1. $k_1 = \frac{1}{3}(0-1)^2 = \frac{1}{3}$

$$w_0 + k_1 = \frac{4}{3}$$

$$k_2 = \frac{1}{3}\left(\frac{1}{3} - \frac{4}{3}\right)^2 = \frac{1}{3}$$

$$w_1 = 1 + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{3} = \frac{4}{3}$$

2. $\rho(z) = z - 1, \quad \sigma(z) = \theta z + (1-\theta)$

(a) $\rho(1) = 0, \quad \rho'(1) = 1, \quad \sigma(1) = \theta + 1 - \theta = 1$ consistent

$\rho(z) = 0 \Rightarrow z = 1$ stable

(b) $\theta = \frac{1}{2}$ gives the trapezoid rule

(c) $$w_{i+1} = w_i + \frac{3}{4}h(-40w_i) + \frac{1}{4}h(-40w_{i+1})$$

$$w_{i+1} = \frac{1-30h}{1+10h}\, w_i \qquad |1-30h| \le 1 + 10h$$

$$-1-10h \le 1-30h \le 1+10h$$

$$20h \le 2 \quad \text{and} \quad 20h \ge 0$$

$$h \le \frac{1}{10} \quad \text{and} \quad \cancel{h \ge 0}$$

3. (a) $$Q(t) = w_{i+1}\frac{(t-t_i)(t-t_{i-1})}{2h^2} + w_i\frac{(t-t_{i+1})(t-t_{i-1})}{-h^2} + w_{i-1}\frac{(t-t_{i+1})(t-t_i)}{2h^2}$$

$$Q'(t_{i+1}) = w_{i+1}\frac{2t_{i+1}-t_i-t_{i-1}}{2h^2} + w_i\frac{t_{i+1}-t_{i-1}}{-h^2} + w_{i-1}\frac{t_{i+1}-t_i}{2h^2}$$

$$f_{i+1} = Q'(t_{i+1}) = \frac{3}{2h}w_{i+1} - \frac{2}{h}w_i + \frac{1}{2h}w_{i-1}$$

$$\frac{2}{3}h f_{i+1} = w_{i+1} - \frac{4}{3}w_i + \frac{1}{3}w_{i-1}$$

4. $x_1 = y$, $x_2 = y'$

$$x_1' = x_2$$
$$x_2' = x_1 - x_1^3$$

$$J = \begin{bmatrix} 0 & 1 \\ 1-3x_1^2 & 0 \end{bmatrix}$$

For $x_1 = \pm 1$, $J = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$ eigenvalues $\lambda = \pm i\sqrt{2}$

$\lambda h = \pm i \frac{\sqrt{2}}{20}$ in stability region of Midpoint, not Euler