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$$S^{(w)} = \sum_{y=1}^{c} \sum_{i:y_i=y} (x_i - \mu_y)(x_i - \mu_y)^T$$

$$= \sum_{y=1}^{c} \sum_{i:y_i=y} x_i x_i^T - \mu_y x_i^T - x_i \mu_y^T + \mu_y \mu_y^T$$

$$= \sum_{y=1}^{c} \sum_{i:y_i=y} x_i x_i^T - \frac{\sum x_{i'}}{n_y} x_i^T - x_i \frac{\sum x_{i'}^T}{n_y} + \frac{\sum x_{i'} \sum x_{i'}^T}{n_y^2} = 0$$

$$= \sum_{y=1}^{c} \sum_{i:y_i=y} x_i x_i^T - \frac{x_i x_i^T}{n_y} - \frac{x_i x_i^T}{n_y} - \frac{\sum_{j\neq i} x_i x_i^T}{n_y} - \frac{\sum_{j\neq i} x_i x_i^T}{n_y}$$

$$= \frac{1}{2} \sum Q \quad - x_{i'} x_i^T - x_i x_{i'}^T$$

$$= \frac{1}{2} \sum Q \quad x_i x_i^T - x_{i'} x_i^T - x_i x_{i'}^T + x_{i'} x_{i'}^T \qquad x_i x_i^T = x_{i'} x_{i'}^T \ @ \ i = i'$$

$$= \frac{1}{2} \sum Q \, (x_i - x_{i'})(x_i - x_{i'})^T$$

$$C = \sum_{i=1}^{n} x_i x_i^T$$

$$= \sum_{i,i'=1}^{n} \frac{1}{n} (x_i x_i^T)$$

$$= \sum_{i,i'=1}^{n} \frac{1}{n} \left(x_i x_i^T - x_{i'} x_i^T \right)$$

$$= \frac{1}{2} \sum_{i,i'=1}^{n} \frac{1}{n} \left(x_i x_i^T - x_i x_i^T - x_i x_{i'}^T + x_{i'} x_{i'}^T \right)$$

$$= \frac{1}{2} \sum_{i,i'=1}^{n} \frac{1}{n} (x_i - x_{i'})(x_i - x_{i'})^T$$

$$S^B = C - S^w$$

$$Q^c_{i,i'} = \frac{1}{n}$$

$$= \frac{1}{2} \sum_{i,i'=1} \left(Q_{ii'} - Q_{ii'} \right) (x_i - x_{i'})(x_i - x_{i'})'$$

$$= \frac{1}{2} \sum_{i,i'=1}^{n} Q^{B}_{ii'} (x_i - x_{i'})(x_i - x_{i'})^T \qquad Q^{B}_{i,i'} = \begin{cases} \frac{1}{n} - \frac{1}{n_y} & i = i' \\ \frac{1}{n} & i \neq i' \end{cases}$$