FORM B

MATH 241, Spring 2009
Exam 3: May 13

NAME ______________________ **Tue/Thurs discussion time** __________

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (10 points) Find all inflection points and the intervals of concavity in $0 \leq \theta \leq \pi$ for $f(\theta) = \cos(\theta) + \frac{1}{4}\theta^2$.

$f'(\theta) = -\sin\theta + \frac{1}{2}\theta$

$f''(\theta) = -\cos\theta + \frac{1}{2}$

$f''(\theta) = 0 \Rightarrow \cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$ in $[0, \pi]$

inflection point.

$\cos\theta > \frac{1}{2}$ for $0 \leq \theta < \frac{\pi}{3}$, so $f'' < 0$ concave down

$\cos\theta < \frac{1}{2}$ for $\frac{\pi}{3} < \theta \leq \pi$, so $f'' > 0$ concave up

2. (15 points) Find the minimum and maximum values of $f(t) = t\sqrt{18-t^2}$ on the interval $[0,4]$.

$f'(t) = \sqrt{18-t^2} + \frac{1}{2}t(18-t^2)^{-1/2}(-2t)$

critical number : $0 = (18-t^2)^{+1/2} - t^2(18-t^2)^{-1/2}$

$0 = (18-t^2)^1 - t^2 = 18-2t^2$

$t = 3$ or $\cancel{t = -3}$

(not in $[0,4]$)

$f(0) = 0$ ← min value

$f(3) = 3\sqrt{9} = 9$ ← max value

$f(4) = 4\sqrt{2} < 9$

3. (10 points) Find all local minimum and local maximum points of $g(x) = 100 - 18x^2 + x^4$.

$$g'(x) = -36x + 4x^3 = 4x(x^2-9) = 4x(x-3)(x+3)$$

critical numbers $x=0$, $x=3$, $x=-3$

$$g''(x) = -36 + 12x^2$$

$g''(0) = -36 < 0$ local max

$g''(3) = -36 + 12\cdot 9 > 0$ local min

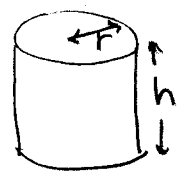
$g''(-3) = -36 + 12\cdot 9 > 0$ local min

OR

	$4x$	$x-3$	$x+3$	$g'(x)$	
$x<-3$	$-$	$-$	$-$	$-$	
					$\leftarrow$ min
$-3<x<0$	$-$	$-$	$+$	$+$	
					$\leftarrow$ max
$0<x<3$	$+$	$-$	$+$	$-$	
					$\leftarrow$ min
$x>3$	$+$	$+$	$+$	$+$	

4. (15 points) A cylindrical metal can with no lid is supposed to hold 1000π cm^3 of liquid. Find the dimensions of the can that minimizes the amount of material used.

$1000\pi = V = \pi r^2 h$ (Constraint)



material used $\rightarrow$ surface area

$S = 2\pi r h + \pi r^2$ (minimize)

(sides) (bottom)

$$S = 2\pi r\left(\frac{1000\pi}{\pi r^2}\right) + \pi r^2 = \frac{2000\pi}{r} + \pi r^2$$

$$S'(r) = -\frac{2000\pi}{r^2} + 2\pi r$$

$$0 = S'(r) \Rightarrow 2\pi r = \frac{2000\pi}{r^2}$$

$$r^3 = 1000$$

$$r = 10 \text{ cm}$$

$$h = \frac{1000}{r^2} = \frac{1000}{100} = 10 \text{ cm}$$

5. (10 points) A car traveling 99 ft/sec begins to experience deceleration of e^t starting at $t = 0$. How far will it have traveled between $t = 0$ and $t = 1$? (No need to simplify the number.)

$$v(t) = \int a(t)\,dt = \int -e^t\,dt = -e^t + C$$

$$99 = v(0) = -1 + C, \quad \text{so} \quad C = 100$$

$$s(t) = \int v(t)\,dt = -e^t + 100t + B$$

$$s(1) - s(0) = (-e^1 + 100 + B) - (-e^0 + 0 + B)$$

$$= -e + 101 \text{ ft.}$$

6. (10 points) Write down the Riemann sum R_5 that approximates $\int_{-5}^{10} \cos(x)\,dx$, using right endpoints of five intervals. **Do not try to simplify or evaluate the number.**

$$\Delta x = \frac{10-(-5)}{5} = 3$$

$x_0 = -5$ $\quad x_1 = -2$ $\quad x_2 = 1$ $\quad x_3 = 4$ $\quad x_4 = 7$ $\quad x_5 = 10$

$$R_5 = \sum_{i=1}^{5} f(x_i)\,\Delta x = 3\left(\cos(-2) + \cos(1) + \cos(4) + \cos(7) + \cos(10)\right)$$

7. (10 points) Find $\frac{d}{dx}\left[\int_1^{1/x} \sinh^3(s)\,ds\right]$.

$u = \frac{1}{x}$

$$\left(\frac{d}{du}\int_1^u \sinh^3(s)\,ds\right)\left(\frac{du}{dx}\right)$$

$$= \sinh^3(u)\cdot\left(-\frac{1}{x^2}\right)$$

$$= -\frac{\sinh^3(1/x)}{x^2}$$

8. (10 points) Evaluate $\int_0^{\pi/6} 2\cos(\theta)\, d\theta$.

$$\left[2 \sin \theta\right]_0^{\pi/6} = \left(2 \sin \tfrac{\pi}{6} - 2 \sin 0\right) = 1$$

9. (10 points) Evaluate $\int x(\sqrt{x} - 1)\, dx$.

$$\int (x^{3/2} - x)\, dx = \frac{2}{5} x^{5/2} - \frac{1}{2} x^2 + C$$