Rational filter functions for solving eigenvalue problems by contour integration

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In this talk, the following *eigenvalue problem* is considered. Given an integer $m \geq 1$, a (bounded) domain $\Omega \subset \mathbb{C}$ and a matrix-valued function $T : \Omega \to \mathbb{C}^{m\times m}$ analytic in Ω, we want to compute the values $\lambda \in \Omega$ (eigenvalues) and $v \in \mathbb{C}^m$, $v \neq 0$ (eigenvectors) such that

$$T(\lambda)v = 0.$$

Note that this formulation reduces to the linear eigenvalue problem in case $T(z) = A - zB$, and to the polynomial eigenvalue problem when $T(z)$ is a polynomial matrix. If the problem size m is equal to 1, then the problem reduces to that of computing all the zeros λ of the analytic scalar function T in the domain Ω.

The number of eigenvalues could be large, e.g., when m is large, or in case of a polynomial eigenvalue problem when the degree of the polynomial matrix is large. In several applications, one is not interested in *all* eigenvalues but only in those lying in a certain region(s) of the complex plane. Therefore, we can reduce the original problem of finding *all* eigenvalues into one where we are only interested in those eigenvalues (and corresponding eigenvectors) lying within (or in the neighborhood) of a given closed contour $\Gamma \subset \Omega$. The relevant information to approximate these eigenvalues (and corresponding eigenvectors) can be extracted from the function $T(z)$ by using (approximate) contour integrals to the resolvent operator $T(z)^{-1}$ applied to a rectangular matrix $\hat{V}$:

$$\frac{1}{2\pi i}\int_{\Gamma} f(z)T(z)^{-1}\hat{V}dz \quad \in \mathbb{C}^{m\times q}$$

for different choices of the function $f(z)$, e.g., $f(z) = z^0, z^1, z^2, \ldots$. Here, $f : \Omega \to \mathbb{C}$ is analytic in Ω and $\hat{V} \in \mathbb{C}^{m\times q}$ is a matrix chosen randomly or in another specified way, with $q \leq m$.

In [2], we presented an algorithm based on contour integration to solve the corresponding eigenvalue problem. We showed that the so-called filter function plays an important role in the effectiveness of the contour integration approach. To compute the eigenvalues in the neighborhood of a branch cut, we used in [2] a filter function developed in [1] by Hale, Higham and Trefethen using detailed knowledge of complex analysis. Because this knowledge is not always readily available for someone who wants to solve a specific eigenvalue problem, we will design in this talk effective filter functions using an optimization algorithm.

References

[1] N. Hale, N. J. Higham, and L. N. Trefethen. Computing A^α, $\log(A)$, and related matrix functions by contour integrals. *SIAM Journal on Numerical Analysis*, 46(5):2505–2523, 2008.

[2] M. Van Barel and P. Kravanja. Nonlinear eigenvalue problems and contour integrals. Technical Report TW656, Department of Computer Science, KU Leuven, October 2014.