HW 8
12.1: 2,7,12,16,19,22,28,29,31,33

2. a) A convergent sequence is one for which $\lim_{n\to\infty} a_n$, like $\{1/n\}$ or $\{1/3^n\}$

b) A divergent sequence is one for which $\lim_{n\to\infty} a_n$ does not exist, like $\{n\}$ or $\{\sin(n)\}$

7. $\{3, 5, 9, 17, 33....\}$

12. Looking at just the numerators, they are each defined by n (for each term the numerators are 1, 2, 3…). Also notice that the signs change for each term, and when n is odd the term is negative, and when n is even the term is positive. The denominator has the pattern $(n+1)^2$. So the whole sequence is defined by $a_n = (-1)^n \dfrac{n}{(n+1)^2}$.

16. Dividing each term in the numerator and denominator by n gives $a_n = \dfrac{1+1/n}{3-1/n}$ so

$\lim_{n\to\infty} a_n = \dfrac{1+1/n}{3-1/n} = \dfrac{1+0}{3-0} = \dfrac{1}{3}$ (Converges).

19. Rewriting the sequence as $a_n = \dfrac{2^n}{3^1 3^n} = \dfrac{1}{3}\left(\dfrac{2}{3}\right)^n$ now $\lim_{n\to\infty} a_n = \dfrac{1}{3}\lim_{n\to\infty}\left(\dfrac{2}{3}\right)^n$ and by formula (8) in this section (pg 743) $\lim_{n\to\infty}\left(\dfrac{2}{3}\right)^n = 0$ since 2/3 is less than 1. So $\lim_{n\to\infty} a_n = 0$ (Converges).

22. Looking at $|a_n| = \dfrac{n^3}{n^3+2n^2+1}$ and dividing through by an n^3 then $a_n = \dfrac{1}{1+(2/n)+(1/n^3)}$ so $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{1}{1+(2/n)+(1/n^3)} = 1$. However, without the absolute value, the terms in the sequence alternate sign. So for even n values, the sequence converges to 1 and for odd n values the sequence converges to -1. So overall, the sequence diverges.

28. $a_n = \dfrac{\ln n}{\ln 2 + \ln n}$ (by rules of natural logarithms). Now divide each term by ln(n). Then $a_n = \dfrac{1}{(\ln 2)/(\ln n)+1}$. So $\lim_{n\to\infty} a_n = \dfrac{1}{(\ln 2)/(\ln n)+1} = 0$ so the sequence converges.

29. $a_n = \frac{n^2}{e^n}$. Using L'Hopital's rule, then $\lim_{x\to\infty}\frac{x^2}{e^x} = \lim_{x\to\infty}\frac{2x}{e^x} = \lim_{x\to\infty}\frac{2}{e^x} = 0$. So by theorem 3, the sequence converges to 0.

31. Since $0 \le \cos^2(n) \le 1$, and $2^n > 0$ then $0 \le \frac{\cos^2(n)}{2^n} \le \frac{1}{2^n}$ and since $\lim_{n\to\infty}\frac{1}{2^n} = 0$ then $\frac{\cos^2(n)}{2^n}$ converges to 0 by the squeeze theorem.

33. Rewriting $a_n = \frac{\sin(1/n)}{1/n}$, now consider $\lim_{x\to\infty}\frac{\sin(1/x)}{1/x}$ now let $t = 1/x$ then $\lim_{t\to 0^+}\frac{\sin t}{t} = 1$. Thus it follows from Theorem 3 that a_n converges to 1.