Toward integration of high-frequency oscillators at logarithmic cost: QTT tensor approximation of discretized functions

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This talk is dedicated to Prof. Nick Trefethen on occasion of his 60th birthday.

The well established approach to the efficient approximation and integration of "complicated" functions, based on the Chebyshev interpolation (Chebfun1, Chebfun2), was developed in the last decade by Nick Trefethen and his group [1].

The recent alternative approach to low complexity representation of multivariate functions is based on the principle of separation of variables [2]. The novel method of quantized tensor approximation (QTT) is proven to provide the compressed representation of logarithmic complexity to the grid-based discretizations of functions and operators [3]. This enables reformulation of the standard discretization schemes for steady-state and dynamical problems by using the low-rank quantized tensor approximation and then solving the resultant multi-dimensional systems with the logarithmic complexity in the initial problem size. Several examples of successful applications of the QTT-based tensor numerical methods will be discussed.

We demonstrate how the grid-based QTT tensor approximation applies to the integration of multidimensional or/and highly-oscillating functions, say, many-electron integrals, integrals with the large lattice sum of interaction potentials [4] or with the complicated high-frequency oscillators [5] including some examples from Chebfun demonstrations. Other topic addresses the rank-structured QTT tensor representation of the electron density and the Fock matrix in electronic structure calculations for large 3D $L \times L \times L$ lattice systems [6].

References

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