HW 12

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1) $L = D - W$

もし L は PSD

ならば $\alpha L \alpha^T \geq 0$

$\alpha (D - W) \alpha^T \geq 0$

$\alpha \left(diag \sum_{i'=1}^{n} W_{i,i'} - W \right) \alpha^T \geq 0$

$L_{ii'} = \left(\sum_{i'=1}^{n} W_{i,i'} \right) - W_{i,i'}$ for $i = i'$

$- W_{i,i'}$ for $i \neq i'$

$\alpha L \alpha^T = \left(\sum_i \alpha_i L_{i,0} , \sum_i \alpha_i L_{i,1} \cdots \sum_i \alpha_i L_{i,n} \right) \alpha^T$

$= \alpha_0 \sum \alpha_i L_{i,0} + \alpha_1 \sum_i \alpha_i L_{i,1} + \cdots + \alpha_n \sum_i \alpha_i L_{i,n}$

$= \alpha_0^2 L_{0,0} + \alpha_1^2 L_{1,1} + \cdots + \alpha_n^2 L_{n,n}$

$+ \alpha_0 \alpha_1 L_{0,1} + \cdots + \alpha_0 \alpha_n L_{0,n}$

$\vdots$

$+ a_n \alpha_0 L_{n,0} + \cdots \alpha_n \alpha_{n-1} L_{n,n-1}$

$= \alpha_0^2 \left(\sum W_{0,i} - W_{0,0} \right) + \alpha_1^2 \left(\sum W_{1,i} - W_{1,1} \right) + \cdots$

$+ \alpha_0 \alpha_1 L_{0,1} + \cdots + \alpha_0 \alpha_n L_{0,n}$

$\vdots$

$+ a_n \alpha_0 L_{n,0} + \cdots \alpha_n \alpha_{n-1} L_{n,n-1}$

$$= a_0\left((a_0-a_1)W_{0,1} + (a_0-a_2)W_{0,2} + \cdots (a_0-a_n)W_{0,n} - a_0 W_{0,0}\right)$$

$$\vdots$$

$$a_n(a_n-a_0)W_{n,0} + \cdots + (a_n-a_{n-1})W_{n,n-1} - a_n W_{n,n})$$

Wは対称, $W_{i,j} = W_{j,i}$

$$= \sum_{i,j,\ i\neq j} \underbrace{(a_i-a_j)^2}_{\geq 0} \underbrace{W_{i,j}}_{\geq 0} - \underbrace{\sum_i a_i^2 W_{i,i}}_{=0}$$

$$\geq 0 \ //$$

固有値 $\gamma = 0$ のとき

$$L\varphi = 0$$

Lの定義から

$$L_{i,i'} = \left(\sum_{i'=1}^{n} W_{i,i'}\right) - W_{i,i'} \quad \text{for } i = i'$$

$$-W_{i,i'} \quad \text{for } i \neq i'$$

L(1)の場合が 0 となる