Legendre polynomials in scientific computing

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Polynomial expansions and polynomial interpolation are ubiquitous in algorithms

of numerical analysis and scientific computing. To quote Nick Trefethen himself:

"In almost every area of numerical analysis it is a fact that, sooner or later,

the discussion comes down to approximation theory.'

However, if you've read Nick's approximation theory book, ATAP, you'll know that

whilst monomial series and/or equally-spaced interpolation points are

conceptually the most straightforward, these usually lead to ill-conditioned and

unstable algorithms. Instead, Chebyshev polynomials and Chebyshev points are

typically those of choice, in no small part because of their close connection

with the discrete Cosine transform (DCT) and hence the FFT. It's no surprise

then that Chebyshev interpolation forms the basis of the Chebfun software

system. (And if you need me to tell you what `Chebfun' is, then you're at the

wrong conference.)

But are Chebyshev expansions always the best choice? For one thing, they are

associated with the weighted inner-product space,

      $$<f, g> = \int\_{-1}^1\frac{f(x)g(x)}{\sqrt{1-x^2}}dx,$$

whereas the standard $L\_2$ inner-product may be more natural in many situations.

Conversely, the Legendre polynomials are orthogonal in $L\_2$, but lack the

connection to the FFT.

In this talk we shall take a brief look at some algorithms made possible by the

recent development of fast and stable computation of Legendre series expansions,

including: fast $L\_2$ projection; convolution on finite domains; computation of

fractional derivatives; and computation of the discrete Legendre transform (DLT).