10-10

Note Title 10/10/2007

**Ex** Find the line tangent to $x = e^t$, $y = (t-1)^2$ at point $(1,1)$.

1. Find $dy/dx$ by formula. $\quad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2(t-1)}{e^t}$

2. Find value of $t$ that gives correct $(x,y)$ at point. $\quad 1 = e^t$ and $1 = (t-1)^2$

$$t = 0$$

3. Plug in to get the slope, then write down the line. $\quad m = \dfrac{2(0-1)}{e^0} = -2$

(P.S. form of line)

$$(y - y_0) = m(x - x_0)$$

$$\boxed{y - 1 = -2(x-1)}$$

Ex Find all points on $x = t^3 - 3t^2$, $y = t^3 - 3t$ where the tangent is vertical or horizontal.

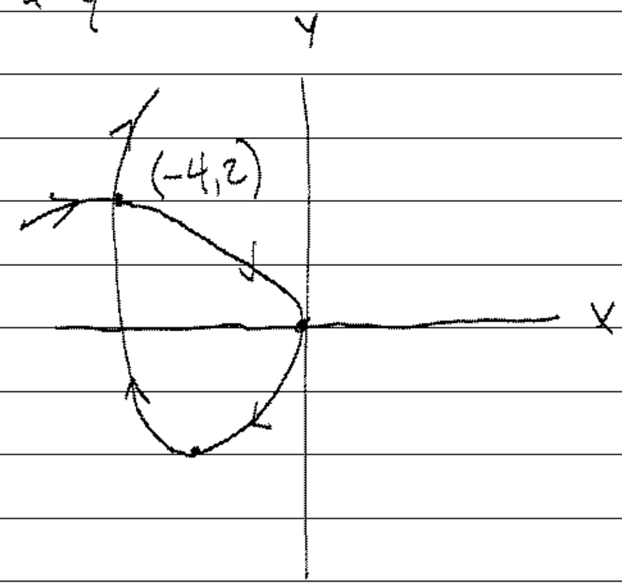
Vertical : $\frac{dx}{dt} = 0 \implies 3t^2 - 6t = 0 \Rightarrow t = 0$ or $t = 2$

Horizontal : $\frac{dy}{dt} = 0 \implies 3t^2 - 3 = 0 \Rightarrow t^2 - 1 = 0 \Rightarrow t = 1$ or $t = -1$

To find points, plug in $t$ to $x$ and $y$ :

| | $t$ | $x$ | $y$ |
|---|---|---|---|
| H | -1 | -4 | 2 |
| V | 0 | 0 | 0 |
| H | 1 | -2 | -2 |
| V | 2 | -4 | 2 |

same point!



<u>Ex</u> $x = \sin t$
$y = \sin t\,(1-\cos t)$

Show that the curve crosses itself at $(0,0)$, and find equations for all tangent lines there.

$$\left.\begin{array}{l} x=0 \\ \text{and} \\ y=0 \end{array}\right\} \quad \left.\begin{array}{ll} \sin t = 0 & t = 0, \pm\pi, \pm 2\pi, \cdots \quad t = n\pi \\ \sin t\,(1-\cos t) = 0 & t = 0, \pm\pi, \pm 2\pi, \ldots \\ & \text{OR} \\ & t = 0, \pm 2\pi, \pm 4\pi, \ldots \end{array}\right\} \begin{array}{l} t = n\pi \\ \text{gives all} \\ \text{simultaneous} \\ \text{solutions} \end{array}$$

We need all possible slopes at these values of $t$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t\,(1-\cos t) + \sin t\,(\sin t)}{\cos t} = \frac{\cos t\,(1-\cos t) + (1-\cos^2 t)}{\cos t}$$

$t=0$: $m = 0$

$t=\pi$: $m = \frac{(-1)(2)}{-1} = 2$

$t=2\pi$: $m = 0$

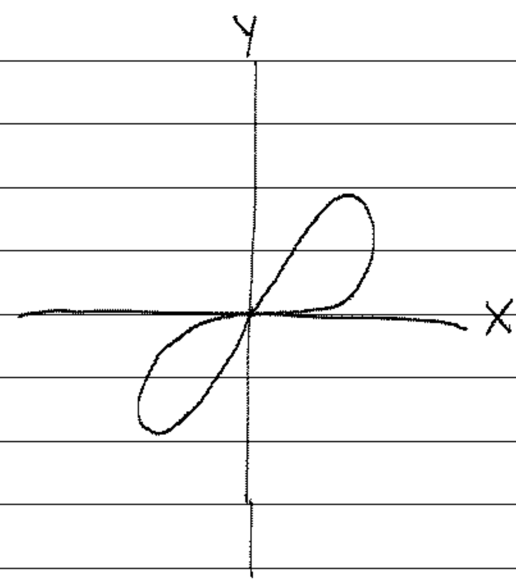
$t=n\pi$: $m = 1-(-1)^n$

Curve at $(0,0)$ at $t=n\pi$.

If $n$ is even, slope $=0$. If $n$ is odd, slope $=2$.

$y-0=0(x-0)$ $\quad\quad$ $y-0=2(x-0)$

$y=0$ $\quad\quad$ $y=2x$



<u>Ex</u> Find all lines tangent to $x = 3t^2+1, \ y = 2t^3+1$ that pass through $(4,3)$.

not a point on the curve, necessarily.

Slope: $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{6t^2}{6t} = t$

All such lines have the form $(y-3) = m(x-4)$

Plug in curve info: $(2t^3+1-3) = t\,(3t^2+1-4)$

$$2t^3-2 = 3t^3-3t$$

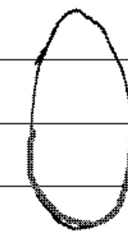
$$t^3-3t+2 = 0$$

$t=1$
$t=-2$
$t=?$

exercise left to the reader

<u>Ex</u> $x = 2\sin t$ $\quad 0 \le t < 2\pi$ $\quad$ Find all $t$ for which the curve is concave up.

$y = 4\cos t$



$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-4\sin t}{2\cos t} = -2\tan t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(-2\tan t\right) = \frac{\frac{d}{dt}\left(-2\tan t\right)}{dx/dt} = \frac{-2\sec^2 t}{2\cos t}$$

$$= -\sec^3 t$$

$$= -\frac{1}{\cos^3 t}$$

$$\frac{d^2y}{dx^2} > 0$$

$$\Updownarrow$$

$$\frac{1}{\cos^3 t} < 0$$

$$\Updownarrow$$

$$\cos t < 0 \iff \frac{\pi}{2} < t < \frac{3\pi}{2}$$