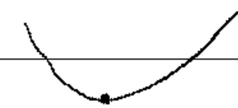
Note Title 09-12 9/12/2007

Ex Minimize $f(x) = e^x - 2x$.

$f'(x) = e^x - 2$ $\quad f'(x) = 0 \Rightarrow \quad e^x = 2$, or $x = \ln 2$

For $x < \ln 2$, $e^x < 2$ and $f'(x) < 0$
$x > \ln 2$, $e^x > 2$ and $f'(x) > 0$



$x = \ln 2$ is a minimizer

$f(\ln 2) = e^{\ln 2} - 2\ln 2 = 2 - \ln 4$ is the min of f.

Ex Find $\frac{d}{dx}\left(e^{-x^2}\right)$. Let $u = -x^2$, so

$$\frac{d}{dx}\left(e^{-x^2}\right) = \frac{d}{du}\left(e^u\right)\frac{du}{dx} = e^u(-2x) = -2x\,e^{-x^2}$$

Ex $f(x) = xe^{-x}$ Find $f^{(1000)}(x)$.

$$f'(x) = (1)(e^{-x}) + (x)(e^{-x} \cdot (-1)) = e^{-x}(1-x)$$

$$f''(x) = (-e^{-x})(1-x) + (e^{-x})(-1) = e^{-x}(-2+x)$$

$$f'''(x) = e^{-x}(3-x)$$

$$\vdots$$

$$f^{(1000)}(x) = e^{-x}(x-1000)$$

$$f^{(n)}(x) = e^{-x}(x-n)(-1)^n$$

Ex $\displaystyle\int e^{4x-3}\,dx = \int e^u\left(\tfrac{1}{4}du\right) = \tfrac{1}{4}\int e^u\,du = \tfrac{1}{4}\left[e^{4x-3} + C\right]$

$\begin{bmatrix} u = 4x-3 \\ du = 4dx \end{bmatrix}$ $\qquad = \tfrac{1}{4}e^{4x-3} + C$

Ex $\displaystyle\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt = \int \frac{e^u}{\cancel{\sqrt{t}}}\left(2\cancel{\sqrt{t}}\,du\right) = 2\int e^u\,du = 2e^{\sqrt{t}} + C$

$\begin{bmatrix} u = \sqrt{t} = t^{1/2} \\ du = \frac{1}{2}t^{-1/2}\,dt \\ dt = 2\sqrt{t}\,du \end{bmatrix}$

__Ex__ $\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x+1) + C$

$\boxed{\begin{aligned} u &= e^x+1 \\ du &= e^x\,dx \end{aligned}}$

__Ex__ $\int \frac{e^x+1}{e^x}\,dx = \int \left(\frac{e^x}{e^x} + \frac{1}{e^x}\right) dx$

$\cancel{u = e^x+1}$ $\quad$ $\cancel{u = e^{-x}}$

$\cancel{du = e^x\,dx}$ $\quad$ $\cancel{du = -e^{-x}\,dx}$

__Ex__ $\int \frac{1+e^x-e^x}{e^x+1}\,dx$ $\qquad$ $\frac{1}{e^x+1} \cdot \frac{e^x-1}{e^x-1} = \frac{e^x-1}{e^{2x}-1}$

$\int \frac{1+e^x}{e^x+1} - \frac{e^x}{e^x+1}\,dx = \int \left(1 - \frac{e^x}{e^x+1}\right) dx$ ✓

__Ex__ $\int e^{-x^2}\,dx$

not elementary

__Ex__ $\frac{d}{dx}\left(2^{x^2}\right) = \frac{d}{dx}\left(e^{x^2 \ln 2}\right) = \left(e^{x^2 \ln 2}\right)\left(2x \ln 2\right)$

$$= 2x \ln 2 \, e^{x^2 \ln 2}$$

$$= 2x \ln 2 \; 2^{x^2} = x(\ln 2) 2^{x^2+1}$$

__Ex__ $\frac{d}{dx}\left(x^x\right) = \frac{d}{dx}\left(e^{x \ln x}\right) = e^{x \ln x}\left(1 \cdot \ln x + x \cdot \frac{1}{x}\right)$

$$= x^x\left(1 + \ln x\right)$$