On Interpolation Approximation: Convergence rates on interpolation for functions of limited regularity[1]

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Abstract. The convergence rates on polynomial interpolation in most cases are estimated by Lebesgue constants. These estimates may be overestimated for some special points of sets for functions of limited regularities. In this paper, by applying the Peano kernel theorem and Wainerman's lemma, new formulas on the convergence rates are considered. Based upon these new estimates, it shows that the interpolation at strongly normal pointsystems can achieve the optimal convergence rate, the same as the best polynomial approximation. Furthermore, by using the asymptotics on Jacobi polynomials, the convergence rates are established for Gauss-Jacobi or Jacobi-Gauss-Lobatto pointsystems. From these results, we see that the interpolations at the Gauss-Legendre, Legendre-Gauss-Lobatto pointsystem, or at strongly normal pointsystems, has essentially the same approximation accuracy compared with those at the two Chebyshev piontsystems, which illustrates the equally accuracy of the Gauss and Clenshaw-Curtis quadrature. In addition, numerical examples illustrate the perfect coincidence with the estimates, which means the convergence rates are optimal.

Keywords. polynomial interpolation, Peano kernel, convergence rate, limited regularity, strongly normal pointsystem, Gauss-Jacobi point, Jacobi-Gauss-Lobatto point, Chebyshev point.

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