7.7: 12, 15, 17, 24, 44, 49, 58, 80

12. $\lim_{x\to 0}\frac{e^{3x}-1}{x}=\frac{0}{0}$ so use L'hopitals Rule to get: $\lim_{x\to 0}\frac{e^{3x}-1}{x}=\lim_{x\to 0}\frac{3e^{3x}}{1}=3$

15. $\lim_{x\to\infty}\frac{\ln x}{x}=\frac{\infty}{\infty}$ so use L'hopitals Rule to get: $\lim_{x\to\infty}\frac{\ln x}{x}=\lim_{x\to\infty}\frac{1/x}{1}=0$

17. $\lim_{x\to 0^+}\frac{\ln x}{x}=-\infty$ since dividing by a small x makes the quotient bigger, L'hopitals rule doesn't apply.

24. $\lim_{x\to 0}\frac{\sin x}{\sinh x}=\frac{0}{0}$ so use L'hopitals Rule to get: $\lim_{x\to 0}\frac{\sin x}{\sinh x}=\lim_{x\to 0}\frac{\cos x}{\cosh x}=1$

44. $\lim_{x\to\infty}x\tan(1/x)=\infty(\infty)$ so use L'hopitals Rule to get

$$\lim_{x\to\infty}\frac{\tan x}{1/x}=\lim_{x\to\infty}\frac{\sec^2(1/x)(-1/x^2)}{(-1/x^2)}=\lim_{x\to\infty}\sec^2 x=1$$

49. $\lim_{x\to\infty}x-\ln x=\infty-\infty$ so factor out an x to use L'Hopitals Rule: $\lim_{x\to\infty}x(1-\frac{\ln x}{x})$ looking just at the particular limit $\lim_{x\to\infty}\frac{\ln x}{x}=\lim_{x\to\infty}\frac{1/x}{1}=0$ then $\lim_{x\to\infty}x(1-\frac{\ln x}{x})=\infty$.

58. Let $y=\lim_{x\to\infty}(e^x+x)^{1/x}$, Taking the natural log of both sides gives:

$\ln y=\lim_{x\to\infty}\frac{\ln(e^x+x)}{x}$ using L'Hopitals Rule: $\ln y=\lim_{x\to\infty}\frac{e^x+1}{e^x+x}=\lim_{x\to\infty}\frac{e^x}{e^x+1}=\lim_{x\to\infty}\frac{e^x}{e^x}=1$

Now since $\ln y=1$ then this means $y=e^1=e$ so $\lim_{x\to\infty}(e^x+x)^{1/x}=e$.

80.

a) $\lim_{t\to\infty}v=\frac{mg}{c}\lim_{t\to\infty}\left(1-e^{-ct/m}\right)=\frac{mg}{c}$

b) $\lim_{m\to\infty}=\frac{g}{c}\lim_{m\to\infty}\left(\frac{1-e^{-ct/m}}{1/m}\right)=\frac{g}{c}(ct)\lim_{m\to\infty}\left(\frac{e^{-ct/m}(-1/m^2)}{-(1/m^2)}\right)=gt\lim_{m\to\infty}(e^{-ct/m})=gt$

So the velocity of a heavy falling object is approximately proportional to the time t.

8.1: 2, 9, 18, 22, 42

2. Let $u=\theta, dv=\sec^2\theta$ then $du=d\theta, v=\tan\theta$

so $\int \theta\sec^2\theta = \theta\tan\theta - \int \tan\theta d\theta = \theta\tan\theta - \ln|\sec\theta| + C$

9. Let $u=\ln(2x+1), dv=dx$ then $du=\dfrac{2}{2x+1}, v=x$

so $\int \ln(2x+1)dx = \ln(2x+1)x - \int \dfrac{2x}{2x+1}dx$

Now, $\int \dfrac{2x}{2x+1}dx = \int \dfrac{(2x+1)-1}{(2x+1)}dx = \int 1-\dfrac{1}{2x+1}dx = x-\dfrac{1}{2}\ln|2x+1|+C$

So $\int \ln(2x+1)dx = \ln(2x+1)x - x + \dfrac{1}{2}\ln|2x+1|+C$

18. Let $u=y, dv=\cosh(ay)dy$ then $du=dy, v=\dfrac{\sinh(ay)}{a}$

so $\int y\cosh(ay)dy = \dfrac{y\sinh(ay)}{a} - \dfrac{1}{a}\int \sinh(ay)dy = \dfrac{y\sinh(ay)}{a} - \dfrac{1}{a^2}\cosh(ay) + C$

22. Let $u=\ln t, dv=\sqrt{t}dt$ then $du=\dfrac{1}{t}dt, v=\dfrac{2}{3}t^{3/2}$

so $\int_1^4 \sqrt{t}\ln t dt = \left[\dfrac{2}{3}t^{3/2}\ln t\right]_1^4 - \dfrac{2}{3}\int_1^4 t^{1/2}dt = \left[\dfrac{2}{3}t^{3/2}\ln t\right]_1^4 - \dfrac{4}{9}\left[t^{3/2}\right]_1^4 = \dfrac{16}{3}\ln 4 - \dfrac{28}{9}$

42.

a) $u=\cos^{n-1}, dv=\cos x dx$ then $du=-(n-1)(\cos^{n-2}x)(\sin x), v=\sin x$ so

$$\int \cos^n dx = \cos^{n-1}x\sin x + \int \sin x(n-1)\cos^{n-2}xdx$$
$$= \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}x(1-\cos^2 x)dx$$
$$= \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}xdx - (n-1)\int \cos^n xdx$$

Now, adding $(n-1)\int \cos^n xdx$ to both sides gives

$n\int \cos^n xdx = \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}xdx$, and now dividing thru by n gives:

$$\int \cos^n dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}xdx$$

b) let n=2:

$\int \cos^2 xdx = \dfrac{1}{2}\cos x\sin x + \dfrac{1}{2}\int dx = \dfrac{x}{2} + \dfrac{\sin 2x}{4} + C$ now using the

c) $\int \cos^4 x dx = \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\int \cos^2 x dx$

now using the result in part (b), this gives:

$$\int \cos^4 x dx = \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}x + \frac{3}{16}\sin(2x) + C$$