MATH 428/CISC 411, Spring 2007
Exam 2

Write all solutions on these sheets. Please clearly erase or cross out irrelevant work; otherwise it will be part of the graded material. **You must justify answers to receive full credit.** You may not use calculators or the computer.

1. (25 points) The modified Euler method is given by the tableau

$$\begin{array}{c|cc} 0 & & \\ 1 & 1 & \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

   In the initial value problem $y' = (t-y)^2$, $y(0) = 1$, find w_1 if $h = 1/3$.

2. (30 points) Consider the multistep method

$$w_{i+1} = w_i + (1-\theta)hf_i + \theta h f_{i+1},$$

   where $0 \le \theta \le 1$ is a constant.

   (a) Show that the method is convergent as $h \to 0$ for any value of θ.

   (b) Find a value of θ such that the order of accuracy is greater than one.

   (c) Suppose $\theta = 1/4$ and you wish to solve $y' = -40y$. Find a timestep restriction on h due to absolute stability.

3. (a) (15 points) Derive the BDF2 method by interpolating three values of w by a polynomial, differentiating, evaluating at a value of t, and equating to f_{i+1}.

   (b) (10 points) Prove that BDF2 is stable.

4. (20 points) The ODE $y'' = y - y^3$ has two stable constant solutions, $y(t) \equiv -1$ and $y(t) \equiv 1$. (That is, perturbations to these solutions do not grow with time.) Which IVP method would give stable long-time approximations to these solutions using $h = 1/20$, Euler or Midpoint? Explain your answer carefully.