NAME _____

MATH341, Spring 2007
Exam 2: April 6

Please clearly erase or cross out irrelevant work; otherwise it will be part of the graded material. **You must justify answers to receive full credit.** You may not use calculators or notes.

$mx'' + cx' + kx = F_0 \cos \omega t$ has solution $x = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t + C \cos(\omega t - \alpha)$, where

$$C = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}, \qquad \tan \alpha = \frac{c\omega}{k - m\omega^2}, \quad 0 < \alpha < \pi,$$

unless $c = 0$ and $\omega = \omega_0$.

1. (10 points) Determine whether the functions $f(x) = 1$, $g(x) = \cos^2 x$, and $h(x) = \cos 2x$ are linearly dependent or independent on the real line.

2. (20 points) Solve the initial-value problem $y''' - 2y'' + y' = 0$, $y(0) = 1$, $y'(0) = 1$, $y''(0) = 2$.

3. (20 points) Write down the form for a particular solution y_p, but **do not try to find the coefficients**, for the equation $y''' - 2y'' + y' = 5 + 3e^x + e^{-x} - e^x \cos 3x$. (This has the same left-hand side as problem #2.)

4. (20 points) Find the general solution of $y'' + 2y' + 2y = 10 \sin x$.

5. (20 points) A mass of 2 kg is attached to a spring and allowed to hang freely, causing a static displacement of $0.05g$ m, where g is gravitational acceleration. Assume a damping coefficient $c = 16$ kg/s.

   (a) Is this oscillator overdamped, critically damped, or underdamped? Explain.

   (b) Suppose the system is subjected to a driving force $F_0 \cos \omega t$. Find the practical resonant frequency ω, or show that none exists.

6. (10 points) Write $xy''' - yy' = e^x$, where $y = y(x)$, as an equivalent first-order system of differential equations.