7.1: 10, 12, 22, 27, 38, 40, 43

10. Not one-to-one by either horizontal line test, or by an example of a repeating y value, like $y = 4$ when either $x = 1$ or $x = 3$.

12. Not one-to-one by either the horizontal line test. Consider $x = 2$ and $x = -2$, both points give $y = 4$.

22. a) Satisfies the horizontal line test.
b) Range is (-3,3), Domain is (-2,2)
c) $x = -2$.

27.

$$y^2 = 10 - 3x$$
$$\Rightarrow \frac{10 - y^2}{3} = x$$
$$\Rightarrow \frac{10 - x^2}{3} = f^{-1}(x)$$

38. a)

Assume $x_1 \neq x_2 \Rightarrow x_1 - 1 \neq x_2 - 1$

$$\Rightarrow \frac{1}{x_1 - 1} \neq \frac{1}{x_2 - 1}$$

$\Rightarrow f(x_1) \neq f(x_2)$, so the function is one-to-one

b) First find when $2 = \dfrac{1}{x - 1}$, which is when $x = 3/2$ so $f^{-1}(2) = 3/2$.

Then find f' by using either the chain or product rule. $f'(x) = \dfrac{-1}{(x-1)^2}$ and $f'(3/2) = -4$.

By a theorem in 7.1 then $\left(f^{-1}\right)'(a) = \dfrac{1}{f'(f^{-1}(a))}$. So $\left(f^{-1}\right)'(2) = \dfrac{1}{f'(3/2)} = -\dfrac{1}{4}$.

c) $y = \dfrac{1}{x-1} \Rightarrow (x-1)y = 1 \Rightarrow x = \dfrac{1+y}{y}$ so $f^{-1} = g = \dfrac{1+x}{x}$

d) $g'(x) = -\dfrac{1}{x^2}$ so $g'(2) = -\dfrac{1}{4}$ which matches the result in (b).

40) To use the formula $\left(f^{-1}\right)'(a) = \dfrac{1}{f'(f^{-1}(a))}$ first find when $2 = x^5 - x^3 + 2x$. This gives that $f^{-1}(2) = 1$. And $f'(x) = 5x^4 - 3x^2 + 2$, with $f'(1) = 4$. So $g'(a) = \dfrac{1}{f'(1)} = \dfrac{1}{4}$

43. Since $g = f^{-1}$ then $g'(5) = \dfrac{1}{f'(g(5))} = \dfrac{1}{f'(4)} = \dfrac{1}{2/3} = 3/2$

7.2* 19, 34, 37, 44, 55, 56, 59, 61, 66

19. $g'(x) = \frac{a+x}{a-x}\left(\frac{(a+x)(-1)-(a-x)(1)}{(a+x)^2}\right) = \frac{-2a}{(a-x)(a+x)}$

34. Domain: $x > 0$ and $\ln x \neq -1 \Rightarrow x \neq \frac{1}{e}$ so the domain is $(0,1/e) \cup (1/e,\infty)$.

$f' = -(1+\ln x)^{-2}(1/x) = \frac{-1}{x(1+\ln x)^2}$.

37. $f' = \frac{(\ln x)(1) - x(1/x)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$ and $f'(e) = \frac{1-1}{1^2} = 0$

44. $\frac{1}{xy}(xy' + y) = y\cos x + y'\sin x$

$$\Rightarrow y' + \frac{1}{x}y = y^2\cos x + yy'\sin x$$

$$\Rightarrow y' = \frac{y^2\cos x - y/x}{1 - y\sin x}$$

55. $\ln y = \ln(2x+1)^5 + \ln(x^4-3)^6$

$$\Rightarrow \ln y = 5\ln(2x+1) + 6\ln(x^4-3)$$

$$\Rightarrow \frac{1}{y}y' = \frac{5}{2x+1}(2) + \frac{6}{x^4-3}(4x^3)$$

$$\Rightarrow y' = (2x+1)^5(x^4-3)^6\left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3}\right)$$

56. $\ln y = \ln(x^3+1)^4 + \ln\sin^2 x - \ln x^{1/3}$

$$\Rightarrow \frac{1}{y}y' = \frac{4}{x^3+1}(3x^2) + \frac{2}{\sin x}(\cos x) - \frac{1}{3x}$$

$$\Rightarrow y' = \left(\frac{(x^3+1)^4(\sin^2 x)}{\sqrt[3]{x}}\right)\left(\frac{12x^2}{x^3+1} + 2\cot x - \frac{1}{3x}\right)$$

59. $3\ln 4 - 3\ln 2 = 3\ln 2$

61. Let $u = 8 - 3t$ then the evaluated integral is $\frac{-1}{3}(\ln 2 - \ln 5) = \frac{1}{3}(\ln 5 - \ln 2) = \frac{\ln(5/2)}{3}$

66. Let $u = 2 + \sin x$ then $du = \cos x dx$ so plugging in for dx gives you:

$$\int \frac{\cos x}{u}\frac{1}{\cos x}du = \int \frac{1}{u}du = \ln|u| + C = \ln|2+\sin x| + C$$

7.3* 3,7,10,20,35,41,59,69,73,76

3. a. $\sqrt{2}$ b. $2^3 = 8$

7. $\ln(\ln x) = 1 \Rightarrow e^1 = \ln x \Rightarrow x = e^e$

10. $e^{2-\ln x} = 2x+1$

$\Rightarrow e^2 e^{\ln x^{-1}} = 2x+1$

$\Rightarrow e^2 / x = 2x+1 \Rightarrow 0 = 2x^2 + x - e^2$ using the quadratic formula gives

$$x = \frac{-1 \pm \sqrt{1-4(2)(-e^2)}}{2(2)} = \frac{-1 \pm \sqrt{1+8e^2}}{4}$$

20. The graph was moved up by 1 unit and stretched vertically by a factor of 2.

35. $f'(u) = e^{1/u}(-u^{-2}) = \dfrac{-e^{1/u}}{u^2}$

41. $y' = e^{e^x}e^x = e^{e^x + x}$

59. $f'(x) = 1 - e^x = 0$ when $e^x = 1$, so when $x = 0$ and $f(x) = 0 - 1 = -1$.

69. $u = -3x \Rightarrow dx = -1/3du$ so $-\dfrac{1}{3}\displaystyle\int_5^0 e^u du = -\dfrac{1}{3}(e^{-15} - e^0) = \dfrac{1}{3} - e^{-15}$

73. Rewrite the integral as $\displaystyle\int \frac{e^x}{e^x} + \frac{1}{e^x}dx$. Then the integral becomes:

$$\int 1 + e^{-x}dx = x - e^{-x} + C$$

76. Let $u = e^x$ then $dx = \dfrac{1}{e^x}du$. Plugging this into the integral gives the new integral equation:

$$\int e^x \sin(u)\frac{1}{e^x}du = \int (\sin u)du = -\cos u + C = -\cos(e^x) + C$$