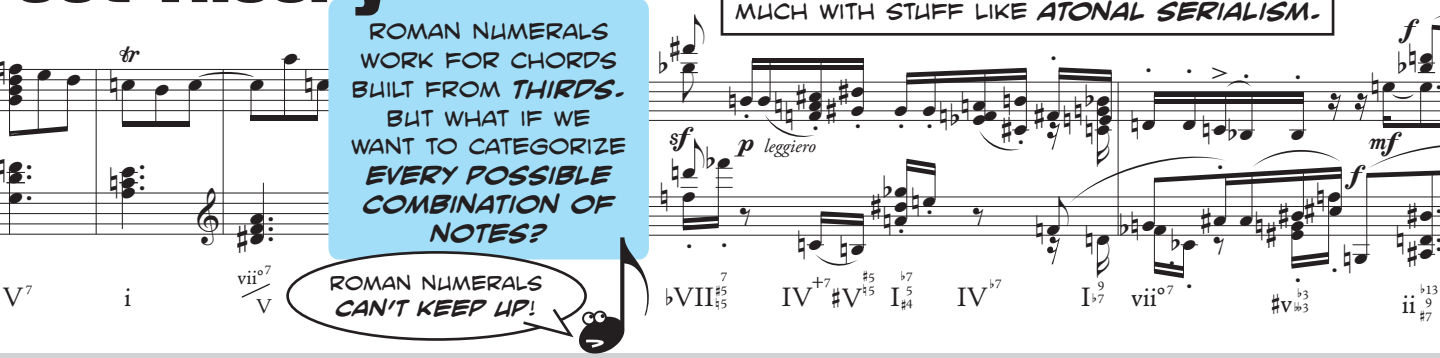
Set Theory

ROMAN NUMERALS WORK FOR CHORDS BUILT FROM **THIRDS.** BUT WHAT IF WE WANT TO CATEGORIZE **EVERY POSSIBLE COMBINATION OF NOTES?**

ROMAN NUMERALS **CAN'T KEEP UP!**

ROMAN NUMERAL ANALYSIS IS **GREAT** FOR **TONAL MUSIC,** BUT IT DOESN'T HELP MUCH WITH STUFF LIKE **ATONAL SERIALISM.**



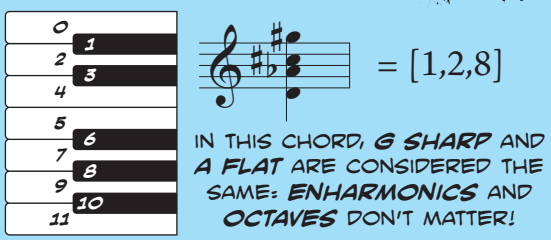
ONE OF THE MOST BASIC CHARACTERISTICS OF ANY CHORD IS HOW **CONSONANT** OR **DISSONANT** IT IS... SOMETHING THAT DEPENDS ENTIRELY ON WHICH **INTERVALS** ARE PRESENT IN THAT CHORD!

THE GOOD NEWS IS THAT **SET THEORY** DOES EXACTLY THAT! THE **BAD NEWS:** SET THEORY IS **MATH!**

PITCH CLASS SET

THE FIRST STEP TO ANALYZE A CHORD USING SET THEORY IS TO THINK ABOUT THE **PITCHES** IT CONTAINS. THIS IS **MATH,** SO INSTEAD OF USING LETTER NAMES WE'LL USE **NUMBERS...** WHERE **C** IS ALWAYS **ZERO.**

TAKE THOSE NUMBERS, REMOVE ANY **DUPLICATES,** AND LIST THEM IN **BRACKETS** LIKE THIS: **[1,2,3].**

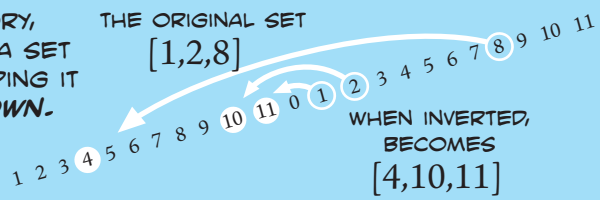


IN THIS CHORD, **G SHARP** AND **A FLAT** ARE CONSIDERED THE SAME: **ENHARMONICS** AND **OCTAVES** DON'T MATTER!

INVERSION

IN SET THEORY, **INVERTING** A SET MEANS FLIPPING IT **UPSIDE-DOWN.**

THE ORIGINAL SET [1,2,8]



WHEN INVERTED, BECOMES [4,10,11]

WE CAN DO THIS WITH **MATH** BY TAKING ALL **NON-ZERO** NUMBERS AND **SUBTRACTING THEM FROM 12.**

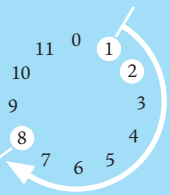
ORIGINAL:	[12 − 1	12 − 2	12 − 8]
INVERSION:	[11	10	4]

NORMAL FORM

THE **NORMAL FORM** OF A SET IS THE **MOST COMPACT ORDERING** OF THE SET. WE DEFINE **"MOST COMPACT"** AS THE ARRANGEMENT WITH THE **SMALLEST INTERVALS!**

IT'S EASIEST TO DO THIS BY THINKING OF THE PITCHES IN A **CIRCLE** AND MEASURING THE **DISTANCE AROUND!**

JUST MAKE SURE TO ALWAYS MEASURE GOING **CLOCKWISE.**



[1,2,8]: 1 2 3 4 5 6 7 8

[2,8,1]: 2 3 4 5 6 7 8 9 10 11 0 1

[8,1,2]: 8 9 10 11 0 1 2 ← **NORMAL FORM!**

PRIME FORM

TO FIND A SET'S **PRIME FORM,** FIND THE **MOST COMPACT** OF A SET'S **NORMAL FORM** AND THE NORMAL FORM OF **ITS INVERSION.** THEN **TRANSPOSE** THAT SET SO IT STARTS ON **ZERO!**

NORMAL FORM: [8,1,2]: 8 9 10 11 0 1 2

NORMAL FORM OF INVERSION: [10,11,4]: 10 11 0 1 2 3 4

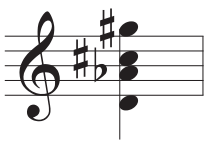
THESE SETS SPAN THE **SAME DISTANCE...** SO TO DECIDE WHICH IS **MOST COMPACT,** WE COMPARE THE **NEXT LARGEST INTERVAL** IN EACH SET!

LASTLY, WE **TRANSPOSE IT** SO IT STARTS ON **ZERO:** 0 1 2 3 4 5 6

SO THE **PRIME FORM** OF [1,2,8] IS **[0,1,6]!**

SO **SET THEORY** IS TELLING US THAT THESE TWO SETS HAVE SOMETHING **IMPORTANT** IN COMMON. **WHAT IS IT?**

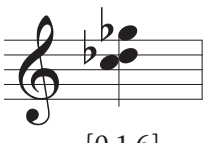
LET'S TALLY UP ALL THE **INTERVALS** IN OUR ORIGINAL SET. (AND **INVERT** ANY INTERVALS LARGER THAN A **TRITONE** AND **SIMPLIFY** ANY **ENHARMONICS!**)



[1,2,8]

D–A♭	d5	TT
D–C♯	M7	m2
D–G♯	A4	TT
A♭–C♯	A3	P4
A♭–G♯	A7	P1
C♯–G♯	P5	P4

NOW LET'S DO THE SAME THING FOR THE SET IN ITS **PRIME FORM!**



[0,1,6]

C–D♭	m2	m2
C–G♭	d5	TT
D♭–G♭	P4	P4

PRIME FORM IS A WAY TO DESCRIBE **ANY SET** BY ITS **BASIC INTERVALS!**