Note Title: 10-01

10/1/2007

Ex $\int \frac{2x+5}{x^2-x-2}\,dx$

1. Factor denominator.

$$(x-2)(x+1) = x^2-x-2$$

2. Set up the PFD.

$$\frac{2x+5}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

3. Clear the denominator.

$$2x+5 = \frac{A}{\cancel{x-2}}\cancel{(x-2)}(x+1) + \frac{B}{\cancel{x+1}}(x-2)\cancel{(x+1)}$$

$$2x+5 = A(x+1) + B(x-2)$$

4. Set x equal to each root of denominator.

$x=2$: $2(2)+5 = A(2+1) + B(0) \Rightarrow 9 = 3A$, or $A=3$

$x=-1$: $2(-1)+5 = A(0) + B(-1-2) \Rightarrow 3 = -3B$, or $B=-1$

Now $$\frac{2x+5}{x^2-x-2} = \frac{3}{x-2} + \frac{-1}{x+1}$$

5. Integrate:

$$\int \frac{2x+5}{x^2-x-2}\,dx = 3\ln|x-2| - \ln|x+1| + C$$

$$= \ln\left|\frac{(x-2)^3}{x+1}\right| + C$$

Ex $\int \frac{6}{x^3-x}\,dx$

1. Factor denominator.

$$x^3-x = x(x^2-1) = x(x+1)(x-1)$$

2. $\frac{6}{x^3-x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$

3. $6 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$

4. $x=0: \quad 6 = A(1)(-1) + B\cdot 0 + C\cdot 0 \Rightarrow A=-6$

$x=-1: \quad 6 = A\cdot 0 + B(-1)(-2) + C\cdot 0 \Rightarrow 6=2B, \quad B=3$

$x=1: \quad 6 = A\cdot 0 + B\cdot 0 + C(1)(2) \Rightarrow C=3$

5. $\int \frac{6}{x^3-x}\,dx = \int\left(\frac{-6}{x} + \frac{3}{x+1} + \frac{3}{x-1}\right)dx = -6\ln|x| + 3\ln|x^2-1| + C$

<u>Ex</u> $\int e^{\sqrt{x+1}}\,dx$

$$t = \sqrt{x+1}$$

$$dt = \frac{1}{2}(x+1)^{-1/2}\,dx$$

$$dx = 2\sqrt{x+1}\,dt = 2t\,dt$$

$$= \int e^{u}(2u\,du) = 2\int e^{t}\,t\,dt$$

$$u = t \qquad dv = e^{t}\,dt$$

$$du = dt \qquad v = e^{t}$$

$$= 2\left[te^{t} - \int e^{t}\,dt\right] = 2\left[te^{t} - e^{t} + C\right]$$

$$= 2e^{\sqrt{x+1}}\left(\sqrt{x+1} - 1\right) + C$$

Ex

$$\int \frac{e^{-2s}}{1+e^{-4s}}\,ds \qquad u = e^{-2s}, \text{ then } u^2 = e^{-4s}$$
$$du = -2e^{-2s}\,ds$$

$$= \int \frac{-\frac{1}{2}du}{1+u^2} = -\frac{1}{2}\int \frac{du}{1+u^2} = -\frac{1}{2}\tan^{-1}\left(e^{-2s}\right) + C$$

Ex

$$\int e^{x+e^x}\,dx = \int e^x e^{e^x}\,dx \qquad u = e^x, \quad du = e^x\,dx$$

$$= \int e^u\,du = e^{e^x} + C$$

Ex

$$\int \frac{1+e^x}{1-e^x}\,dx = \int \frac{1-e^x+e^x}{1-e^x}\,dx + \int \frac{e^x}{1-e^x}\,dx$$

$$= \int 1\,dx + \int \frac{e^x\,dx}{1-e^x} + \int \frac{e^x}{1-e^x}\,dx = x + \int \frac{2e^x}{1-e^x}\,dx$$