09-26

Note Title 9/26/2007

Ex $\int \sin^4\theta\, d\theta$ $m=4$, $n=0$ both even

$= \int (\sin^2\theta)^2\, d\theta = \int \left[\frac{1}{2}(1-\cos 2\theta)\right]^2 d\theta = \frac{1}{4}\int (1 - 2\cos 2\theta + \cos^2 2\theta)\, d\theta$

$= \frac{1}{4}\theta - \frac{1}{2}\left(\frac{1}{2}\sin 2\theta\right) + \frac{1}{4}\int \cos^2 2\theta\, d\theta$ → $m=0$, $n=2$ both even

$+ \frac{1}{4}\int \frac{1}{2}(1+\cos 4\theta)\, d\theta$

$+ \frac{1}{8}\theta + \frac{1}{8}\left(\frac{1}{4}\sin 4\theta\right) + C$

$= \frac{3}{8}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta + C$

$\cos^2 2\theta = \frac{1}{2}(1+\cos 4\theta)$.

Sometimes you can do this with negative powers.

Ex $\int \frac{\sin^3 x}{\cos^6 x}\,dx = \int \frac{\sin^2 x}{\cos^6 x}\sin x\,dx = \int \frac{1-\cos^2 x}{\cos^6 x}\sin x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$= \int \frac{1-u^2}{u^6}(-du) = \int \left(\frac{u^2}{u^6} - \frac{1}{u^6}\right) du = \frac{1}{-3}u^{-3} - \frac{1}{-5}u^{-5} + C$

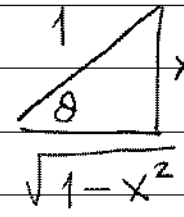
$= \frac{1}{5}\cos^{-5} x - \frac{1}{3}\cos^{-3} x + C$

Ex $\int \frac{1}{(1-x^2)^{3/2}}\,dx$ $\qquad (1-x^2)^{1/2}$: $x = 1\sin\theta$

$dx = \cos\theta\,d\theta$

$$= \int \frac{1}{(1-\sin^2\theta)^{3/2}}(\cos\theta\,d\theta) = \int \frac{1}{\cos^3\theta}\cos\theta\,d\theta = \int \frac{d\theta}{\cos^2\theta}$$

$$= \int \sec^2\theta\,d\theta = \tan\theta + C$$



$x = \sin\theta$

$$= \frac{x}{\sqrt{1-x^2}} + C$$

Ex $\int \frac{dx}{(a^2+x^2)^{1/2}}$: $x = a\tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$

$dx = a\sec^2\theta\, d\theta$

$$= \int \frac{a\sec^2\theta\, d\theta}{a^2(1+\tan^2\theta)} = \int \frac{a\sec^2\theta\, d\theta}{a^2\sec^2\theta} = \int \frac{d\theta}{a} = \frac{\theta}{a} + C$$

$$= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Ex $\displaystyle\int_2^{2\sqrt{3}} \frac{1}{x^2\sqrt{x^2+4}}\,dx$

$\sqrt{x^2+4}$: $x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

If $x=2$, $1=\tan\theta \Rightarrow \theta = \frac{\pi}{4}$

If $x=2\sqrt{3}$, $\sqrt{3}=\tan\theta \Rightarrow \theta = \frac{\pi}{3}$

$$\int_{\pi/4}^{\pi/3} \frac{1}{4\tan^2\theta\sqrt{4+4\tan^2\theta}}\cdot 2\sec^2\theta\,d\theta = \int_{\pi/4}^{\pi/3} \frac{\sec^2\theta}{4\tan^2\theta\sec\theta}\,d\theta$$

$$= \int_{\pi/4}^{\pi/3} \frac{\cos^2\theta}{4\sin^2\theta\cos\theta}\,d\theta = \frac{1}{4}\int_{\pi/4}^{\pi/3} \frac{\cos\theta}{\sin^2\theta}\,d\theta = \frac{1}{4}\left[-\frac{1}{1}(\sin\theta)^{-1}\right]_{\pi/4}^{\pi/3}$$

$$= \frac{1}{4}\left[-\frac{2}{\sqrt{3}} + \sqrt{2}\right]$$