

MATH 241, Fall 2008

Exam 2: November 3

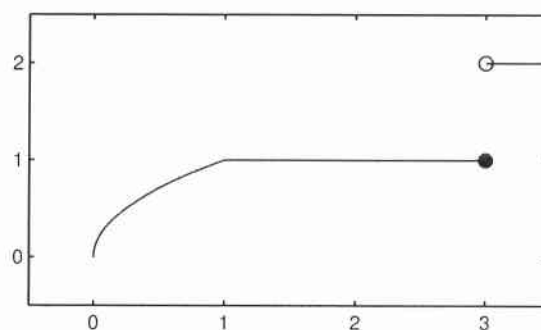
NAME \_\_\_\_\_

Discussion section \_\_\_\_\_

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (12 points) At which value(s) of  $x$  is this function not differentiable? Give short reasons for each value you state.



$x = 0$  — vertical tangent

$x = 1$  — corner

$x = 3$  — discontinuous

2. (12 points) Find  $y'$  if  $y = \frac{2^x}{1-x^2}$ .

$$y' = \frac{(1-x^2)(\ln 2)2^x - 2^x(-2x)}{(1-x^2)^2} = 2^x \frac{\ln(2)(1-x^2) + 2x}{(1-x^2)^2}$$

3. (12 points) Find  $y''$  if  $y = x^5 + \frac{2}{e^{2x}}$ .

$$y = x^5 + 2e^{-2x}$$

$$y' = 5x^4 - 4e^{-2x}$$

$$y'' = 20x^3 + 8e^{-2x}$$

4. (12 points) Find the line tangent to the hyperbola  $x^2 + 2xy - y^2 + x = 2$  at the point  $(1, 2)$ .

$$2x + 2y + 2xy' - 2yy' + 1 = 0$$

$$y' = \frac{-2x - 2y - 1}{2x - 2y} \quad @ \quad x=1, y=2 : y' = \frac{-2-4-1}{2-4} = \frac{7}{2}$$

$$y - 2 = \frac{7}{2}(x - 1)$$

5. (12 points) Find  $\frac{d}{dx}[(\cos x)^x]$ .

$$y = (\cos x)^x \Rightarrow \ln y = x \ln(\cos x)$$

$$\Rightarrow \frac{y'}{y} = \ln(\cos x) + x \cdot \frac{-\sin x}{\cos x}$$

$$\Rightarrow y' = (\cos x)^x [\ln(\cos x) - x \tan x]$$

6. (12 points) At what point does the curve  $y = [\ln(x+3)]^2$  have a horizontal tangent?

$$y' = 2 [\ln(x+3)] \frac{1}{x+3}$$

$$\text{If } y' = 0, \text{ then } \ln(x+3) = 0$$

$$x+3 = e^0 = 1$$

$$x = -2 \Rightarrow y = [\ln(1)]^2 = 0$$

$$@ (-2, 0)$$

7. (14 points) A cup of tea sits in a room kept at  $20^{\circ}\text{C}$  and cools from  $80^{\circ}\text{C}$  to  $60^{\circ}\text{C}$  in half an hour. What will be the temperature of the tea after another half hour? Simplify your answer for full credit.

$$\left. \begin{array}{l} y(0) = 80 - 20 = 60 \\ y(30) = 60 - 20 = 40 \end{array} \right\} \text{ given} \quad \text{Find } y(60) + 20.$$


$$y(t) = y(0) e^{kt} = 60 e^{kt}$$

$$40 = 60 e^{30k} \Rightarrow k = \frac{1}{30} \ln\left(\frac{2}{3}\right) \quad (\text{per min})$$

$$y(60) = 60 e^{2 \ln(2/3)} = 60 e^{\ln(4/9)} = 240/9$$

$$T(60) = 20 + \frac{240}{9}$$

8. (14 points) Gravel is dumped onto a conical pile at a rate of  $36 \text{ ft}^3$  per second. The gravel always shifts so that the base diameter of the pile equals its height. At what rate is the height of the pile increasing when it is 6 ft high? (The volume of a cone with base radius  $r$  and height  $h$  is  $\frac{1}{3}\pi r^2 h$ .)

  $h=2r$  given

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{36}{\pi/4 \cdot 6^2} = \frac{4}{\pi} \text{ ft/sec}$$