Math 242 Homework Set #3
Due: 9/21/07

Section 7.4*

3. $5^{\sqrt{7}} = (e^{\ln 5})^{\sqrt{7}} = e^{\sqrt{7}\ln 5}$

4. $10^{x^2} = (e^{\ln 10})^{x^2} = e^{x^2 \ln 10}$

10. a.) $\log_a \frac{1}{a} \Rightarrow a^x = \frac{1}{a} \Rightarrow x = -1$

    b.) $10^{(\log_{10} 4 + \log_{10} 7)} = (10^{\log_{10} 4}) \cdot (10^{\log_{10} 7}) = 4 \cdot 7 = 28$

28. $y = 2^{3^{x^2}}$, $y' = 2^{3^{x^2}} \cdot \ln(2) \cdot 3^{x^2} \cdot \ln(3) \cdot 2x$

32. $y = x^{1/x}$

$$\ln(y) = \frac{1}{x}\ln(x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \cdot \frac{1}{x} + \ln(x) \cdot \frac{-1}{x^2}$$

$$\frac{dy}{dx} = x^{1/x}\left(\frac{1-\ln(x)}{x^2}\right)$$

46. $\int \frac{2^x}{2^x+1}\,dx$

$$u = 2^x + 1$$

$$du = 2^x \ln(2)\,dx$$

Therefore the integral becomes:

$$\frac{1}{\ln(2)}\int \frac{du}{u} = \frac{1}{\ln(2)}\ln(u) + C = \frac{1}{\ln(2)} \cdot \ln(2^x + 1) + C$$

51. To find the inverse of $y = \frac{10^x}{10^x + 1}$ we want to switch the x and y and then solve for y.

$$x = \frac{10^y}{10^y + 1}$$
$$\Rightarrow x(10^y + 1) = 10^y$$
$$\Rightarrow x10^y + x = 10^y$$
$$\Rightarrow x = 10^y(1 - x)$$
$$\Rightarrow 10^y = \frac{x}{1 - x}$$
$$\Rightarrow \log_{10}(10^y) = \log_{10}(\frac{x}{1-x})$$
$$\Rightarrow y = \log_{10}(x) - \log_{10}(1 - x)$$

Section 7.5

2. a.) $\arctan(-1) = -\frac{\pi}{4}$ since $\tan(\frac{-\pi}{4}) = -1$ and $-\frac{\pi}{4}$ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

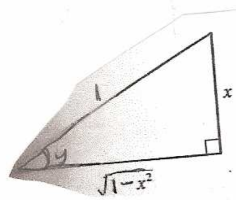
   b.) $\csc^{-1}(2) = \frac{\pi}{6}$ since $\csc(\frac{\pi}{6}) = 2$ and $\frac{\pi}{6}$ is in $\left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$.

6. a.) $\tan^{-1}(\tan(3\pi/4)) = \tan^{-1}(-1) = -\pi/4$

   b.) $\cos(\arcsin(1/2)) = \cos \pi/6 = \sqrt{3}/2$

12. $\tan(\sin^{-1}(x))$

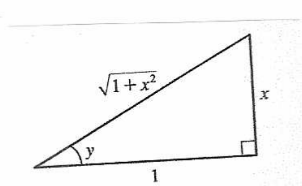
    To solve this, let $y = \sin^{-1}(x)$. Then we get the following triangle:



Then we get $\tan(\sin^{-1}(x)) = \tan(y) = \frac{x}{\sqrt{1-x^2}}$ from the triangle.

13. $\sin(\tan^{-1}(x))$

    To solve this, let $y = \tan^{-1}(x)$. Then we get the following triangle:



Then we get $\sin(\tan^{-1}(x)) = \sin(y) = \dfrac{x}{\sqrt{1+x^2}}$ from the triangle.

22. $y = \sqrt{\tan^{-1}(x)}$

$$y' = \frac{1}{2}(\tan^{-1}(x))^{-1/2} \cdot \frac{1}{1+x^2}$$

23. $y = \tan^{-1}(\sqrt{x})$

$$y' = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2} x^{-1/2}$$

$$= \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$$

46. $\lim\limits_{x \to 0^+} \tan^{-1}(\ln(x))$

Let $t = \ln(x)$. As $x \to 0^+, t \to -\infty$, so $\lim\limits_{t \to -\infty} \tan^{-1}(t) = -\dfrac{\pi}{2}$.

61. $\displaystyle\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2}$

Let $u = 4x$, then $du = 4dx$. The integral then becomes

$$\frac{1}{4}\int_0^{\sqrt{3}} \frac{1}{1+u^2}\,du = \frac{1}{4}(\tan^{-1}(u))\Big|_0^{\sqrt{3}} = \frac{1}{4}(\tan^{-1}(\sqrt{3}) - \tan^{-1}(0)) = \frac{1}{4}(\frac{\pi}{3} - 0) = \frac{\pi}{12}.$$

62. $\displaystyle\int \frac{dt}{\sqrt{1-4t^2}}$: Let $u = 2t$ and $du = 2dt$. Therefore the integral becomes

$$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\sin^{-1}(u) + C = \frac{1}{2}\sin^{-1}(2t) + C.$$

65. $\displaystyle\int \frac{x+9}{x^2+9}\,dx = \int \frac{x}{x^2+9}\,dx + \int \frac{9}{x^2+9}\,dx = \frac{1}{2}\int \frac{1}{u}\,du + 9\int \frac{1}{x^2+9}\,dx$, where $u = x^2+9$ and $du = 2x\,dx$. Evaluating these integrals we get $\dfrac{1}{2}\ln(x^2+9) + 3\tan^{-1}(\dfrac{x}{3}) + C$.

Section 7.6

2. a.) $\tanh(0) = \dfrac{\sinh(0)}{\cosh(0)} = \dfrac{e^0 - e^0}{e^0 + e^0} = 0$

b.) $\tanh(1) = \dfrac{e^1 - e^{-1}}{e^1 + e^{-1}} = \dfrac{e - 1/e}{e + 1/e} \approx .76159$

7. $\sinh(-x) = \dfrac{e^{-x} - e^{x}}{2} = -\dfrac{e^{x} - e^{-x}}{2} = -\sinh(x)$

11.

$$\sinh(x)\cosh(y) + \cosh(x)\sinh(y) = \frac{1}{2}(e^x - e^{-x}) \cdot \frac{1}{2}(e^y + e^{-y}) + \frac{1}{2}(e^x + e^{-x}) \cdot \frac{1}{2}(e^y - e^{-y})$$

$$= \frac{1}{4}\left[e^{x+y} + e^{x-y} - e^{y-x} - e^{-(x+y)}\right] + \frac{1}{4}\left[e^{x+y} - e^{x-y} + e^{y-x} - e^{-(x+y)}\right]$$

$$= \frac{1}{4}\left[2e^{x+y} - 2e^{-(x+y)}\right]$$

$$= \frac{e^{x+y} - e^{-(x+y)}}{2} = \sinh(x+y)$$

23. a.) $\lim\limits_{x\to\infty} \tanh(x) = \lim\limits_{x\to\infty} \dfrac{\sinh(x)}{\cosh(x)} = \lim\limits_{x\to\infty} \dfrac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \dfrac{e^{-x}}{e^{-x}} = \lim\limits_{x\to\infty} \dfrac{1 - e^{-2x}}{1 + e^{-2x}} = 1$

d.) $\lim\limits_{x\to-\infty} \sinh(x) = \lim\limits_{x\to-\infty} \dfrac{e^x - e^{-x}}{2} = -\infty$

31. $f(x) = x\cosh(x)$
$f'(x) = x\sinh(x) + \cosh(x)$

34. $F(x) = \sinh(x)\tanh(x)$
$F'(x) = \sinh(x) \cdot \operatorname{sec} h^2(x) + \tanh(x)\cosh(x)$

60. $\displaystyle\int \frac{\operatorname{sec} h^2(x)}{2 + \tanh(x)} dx$: Let $u = 2 + \tanh(x)$ and $du = \operatorname{sec} h^2(x) dx$. Then the integral becomes $\displaystyle\int \frac{du}{u} = \ln(u) + C = \ln(2 + \tanh(x)) + C$.