Exercises

1. Play the "chaos game." In this game, let $x = [0, 0]^T$ and then repeatedly replace x by $Ax + b$, where $A = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$, and b is chosen randomly from the three vectors

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} 1/4 \\ \sqrt{3}/4 \end{bmatrix}.$$

   Add a dot for each value of x into a plot in the x_1–x_2 plane.

2. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}.$$

   (a) Do elimination manually in MATLAB for this system (as in the demo). What does the result tell you about the linear system?

   (b) Use backslash to solve the original linear system. Is the result valid?

3. The eigenvalues of Toeplitz matrices are interesting.

   (a) Let A be the 10×10 matrix

$$\begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ -2 & 1 & -1 & \cdots & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & 0 & \cdots & -2 & 1 \end{bmatrix}$$

   . Plot its eigenvalues as crosses (`'x'`) in the complex plane.

   (b) Repeat (a) for the same matrix at size 50, 100, and 200. Superimpose all the plots.

   (c) Repeat (a)–(b) for the Toeplitz matrix whose first row starts with $4, -1, 1$, and whose first column starts with $4, -2, 1$.

4. Reconsider interpolation. Interpret the problem of interpolating data $(x_1, y_1), \ldots,)(x_n, y_n)$ by a linear combination of functions $f_1, \ldots, f_n$ as a linear system of equations.

   (a) Use this connection to interpolate some made-up nonlinear data to the functions 1, x, x^2, $\cos(x)$, $\cos^2(x)$. Make a convincing plot of the interpolant and data.

   (b) Now add $\sin^2(x)$ to the list of functions and do another example. Can you figure out (mathematically) why you get a warning from MATLAB?