Polarisation of Light

PHYS3114 - Electrodynamics

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Contents

1 Introduction

In this report we will explore the wave nature of light. Part I investigates the scattering of light and different methods of polarising light through the model of Malus' Law. Part II will look at the interaction of incident light on dielectric material and metals.

Scattering of light is a special case of diffraction, ocurring when the reflector is much smaller than the wavelength of light. The wave scattered from the reflector will be then spherical, this is because there will be no interference between the wavelets emitted by the several points on the surface of the scattering particle. Unpolarised light hitting a molecule will scatter polarised light depending on the angle observed as seen in Figure 1.

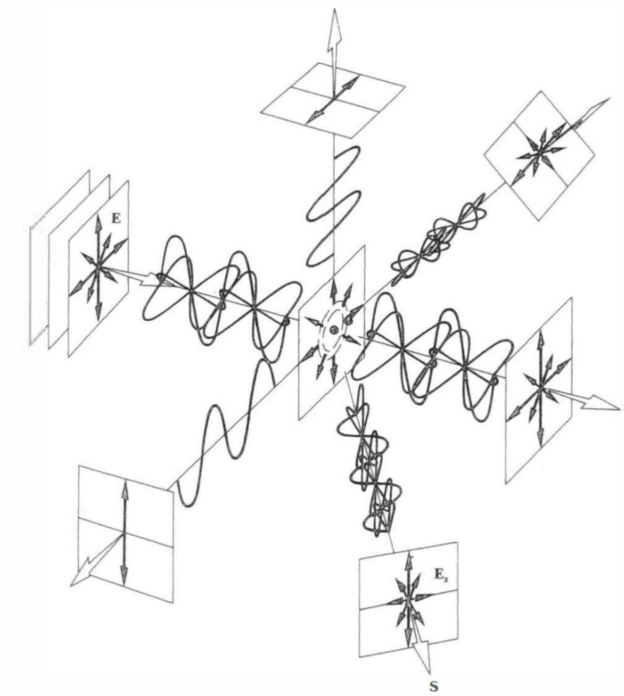


Figure 1: Scattering of unpolarised light by a molecule.

The scattering mechanism of light on a molecule results from the electric-dipole radiation from the now energised molecule. The direction of oscillation of the dipole will be aligned with the direction of linearly polarised light. Due to the pattern of electric field found from an oscillating electric dipole, an example of which can be found in Figure 2below

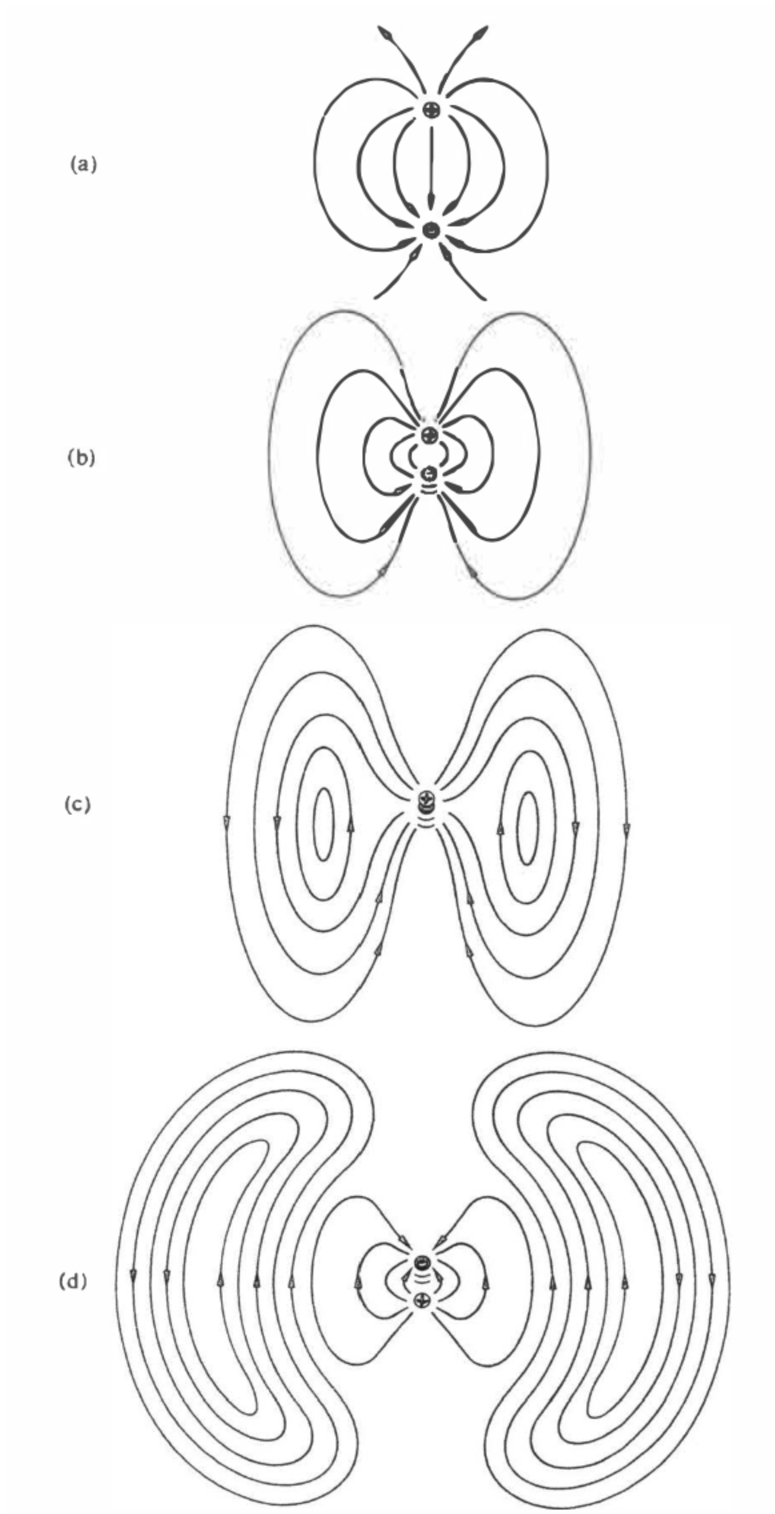


Figure 2: Scattering of unpolarised light by a molecule.

It is important to note that the dipole will not radiate in the direction of oscillation.

In liquids, the spatial distribution of the molecules will impact the scattering of light. Because of the forces between the molecules and the fact that the molecules are spaced out more regularly, scattering of molecules in any other direction than forward tends to be quite weak.

Polarised light can also be replicated using polarisers and analysers. Law of Malus states that the intensity transmitted by the analyser varies as the square of the cosine of the angle between the analyser and polariser.

Unpolarised light incident on a dielectric material like glass will produce a transmitted or refracted ray as well as a reflected ray as seen in Figure 3 below.

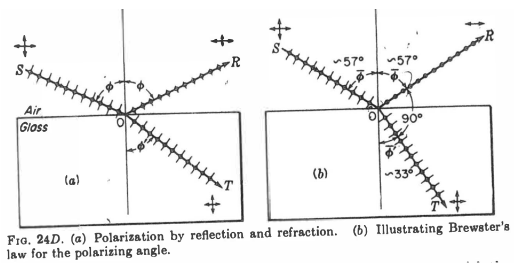


Figure 3: Scattering of unpolarised light by a molecule.

As the angle of incidence approaches a special angle, dubbed the Brewster's angle, the polarisation of the reflected light approaches linearly polarised in one direction and the angle between the reflected light and the refracted light becomes a right angle. The reflected light being maximally polarised in the plane perpendicular to the plane of incidence arises due to the fact that as the incident light sets the electrons in the atoms of the glass into oscillation, the reradiation generates the reflected beam. Thus it becomes polarised in one direction as the oscillating dipoles will only radiate in that direction.

In glass, at a normal incidence, the reflected intensity of light will be 4 per cent its total intensity, so 96 per cent of it is transmitted. This will be useful when we calculate the intensities to verify Fresnel's equations. For polarised light perpendicular to the plane of the incidence, subscripted s, the corresponding Fresnel equation is,

$$\frac{R_s}{E_s} = \frac{\sin(\phi - \phi')}{\sin(\phi + \phi')}. \tag{1}$$

Similarly, for polarised light parallel to the plane of incidence, subscripted p,

$$\frac{R_p}{E_p} = \frac{\tan(\phi - \phi')}{\tan(\phi + \phi')}. \tag{2}$$

Geometrically we can define an azimuthal angle, ψ, so that

$$\tan\psi = \frac{R_p}{R_s}. \tag{3}$$

At small angles, we can set the sines equal to the tangents and so $\frac{R_s}{E_s} \approx \frac{R_p}{E_p} = \frac{R}{E}$. We can then define $R_{12} = \frac{R^2}{E^2}$.

For metals, another constant, k, that describes the absorption of light as it enters the metal must be used. This is because metals contain many free electrons. We can now define two general equations for any material, dielectric and metal.

$$R_{12} = \frac{(1-n)^2 + k^2}{(1+n)^2 + k^2}. \tag{4}$$

The associated phase delay is then,

$$\phi = \arctan\left(\frac{2k}{1 - n^2 - k^2}\right). \tag{5}$$

2 Experimental Setup

For the experiments explored in this report, the general setup is shown in the diagram below in Figure 4.

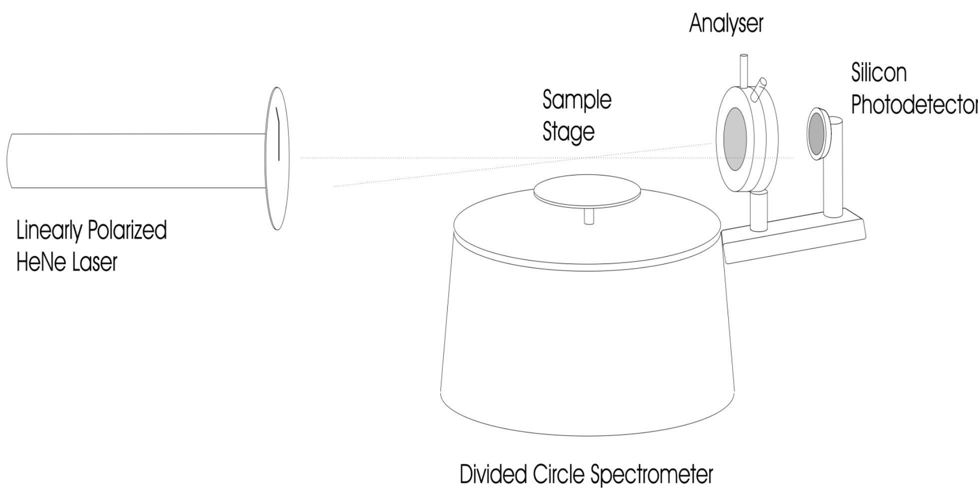


Figure 4: Schematic drawing of a basic experimental layout

By using a linear polariser filter, we measure the angle, $\theta°$, for which the polariser infront of the laser makes to make the light vertically polarised is $199 \pm 0.5°$. Likewise, the horizontal angle is $244 \pm 0.5°$. Throughout the report, we will take the azimuthal angle, ϕ, to be the angle that is read off the spectrometer with an absolute uncertainty of $\pm 0.25°$.

3 Part I

3.1 Experiment 1: Scattering

3.1.1 Method

In this first experiment, we placed solutions of Ajax and Dettol on the circle spectrometer. With the laser pointed at the solution, illuminating the liquid and then scattering light around the bottle, we moved the photodetector around the circle spectrometer, measuring the intensity of light at 3 different angles to the laser emitter, i.e parallel, 45° to and perpendicular to the emitter.

The detector was then kept 90 degrees to the beam. The half wave plate was aligned to begin recording data from a horizontal starting orientation. The analyser in front of the detector was set to produce a maximum reading of intensity.

We then further tested the orientation of scattered light by comparing horizontally and vertically polarised light incident on Dettol and Ajax particles. By placing the solution on the spectrometer and aligned the vertically polarised laser into the beaker. The detector was then spun around the spectrometer arm to capture the intensity of light at different azimuthal angle, ϕ.

3.1.2 Results

The tentative results are shown in the table below in Figure 1.

Angle/Intensity	Ajax (μA)	Dettol (μA)
Parallel	0.398	0.463
45°	0.18	0.435
Perpendicular	0.026	0.304

Table 1: Table displays the relationship between the angle observed and the intensity of the scattering. It is important to note that the intensity falls as the angle increases but due to low sample of points, hard to describe proper trendline.

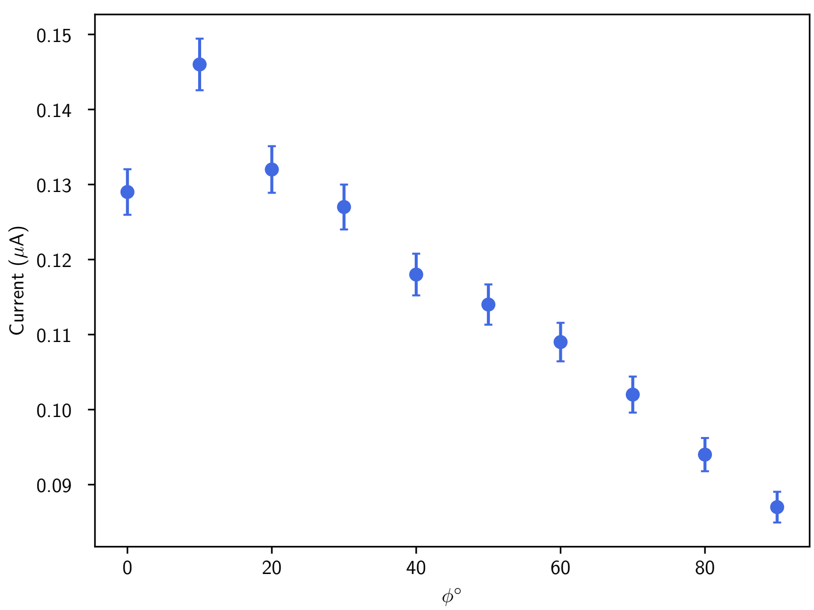


Figure 5: Hysteresis Loop found experimentally for I = 1.6A.

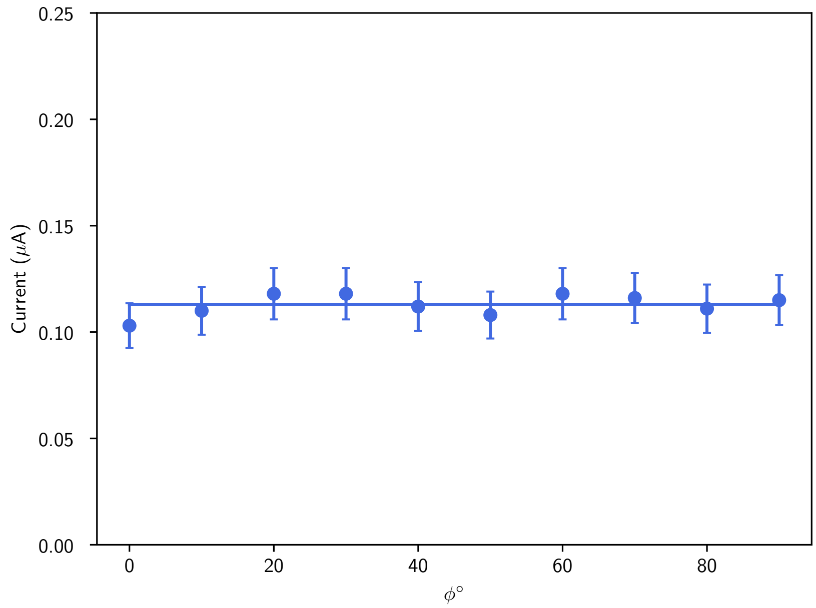


Figure 6: Hysteresis Loop found experimentally for I = 1.6A.

3.1.3 Analysis

3.2 Experiment 2: Linear Polarisation

3.2.1 Method

In this second experiment, we want to verify Malus' law. We will do this by keeping the vertical analyser upright and changing angle of the polariser on the laser. In this setup, the analyser and photodiode is parallel to the laser.

Using Jones' vector notation, we can represent the transmission of light through the polariser and analyser as matrix states. Polarisation is represented by a 2 by 1 Jones vector, since we are dealing with linear polarisation, we assume the phase of the orthogonal components to be equal,

$$\mathbf{J} = \begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix}. \tag{6}$$

The initial polarisation on the laser is vertically orientated so we can represent this using the Jones vector,

$$\mathbf{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{7}$$

Or more specifically,

$$\mathbf{J} = \begin{pmatrix} 0 \\ E_{oy} \end{pmatrix}. \tag{8}$$

To find the Jones' matrix representation of an analyser at some angle θ to the plane of polarisation of the incoming light, we first consider some arbitrary reference frame for which the polariser is aligned with, at an angle θ. So we have to introduce a rotation matrix with angle θ to transform the coordinates into the reference frame of the polariser,

$$R_{+} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{9}$$

We can now just apply the follow Jones matrix to remove any orthogonal components,

$$P_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{10}$$

Now we can transform the coordinates back into the reference frame of the lab by applying the rotation matrix again but with $-\theta$,

$$R_{-} = \begin{pmatrix} \cos -\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{11}$$

Putting all of this together,

$$\mathbf{J'} = R_{-}P_yR_{+}\mathbf{J}. \tag{12}$$

Computing the matrix multiplication will result in,

$$\mathbf{J'} = \begin{pmatrix} \sin^2\theta & -\cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{pmatrix}\mathbf{J}. \tag{13}$$

Substituting in Equation 8, we find,

$$\begin{aligned} \mathbf{J'} &= \begin{pmatrix} \sin^2\theta & -\cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta^2 \end{pmatrix}\begin{pmatrix} 0 \\ E_{0y} \end{pmatrix} && (14) \\ &= E_{0y}\begin{pmatrix} \cos\theta\sin\theta \\ \cos^2\theta \end{pmatrix} && (15) \end{aligned}$$

Taking the magnitude of the Jones vector to represent its magnitude,

$$I = |\mathbf{J'}|^2 = E_{0y}(\sin^2\theta\cos^2\theta + \cos^4\theta). \tag{16}$$

Simplifying the equation and then replacing E_{0y} with I_0,

$$\begin{aligned} I &= I_0(\cos^2\theta(1-\cos^2\theta) + \cos^4\theta) && (17) \\ &= I_0\cos^2\theta. && (18) \end{aligned}$$

We recover Malus' law.

3.2.2 Results

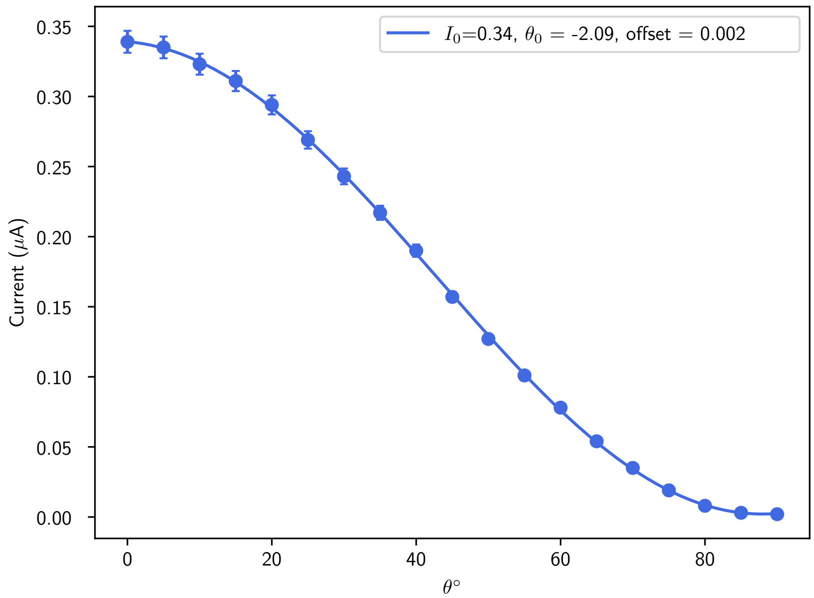


Figure 7: Hysteresis Loop found experimentally for I = 1.6A.

3.2.3 Analysis

3.3 Experiment 3: Circular Polarisation

3.3.1 Method

In this third experiment, the same setup from Experiment 2 is used except that the analyser is replaced with a filter of either quarter wave plate or circular polariser.

Using Jones' vector notation we find the matrix associated with quarter wave plates to be,

$$\mathbf{P}_{quarter} = e^{\frac{i\pi}{4}} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \tag{19}$$

assuming the fast axis is the vertical axis in this case, i.e E_y leads E_x.

3.3.2 Results

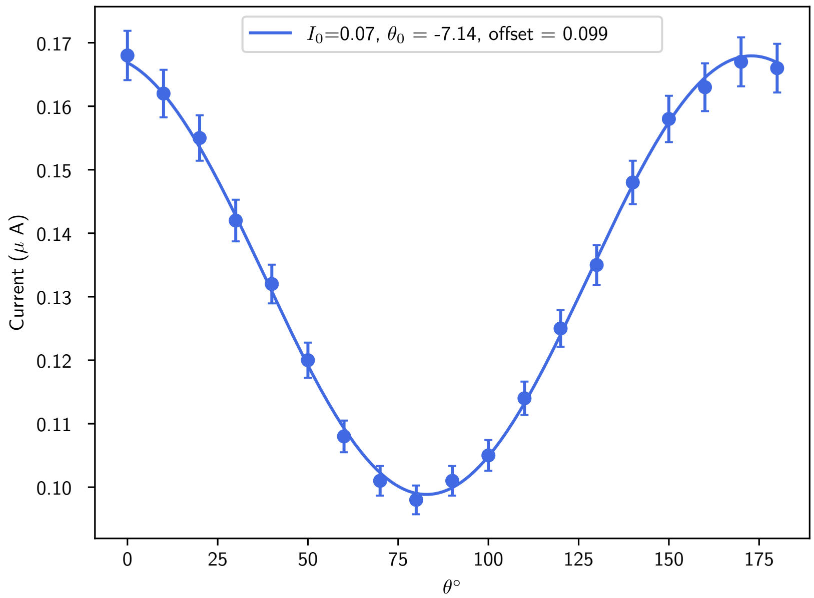


Figure 8: Hysteresis Loop found experimentally for I = 1.6A.

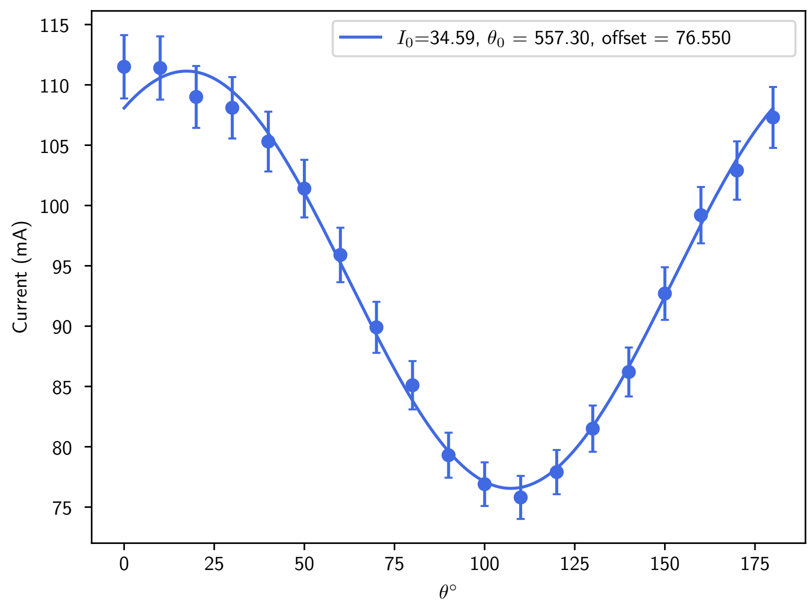


Figure 9: Hysteresis Loop found experimentally for I = 1.6A.

3.3.3 Analysis

4 Part II

4.1 Experiment 1: Reflection from Dielectrics

In these experiments we want to determine the optical constants, n, and k for the glass prism.

4.1.1 Method

Firstly, we need to determine Brewster's angle for the glass prism in order to determine its refractive index. Brewster's angle is related to the refractive index, n, through the following formula,

$$\tan \theta_B = n. \tag{20}$$

A prism is now mounted on the spectrometer table with its reflecting face directed towards the laser to produce a normal incidence. Rotating the table to produce an angle of incidence of 45 degrees to the normal we can now find the reflected beam visible as a spot on the screen on the spectrometer arm. By rotating the half-wave plate infront of the laser, and the table we can begin search for Brewster's angle.

We know that at Brewster's angle, the reflected light will be maximally polarised in the direction perpendicular to the plane of incidence. This will correspond to a minimum intensity on the screen.

Once Brewster's angle is found, we can determine the polarisation of the laser.

Next, we will find the reflectance intensities, Rp and R_s. We know Es and Ep by simply measuring the intensity of horizontally and vertically polarised light without any reflection. Therefore we can find the reflectance coefficient, which we expect to be 0.04 at the normal incidence.

In Task C, we will explore the effect of angle of incidence on the polarisation of the reflected light. The polarisation of the reflected light parallel to the plane of incidence will flip when the angle changes past the critical angle i.e the angle at total internal reflection.

4.1.2 Results

4.1.3 Analysis

4.2 Experiment 2: Reflection from Metals

4.2.1 Method

We can find the optical constants n and k through using Equations 4 and 5. The half wave plate is rotated so that the polarised light is 45 degrees to the vertical. First, the table is adjusted so that the beam hits the mirror at 20 degrees to the normal. The spectrometer arm is moved to detect the reflected beam and the analyser is rotated such that a maximum intensity is achieved.

4.2.2 Results

4.2.3 Analysis

5 Appendix

5.1 Pre-Work Theoretical Questions

5.1.1 Question 2a

What is the condition for linearly polarised light to remain unaltered as it passes through a crystal?

For the linearly polarised light to remain unaltered, the light incidence needs to experience no relative phase changes. Crystals can either be isotropic or anisotropic. In the case of isotropic crystals, the linearly polarised light has to be pass through the crystal along its polarisation axis. Anisotropic materials have a property known as birefringence, where the crystal has a different refractive index for each polarisation and propagation of light. The linearly polarised light must enter the crystal along the optical axis. This means that the light will only experience the refractive index of ordinary rays. If light were to enter the crystal in any other direction, then it would experience two refractive indexes which will alter the polarisation of the wave. Similarly, in biaxial crystals, there are two optical axes (binormals) that allow light to travel through without birefringence.

5.1.2 Question 2b

What are the conditions to create circularly polarised light?

You can use a linear polariser to get linearly polarised light first and then use a quarter wave layer to create a circular polariser. The corresponding phase difference needs to be 90 degrees. To achieve this, the formula for phase difference in uniaxial crystals can be used,

$$\Delta\phi = \frac{2\pi}{\lambda}(n_e - n_o)d. \tag{21}$$

By adjusting the thickness, d, of the crystal appropriately as n_e and n_o are fixed, we can obtain a phase difference of 90 degrees. Then the quarter wave plate needs to placed at an angle of 45 degrees with the plane of the incident polarised light.

5.1.3 Question 2c

What is the effect of a quarter wave plate on circularly polarised light?

The quarter wave plate will introduce a 90 degree phase shift which will convert circularly polarised light back to linearly polarised.

5.1.4 Question 2d

What is the effect of a half wave plate on circularly polarised light?

A half wave plate will introduce a phase difference of 180 degrees, meaning that the handedness of the circular polarisation will switch i.e right handed circular polarisation will turn into left handed circular polarisation and vice versa.

5.1.5 Question 2e

Sometimes the scattering of light off particles may lead to polarisation. Explaining in terms of scattering, why is the sky blue and why clouds are grey?

In 1871, Rayleigh discovered that when light is scattered by molecules or particles much smaller than the wavelength of the light, the intensity of the scattered light has the following property,

$$I \propto \frac{1}{\lambda^4}. \tag{22}$$

This is why the sky is blue as the lower wavelengths of light, i.e bluer, are scattered much more. At sunset, when the sun is low of the horizon, the blue and violet light is scattered out of the direct line of sight, giving the sky its redder tinge. In clouds, the particles are no longer small enough for Rayleigh scattering and instead we look at Mie scattering. Here we find that the droplets of water will scatter all wavelengths equally and so clouds are perceived as white. As the clouds become thicker, more light is absorbed and less sunlight can penetrate, dimming the sky and the appearance of the cloud.

When light strikes a particle, it will induce an oscillation in the charges of the particle, causing it to re-emit light but they do so more strongly in directions where the electric field is perpendicular to the direction of observation. For the original incoming light, it becomes polarised as the electric field in the direction of scattering is minimised.

5.1.6 Question 2f

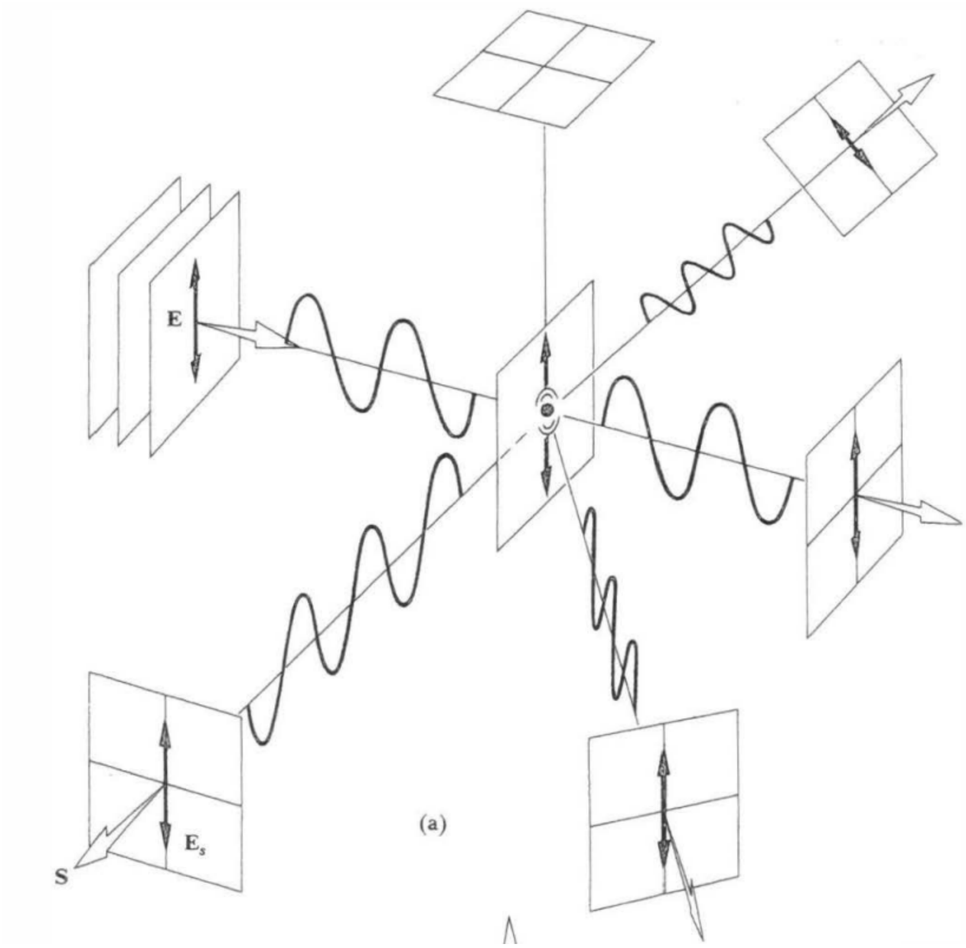


Figure 10: Scattering of polarised light by a molecule

Scattered light will only propagate in forward directions and not in the direction of the electric field. The magnitude will be equal in the directions orthogonal to the propagation of the electric field and decrease as the angle between the propagation of the scattered wave and the propagation of the original wave's electric field is smaller.

5.1.7 Question 2g

Using

$$I_1 = I_0 \cos^2 \theta. \tag{23}$$

a. 100% b. 50% c. 0%

5.2 Lab Book

5.2.1 Proper Attempt

Final lab attempt

Reading on half wave plate to make the laser vertically polarised: 199°

Horizontal reading: 244°

Reading on table to obtain normal incidence with detector: $259\frac{1}{3} \pm 0.25°$

Experiment 2

Background intensity: 5.64 (10nA)

Reading for peak analyser: $151 \pm 0.5°$

Experiment 3

Task A – Light is internally vertically polarised

Reading on analyser to give a minimum:

Reading on analyser to give a maximum: $151 \pm 0.5°$

Background intensity:

~~Vernier table reading: $330\frac{1}{3}° \pm 0.25°$ for normal incidence~~

Task B

Reading on analyser to give a maximum :

Background intensity:

<u>Experiment 4</u>

Task A

Reflectance angle reading on table : $330 \pm 0.25°$

Brewster's angle:

↳ Table position: $21 \pm 0.25°$

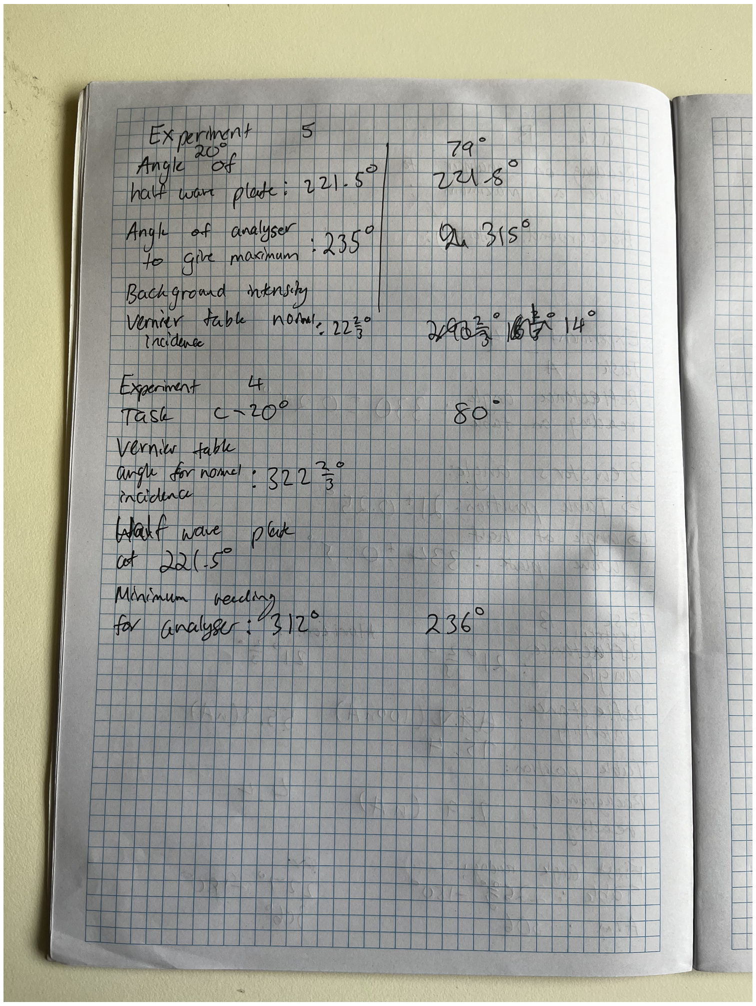
↳ angle of half wave plate : $334 \pm 0.5°$

Task B

	vertical	Horizontal
Reflectance angle :	$21° \frac{2}{3}°$	$21° \frac{2}{3}°$
Reflectance intensity :	~~69.2~~ (100nA) 75.4	55.3(nA)
Table position :		
Background reading :	7.9 (nA)	4.4

First angle reading:

Table :	$225\frac{2}{3}° - 180°$	$227° - 180°$
Arm :	$306°$	$306°$



Experiment 5

Angle of (20°)
half wave plate: 221.5° | 79°
221.8°

Angle of analyser to give maximum: 235° | ~~2~~ 315°

Background intensity

Vernier table normal incidence: $22\frac{2}{3}°$ | ~~290~~ $\frac{2}{3}°$ ~~16~~ $\frac{2}{3}°$ 14°

Experiment 4

Task c~20° | 80°

Vernier table angle for normal incidence: $322\frac{2}{3}°$

Half wave plate at 221.5°

Minimum reading for analyser: 312° | 236°

5.3 Failed Attempts

Lab 1

Part 1

Experiment 1 - Scattering (diode on μA)

"~~Analyser~~ Protractor" - At 290° (287°) for source to be vertically polarised

Orientation of photodiode relative to the laser (Assume light is vertically linearly polarised)

~~Dettol~~ Ajax (μA)

Parallel: 0.398

45°: 0.18

Perpendicular: 0.026

~~Ajax~~

Dettol

Perpendicular: 0.304

45°: 0.463

Parallel: 0.435

Lab 1

Part 1

Experiment 1 - Scattering (diode on μA)

"~~Analyser~~ protractor" - At 290° (287°) for source to be vertically polarised

Orientation of photodiode relative to the laser (Assume light is vertically linearly polarised)

~~Dettol~~ Ajax (μA)

Parallel: 0.398

45°: 0.18

Perpendicular: 0.026

~~Ajax~~

Dettol

Perpendicular: 0.304

45°: 0.463

Parallel: 0.435

Experiment 2

(Analyser: 45° as base)

No IBG)

~~[Protractor~~:

Experiment 3:

Protractor: 289° Analyser: 269.5° (minimum)

Analyser: 17.5° (maximum)

Circular Analyser: used 10mA scale

Analyser: 112.5° (maximum)

Checking circular polariser

Side 2 facing laser = 0.159

Side 1 facing laser = 0.029

Ratio = 5.48

Side 1: 0.003 / 0.387

Side 2: 0.177 / 0.220

~~Part 2~~ Lab 2

Experiment 1

$0^\circ \Rightarrow 264\frac{2}{3}^\circ$ ~~For Dettol~~

Increments of $10^\circ \rightarrow +10$

~~$0^\circ \Rightarrow 267$~~ ~~for Ajax~~

→ spectrometer

Dettol scatters but Ajax only refracts

→ non zero current at $\theta = 267^\circ$ for vertical polarised light and $\theta = 274\frac{2}{3}^\circ$ for horizontal polarised light.

For horizontally polarised light, the Dettol is measured from max anticlockwise spectrometer arm i.e. $\theta = 234^\circ$ to min, $\theta = 58\frac{1}{3}^\circ$, going from $234 \rightarrow 58\frac{1}{3}^\circ$ or

$\theta^\circ = 0 \rightarrow 184\frac{1}{3}^\circ$

Experiment 3

Task A

The laser is kept linearly polarised.

Analyser caused minimum at ~~267°~~ 270°

Background reading for all experiments is 3.1 nA ~ 0

Task B

The peak analyser angle is 101° for a current reading of 8.612 (10 μA). Polariser sent at vertical (199°)

Without any plates the laser emits 12.5 (100 μA) at vertically polarised.

Sources of error

Ammeter - negligible
Laser - negligible & large variation at 1% of (10 μA reading) ie. 0.01 varies
Goes from 12.45 → 12.55 undisturbed

Polariser - Increment of 1°
Spectrometer - Increment of $\frac{1}{3}$° + more bc wear
Analyser - Increment of 1°

Circular polariser check
Laser is initially vertically polarised
With side 1 facing the laser
= 0.524 (μA)
Side 2
= 155.4 (100 μA)

Background = ~4 (10 μA)

PART II 31st October

Experiment 4

Task A

[264° → 219° = 45° clockwise]

269° → 224° = 45° anticlockwise

(half wave plate = linear polarisor)

laser is polarised but you don't know where laser is polarised

Brewsters angle − dimmest

rotate half-wave plate to min, then table
laser dimmest

Brewsters = 221.67

10 Brewsters angle = refractive index

Task B

centre: of detector
200mV = max

check linearly polarised by turning half wave plate til no transmission (detector)

199° → iii) vertical direction (vertical polarisation on waveplate)

laser at max

incorrect
[295° = table position
~~8.9~~ = 18.9 100 micron Amps = ~~each~~ detector signal
0.1 100 micron = background reading]

when change half wave plate by 1°, polarisation changes by 2°

~~Task B again~~

~~use analyser to vertically polarise laser or use linear polariser~~

horizontally polarised
rotate waveplate by 45° and check with linear polariser (on its side)

244° → half wave plate for horizontal polarisation

table position = 205.6° reflected beam back on itself

iv) table position = 184.8°

detector signal = ~~16.4~~ 15.1 vertically

Background ≈ 0.00

table position = 191° horizontally
(our zero)

~~Task C~~

~~340.5° (normal)~~

~~→ 20° 320.5°~~

~~analysing angle at minimum = 52°~~ incorrect

~~→ 80° ~~280.5°~~ ~~240.5°~~ 260.5°~~

~~(360.5) 000.5° → 20° 52°~~

~~060.5° → 80°~~

5.4 Data Analysis in Python

5.4.1 Curve Fitting

Python was used to create theoretical models to best fit the experimental data. First the cosine squared function was defined with parameters: I_0, theta0 and offset with variable theta. The amplitude is given by the initial intensity, I_0, the phase shift is given by theta0 and the vertical displacement of the funtion is given by the parameter offset.

```
def curve(theta,I_0,theta0,offset):
    return I_0 * np.cos(np.radians((theta - theta0)))**2 + offset
```

Using the scipy optimize package, the curve fit function can be used which uses a least-squares algorithm to guess the parameters that best fits the data input. An example can be seen below,

```
x = data['Angle (degrees)']
y = data['Current (micro amps)']

param,covariance = sp.curve_fit(curve,x,y)
```

The return of the function is of course an array, param, of the best fitting parameters.

Rather than plotting the model as a function of the experimental data, the model was plotted over a a higher resolution of x values to provide a smoother curve.

```
x_smooth = np.linspace(x.min(),x.max(),1000)
y_smooth = curve(x_smooth,*param)
plt.plot(
    x_smooth,
    y_smooth,
    label=rf'$I_0$={param[0]:.2f}, $\theta_0$ = {param[1]:.2f}, offset = {param[2]:.3f}'
)
```