FORM B

MATH 241, Spring 2009
Exam 1: March 11

NAME _____ **Discussion section time** _____

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

For this exam, you must use a limit formula to compute any derivative.

1. (6 points each) Let $f(x) = e^{2x} - 1$.

   (a) Find a formula for $f^{-1}(x)$. (b) Find the domain of f and the domain of f^{-1}.

(a) $y = e^{2x} - 1$

$e^{2x} = y + 1$

$x = \frac{1}{2}\ln(y+1) = f^{-1}(y)$

$f^{-1}(x) = \frac{1}{2}\ln(x+1)$

(b) e^{2x} defined for all x

domain of $f = (-\infty, \infty)$

$\ln(x+1)$ defined only if $x+1 > 0$

domain of $f^{-1} = (-1, \infty)$

2. (12 points) Solve $\ln(x-1) - \ln(x) = \ln(2)$ for x.

$$\ln\left(\frac{x-1}{x}\right) = \ln(2)$$

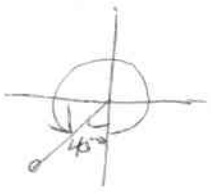
$$\frac{x-1}{x} = 2$$

$$x-1 = 2x$$

$$x = -1$$

3. (12 points) Find the exact value of $\arccos\left[\cos\left(\frac{5\pi}{4}\right)\right]$.

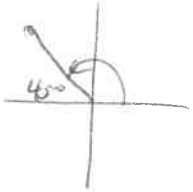
$$\cos\left(\frac{5\pi}{4}\right) = \frac{1}{\sqrt{2}}$$



$$\cos\theta = \frac{1}{\sqrt{2}}$$

and

$$0 \le \theta \le \pi$$



$$\theta = \frac{3\pi}{4}$$

Reminder: **For this exam, you must use a limit formula to compute any derivative.**

4. (4 points each) Let $f(x) = \begin{cases} \sqrt{|1+x|} & \text{if } x < 0, \\ 1 - x & \text{if } x \geq 0. \end{cases}$

Evaluate each limit, or write DNE if it does not exist.

(a) $\lim_{x \to 0^-} f(x)$ (b) $\lim_{x \to 0^+} f(x)$ (c) $\lim_{x \to 0} f(x)$

(a) $\lim_{x \to 0^-} \sqrt{|1+x|} = \lim_{x \to 0^-} \sqrt{1+x} = 1$

(b) $\lim_{x \to 0^+} 1 - x = 1$

(c) 0^- and 0^+ agree, so $\lim_{x \to 0} f(x) = 1$

Reminder: **For this exam, you must use a limit formula to compute any derivative.**

5. (12 points) Find the limit, or write DNE if it does not exist.

$$\lim_{x\to 1^-} \frac{x^2-16}{x^2-5x+4}$$

$$\lim_{x\to 1^-} \frac{(x+4)(x-4)}{(x-4)(x-1)} = \frac{4^-}{0^-} = -\infty$$

Reminder: **For this exam, you must use a limit formula to compute any derivative.**

6. (8 points each) Find each limit, or write DNE if it does not exist.

(a) $\lim_{x\to\infty} \frac{1}{\ln(x)}$ (b) $\lim_{x\to\infty} \frac{4x^2-16}{x^4+1}$

(a) $\ln(x) \to +\infty$ as $x \to +\infty$,

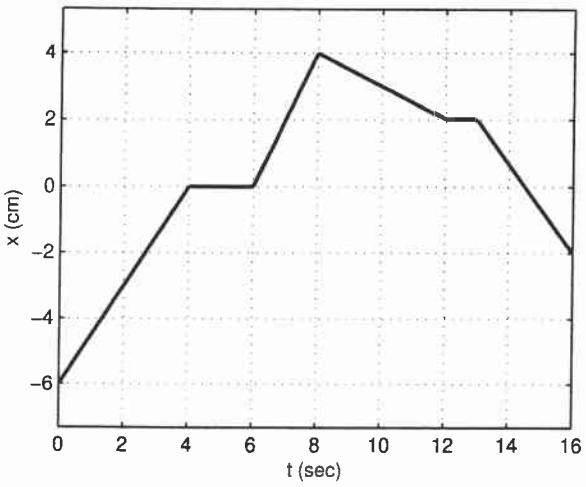
so $\frac{1}{\ln(x)} \to 0$ as $x \to +\infty$

(b) $\lim_{x\to\infty} \frac{4x^2-16}{x^4+1} \cdot \frac{x^{-4}}{x^{-4}}$

$= \lim_{x\to\infty} \frac{4x^{-2}-16x^{-4}}{1+x^{-4}} = \frac{0}{1} = 0$

Reminder: **For this exam, you must use a limit formula to compute any derivative.**

7. (4 points each) A particle moves horizontally in a straight line according to the position function $x(t)$, whose graph is shown here.



(a) What is the average velocity over $0 \leq t \leq 16$?

(b) At what time(s), if any, is the particle moving to the right?

(c) At what time(s), if any, is the instantaneous velocity undefined?

(a) $\frac{\Delta x}{\Delta t} = \frac{-2-(-6)}{16-0} = \frac{1}{4}$

(b) slope > 0 for $(0,4)$, $(6,8)$

(c) corners show no differentiability at $t = 4, 6, 8, 12, 13$

Reminder: **For this exam, you must use a limit formula to compute any derivative.**

8. (12 points) Find the equation of the line tangent to $y = 2x^2$ at the point $(1,2)$.

$f(x) = 2x^2$, slope $= f'(1)$

$$f'(1) = \lim_{h \to 0} \frac{2(1+h)^2 - 2(1)^2}{h}$$

$$= \lim_{h \to 0} \frac{2 + 4h + 2h^2 - 2}{h}$$

$$= \lim_{h \to 0} 4 + 2h = 4$$

line is $(y-2) = 4(x-1)$

Reminder: **For this exam, you must use a limit formula to compute any derivative.**