

DEPARTMENT OF ACTUARIAL MATHEMATICS AND STATISTICS

F70LA Life Insurance Mathematics A

Coursework: November 2020

Part 1:

- a)  ${}_{20}P_{74}$ : **0.03882** (to 5dp).

*This value is found using the equation derived from Makeham's law:  ${}_tP_x = s^t g^{c^x(c^t-1)}$ ,  $s = \exp(-A)$ ,  $g = \exp(-\frac{B}{\ln c})$*

- b)  $e_{74}$ : **8.5357** (to 5sf).

*This is found by using  $e_k = \sum_{n=1}^{\infty} {}_nP_k$ .*

- c)  $\bar{a}_{74}$ : **3.4008** (to 5sf).

*Found by using the repeated Simpsons rule to integrate  $\int_0^{\infty} v^t {}_tP_x dt$  with step size set at 0.125 to ensure maximum accuracy to 5 decimal places.*

- d) **Standard Deviation: 9.2762** (to 5sf).

*The standard deviation of the present value of the annuity is found by using the equation  $Var[\bar{a}_{T_x}] = \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2}$ . To find  $\bar{A}_x$ , we use the repeated Simson's rule again to integrate  $\int_0^{\infty} v^t {}_tP_x \mu_{x+t} dt$  with  $i=5\%$ . To find  ${}^2\bar{A}_x$  we integrate the same but with  $i_2=2i+i^2$ . Once we have found the variance, we can square root it to find the standard deviation.*

Part 2:

- a) **B:  $3.8335 \times 10^{-6}$**  (to 5sf)

*Here we can use GOALSEEK on our working in part 1 to set  $e_k=18$  by changing the value for B.*

- b) The survival model seems to be an appropriate model for men in the 21<sup>st</sup> century since the probability of a person surviving as time goes on gets smaller, which we would expect. Also, the limit of the probability of surviving is 0 and it appears to reach this at the limiting age of 122, this also makes sense as you wouldn't expect men to live any longer than that. Additionally, the probability that someone aged x survives to age x is 1. This tells us that our survival model fits all the criterion to be a survival model. Additionally, by comparing the estimated values for  $\mu_x$  to other models (e.g AM92) it would appear that this model is suitable for modelling mortality rates for men in the UK in the 21<sup>st</sup> century.

References:

- Tutorial 2 – Question 3
- Formulae and Tables for Examinations of the faculty of actuaries and the institute of actuaries (2002)