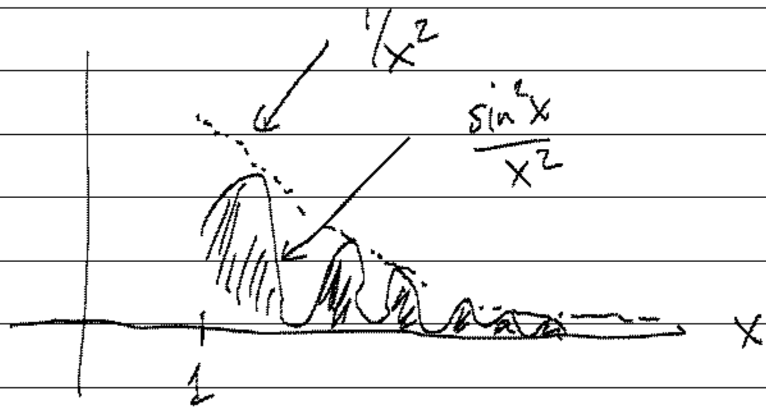
10-08

Note Title 10/8/2007

<u>Ex</u> $\int_1^\infty \frac{\sin^2 x}{x^2}\,dx = \lim_{b\to\infty} \int_1^b \frac{\sin^2 x}{x^2}\,dx$ no antiderivative is available.

But $0 \le \sin^2 x \le 1$ for all x, so



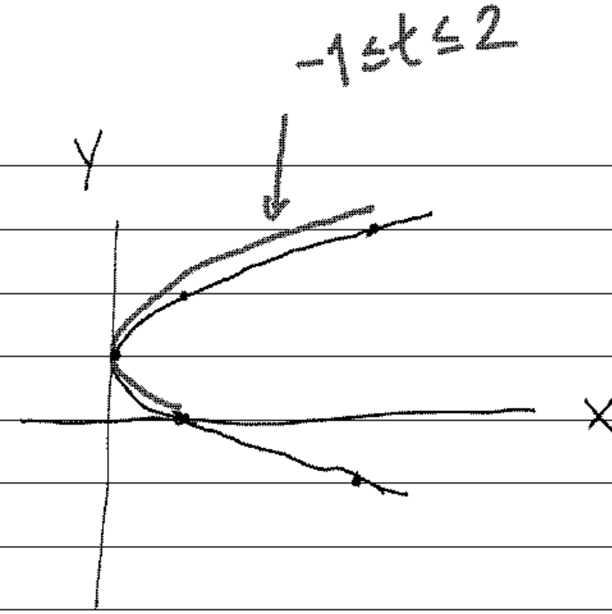
area under $\frac{\sin^2 x}{x^2}$ $\le$ area under $\frac{1}{x^2}$

And $\int_1^\infty \frac{1}{x^2}\,dx < \infty$, so

$\int_1^\infty \frac{\sin^2 x}{x^2}\,dx$ converges.

<u>Ex</u> $x = t^2$, $y = t+1$

t	-2	-1	0	1	2
x	4	1	0	1	4
y	-1	0	1	2	3



Note that $y-1=t$, so $x = t^2 = (y-1)^2$ → $x=(y-1)^2$ parabola

eliminating the parameter

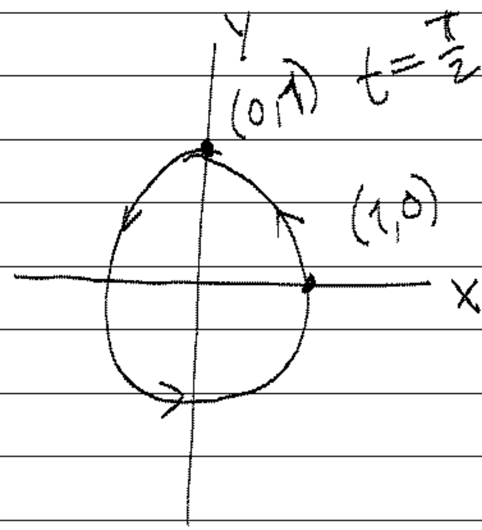
Assumed that t takes on all values $-\infty < t < \infty$.
If also specified $-1 \le t \le 2$, then we get only part of the parabola.

Ex $x = \cos t$, $y = \sin t$ Fundamentally important!

Eliminating t, $x^2 + y^2 = \cos^2 t + \sin^2 t = \underline{1}$

$x^2 + y^2 = 1$ circle centered at origin with radius $= 1$ <u>unit circle</u>

If $-\infty < t < \infty$, x goes -1 and 1, and so does y: get whole circle, traced infinitely often



$t = \frac{\pi}{2} + 2n\pi$

at $t = 0, \pm 2\pi, \pm 4\pi, \ldots$

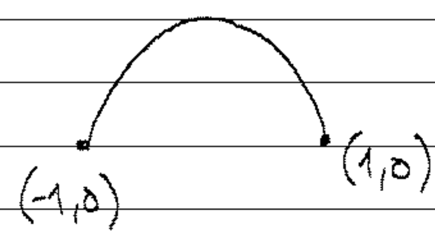
t is identical to the angle made with positive x-axis

<u>Ex</u> $x = \cos(2t)$, $y = \sin(2t)$, $0 \le t \le \frac{\pi}{2}$

Eliminate t: $x^2 + y^2 = \cos^2(2t) + \sin^2(2t) = 1$ on the unit circle

Restriction: As t goes from 0 to $\frac{\pi}{2}$, x goes from 1 to -1
y goes from 0 to 1 to 0 again

$\therefore$ Upper semicircle:

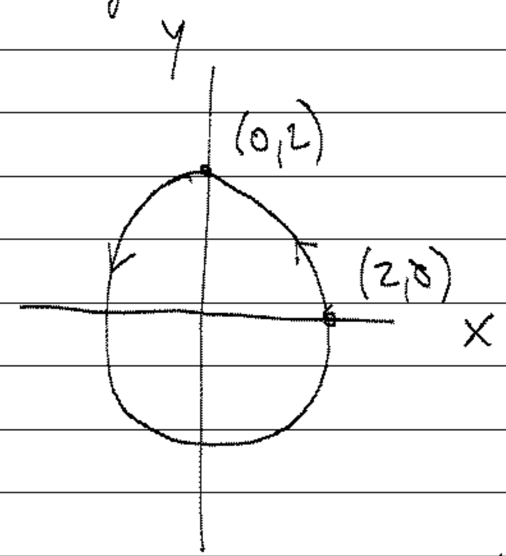


<u>Ex</u> $x = 2\cos t, \quad y = 2\sin t$

$$x^2 + y^2 = 4\cos^2 t + 4\sin^2 t = 4(\cos^2 t + \sin^2 t) = 4$$

circle centered at $(0,0)$ with radius $= 2$.

t unrestricted: get the entire circle.



$x = a\cos(bt) + c$

$y = a\sin(bt) + d$

a = scale

$b \leftrightarrow$ speed

(c,d) center

$$(x-c)^2 + (y-d)^2 = a^2$$

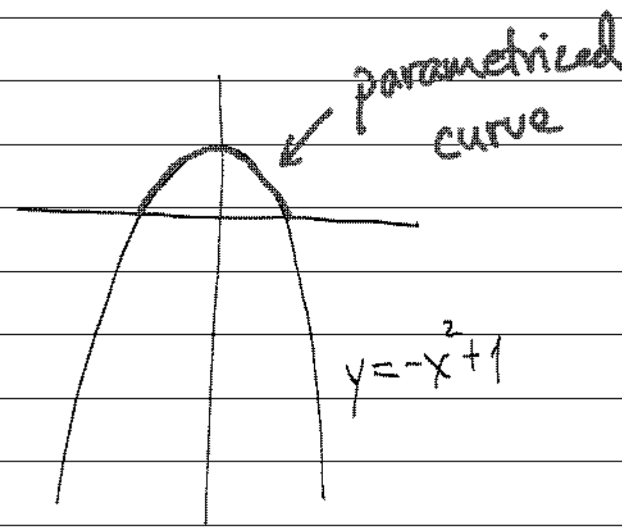
Ex $x = \cos t, \quad y = \sin^2 t$

$$x^2 + y = \cos^2 t + \sin^2 t = 1$$

$x^2 + y = 1$

$y = -x^2 + 1$ on parabola opening downward

But even though t is unrestricted, $\left.\begin{array}{l} x \in [-1,1] \\ y \in [0,1] \end{array}\right\}$ only get part of the parabola



Ex $x = e^t, \quad y = e^{-t}$

Eliminate : $y = \frac{1}{x}$

Since $x = e^t, \quad x > 0$

$y = e^{-t}, \quad y > 0$