Practice Final

① $\frac{dP}{dt} = kP$ let $1990 = t = 0$

$P = P_0 e^{kt}$

$P(0) = 20$

$P(8) = 80$

$\Rightarrow P_0 = 20$

$80 = 20e^{k(8)}$

$4 = e^{k(8)} \Rightarrow \ln(4) = 8k \Rightarrow k = \frac{\ln(4)}{8}$

$\boxed{P(20) = 20e^{\left(\frac{\ln 4}{8}\right)(20)}}$

② $\sin^{-1}(\sin(\frac{3\pi}{2})) = \sin^{-1}(-1) = -\frac{\pi}{2}$ or $\frac{3\pi}{2}$

③ $r = 4\sin\theta$

$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$

$y^2 - 4y = -x^2$

$(y-2)^2 = -x^2 + 4$

$(y-2)^2 + x^2 = 4$

circle, $r = 2$

center $= (0, 2)$

(4) $e^{\sinh(5x)}$

$$\frac{d}{dx}\left(e^{\sinh(5x)}\right) = e^{\sinh(5x)} \cdot \cosh(5x) \cdot 5$$

$$\frac{d}{dx}(x=0) = e^{\sinh(0)} \cdot \cosh(0) \cdot 5$$
$$= \boxed{5}$$

(5) $\displaystyle\int \frac{2^x}{1+2^x}\,dx$

$u = 1+2^x$

$du = 2^x \cdot \ln(2)\,dx$

$$= \frac{1}{\ln(2)} \int \frac{du}{u} = \frac{1}{\ln(2)} \ln(u) + C$$

$$= \boxed{\frac{1}{\ln(2)} \ln(1+2^x) + C}$$

(6) $\displaystyle\int_1^\infty \frac{ds}{(s+1)(s+2)}$ $\qquad \dfrac{A}{s+1} + \dfrac{B}{s+2} = \dfrac{1}{(s+1)(s+2)}$

$A(s+2) + B(s+1) = 1$

$As + 2A + Bs + B = 1 \Rightarrow (A+B)s + 2A + B = 1$

$\Rightarrow A = -B \quad , 2A+B = 1 \Rightarrow -2B + B = 1$

$\Rightarrow -B = 1 \Rightarrow \boxed{B = -1}$

$\boxed{A = 1}$

$$\lim_{t\to\infty} \int_1^t \frac{1}{s+1} - \frac{1}{s+2}\,ds$$

$$= \lim_{t\to\infty} \left[\ln(s+1) - \ln(s+2)\right]\Big|_1^t = \lim_{t\to\infty} \left(\ln\left(\frac{t+1}{t+2}\right) - \ln\left(\frac{3}{2}\right)\right)$$

$$= \ln(1) - \ln\left(\frac{3}{2}\right) = \boxed{-\ln\left(\frac{3}{2}\right)}$$

(7) $x = t^2$ At $(4,0)$ $t = -2$ or 2
$y = t^3 - 4t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-4}{2t}$$

$$\frac{dy}{dx}(t=-2) = \frac{3(-2)^2-4}{2(-2)} = \frac{3\cdot4-4}{-4} = \frac{8}{-4} = -2$$

$$\frac{dy}{dx}(t=2) = \frac{3(2^2)-4}{2(2)} = \frac{8}{4} = 2$$

Two tangent lines:

$$y - 0 = -2(x-4)$$

$$\Rightarrow \boxed{y = -2x + 8}$$

and

$$y - 0 = 2(x-4)$$

$$\Rightarrow \boxed{y = 2x - 8}$$

(8) $$\sum_{n=1}^{\infty} \frac{1}{3^{2n-1}} = \sum_{n=1}^{\infty} \frac{1}{3^{2n}\cdot 3^{-1}} = \sum_{n=1}^{\infty} \frac{3}{3^{2n}} = 3\sum_{n=1}^{\infty} \frac{1}{(3^2)^n} = 3\sum_{n=1}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= \frac{3\cdot 1/9}{1-1/9} = \frac{1/3}{1-1/9} = \frac{3}{8}$$

(9) $T_2(x)$ at $a=-1$ for x^3

$f(x)=x^3 \quad f(-1)=-1$
$f'(x)=3x^2 \quad f'(-1)=3(-1)^2=3$
$f''(x)=6x \quad f''(-1)=-6$

$$T_2(x) = -1 + \frac{3(x+1)}{1!} - \frac{6(x+1)^2}{2!}$$

$$= -1 + 3(x+1) - 3(x+1)^2$$

(10) $$\lim_{x\to 0} \frac{e^{4x}-1-4x}{x^2} \overset{LH}{=} \lim_{x\to 0} \frac{4e^{4x}-4}{2x} \overset{LH}{=} \lim_{x\to 0} \frac{16e^{4x}}{2} = \frac{16}{2}=8$$