NAME _____

T.A. _____

MATH 242, Fall 2007
Exam 3: November 30

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (5 points each) For the sequence $\left\{\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \dots\right\}$:

   (a) Find a formula for a_n, the nth term of the sequence. (Assume $n = 1$ is the first term.)
   (b) Find the limit, or show that the sequence diverges.

2. (15 points) Determine the convergence/divergence of the series $\sum_{n=1}^{\infty} \frac{2^{n+1} + \pi^n}{3^n}$. State what test or rules you are using. If it converges, determine the value of the sum.

3. (15 points each) Determine the convergence/divergence of each series. State what test or rules you are using.

   (a) $\sum_{n=1}^{\infty} \ln(n)$

   (b) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

4. (15 points) Find the radius and interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}\, 2^n}.$$

5. (15 points) Find the Taylor series of $\sin(3x)$ at $a = 2\pi/3$.

6. (15 points) Find the first three nonzero terms in the Maclaurin series of $\frac{xe^{-x}}{1+x}$.

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, \quad 1 - \sin^2\theta = \cos^2\theta, \quad \sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$$
$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}, \quad 1 + \tan^2\theta = \sec^2\theta, \quad \cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta)$$