09-19

Note Title 9/19/2007

__Ex__ $$\int \tanh x \, dx = \int \frac{\sinh x}{\cosh x} dx = \int \frac{du}{u} = \ln|u| + C$$

$$\begin{bmatrix} u = \cosh x \\ du = \sinh x \, dx \end{bmatrix} \qquad = \ln(\cosh x) + C$$

__Ex__ $$\lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$$ indeterminate

$$\overset{L'H}{=} \lim_{x \to 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$

__Ex__ $\lim_{x \to 0} \frac{\sin x}{\sqrt{x}} = \frac{0}{0}$ indeterminate

$\overset{L'H}{=} \lim_{x \to 0} \frac{\cos x}{\frac{1}{2} x^{-1/2}} = \lim_{x \to 0} 2\sqrt{x} \cos x = 0$

__Ex__ $\lim_{x \to 0} \frac{\cos(x) - 1}{x^2} = \frac{1-1}{0^2} = \frac{0}{0}$ indeter.

$\overset{L'H}{=} \lim_{x \to 0} \frac{-\sin x}{2x} = \frac{0}{0}$ indeter.

$\overset{L'H}{=} \lim_{x \to 0} \frac{-\cos x}{2} = -\frac{1}{2}$

<u>Ex</u> $\lim\limits_{x \to \infty} \dfrac{e^x}{x^n}$ n is any integer ≥ 1

$$= \frac{\infty}{\infty} \quad \text{indeter.}$$

$$\overset{L'H}{=} \lim_{x \to \infty} \frac{e^x}{n x^{n-1}} = \begin{cases} \dfrac{\infty}{1} & \text{if } n=1 \longrightarrow \infty \\ \dfrac{\infty}{\infty} & \text{if } n>1 \longrightarrow \text{indeter.} \end{cases}$$

$$\overset{L'H}{=} \lim_{x \to \infty} \frac{e^x}{n(n-1) x^{n-2}} = \begin{cases} \dfrac{\infty}{2} & \text{if } n=2 \longrightarrow \infty \\ \dfrac{\infty}{\infty} & \text{if } n>2 \end{cases}$$

... (n times L'Hopital) limit $= \infty$ for any $n \geq 1$

Ex $\lim_{x\to\infty} \frac{3x^3+x}{x^3-x}$ can use highest powers, or:

$$= \frac{\infty}{\infty}$$

$$\stackrel{LH}{=} \lim_{x\to\infty} \frac{9x^2+1}{3x^2-1} = \frac{\infty}{\infty}$$

$$\stackrel{LH}{=} \lim_{x\to\infty} \frac{18\cancel{x}}{6\cancel{x}} = 3$$

<u>Ex</u> $\lim\limits_{x\to\infty} x^2 e^{-x} = \infty \cdot 0$ indeterminate

$$= \lim_{x\to\infty} \frac{x^2}{e^x} = \frac{\infty}{\infty} \qquad \left(\begin{array}{l} e^{-x} \to 0 \\ \dfrac{1}{e^x} \to \dfrac{1}{0} = \infty \end{array} \right)$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} = \frac{\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$$

Ex $\lim_{x \to 0^+} x \ln x = 0 \cdot (-\infty)$ indeterminate

$$= \lim_{x \to 0^+} \frac{\ln x}{x^{-1}} = \frac{-\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x \to 0^+} \frac{1/x}{-1/x^2} = \lim_{x \to 0^+} -\frac{x^2}{x} = \lim_{x \to 0^+} -x = 0$$

(Same result for $x^\alpha \ln(x)$, any $\alpha > 0$.)