11-05

Note Title 11/5/2007

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2+1} = (-1)^2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n^2+1}$ $\qquad b_n = \frac{1}{4n^2+1}$ alternating

$b_n \geq 0$

- $\displaystyle\lim_{n\to\infty} b_n = 0$

- $f(x) = \dfrac{1}{4x^2+1}$, so $b_n = f(n)$. $\qquad f'(x) = (-1)(4x^2+1)^{-2}(8x)$

$(x \geq 1)$ $\qquad$ N $\qquad$ P $\qquad$ P

f decreases, so $\qquad f'(x) < 0$ for $x \geq 1$

$b_{n+1} \leq b_n$, all n

By Alternating Series Test, series converges.

Ex $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n+1}$ Try $b_n = \frac{n}{n+1}$ alternating series

But $\lim_{n\to\infty} b_n = 1$, not zero. Can't use A.S. Test.

However, $a_n \to 1$ if n is odd and $a_n \to -1$ if n is even

$\{a_n\}$ does not converge to zero

Divergence Test : series diverges

Ex $\displaystyle\sum_{n=1}^{\infty} \cos(n\pi) \frac{e^{1/n}}{n}$ Remember $\cos(n\pi) = (-1)^n$

$= (-1) \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{1/n}}{n}$ alternating, $b_n = \dfrac{e^{1/n}}{n}$

- $\displaystyle\lim_{n\to\infty} \frac{e^{1/n}}{n} = \frac{1}{\infty} = 0$
- $f(x) = \dfrac{e^{1/x}}{x}$, $f'(x) = \dfrac{x e^{1/x}\left(-\frac{1}{x^2}\right) - e^{1/x}(1)}{x^2}$ $\dfrac{N + N}{P}$

$f'(x) < 0$ for $x \geq 1 \Rightarrow b_{n+1} \leq b_n$

By Alt. Series Test, converges.

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2} = \frac{\cos(1)}{1} + \frac{\cos(2)}{4} + \frac{\cos(3)}{9} + \cdots$

$+ \quad - \quad - \quad - \quad + \quad +$

no regular pattern of signs

Note $\displaystyle\sum_{n=1}^{\infty} \left|\frac{\cos(n)}{n^2}\right|$ is a sum of positive terms.

We can use Comparison, because $\dfrac{|\cos(n)|}{n^2} \le \dfrac{1}{n^2}$ → convergent p-series

$\therefore \displaystyle\sum \left|\frac{\cos(n)}{n^2}\right|$ converges by Comparison, and

$\displaystyle\sum \frac{\cos(n)}{n^2}$ is absolutely convergent, hence Convergent.