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Article

Discreteness from Identifiability: Why Continuous Physics Produces Integers

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Abstract - Discrete phenomena pervade physics: quantum numbers, particle species, phase transitions, symbolic codes. Yet the underlying state spaces are typically continuous. We identify a unifying mechanism: *discrete transitions emerge from integer-valued invariants of observation maps*. When a continuous high-dimensional system is observed through a constraint, symmetry, or projection, the observation map carries integer-valued invariants—rank, dimension, index—that can change only discretely. The *identifiable dimension* (rank of the observation differential) controls how many effective degrees of freedom are accessible; when this rank changes, qualitative transitions occur. We demonstrate this mechanism in three domains: (1) information geometry, where Fisher rank activation under coupling produces emergent statistical coordinates; (2) quantum mechanics, where measurement partitions state space and integer invariants (degeneracies, quantum numbers) label the structure; (3) symbol formation, where tokens emerge as stable equivalence classes in phase space under finite-resolution observation. We discuss connections to anthropic reasoning: observers require a separation between high-dimensional microstates and lower-dimensional accessible interfaces, suggesting that *effective dimensionality gradients* are necessary for observation. Discreteness is not fundamental but *observational*—a necessary consequence of finite-capacity observers accessing high-dimensional substrates through maps with integer-valued invariants.

Keywords - Discreteness; Identifiability; Quotient geometry; Fisher information; Emergence; Observation

1 Introduction

Physics repeatedly produces integers from continuous substrates. Quantum mechanics yields discrete spectra. Statistical mechanics produces phase transitions at critical points. Biological systems generate symbolic codes. Digital computation emerges from analog circuits. In each case, the underlying dynamics are continuous, yet the observed phenomena are discrete.

The standard explanations are domain-specific: quantum discreteness comes from eigenvalue equations; phase transitions from symmetry breaking; codes from evolutionary selection. But the pattern is too universal to be coincidental. We propose a common mechanism: **discreteness emerges from identifiability**.

The argument is geometric. Consider a smooth observation map $h : X \to Y$ from a high-dimensional state space X to an observation space Y. The differential dh determines which directions in X produce distinguishable outputs in Y. The rank of dh—the dimension of the identifiable subspace—can only take integer values: $\mathrm{rank}(dh) \in \{0, 1, 2, \ldots, \min(\dim X, \dim Y)\}$.

When parameters or state change the structure of dh, the rank can increase or decrease. But it can only jump by integers. Smooth variation in the underlying physics produces discrete transitions in what can be observed.

This paper develops this insight across three domains and connects it to cosmological structure. The goal is not to replace existing explanations but to identify the common geometric mechanism that makes discrete phenomena *inevitable* whenever continuous systems are observed through finite-capacity channels.

2 The Quotient-Geometric Mechanism

2.1 Observation as Projection

Let X be a smooth manifold representing the full state space of a physical system, and let $h : X \to Y$ be a smooth observation map. Two states $x_1, x_2 \in X$ are *observationally equivalent* if $h(x_1) = h(x_2)$. This equivalence relation partitions X into fibers:

$$[x] := \{x' \in X : h(x') = h(x)\}. \tag{1}$$

The quotient space $X/\sim$ is the space of distinguishable observations. When h is a submersion onto its image (constant rank equal to $\dim h(X)$), the quotient inherits a manifold structure locally diffeomorphic to the image. More generally, the quotient may be a stratified space, with smooth strata corresponding to regions of constant rank.

2.2 Rank as Identifiable Dimension

The differential $dh_x : T_xX \to T_{h(x)}Y$ determines which infinitesimal variations at x produce distinguishable changes in observation. The *identifiable dimension* at x is:

$$r(x) := \mathrm{rank}(dh_x). \tag{2}$$

This is the dimension of the subspace of state-space directions that "matter" for observation.

Key property. The rank function $r : X \to \mathbb{Z}_{\geq 0}$ is integer-valued by definition. It is lower semicontinuous: under limits, rank can *drop* but cannot increase. (The set of points with rank $\leq k$ is closed.) Rank deficiencies occur on closed, typically nongeneric sets; changes correspond to structural singularities such as bifurcations, symmetry breaking, or coupling activation.

2.3 Why Integer Invariants Matter

The appearance of integers is not contingent but geometric. Any smooth map between manifolds carries integer-valued invariants: rank, dimension of fibers, dimension of image, indices. These can change only discretely.

What this does NOT imply. A quotient structure does not automatically produce a discrete set of equivalence classes. Projecting $\mathbb{R}^2 \to \mathbb{R}$ via $(x, y) \mapsto x$ yields equivalence classes (vertical lines) indexed by $\mathbb{R}$—a continuous quotient. Discreteness of *observed outcomes* requires additional structure: finite resolution, compactness, spectral gaps, or stability constraints. What *is* always discrete are the integer-valued invariants (rank, dimension, index) that characterize the observation map itself.

The physically significant claim is this: if the identifiable dimension $r(x)$ controls how many effective degrees of freedom are accessible to an observer, then any qualitative change in what can be represented, encoded, or controlled will occur when $r(x)$ changes—and $r(x)$ changes only in integer steps.

Proposition (Capacity Scaling). Let $h : X \to Y$ be a smooth observation map, and let $U \subseteq X$ be an open region where $\mathrm{rank}(dh) = r$ is constant. Equip Y with a metric. Then:

(a) The image $h(U)$ is locally an r-dimensional submanifold of Y.

(b) If additionally $h(U)$ is bounded with finite r-dimensional volume V_r, then the ε-covering number satisfies:

$$N(\varepsilon) \sim C \cdot V_r \cdot \varepsilon^{-r} \tag{3}$$

for small ε, where C depends on the local geometry.

(c) Changes in r produce discrete changes in capacity scaling exponents.

Proof sketch. Part (a) follows from the **constant rank theorem**: if $\text{rank}(dh) = r$ on U, then locally h is diffeomorphic to the standard projection $\mathbb{R}^n \to \mathbb{R}^r \times \{0\}^{m-r}$. The image is thus locally an r-dimensional embedded submanifold.

Part (b) is standard from metric geometry: an r-dimensional Riemannian manifold of volume V_r requires $\Theta(\varepsilon^{-r})$ balls of radius ε to cover. The scaling exponent equals the dimension.

Part (c) follows because the exponent in (b) is exactly r, which is integer-valued. A transition from rank r to rank $r+1$ changes the capacity scaling from $O(\varepsilon^{-r})$ to $O(\varepsilon^{-(r+1)})$—a multiplicative factor of ε^{-1} that diverges as resolution improves. □

Key assumptions. The proposition requires: (1) constant rank on U (if rank varies, the image is stratified and the dominant stratum determines scaling); (2) boundedness for finite covering numbers; (3) sufficient regularity (Lipschitz bounds) linking ε-balls in Y to preimages in X. These assumptions are satisfied in typical physical applications where phase space is bounded and maps are smooth.

This connects integer ranks to observable phenomena: number of order parameters, identifiable couplings, capacity scaling laws, and effective degrees of freedom.

3 Information Geometry: Rank Activation Under Coupling

The first application is to coupled dynamical systems, using the information-geometric framework developed in [1].

3.1 Fisher Rank as Identifiable Dimension

Consider a family of probability distributions $\{p_\theta : \theta \in \Theta\}$ parameterized by θ. The Fisher information matrix is:

$$g_{ij}(\theta) = \mathbb{E}_{p_\theta}\left[\frac{\partial \log p_\theta}{\partial \theta^i}\frac{\partial \log p_\theta}{\partial \theta^j}\right]. \tag{4}$$

The *Fisher rank* $R(\theta) := \text{rank}(g(\theta))$ counts identifiable parameters—directions in parameter space that produce statistically distinguishable distributions.

3.2 Coupling Activates Rank

Fisher rank deficiency—where $g_{\kappa\kappa} = 0$ for a coupling parameter κ—arises under specific structural conditions:

1. **Structural absence**: κ is not a parameter in the uncoupled model (different model classes at $\kappa = 0$ vs $\kappa > 0$).

2. **Symmetry forcing**: The distribution has a symmetry (e.g., $\kappa \leftrightarrow -\kappa$) that forces $\partial p/\partial\kappa|_{\kappa=0} = 0$.

3. **Observation restriction**: The observable channel cannot distinguish κ-dependent effects below threshold.

Clarification. Factorization at $\kappa = 0$ does *not* automatically imply $g_{\kappa\kappa}(0) = 0$. A model can factor at $\kappa = 0$ while still having nonzero first derivative there—the coupling "turns on" smoothly. Rank deficiency requires one of the conditions above.

When coupling activates and rank deficiency is lifted, new directions in parameter space become identifiable. The Fisher rank increases:

$$R(\kappa > 0) > R(0). \tag{5}$$

This is **manifold expansion**: coupling reveals new statistical coordinates that were hidden at $\kappa = 0$. The rank jump is discrete—from $R(0)$ to $R(\kappa)$ without intermediate values. A continuous change in coupling produces a discrete change in identifiable dimension.

3.3 Example: Coupled Oscillators

In Kuramoto oscillators with coupling K, the order parameter (r, Ψ) becomes identifiable only above a critical coupling K_c. Below threshold, all coupling values produce statistically indistinguishable outputs (for observations restricted to (r, Ψ)). Above threshold, the Fisher rank jumps from 0 to 2. This discrete transition—the synchronization transition—emerges from continuous dynamics through the rank mechanism.

4 Quantum Mechanics: Integer Invariants from Measurement

4.1 Measurement as Quotient

In quantum mechanics, measurement of observable $\hat{A}$ with spectral decomposition $\hat{A} = \sum_\lambda \lambda P_\lambda$ induces an observation map on density matrices:

$$h(\rho) = \{\mathrm{Tr}(\rho P_\lambda)\}_\lambda \tag{6}$$

which assigns to each state ρ its *outcome probability distribution*. Two states are observationally equivalent with respect to $\hat{A}$ if they produce the same probability distribution over outcomes:

$$\rho_1 \sim \rho_2 \iff \mathrm{Tr}(\rho_1 P_\lambda) = \mathrm{Tr}(\rho_2 P_\lambda) \text{ for all } \lambda. \tag{7}$$

This equivalence relation partitions state space; measurement induces a quotient structure.

4.2 When Spectra Are Discrete (and When They Are Not)

The quotient structure itself does not guarantee discrete outcomes. Many observables have *continuous spectrum*: position, momentum, and the free-particle Hamiltonian all yield continuous ranges of eigenvalues. The quotient labels form a continuum.

(For continuous spectrum, the spectral decomposition uses projection-valued measures rather than discrete sums, and "eigenspaces" are generalized; the quotient formalism extends via spectral measures.)

Discrete spectra arise from additional structure:

- **Boundary conditions**: confinement to a bounded region quantizes energy levels.
- **Decay conditions**: requiring square-integrability at infinity selects discrete bound states.
- **Compactness**: the hydrogen atom's discrete spectrum reflects the compact effective configuration space.
- **Finite resolution**: even continuous spectra become discrete when binned by finite-precision measurement.

What *is* always integer-valued are the invariants characterizing the spectral structure: degeneracies of eigenspaces, dimensions of irreducible representations, quantum numbers labeling symmetry multiplets. These are the integer invariants of quantum mechanics.

4.3 Collapse as Projection

Quantum “collapse” is projection from the full state to an equivalence class. This is the same geometric move as dimensional reduction: high-dimensional continuous dynamics, viewed through a projection, produce outputs labeled by equivalence classes.

The probabilistic aspect (Born rule) determines *which* class; the quotient structure determines *that* classes exist. Whether the labels form a discrete or continuous set depends on the observable and boundary conditions, not on the quotient structure alone.

5 Symbol Formation: Tokens as Equivalence Classes

5.1 Codes from Dynamics

Biological and cognitive systems produce discrete symbols (genetic codons, phonemes, concepts) from continuous underlying dynamics. The standard explanation invokes evolutionary selection for error-robustness. But this begs the question: why are discrete codes *possible*?

The answer involves quotient geometry plus finite resolution. A symbol is a stable equivalence class in phase space: a region where many different microstates produce the same functional output under a finite-resolution observation channel.

5.2 Stability and Finite Resolution

For a symbol to be reliably transmitted, it must be robust to perturbation. This requires that small changes in the underlying state do not change the symbol—i.e., that the symbol labels an *open* set in state space.

Required assumptions. Discrete symbol alphabets require: (1) finite resolution $\varepsilon > 0$ in the observation channel, (2) bounded energy or phase space volume, and (3) stability constraints that collapse continuous microvariation into finitely many robust macrostates.

Under these assumptions, the number of distinguishable stable classes is finite. Without them, the quotient could be continuous.

5.3 The Alphabet Size

Given finite resolution ε and an observation map $h : X \to Y$ with identifiable dimension $r = \mathrm{rank}(dh)$, the number of distinguishable symbols scales as:

$$|\text{alphabet}| \sim O(\varepsilon^{-r}). \tag{8}$$

This is a packing-number argument: an r-dimensional manifold admits $O(\varepsilon^{-r})$ ε-separated points. Higher rank admits larger alphabets; lower rank forces coarser coding.

This connects to the “minimal embedding dimension” results [2]: cyclic processes require $k \geq 3$ for continuous dynamics; below this threshold, dynamics collapse into discrete categories. The discreteness is not chosen but *forced* by dimensional constraints combined with finite resolution.

6 Observation Requires Dimensional Gradients

We now prove that useful observation requires a gap between environment dimension and representation dimension. This upgrades the anthropic discussion from conjecture to theorem.

6.1 Functional Definition of Observer

We define observers by what they do, not what they are made of:

Definition (Observer). An *observer* is a physical subsystem S with state space $\mathcal{S}$ equipped with:

1. An *observation map* $h : X \to \mathcal{S}$ from environment states to internal states
2. *Stability*: small perturbations of $x \in X$ do not change the representation $h(x)$ (equivalently: h has nontrivial fibers)
3. *Utility*: the representation $h(x)$ enables prediction or control of future environment states

This definition is substrate-independent: it applies to brains, measuring devices, thermostats, or any system that forms stable, useful representations.

6.2 The Dimensional Gradient Theorem

Theorem. If $h : X \to \mathcal{S}$ satisfies the observer definition, then:

$$0 < \operatorname{rank}(dh) < \dim X. \tag{9}$$

Proof.

(i) Rank > 0:

Suppose $\operatorname{rank}(dh_x) = 0$ for all $x \in X$. Then $dh_x = 0$ everywhere, which means h is locally constant on each connected component of X.

Let $U \subseteq X$ be a connected open set. For any $x_1, x_2 \in U$, we can connect them by a path $\gamma : [0, 1] \to U$. Then:

$$h(x_2) - h(x_1) = \int_0^1 dh_{\gamma(t)}(\dot{\gamma}(t))\, dt = 0 \tag{10}$$

since $dh = 0$ everywhere. Thus h is constant on U.

If h is constant on connected components, then for any x_1, x_2 in the same component, $h(x_1) = h(x_2)$. The representation provides no discrimination between states in the same component. But Utility requires that $h(x)$ enables prediction or control, which is impossible if h cannot distinguish states.

Formally: let $f : X \to \mathbb{R}$ be any predictable quantity (e.g., future reward, system evolution). For h to enable prediction of f, there must exist a map $\hat{f} : \mathcal{S} \to \mathbb{R}$ such that $\hat{f}(h(x)) \approx f(x)$. But if $h(x_1) = h(x_2)$ for all x_1, x_2 in a component, then $\hat{f}(h(x))$ is constant on that component, hence cannot approximate any non-constant f.

$\Rightarrow$ Utility is violated $\Rightarrow$ $\operatorname{rank}(dh) > 0$ somewhere. By continuity and the definition requiring this to hold throughout the observer's domain, we need $\operatorname{rank}(dh) > 0$. ✓

(ii) Rank $< \dim X$:

Suppose $\operatorname{rank}(dh_x) = \dim X$ for all $x \in X$ (and $\dim \mathcal{S} \geq \dim X$). Then by the inverse function theorem, h is a local diffeomorphism onto its image—in particular, h is locally injective.

Local injectivity means: for each x, there exists a neighborhood U_x such that $h|_{U_x}$ is injective. Equivalently, the fiber $h^{-1}(h(x)) \cap U_x = \{x\}$ is a singleton.

But Stability requires that small perturbations of x do not change the representation. Formally, Stability means there exists $\varepsilon > 0$ such that for all $\|x' - x\| < \varepsilon$, we have $h(x') = h(x)$—i.e., the fiber $h^{-1}(h(x))$ contains a neighborhood of x.

These are contradictory:

- Local injectivity $\Rightarrow$ fiber $\cap$ neighborhood $= \{x\}$
- Stability $\Rightarrow$ fiber $\supseteq$ neighborhood of x

Therefore, Stability and $\operatorname{rank}(dh) = \dim X$ are incompatible.

$\Rightarrow$ Stability requires $\operatorname{rank}(dh) < \dim X$. ✓

Combining (i) and (ii): Any h satisfying the observer definition must have $0 < \operatorname{rank}(dh) < \dim X$. □

Remark. The inequality is strict on both sides. The proof shows that the boundary cases (rank $= 0$ or rank $= \dim X$) violate the core requirements of Utility and Stability respectively. The "sweet spot" for observation is $0 < r < n$, which is precisely what we call a dimensional gradient.

6.3 The Anthropic Implication

The theorem implies:

Corollary (Dimensional Gradient Necessity). Any observer, in any physical substrate, requires:

1. A high-dimensional environment X (source of information)
2. A lower-dimensional representation space $\mathcal{S}$ (compressed model)
3. An observation map h with $0 < \text{rank}(dh) < \dim X$

This is the "dimensional gradient" that observation requires.

Note what this does *not* require:

- Specific physical constants
- Carbon-based chemistry
- Particular cosmological conditions

It requires only that *some* dimensional gradient exists. The traditional fine-tuning puzzles (why these constants?) are reframed: constants must permit the formation of stable, compressive interfaces. This is a weaker requirement than "constants must permit carbon life."

6.4 A Philosophical Observation (Speculative)

The theorem suggests—but does not prove—a dissolution of "why is there something rather than nothing":

1. "Something" operationally means "identifiable structure."
2. To ask the question is to observe.
3. To observe requires $0 < \text{rank}(dh) < \dim X$ (by the theorem).
4. If $\text{rank}(dh) > 0$, the observer can make distinctions.
5. Therefore: any observer will find identifiable structure.

The alternative—$\text{rank}(dh) = 0$—is not "nothing"; it is the absence of observation. This is a *selection effect* argument, not an ontological claim. The theorem shows that observers require dimensional gradients; it does not show that dimensional gradients require observers, nor that "nothing" is impossible.

6.5 Remaining Gaps

The theorem proves that dimensional gradients are *necessary* for observation. It does not prove:

- That dimensional gradients are *sufficient* (other conditions may also be required)
- That our universe is the only way to achieve such gradients
- Any specific connection between rank and cosmological parameters

The theorem is local (about dh at a point) and functional (about what observers do). Connecting it to global cosmological structure remains open.

7 Worked Example: Rank Drop on a Singular Locus

To ground the abstract framework, we present a canonical example where rank changes genuinely within a single smooth map.

7.1 Setup

Consider the observation map $h : \mathbb{R}^2 \to \mathbb{R}^2$ defined by:

$$h(x, y) = (x,\, xy). \tag{11}$$

Here x can be interpreted as a coupling strength and y as an amplitude or internal coordinate.

7.2 Rank Analysis

The Jacobian of h is:

$$dh_{(x,y)} = \begin{pmatrix} 1 & 0 \\ y & x \end{pmatrix}. \tag{12}$$

The determinant is $\det(dh) = x$. Therefore:

- $\operatorname{rank}(dh) = 2$ when $x \neq 0$ (generic points)
- $\operatorname{rank}(dh) = 1$ when $x = 0$ (the singular locus)

This is a *genuine* rank change within a single smooth map. The rank drops discretely—from 2 to 1—on the singular set $\{x = 0\}$.

7.3 Identifiability Interpretation

Away from the singular locus ($x \neq 0$): Both coordinates of h vary independently. From observing (x, xy), we can recover x directly and compute $y = (xy)/x$. The full state (x, y) is identifiable; the identifiable dimension equals $\dim X = 2$.

On the singular locus ($x = 0$): The observation is $h(0, y) = (0, 0)$ for all y. The entire y-axis collapses to a single point. The amplitude y becomes completely unidentifiable; the identifiable dimension drops to 1.

7.4 The Discrete Transition

Consider a path crossing the singular locus: $\gamma(t) = (t, 1)$ for $t \in [-1, 1]$.

- At $t < 0$: $\operatorname{rank}(dh) = 2$, both coordinates identifiable
- At $t = 0$: $\operatorname{rank}(dh) = 1$, amplitude y unidentifiable
- At $t > 0$: $\operatorname{rank}(dh) = 2$, both coordinates identifiable

The rank function $t \mapsto \operatorname{rank}(dh_{\gamma(t)})$ is integer-valued and lower semicontinuous: it can drop (at $t = 0$) but such drops occur on measure-zero sets. Moving away from the singular locus, the rank jumps back up discretely.

7.5 Physical Interpretation

This structure appears whenever a coupling parameter "turns on" an interaction:

- **Phase transitions**: An order parameter y becomes observable only when symmetry-breaking coupling x is nonzero.
- **Synchronization**: Individual phases become distinguishable only above critical coupling.
- **Measurement**: A quantum amplitude becomes resolvable only when the detector couples to it.

The key point: the discrete rank change is not an artifact of changing the parameter space or model class. It is a property of the single map h evaluated at different points in its domain.

8 The Structural Spine

The preceding applications share a common structure:

Domain	**Observation map h**	**Integer invariant**
Information geometry	Score function	Fisher rank
Quantum mechanics	Measurement projection	Degeneracy, quantum numbers
Symbol formation	Output function	Alphabet size (at finite ε)

In each case:

1. The substrate is continuous and high-dimensional.
2. Observation defines an equivalence relation via projection.
3. The map carries integer-valued invariants (rank, dimension, index).
4. Structural changes (coupling, measurement, bifurcation) produce discrete transitions in these invariants.

Note: we have removed "cosmology" from this table, as the connection between local rank and cosmological structure is conjectural (see Discussion).

This is not a claim that all discrete phenomena reduce to one mechanism. It is a claim that *integer-valued invariants of observation maps* underlie diverse manifestations of discreteness.

9 Discussion

9.1 Discreteness Is Observational, Not Ontological

We propose a shift in perspective: from "the world is discrete" to "observation produces discreteness." The integers in physics are not stamped into reality at the Planck scale; they emerge whenever continuous dynamics are accessed through finite-capacity channels.

This does not make discreteness illusory. The quotient structure is real, objective, and measurable. But it is a property of the observation relation, not of the substrate alone.

9.2 Relation to the Infodynamics Framework

This paper extends the infodynamics framework developed in [3,4,5]. The geometric maintenance bound [3] shows that asymmetric states cost work. The aperture theory [4] shows that time is observer-relative information rate. The relaxation framework [5] grounds both in cosmological dynamics.

The present paper adds a complementary insight: the *discreteness* of structure is also observation-dependent. Integers emerge from the rank of the observation map, just as time dilation emerges from aperture contraction.

9.3 Implications for Quantization

If discreteness arises from observation rather than substrate, then "quantization" may be misnamed. We are not discovering that reality is made of discrete chunks. We are discovering that our observation channels have finite rank, and finite rank produces integer-valued invariants.

This suggests a different approach to quantum gravity: rather than quantizing the metric (making spacetime discrete), we might characterize the observation interface through which the metric becomes accessible [6]. Discreteness would then be a property of the channel, not of spacetime itself.

10 Conclusion

We have identified a unifying mechanism for discrete transitions in physics: **observation maps carry integer-valued invariants that can change only discretely**.

The key invariant is the *identifiable dimension*—the rank of the observation differential—which controls how many effective degrees of freedom are accessible. When this rank changes, qualitative transitions occur in what observers can represent, encode, or control.

This mechanism operates across domains:

- In information geometry, Fisher rank counts identifiable parameters; coupling activation produces rank jumps.
- In quantum mechanics, degeneracies and quantum numbers are integer invariants of the measurement structure.
- In symbol formation, alphabet size at finite resolution scales as ε^{-r} where r is the identifiable dimension.

What we claim. Observation maps carry integer-valued invariants (rank, dimension, index). Discrete transitions in observed phenomena correspond to discrete changes in these invariants.

What we do NOT claim. Quotients automatically produce discrete equivalence classes. (They don't—see §2.3.) Continuous spectra don't exist. (They do—see §4.2.) Dimensional gradients are sufficient for observers. (They're necessary, not sufficient—see §6.5.)

The ubiquity of integers in physics is not mysterious once we recognize that observation maps have integer-valued invariants by geometric necessity. The "discreteness problem" shifts from "why is reality discrete?" to "why do observation channels have finite rank?"—and the latter has a straightforward answer: finite-capacity systems cannot resolve infinite-dimensional structure.

The Dimensional Gradient Theorem (§6) proves that any useful observer requires $0 < \mathrm{rank}(dh) < \dim X$. This is the minimal anthropic condition: not carbon, not water, not specific constants—just a gap between environment complexity and representation complexity. The traditional fine-tuning puzzles are reframed as questions about whether stable, compressive interfaces can form.

Discreteness is not fundamental; identifiability is.

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