MATH 241, Fall 2008
Exam 1: October 3

NAME SOLUTIONS Discussion section ____________

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (10 points) If $f(x) = 2e^{-x}$, find a formula for $f^{-1}(x)$.

$$y = 2e^{-x}$$

$$\frac{y}{2} = e^{-x}$$

$$\ln\left(\frac{1}{2}y\right) = -x$$

$$x = -\ln\left(\frac{1}{2}y\right)$$

swap

$$y = -\ln\left(\frac{1}{2}x\right)$$

$$= \ln\left(\frac{2}{x}\right) \quad \text{or} \quad \ln(2) - \ln(x)$$

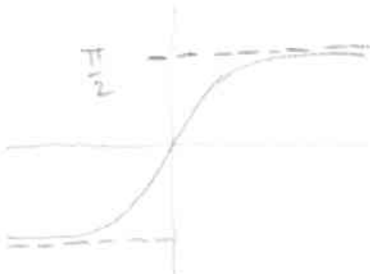
$$f^{-1}(x) = \ln\left(\frac{2}{x}\right)$$

2. (5 points each) State the domain and range of each function.

(a) $f(x) = \tan^{-1}(\sqrt{x})$

$\sqrt{x}$ maps $[0,\infty)$ to $[0,\infty)$

$\tan^{-1}$ maps $[0,\infty)$ to $[0,\frac{\pi}{2})$



$$\boxed{\begin{array}{l} \text{domain} = [0,\infty) \\ \text{range} = [0,\frac{\pi}{2}) \end{array}}$$

(b) $g(x) = \ln(2^x - 1)$

ln requires that

$2^x - 1 > 0$

$2^x > 1$

$x \ln(2) > \ln(1) = 0$

$x > 0$

2^x maps $(0,\infty)$ to $(1,\infty)$

$2^x - 1$ maps $(0,\infty)$ to $(0,\infty)$

ln maps $(0,\infty)$ to $(-\infty,\infty)$

$$\boxed{\begin{array}{l} \text{domain} = (0,\infty) \\ \text{range} = (-\infty,\infty) \end{array}}$$

3. (6 points each) Simplify each expression into a simple number.

(a) $\log_2(6) - \log_2(15) + \log_2(10)$

(b) $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$

$= \log_2\left(\frac{6}{15}\right) + \log_2(10)$

$= \log_2\left(\frac{60}{15}\right)$

$= \log_2(4) = \boxed{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\Downarrow$

$\cos\theta = \frac{1}{2}$ and

$0 \le \theta \le \pi$

$\Downarrow$

$\theta = \boxed{\frac{\pi}{3}}$

4. (12 points) Find $\lim_{t \to 2} \frac{t^2 - 2t}{|2 - t|}$, or show it does not exist.

$$\frac{t^2-2t}{|2-t|} = \begin{cases} \dfrac{t^2-2t}{2-t} & \text{if } 2-t>0, \text{ or } t<2 \\[2ex] \dfrac{t^2-2t}{-(2-t)} & \text{if } 2-t \le 0, \text{ or } t \ge 2 \end{cases}$$

$$= \begin{cases} \dfrac{t\cancel{(t-2)}}{(-1)\cancel{(t-2)}} & \text{if } t<2 \\[2ex] \dfrac{t(t-2)}{t-2} & \text{if } t \ge 2 \end{cases}$$

So $\lim_{t \to 2^-} f(t) = \frac{2}{(-1)} = -2$, $\lim_{t \to 2^+} f(t) = 2$

different, so $\boxed{\text{DNE}}$

5. (6 points each) Find all points where the given function is discontinuous. If the function is continuous everywhere, write NONE. (Remember to justify your answers.)

(a) $f(x) = \dfrac{\sin(x)}{x}$

(b) $g(x) = \begin{cases} 1, & \text{if } x < 0, \\ \cos(x), & \text{if } 0 \le x < \pi, \\ \sin(x), & \text{if } x \ge \pi. \end{cases}$

$f(0)$ not defined so

f not continuous at $\boxed{x=0}$

$\lim_{x\to 0^-} g(x) = 1 = \lim_{x\to 0^+} g(x)$

and $g(0) = 1$

$\lim_{x\to \pi^-} g(x) = -1$

$0 = \lim_{x\to \pi^+} g(x)$

$\lim_{x\to \pi} g(x)$ DNE

$\boxed{x = \pi}$

6. (15 points) Find all of the vertical and horizontal asymptotes to the graph of $y = \frac{x^3 - 3}{2x(x^2 - 1)}$. It is not necessary to draw the graph.

vertical: denom $= 0$ $\quad 2x(x+1)(x-1) = 0$

$$\boxed{x = 0, \quad x = -1, \quad x = 1}$$

horizontal: $\displaystyle\lim_{x \to \infty} \frac{x^3 - 3}{2x^3 - 2x} \cdot \frac{x^{-3}}{x^{-3}}$

$$= \lim_{x \to \infty} \frac{1 - 3x^{-3}}{2 - 2x^{-2}} = \frac{1}{2}$$

$\boxed{y = \frac{1}{2}}$ $\quad$ (same as $x \to -\infty$)

7. (15 points) Using one of our limit formulas for the derivative, find the line tangent to $y = 3 + 4x^2$ at the point $(-1, 7)$.

$f(x) = 3+4x^2$ find $f'(-1)$

$$f'(-1) = \lim_{x \to -1} \frac{3+4x^2 - 7}{x+1} = \lim_{x \to -1} \frac{4(x^2-1)}{x+1}$$

$$= \lim_{x \to -1} 4(x-1) = -8$$

$\boxed{(y-7) = -8(x+1)}$ or $y = -8x - 1$

8. (14 points) A falling particle has height described by the equation $s = 1/t$. Find its velocity when $t = 3$.

$$s'(3) = \lim_{t \to 3} \frac{\frac{1}{t} - \frac{1}{3}}{t-3} \cdot \frac{3t}{3t}$$

$$= \lim_{t \to 3} \frac{3-t}{3t(t-3)}$$

$$= \lim_{t \to 3} \frac{-1}{3t} = \boxed{-\frac{1}{9}}$$

(units not given)