M242 HW 9

12.1 51, 60

51. Consider two cases. First, when $|r| \geq 1$ then $\{r^n\}$ diverges, so does $\{nr^n\}$ (since $|nr^n| = n|r^n| \geq |r^n|$. Now, if $|r| < 1$ then $\lim_{x\to\infty} xr^x = \lim_{x\to\infty} \frac{x}{r^{-x}} = \lim_{x\to\infty} \frac{1}{(-\ln r)r^{-x}} = \lim_{x\to\infty} \frac{r^x}{-\ln r} = 0$. So, $\lim_{n\to\infty} nr^n = 0$ and thus $\{nr^n\}$ converges for $|r| < 1$.

60. Consider $f(x) = x + \frac{1}{x}$, Now $f'(x) = 1 - \frac{1}{x^2}$ which is greater than 0 if x>1, so f is increasing if x is greater than 1. This is not bounded since $\lim_{n\to\infty}\left(n + \frac{1}{n}\right) = \infty$.

12.2 10, 17, 18, 21, 23, 27

10. a) These are both the sum of the first n terms of the sequence.

b) $\sum_{i=1}^{n} a_i$ is the sum of the first n terms of the sequence, where $\sum_{i=1}^{n} a_j$ is just $a_j + a_j + \cdots + a_j$

17. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} = \sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{(4)(4^{n-1})} = \frac{1}{4}\sum_{n=1}^{\infty}\left(\frac{-3}{4}\right)^{n-1}$ now this is a geometric series with $a = 1, r = -\frac{3}{4}$, since $|r| < 1$, the series converges to $\frac{1}{4}\left(\frac{1}{1-(-3/4)}\right) = \frac{1}{7}$.

18. $\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^n$ this is a geometric series with $|r| = \frac{1}{\sqrt{2}} \approx .707 < 1$ so the series converges to $\frac{1}{1-1/\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}-1} = \sqrt{2}(\sqrt{2}+1) = \sqrt{2}+2$.

21. $\lim_{n\to\infty} \frac{n}{n+5} = 1 \neq 0$ so the series diverges.

23. Using partial fractions, we know that $\frac{2}{(i+1)(i-1)} = \frac{A}{(i+1)} + \frac{B}{(i-1)}$, so solving for A and B gives A=-1, B=1. Consider the nth sum,

$s_n = \sum_{i=2}^{n}\left(\frac{1}{i-1}-\frac{1}{i+1}\right) = \left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{n-3}-\frac{1}{n-1}\right)+\left(\frac{1}{n-2}-\frac{1}{n}\right)$ This is a telescoping series with $s_n = 1+\frac{1}{2}-\frac{1}{n-1}-\frac{1}{n}$. So $\sum_{n=2}^{\infty}\frac{2}{n^2-1} = \lim_{n\to\infty}\left(1+\frac{1}{2}-\frac{1}{n-1}-\frac{1}{n}\right) = \frac{3}{2}$

27. $\sum_{n=1}^{\infty}\frac{3^n+2^n}{6^n} = \sum_{n=1}^{\infty}\left(\frac{3^n}{6^n}+\frac{2^n}{6^n}\right) = \sum_{n=1}^{\infty}\left(\left(\frac{1}{2}\right)^n+\left(\frac{1}{3}\right)^n\right) = \frac{1/2}{1-1/2}+\frac{1/3}{1-1/3} = \frac{3}{2}$