09-10

Note Title 9/10/2007

Ex Simplify $\frac{1}{2}\ln(4z^4) - \ln(2)$

$= \frac{1}{2}\left[\ln 4 + \ln(z^4)\right] - \ln 2$

$= \frac{1}{2}\ln 4 + \frac{1}{2}\cdot 4 \ln z - \ln 2$

$= \frac{1}{2}(2\ln 2) + 2\ln z - \ln 2 = 2\ln z$

Ex $\frac{d}{dx}\ln(ax) = \frac{d}{dx}\left[\ln a + \ln x\right] = 0 + \frac{1}{x} = \frac{1}{x}$ (version A)

$\frac{d}{dx}\ln(ax) = \frac{d}{dx}\ln u = \left(\frac{d}{du}\ln u\right)\frac{du}{dx} = \frac{1}{u}a = \frac{a}{ax} = \frac{1}{x}$ (version B)

$[u = ax]$

Ex $\quad \frac{d}{dx}\left[\ln(\cos x)\right] = \frac{-\sin x}{\cos x} = -\tan x$ (antiderivative of $\tan x$)

Ex (logarithmic differentiation) $\quad y = \frac{(x^2+1)(x+3)^{1/2}}{(x-2)^{2/3}}$, find $\frac{dy}{dx}$.

First, take log, then differentiate.

$$\ln y = \ln\left[\sim\right] = \ln(x^2+1) + \ln\left[(x+3)^{1/2}\right] - \ln\left[(x-2)^{2/3}\right]$$

$$= \ln(x^2+1) + \frac{1}{2}\ln(x+3) - \frac{2}{3}\ln(x-2)$$

$$\frac{d}{dx}(\ln y) = \frac{2x}{x^2+1} + \frac{1}{2}\,\frac{1}{x+3} - \frac{2}{3}\,\frac{1}{x-2}$$

$$\frac{1}{y}\frac{dy}{dx} = \sim$$

$$\frac{dy}{dx} = \frac{(x^2+1)(x+3)^{1/2}}{(x-2)^{2/3}} \cdot \left[\frac{2x}{x^2+1} + \frac{1}{2x+6} - \frac{2}{3x-6}\right]$$

<u>Ex</u> $$\int_1^5 \frac{2}{s}\,ds = 2\int_1^5 \frac{1}{s}\,ds = 2\Big[\ln|s|\Big]_1^5 = 2(\ln 5 - \cancelto{0}{\ln 1}) = 2\ln 5$$

<u>Ex</u> $$\int \frac{x^2}{x^3-5}\,dx = \int \frac{\frac{1}{3}du}{u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3-5| + C$$

$$\begin{bmatrix} u = x^3-5 \\ du = 3x^2\,dx \end{bmatrix}$$

ln(abs(x^3-5))/3

<u>Ex</u> $$\int_{-\pi/2}^{\pi/2} \frac{4\cos t}{3+2\sin t}\,dt = \int_1^5 \frac{2du}{u}\,du = 2\ln 5$$

(as above)

$$\begin{bmatrix} u = 3+2\sin t \\ du = 2\cos t\,dt \end{bmatrix}$$

Ex Find t if $e^{2t} = 9$.

$$\ln(e^{2t}) = \ln 9 = \ln(3^2)$$

$$2t = 2\ln 3$$

$$t = \ln 3$$

Ex Simplify $e^{x+\ln 2} = e^x e^{\ln 2} = 2e^x$

Ex $\displaystyle\lim_{x \to -\infty} \frac{e^{2x}}{2-e^{2x}} = \frac{0}{2-0} = 0$

$$\lim_{x\to\infty} \frac{e^{2x}}{2-e^{2x}} \cdot \frac{e^{-2x}}{e^{-2x}} = \lim_{x\to\infty} \frac{e^0}{2e^{-2x}-e^0} = \frac{1}{0-1} = -1$$