MATH341-011, Spring 2007
Final Exam

Please clearly erase or cross out irrelevant work; otherwise it will be part of the graded material. **You must justify answers to receive full credit.** You may not use calculators or notes. There are 200 points total.

1. (20 points) After its motor is shut off, a boat moves through the water and encounters a resistance proportional to the square of its speed. Show that as $t \to \infty$, its speed approaches zero but its travel distance approaches infinity.

2. (25 points) Solve the initial-value problem $xy' - y = 3x^2$, $y(1) = 1$.

3. (25 points) An RLC circuit having $R = 100\,\Omega$, $L = 5$ H, $C = 0.001$ F, is connected to a generator with EMF $E(t) = 100\sin(10t)$ volts. Find the steady periodic current $I_{\text{sp}}(t)$, and find its amplitude.

4. (a) (15 points) Find the general solution of the equations $x_1' = x_1 - 2x_2$, $x_2' = 2x_1 + x_2$.
   (b) (15 points) Find a particular solution of $x_1' = x_1 - 2x_2 + 4e^t$, $x_2' = 2x_1 + x_2 - 2e^t$.

5. Let $A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
   (a) (10 points) Show that $A^n = 0$ for all $n \geq 4$.
   (b) (15 points) Find e^A.

6. (25 points) Find the dimension of the subspace spanned by the vectors $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \end{bmatrix}$.

7. (30 points) Factor the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$ as XDX^{-1}, for a diagonal D. (Find *all three* matrices in the product.)

8. (20 points) Let
$$A = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ c & 11 & 1 \end{bmatrix} \begin{bmatrix} 2 & 7 & c \\ 0 & -1 & -9 \\ 0 & 0 & 2 \end{bmatrix}.$$
   Find all values for c, if any, that make A singular. (Hint: Use determinants.)