Optimal discretization in Banach spaces: Residual minimization, nonlinear Petrov–Galerkin, and monotone mixed methods[1]

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Is it possible to obtain numerically the best approximation to solutions of linear operator equations in general Banach spaces? In this contribution, we address this question by considering nonstandard Petrov–Galerkin discretisations. We build on ideas of residual minimization and the recent theory of optimal Petrov–Galerkin methods in Hilbert space settings [1–4]. We propose an optimal discretization method in the setting of certain Banach spaces, proof a priori error estimates, and present some numerical experiments [5].

References

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[1]Abstract for "New directions in numerical computation: In celebration of Nick Trefethen's 60th birthday", 25–28 August 2015, Oxford, UK