Question	1	2	3	4	5	6	7	8	Total	%
Available	7	4	6	7	7	7	7	8		
Score										

Q1. The discrete random variable X has the probability distribution

x	1	2	3	4
$P(X = x)$	k	$2k$	$3k$	$4k$

(a) Show that $k = 0.1$

(1)

Two independent observations X_1 and X_2 are made of X.

(b) Show that $P(X_1 + X_2 = 4) = 0.1$

(2)

(c) Complete the probability distribution table for $X_1 + X_2$

y	2	3	4	5	6	7	8
$P(X_1 + X_2 = y)$	0.01	0.04	0.10		0.25	0.24	

(2)

(d) Find $P(1.5 < X_1 + X_2 \leq 3.5)$

(2)

(Total 7 marks)

Q2. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} a(3 - x), & x = 0, 1, 2 \\ b, & x = 3 \end{cases}$$

(a) Find P(X = 2) and complete the table below.

x	0	1	2	3
$P(X = x)$	$3a$	$2a$		b

(1)

Given that b = 0.4

(b) Find the value of a **(1)**

(c) Find P(0.5 < X < 3), **(2)**

(Total 4 marks)

Q3. A fair 5-sided spinner has sides numbered 1, 2, 3, 4 and 5

The spinner is spun once and the score of the side it lands on is recorded.

(a) Write down the name of the distribution that can be used to model the score of the side it lands on.

(1)

The spinner is spun 28 times.

The random variable X represents the number of times the spinner lands on 2

(b) (i) Find the probability that the spinner lands on 2 at least 7 times.

(ii) Find P(4 ≤ X < 8)

(5)

(Total for question = 6 marks)

Q4. Naasir is playing a game with two friends. The game is designed to be a game of chance so that the probability of Naasir winning each game is $\frac{1}{3}$

Naasir and his friends play the game 15 times.

(a) Find the probability that Naasir wins

(i) exactly 2 games

(ii) more than 5 games.

(3)

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim, the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

(4)

(Total for question = 7 marks)

Q5. Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm.

She tries a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm.

Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.

Test Dhriti's claim at the 5% level of significance. State your hypotheses clearly.

(Total 7 marks)

Q6. A manufacturer of sweets knows that 8% of the bags of sugar delivered from supplier *A* will be damp. A random sample of 35 bags of sugar is taken from supplier *A*.

(a) Using a suitable model, find the probability that the number of bags of sugar that are damp is

(i) exactly 2

(ii) more than 3

(3)

Supplier *B* claims that when it supplies bags of sugar, the proportion of bags that are damp is less than 8%

The manufacturer takes a random sample of 70 bags of sugar from supplier *B* and finds that only 2 of the bags are damp.

(b) Carry out a suitable test to assess supplier *B*'s claim.
You should state your hypotheses clearly and use a 10% level of significance.

(4)

(Total for question = 7 marks)

Q7. A single observation x is to be taken from a Binomial distribution B(20, p).

This observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be **as close as possible to 2.5%.**

(3)

(b) State the actual significance level of this test.

(2)

The actual value of x obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.

(2)

(Total 7 marks)

Q8. A biased spinner can only land on one of the numbers 1, 2, 3 or 4. The random variable X represents the number that the spinner lands on after a single spin and $P(X = r) = P(X = r + 2)$ for $r = 1, 2$

Given that $P(X = 2) = 0.35$

(a) find the complete probability distribution of X.

(2)

Ambroh spins the spinner 60 times.

(b) Find the probability that more than half of the spins land on the number 4

Give your answer to 3 significant figures.

(3)

(c) The random variable $Y = \frac{12}{X}$

Find $P(Y - X \leq 4)$

(3)

(Total for question = 8 marks)

Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$k+2k+3k+4k=1$ or $10k=1$ $k=0.1$ (*) [allow verification with a comment e.g. "so $k=0.1$"]	B1cso (1)
(e)	$P(1,3)+P(2,2)=2\times0.1\times0.3+0.2\times0.2=0.1$ (*)	M1 A1cso (2)
(f)	X_1+X_2: 2 | 3 | 4 | 5 | 6 | 7 | 8 p: 0.01 | 0.04 | 0.1 | **0.2** | 0.25 | 0.24 | **0.16**	B1 B1 (2)
(g)	$P(2)+P(3)=0.05$	M1A1 (2) **[14]**

Question Number	Scheme	Marks
	Notes	
(a)	B1 for a clear attempt to use sum of probabilities = 1. Must see previous line as well as $k=0.1$	
(e)	M1 for correctly identifying (1, 3) or (3, 1) and (2, 2) as required cases ($3k^2+4k^2$ or better) A1 cso for 0.1 only but must see evidence for M1	
(f)	1st B1 for 0.2 correctly assigned. May be in table. 2nd B1 for 0.16 correctly assigned. May be in table	
(g)	M1 for P(2) + P(3). May be implied by correct answer of 0.05 A1 for 0.05 only. **Correct answer only can score full marks in parts (b), (c), (f) and (g)**	

Q2.

Question Number	Scheme	Marks
(a)	0 | 1 | 2 | 3 $3a$ | $2a$ | a | b	B1 (1)
	$a=0.1$ cao	B1
Notes	6(a) Condone a clearly stated in text but not put in table.	
(c)	$P(0.5<x<3)=P(1)+P(2)$ 3a or their $2a$+their a $=0.2+0.1$ $=0.3$ Require $0<3a<1$ to award follow through	M1 A1 ft (2)

Q3.

Question	Scheme	Marks	AOs
(a)	(Discrete) uniform (distribution)	B1	1.2
		(1)	
(b)	B(28, 0.2)	B1	3.3
(i)	$P(X \geq 7) = 1 - P(X \leq 6)$ [$= 1 - 0.6784\ldots$]	M1	3.4
	awrt **0.322**	A1	1.1b
(ii)	$P(4 \leq X < 8) = P(X \leq 7) - P(X \leq 3)$ [$= 0.818\ldots - 0.160\ldots$]	M1	3.1b
	awrt **0.658**	A1	1.1b
		(5)	
			(6 marks)

Notes	
(a)	Continuous uniform is B0
(b)	**B1:** for identifying correct model, B(28, 0.2) allow B, bin or binomial may be implied by one correct answer or sight one correct probability i.e. awrt 0.678, awrt 0.818 or awrt 0.160 B(0.2, 28) is B0 unless it is used correctly
(i)	**M1:** Writing or using $1 - P(X \leq 6)$ or $1 - P(X < 7)$ **A1:** awrt 0.322 (correct answer only scores M1A1)
(ii)	**M1:** Writing or using $P(X \leq 7) - P(X \leq 3)$ or $P(X < 8) - P(X < 4)$ or $P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$ Condone P(4) as $P(X = 4)$, etc. **A1:** awrt 0.658 (correct answer only scores M1A1)

Q4.

Qu	Scheme	Marks	AO
(a)	Let N = the number of games Naasir wins $N \sim B(15, \frac{1}{3})$	M1	3.3
(i)	$P(N = 2) = 0.059946\ldots$ **awrt 0.0599**	A1	1.1b
(ii)	$P(N > 5) = 1 - P(N \leqslant 5) = 0.38162\ldots$ **awrt 0.382**	A1	1.1b
		(3)	
(b)	$H_0 : p = \frac{1}{3}$ $H_1 : p > \frac{1}{3}$	B1	2.5
	Let X = the number of games Naasir wins $X \sim B(32, \frac{1}{3})$	M1	3.3
	$P(X \geqslant 16) = 1 - P(X \leqslant 15) = 0.03765$ (< 0.05)	A1	3.4
	[Significant result so reject H_0 (the null model) and conclude:] There is evidence to support Naasir's claim (o.e.)	A1	3.5a
		(4)	
		(7 marks)	

(a)	M1 for selecting a binomial model with correct n and p Award for sight of $B(15, \frac{1}{3})$ (o.e. e.g. in words) or implied by 1 correct answer 1st A1 for awrt 0.0599 (from a calculator). Allow 0.05995 2nd A1 for awrt 0.382 (from a calculator)
(b)	B1 for correctly stating both hypotheses in terms of p or π Accept $p = 0.\dot{3}$ or any exact equivalent. $H_1 : p \geqslant \frac{1}{3}$ is B0 M1 for selecting a suitable model to use for the test. Award for sight of $B(32, \frac{1}{3})$ (o.e. e.g. in words) or implied by 0.03765 Can also allow M1 for $P(X \leqslant 15) = 0.962$ or better or $P(X \leqslant 14) = 0.922$ or better 1st A1 for use of the model to calculate an appropriate probability using calc. Sight of $P(X \geqslant 16)$ **and** answer awrt 0.0377
ALT	**CR** May use CR so award 1st A1 for CR of $X \geqslant 16$ must have seen some probabilities though: 1 of $P(X \leqslant 15) = 0.9623$ or $P(X \leqslant 14) = 0.9224$ or 0.9223 2nd A1 for conclusion in context that there is support for Naasir's claim Must mention "Naasir" or "his" and "claim" or "method" (o.e.) or e.g. probability of winning a game is $> \frac{1}{3}$ or has increased Dependent on M1 and 1st A1 but can ignore hypotheses but see below If you see $P(X \geqslant 16) = 0.0376$ followed by a correct contextualised conclusion then please award A0A1
SC	**Use of 0.3 for $\frac{1}{3}$** If used 0.3 instead of $\frac{1}{3}$ in (a) and score M0A0A0 can condone use of 0.3 in (b) 1st A1 ft needs $P(X \geqslant 16) = 0.0138$ or CR of $X \geqslant 15$ and sight of 1 of $P(X \geqslant 15) = 0.0327$ or $P(X \geqslant 14) = 0.0694$ 2nd A1 as before with 0.3 instead $\frac{1}{3}$ (if appropriate)

Q5.

Question Number	Scheme Marks	
	$H_0 : p = 0.3$; $H_1 : p > 0.3$	B1 B1
	Let X represent the number of tomatoes greater than 4 cm : X~ B(40, 0.3)	B1
	$P(X \geq 18) = 1 - P(X \leq 17)$ = 0.0320 $P(X \geq 18)$ $1 - P(X \leq 17) = 0.0320$ $P(X \geq 17) = 1 - P(X \leq 16) = 0.0633$ CR $X \geq 18$	M1 A1
	$0.0320 < 0.05$ $\quad$ $18 \geq 18$ or 18 in the critical region	
	no evidence to Reject H_0 or it is significant	M1
	New fertiliser has <u>increased</u> the probability of a <u>tomato</u> being greater than 4 cm **Or** Dhriti's claim is true	B1d cao (7)

	B1 for correct H_0 . must use p or pi **B1** for correct H_1 must use p and be one tail. **B1** using B(40, 0.3). This may be implied by their calculation **M1** attempt to find $1 - P(X \leq 17)$ or get a correct probability. For CR method must attempt to find $P(X \geq 18)$ or give the correct critical region **A1** awrt 0.032 or correct CR. **M1** correct statement based on their probability , H_1 and 0.05 or a correct contextualised statement that implies that. **B1** this is not a follow through .conclusion in context. Must use the words increased, tomato and some reference to size or diameter. This is dependent on them getting the previous M1 If they do a **<u>two tail test</u>** they may get B1 B0 B1 M1 A1 M1 B0 For the second M1 they must have accept Ho or it is not significant or a correct contextualised statement that implies that. .	

Q6.

Qu	Scheme	Mark	AO
(a)	[D = number of bags that are damp] $D \sim B(35, 0.08)$ NB $0.08 = \frac{2}{25}$	M1	3.3
(i)	$P(D = 2) = 0.2430497\ldots$ awrt **0.243**	A1	3.4
(ii)	$P(D > 3) = [1 - P(D \leq 3) = 1 - 0.69397\ldots] = 0.30602\ldots$ awrt **0.306**	A1	1.1b
		(3)	
(b)	$H_0 : p = 0.08$ $H_1 : p < 0.08$	B1	2.5
	[$X \sim$] B(70, 0.08)	M1	2.1
	$[P(X \leq 2)] = 0.0739756\ldots$ awrt **0.074**	A1	1.1b
	[0.074 < 0.10 so significant, reject H0 so…] there is evidence to support supplier B's claim (o.e.)	A1	2.2b
		(4)	
		(7 marks)	

	Notes
(a)	M1 for selecting a correct model: sight of or use of B(35, 0.08) [Condone B(0.08, 35)] May be implied by one correct answer or sight of $P(D \leq 3)$ = awrt 0.694 (or allow 0.693) or seeing $\binom{35}{2}0.08^2 \times (1-0.08)^{35-2}$ Saying B(35, 8%) without a correct calculation would score M0
(i)	1st A1 for awrt 0.243
(ii)	2nd A1 for awrt 0.306 (Condone poor use of notation e.g. $P(D = 3) = 0.306\ldots$ i.e. just mark ans)
NB	$P(D \ldots 3) = 0.539$ scores 2nd A0 but would of course score M1
(b)	B1 for both hypotheses correct in terms of p or π [Condone 8% for 0.08]
	M1 for sight or correct use of B(70, 0.08) [Condone B(0.08, 70)] May be implied by prob of 0.074 or better
	1st A1 for final answer awrt 0.074 can condone poor notation e.g. $P(X = 2)$ = awrt 0.074 Can allow this mark for CR of $X \leq 2$ provided $[P(X \leq 2)] = 0.074$ (or better) is seen [Can allow 0.07 if $X \sim$B(70, 0.08) and $P(X \leq 2)$ are both seen]
	2nd A1 (**dep on** M1A1 but independent of hypotheses) for a correct inference in context Must mention claim or B **and** idea of support for … or proportion/probability (of damp bags) **and** idea of less than 8% or A 2nd A0 for contradictory statements e.g. "accept H0 so evidence to support B's claim" 2nd A0 if you see 0.0739… < 0.08 so significant/ reject H0 etc
MR	**0.8 for 0.08** In (a) allow M1 for B(35, 0.8) then A0A0 In (b) allow B1 for Hypotheses and M1 for B(70, 0.8) seen, then A0A0

Q7.

Question Number	Scheme	Marks
(a)	$X \sim B(20, 0.3)$	M1
	$P(X \leq 2) = 0.0355$	
	$P(X \geq 11) = 1 - 0.9829 = 0.0171$	
	Critical region is $(X \leq 2) \cup (X \geq 11)$	A1 A1 (3)
(b)	Significance level = 0.0355 + 0.0171, = 0.0526 or 5.26%	M1 A1 (2)
(c)	Insufficient evidence to reject H_0 **Or** sufficient evidence to accept H_0 /not significant	B1 ft
	$x = 3$ (or the value) is not in the critical region or 0.1071> 0.025	B1 ft (2)
	Do not allow inconsistent comments	

Q8.

Qu	Scheme	Marks	AO
(a)	$P(X=4)=P(X=2)$ so $P(X=4)=0.35$ $P(X=1)=P(X=3)$ and $P(X=1)+P(X=3)=1-0.7$ So x: 1, 2, 3, 4 $P(X=x)$: 0.15, 0.35, 0.15, [0.35]	M1 A1 (2)	2.1 1.1b
(b)	Let A = number of spins that land on 4 $A \sim B(60, "0.35")$	B1ft	3.3
	[$P(A>30)=$] $1-P(A \leqslant 30)$ $=1-0.99411\ldots$ = **awrt 0.00589**	M1 A1 (3)	3.4 1.1b
(c)	$Y-X \leqslant 4 \Rightarrow \frac{12}{X}-X \leqslant 4$ or $12-X^2 \leqslant 4X$ (since $X>0$) o.e.	M1	3.1a
	i.e. $0 \leqslant X^2+4X-12 \Rightarrow 0 \leqslant (X+6)(X-2)$ so $X \geqslant 2$	M1	1.1b
	$P(Y-X \leqslant 4)=P(X \geqslant 2)=0.35+0.15+0.35$ = **0.85**	A1 (3)	3.2a
		(8 marks)	

Notes

(a)
M1 for using the given information to obtain $P(X=4)$
Award for statement $P(X=4)=P(X=2)$ or writing $P(X=4)=0.35$
A1 for getting fully correct distribution (any form that clearly identifies probs)
e.g. can be list $P(X=1)=0.15$, $P(X=3)=\ldots$etc or as a probability function $P(X=x)=\begin{cases}0.15 & x=1,3\\0.35 & x=2,4\end{cases}$
[Condone missing $P(X=2)$ as this is given in QP]

(b)
B1 for selecting a suitable model, sight of B(60, their 0.35) o.e. in words f.t. their $P(X=4)$ from part (a).
Can be implied by $P(A \leqslant 30)$ = awrt 0.9941 or final answer = awrt 0.00589
M1 for using their model and interpreting "more than half"
Need to see $1-P(A \leqslant 30)$. Can be implied by awrt 0.00589
Can ignore incorrect LHS such as $P(A \geqslant 30)$
A1 for awrt 0.00589

(c)
1st M1 for translating the prob. problem into a correct mathematical inequality
Just an inequality in 1 variable. May be inside a probability statement.
ALT **Table of values:**

X	1	2	3	4
Y	12	6	4	3

or values of $Y-X=11, 4, 1, -1$

2nd M1 for solving the inequality leading to a range of values, allow 1 or 2 slips
May be a quadratic or cubic but must lead to a set of values of X or $Y-X$
ALT **Table or values:** They must state clearly which values are required
Both Ms can be implied by a correct answer (or correct ft of their distb'n)
A1 for interpreting the inequality and solving the problem i.e. 0.85 cao