Componentwise and Mixed Condition Numbers for Matrix Functions

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In recent years there has been a significant amount of research into the conditioning of matrix functions $f\colon \mathbb{C}^{n\times n} \to \mathbb{C}^{n\times n}$. This work has mainly focused on the normwise condition number given by

$$\mathrm{cond}(f, A) = \lim_{\epsilon\to 0} \sup_{\|\Delta A\|<\epsilon\|A\|} \frac{\|L_f(A,E)\|\,\|A\|}{\|f(A)\|},$$

where $L_f(A,E) \in \mathbb{C}^{n\times n}$ denotes the Fréchet derivative of the function f at the point A in the direction E.

There has been comparatively little research on the componentwise and mixed condition numbers. Identifying the individual components of the input A contributing to ill-conditioning can be extremely useful when the entries of A have some physical interpretation.

For example, suppose we are solving a differential equation $\frac{dx}{dt} = Ax(t)$, with $x(0) = x_0$. The solution of this equation is $x(t) = e^{tA}x_0$. In some branches of chemistry this equation, where the entries of A are flux coefficients of molecules, helps predict chemical reactions over time. The flux coefficients are measured experimentally and are therefore subject to small error: using the componentwise condition number we can find which flux coefficients are most important to the chemical reaction so that their values can be determined with greater accuracy via further experimentation.

In this talk we will define the componentwise and mixed condition numbers of a matrix function and introduce some novel algorithms for their computation.