Zeros of rational harmonic functions and their applications

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This talk will survey recent results on the zeros of rational harmonic functions $f(z) = r(z) - \overline{z}$ [1], which have fascinating applications ranging from numerical linear algebra [2] to astrophysics [3,4]. A particular focus will be on "extremal" functions, where $r(z)$ is of degree $n \geq 2$ and $f(z)$ has the maximal possible number of $5n - 5$ zeros. Examples of such functions will be visualized using phase portraits, and the implication of our theoretical results in the theory of graviational lensing will be discussed.

References

[1] O. Sète, R. Luce and J. Liesen, *Perturbing rational harmonic functions by poles*, Comput. Methods Funct. Theory, 2014.

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[3] R. Luce, O. Sète and J. Liesen, *Sharp parameter bounds for certain maximal point lenses*, Gen. Relativity Gravitation, 46:1736, 2014.

[4] O. Sète, R. Luce and J. Liesen, *Creating images by adding masses to gravitational point lenses*, arXiv:1405.2785, submitted, 2014.