Sydney Medical School
University of Sydney
Sydney, NSW 2006
Australia
itod2305@uni.sydney.edu.au

December 24, 2025

Editorial Office
Information Geometry
Springer

Dear Editors,

I am pleased to submit the manuscript entitled "Quotient Geometry of Statistical Manifolds Under Dimensional Collapse" for consideration in *Information Geometry*.

This paper develops the differential geometry of statistical manifolds under collapse maps—smooth maps $\pi : \mathcal{M} \to R^k$ with $k < \dim(\mathcal{M})$. The work generalizes the minimal embedding theorem for recurrent processes (manuscript INGE-D-25-00099, currently under review at this journal) to arbitrary dimensional reduction on statistical manifolds.

The main contributions are:

1. **Fiber Structure Theorem:** We show that collapse maps foliate $\mathcal{M}$ into fibers along which the Fisher metric degenerates. Points on the same fiber are statistically non-identifiable, providing a differential-geometric interpretation of Watanabe's singular learning theory.

2. **Quotient Metric Theorem:** We characterize when the Fisher metric descends to a well-defined Riemannian metric on the quotient $\mathcal{M}/\sim_\pi$, establishing that fibers must be totally geodesic. We show how α-connections transform under collapse and identify conditions for dual flatness to survive.

3. **Covering Number Bounds:** We prove that the number of ε-distinguishable equivalence classes scales as $N(\varepsilon) \sim \varepsilon^{-r}$ where r is the projection rank, quantifying the "discretization" induced by dimensional collapse.

The paper connects to core themes of information geometry: Chentsov's uniqueness theorem (the Fisher metric is invariant under sufficient statistics; our quotient construction respects this), Amari's α-geometry (we characterize when dual connections descend), and Watanabe's singular learning theory (our fiber structure provides the differential-geometric setting for his algebraic singularities).

This work is appropriate for *Information Geometry* because it develops fundamental Riemannian and affine geometry on statistical manifolds, extending classical results on submersions and quotients to the information-geometric setting.

The manuscript has not been submitted elsewhere. There are no conflicts of interest to declare.

Thank you for considering this submission.

Sincerely,

Ian Todd
Sydney Medical School
University of Sydney