Let $f(x) = x_1^2 - x_2^2$.

(a) Show that the origin is a saddle point (i.e., stationary with indefinite Hessian).

(b) Define $H(x) = \nabla^2 f(x) + \mu I$ for constant $\mu > 0$. Find a condition on μ that guarantees that $H(x)$ is positive definite for all x.

(c) Consider a modified Newton method based on the quadratic model $q(x) = f(x_c) + g(x_c)^T(x - x_c) + \frac{1}{2}(x - x_c)^T H(x_c)(x - x_c)$, where g is the gradient of f and H is as above with your condition from part (b). Show that the iteration will not converge to the saddle point unless the second component of the initial guess point x_0 is zero.