MATH341, Spring 2007
Exam 3: May 4

Please clearly erase or cross out irrelevant work; otherwise it will be part of the graded material. **You must justify answers to receive full credit.** You may not use calculators or notes.

1. (15 points) Given $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 3 & 4 & 1 \end{pmatrix}$, find three elementary matrices such that
$$E_3 E_2 E_1 A = U$$
for an upper triangular U.

2. (15 points) Consider a homogeneous linear system $A\mathbf{x} = \mathbf{0}$ with coefficient matrix
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & 3 \\ -1 & 1 & c \end{pmatrix}.$$
   (a) Find values of c, if any, for which the system is inconsistent.
   (b) Find values of c, if any, for which the system has infintely many solutions.

3. (10 points) Let A and B be 2×2 matrices with $\det A = 2$ and $\det B = -1$. Find
   (a) $\det(-3A)$
   (b) $\det(A^{-1}B)$

4. (15 points) Determine whether the following sets form subspaces of P_3.
   (a) $\{ax^2 + bx + c \mid a = 0\}$
   (b) $\{ax^2 + bx + c \mid a = 0 \text{ and } c = 0\}$
   (c) $\{ax^2 + bx + c \mid a = 0 \text{ or } c = 0\}$

5. (25 points) Let $\mathbf{a}_1$, $\mathbf{a}_2$, ... $\mathbf{a}_5$ be, respectively,
$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}.$$
   Also let A be the 3×5 matrix whose columns are $\mathbf{a}_1$, $\mathbf{a}_2$, ... $\mathbf{a}_5$.
   (a) Find a basis for and the dimension of the nullspace $N(A)$.
   (b) Find a basis for the column space of A.
   (c) Find a basis for the row space of A.
   (d) Find the rank of A. Is $\{\mathbf{a}_1, ..., \mathbf{a}_5\}$ a spanning set for R^3? Is $\{\mathbf{a}_1, ..., \mathbf{a}_5\}$ linearly independent?

6. (20 points) Given
$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ \mathbf{u}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$
   (a) Find the transition matrix from the standard basis $[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]$ to the basis $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$.
   (b) Given $\mathbf{x} = \begin{pmatrix} 10 \\ 0 \\ 10 \end{pmatrix}$, find the coordinates of $\mathbf{x}$ with respect to $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$.