MATH 508 Challenge problem #8
Turn in no later than Apr. 26 for credit

Define

$$f(z) = \sum_{n=0}^{\infty} z^{2^n}$$

which converges for $|z| < 1$. First, show that $f(1) = \infty$ (so that f is singular there). Then show that $f(z^2) = f(z) - z$ and explain why this implies that f is singular at -1 as well. Continue this line of argument inductively to show that every arc of the unit circle contains points at which f is singular. (The unit circle is called a **natural barrier** for f.)