Stochastics (Probability and Statistics) hw 2

406-0603-AAL

Toby LAW (19-945-484)

1.1.4 a) All the possible outcomes for the event $y=6$ =

Red die =	1	2	3	4	5
Green die =	5	4	3	2	1

$$P(y=6) = \frac{\text{Number of occurrences where } y=6}{\text{number of possible outcomes}}$$

$$= \frac{5}{6\times 6} = \frac{5}{36} \text{ //}$$

b) All the possible outcomes for the event $y=8$

Red die =	2, 3, 4, 5, 6
Green die =	6, 5, 4, 3, 2

$$P(y=8) = \frac{5}{6\times 6} = \frac{5}{36} \text{ //}$$

c) All possible outcomes for the event =

	$y=7$	$y=11$
Red die =	1, 2, 3, 4, 5, 6	5, 6
Green die =	6, 5, 4, 3, 2, 1	6, 5

$$P(y=7) = \frac{6}{36} = \frac{1}{6} \qquad P(y=11) = \frac{2}{36} = \frac{1}{18}$$

$$P(y=7 \text{ or } y=11) = \frac{1}{6} + \frac{1}{18} = \frac{4}{18} = \frac{2}{9} \text{ //}$$

d) All possible outcomes for the event =

	$y=2$	$y=3$	$y=12$
Red die =	1	1 , 2	6
Green die =	1	2 , 1	6

$P(y=2) = \frac{1}{36}$
$P(y=3) = \frac{2}{36} = \frac{1}{18}$
$P(y=12) = \frac{1}{36}$

$P(y=2 \text{ or } 3 \text{ or } 12) = \frac{1}{36} + \frac{1}{18} + \frac{1}{36}$
$= \frac{4}{36}$
$= \frac{1}{9}$ //

1.2.6 a) Null hypothesis $H_0 = \pi = 0.25$, where π is the proportion of consumers that prefer the new product

Alternative hypothesis: $H_a = \pi > 0.25$, where there is a preference for the new product

b) As we reject H_0 when $P < \alpha$, the region of rejection is when more than or equal to 9 out of 20 consumers indicate that the new product has better taste. Hence, the region of acceptance would be when less than 9 out of 20 consumers indicate that the new product has better taste.

c) Statement iv is correct.

2.1.1 a) Population = All students in a large university
Sample = 150 student accounts
Research variable = the total amount of error in all students' bills

b) Population = 28 surviving California condors
Sample = 28 surviving California condors
Research variable = The sex of the 28 surviving California condors

c). Population = All the instances of the synthesis of a certain compound
Sample = 5 synthesis attempts of the certain compound
Research variable = the percentage of yield of the certain compound

d) Population = All of the households in the census

Sample = 5% of the households in the census

Research Variable = The number of rooms in the dwelling for all households

e) Population = All of the employees in the company

Sample = The records of all of the employees in the company

Research variable = The length of employment of all employees in the company

2.4.2 (a)

y	1	2	3	4	5
$P(y)$ = probability of occurence of each possible value of y	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

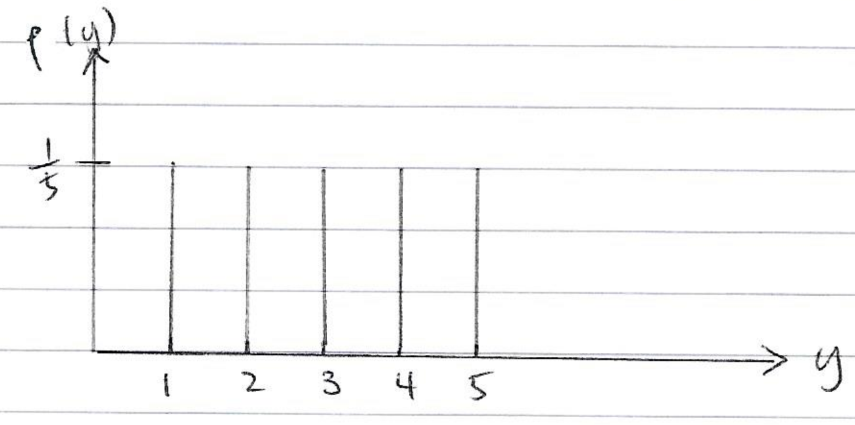
$\because$ For all values of y,

$$0 \le p(y) \le 1$$

$\because$ $\sum_y p(y) = \frac{1}{5} \times 5 = 1$

$\therefore$ $p(y) = \frac{1}{5}$ is a probability distribution.

(b)



(c) $P(y>3) = P(y=4) + P(y=5) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$ //

$P(y=3) = \frac{1}{5}$ //

$P(y \le 3) = P(y=3) + P(y=2) + P(y=1)$

$= \frac{1}{5} \times 3 = \frac{3}{5}$ //

$P(y<3) = P(y=2) + P(y=1) = \frac{1}{5} \times 2 = \frac{2}{5}$ //

2.5.2 a) Expected value $E(y) = \sum y\,p(y)$

$$= 0\left(\frac{1}{256}\right) + 1\left(\frac{12}{256}\right) + 2\left(\frac{54}{256}\right) + 3\left(\frac{108}{256}\right) + 4\left(\frac{81}{256}\right)$$
$$= 3 \,//$$

b) Variance $V(y) = \sum [y - E(y)]^2\, p(y)$

$$= (0-3)^2\left(\frac{1}{256}\right) + (1-3)^2\left(\frac{12}{256}\right) + (2-3)^2\left(\frac{54}{256}\right) + (3-3)^2\left(\frac{108}{256}\right) + (4-3)^2\left(\frac{81}{256}\right)$$

$$= \frac{9}{256} + \frac{48}{256} + \frac{54}{256} + \frac{81}{256} = 0.75 \,//$$

c) $E(y) - 2[sd(y)] = E(y) - 2\sqrt{V(y)}$

$$= 3 - 2 \cdot \sqrt{0.75}$$
$$= 1.26$$

$\therefore$ The probability that the random variable will be < 1.26 :

$$= P(y=1) + P(y=0)$$
$$= \frac{1}{256} + \frac{12}{256}$$
$$= \frac{13}{256} \,//$$

d) $E(y) + 2[sd(y)] = E(y) + 2\sqrt{V(y)}$

$$= 3 + 2 \cdot \sqrt{0.75}$$
$$= 4.73$$

$\therefore$ Probability that the random variable > 4.73

$= 0 \,//$

3.2.6 Refer to Table 3.2. Four Binomial Distributions

y	$b(y;20,0.3)$	$b(y;20,0.50)$	$b(y;20,0.70)$
0	0.001	0.000	0.000
1	0.007	0.000	0.000
2	0.028	0.000	0.000
3	0.072	0.001	0.000
4	0.130	0.005	0.000
5	0.179	0.015	0.000
6	0.192	0.037	0.000
7	0.164	0.074	0.001
8	0.114	0.120	0.004
9	0.065	0.160	0.012
10	0.031	0.176	0.031
11	0.012	0.160	0.065
12	0.004	0.120	0.114
13	0.001	0.074	0.164
14	0.000	0.037	0.192
15	0.000	0.015	0.179
16	0.000	0.005	0.130
17	0.000	0.001	0.072
18	0.000	0.000	0.028
19	0.000	0.000	0.007
20	0.000	0.000	0.001

a) let $\alpha = 0.05$, For $H_a = \pi > 0.3$

Region of rejection is $C_u \leq y \leq n$ such that $\sum_{C_u}^{n} b(y;20,0.3)$ is as close as possible to α.

$P(10)+P(11)+P(12)+P(13)+P(14)+P(15)+P(16)+$
$P(17)+P(18)+P(19)+P(20) = 0.048$

PTO →

$\therefore$ The region of rejection of the null hypothesis H_0 is $10 \leq y \leq 20$.

b) Find β for $(y; 20, 0.5)$, with $\alpha = 0.05$

Region of acceptance = $0 \leq y \leq 13$

$$\beta = \sum_{y=0}^{13} P(y) = 0.942$$

Power = $1 - 0.942$

$= 0.058$ //

c) Find β for $(y; 20, 0.7)$, with $\alpha = 0.05$

Region of acceptance = $0 \leq y \leq 17$

$$\beta = \sum_{y=0}^{17} P(y) = 0.964$$

Power = $1 - 0.964$

$= 0.036$ //

3.3.2

a) Best estimate $\hat{\pi} = \frac{175}{250} = 0.7$

where $\hat{\pi}$ is the proportion of federal offenders that have committed non-violent crimes.

b) Refer to Table A.5.d

Confidence intervals on the binomial parameter π, sample size $n = 250$, $\alpha = 0.10$

We enter the table at $y/n = 1 - 0.7$
$= 0.3$

The 95% confidence interval on the proportion of all federal prisoners convicted of nonviolent crimes:

$U = 1 - 0.252 = 0.748$
$L = 1 - 0.351 = 0.649$

$0.649 \leq \pi \leq 0.748$

c) $H_0 : \pi = 0.5$ (minority of inmates convicted)
$H_a : \pi > 0.5$ (majority of inmates convicted)

$\because \pi = 0.5$ does not fall in the 95% confidence interval for π,
where $0.649 \leq \pi \leq 0.748$

Therefore the null hypothesis can be rejected at 5% significance level. Also the lower bound of π is 0.649, which is greater than 0.5,

PTO

this tells us that the majority of inmates of all majority of all federal prisoners have been convicted of non-violent crimes.

6.1.4 Population mean $\mu = \sum yf$

$$\mu = 0(0.870) + 1(0.071) + 2(0.031) + 3(0.012) + 4(0.011) + 5(0.005)$$
$$= 0.238 \text{ courses}$$

6.2.6 a) Population mean $\mu = 6$ eggs

Sample mean $\bar{y} = \dfrac{8+2+5+7+4+10+6}{7}$
$= 6$ eggs

$\because \mu = \bar{y}$
$\therefore$ There is no evidence that the alkaloid causes the birds to lay fewer eggs than usual.

b) $s^2 = \dfrac{\sum y^2 - (\sum y)^2/n}{n-1}$

$\sum y^2 = 8^2 + 2^2 + 5^2 + 7^2 + 4^2 + 10^2 + 6^2$
$= 294$

$\sum y = 8+2+5+7+4+10+6$
$= 42$

$s^2 = \dfrac{294 - (42)^2/7}{6} = 7$ eggs

6.3.4 Referring to the table in Exercise 2.2.4 :

y	64	65	66	67	68	69	70	71	72	73	74	75
f	1	2	2	3	3	8	10	7	7	3	3	1

$\because$ we know for sampling distribution of averages :

1. $\mu_{\bar{y}} = \mu_y$

2. $\sigma^2_{\bar{y}} = \sigma^2_y / n$, where n = sample size

$\therefore E(\bar{y}) = E(y) = \mu_y$ and $V(\bar{y}) = \dfrac{V(y)}{n}$

a) $$E(\bar{y}) = \frac{64 + (65\times2) + (66\times2) + (67\times3) + (68\times3) + (69\times8) + (70\times10) + (71\times7) + (72\times7) + (73\times3) + (74\times3) + 75}{50}$$

$= 3500 \div 50 = 70$ inches //

b) $$V(y) = \frac{\sum y^2 - (\sum y)^2/N}{N}$$

$$\sum y^2 = 64^2 + 2(65)^2 + 2(66)^2 + 3(67)^2 + 3(68)^2 + 8(69)^2 + 10(70)^2 + 7(71)^2 + 7(72)^2 + 3(73)^2 + 3(74)^2 + 75^2$$
$= 245300$

$$(\sum y)^2/N = \frac{3500^2}{50} = 245000$$

$$V(y) = \frac{245300 - 245000}{50} = 6 \qquad \therefore V(\bar{y}) = \frac{6}{10} = 0.6 \text{ inches}$$ //