$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$\sin 2A \equiv 2\sin A\cos A$

$\cos(2A) \equiv \cos^2 A - \sin^2 A$
$\equiv 2\cos^2 x - 1$
$\equiv 1 - 2\sin^2 x$

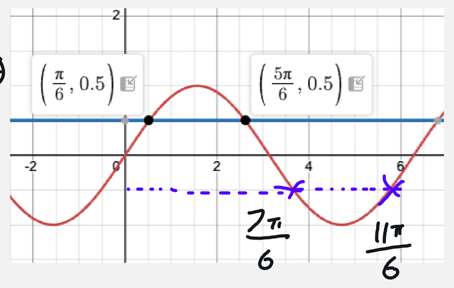
$\tan(2x) \equiv \frac{2\tan x}{1-\tan^2 x}$

1	Solve each equations for θ in the interval $0 \le \theta \le 2\pi$. Give your answers to 3s.f. where appropriate a) $cosec^2\theta - 4 = 0$ b) $cot\theta cosec\theta = 6cot\theta$ c) $cosec\theta = 4sec\theta$ d) $2cos\theta = cot\theta$ e) $5sin\theta - 2cosec\theta = 3$	TOTAL 15 marks
2	Prove each identity a) $secx - cosx \equiv sinxtanx$ b) $(1+cosx)(cosecx - cotx) \equiv sinx$ c) $\frac{cotx - cosx}{1 - sinx} \equiv cotx$ d) $(sinx + tanx)(cosx + cotx) \equiv (1 + sinx)(1 + cosx)$	TOTAL 12 marks
3	Show that: **a** $\text{cosec}^2 x + \tan^2 x \equiv \sec^2 x + \cot^2 x$ **b** $\cot^2 x + \cos^2 x \equiv (\text{cosec}\, x - \sin x)(\text{cosec}\, x + \sin x)$ **c** $\frac{1}{1+\sin x} + \frac{1}{1-\sin x} \equiv 2 + 2\tan^2 x$	TOTAL 9 marks
4	Given that $sinx = \frac{4}{5}$ and that $90° \le x \le 180°$, find the exact values of a) $cosx$ b) $cotx$ c) $cosecx$	TOTAL 5 marks
5	Solve $2cosec^2x + 5cotx = 5$ in the interval $-\pi \le \theta \le \pi$, giving your answers to 2 d.p.	TOTAL 4 marks
6	**a** Prove that $\sec^4\theta - \tan^4\theta = \sec^2\theta + \tan^2\theta$. **(3 marks)** **b** Hence solve, in the interval $-180° \leqslant \theta \leqslant 180°$, $\sec^4\theta = \tan^4\theta + 3\tan\theta$. **(4 marks)**	TOTAL 7 marks

1a) $\frac{1}{\sin^2\theta} - 4 = 0$

$1 = 4\sin^2\theta$

$\frac{1}{4} = \sin^2\theta \rightarrow \pm\frac{1}{2} = \sin\theta$



$x = \frac{\pi}{6}$ $\quad \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $\frac{1}{\sin\theta\tan\theta} = \frac{6}{\tan\theta} \rightarrow \frac{1}{\sin\theta} = 6 \rightarrow \boxed{\sin\theta = \frac{1}{6}}$

$x = 0.167$ $\quad \pi - 0.167 = 2.97$

$x = 0.167, 2.97$

$\sin\theta\tan\theta = \frac{\tan\theta}{6}$

$\sin\theta\tan\theta - \frac{\tan\theta}{6} = 0$

$\tan\theta\left(\sin\theta - \frac{1}{6}\right) = 0$

$\tan\theta = 0$ or $\boxed{\sin\theta - \frac{1}{6} = 0}$

$= 0, \pi, 2\pi$

c) $\frac{1}{\sin\theta} = \frac{4}{\cos\theta}$

$\cos\theta = 4\sin\theta$

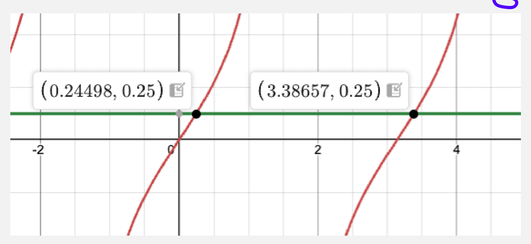
$4\tan\theta = 1$

$\tan\theta = \frac{1}{4}$

$x = 0.245$

$\pi + 0.245 = 3.387$

$x = 0.245, 3.39$



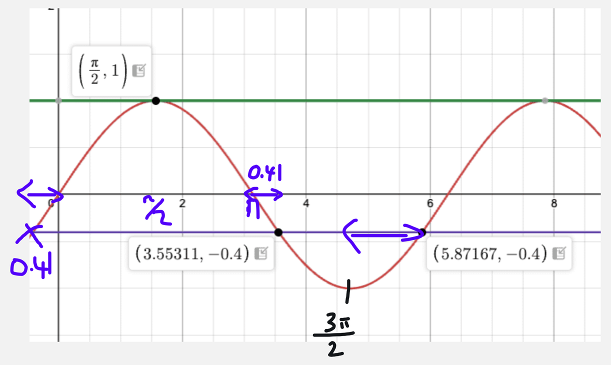
d) $2\cos\theta = \frac{1}{\tan\theta}$ $\quad 2\cos\theta\tan\theta = 1 \Rightarrow 2\cos\left(\frac{\sin\theta}{\cos\theta}\right) = 1 \rightarrow 2\sin\theta = 1 \rightarrow \sin\theta = \frac{1}{2}$

$x = \frac{\pi}{6}, x = \frac{5\pi}{6}$

e) $5\sin\theta - \frac{2}{\sin\theta} = 3 \rightarrow 5\sin^2\theta - 2 = 3\sin\theta \rightarrow 5\sin^2\theta - 3\sin\theta - 2 = 0$

$S = \sin\theta$

$(5s + 2)(s - 1)$



$\sin\theta = 1$ $\quad \sin\theta = -\frac{2}{5}$

$\theta = \frac{\pi}{2}$

$\theta = 3.55$

$\pi + 0.41$ $\quad -0.41 + 2\pi = 5.87$

$\theta = \frac{\pi}{2}, 3.55, 5.87$

2) a) $\sec x - \cos x = \sin x\tan x$

$\frac{1}{\cos x} - \cos x = \sin x\tan x \rightarrow 1 - \cos^2 x = \sin x\tan x\cos x$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = \sin x\tan x\cos x$

$\sin^2 x = \sin x\frac{\sin x}{\cos x}\cos x$ $\quad \tan x = \frac{\sin x}{\cos x}$

$\sin^2 x = \sin^2 x$

b) $(1+\cos x)(\text{cosec}\,x - \cot x) \equiv \sin x \Rightarrow \text{cosec}\,x - \cot x + \cos x\,\text{cosec}\,x - \cot x\cos x \rightarrow \frac{1}{\sin x} - \frac{1}{\tan x} + \frac{\cos x}{\sin x} - \frac{\cos x}{\tan x} = \sin x \rightarrow \frac{1}{\sin x} - \frac{\cos x}{\sin x} + \frac{\cos x}{\sin x} - \frac{\cos^2 x}{\sin x}$

$\frac{\cos}{\sin}$ $\quad \frac{\cos^2 x}{\sin x}$

$\frac{1 - \cancel{\cos x} + \cancel{\cos x} - \cos^2 x}{\sin x} = \sin x$

$\frac{1 - \cos^2 x}{\sin x} = \sin x$

$1 - \cos^2 x = \sin^2 x$

$\frac{\sin^2 x}{\sin x} = \sin x \rightarrow \sin x = \sin x$

c) $\frac{\cot x - \cos x}{1 - \sin x} = \cot x \Rightarrow \cot x - \cos x = \cot x - \cot x\sin x \rightarrow \frac{\cancel{1}}{\tan x} - \cos x = \frac{\cancel{1}}{\tan x} - \frac{1}{\tan x}\sin x \rightarrow -\cos x = -\frac{\cos x}{\sin}(\sin x) \rightarrow \cos x = \cos x$

d) $(\sin x + \tan x)(\cos x + \cot x) = (1 + \sin x)(1 + \cos x)$

$\sin x\cos x + \sin x\cot x + \tan x\cos x + \tan x\cot x = 1 + \cos x + \sin x + \sin x\cos x$

$\sin x\cos x + \sin x\left(\frac{\cos x}{\sin x}\right) + \cot x\left(\frac{\sin x}{\cos x}\right) + \frac{\tan x}{\tan x} = 1 + \cos x + \sin x + \sin x\cos x$

$\sin x\cos x + \cos x + \sin x + 1 = 1 + \cos x + \sin x + \sin x\cos x$

$= (1 + \sin x)(1 + \cos x)$

LHS = RHS