January 5, 2026

Editorial Board
Information Geometry

Dear Editors,

I submit **"Alignment Probabilities on Product Statistical Manifolds"** for consideration.

Summary. For M independent von Mises random variables, the probability that all phases fall within ε of each other scales as p_1^{M-1}, where p_1 is the single-variable window probability. The exponent $(M-1)$ rather than M arises from quotient geometry: alignment is invariant under global rotation, so one phase serves as reference.

Contributions.

1. Rigorous proof of the $(M-1)$ exponent via quotient space analysis
2. Characterization of p_1 via Fisher information on the circle
3. Hitting time scaling $\tau \propto p_1^{-(M-1)}$, exponential in coordination depth
4. Perturbative analysis showing coupling yields effective exponent $\alpha(M-1)$

Why Information Geometry. The $(M-1)$ exponent equals the codimension of the alignment constraint after quotienting by the diagonal S^1 action. The window probability p_1 is characterized via Fisher information of the von Mises family. These are fundamentally information-geometric results independent of application domain.

Companion work. A separate paper (under review at BioSystems) applies this framework to neural synchronization. The present paper contains the pure mathematics; the companion focuses on empirical validation. The submissions are independent.

Potential reviewers: S.-I. Amari (RIKEN), K. Mardia (Leeds/Oxford), F. Nielsen (Sony CSL).

Sincerely,

Ian Todd
Sydney Medical School, University of Sydney
itod2305@uni.sydney.edu.au