

# Seizure as Critical Transition: Dimensional Signatures of Approaching Collapse in EEG

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## Abstract

Seizures represent a critical phase transition in neural dynamics—from the flexible, multi-scale activity of healthy brain states to the hypersynchronous collapse of ictal discharge. We propose that the approach to seizure follows the signature pattern of critical transitions: **increasing coordination with decreasing variability**. Using the CHB-MIT scalp EEG database (subject chb01: 42 recordings, 7 seizures, 131,584 analysis windows), we show that pre-ictal periods exhibit (1) *higher* participation ratio  $D_{PR}$  than interictal baseline (5.69 vs. 5.05,  $p < 10^{-270}$ ), indicating more coordinated oscillatory modes, but (2) dramatically *reduced* variance ( $4.9\times$  reduction in  $D_{PR}$  variance,  $7.0\times$  in  $\lambda_1$  variance), indicating the system has settled into a more stereotyped attractor. The ictal state then shows collapse: lower  $D_{PR}$  (5.04), steeper eigenvalue slope ( $-3.19$  vs.  $-2.76$ ), and fewer components required for variance thresholds ( $K_{95} = 9.4$  vs.  $10.6$ ). This pattern—organization preceding collapse—mirrors critical phenomena in physics, where correlation length diverges before phase transitions. We propose that “brain rate variability” (the variance of dimensional metrics over time) may be a more sensitive seizure warning than the metrics themselves, analogous to how loss of heart rate variability predicts cardiac events. Classification of pre-ictal vs. interictal windows achieves  $AUC = 0.68$  using dimensional features alone.

**Keywords:** seizure prediction, critical transition, EEG complexity, effective dimensionality, participation ratio, neural criticality

## 1 Introduction

### 1.1 The Clinical Problem

Epilepsy affects approximately 50 million people worldwide, with roughly one-third of patients experiencing seizures that remain uncontrolled despite medication [World Health Organization, 2019]. The unpredictability of seizures is often more disabling than the seizures themselves—patients cannot drive, swim alone, or live without constant vigilance. A reliable early warning system would transform quality of life.

Decades of research have sought EEG biomarkers that predict seizures minutes to hours in advance. While many features show statistical differences between interictal (between-seizure) and pre-ictal (before-seizure) periods, clinical translation has been limited [Mormann et al., 2007, Kuhlmann et al., 2018]. We argue this reflects a conceptual gap: most approaches seek features that *decrease* before seizures, assuming pre-ictal states are “simpler” or “more pathological.” Our data suggest the opposite pattern.

## 1.2 The Critical Transition Hypothesis

Critical phase transitions in physical systems exhibit a characteristic signature: **increasing correlation with decreasing fluctuation**. As a system approaches a critical point:

- Correlation length increases—distant parts of the system become more coordinated
- Fluctuations around the mean decrease—the system settles into a more stereotyped state
- Then the transition occurs—an avalanche, a phase change, a collapse

We propose that seizure follows this pattern:

**Seizure is a critical transition.** The pre-ictal state is not “simpler” than interictal—it is *more organized*. Neural oscillators that normally operate semi-independently become increasingly coordinated. The system settles into a narrower attractor with reduced variability. This organization is the buildup; seizure is the avalanche.

This reframes the warning signs. The signature of impending seizure is not “things getting simpler” but “things getting too organized”—more coordination, less wandering, reduced variability in complexity metrics themselves.

## 1.3 Effective Dimensionality as a Window into Criticality

We operationalize “coordination” through the eigenvalue spectrum of the EEG channel covariance matrix. The participation ratio  $D_{\text{PR}}$  measures how many independent modes contribute to the signal:

$$D_{\text{PR}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2} \quad (1)$$

Counterintuitively, *higher*  $D_{\text{PR}}$  can indicate *more* coordination—when previously independent oscillators begin to synchronize, they form coherent modes that become visible in the covariance structure. The key insight is that we must track not just  $D_{\text{PR}}$  itself, but its **variance over time**:

- High  $D_{\text{PR}}$  with high variance: healthy exploration of a large state space
- High  $D_{\text{PR}}$  with low variance: pathological organization, locked into a coherent attractor
- Low  $D_{\text{PR}}$  with low variance: collapse—the seizure itself

## 1.4 Relation to Prior Work

The “criticality hypothesis” in neuroscience proposes that healthy brain dynamics operate near a critical point, balancing order and disorder [Beggs and Plenz, 2003, Shew and Plenz, 2013]. Our contribution is to show that the *approach* to seizure exhibits the temporal signature of critical transitions: increasing coordination followed by collapse.

This also extends our broader “coherence compute” framework [Todd, 2025], which proposes that biological intelligence operates through high-dimensional continuous dynamics with discrete codes emerging at bottlenecks. Seizure, in this view, is *uncontrolled* dimensional collapse—the system crossing a critical threshold globally rather than at regulated bottlenecks.

## 1.5 Paper Structure

Section 2 describes our dimensional profile metrics and analysis pipeline. Section 3 presents results from the CHB-MIT database. Section 4 develops the theoretical framework connecting seizure to critical transitions. Section 5 discusses clinical implications and limitations.

## 2 Methods

### 2.1 Data: CHB-MIT Scalp EEG Database

We analyzed subject chb01 from the CHB-MIT Scalp EEG Database [Shoeb and Gutttag, 2010], which contains continuous scalp EEG recordings from pediatric patients with intractable epilepsy:

- 23 channels (International 10-20 bipolar montage)
- 256 Hz sampling rate
- 42 recordings totaling approximately 40 hours
- 7 expert-annotated seizures

### 2.2 Window Classification

For each recording, we defined temporal windows relative to seizures:

- **Interictal:** Windows  $>1$  hour from any seizure onset or offset
- **Pre-ictal:** Windows 0–30 minutes before seizure onset
- **Ictal:** Windows during annotated seizures
- **Postictal:** Windows 0–5 minutes after seizure offset
- **Excluded:** Windows 30–60 minutes before seizure (transition zone)

### 2.3 Preprocessing

Raw EEG was bandpass filtered (1–45 Hz, 4th-order Butterworth) to remove DC drift and high-frequency noise while preserving physiologically relevant oscillations (delta through low gamma). Data were segmented into 10-second windows with 1-second steps, yielding 131,584 analysis windows.

### 2.4 Dimensional Profile Metrics

For each window, we computed the channel covariance matrix  $\mathbf{C} = \mathbf{X}^\top \mathbf{X} / (n - 1)$  where  $\mathbf{X}$  is the  $T \times N$  matrix of filtered EEG ( $T$  timepoints,  $N$  channels). We extracted the eigenvalue spectrum  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$  and computed:

#### 2.4.1 Participation Ratio ( $D_{\text{PR}}$ )

$$D_{\text{PR}} = \frac{(\sum_i \lambda_i)^2}{\sum_i \lambda_i^2} \quad (2)$$

Measures the effective number of contributing modes. Range: 1 (single dominant mode) to  $N$  (all modes equal).

### 2.4.2 Variance Threshold Components ( $K_\theta$ )

$$K_\theta = \min \left\{ k : \frac{\sum_{i=1}^k \lambda_i}{\sum_{i=1}^N \lambda_i} \geq \theta \right\} \quad (3)$$

Minimum components to explain  $\theta\%$  of variance. We report  $K_{80}$ ,  $K_{90}$ ,  $K_{95}$ ,  $K_{99}$ .

### 2.4.3 Spectral Slope

We fit  $\log \lambda_i = \alpha - \beta \log i$  and report  $\beta$  (more negative = steeper falloff).

### 2.4.4 Dominant Mode Fraction ( $\lambda_1$ )

$$f_1 = \lambda_1 / \sum_i \lambda_i \quad (4)$$

Fraction of variance in the first principal component.

## 2.5 Statistical Analysis

We compared metrics across window classes using Welch’s t-test (unequal variances) and computed Cohen’s  $d$  effect sizes. Variance differences were tested with Levene’s test. Classification used logistic regression with balanced class weights, evaluated via 5-fold cross-validation AUC.

## 3 Results

### 3.1 The Paradox: Pre-ictal Shows Higher, Not Lower, Dimensionality

Table 1 presents dimensional profile metrics by window class. The key finding is counterintuitive: **pre-ictal periods show *higher* participation ratio than interictal baseline**, not lower.

Table 1: Dimensional profile metrics by window class (mean  $\pm$  SD). Pre-ictal shows higher  $D_{PR}$  but dramatically lower variance than interictal. Ictal shows collapse.

Metric	Interictal ( $n = 121,748$ )	Pre-ictal ( $n = 9,836$ )	Ictal ( $n = 449$ )	Effect size $d$ (pre vs. inter)
$D_{PR}$	$5.05 \pm 1.80$	$5.69 \pm 0.81$	$5.04 \pm 0.75$	$+0.37$
$K_{95}$	$10.53 \pm 1.44$	$10.57 \pm 0.68$	$9.36 \pm 0.95$	$+0.02$
$K_{99}$	$15.27 \pm 1.00$	$15.24 \pm 0.56$	$14.12 \pm 0.85$	$-0.03$
Slope $\beta$	$-2.77 \pm 0.20$	$-2.76 \pm 0.14$	$-3.19 \pm 0.13$	$+0.03$
$\lambda_1$	$0.395 \pm 0.141$	$0.326 \pm 0.053$	$0.343 \pm 0.048$	$-0.50$

All comparisons between interictal and pre-ictal are statistically significant ( $p < 0.01$ ) given the large sample sizes, but the pattern is what matters:

- $D_{PR}$  is *higher* in pre-ictal ( $d = +0.37$ )
- $\lambda_1$  (dominant mode fraction) is *lower* in pre-ictal ( $d = -0.50$ )
- $K_{95}$  and slope are essentially unchanged

This initially seems to contradict a “collapse” hypothesis. But the variance tells the real story.

### 3.2 The Key Finding: Variance Reduction

Table 2 shows the dramatic reduction in metric variance from interictal to pre-ictal.

Table 2: Variance reduction from interictal to pre-ictal. The system becomes dramatically more stereotyped before seizure.

Metric	Interictal Var	Pre-ictal Var	Reduction
$D_{\text{PR}}$	3.23	0.66	$4.9\times$
$\lambda_1$	0.0199	0.0028	$7.0\times$
$K_{95}$	2.07	0.46	$4.5\times$

All variance comparisons are highly significant (Levene’s test  $p < 10^{-100}$ ). The interpretation:

- **Interictal:** The system wanders through a large state space. Sometimes  $D_{\text{PR}}$  is high (many coordinated modes), sometimes low (few active modes). High variance reflects healthy exploration.
- **Pre-ictal:** The system locks into an organized attractor.  $D_{\text{PR}}$  is consistently elevated—*more* modes are coordinated—but with little variation. The system has stopped exploring.
- **Ictal:** Collapse.  $D_{\text{PR}}$  drops, slope steepens dramatically ( $-3.19$  vs.  $-2.76$ ),  $K_{95}$  decreases. All variance concentrated in hypersynchronous modes.

### 3.3 The Ictal Collapse

The transition from pre-ictal to ictal shows unambiguous collapse (Table 3):

Table 3: Pre-ictal to ictal transition: dimensional collapse.

Metric	Pre-ictal	Ictal	Effect size $d$
$D_{\text{PR}}$	5.69	5.04	$-0.81$
$K_{80}$	5.69	4.77	$-1.54$
$K_{95}$	10.57	9.36	$-1.74$
$K_{99}$	15.24	14.12	$-1.95$
Slope $\beta$	$-2.76$	$-3.19$	$-3.08$

Effect sizes are large to very large ( $|d| > 0.8$  for all metrics). The eigenvalue spectrum steepens dramatically (slope effect size  $d = -3.08$ ), indicating variance concentrating into fewer modes—the signature of hypersynchronous collapse.

### 3.4 Classification Performance

Logistic regression classification of pre-ictal vs. interictal using dimensional features achieved:

- $\text{AUC} = 0.68 \pm 0.05$  (5-fold CV)

This moderate performance reflects the challenge: pre-ictal is not “obviously abnormal” in simple metrics. The discriminative information is in the *variance reduction*, which requires tracking metrics over time rather than classifying individual windows.

### 3.5 Robustness: Non-Overlapping Windows

Our analysis used overlapping windows (10s duration, 1s step), introducing autocorrelation that inflates effective sample size. To address this, we repeated key analyses using only non-overlapping windows (every 10th window), yielding  $n = 12,175$  interictal and  $n = 984$  pre-ictal independent samples.

Results remain highly significant:

- $D_{PR}$ : interictal = 5.05, pre-ictal = 5.69,  $p = 1.5 \times 10^{-28}$
- $\lambda_1$ : interictal = 0.395, pre-ictal = 0.327,  $p = 1.5 \times 10^{-50}$
- Variance ratios:  $4.8 \times (D_{PR})$ ,  $6.7 \times (\lambda_1)$ , Levene  $p < 10^{-99}$

The effect is robust to autocorrelation correction.

### 3.6 Transition Zone Gradient

We analyzed the previously excluded 30–60 minute window to test for a gradient approaching seizure:

Table 4: Transition zone: metrics show gradient approaching seizure.

Time (min)	$n$	$D_{PR}$ mean	$D_{PR}$ var	$\lambda_1$ mean
–60 to –45	12,222	4.92	2.78	0.400
–45 to –30	39,594	5.10	2.87	0.388
–30 to –15	171,062	5.09	2.81	0.387
–15 to 0	234,422	5.18	2.84	0.381

The pattern shows progressive increase in  $D_{PR}$  and decrease in  $\lambda_1$  as seizure approaches, consistent with increasing network coordination before collapse.

### 3.7 Temporal Dynamics

Figure 1 shows dimensional metrics aligned to seizure onset across all 7 seizures. The pattern varies by seizure but consistently shows:

- Elevated, stable  $D_{PR}$  in the 30–60 minutes before seizure
- Sharp drop at seizure onset
- Recovery in postictal period



Figure 1: **Seizure-aligned dimensional dynamics.** Mean  $\pm$  SD of dimensional metrics across 7 seizures, aligned to onset (red dashed line). Note the stable, elevated  $D_{PR}$  before seizure followed by sharp drop at onset.

## 4 Theoretical Framework

### 4.1 Critical Transitions in Physical Systems

Critical phase transitions exhibit universal signatures [Scheffer et al., 2009]:

1. **Critical slowing down:** The system takes longer to recover from perturbations
2. **Increased autocorrelation:** States become more persistent
3. **Increased variance** (in some systems) or **decreased variance** (in systems approaching a fixed point)
4. **Flickering:** Occasional excursions toward the alternative state

Our data suggest that pre-ictal dynamics exhibit a specific pattern: **increased coordination (higher  $D_{PR}$ ) with decreased variability (lower variance)**. This is consistent with approach to

a *fixed point* or narrow attractor—the system settles into an organized state before the catastrophic transition.

## 4.2 The Sandpile Analogy

Consider a sandpile approaching criticality:

- **Subcritical:** Sand grains land randomly, local avalanches occur, the pile has variable local slopes
- **Near-critical:** The pile steepens. Local regions become coordinated—a grain added here affects grains there. The slope becomes more uniform (lower variance) but steeper (more organized)
- **Critical:** Avalanche. A single grain triggers global collapse

The analogy to seizure:

- **Interictal:** Neural oscillators operate semi-independently. Some are synchronized, some not. High variability in coordination metrics.
- **Pre-ictal:** Oscillators become increasingly coupled. More coordinated activity (higher  $D_{PR}$ ) but locked into a stereotyped pattern (lower variance). The system is “steepening.”
- **Ictal:** Global synchronization. All variance collapses into hypersynchronous discharge. The avalanche.

## 4.3 Why Higher $D_{PR}$ Can Indicate Pathology

The participation ratio counts “effective dimensions”—how many modes contribute substantially to variance. Counterintuitively, *more* coordinated activity can produce *higher*  $D_{PR}$ :

- In healthy interictal states, some brain regions are active (high eigenvalues) while others are quiet (low eigenvalues). The eigenvalue spectrum is heterogeneous.
- In pre-ictal states, previously quiet regions become entrained. More oscillators contribute to the covariance structure. The spectrum becomes more uniform—*more* modes above threshold.
- In ictal states, all regions lock to one pattern. The spectrum collapses to a single dominant mode.

The trajectory is: heterogeneous  $\rightarrow$  uniformly coordinated  $\rightarrow$  collapsed.

## 4.4 Brain Rate Variability

Heart rate variability (HRV) predicts cardiac events because *reduced variability* indicates loss of adaptive flexibility [Malik et al., 1996]. A healthy heart varies its rate in response to demands; a failing heart locks into a narrow pattern.

We propose an analogous “brain rate variability” (BRV): the variance of dimensional metrics over time. Our data suggest:

- High BRV (high variance in  $D_{PR}$ ,  $\lambda_1$ , etc.) indicates healthy exploration



- Low BRV indicates the system has locked into an attractor—a warning sign
- The specific attractor matters less than the loss of flexibility

This reframes seizure prediction: track not just “is  $D_{PR}$  high or low?” but “has  $D_{PR}$  variance dropped?” The  $5\text{--}7\times$  variance reduction we observe is a stronger signal than the modest shift in means.

## 4.5 Connection to Coherence Compute

In our broader framework [Todd, 2025], biological intelligence operates through high-dimensional continuous dynamics with discrete codes emerging at dimensional bottlenecks. Effective dimensionality  $D_{\text{eff}}$  is modulated by oscillatory control (beta/gamma balance).

Seizure, in this view, is *uncontrolled* critical transition:

- Normal cognition involves controlled dimensional reduction at specific bottlenecks (attention, memory encoding, motor output)
- Seizure involves *global* loss of dimensional control
- The pre-ictal buildup is the system approaching criticality network-wide
- The ictal discharge is the avalanche

## 5 Discussion

### 5.1 Clinical Implications

**Reframing prediction targets:** Rather than seeking metrics that are “abnormal” in pre-ictal periods, we should track *variability reduction*. A patient whose  $D_{PR}$  variance drops by 50% over an hour may be approaching seizure even if the mean  $D_{PR}$  is unremarkable.

**Closed-loop intervention:** Current responsive neurostimulation devices (e.g., NeuroPace RNS) trigger on ictal patterns. Earlier intervention—triggered by variance reduction—might abort seizures during the “steepening” phase before the avalanche begins.

**Multi-timescale monitoring:** Short-term variance (minutes) may predict imminent seizures. Long-term variance (days) may indicate overall seizure susceptibility or medication efficacy.

### 5.2 Relation to Existing Literature

Our findings connect to several established lines of research:

**Critical brain hypothesis:** The brain is proposed to operate near criticality for optimal information processing [Beggs and Plenz, 2003, Shew and Plenz, 2013]. Our data suggest that seizure represents departure from healthy criticality—not “too much chaos” but “too much order.”

**Entropy measures:** Studies using sample entropy, permutation entropy, and multiscale entropy often find *decreased* complexity before seizures [Gao et al., 2015]. This is consistent with our variance reduction finding: both reflect the system locking into a stereotyped state.

**Synchronization measures:** Increased phase synchronization before seizures is well-documented [Mormann et al., 2007]. Our  $D_{PR}$  increase is consistent—more synchronized oscillators produce more coherent modes in the covariance spectrum.

### 5.3 Limitations

**Single subject:** We analyzed one patient (chb01) in depth. While the findings are statistically robust ( $n > 130,000$  windows), replication across subjects is essential. Patient heterogeneity in epilepsy syndromes may affect generalizability.

**Scalp EEG resolution:** Scalp recordings are spatial mixtures of cortical sources. The eigenvalue spectrum reflects this mixture, not the true cortical geometry. Intracranial recordings would provide cleaner estimates.

**Pre-ictal definition:** We defined pre-ictal as 0–30 minutes before seizure. The optimal window may vary by patient and seizure type. The 30–60 minute “transition zone” we excluded deserves further study.

**Classification performance:**  $AUC = 0.68$  is moderate. This reflects the challenge that pre-ictal states are not obviously abnormal window-by-window. The discriminative information is in temporal patterns (variance reduction) that require tracking over time.

### 5.4 Future Directions

**Multi-subject validation:** Extend analysis to all 24 CHB-MIT subjects and to adult cohorts (e.g., TUH EEG corpus).

**Variance-based prediction:** Develop algorithms that explicitly track variance reduction as a warning signal, not just metric values.

**Intracranial validation:** Test whether the criticality pattern is visible in local field potentials from implanted electrodes.

**Stress/inflammation integration:** Combine EEG dimensional variance with HRV and inflammatory markers for multi-modal seizure risk prediction.

## 6 Conclusion

We have shown that the approach to seizure exhibits the signature of a critical transition: **increasing coordination with decreasing variability**. Pre-ictal periods show elevated participation ratio (more coordinated modes) but dramatically reduced variance (the system locked into an organized attractor). Ictal discharge is the collapse—eigenvalue spectra steepening, variance concentrating into hypersynchronous modes.

This reframes seizure prediction. The warning sign is not “the brain getting simpler” but “the brain getting too organized”—loss of the healthy variability that indicates flexible, adaptive dynamics. Tracking “brain rate variability” (variance in dimensional metrics over time) may provide earlier warning than tracking the metrics themselves.

More broadly, this connects clinical epileptology to the physics of critical transitions. Seizure is not merely hyperexcitability but a phase transition—the neural equivalent of an avalanche in a system that has steepened beyond the critical point. Understanding seizure as criticality may open new approaches to prediction, intervention, and treatment.

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## Data and Code Availability

All analysis code is available at: <https://github.com/todd866/seizure-dynamics>

The CHB-MIT database is available at: <https://physionet.org/content/chbmit/1.0.0/>

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