A Reduced Basis approach to large-scale pseudospectra computation

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When studying spectral properties of a non-normal matrix $A \in \mathbb{C}^{n\times n}$ information about its spectrum alone is usually not enough to provide complete information. Effects of small perturbations on $\sigma(A)$ can be studied by computing so-called ϵ-pseudospectra. For $\epsilon > 0$ the ϵ-pseudospectrum of A is usually defined as

$$\sigma_\epsilon(A) = \{z \in \mathbb{C} : \|(zI - A)^{-1}\|_2 > \epsilon^{-1}\}, \tag{1}$$

or

$$\sigma_\epsilon(A) = \{z \in \mathbb{C} : \sigma_{\min}(zI - A) < \epsilon\}. \tag{2}$$

Computation of $\sigma_\epsilon(A)$ requires the evaluation of the function $g(z) = \sigma_{\min}(zI - A)$ on a portion of the complex plane. For $z = x + iy$, the computation of $g(z)$ can be seen as a 2-dimensional parameter-dependent eigenvalue problem:

$$g(x + iy)^2 = \lambda_{\min}\left(((x + yi)I - A)^*((x + yi)I - A)\right). \tag{3}$$

In this work, we propose a Reduced Basis approach to pseudospectra computation that provides highly accurate estimates of pseudospectra in the region of interest. It incorporates the sampled singular vectors of $zI - A$ and implicitly exploits their smoothness properties. It provides rigorous bounds, both upper and lower, on pseudospectra in the region of interest.