Contents

Preface					
Prologue					
1	Num	bers, problems, and algorithms	7		
	1.1	Floating point numbers	8		
	1.2	Problems and conditioning	14		
	1.3	Stability of algorithms	20		
2	Square linear systems				
	2.1	Polynomial interpolation	32		
	2.2	Computing with matrices	37		
	2.3	Linear systems	45		
	2.4	LU factorization	53		
	2.5	Efficiency of matrix computations	63		
	2.6	Row pivoting	71		
	2.7	Vector and matrix norms	78		
	2.8	Conditioning of linear systems	84		
	2.9	Exploiting matrix structure	89		
3	Over	determined linear systems	99		
	3.1	Fitting functions to data	100		

Contents

	3.2	The normal equations	108
	3.3	The QR factorization	113
	3.4	Computing QR factorizations	119
4	Roots	of nonlinear equations	127
	4.1	The rootfinding problem	128
	4.2	Fixed point iteration	135
	4.3	Newton's method in one variable	143
	4.4	Interpolation-based methods	152
	4.5	Newton for nonlinear systems	161
	4.6	Quasi-Newton methods	169
	4.7	Nonlinear least squares	176
5	Piecew	vise interpolation and calculus	185
	5.1	The interpolation problem	186
	5.2	Piecewise linear interpolation	193
	5.3	Cubic splines	201
	5.4	Finite differences	208
	5.5	Convergence of finite differences	214
	5.6	Numerical integration	221
	5.7	Adaptive integration	231
6	Initial-	value problems for ODEs	241
	6.1	Basics of IVPs	242
	6.2	Euler's method	249
	6.3	Systems of differential equations	256
	6.4	Runge–Kutta methods	264
	6.5	Adaptive Runge–Kutta	271
	6.6	Multistep methods	277

Contents

	6.7	Implementation of multistep methods	283
	6.8	Zero-stability of multistep methods	291
7	Matrix	k analysis	297
	7.1	From matrix to insight	298
	7.2	Eigenvalue decomposition	303
	7.3	Singular value decomposition	311
	7.4	Symmetry and definiteness	317
	7.5	Dimension reduction	322
8	Krylov	methods in linear algebra	329
	8.1	Sparsity and structure	330
	8.2	Power iteration	338
	8.3	Inverse iteration	346
	8.4	Krylov subspaces	353
	8.5	GMRES	360
	8.6	MINRES and conjugate gradients	366
	8.7	Matrix-free iterations	372
	8.8	Preconditioning	377
9	Global	function approximation	385
	9.1	Polynomial interpolation	386
	9.2	The barycentric formula	392
	9.3	Stability of polynomial interpolation	397
	9.4	Orthogonal polynomials	406
	9.5	Trigonometric interpolation	413
	9.6	Spectrally accurate integration	420
	9.7	Improper integrals	429

xii Contents

10	Bound	lary-value problems	441
	10.1	Shooting	442
	10.2	Differentiation matrices	451
	10.3	Collocation for linear problems	459
	10.4	Nonlinearity and boundary conditions	466
	10.5	The Galerkin method	474
11	Diffusi	on equations	483
	11.1	Black–Scholes equation	484
	11.2	The method of lines	491
	11.3	Absolute stability	499
	11.4	Stiffness	507
	11.5	Method of lines for parabolic PDEs	514
12	Advec	tion equations	521
	12.1	Traffic flow	522
	12.2	Upwinding and stability	527
	12.3	Absolute stability for advection	533
	12.4	The wave equation	538
13	Two-dimensional problems		
	13.1	Tensor-product discretizations	546
	13.2	Two-dimensional diffusion and advection	554
	13.3	Laplace and Poisson equations	564
	13.4	Nonlinear elliptic PDEs	572
A	Reviev	v of linear algebra	581
Bibli	ography	y	587
Inde	x		592

List of functions

xiv LIST OF FUNCTIONS

10.3.1(bvplin) Solve a linear boundary-value problem	461
10.4.1(bvp) Solve a nonlinear boundary-value problem	468
10.5.1(fem) Piecewise linear finite elements for a linear BVP	479
11.2.1(diffper) Differentiation matrices for periodic end conditions	492
13.2.1(rectdisc) Discretization on a rectangle	558
13.3.1(poissonfd) Solve Poisson's equation by finite differences	569
13.4.1(newtonpde) Newton's method to solve an elliptic PDE	578