

HW9

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$$D_E^2(p_{\text{test}}, q_\pi) = 2 E_{x' \sim p_{\text{test}}, x \sim q_\pi} \|x' - x\| - \overset{\text{constant}}{E_{x', \tilde{x}' \sim p_{\text{test}}} \|x' - \tilde{x}'\|} - E_{x, \tilde{x} \sim q} \|x - \tilde{x}\|$$

for simplicity, $E_{x \sim p, x' \sim p'} := E_{x \sim p, x' \sim p'} \|x - x'\|$

$$D_E^2(p_{\text{test}}, q_\pi) = 2 \left(\pi E_{x' \sim p_{\text{test}}, x \sim p_{\text{train}}(x|+)} + (1-\pi) E_{x' \sim p_{\text{test}}, x \sim p_{\text{train}}(x|-)} \right) - \left(\pi^2 E_{x, \tilde{x} \sim p_{\text{train}}(x|+)} + 2\pi(1-\pi) E_{x \sim p_{\text{train}}(x|+), \tilde{x} \sim p_{\text{train}}(x|-)} + (1-\pi)^2 E_{x, \tilde{x} \sim p_{\text{train}}(x|-)} \right) + C$$

$$= 2(\pi b_{+1} - \pi b_{-1} + \overset{\text{constant}}{b_{-1}}) - (\pi^2 A_{+,+1} + (2\pi - \pi^2) A_{+,-1} + \pi^2 A_{-,-1} - 2\pi A_{-,+1} + \overset{\text{constant}}{A_{+,-1}}) + C$$

$$= \pi(2A_{+,-1} - A_{+,+1} - A_{-,-1}) - 2(A_{+,-1} - A_{-,-1} - b_{+1} + b_{-1})\pi + C$$