CONVERGENCE OF THE LEAST SQUARES SHADOWING METHOD FOR COMPUTING THE DERIVATIVE OF ERGODIC AVERAGES

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My talk would actually be the continuation of a first talk given by Pr.Qiqi Wang in which he exposes the motivation behind the *Least Squares Shadowing* method (LSS). Mine would present the theoretical foundations of LSS. Just as a reminder of the subject:

Many applications involve simulation of nonlinear dynamical systems that exhibit chaos. Examples include weather and climate, turbulent combustion and nuclear reactor physics. The quantities that are to be predicted (the so-called quantities of interest) are often time averages $\langle J \rangle$:

$$\langle J \rangle(s) = \frac{1}{T} \int_0^\infty J(u(t), s) dt$$

where $u(t)$ satisfies a parametrized hyperbolic dynamical system:

$$\begin{cases} \frac{du}{dt} = f(u, s) \\ u(0) = u_0 \qquad u_0 \in \mathbb{R}^m \end{cases}$$

The computation of the derivative of the time average with respect to the parameter s represents a class of important problems in computational science and engineering (numerical optimization, uncertainty quantification). Traditional transient sensitivity analysis methods fail to compute $\frac{d\langle J \rangle}{ds}$ in chaotic systems.
Least Squares Shadowing is a method that efficiently computes $\frac{d\langle J \rangle}{ds}$ by solving a constrained least squares problem. I will expose the underlying theory behind the convergence of this method for a particular case : the one where the dynamical system is ergodic and uniformly hyperbolic.

Aware of the limited number of contributed talks, I would like mine, if selected, to follow the one given by Pr.Wang since it is a natural continuation of his.