Stable polefinding and rational least-squares fitting via eigenvalues

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Abstract

A common way of finding the poles of a meromorphic function f, whose explicit expression is unknown but $f(z)$ can be evaluated at any given z, is to interpolate f by a rational function $\frac{p}{q}$ such that $\frac{p(\gamma_i)}{q(\gamma_i)} = f(\gamma_i)$ at sample points $\{\gamma_i\}$, and then find the roots of q [1, 3]. This is a two-step process, and the type of the rational interpolant is usually required as inputs. Many other algorithms for polefinding and rational interpolation have been proposed [2, 4, 5, 6], but their numerical stability has remained largely uninvestigated; for example, many methods become unstable when a pole lies very close to a sample point.

This talk describes an algorithm with the following three features: (i) it automatically detects the appropriate type of the rational approximant, thereby allowing the user to input just the function f, (ii) it finds the poles in a one-step fashion via a generalized eigenvalue problem involving matrices constructed directly from the sampled values $f(\gamma_i)$, and (iii) it computes rational approximants $\hat{p}, \hat{q}$ in a numerically stable manner, in that $(\hat{p} + \Delta p)/(\hat{q} + \Delta q) = f$ at the sample points with small $\Delta p, \Delta q$, making it the first rational interpolation algorithm with guaranteed numerical stability. Moreover, our algorithm executes an implicit polynomial change-of-basis by the QR factorization for improved accuracy, and allows for oversampling combined with a least-squares fitting as in [3]. Through experiments we illustrate the resulting accuracy and stability, which can significantly outperform existing algorithms.

References

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