimes in modern acousto-optic devices.

ACKNOWLEDGMENTS

This work was supported by the Russian Science Foundation, project no. 14-12-00380.

REFERENCES

1. Veselago, V.G., *Phys. Usp.,* 2011, vol. 54, p. 1161.
2. Burov, V.A., Voloshinov, V.B., Dmitriev, K.V., Polikarpova, N.V., *Phys. Usp.*, 2011, vol. 54, p. 1165.
3. Voloshinov, V.B., Polikarpova, N.V., and Declercq, N.F., *J. Acoust. Soc. Amer.*, 2009, vol. 125, p. 772.
4. Voloshinov, V.B. and Polikarpova, N.V., *Appl. Opt.*, 2009, vol. 48, p. C55.
5. Zakharov, A.V., Voloshinov, V.B., and Blomme, E., *Ultrasonics*, 2011, vol. 51, p. 745.
6. Balakshy, V.I. and Voloshin, A.S., *Opt. Spectrosc.*, 2011, vol. 110, no. 5, p. 788.
7. Kastelik, J.-C., Dupont, S., Yushkov, K.B., and Gazalet, J., *Ultrasonics,* 2013, vol. 53, p. 219.
8. Dieulesaint E. and Royer, D., *Ondes elastiques dans les solides,* Paris: Masson, 1974.
9. Voloshinov, V.B., Balakshy, V.I., Kulakova, L.A., and Gupta, N., *J. Opt. A: Pure Appl. Opt.*, 2008, vol. 10, p. 095002.
10. Mil'kov, M.G., Volnyanskii, M.D., Antonenko, A.M., and Voloshinov, V.B., *Acoust. Phys.*, 2012, vol. 58, no. 2, p. 172.
11. Balakshy, V.I., Parygin, V.N., and Chirkov, L.E., *Fizicheskie osnovy akustooptiki* (Physical Fundamentals of Acousto-optics), Moscow: Radio i svyaz', 1985.
12. Korpel, A., *Acousto-Optics,* New York: Marcel Dekker, 1988.
13. Yariv, A. and Yeh, P., *Optical Waves in Crystals*, New York: Wiley, 1984.
14. Zadorin, A.S., *Dinamika akustoopticheskogo vzaimodeistviya* (Dynamics of Acousto-optic Interaction), Tomsk: Tomsk State Univ., 2004.
15. Klein, W.R. and Cook, B.D., *IEEE Trans. Sonics Ultrason.*, 1967, vol. 14, p. 123.

Translated by O. Maslova