09-24

Note Title 9/24/2007

Ex $\int \ln x \, dx$ ILATE $\longrightarrow$ Try $u = \ln x$ $du = \frac{1}{x}\,dx$

$dv = dx$ $v = x$

$$\int u\,dv = uv - \int v\,du = x \ln x - \int x \frac{1}{x}\,dx = x\ln x - x + C$$

Ex $\int x^2 e^{2x}\,dx$ $\qquad \cancel{I}\cancel{L}ATE$ $\qquad u = x^2 \quad du = 2x\,dx$

$\qquad dv = e^{2x}\,dx \quad v = \frac{1}{2}e^{2x}$

$$= (x^2)\left(\tfrac{1}{2}e^{2x}\right) - \int \left(\tfrac{1}{2}e^{2x}\right)(2x\,dx)$$

$$\int x^2 e^{2x}\,dx = \tfrac{1}{2}x^2 e^{2x} - \underline{\int x e^{2x}\,dx}$$

Use IBP again : ILATE $\qquad u = x \quad du = dx$

$\qquad dv = e^{2x}\,dx \quad v = \frac{1}{2}e^{2x}$

$$\int x e^{2x}\,dx = \tfrac{1}{2}x e^{2x} - \int \tfrac{1}{2}e^{2x}\,dx = \tfrac{1}{2}x e^{2x} - \tfrac{1}{2}\tfrac{1}{2}e^{2x} + C$$

So the original integral is

$$\int x^2 e^{2x}\,dx = \tfrac{1}{2}x^2 e^{2x} - \left[\tfrac{1}{2}x e^{2x} - \tfrac{1}{4}e^{2x} + C\right] = \tfrac{1}{2}x^2 e^{2x} - \tfrac{1}{2}x e^{2x} + \tfrac{1}{4}e^{2x} + C$$

Ex $\int_0^{1/2} \cos^{-1} x \, dx$

ILATE

$u = \cos^{-1} x \qquad du = \frac{-1}{\sqrt{1-x^2}} \, dx$

$dv = dx \qquad v = x$

$$= \left[x \cos^{-1} x \right]_0^{1/2} + \int_0^{1/2} \frac{x}{\sqrt{1-x^2}} \, dx$$

Let $u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2} du = x\,dx$

$$= \boxed{\frac{1}{2} \cos^{-1}\left(\frac{1}{2}\right) - 0} + \int_1^{3/4} \frac{\left(-\frac{1}{2} du\right)}{u^{1/2}}$$

$$= \frac{1}{2} \cdot \frac{\pi}{3} - \frac{1}{2} \int_1^{3/4} u^{-1/2} du = \frac{\pi}{6} - \left[u^{1/2} \right]_1^{3/4} = \frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1$$

Ex (Sneaky!) $\int e^x \cos x \, dx$

ILATE $\quad u = \cos x \quad du = -\sin x \, dx$

$dv = e^x dx \quad v = e^x$

$$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$$

ILATE $\quad u = \sin x \quad du = \cos x \, dx$

$dv = e^x dx \quad v = e^x$

$$= e^x \cos x + \left[e^x \sin x - \int e^x \cos x \, dx \right]$$

We have $\underline{I} = e^x \cos x + e^x \sin x - \underline{I} \implies \underline{I} = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C$

Ex $\int [\ln x]^n \, dx$, n integer $n \geq 1$

ILATE. $u = (\ln x)^n$ $du = n(\ln x)^{n-1}\left(\frac{1}{x}\right)dx$

$dv = dx$ $v = x$

$$\int (\ln x)^n \, dx = x(\ln x)^n - \int x \, n(\ln x)^{n-1}\left(\frac{1}{x}\right)dx$$

$$\int (\ln x)^n \, dx = x(\ln x)^n - n\int (\ln x)^{n-1} \, dx$$ <u>reduction formula</u>

$$I_n = x(\ln x)^n - n \, I_{n-1}$$

Ex $\int \sin^3 x\, dx = \int \sin^2 x \sin x\, dx$

$$= \int (1-\cos^2 x) \sin x\, dx$$

$$= \int \sin x\, dx - \int \cos^2 x \sin x\, dx$$

$$\left[\begin{array}{l} u = \cos x \\ du = -\sin x\, dx \end{array}\right]$$

$$= -\cos x - \int u^2(-du) = -\cos x + \frac{\cos^3 x}{3} + C$$

Ex $\int \sin^5 x \, dx = \int \sin^4 x \sin x \, dx$

$$= \int (\sin^2 x)^2 \sin x \, dx$$

$$= \int (1-\cos^2 x)^2 \sin x \, dx$$

$$= \int (1 - 2\cos^2 x + \cos^4 x) \sin x \, dx$$

$$\begin{bmatrix} u = \cos x \\ du = -\sin x \, dx \end{bmatrix}$$

$$= \int \sin x \, dx - 2\int \cos^2 x \sin x \, dx + \int \cos^4 x \sin x \, dx$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$$

Ex $\int_0^{\pi/4} \cos^2 x \, dx = \int_0^{\pi/4} \frac{1}{2}(1+\cos 2x)\, dx$

$$= \frac{1}{2}\int_0^{\pi/4} dx + \frac{1}{2}\int_0^{\pi/4} \cos 2x \, dx$$

$$= \frac{1}{2}\left[\boxed{x}\right]_0^{\pi/4} + \frac{1}{4}\left[\sin 2x\right]_0^{\pi/4}$$

$$= \frac{1}{2}\cdot\frac{\pi}{4} + \frac{1}{4}\left[1-0\right] = \frac{\pi}{8} + \frac{1}{4}$$