Math 242 Homework Set #5
Due: 10/5/07

Section 8.2

6. $\int \sin^3(mx)dx = \int \sin(mx)\sin^2(mx)dx = \int \sin(mx)(1-\cos^2(mx))dx$. So, let $u = \cos(mx),\ du = -m\sin(mx)$, therefore we have

$$-\frac{1}{m}\int(1-u^2)du = -\frac{1}{m}(u-\frac{u^3}{3})+C = -\frac{1}{m}(\cos(mx)-\frac{\cos^3(mx)}{3})+C$$

$$=\frac{1}{3m}\cos^3(mx)-\frac{1}{m}\cos(mx)+C$$

8. $\int_0^{\pi/2}\sin^2(2\theta)d\theta = \frac{1}{2}\int_0^{\pi/2}(1-\cos(4\theta))d\theta = \frac{1}{2}[\theta-\frac{\sin(4\theta)}{4}]\Big|_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2}\right)=\frac{\pi}{4}$

14. $\int_0^{\pi/2}\sin^2(x)\cos^2(x)dx = \int_0^{\pi/2}\frac{1}{4}(4\sin^2(x)\cos^2(x))dx = \frac{1}{4}\int_0^{\pi/2}(2\sin x\cos x)^2dx$

$= \frac{1}{4}\int_0^{\pi/2}(\sin(2x))^2dx$ since $\sin x\cos x = \frac{1}{2}\sin(2x)$.

$$=\frac{1}{4}\int_0^{\pi/2}\frac{1}{2}(1-\cos(4x))dx = \frac{1}{8}\int_0^{\pi/2}(1-\cos(4x))dx = \frac{1}{8}[x-\frac{\sin(4x)}{4}\Big|_0^{\pi/2} = \frac{1}{8}\left(\frac{\pi}{2}\right)=\frac{\pi}{16}$$

Section 8.3

6. $\int_0^2 x^3\sqrt{x^2+4}dx$, let $x = 2\tan(\theta),\ dx = 2\sec^2(\theta)d\theta$, $x=0\Rightarrow\theta=0,\ x=2\Rightarrow\theta=\pi/4$.

So therefore we have

$$\int_0^{\pi/4}8\tan^3(\theta)\cdot\sqrt{4\tan^2\theta+4}\cdot 2\sec^2(\theta)d\theta = \int_0^{\pi/4}8\tan^3(\theta)\cdot 2\sec\theta\cdot 2\sec^2\theta\,d\theta$$

$$=32\int_0^{\pi/4}\tan^2\theta\sec^2\theta\sec\theta\tan\theta\,d\theta = 32\int_0^{\pi/4}(\sec^2-1)\sec^2\theta\sec\theta\tan\theta\,d\theta.$$

Now, let $u=\sec\theta,\ du=\sec\theta\tan\theta$, $\theta=0\Rightarrow u=1,\ \theta=\pi/4\Rightarrow u=\sqrt{2}$, so we have:

$$=32\int_1^{\sqrt{2}}(u^2-1)u^2du = 32\left(\frac{u^5}{5}-\frac{u^3}{3}\right)\Bigg|_1^{\sqrt{2}} = 32\left(\left(\frac{(\sqrt{2})^5}{5}-\frac{(\sqrt{2})^3}{3}\right)-\left(\frac{1}{5}-\frac{1}{3}\right)\right)=\frac{64}{15}(\sqrt{2}+1)$$

7. $\int \frac{1}{x^2\sqrt{25-x^2}}dx$, let $x = 5\sin\theta,\ dx = 5\cos\theta\, d\theta$, then we have

$$= \int \frac{1}{25\sin^2\theta \cdot 5\cos\theta}\cdot 5\cos\theta\, d\theta = \frac{1}{25}\int \csc^2\theta\, d\theta = -\frac{1}{25}\cot\theta + C = -\frac{1}{25}\cdot\frac{\sqrt{25-x^2}}{x} + C$$

Section 8.4

12. $\int_0^1 \frac{x-1}{x^2+3x+2}dx$ We want to use partial fractions in order to solve this integral.

$$\frac{x-1}{x^2+3x+2} = \frac{x-1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)} = \frac{x(A+B)+2A+B}{(x+1)(x+2)}$$

$\Rightarrow A+B=1,\ 2A+B=-1 \Rightarrow B=3,\ A=-2$. So we now can rewrite the integral as

$$\int_0^1 \frac{-2}{x+1} + \frac{3}{x+2}dx = -2\ln(x+1) + 3\ln(x+2)\Big|_0^1 = -2\ln(2) + 3\ln(3) - 3\ln(2) = 3\ln(3) - 5\ln(2)$$

or $\ln(\frac{27}{32})$.

48. $\int \frac{\cos x}{\sin^2 x + \sin x}dx$, let $u = \sin x,\ du = \cos x\, dx$, then we have the integral

$\int \frac{du}{u^2+u} = \int \frac{1}{u(u+1)}du$. Using partial fractions we have

$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1} = \frac{A(u+1)+Bu}{u(u+1)} = \frac{(A+B)u+A}{u(u+1)} \Rightarrow A=1,\ B=-1$. So, the integral

becomes $\int \frac{1}{u} - \frac{1}{u+1}du = \ln u - \ln(u+1) + C = \ln\left|\frac{u}{u+1}\right| + C = \ln\left|\frac{\sin x}{1+\sin x}\right| + C$.