9-5

Note Title 9/5/2007

Ex Find y' if $x + 4xy^2 - y^3 = 1$ (No $y = f(x)$ given!)

Implicit

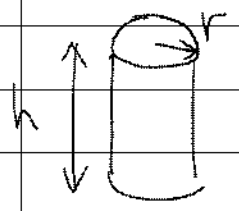
$$\frac{d}{dx}\left(x + 4xy^2 - y^3\right) = \frac{d}{dx}(1)$$

$$1 + 4\frac{d}{dx}(xy^2) - \frac{d}{dx}(y^3) = 0$$

$$1 + 4(1)(y^2) + 4(x)\left(2y\frac{dy}{dx}\right) - 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-1 - 4y^2}{8xy - 3y^2}$$

Ex Find the can of volume $V = 1$ that has minimal surface area.



$V = (\pi r^2)h = 1$ constraint on r, h

$S = h(2\pi r) + 2(\pi r^2)$ surface area

Get rid of h via constraint: $h = 1/\pi r^2$, so $S(r) = \frac{2\pi r}{\pi r^2} + 2\pi r^2$

$= \frac{2}{r} + 2\pi r^2$

$\frac{dS}{dr} = -\frac{2}{r^2} + 4\pi r$ if $\frac{dS}{dr} = 0$, then $\frac{2}{r^2} = 4\pi r$

$r^3 = \frac{1}{2\pi}$

$r = (2\pi)^{-1/3}$

Ex $\int_0^{\pi/2} \cos x \sin(\sin x)\, dx = \int_0^1 \sin(u)\, du$

$\begin{bmatrix} u = \sin x \\ du = \cos x\, dx \end{bmatrix}$

$= \left[-\cos u\right]_{u=0}^{1} = -\cos 1 + \cos 0$

$= 1 - \cos 1$

Ex $\dfrac{d}{dx}\displaystyle\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du = \frac{d}{dx}\left[\int_{2x}^{0} \frac{u^2-1}{u^2+1}\,du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\,du\right]$

$= \dfrac{d}{dx}\left[-\underbrace{\int_{0}^{2x} \sim}_{s=2x} + \int_{0}^{3x} \sim\right] = -2\frac{4x^2-1}{4x^2+1} + 3\frac{9x^2-1}{9x^2+1}$

$\dfrac{d}{dx}\displaystyle\int_0^s \frac{u^2-1}{u^2+1}\,du = \left(\frac{d}{ds}\int_0^s \frac{u^2-1}{u^2+1}\,du\right)\frac{ds}{dx} = 2\frac{s^2-1}{s^2+1} = 2\frac{4x^2-1}{4x^2+1}$

$t=3x$

$3\dfrac{t^2-1}{t^2+1} = 3\dfrac{9x^2-1}{9x^2+1}$