

BIMODAL POLYNOMIALS

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Abstract

Bimodal polynomials arise in spectral methods for the Fokker–Planck equation involving bistable potentials. They are orthogonal with respect to a symmetric weight function $w(\cdot; \varepsilon)$ on \mathbb{R} depending on a parameter $\varepsilon > 0$. When ε is small (which is a case of interest), w has sharp peaks at ± 1 of height 1. Two methods are developed to compute these polynomials, i.e., their three-term recurrence relation. Both methods use the discretized Stieltjes procedure, the discretization being accomplished by one-component Gauss–Freud quadrature in the first method, and by multi-component Fejér quadrature in the second.