An iterative implementation of the Tau method based on Schur complements

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Abstract: In this work we propose an iterative implementation of the Tau method for the solution of ordinary differential equations. This spectral method, originally proposed by C. Lanczos [1] in 1938, was expressed in matricial form by E. Ortiz [3] in 1981. One of the main advantages of our implementation is that the approximate solution is iteratively refined at the same time that an error estimate is being computed. Noteworthy, this estimate is used as a stopping criteria for the iterative scheme. The iterative process built relies on the Schur complement of the matrix representing the effect of the differential operator over an orthogonal polynomial basis jointly with all boundary conditions. LU factorization is computed for a moderate size basis, and richer approximations can be built from higher dimensional basis without the need to explicitly recompute other factorizations. Our step length, in the iterative process, is related to properties of the residual in the differential equation, and is established in an way allowing that an error estimate and a new approximate solution are recalculated with the same matrix.

This method delivers polynomial approximate solutions, expressed in any orthogonal polynomial basis, to a linear ordinary differential equation, with initial, boundary or any type of conditions; thus, some functions of *Chebfun* package [6, 7] were applied to approximate a transcendent function on the Chebyshev basis. Extensions to generalize our procedure to nonlinear ordinary systems of differential equations and to partial differential equations [5, 2] are underway.

Numerical examples illustrating the efficiency of the code and comparisons with results obtained in [4] are presented.

Keywords: Iterative spectral method, differential equations, error estimative.

References

[1] Lanczos C., *Trigonometric Interpolation of Empirical and Analytical Functions*, Journal Maths Phis, 17, (1938) pp. 123–199.

[2] Matos J., Rodrigues M.J., Vasconcelos P.B, *New implementation of the Tau method for PDEs*, Journal of Computational and Applied Mathematics, 164165 (2004) pp. 555–567.

[3] Ortiz E.L., Samara H., *An operational approach to the tau method for the numerical solution of nonlinear differential equations*, Computing, 27 (1981) pp. 15–25.

[4] Olver S., Townsend A., *A Fast and Well-Conditioned Spectral Method*, SIAM Review, 55-3 (2013), pp. 462–489.

[5] Rodrigues M. J., Matos J., *A tau method for nonlinear dynamical systems*, Numerical Algorithms, 62-4 (2012) pp. 583–600.

[6] Trefethen L. N., *Approximation Theory and Approximation Practice*, SIAM, 2013.

[7] Trefethen L. N., *Spectral Methods in MATLAB*, SIAM, Philadelphia, 2000.