

NAME \_\_\_\_\_

**MATH 428/CISC 411, Spring 2007**  
**Exam 1**

Write all solutions on these sheets. Please clearly erase or cross out irrelevant work; otherwise it will be part of the graded material. **You must justify answers to receive full credit.** You may not use calculators or the computer.

1. Suppose that

$$c_1 + c_2 \cos(\pi x) + c_3 \sin(\pi x)$$

is used to interpolate the function  $f(x) = 1 - 4x$  at the nodes  $x_0 = 0$ ,  $x_1 = 1/2$ , and  $x_2 = 1$ . Find  $c_1$ ,  $c_2$ , and  $c_3$ .

2. Suppose in the barycentric interpolation formulas for  $n = 2$  you are given that  $x_0 = 0$ ,  $x_2 = 1$ ,  $w_0 = 4$ ,  $w_2 = 4/3$ . Find  $x_1$  and  $w_1$ .
3. Find the constant  $C$  and order  $p$  in the error term for this finite difference formula:

$$f'(x) = \frac{-2f(0) - 3f(h) + 6f(2h) - f(3h)}{6h} + C f^{(p+1)}(\xi) h^p.$$

4. Derive the 2-point “half-open” Newton–Cotes formula that results from interpolation of the values  $f(0)$  and  $f(h)$ , followed by integration from  $-h/2$  to  $h$ .
5. Verify that  $P_3(x) = x^3 - \frac{3}{5}x$  is the monic degree-3 Legendre polynomial. (Hint: What condition has to be checked?)