NAME_______________________________________

MATH349-010, Fall 2005
Exam 1: October 18, 09:30-10:45

Write all solutions on these sheets. Please circle your final answers. **Except where noted, you must justify answers to receive full credit.** You may not use calculators, notes, or any kinds of aids.

1. (12 points) Suppose that

$$A = \begin{bmatrix} 1 & 4 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 0 \\ 3 & -2 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 1 & 1 \\ 0 & 2 & -2 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

Only one of these products is defined. Circle which one, and find its value.

(a) $A^T B A$ (b) $\mathbf{x}^T B \mathbf{x}$ (c) $A^2 \mathbf{x}$

2. This problem is about the linear system

$$\begin{aligned} x + 2y + 2z &= -2 \\ y + 2z &= -5 \\ x + y + z &= 1 \\ 3x + 5y + 4z &= -1 \end{aligned}$$

(a) (2 points) Write the system in the form $A\mathbf{x} = \mathbf{b}$ (that is, say what A and $\mathbf{b}$ are).

(b) (16 points) Find all solutions, or show that none exist.

(c) (2 points) Is A nonsingular?

3. (16 points) Prove that a diagonal $n \times n$ matrix is singular if and only if at least one of its diagonal elements is zero. (Recall that a diagonal matrix D is one for which $d_{ij} = 0$ whenever $i \neq j$.)

4. (16 points) Suppose that A is row-equivalent to the matrix

$$\begin{bmatrix} 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find the nullspace of A.

5. (18 points) Let W be the set of all polynomials $p(t)$ in P_3 such that $p(1) = 0$. Is W a subspace of P_3? Justify your answer.

6. (18 points) For what values of c are the vectors $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -3 \\ c(c-1) \end{bmatrix}$ independent?

Extra credit (10 points) Prove that if A is symmetric and nonsingular, then A^{-1} is symmetric also.