The RKFIT algorithm for nonlinear rational approximation and RKFUNs

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For given matrices $\{A, F\} \subset \mathbb{C}^{N \times N}$ and a vector $\boldsymbol{b} \in \mathbb{C}^N$, we consider the problem of finding a rational function R_m of type (m, m) such that

$$\|F\boldsymbol{b} - R_m(A)\boldsymbol{b}\|_2^2 \to \min.$$

We propose an iterative algorithm called RKFIT for its approximate solution [1]. At each iteration RKFIT constructs a rational Krylov space [4] and manipulates an associated Arnoldi decomposition to find better approximations to the poles of R_m. In the special case when A and F are diagonal matrices, RKFIT is closely related to the popular vector fitting algorithm by Gustavsen and Semlyen [3]. However, vector fitting uses partial fractions whereas RKFIT works with discrete-orthogonal rational basis functions and hence reduces numerical instabilities often encountered with vector fitting.

RKFIT is part of the MATLAB Rational Krylov Toolbox available for download from

`http://www.guettel.com/rktoolbox`

The rational functions computed by RKFIT are represented as objects of class `rkfun`. Inspired by Chebfun's capabilities and syntax [2] we have implemented various methods for `rkfun`s, such as pole-finding (`poles`), root-finding (`roots`), differentiation (`diff`), conversion to partial fraction form (`residue`), and plotting (`plot`). Different from Chebfun we are working here with rational functions being orthogonal on a "discrete domain" specified via $(A, \boldsymbol{b})$. We will discuss some potentials and challenges of computing with rational functions.

This is joint work with Mario Berljafa (Manchester).

References

[1] M. Berljafa and S. Güttel, *Generalized rational Krylov decompositions with an application to rational approximation,* to appear in SIAM J. Matrix Anal. Appl., 2015. MIMS Eprint available at `http://eprints.ma.man.ac.uk/2278/`

[2] T. A. Driscoll, N. Hale, and L. N. Trefethen, editors, *Chebfun Guide,* Pafnuty Publications, Oxford, 2014.

[3] B. Gustavsen and A. Semlyen, *Rational approximation of frequency domain responses by vector fitting,* IEEE Trans. Power Del., 14 (1999), pp. 1052–1061.

[4] A. Ruhe, *Rational Krylov sequence methods for eigenvalue computation,* Linear Algebra Appl., 58 (1984), pp. 391–405.