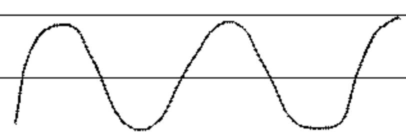
09-07

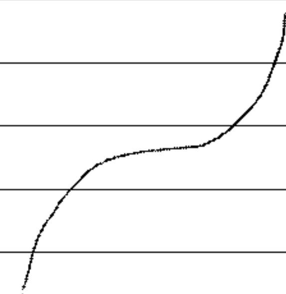
Note Title 9/7/2007

<u>Ex</u> $\cos(x)$ $\cos(0) = \cos(2\pi)$ but $0 \neq 2\pi$ <u>not one to one</u>

<u>Ex</u> x^3 If $s^3 = t^3$, take $1/3$ power of both sides

then $s = t$. <u>one to one</u>





Ex $f(x) = \frac{2x+1}{2x-3}$, find $f^{-1}(x)$

1. $y = \frac{2x+1}{2x-3}$, $2xy - 3y = 2x + 1$, $x = \frac{1+3y}{2y-1}$

2. $y = \frac{1+3x}{2x-1} = f^{-1}(x)$

Ex $f(x) = x^3 - 2$, $g = f^{-1}$, find $g'(6)$.

First method: Find formula for $g(x)$, then take derivative.

$y = x^3 - 2$, $x^3 = y + 2$, $x = \underbrace{(y+2)^{1/3}}_{g(y)}$, so $g(x) = (x+2)^{1/3}$

$g'(6) = \frac{1}{3}(x+2)^{-2/3}\Big|_{x=6} = \frac{1}{3} 8^{-2/3} = \frac{1}{12}$

Second method $y = x^3 - 2$, find $\frac{dx}{dy}$ when $y = 6$.

$\frac{dy}{dx} = 3x^2$, so $\frac{dx}{dy} = \frac{1}{3x^2}$

Must find x when $y = 6$: $6 = x^3 - 2$, $x^3 = 8$, $x = 2$

$\frac{dx}{dy}\Big|_{y=6} = \frac{dx}{dy}\Big|_{x=2} = \frac{1}{12}$

Ex $f(x) = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$, $g = f^{-1}$, find $g'(3)$.

First method $y = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$, solve for x

$$x^2 + \tan\left(\frac{\pi x}{2}\right) = y - 3 \qquad \text{STUCK!}$$

Second method $\left.\frac{dx}{dy}\right|_{y=3}$

$\frac{dy}{dx} = 2x + \frac{\pi}{2}\sec^2\left(\frac{\pi x}{2}\right)$, so $\frac{dx}{dy} = \left[2x + \frac{\pi}{2}\sec^2\left(\frac{\pi x}{2}\right)\right]^{-1}$

Find x when $y=3$: $3 = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$

$0 = x^2 + \tan\left(\frac{\pi x}{2}\right)$ $\quad x = 0$ works!

$$\left.\frac{dx}{dy}\right|_{y=3} = \left.\frac{dx}{dy}\right|_{x=0} = \left[0 + \frac{\pi}{2}(1)^2\right]^{-1} = \frac{2}{\pi}$$