M242 HW 13

10.1:

1. First find y'. $y' = 1 + x^{-2}$.
   Now, $xy' + y = x(1+x^{-2}) + x - x^{-1} = 2x$ so this is a solution.

5. a) $y' = e^t$, $y'' = e^t$ so $y'' + 2y' + y = e^t + 2e^t + e^t = 4e^t \neq 0$. Not a solution.
   b) $y' = -e^{-t}$, $y'' = e^{-t}$ so $y'' + 2y' + y = e^{-t} + 2(-e^{-t}) + e^{-t} = 0$. This is a solution.
   c) $y' = t(-e^{-t}) + e^{-t}$, $y'' = -t(-e^{-t}) + e^{-t} - e^{-t} = te^{-t}$.
   So $y'' + 2y' + y = te^{-t} + 2e^{-t} - 2te^{-t} + te^{-t} = 0$. This is a solution.
   d) $y' = t^2(-e^{-t}) + 2te^{-t}$, $y'' = -t^2(-e^{-t}) + e^{-t}(-2t) + 2t(-e^{-t}) + e^{-t}(2) = t^2e^{-t} - 4te^{-t} + 2e^{-t}$
   So $y'' + 2y' + y = t^2e^{-t} - 4te^{-t} + 2e^{-t} - 2t^2e^{-t} + 4te^{-t} + t^2e^{-t} = 2e^{-t} \neq 0$. Not a solution.

9. a) $P > 0$, so just need when $1 - \frac{P}{4200} > 0 \to P < 4200$
   b) $1 - \frac{P}{4200} < 0 \to P > 4200$
   c) $\frac{dP}{dt} = 0$ when P=0, and when $1 - \frac{P}{4200} = 0 \to P = 4200$

10.4:

1. P(0)=2, what is P(6)? The relative growth rate is given by $\frac{1}{P}\frac{dP}{dt} = .7944$ so

$$\frac{dP}{dT} = .7944P \Rightarrow P(t) = P(0)e^{.7944t} = 2e^{.7944t}$$

   So P(6) is given by $P(6) = 2e^{.7944*6} \approx 234.99$ (About 235 members)

2. P(0)=60 cells at 20 minutes the cells double, this is equivalent to 1/3 of an hour, so $P(1/3) = 2(60) = 120$.
   a) $P(t) = 60e^{kt}$, so $P(1/3) = 120 = 60e^{(1/3)k} \Rightarrow k = \ln(8)$.
   b) $P(t) = 60e^{(\ln 8)t} = (60)(8^t)$
   c) $P(8) = 1{,}006{,}632{,}960$
   d) $\frac{dP}{dt} = kP, \frac{dP}{dt}(8) = kP(8) = \ln(8)P(8) \approx 2.093$ billion cells
   e) $P(t) = 20{,}000 = (60)(8^t) \Rightarrow t \approx 2.79$ hours

7. a) $y = [N_2O_5]$ and by theorem 2, $\frac{dy}{dt} = -.0005y \Rightarrow y(t) = y(0)e^{-.0005t} = Ce^{-.0005t}$
   b) $y(t) = Ce^{-.0005t} = .9C \Rightarrow e^{-.0005t} = .9 \Rightarrow t \approx 211$

9. Let y(t) be the mass after t years

a) $y(t) = y(0)e^{kt} = 100e^{kt}$ so using the half life value given:

$y(30) = 100e^{30k} = .5(100) \Rightarrow k = \frac{-\ln 2}{30}$ Plugging this k in gives $y(t) = (100)2^{-t/30}$

b) $y(100) = 100(2^{-100}) \approx 9.92$

c) $100e^{(-\ln 2)t/30} = 1 \Rightarrow t \approx 199.3$

13. a) Referring to Newton's Law of Cooling, then $\frac{dT}{dt} = k(T - T_s)$. Now let $y(t) = T(t) - 75$ so that $y(0) = T(0) - 75 = 185 - 75 = 110$. So now y is a solution to $\frac{dy}{dt} = ky$ with $y(0) = 110$.

By theorem 2, $y(t) = y(0)e^{kt} = 110e^{kt}$ and we know that $y(30) = 75 = 110e^{30k}$ solving this for k gives $k = \frac{1}{30}\ln\left(\frac{15}{22}\right)$. So plugging this in to y(t) gives $y(t) = 110e^{(1/30)t\ln(15/22)}$ and using t=45 gives that $y(45) \approx 62^\circ F$

Lastly, $y(45) = T(45) - 75$ so $T(45) = y(45) + 75 = 62 + 75 = 137^\circ F$

b) $T(t) = 100, y(t) = 25$ so $y(t) = 110e^{(1/30)t\ln(15/22)} = 25 \Rightarrow t \approx 116$ minutes