Accuracy and stability of inversion of power series

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Abstract

We consider the numerical inversion of the power series $p(x) = 1+b_1x+b_2x^2+\cdots$ to compute the inverse series $q(x)$ satisfying $p(x)q(x) = 1$. Numerical inversion is a special case of triangular back-substitution, which has been known for its beguiling numerical stability since the classic work of Wilkinson (1961). We prove the numerical stability of inversion of power series and obtain bounds on numerical error. A range of examples show these bounds to be quite good. When $p(x)$ is a polynomial and $x = a$ is a root with $p(a) = 0$, we show that root deflation via the simple division $p(x)/(x-a)$ can trigger instabilities relevant to polynomial root finding and computation of finite-difference weights. When $p(x)$ is a polynomial, the accuracy of the computed inverse $q(x)$ is connected to the pseudozeros of $p(x)$.