

NAME _____

MATH341-011, Spring 2007
Exam 1: March 9

Write all solutions on these sheets. Please clearly erase or cross out irrelevant work; otherwise it will be part of the graded material. **You must justify answers to receive full credit.** You may not use calculators or notes.

1. (15 points) Find the position function $x = x(t)$ given the acceleration $a(t) = 6 + \cos t$, and $v(0) = 2$, $x(0) = -1$.

2. (15 points) For each initial-value problem, determine whether the existence and uniqueness theorem guarantees a local solution.

(a) $\frac{dy}{dx} = x^{-1}y^{1/2}, \quad y(1) = 1$

(b) $\frac{dy}{dx} = x^{-1}y^{1/2}, \quad y(1) = 0$

(c) $\frac{dy}{dx} = x^{-1}y^{1/2}, \quad y(0) = 1$

3. (20 points) Find the general solution of $y \frac{dy}{dx} = x(y^2 + 1)$.

4. (15 points) Use the substitution $v = y/x$ to change $xy' = \sqrt{x^2 + y^2}$ into a separable, first-order equation for v . Show clearly that the equation for v is separable, but **do not try to solve the new equation.**

5. (20 points) Solve $xy' - 3y = x^3$, $y(1) = 10$.

6. (15 points) Suppose the fish population $P(t)$ in a lake is afflicted with a disease starting at $t = 0$, such that the birth rate β is zero and the death rate δ is inversely proportional to \sqrt{P} . Show that the fish population dies out in a finite amount of time.