Fast algorithms for Fourier Extensions

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Fourier series are standard for periodic approximation problems. The fast convergence coupled with a fast transform and straightforward manipulation makes them an obvious choice. For non-periodic problems, convergence slows down due to the Gibbs phenomenon. This leaves users with either low accuracy or a convoluted post-processing step.

Fourier extensions transfer ease of use and fast convergence of Fourier Series to non-periodic problems. In it's most basic form a function defined on a domain $[-1, 1]$ is approximated by a Fourier series on an 'extended' domain $[-T, T]$. In higher domains functions on arbitrary domains are approximated by Fourier series on some bounding box. The unspecified behaviour outside of the function domain allows for a lot of freedom in the extension. A least-squares solution to the extension problem can therefore converge to the approximant at least superalgebraically, and often exponentially [1]. The difficulty lies in determining the extension in a fast way, as the least-squares system to be solved is severely ill-conditioned.

In this talk, we present a novel approach to solving the Fourier extension problem. Our approach depends solely on the singular value profile of the least-squares matrix, and thereby extends naturally to other bases that exhibit a similar profile. We achieve $O(N \log^2 N)$ complexity in the 1D case, and elaborate on the difficulties associated with extensions to higher dimensions.

References

[1] Ben Adcock, D Huybrechs, and J Martín-Vaquero. On the numerical stability of Fourier extensions. *Found. Comp. Math.*, 14:635–687, 2014.