Toby LAW (19-945-484) 1 December 2019

7.2.2 (a) $z = \frac{y - \mu}{\sigma}$

$= \frac{100.5 - 99}{1}$

$= 1.5$

$H_0 : \mu = 99$
$H_a : \mu \neq 99$
Significance level $\alpha = 0.05$

Region of rejection $= |z| \geq z_{\alpha/2}$

$\because z_{0.025} = 1.96 \geq z$

$\therefore$ The temperature falls within the normal range, and $H_0 : \mu = 99$ is not rejected.

(b) p-value = probability that a sample in this distribution has a more extreme result assuming the null hypothesis is true.

$\therefore$ p-value $= 0.05$ //

7.3.2 (a) Probability that 4 random samples will all have Score above 88 =

$$P(\bar{y} > 88)^4$$

$$z = \frac{88-80}{16}$$
$$= 0.5$$

$$P(\bar{y} > 88) = P(z > 0.5) = 0.309$$

$$\therefore P(\bar{y} > 88)^4 = (0.309)^4 = 0.00912 \text{ (3 s.f.)}$$

(b) To find probability of an average score of sample size = 4. =

From the sampling distribution of averages.
where $\mu_{\bar{y}} = 80$ and $\sigma^2_{\bar{y}} = 16^2/4$
$= 64$

$$z = \frac{88-80}{\sqrt{64}/\sqrt{4}} = \frac{8}{4} = 2$$

$$P(\text{average score for sample of } 4 > 88) = P(z = 2) = 0.023$$

7.4.6 (a) Mean ppm of the compound found in amniotic fluid

$$\bar{y} = \frac{\Sigma y}{n} = \frac{320.00}{64} = 5.00\text{ppm}$$ //

(b) Sample variance s^2 =

$$= \frac{\Sigma y^2 - (\Sigma y)^2/n}{n-1}$$

$$= \frac{1761.28 - (320.00)^2/64}{64-1}$$

$$= 2.56 \text{ ppm}^2$$ //

(c) $\because CI_{0.95} = \bar{y} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

$$5.00 - 1.96 \times \frac{\sqrt{2.56}}{\sqrt{64}} \le \mu \le 5.00 + 1.96 \times \frac{\sqrt{2.56}}{\sqrt{64}}$$

$$CI_{0.95} = 4.608 \le \mu \le 5.392$$ //

(d) $CI_{0.95} : \frac{(n-1)s^2}{\chi^2_{0.025,63}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{0.975,63}}$

$$\frac{63(2.56)}{86.830} \le \sigma^2 \le \frac{63(2.56)}{42.950}$$

$$1.86 \le \sigma^2 \le 3.76 \text{ (3s.f.)}$$ //

7.5.8 $CI_{1-\alpha} = \hat{\phi} \pm e^{Z_{\alpha/2}\sqrt{\frac{1}{n_1\hat{\pi}_1(1-\hat{\pi}_1)} + \frac{1}{n_2\hat{\pi}_2(1-\hat{\pi}_2)}}}$

Estimated odds ratio $\hat{\phi}$ =

odds for colds in treated group

$$= \frac{155}{300} \times \frac{300}{145} = 1.068$$

odds for colds in untreated group

$$= \frac{120}{80} = 1.5$$

Odds ratio $\hat{\phi} = \frac{1.5}{1.068} = 1.404$

$Z_{\alpha/2} = 1.96$

$$\frac{1}{n_1\hat{\pi}_1(1-\hat{\pi}_1)} = \frac{1}{300\left(\frac{145}{300}\right)\left(\frac{155}{300}\right)} = 0.0133$$

$$\frac{1}{n_2\hat{\pi}_2(1-\hat{\pi}_2)} = \frac{1}{200\left(\frac{80}{200}\right)\left(\frac{120}{200}\right)} = 0.0208$$

$$\therefore CI_{0.95} = \hat{\phi} \pm e^{1.96\sqrt{0.0133+0.0208}}$$
$$= 1.404 \pm 1.436$$

$\because$ ϕ odds ratio cannot be negative.

$CI_{0.95}: 0 \le \phi \le 2.84$ //

8.2.4 (a) The average response time of the offspring of the male and female flies with the most rapid response time should be less than that of the previous generation.

(b) $H_0 : \mu = 80$

$H_a : \mu < 80$

(c) $\bar{y} = \frac{\sum y}{n} = \frac{2136}{30} = 71.2$ seconds

$s^2 = \frac{\sum y^2 - (\sum y)^2/n}{n-1} = \frac{155225.2 - (2136)^2/30}{29}$

$= 108.34$ seconds^2

$s = \sqrt{108.34} = 10.4$ seconds

$t = \frac{\bar{y} - \mu_0}{s/\sqrt{n}} = \frac{71.2 - 80}{10.4/\sqrt{30}}$

$= -4.63$

If $\alpha = 0.05$:

Region of rejection $= t \leq -t_{0.05, 29}$

$-t_{0.05,29} = -1.699$

$\because t \leq -t_{0.05, 29}$

The null hypothesis is rejected, and the average phototatic response time of the offspring is less than that of the previous generation.

8.3.2

By hand

$\bar{y} = 3.5ppm$

$\sum y = 3.5 \times 16$
$= 56$

$\sum y^2 = 200.5$

By machine

$\bar{y} = \frac{48}{16} = 3ppm$

$\sum y = 48ppm$

$\sum (y-\bar{y})^2 = 5.1$

For apples washed by hand,

$$s^2 = \frac{\sum y^2 - (\sum y)^2/n}{n-1}$$

$$s^2 = \frac{200.5 - (56)^2/16}{15}$$

$$s^2 = 0.3$$

pooled sample variance $s_p^2 =$

$$\frac{0.3(16-1) + 5.1}{(16 \times 2) - 2}$$

$$s_p^2 = 0.32$$

We carry out the two-tailed test as it is unsure which method would be more effective in removing insecticide residues.

$H_0 : \mu_1 - \mu_2 = (\mu_1 - \mu_2)_0$ against $H_a : \mu_1 - \mu_2 \neq (\mu_1 - \mu_2)_0$

$\alpha = 0.01$

The critical value $t_{0.005, 30} = 2.750$

Region of rejection :

$|t| \geq 2.750$

continued →

Continued...

8.3.2

$$|t| = \frac{(3.5-3)-0}{\sqrt{\frac{0.32}{16}+\frac{0.32}{16}}} = \frac{0.5}{0.2} = 2.5$$

$\because |t| < 2.750$

$\therefore$ The null hypothesis is not rejected, and there is no evidence that different washing methods cause a difference of insecticide residue at 0.01 level of significance.

9.1.6 (a) Slope of the regression line $= \frac{2}{5} = 0.4$

(b) Number of grams the mother's weight is above the mean $= 28 - 23 = 5g$

Predicted mature weight of the daughter =
$(5 \times \frac{2}{5}) + 20 = 22g$ //

(c) Number of grams the mother's weight is above the mean $= 23 - 20 = 3g$

Predicted mature weight of the daughter =
$22 + (\frac{2}{5} \times 3) = 23.2g$ //

let $\bar{y}$ be the mean mature weight of the daughter mice, and $\bar{x}$ be that for maternal mice

$$\bar{y} = m\bar{x} + c \qquad - (1)$$

$$\bar{y} + \frac{2}{5} = m(\bar{x}+1) + c \qquad - (2)$$

$$(2) - (1) = \bar{y} - \bar{y} + \frac{2}{5} = m\bar{x} + m + c - m\bar{x} - c$$

$$m = \frac{2}{5}$$

9.2.6 (a) The null hypothesis could be the level of blood copper does not increase with the administration of increasing dosages of LSD. The slope of this regression $\beta = 0$.

$$\therefore H_0 : \beta = 0$$

(b) The alternative hypothesis could be that the level of blood increases with administration of increasing dosages of LSD. The slope of this regression $\beta > 0$

$$\therefore H_a : \beta > 0$$

(c) (i) Slope of the least-squares trend line b =

$$b = \frac{S_{xy}}{S_{xx}}$$

where $S_{xx} = \sum x^2 - (\sum x)^2/n$

and

$$S_{xy} = \sum xy - (\sum x)(\sum y)/n$$

$\sum x = 0 + 0.25 + 0.5 + 0.75 + 1 = 2.5$

$\sum x^2 = 0 + 0.25^2 + 0.5^2 + 0.75^2 + 1 = 1.875$

$\sum y = 0.87 + 0.98 + 0.7 + 0.9 + 1.05 = 4.5$

$\sum xy = 0(0.87) + 0.25(0.98) + 0.5(0.7) + 0.75(0.9) + 1.05 = 2.32$

continued →

$\therefore S_{xx} = 1.875 - (2.5)^2/5 = 0.625$

$S_{xy} = 2.32 - (2.5)(4.5)/5 = 0.07$

continued ...

9.2.6 $\therefore b = \dfrac{0.07}{0.625}$

$b = 0.112$ //

(c) (ii)

$H_0 = \beta = 0$

$H_a = \beta > 0$

$\alpha = 0.5$

A one-tailed t-test is carried out as it is expected that increased dosages of LSD would increase blood copper levels, if it does have an effect.

Region of rejection =

$t > t_{0.5,3} = 0$

$$t = \frac{b - \beta_0}{S_{y.x} / \sqrt{S_{xx}}}$$

$b = 0.112$, $\beta_0 = 0$, $S_{xx} = 0.625$

$$S_{y.x}^2 = \frac{S_{yy} - bS_{xy}}{n-2}$$

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Continued...

9.2.6 $S_{yy} = \Sigma y^2 - (\Sigma y)^2/n$

$\Sigma y^2 = 0.87^2 + 0.98^2 + 0.7^2 + 0.9^2 + 1.05^2$
$= 4.1198$

$(\Sigma y)^2 = 4.5^2 = 20.25$

$\therefore S_{yy} = 4.1198 - 20.25/5$
$= 0.0698$

$\therefore S^2_{y.x} = \dfrac{0.0698 - (0.112)(0.07)}{5-2}$

$= 0.0206$

$S_{y.x} = \sqrt{0.0206} = 0.1437$

$t = \dfrac{0.112 - 0}{0.1437/\sqrt{0.625}}$

$t = 0.616$

$\because t > 0$

$\therefore$ The null hypothesis can be rejected, and at $\alpha = 0.5$, it seems that increased dosages of LSD would increase blood copper levels.

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continued ...

9.2.b

(c) (iii)

At $\alpha = 0.5$, the null hypothesis would be rejected as long as b is not 0 or i.e. $\hat{y} = \bar{y}$

Under these conditions, the confidence level of our result is basically 0%, therefore with this test we cannot say there is a significant increase in blood copper level with increasing dosages of LSD, neither could we say that there is significant evidence that LSD may be a chemical cause of schizophrenia.

9.3.2 (a) $n = 27$, $\Sigma x = 675$, $\Sigma y = 810$, $b = 1.2$

$S_{xx} = 25$, $S_{yy} = 136$

$\because b = \dfrac{S_{xy}}{S_{xx}}$

$\therefore S_{xy} = 1.2 \times 25$
$= 30$

$\therefore$ Variance around the sample regression line =

$$S^2_{y.x} = \frac{S_{yy} - S_{xy}^2 / S_{xx}}{n-2}$$

$$= \frac{136 - 30^2/25}{27-2}$$

$$= 4$$

we could conduct a one-tailed t-test for

(b) $H_0 = \beta = 0$ against $H_a : \beta > 0$

The alternative hypothesis is defined as $\beta > 0$ as we expect legs to be longer than arms, so the slope of the regression line should be positive.

(c) $H_0 = \beta = 0$

$H_a = \beta > 0$

$\alpha = 0.05$

Region of rejection $t > t_{0.05, 25} = 1.708$ continued →

continued ...

Q.3.2

$$t = \frac{1.2 - 0}{\sqrt{4} / \sqrt{25}}$$

$$t = 3$$

$\because t > t_{0.05, 25} = 1.708$

$\therefore$ Ho is rejected, and there is a positive linear relationship between arm length and leg length.

(d) The y-intercept a of the sample regression line is $= \bar{y} - b\bar{x} = \left(\frac{810}{27}\right) - 1.2\left(\frac{675}{27}\right)$

$= 0$

$\therefore$ The least-squares trend line $\hat{y} = 1.2x$

$\therefore$ Predicted leg length of a man with arm length of 25 inches =

$= 1.2 \times 25$

$= 30$ inches //

continued →

continued...

9.3.2 (e) $\alpha = 0.05$

$$CI_{0.95} = \hat{y} \pm t_{0.025, 25} \cdot S_{y \cdot x} \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$$

$$= 30 \pm 2.06 \cdot 2 \sqrt{\frac{1}{27} + \frac{(25 - \bar{x})^2}{25}}$$

$$\because \bar{x} = \frac{\sum x}{n} = \frac{675}{27} = 25$$

$$CI_{0.95} = 30 \pm 2.06 \times 2 \sqrt{\frac{1}{27}}$$

$$= 30 \pm 0.79$$

The 95% central confidence interval of $\hat{y}$ when $x = 25$ inches is 30 ± 0.79.

9.44 (a) As we do not know how the milk yield from first and second lactations are positively or negatively associated, we carry out a two-tailed t-test.

$H_0 = \rho = 0$

$H_a : \rho \neq 0$

$\alpha = 0.05$

$r = 0.42$

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

$$t = \frac{0.42}{\sqrt{\frac{1-0.42^2}{23}}}$$

$t = 2.219$

$\because$ Region of rejection $|t| \geq t_{0.025, 23} = 2.069$

$\because t \geq 2.069$

$\therefore$ H_0 is rejected, and we can conclude that there is a significant linear relationship between the milk yield from the first & second lactation.

(b) The relatively small value of $r^2 = 0.1764$ indicates that there is little linear association between the first and second

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9.44 laclation, and the relationship is not very useful in predicting milk yield for the second lactation if we have information about that for the first lactation.

(End)