11-02

Note Title 11/2/2007

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$ sum of positive terms

Choose $\sum b_n$ for comparison : $b_n = \dfrac{1}{\sqrt{n \cdot n \cdot n}} = \dfrac{1}{n^{3/2}}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ is a p-series with $p = 3/2$, so convergent.

Check the limit:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^{3/2}}{\sqrt{n(n+1)(n+2)}} \cdot \frac{n^{-3/2}}{n^{-3/2}} = \lim_{n\to\infty} \frac{1}{\sqrt{\underbrace{n(n+1)(n+2)n^{-3}}_{1\cdot(1+n^{-1})(1+2n^{-1})}}} = 1$$

Limit Comparison implies original series converges.

<u>Ex</u> $\sum_{n=1}^{\infty} \frac{2^n}{3^n+2}$ series of positive terms

$a_n = \frac{2^n}{3^n+2}$ Try $b_n = \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n$ $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \underbrace{\frac{2}{3}\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n-1}}_{\text{geometric, convergent}}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3^n}{\cancel{2^n}} \cdot \frac{\cancel{2^n}}{3^n+2} = \lim_{n\to\infty} \frac{3^n}{3^n+2} \cdot \frac{3^{-n}}{3^{-n}}$$

$$= \lim_{n\to\infty} \frac{1}{1+2\cdot 3^{-n}} = 1$$ nonzero finite

By Limit Comparison, original series converges.

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{2n(n+5)}{n(4n^2-1)}$ positive terms

Compare to $b_n = \dfrac{n \cdot n}{n \cdot n^2} = \dfrac{1}{n}$ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ p-series with $p=1$ (harmonic) diverges

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} n \frac{2n(n+5)}{n(4n^2-1)} = \lim_{n\to\infty} \frac{2n^3+10n^2}{4n^3-n} = \frac{1}{2}$$ nonzero finite

By Limit Comparison, original series diverges.

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{5/2}}$ positive terms

Try $b_n = \dfrac{1}{n^{5/2}}$, since $\overbrace{\dfrac{\sin^2 n}{n^{5/2}}}^{a_n} \le \overbrace{\dfrac{1}{n^{5/2}}}^{b_n}$ for all n.

$\sum b_n$ is p-series with $p = 5/2$, so converges.

By Comparison Test, $\sum \dfrac{\sin^2 n}{n^{5/2}}$ also converges.

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$ Recall $n! = n(n-1)(n-2)\cdots(1)$

positive terms

(factorials make limits hard)

$a_1 = \frac{1}{1} = \frac{1}{2^0}$ If $b_n = \frac{1}{2^{n-1}}$, then $a_n \le b_n$.

$a_2 = \frac{1}{2\cdot 1} = \frac{1}{2^1}$ Also, $\displaystyle\sum b_n = \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}}_{\text{geometric}}$

$a_3 = \frac{1}{3\cdot 2\cdot 1} < \frac{1}{2\cdot 2} = \frac{1}{2^2}$ $r = \frac{1}{2}$

$a_4 = \frac{1}{4\cdot 3\cdot 2\cdot 1} < \frac{1}{2\cdot 2\cdot 2} = \frac{1}{2^3}$ $\therefore$ convergent

$\vdots$

By Comparison Test, $\sum \frac{1}{n!}$ converges