MATH 508 Challenge problem #6

Turn in no later than Apr. 5 for credit

Suppose you have two coins, one of radius R and another of radius nR for $n \geq 1$. Let the larger coin be fixed with its center at the origin, and rotate the smaller coin around it (without slipping). If n is an integer, a fixed point on the rim of the smaller coin (say, a dab of ink) traces out a closed curve called an **epicycloid**.

Use Maple to make a plot of the epicycloid when $R = 1$ and $n = 5$. Then, use (without proof) the formula

$$\int_\Gamma \overline{z}\, dz = 2i(\text{area of } D),$$

where Γ is a simple closed curve enclosing domain D, to find the area of the domain enclosed by a general epicycloid. (You may use Maple to assist with the calculation, if you like.)