Equivalence of Lanczos Tridiagonalization, Golub-Kahan Bidiagonalization, and some solution of equations algorithms, for Skew Symmetric matrices.

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We prove the numerical equivalence of Lanczos tridiagonalization and Golub-Kahan bidiagonalization (GKB) for any skew symmetric matrix A. We give a short derivation of a Lanczos-Galerkin algorithm for solving $Ax = b$ with skew symmetric A that appeared in [C. Greif and J. M. Varah, SIAM J. Matrix Anal. Appl., 31 (2009) 584–601], and use the above equivalence to show that this is numerically equivalent to the GKB variant of CGNE, i.e., of Craig's method. We point out that this approach can be used to show for skew symmetric A that a Lanczos process based minimum residual solution is numerically equivalent to LSQR, the GKB variant of CGNR. These solution of equations *numerical* equivalences add to the *theoretical* equivalences of solutions proven in [S. C. Eisenstat, "Equivalence of Krylov Subspace Methods for skew-symmetric linear systems", Unpublished Manuscript, Yale University, 2014]. Finally, we point out that this approach can also be used to show that a Lanczos process based method for minimizing $\|A^T(b-Ax_k)\|_2$ each step is numerically equivalent to LSMR (another GKB based algorithm) from [D. Fong and M. A. Saunders, SIAM J. Scientific Computing 33 (2011) 2950-2971] when A is skew symmetric. Time permitting, we will discuss preconditioning for such problems.