MATH 611, Fall 2007
First midterm exam
October 15, 2007

Please start each problem on a new page. Remember to justify your answers to receive full credit.

1. Suppose Q is a $2m \times 2m$ unitary matrix. Let $\hat{Q}$ be the $2m \times m$ matrix consisting of the first m columns of Q.
   (a) Write out a full (not reduced) SVD of $\hat{Q}$, in terms of Q and any other matrices you wish to define.
   (b) What is $\kappa_2(\hat{Q})$?

2. Suppose $A \in \mathbf{C}^{m \times n}$ has full rank, and A is diagonal:

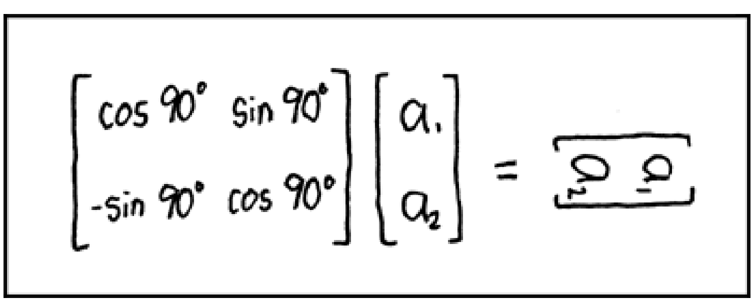
$$A = \begin{bmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \ddots & \\ & & & a_{nn} \\ 0 & \cdots & & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & & 0 \end{bmatrix}$$

   (a) Find A^+, the pseudoinverse of A.
   (b) Find $\|A^+A - I\|_F$ and $\|AA^+ - I\|_F$, for appropriately sized identity matrices.

3. Let D be a diagonal $m \times m$ matrix with positive numbers on the diagonal. Then we can define a norm for all vectors $u \in \mathbf{C}^m$ by $\|u\|_D = (u^* D u)^{1/2}$.
   (a) Show that $\|u\|_D = \|v\|_2$ for an appropriately defined v.
   (b) Suppose also that $b \in \mathbf{C}^m$, $A \in \mathbf{C}^{m \times n}$, $m \geqslant n$, and A has full rank. Find an x that minimizes $\|Ax - b\|_D$.

4. Find the 1-norm condition number for the problem of computing e^{x+y} given the scalar values x and y.

5. Consider the problem of finding the square root of a positive number. Suppose a computer returns exactly $\mathrm{fl}\left(\sqrt{\mathrm{fl}(x)}\right)$ in every case. Is this algorithm backward stable?



(From xkcd.com)