2013 Summer Entrance Examination

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Creative Informatics

Instructions

1. Do not open this brochure until the signal to begin is given.

2. Write your examinee ID below on this cover page.

3. Answer <u>three problems out of the four.</u>

4. Three answer sheets are given. Use a separate sheet of paper for each problem. You may write on the back of the sheet.

5. Write down the examinee ID and the problem ID inside the top blanks of each sheet.

6. Do not take out the answer sheets or this brochure from this room.

Examinee ID _____

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Problem 1

An English conversation school plans to make pairs of students and teachers for private lessons. Given a set $S = \{s_1, s_2, ..., s_n\}$ of students and a set $T = \{t_1, t_2, ..., t_n\}$ of teachers, we make disjoint n pairs of a student and a teacher, which we call a *p-match*. Answer the following questions:

(1) How many p-matches exist?

(2) For $n = 5$, given a list of preferable teachers by students (Table 1)

$$V = S \cup T$$
$$E = \{xy \mid x \in S, y \in T, x \text{ prefers } y.\}$$

draw a graph $G = (V, E)$, and show a p-match which maximizes the number of students who are fulfilled.

Student	Teachers
s_1	t_1, t_3
s_2	t_2, t_4, t_5
s_3	t_1, t_3
s_4	t_3, t_5
s_5	t_1, t_3

Table 1: List of Preferences

(3) Let the size of a set $|E| = m$. Show an algorithm to get a p-match which maximizes the number of students who are fulfilled and its complexity.

(4) For $n = 7$, given a ranked list of teachers by students (Table 2), show a p-match which minimizes the total sum of ranks.

(5) In addition to the ranked list of teachers by students, consider a ranked list of students by teachers. Given a p-match, if there exists no pair of student and teacher who would both get the higher rank than that of the current partner of the given p-match, then, this p-match is called an *s-match*. For $n = 7$, given a ranked list of students by teachers (Table 3) in addition to the Table 2, show an s-match.

Ranking	1	2	3	4	5	6	7
s_1	t_7	t_1	t_2	t_6	t_5	t_4	t_3
s_2	t_7	t_1	t_2	t_3	t_4	t_5	t_6
s_3	t_7	t_5	t_2	t_6	t_1	t_4	t_3
s_4	t_1	t_4	t_3	t_6	t_2	t_5	t_7
s_5	t_7	t_3	t_1	t_2	t_4	t_5	t_6
s_6	t_3	t_7	t_2	t_1	t_5	t_4	t_6
s_7	t_7	t_3	t_2	t_6	t_5	t_4	t_1

Table 2: Rank of teachers by students

Ranking	1	2	3	4	5	6	7
t_1	s_1	s_2	s_3	s_4	s_5	s_6	s_7
t_2	s_1	s_2	s_4	s_3	s_7	s_6	s_5
t_3	s_3	s_2	s_1	s_5	s_6	s_4	s_7
t_4	s_2	s_3	s_1	s_7	s_4	s_6	s_5
t_5	s_1	s_3	s_4	s_2	s_5	s_6	s_7
t_6	s_1	s_5	s_3	s_4	s_2	s_6	s_7
t_7	s_3	s_5	s_1	s_4	s_2	s_7	s_6

Table 3: Rank of students by teachers

(6) For n, show an algorithm to get an s-match and its complexity.

(7) We will develop a real software system for private lessons for an English conversation school. List possible study items (example: web reservation), and describe each item in two lines.

Problem 2.

Design a logic circuit to light an LED satisfying the following conditions. Figure 1 shows the connection diagram of the circuit, input signals and an output signal to the LED.

Condition 1: Inputs of the circuit to design are LEDSTR and CLOCK INPUT (100 kHz clock input signal). The output of the circuit to design is LEDOUT. LEDOUT is directly connected to the LED driver and lights the LED. When LEDOUT is H, LED is on, and when LEDOUT is L, LED is off.

Condition 2: LEDSTR (INPUT 0, INPUT1, INPUT2) specifies the strength of the LED lighting. When LEDSTR is 0, the LED is completely turned off. When LEDSTR is 5, the LED is continuously turned on. When $1 \leqq$ LEDSTR $\leqq 4$, the strength of the light from the LED is proportional to the value of LEDSTR. When LEDSTR is more than or equal to 6, any behavior of the circuit is allowed.

Condition 3: When the LED turns on and off repeatedly at more than 100Hz, the strength of the LED light is seen as an average value of the time periods when the LED is turned on.

Condition 4: The logic circuit is designed using AND, OR, XOR, NOT and D-type Flip-Flops.

Design the circuit following the questions below.

1) Design a 3-bit counter that count from 0 to 4 repeatedly such as $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 0 \rightarrow 1 \ldots\ldots$

2) Design a circuit that compares two 3-bit numbers.

3) Design a circuit to turn on the LED for 1 clock period when LEDSTR is 1, 2 clock periods when LEDSTR is 2, ..., 5 clock periods (i.e. always on) when LEDSTR is 5. Figure 2 is an example of the output waveform.

4) Using the logic circuit designed in 3), design a logic circuit that turns on and turns off the LED once for more than a second with the strength of the light specified by LEDSTR.
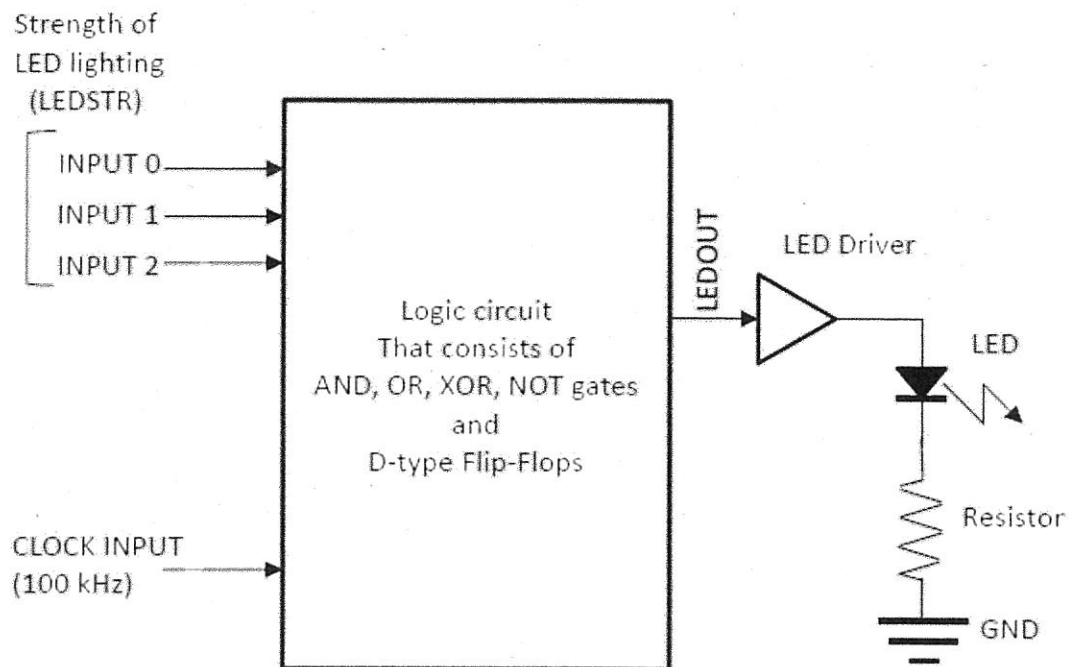
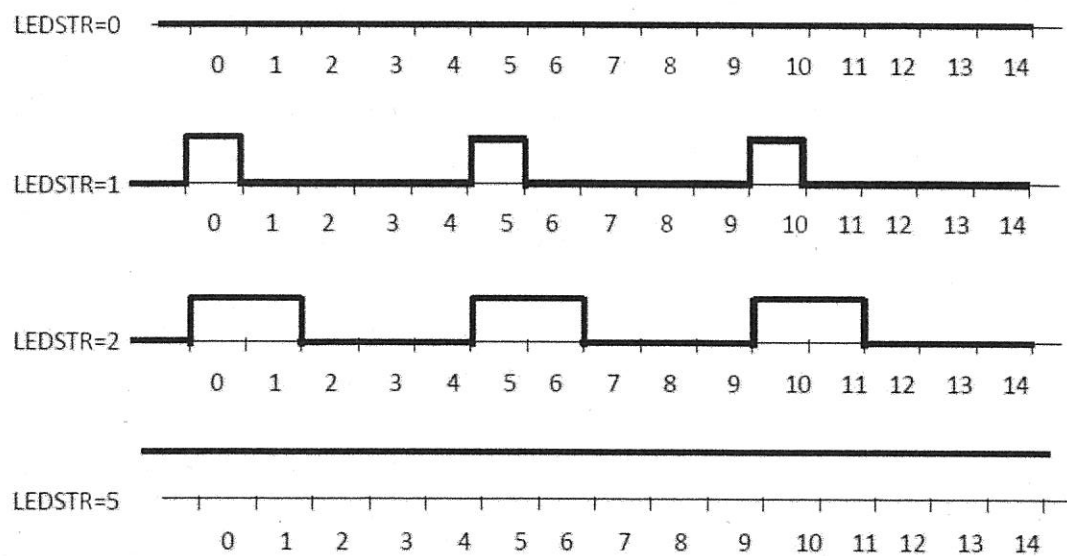Figure 1. The connection diagram of the circuit,

Figure 2.   An example of LEDOUT

Problem 3

In kinematic calculations and visual computations in robotics, vector operations are expressed with matrices sometimes. Answer the following questions on inner-product, outer-product, projection and rotation of three dimensional vectors. I is the 3×3 identity matrix. The three-dimensional vectors x, a, b and n are 3×1 column vectors:

$$x = \begin{bmatrix} x_x \\ x_y \\ x_z \end{bmatrix}, \qquad a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \qquad b = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}, \qquad n = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}.$$

x^{T}, which is a 1×3 row vector, shows the transpose of x.

(1) On the inner product (a, b) between vectors a and b, describe i) the value of (a, b), ii) a 3×3 matrix A which satisfies $(a, b)a = Ab$, and iii) A with vector a and its transpose a^{T}.

(2) On the outer product $a \times b$ from a to b, describe i) 3×1 expression of $a \times b$, ii) 3×3 expression of the matrix A which satisfies $a \times b = Ab$, iii) the 3×3 matrix Q which satisfies $A = a \times Q$ where $x \times Q$ between a vector x and a 3×3 matrix Q means a 3×3 matrix whose column vectors are three outer products from the vector x to each column vector in the matrix Q respectively.

(3) As Figure 1 shows, a vector x is vertically projected to a vector y on a plane whose normal vector is a unit vector n. If the vector y is described as $y = Px$, show that the matrix P becomes $P = I - nn^{\mathrm{T}}$.

(4) Three rotational matrices $R_x(\theta_x)$, $R_y(\theta_y)$ and $R_z(\theta_z)$ are rotational matrices which rotate a vector x around the X-axis, Y-axis and Z-axis with θ_x, θ_y and θ_x respectively, where the direction of the rotation for plus is clock-wise around the axis from the origin to infinity.

i) Describe 3×3 expression of the matrix $R_x(\theta_x)$, ii) As Figure 2 shows, the 3×3 matrix $R_n(\theta_n)$ is defined as the rotation matrix around a unit orientation vector n with θ_n. $R_n(\theta_n)$ is described as

$$R_n(\theta_n) = R_x(-\alpha)R_y(\beta)R_z(\theta_n)R_y(-\beta)R_x(\alpha).$$

Explain what the variables α and β become and explain why the expression is satisfied.
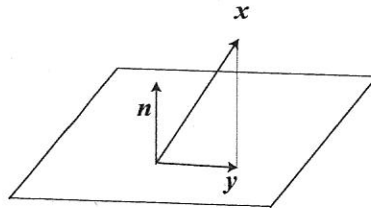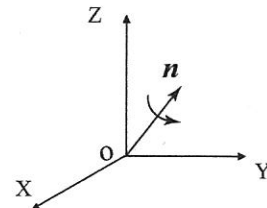

Figure 1

Figure 2

Problem 4

Select <u>four items</u> out of the following eight items concerning information systems, and explain each item in approximately 4~8 lines of text. If necessary, use examples or figures.

    (1)   NP complete

    (2)   Tail recursion

    (3)   Step response and transfer function

    (4)   Discrete cosine transform, DCT

    (5)   Public key cryptosystem

    (6)   DNS (Domain Name Service)

    (7)   TLB (Translation Lookaside Buffer)

    (8)   LL(1) pursing

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