Math 242 Homework Set #7
Due: 10/19/07

Section 11.2

6. $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\cos\theta-2\sin(2\theta)}{-\sin\theta+2\cos(2\theta)}$ at $\theta=0$ it is equal to $\frac{1}{2}$. When $\theta=0$, our (x,y) point is (1,1). Therefore the equation for the tangent line at $\theta=0$ is $y=\frac{1}{2}x+\frac{1}{2}$.

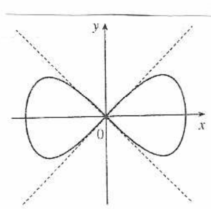
13. $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{1-e^{-t}}{1-e^{t}}=\frac{1-\frac{1}{e^{t}}}{1-e^{t}}=\frac{e^{t}-1}{e^{t}(1-e^{t})}=-e^{-t}$

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}=\frac{e^{-t}}{1-e^{t}}$$

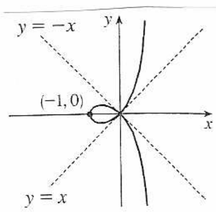
The function is concave up when $e^{t}<1\Rightarrow t<0$ since $e^{-t}>0$.

18. Horizontal tangents occur when $\frac{dy}{dt}=6t^2+6t=0\Rightarrow t=0,-1$, which is at the points (0,1) and (13,2). Vertical tangents occur when $\frac{dx}{dt}=6t^2+6t-12=0\Rightarrow t=-2,1$, which is at the points (20, -3) and (-7, 6).

25. $\frac{dx}{dt}=-\sin(t), \frac{dy}{dt}=-\sin^2(t)+\cos^2(t)=\cos(2t)$. $(x,y)=(0,0)\Rightarrow\cos(t)=0\Rightarrow$ t is an odd multiple of $\frac{\pi}{2}$. When $t=\frac{\pi}{2}, \frac{dx}{dt}=-1$ *and* $\frac{dy}{dt}=-1$, *so* $\frac{dy}{dx}=1$. When $t=\frac{3\pi}{2}, \frac{dx}{dt}=1$ *and* $\frac{dy}{dt}=-1$, *so* $\frac{dy}{dx}=-1$. Therefore, y = x and y = -x are both tangent to the curve at (0,0).



26. $x = 1 - 2\cos^2(t) = -\cos(2t),\ y = (\tan(t))(1 - 2\cos^2(t)) = -\tan(t)\cos(2t)$. We want to look for two values of t (say t_1 and t_2) that give the same point (x,y). The equation for x tells us 1. $\cos^2(t_1) = \cos^2(t_2)$ and the equation for y tells us 2. $\tan(t_1) = \tan(t_2)$ or 3. $\cos^2(t_1) = \cos^2(t_2) = \frac{1}{2}$. We can satisfy 1 and 2 by choosing t_1 arbitrarily and then taking $t_2 = t_1 + \pi$. Therefore the interval is retraced every time t goes an interval of length π. So lets look at the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. It $t_2 = -t_1$, then $\cos^2(t_2) = \cos^2(t_1)$, but $\tan(t_2) = -\tan(t_1)$, so we can try to satisfy 3. Taking $t_1 = \frac{\pi}{4}\ and\ t_2 = -\frac{\pi}{4}$ gives (0,0) for both values of t. So, $\frac{dx}{dt} = 2\sin(2t),\ \frac{dy}{dt} = 2\sin(2t)\tan(t) - \cos(2t)\sec^2(t)$. Therefore, $t = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = \frac{2}{2} = 1,\ and\ t = -\frac{\pi}{4} \Rightarrow \frac{dy}{dx} = \frac{2}{-2} = -1$. So the two tangent lines at (0,0) are y = x and y = -x.

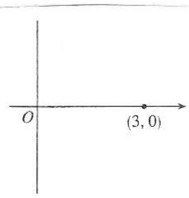


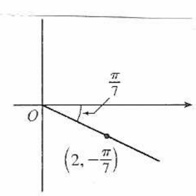
29. $x = -7t,\ y = 12t - 5 \Rightarrow y = -\frac{12}{7}x - 5$, so the slope is $-\frac{12}{7}$. Find when our function has that slope. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{12t}{3t^2 + 4} = -\frac{12}{7} \Rightarrow t = -1, -\frac{4}{3}$. Therefore at these t values, we have $(x, y) = (-5, 6)\ or\ \left(\frac{-208}{27}, \frac{32}{3}\right)$.

Section 11.3

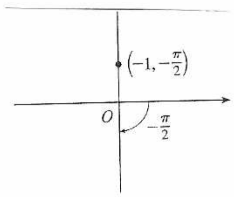
2. a.) $(3,0) \Rightarrow (3, 2\pi), (-3, \pi)$

b.) $(2, -\pi/7) \Rightarrow (2, 13\pi/7), (-2, 6\pi/7)$

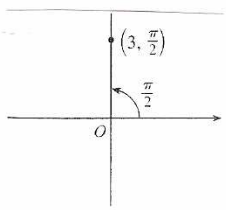




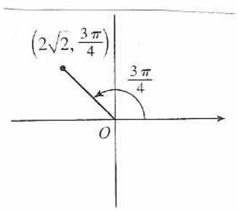
c.) $(-1,-\pi/2) \Rightarrow (1,\pi/2),(-1,3\pi/2)$



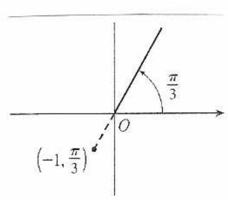
3. a.) $(3,\pi/2) \Rightarrow x = 3\cos(\pi/2) = 0,\ y = 3\sin(\pi/2) = 3 \Rightarrow (0,3)$



b.) $(2\sqrt{2},3\pi/4) \Rightarrow x = 2\sqrt{2}\cos(3\pi/4) = -2,\ y = 2\sqrt{2}\sin(3\pi/4) = 2 \Rightarrow (-2,2)$



c.) $(-1,\pi/3) \Rightarrow x = -1\cos(\pi/3) = -1/2,\ y = -1\sin(\pi/3) = -\sqrt{3}/2 \Rightarrow (-\frac{1}{2},-\frac{\sqrt{3}}{2})$



6. a.) $(-1,-\sqrt{3}) \Rightarrow r = \sqrt{1+3} = 2,\ \tan(\theta) = \sqrt{3} \Rightarrow \theta = 4\pi/3$ (because we need it to be in the 3rd quadrant).

i.) $(2,4\pi/3)$

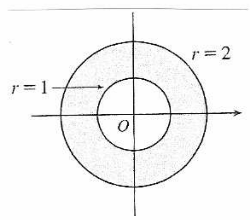
ii.) $(-2,\pi/3)$

b.) $(-2,3) \Rightarrow r = \sqrt{4+9} = \sqrt{13},\ \tan(\theta) = -3/2 \Rightarrow \theta = \tan^{-1}(-3/2)+\pi$ (because we need it to be in the 2nd quadrant).

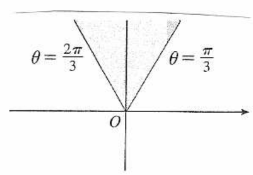
i.) $(\sqrt{13},\theta)$

ii.) $(-\sqrt{13}, \theta+\pi)$

7. $1 \le r \le 2$



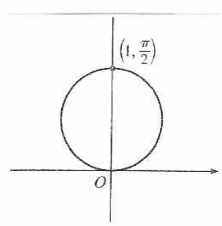
8. $r \ge 0,\ \pi/3 \le \theta \le 2\pi/3$



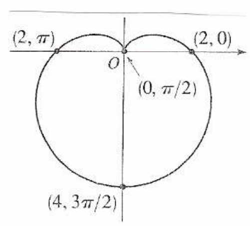
16. $r\cos(\theta)=1 \Rightarrow x=1$

22. $x^2+y^2=9 \Rightarrow r=\sqrt{x^2+y^2}=\sqrt{9}=3 \Rightarrow r=3$ (r = -3 gives the same curve)

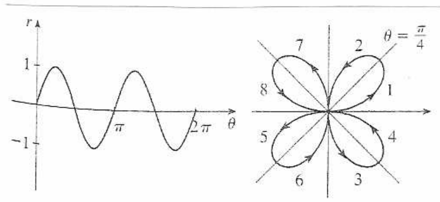
31. $r=\sin(\theta) \Rightarrow r^2=\sin^2(\theta) \Rightarrow x^2+y^2=y \Rightarrow x^2+(y-1/2)^2=(1/2)^2$. This is an equation of a circle with radius ½ centered at (0, ½).



33. $r=2(1-\sin\theta)$ is a cardioid.



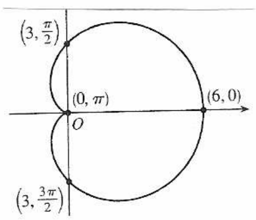
37. $r = \sin(2\theta)$



Section 11.4

5. $r=\theta, 0 \le \theta \le \pi$, $A = \int_0^{\pi} \frac{1}{2} r^2 d\theta = \int_0^{\pi} \frac{1}{2}\theta^2 d\theta = \frac{1}{2}\frac{\theta^3}{3}\Big|_0^{\pi} = \frac{\pi^3}{6}$.

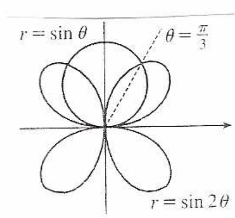
10. $r = 3(1+\cos(\theta))$



$$A = \int_0^{2\pi} \frac{1}{2}(3(1+\cos(\theta))^2 d\theta = \frac{9}{2}\int_0^{2\pi} 1+2\cos(\theta)+\cos^2(\theta) d\theta = \frac{9}{2}\int_0^{2\pi} 1+2\cos(\theta)+\frac{1}{2}(1+\cos(2\theta)) d\theta$$

$$= \frac{9}{2}(\frac{3}{2}\theta + 2\sin(\theta) + \frac{1}{4}\sin(2\theta)\Big|_0^{2\pi} = \frac{27}{2}\pi$$

30. $r = \sin(2\theta)$, $r = \sin(\theta)$

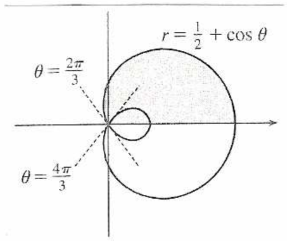


$$\sin(\theta) = \pm\sin(2\theta) = \pm 2\sin(\theta)\cos(\theta) \Rightarrow \sin(\theta)(1 \pm 2\cos(\theta)) = 0 \Rightarrow \theta = \pi/3, 2\pi/3$$

$$A = 2\left[\int_0^{\pi/3} \frac{1}{2}\sin^2(\theta) d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}\sin^2(2\theta) d\theta\right] = \int_0^{\pi/3} \frac{1}{2}(1-\cos(2\theta)) d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}(1-\cos(4\theta)) d\theta$$

$$= \frac{1}{2}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\pi/3} + \frac{1}{2}\left[\theta - \frac{1}{4}\sin(4\theta)\right]_{\pi/3}^{\pi/2} = \frac{4\pi - 3\sqrt{3}}{16}$$

35. $r = \frac{1}{2} + \cos(\theta)$



$$A = 2\left[\int_0^{2\pi/3} \frac{1}{2}(\frac{1}{2} + \cos(\theta))^2\, d\theta - \int_{2\pi/3}^{\pi} \frac{1}{2}(\frac{1}{2} + \cos(\theta))^2\, d\theta\right] = \int_0^{2\pi/3} \frac{1}{4} + \cos(\theta) + \cos^2(\theta) d\theta -$$

$$\int_{2\pi/3}^{\pi} \frac{1}{4} + \cos(\theta) + \cos^2(\theta) d\theta = \int_0^{2\pi/3} \frac{1}{4} + \cos(\theta) + \frac{1}{2}(1 + \cos(2\theta) d\theta - \int_{2\pi/3}^{\pi} \frac{1}{4} + \cos(\theta) + \frac{1}{2}(1 + \cos(2\theta)) d\theta$$

$$= \left[\frac{\theta}{4} + \sin(\theta) + \frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right]_0^{2\pi/3} - \left[\frac{\theta}{4} + \sin(\theta) + \frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right]_{2\pi/3}^{\pi} = \frac{1}{4}(\pi + 3\sqrt{3})$$