(Sheet # 2) – Part [1] – Electrostatic Force

Q1: Compute the electrostatic force between two charges of $5 \times 10^{-9}\ C$ and $-3 \times 10^{-8}\ C$ which are separated by $d = 10\ cm$

Solution

This is the scalar form of electrostatic force $|F|$

$$|F| = k_e \frac{|q_1 q_2|}{d^2} = (9 \times 10^9) \times \frac{|(5 \times 10^{-9}) \times (-3 \times 10^{-8})|}{(10 \times 10^{-2})^2} = 1.35 \times 10^{-4}\ N = 135\ \mu N$$

Q2: Two metallic spheres located at distance of $d = 5\ cm$ attract one another with force of $F = 3\ mN$. If one of them has three times more charges than the other. Determine the charge value of the two spheres

Solution

This is the scalar form of electrostatic force $|F|$

$$q_2 = 3q_1$$

$$|F| = k_e \frac{|q_1 q_2|}{d^2} = (9 \times 10^9) \times \frac{|(q_1) \times (q_2)|}{(5 \times 10^{-2})^2} = 3\ mN = 3 \times 10^{-3}\ N$$

$$\frac{|(q_1) \times (3q_1)|}{(5 \times 10^{-2})^2} = 3.333 \times 10^{-13} \qquad 3{q_1}^2 = 8.333 \times 10^{-16} \qquad {q_1}^2 = 2.778 \times 10^{-16}$$

$$q_1 = 1.667 \times 10^{-8}\ C = 16.67\ nC \qquad q_2 = 3(1.667 \times 10^{-8}) = 5 \times 10^{-8}\ C = 50\ nC$$

Q3: A particle of charge $-40\ \mu C$ is located on $y-axis$ at the point $y = 20\ m$ and the second particle of charge $20\ \mu C$ is placed on $x-axis$ at $x = 30\ m$. Determine the total electrostatic force on a third particle of charge $-10.0\ \mu C$ placed at the origin

Solution

This is the vector form of electrostatic force F

For point charge [1] $-40\ \mu C$ is located at $y-axis$ at point $(0, 20, 0)$

For point charge [2] $20\ \mu C$ is located at $x-axis$ at point $(30, 0, 0)$

For point charge [3] $-10\ \mu C$ is located at $origin$ at point $(0, 0, 0)$

$$F = k_e \frac{|q_1 q_2|}{|R|^3} R = k_e \left[\frac{q_1 q_3}{|R_{13}|^3} R_{13} + \frac{q_2 q_3}{|R_{23}|^3} R_{23}\right]$$

$$F_{13} = k_e \frac{q_1 q_3}{|R_{13}|^3} R_{13} \qquad R_{13} = (0,0,0) - (0,20,0) = -20a_y \qquad |R_{13}| = \sqrt{(-20)^2} = 20$$

$$F_{23} = k_e \frac{q_2 q_3}{|R_{23}|^3} R_{23} \qquad R_{23} = (0,0,0) - (30,0,0) = -30a_x \qquad |R_{23}| = \sqrt{(-30)^2} = 30$$

$$F = (9 \times 10^9)\left[\frac{(-40 \times 10^{-6}) \times (-10 \times 10^{-6})}{(20)^3}(-20a_y) + \frac{(20 \times 10^{-6}) \times (-10 \times 10^{-6})}{(30)^3}(-30a_x)\right]$$

$$F = (9 \times 10^9)[(-1 \times 10^{-12})a_y - (2.22 \times 10^{-13})a_x]$$

$$F = (2 \times 10^{-3})a_x + (-9 \times 10^{-3})a_y\ N = 2a_x - 9a_y\ mN \qquad \text{Vector Form}$$

$$Magnitude\ |F| = \sqrt{(2)^2 + (-9)^2} = 9.22 \times 10^{-3}\ N = 9.22\ mN$$

$$Direction\ a_F = \frac{F}{|F|} = \frac{2a_x - 9a_y}{9.22} = 0.217a_x - 0.976a_y$$

Q4: A point charge $q_1 = 2\ \mu C$ located at origin and another point charge $q_2 = -5\ \mu C$ is on the coordinate $(x = 3, y = 4)\ m$

Determine:

A. **The electrostatic force on charge q_1**
B. **State that this force is attractive or repulsive**

Solution

This is the vector form of electrostatic force F

For point charge $q_1 = +2\ \mu C$ is located at $origin$ at point $(0, 0, 0)$

For point charge $q_1 = -5\ \mu C$ is located at point $(3, 4, 0)$

$$F_{21} = k_e \frac{|q_1 q_2|}{|R_{21}|^3} R_{21}$$

$$R_{21} = (0,0,0) - (3,4,0) = -3a_x - 4a_y \qquad |R_{21}| = \sqrt{(-3)^2 + (-4)^2} = 5$$

$$F = (9 \times 10^9)\left[\frac{(2 \times 10^{-6}) \times (-5 \times 10^{-6})}{(5)^3}(-3a_x - 4a_y)\right] = (2.16 \times 10^{-3})a_x + (2.88 \times 10^{-3})\ a_y\ N$$

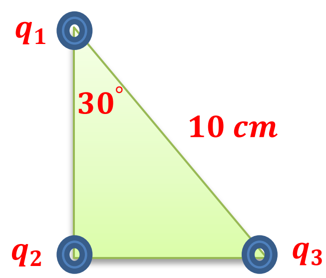
A. $F = 2.16a_x + 2.88a_y\ mN$ **Vector Form**

$$Magnitude\ |F| = \sqrt{(2.16)^2 + (2.88)^2} = 3.6 \times 10^{-3}\ N = 3.6\ mN$$

$$Direction\ a_F = \frac{F}{|F|} = \frac{2.16a_x + 2.88a_y}{3.6} = 0.6a_x + 0.8a_y$$

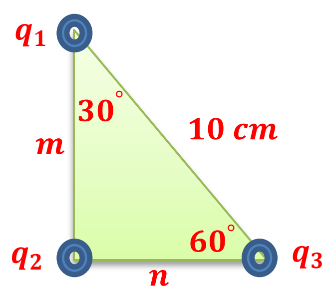
B. **A s the two charges are in opposite sign the net force is attractive**

Q5: Three point charges are fixed in place in the right angle triangle shown below, in which $q_1 = 0.71\ \mu C$ and $q_2 = -0.67\ \mu C$. Determine magnitude and direction of the electrostatic force on the $+1.0\ \mu C$ (let's call this q_3) charge due to the other two charges



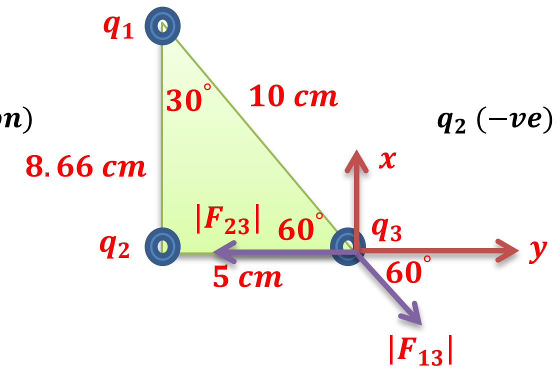
Solution

$m = 10 \cos 30^\circ = 8.66\ cm$

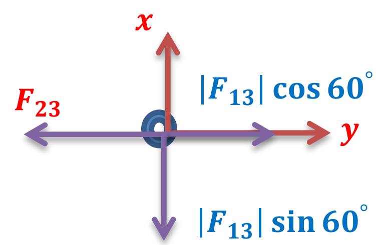


$n = 10 \sin 30^\circ = 5\ cm$

$q_1 \& q_2\ (+ve) \rightarrow F_{13}\ (Repulsion)$



$q_2\ (-ve) \&\ q_3 (+ve) \rightarrow F_{23}\ (Attraction)$



This is the vector form of electrostatic force F

$$F_3 = \sum F_x a_x + \sum F_y a_y = [|F_{13}|\cos 60^\circ - |F_{23}|]a_x + [|F_{13}|\sin 60^\circ](-a_y)$$

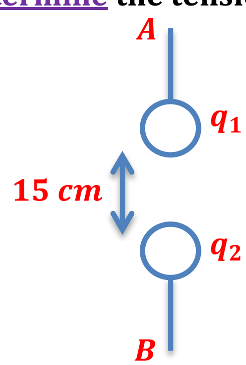
$$|F_{13}| = k_e \frac{|q_1 q_3|}{{d_{13}}^2} \qquad |F_{13}| = (9\times 10^9)\left[\frac{|(0.71\times 10^{-6})(1\times 10^{-6})|}{(10\times 10^{-2})^2}\right] = 0.639\ N$$

$$|F_{23}| = k_e \frac{|q_2 q_3|}{{d_{23}}^2} \qquad |F_{23}| = (9\times 10^9)\left[\frac{|(-0.67\times 10^{-6})(1\times 10^{-6})|}{(5\times 10^{-2})^2}\right] = 2.412\ N$$

$$F_3 = [0.639\cos 60^\circ - 2.412]a_x + [0.639\sin 60^\circ](-a_y) = -2.1a_x - 0.5534a_y$$

$$|F_3| = \sqrt{(-2.1)^2 + (-0.5534)^2} = 2.1717\ N \qquad a_F = \frac{F_3}{|F_3|} = \frac{-2.1a_x - 0.5534a_y}{2.1717} = -0.967a_x - 0.255a_y$$

Q6: Two small insulating spheres are attached to silk threads and aligned vertically as shown in the below figure for stationary position. These spheres have equal masses of $40\ g$ and carry charges q_1 and q_2 of equal magnitude $2.0\ \mu C$ but opposite sign.The spheres are brought into the positions shown in the figure, with a vertical separation of $15\ cm$ between them. Note that you cannot neglect gravity. Determine the tension in the lower threads

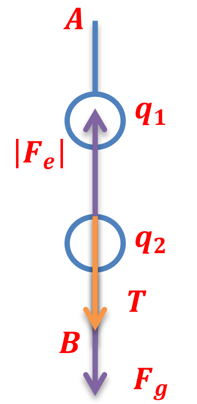


Solution

Lower threads are referred to the sphere B and its corresponding charge q_2

All forces in the $y-$ $axis$ **Tension and weight forces of sphere B in the bottom direction**

As one of the two charges is positive and the other is negative **F_e is attraction to the top**



This is the vector form of electrostatic force F

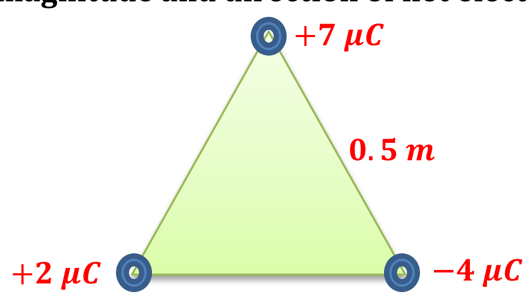
$$F_2 = \sum F_x a_x + \sum F_y a_y = 0 + [|F_e| - T - F_g]a_y$$

$$|F_e| = k_e \frac{|q_1 q_2|}{d^2} \qquad |F_e| = (9 \times 10^9)\left[\frac{|(2 \times 10^{-6})(-2 \times 10^{-6})|}{(15 \times 10^{-2})^2}\right] = 1.6\ N$$

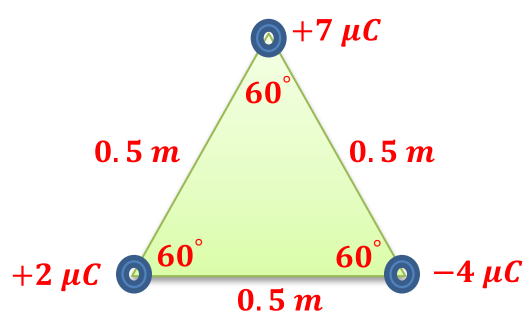
$$F_g = mg = \frac{40}{1000} \times 9.81 = 0.3924\ N \qquad for\ stationary\ position\ F_2 = 0$$

$$T = 1.6 - 0.3924 = 1.208\ N$$

(Report Section) Q7: Three point charges are located at corners of an equilateral triangle as shown below. Determine magnitude and direction of net electrostatic force on $7\ \mu C$ charge

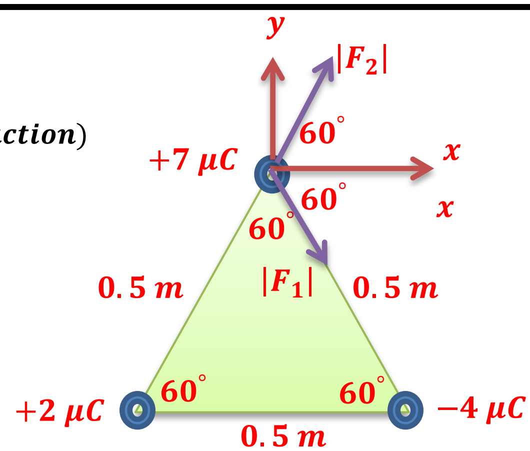


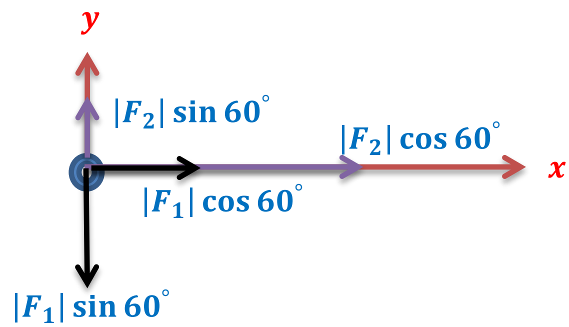
Solution



$+7\ \mu C\ \&\ -4\ \mu C \rightarrow F_1\ (Attraction)$

$+7\ \mu C\ \&\ +2\ \mu C \rightarrow F_2\ (Repulsion)$





This is the vector form of electrostatic force F

$$F = \sum F_x\, a_x + \sum F_y\, a_y = [|F_1|\cos 60^\circ + |F_2|\cos 60^\circ]a_x + [|F_2|\sin 60^\circ - |F_1|\sin 60^\circ]a_y$$

$$|F_1| = (9\times 10^9)\left[\frac{|(7\times 10^{-6})(-4\times 10^{-6})|}{(0.5)^2}\right] = 1.008\ N$$

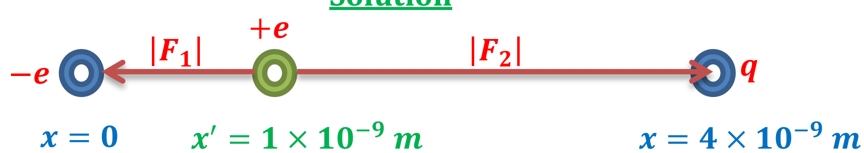
$$|F_2| = (9\times 10^9)\left[\frac{|(7\times 10^{-6})(2\times 10^{-6})|}{(0.5)^2}\right] = 0.504\ N$$

$$F = [1.008\cos 60^\circ + 0.504\cos 60^\circ]a_x + [0.504\sin 60^\circ - 1.008\sin 60^\circ]a_y = 0.756a_x - 0.4365a_y$$

$$|F| = \sqrt{(0.756)^2 + (-0.4365)^2} = 0.873\ N \qquad a_F = \frac{F}{|F|} = \frac{0.756a_x - 0.4365a_y}{0.873} = 0.866a_x - 0.5a_y$$

Q8: An electron is fixed at position $x = 0$ and a second charge q is fixed at $x = 4\times 10^{-9}\ m$ (to the right). A proton is now placed between the two at $x' = 1\times 10^{-9}\ m$. Determine the charge q be (magnitude and sign) so that the proton is in equilibrium

Solution



$as\ proton\ and\ electron\ have\ opposite\ signed\ charges \qquad |F_1|\ is\ an\ attraction\ force$

$for\ equilibrium\ at\ proton\ (+e) \qquad |F_1| = |F_2|$

$$(9\times10^{9})\left[\frac{\left|(1.6\times10^{-19})(-1.6\times10^{-19})\right|}{(1\times10^{-9})^{2}}\right]=(9\times10^{9})\left[\frac{\left|(1.6\times10^{-19})q\right|}{(3\times10^{-9})^{2}}\right]$$

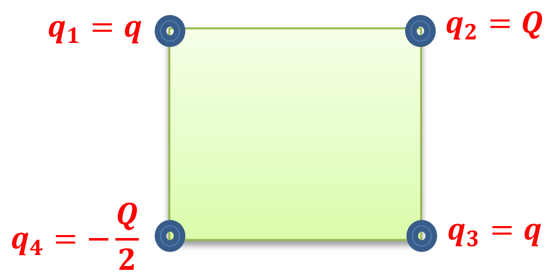
$$2.56\times10^{-20}=0.01778q$$

$$|q|=1.44\times10^{-18}\ C$$

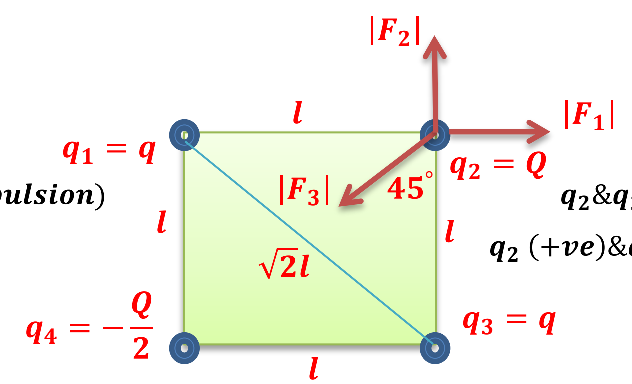
As the $|F_2|$ is attractive force and proton is a positive charge $\quad$ q must be negative charge

$$q=-1.44\times10^{-18}\ C$$

Q9: Four point charges are located on the corners of a square shown in the below figure. If the net Coulomb force on q_2 is zero. Determine is the ratio of Q/q



Solution



$q_1 \& q_2\ (+ve) \rightarrow |F_1|\ (Repulsion)$

$q_2 \& q_3\ (+ve) \rightarrow |F_2|\ (Repulsion)$

$q_2\ (+ve) \& q_4\ (-ve) \rightarrow |F_4|\ (Attraction)$

This is the vector form of electrostatic force F

$$F_2=\sum F_x\,a_x+\sum F_y\,a_y=\left[|F_1|-|F_3|\cos 45^{\circ}\right]a_x+\left[|F_2|-|F_3|\sin 45^{\circ}\right]a_y$$

$$|F_1|=k_e\left[\frac{|qQ|}{(l)^2}\right]\qquad |F_2|=k_e\left[\frac{|qQ|}{(l)^2}\right]\qquad |F_3|=k_e\left[\frac{\left|-\frac{Q}{2}\times Q\right|}{\left(\sqrt{2}l\right)^2}\right]$$

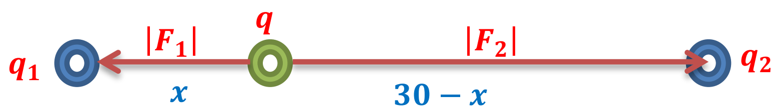
$$for\ \ F_2=0\qquad \left[|F_1|-|F_3|\cos 45^{\circ}\right]a_x=0\qquad k_e\left[\frac{|qQ|}{(l)^2}\right]=k_e\left[\frac{\left|-\frac{Q}{2}\times Q\right|}{\left(\sqrt{2}l\right)^2}\right]\cos 45^{\circ}$$

$$q=\frac{Q}{4}\times0.7071=0.177Q\qquad \frac{Q}{q}=\frac{1}{0.177}=4\sqrt{2}=5.657$$

Note that: **if you utilized the term $\left[|F_2|-|F_3|\sin 45^{\circ}\right]a_y=0$ it will gives the same solution**

Q10: Two point charges $q_1 = +2\ \mu C$ and $q_2 = +8\ \mu C$ are $30\ cm$ apart from each other. Extra charge q is placed so that three charges are brought to a balance. Determine the location of the charge q

Solution



$to\ make\ charges\ balanced\ for\ the\ same\ signed\ q_1 \& q_2 \qquad q\ positioned\ between\ q_1 and\ q_2$

$for\ balance \qquad |F_1| = |F_2|$

$$k_e\left[\frac{|(2\times10^{-6})q|}{(x)^2}\right] = k_e\left[\frac{|(8\times10^{-6})q|}{(30-x)^2}\right]$$

$$\left[\frac{x}{30-x}\right]^2 = \frac{2\times10^{-6}}{8\times10^{-6}} = \frac{1}{4} \qquad \frac{x}{30-x} = \frac{1}{2} \qquad 2x = 30 - x \qquad 3x = 30 \qquad x = 10\ cm$$

$q\ is\ placed\ at\ 10\ cm\ to\ the\ right\ from\ q_1\ or\ at\ 30\ cm\ to\ the\ left\ from\ q_2$

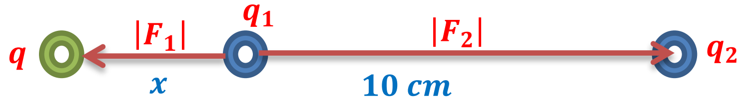
(Report Section)Q11: Two point charges are given as $q_1 = +2\ \mu C$ and $q_2 = -8\ \mu C$ are at distance $d = 10\ cm$. Determine the position of a third charge q_3 to be placed so that net Coulomb force acted upon it is zero

Solution

$for\ a\ balanced\ charges \qquad q\ positioned\ in\ the\ left\ of\ q_1\ or\ in\ the\ right\ of\ q_2$

$for\ balance \qquad |F_1| = |F_2|$

$if\ q\ positioned\ in\ the\ left\ of\ q_1$



$$k_e\left[\frac{|(2\times10^{-6})q|}{(x)^2}\right] = k_e\left[\frac{|(-8\times10^{-6})q|}{(10+x)^2}\right]$$

$$\left[\frac{x}{10+x}\right]^2 = \frac{2\times10^{-6}}{8\times10^{-6}} = \frac{1}{4} \qquad \frac{x}{10+x} = \frac{1}{2} \qquad 2x = 10 + x \qquad x = 10 \qquad x = 10\ cm$$

$q\ is\ placed\ at\ 10\ cm\ to\ the\ left\ from\ q_1\ or\ at\ 20\ cm\ to\ the\ left\ from\ q_2$

Note that: Report (Q7 and Q11) will be delivered in beginning of 5th week section

End of Sheet

Dr. Mohamed Tayseer