M242 HW 6
8.8: 1, 11, 22, 28, 33, 50, 51

1.

a. Improper Integral of Type I because of an infinite interval of integration.

b. Improper Integral of Type II because sec(x) has an infinite discontinuity at $x = \frac{\pi}{2}$.

c. Improper Integral of Type II because of the infinite discontinuity at $x = 2$.

d. Improper Integral of Type I because of an infinite interval of integration.

11. $\int_{-\infty}^{\infty} \frac{x}{(1+x^2)} dx = \int_{-\infty}^{0} \frac{x}{(1+x^2)} dx + \int_{0}^{\infty} \frac{x}{(1+x^2)} dx$. Now consider

$$\int_{-\infty}^{0} \frac{x}{(1+x^2)} dx = \lim_{t\to-\infty} \int_{t}^{0} \frac{x}{(1+x^2)} dx = \lim_{t\to-\infty} \left[\frac{1}{2}(1+x^2)\right]_t^0 = \lim_{t\to-\infty}\left(0+\frac{1}{2}\ln(1+t^2)\right) = -\infty$$

Since this integral diverges, then $\int_{-\infty}^{\infty} \frac{x}{(1+x^2)} dx$ also diverges.

22. $\int_{-\infty}^{\infty} e^{-|x|} dx = \lim_{t\to-\infty} \int_{t}^{0} e^{x} dx + \lim_{t\to\infty} \int_{0}^{t} e^{-x} dx = \lim_{t\to-\infty} \left[e^{x}\right]_t^0 + \lim_{t\to\infty}\left[-e^{-x}\right]_0^t = 1+1=2$ so the integral is convergent.

28. $\int_{0}^{3} \frac{1}{x\sqrt{x}} dx = \lim_{t\to0^+} \int_{t}^{3} \frac{1}{x\sqrt{x}} dx = \lim_{t\to0^+}\left[\frac{-2}{\sqrt{x}}\right]_t^3 = \lim_{t\to0^+}\left(\frac{-2}{\sqrt{3}}+\frac{2}{\sqrt{t}}\right) = \infty$, divergent.

33. $\int_{0}^{33} (x-1)^{-1/5} dx = \lim_{t\to1^-} \int_{0}^{t} (x-1)^{-1/5} dx + \lim_{t\to1+} \int_{t}^{33} (x-1)^{-1/5} dx$

$= \lim_{t\to1^-}\left[\frac{5}{4}(x-1)^{4/5}\right]_0^t + \lim_{t\to1^+}\left[\frac{5}{4}(x-1)^{4/5}\right]_t^{33} = -\frac{5}{4}+20 = \frac{75}{4}$ so the integral is convergent.

50. For $x \geq 1$, $\frac{2+e^{-x}}{x} > \frac{2}{x} > \frac{1}{x}$ (This is because $e^{-x} > 0$.) Now,

$\int_{1}^{\infty} \frac{1}{x} dx$ is divergent by Equation 2 in section 8.8 since the exponent of x is equal to 1.

Thus, by the comparison theorem, the original integral is divergent.

51. For $x \geq 1$ we have that $x + e^{2x} > e^{2x} > 0$ so $\dfrac{1}{x + e^{2x}} \leq \dfrac{1}{e^{2x}} = e^{-2x}$. Now looking at this integral: $\displaystyle\int_1^{\infty} e^{-2x}dx = \lim_{t\to\infty}\int_1^{t} e^{-2x}dx = \lim_{t\to\infty}\left[-\frac{1}{2}e^{-2x}\right]_1^{t} = \lim_{t\to\infty}\left[-\frac{1}{2}e^{-2t} + \frac{1}{2}e^{-2}\right] = \frac{1}{2}e^{-2}$

Thus the original integral is convergent by the comparison theorem.

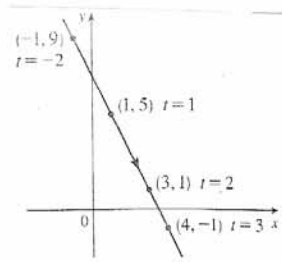
Math 242 Homework Set #6

Due: 10/12/07

Section 11.1

6. a.) $x=1+t,\ y=5-2t,\ -2\le t\le 3$

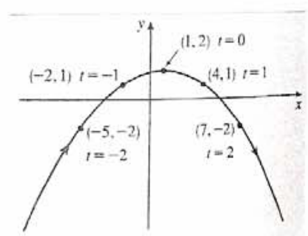
t	-2	-1	0	1	2	3
x	-1	0	1	2	3	4
y	9	7	5	3	1	-1



b.) Since $x=1+t$, we have $t=x-1$, so we can plug this back into the equation for y and we have $y=5-2(x-1)=7-2x$. And we have, $-2\le t\le 3 \Rightarrow -2\le x-1\le 3 \Rightarrow -1\le x\le 4$.

8. a.) $x=1+3t,\ y=2-t^2$

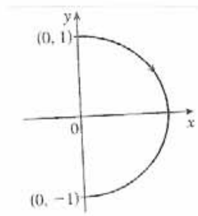
t	-3	-2	-1	0	1	2	3
x	-8	-5	-2	1	4	7	10
y	-7	-2	1	2	1	-2	-7



b.) Since $x=1+3t$, we have $t=\dfrac{x-1}{3}$, and therefore $y=2-\dfrac{(x-1)^2}{9}$.

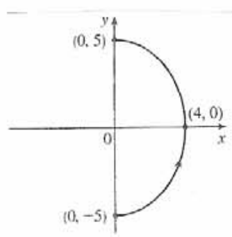
11.a.) $x = \sin\theta,\ y = \cos\theta,\ 0 \le \theta \le \pi$, therefore $x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$, and since $0 \le \theta \le \pi$ we have $\sin\theta \ge 0 \Rightarrow x \ge 0$.

b.)



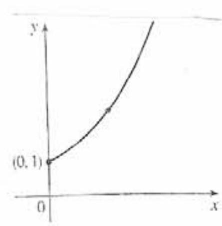
12.a.) $x = 4\cos\theta,\ y = 5\sin\theta,\ {-\pi}/{2} \le \theta \le {\pi}/{2}$, therefore we have $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = \cos^2\theta + \sin^2\theta = 1$, which is an ellipse with x-intercepts $(\pm 4, 0)$ and y-intercepts $(0, \pm 5)$ with $x \ge 0$.

b.)



16.a.) $x = \ln(t),\ y = \sqrt{t},\ t \ge 1$, so we have $x = \ln(t) \Rightarrow e^x = t$, and hence $y = \sqrt{e^x} = e^{x/2}$ and $x \ge 0$.

b.)



24. a.) From the first graph we have $1 \le x \le 2$ and from the second graph we have $-1 \le y \le 1$. The only choice that satisfies either of those conditions is III.

b.) From the first graph, the values of x cycle through the values from -2 to 2 four times. From the second graph, the values of y cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

c.) From the first graph, the values of x cycle through the values from -2 to 2 three times. From the second graph we have $0 \le y \le 2$. Choice IV satisfies these conditions.

d.) From the first graph, the values of x cycle through the values from -2 to 2 two times. From the second graph, the values of y doe the same thing. Choice II satisfies these conditions.