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MATH349-010, Fall 2005
Exam 2: November 22, 09:30-10:45

Write all solutions on these sheets. Please circle your final answers. **You must justify answers to receive full credit.** You may not use calculators, notes, or any kinds of aids.

1. Suppose that A is a 5×3 real matrix whose rank is equal to three.

   (a) (6 points) Are the rows of A linearly dependent or linearly independent? **Explain to receive credit.**

   (b) (6 points) How many independent nonzero solutions are there to $A\mathbf{x} = \mathbf{0}$?

2. (18 points) Find a basis for the subset of $\mathbf{R}^3$ spanned by

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}, \quad \mathbf{v}_5 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

3. (18 points) Let S and T be the bases

$$S = \left\{ \begin{bmatrix} 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}, \qquad T = \left\{ \begin{bmatrix} 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}.$$

Find the coordinate transition matrix $P_{S \leftarrow T}$.

4. (16 points) Using the inner product $(p, q) = \int_0^1 p(t)q(t)\,dt$, compute the distance between the polynomials t and t^2.

5. (18 points) Find an **orthonormal** basis for the column space of $\begin{bmatrix} 1 & 4 \\ 1 & -2 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}$.

6. (a) (10 points) Find the determinant of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & c \\ -1 & 1 & 2 & 0 \\ 1 & -2 & 0 & -1 \\ c & 0 & 0 & 1 \end{bmatrix}$.

(b) (8 points) For what value(s) of c is A singular?

Extra credit (10 points) Find a basis for the subspace of all 2×2 symmetric matrices.

Angle: $\cos\theta = \dfrac{(\mathbf{u}, \mathbf{v})}{\|\mathbf{u}\| \, \|\mathbf{v}\|}$

Cauchy–Schwarz: $|(\mathbf{u}, \mathbf{v})| \leq \|\mathbf{u}\| \, \|\mathbf{v}\|$

Gram–Schmidt: To orthogonalize $\mathbf{u}_1, \ldots, \mathbf{u}_m$, let $\mathbf{v}_k = \mathbf{u}_k - \dfrac{(\mathbf{u}_k, \mathbf{v}_1)}{(\mathbf{v}_1, \mathbf{v}_1)}\mathbf{v}_1 - \cdots - \dfrac{(\mathbf{u}_k, \mathbf{v}_{k-1})}{(\mathbf{v}_{k-1}, \mathbf{v}_{k-1})}\mathbf{v}_{k-1}$.

Projection: If $\mathbf{w}_1, \ldots, \mathbf{w}_n$ are an orthogonal basis of W, then
$\operatorname{proj}_W \mathbf{v} = \dfrac{(\mathbf{v}, \mathbf{w}_1)}{(\mathbf{w}_1, \mathbf{w}_1)}\mathbf{w}_1 + \cdots + \dfrac{(\mathbf{v}, \mathbf{w}_n)}{(\mathbf{w}_n, \mathbf{w}_n)}\mathbf{w}_n$.