Summary/review for exam 2 (Chapters 3,4)

In this unit of the course, there was a greater tendency to filter out some parts of the textbook in favor of emphasizing others. You should assume that this reflects my priorities for the exam.

Definitions Vector spaces and especially subspaces, null space, linear combinations, span, independence, basis, standard basis, dimension, coordinates, transition matrix, row space, column space, rank, inner product, magnitude/length, distance, angle, orthogonality, orthonormality, complement, projection

Properties Vector properties (Theorem 3.1), geometric description of subspaces of R^2 and R^3, Theorem 3.3, Theorems 3.5–3.8, Corollary 3.1, Corollaries 3.4–3.5, Theorem 3.11, Theorem 3.13, Theorems 3.17–3.19, Corollary 3.7, Fundamental Theorem (box on page 211), Theorem 4.1, Theorem 4.3, Corollary 4.1, Theorems 4.4–4.5, Theorems 4.9–4.11, Theorem 4.13

Skills Determine closure of a subspace; determine span and independence of a given set; find an independent subset of a given set; find a basis for a given space; find the dimension of a space; find and transform coordinates, find a transition matrix; find null, column and row spaces; find rank and nullity; compute magnitudes, distances, and angles; determine orthogonality or orthonormality; orthonormalize a set (Gram–Schmidt); find an orthogonal complement; find a projection; find the decomposition of Theorem 4.10 for a given vector

As a rule, exams are biased towards skills, but you can expect some questions to be based on more abstract statements about properties.

There will be no books, notes, or devices allowed for use during the exam. The following formulas will be given to you:

Angle: $\cos\theta = \dfrac{(\mathbf{u}, \mathbf{v})}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$

Gram–Schmidt: To orthogonalize $\mathbf{u}_1, \ldots, \mathbf{u}_m$, let
$\mathbf{v}_k = \mathbf{u}_k - \dfrac{(\mathbf{u}_k, \mathbf{v}_1)}{(\mathbf{v}_1, \mathbf{v}_1)}\mathbf{v}_1 - \cdots - \dfrac{(\mathbf{u}_k, \mathbf{v}_{k-1})}{(\mathbf{v}_{k-1}, \mathbf{v}_{k-1})}\mathbf{v}_{k-1}.$

Projection: If $\mathbf{w}_1, \ldots, \mathbf{w}_n$ are an orthonormal basis of W, then
$\operatorname{proj}_W \mathbf{v} = (\mathbf{v}, \mathbf{w}_1)\mathbf{w}_1 + \cdots + (\mathbf{v}, \mathbf{w}_n)\mathbf{w}_n.$