FORM A

MATH 241, Spring 2009
Exam 3: May 13

NAME ______________________ **Tue/Thurs discussion time** ____________

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (10 points) Find all inflection points and the intervals of concavity in $0 \le \theta \le \pi$ for $f(\theta) = \sin(\theta) - \frac{1}{4}\theta^2$.

$f' = \cos\theta - \frac{1}{2}\theta$

$f'' = -\sin\theta - \frac{1}{2}$

$f''(\theta) = 0 \Rightarrow \sin\theta = -\frac{1}{2}$

no solutions for $0 \le \theta \le \pi$

$\Downarrow$

no inflection points

since $\sin\theta \ge 0$ for $0 \le \theta \le \pi$, $[0, \pi]$ is concave down

2. (15 points) Find the minimum and maximum values of $f(t) = t\sqrt{18-t^2}$ on the interval $[0,4]$.

$f'(t) = \sqrt{18-t^2} + \frac{1}{2}t(18-t^2)^{-1/2}(-2t)$

critical number : $0 = (18-t^2)^{+1/2} - t^2(18-t^2)^{-1/2}$

$0 = (18-t^2)^1 - t^2 = 18 - 2t^2$

$t = 3$ or $\cancel{t = -3}$ (not in $[0,4]$)

$f(0) = 0 \leftarrow$ min value

$f(3) = 3\sqrt{9} = 9 \leftarrow$ max value

$f(4) = 4\sqrt{2} < 9$

3. (10 points) Find all local minimum and local maximum points of $g(x) = 100 - 8x^2 + x^4$.

$g'(x) = -16x + 4x^3 = 4x(x^2-4) = 4x(x-2)(x+2)$

critical numbers $x=0$, $x=2$, $x=-2$

$g''(x) = -16 + 12x^2$

$g''(0) = -16 < 0$ local max

$g''(2) = -16+48 > 0$ local min

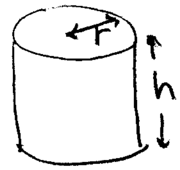
$g''(-2) = -16+48 > 0$ local min

OR

	$4x$	$x-2$	$x+2$	$g'(x)$	
$x<-2$	$-$	$-$	$-$	$-$	
					$\leftarrow$ min
$-2<x<0$	$-$	$-$	$+$	$+$	
					$\leftarrow$ max
$0<x<2$	$+$	$-$	$+$	$-$	
					$\leftarrow$ min
$x>2$	$+$	$+$	$+$	$+$	

4. (15 points) A cylindrical metal can with no lid is supposed to hold 1000π cm^3 of liquid. Find the dimensions of the can that minimizes the amount of material used.

$$1000\pi = V = \pi r^2 h \quad \text{(Constraint)}$$



Material used $\rightarrow$ surface area

$$S = \underset{\text{(sides)}}{2\pi r h} + \underset{\text{(bottom)}}{\pi r^2} \quad \text{(minimize)}$$

$$S = 2\pi r\left(\frac{1000\pi}{\pi r^2}\right) + \pi r^2 = \frac{2000\pi}{r} + \pi r^2$$

$$S'(r) = -\frac{2000\pi}{r^2} + 2\pi r$$

$$0 = S'(r) \Rightarrow 2\pi r = \frac{2000\pi}{r^2}$$

$$r^3 = 1000$$

$$r = 10 \text{ cm}$$

$$h = \frac{1000}{r^2} = \frac{1000}{100} = 10 \text{ cm}$$

5. (10 points) A car traveling 99 ft/sec begins to experience deceleration of e^t starting at $t = 0$. How far will it have traveled between $t = 0$ and $t = 2$? (No need to simplify the number.)

$a(t) = -e^t$

$v(t) = \int a(t)\,dt = -e^t + C$

Also, $99 = v(0) = -1 + C$, so $C = 100$

$s(t) = \int v(t)\,dt = -e^t + 100t + B$

So $s(2) - s(0) = (-e^2 + 200 + B) - (-1 + 0 + B)$

$= 201 - e^2$ ft.

6. (10 points) Write down the Riemann sum R_5 that approximates $\int_{-5}^{10} \cos(x)\,dx$, using right endpoints of five intervals. **Do not try to simplify or evaluate the number.**

$$\Delta x = \frac{10-(-5)}{5} = 3 \qquad x_0 = -5 \quad x_1 = -2 \quad x_2 = 1$$

$$x_3 = 4 \quad x_4 = 7 \quad x_5 = 10$$

$$R_5 = \sum_{i=1}^{5} \Delta x\, f(x_i) = 3\big(\cos(-2) + \cos(1) + \cos(4) + \cos(7) + \cos(10)\big)$$

7. (10 points) Find $\frac{d}{dx}\left[\int_1^{1/x} \cosh^3(s)\,ds\right]$.

$$\left(u = \frac{1}{x}\right) \qquad \left(\frac{d}{du}\int_1^u \cosh^3(s)\,ds\right)\left(\frac{du}{dx}\right)$$

$$= \cosh^3(u)\cdot\left(-\frac{1}{x^2}\right)$$

$$= \frac{-\cosh^3(1/x)}{x^2}$$

8. (10 points) Evaluate $\int_0^{\pi/6} 2\cos(\theta)\,d\theta$.

$$\left[2 \sin\theta\right]_0^{\pi/6} = \left(2 \sin\tfrac{\pi}{6} - 2\sin 0\right) = 1$$

9. (10 points) Evaluate $\int x(\sqrt{x}-1)\,dx$.

$$\int (x^{3/2} - x)\,dx = \frac{2}{5}x^{5/2} - \frac{1}{2}x^2 + C$$