The Leja method for the matrix exponential: backward error analysis and implementation

Abstract

The Leja method is a well established scheme for computing the action of the matrix exponential, see [2]. We present a new backward error analysis that allows for a more efficient method. From a scalar computation in high precision we predict the necessary number of scaling steps based only on a rough estimate of the field of values or norm of the matrix and the desired backward error. We use the convergence behaviour of the scalar case on ellipses in the complex plane to get a bound for the matrix argument.

In numerical experiments we show the behaviour of the estimate as well as a comparison with the truncated Taylor series method presented in [1]. The experiments focus on the necessary matrix vector products for various classes of matrices. We can show that for a wide class of matrices the Leja method saves matrix vector products in comparison to the Taylor method. The numerical experiments include spatial discretization of time dependent partial differential equations and various prototypical test cases.

References

[1] Al-Mohy, A.H., Higham, N.J., 2011. Computing the action of the matrix exponential, with an application to exponential integrators. SIAM J. Sci. Comput. 33 (2), 488-511.

[2] Caliari, M., Kandolf, P., Ostermann, A., Rainer, S., 2014. Comparison of software for computing the action of the matrix exponential. BIT Numerical Mathematics, 54 (1), 113-128.