

MATH 241, Fall 2008

Exam 3: December 3

NAME \_\_\_\_\_

Discussion section \_\_\_\_\_

1	2	3	4	5	6	7	8	Total

Arrange your work as clearly and neatly as possible, and cross out incorrect work. **Unless otherwise noted, you must justify all answers to receive full credit.** You may not use calculators, notes, or any other kinds of aids.

1. (15 points) Find all the local minima and local maxima of  $x^4 - 8x^2 + 8$ .

$$f'(x) = 4x^3 - 16x = 4x(x^2 - 4)$$

critical numbers are  $x=0$ ,  $x=-2$ ,  $x=2$ .

$$f''(x) = 12x^2 - 16$$

$$f''(0) = -16$$

$x=0$  is local max

$$f''(-2) = 32$$

$x=-2$  is local min

$$f''(2) = 32$$

$x=2$  is local min

2. (15 points) Find the absolute minimum and absolute maximum values of  $f(x) = \frac{x}{x^2 + 4}$  on the interval  $[0, 4]$ .

$$f'(x) = \frac{(x^2 + 4) - x(2x)}{(x^2 + 4)^2} = \frac{4 - x^2}{(4 + x^2)^2}$$

Critical numbers  ~~$x = -2$~~ ,  $x = 2$ .  
not in  $[0, 4]$

$$f(0) = 0 \quad \leftarrow \text{abs. min value}$$

$$f(2) = \frac{2}{8} = \frac{1}{4} \quad \leftarrow \text{abs. max value}$$

$$f(4) = \frac{4}{20} = \frac{1}{5}$$

3. (10 points) Evaluate  $\frac{d}{dx} \left[ \int_1^{\cosh(x)} t \sin(t) dt \right]$ . (No need to simplify the result.)

$$\text{(FTC1)} \quad u = \cosh(x), \quad \frac{d}{dx} \left[ \int_1^u t \sin(t) dt \right] = \frac{d}{du} \left[ \int_1^u t \sin(t) dt \right] \frac{du}{dx}$$

$$= u \sin(u) \sinh(x) = \cosh(x) \sin(\cosh(x)) \sinh(x)$$

4. (10 points) Find where the graph of  $y = x + \cos 2x$  is concave up for  $0 \leq x \leq \pi$ .

$$y' = 1 - 2 \sin 2x$$

$$y'' = -4 \cos 2x$$

concave up when

$$-4 \cos 2x > 0$$

$$\cos 2x < 0$$



$$\frac{\pi}{4} < x < \frac{3\pi}{4}$$

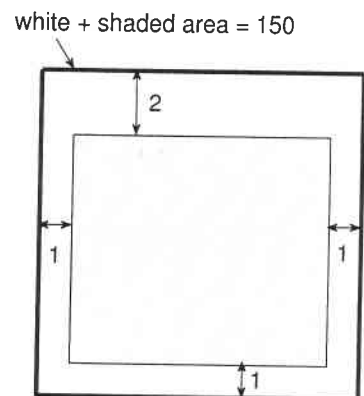
5. (10 points) Evaluate  $\int_{-1}^1 (3t - 6)(t - 2) dt$ .

$$\begin{aligned}\int_{-1}^1 (3t^2 - 12t + 12) dt &= [t^3 - 6t^2 + 12t]_{-1}^1 \\ &= (1 - 6 + 12) - (-1 - 6 - 12) \\ &= 26\end{aligned}$$

6. (10 points) Evaluate  $\int_1^3 \frac{9}{u^4} du$ .

$$\int_1^3 9u^{-4} du = [-3u^{-3}]_1^3 = \left(-\frac{1}{9}\right) - (-3) = \frac{26}{9}$$

7. (15 points) A poster is to have a total area of  $150 \text{ in}^2$ . When printed, it must leave a top margin of 2 in, and margins on the sides and bottom of 1 in each. What are the overall poster dimensions that give the maximum possible printed area? (The diagram does not show the solution!)



$x = \text{width}, y = \text{height}$

maximize  $A = (x-2)(y-3)$

$(x > 2, y > 3)$

constrained by  $xy = 150$  (given)

$$\therefore A(x) = (x-2)\left(\frac{150}{x} - 3\right) = 150 - 3x - \frac{300}{x} + 6$$

$$A' = -3 + \frac{300}{x^2}$$

critical number  $x^2 = 100$

$$x = 10$$

$A'$  changes sign from positive to negative

at  $x = 10$ , so this is an absolute max.

$$x = 10, y = 15$$

8. (15 points) Suppose the height  $h$  of a building 120 meters away is calculated by measuring the angle  $\theta$  it subtends, so that  $h = 120 \tan(\theta)$ . If  $\theta = \pi/6$  with a 5% error, estimate the absolute error in the height.

$$dh = 120 \sec^2 \theta \, d\theta$$

$$\frac{d\theta}{\theta} = 5\% = \frac{1}{20}$$

$$= 120 \left( \cos \frac{\pi}{6} \right)^{-2} \frac{1}{20} \cdot \frac{\pi}{6}$$

$$= \frac{\pi}{(\sqrt{3}/2)^2} = \frac{4\pi}{3} \text{ meters}$$