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Scaling factors for the rates of production of cosmogenic nuclides for geometric shielding and attenuation at depth on sloped surfaces

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Abstract

The decrease in rates of production of cosmogenic nuclides occurs because of shielding of cosmic rays by mountains, sloped surfaces, and local rock formations that block them. When a large part of the sky is blocked, this correction is large and requires detailed model calculations. This paper considers three geometries: a rectangular obstruction, a triangular obstruction, and a sloped surface. Other geometries can be considered as a combination of these. The results are presented in terms of formulas and graphs so that the reader can easily apply them to common field situations. Any use of cosmogenic nuclides in the study of geomorphic processes or forms must consider factors that introduce variations in the production of nuclides.

Keywords: nuclides; geometric shielding; slope

1. Introduction

Trace concentrations of cosmogenic nuclides, measured by accelerator mass spectrometry (AMS) (Elmore and Phillips, 1987), are being used to study a variety of geomorphological processes (Cerling and Craig, 1994). One such study is the determination of surface exposure ages for landforms with simple exposure histories, such as volcanic eruptions, glacial episodes, and other short-lived events (Cerling and Craig, 1994). The exposure age is determined by accounting for the accumulation and the removal of the cosmogenic nuclide through time. Because the removal is usually by radioactive decay, which is very well understood, the accuracy of the exposure age depends primarily on the accuracy to which the rate of production can be determined.

In the development of this method, most calibration data has been collected from flat, level sites with little shielding of the cosmic rays by surrounding obstructions. Whereas the choice of this simple exposure geometry is important for calibration and verification of the procedure, the method must be applicable to a broad range of sampling sites if it is to become a common analytical tool. This requires a procedure for determining the rate of production for

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more complex situations based on the data which has been obtained for the basic sampling geometries. It is common to describe such modifications in terms of scaling factors that multiply the basic production rate. An example is the scaling factor that accounts for changes in the rate of production because of variations in altitude and latitude of sampling sites (Lal and Peters, 1967).

Often a sample of interest is partially shielded from cosmic rays, either because it is buried at some depth beneath the surface or because it lies in the 'shadow' of some obstruction blocking cosmic rays. This paper presents procedures for the development of scaling factors for spallation-based rates of production of cosmogenic nuclides to account for these types of shielding. Section 2 outlines basic ideas concerning the transport of cosmic rays through matter, and specifically addresses changes in cosmic ray intensity, attenuation length, and foreshortening effects due to geometric shielding. Section 3 of this paper provides a review of geometric shielding by thick obstructions that prevent a portion of the cosmic rays from reaching the target. Section 4 examines the effects of attenuation of the flux of cosmic rays to account for shielding by depth on a simple, planar, sloped surface. Scaling factors for spallation-based rates of production are determined, and a semi-empirical formula is presented that closely models the solutions.

2. General effects and results from the shielding of cosmic rays

For a flat, horizontal, unshielded surface being irradiated by cosmic rays, cosmogenic nuclides are produced at a rate that depends on the depth of the sample. Material above the sample will attenuate the cosmic radiation, so deeper material will experience a lower rate of production than material closer to the surface.

Assuming that collimated radiation is attenuated exponentially as it penetrates material (Lal, 1991), the rate of production dependence upon depth for cosmogenic nuclides is described exactly in terms of an incomplete gamma function (see Section 4). This differs from simple exponential decay because of the geometry effects that result from being at depth in a surface. Because the incomplete gamma function is difficult to work with, the rate of production is usually modeled using simple exponential attenuation with depth. The attenuation length used in this model is not the same as the attenuation length for collimated radiation, although it is related as outlined in Section 4. The difference between using a simple exponential and the incomplete gamma function is significantly smaller than other uncertainties inherent to this dating method.

When an obstruction is present the effect on the production of cosmogenic nuclides can be expressed in terms of (1) a decrease in the overall rate of production, and (2) a change in the effective attenuation length. For example, if the obstruction blocks out radiation close to the horizon, a larger fraction of the remaining radiation will consist of rays that are closer to normal incidence. This will result in a longer attenuation length.

In the following section these two effects are addressed independently. In Section 4, the angle of the sampling surface introduces a more complex shielding situation, so the rate of production profiles are provided graphically rather than as scaling factors. In Section 4 these effects are modeled in such a way that the coefficient of the exponential is a surface rate of production scaling factor and the coefficient of (z/Λ) in the exponent of the exponential can be treated as a scaling factor equal for the inverse of the attenuation length.

Radiation is attenuated by passing through mass, not along a length, so attenuation lengths are usually described in terms of grams per square centimeter rather than as an actual distance. For the sake of uniformity, this paper will consistently refer to attenuation length and depth as having the same units. If different units are used, the methodology will still be valid, but additional factors of material density are required to relate physical distances to the attenuating mass.

A foreshortening factor is sometimes used to describe the decrease in absorbed radiation per unit surface area when the incident radiation makes a non-zero angle with the normal to the surface. Attenuated radiation, however, initiates nuclear reactions throughout the length of the path it takes through the attenuating material, not just at the surface. The equivalent to foreshortening in the case of attenuated radiation is the integral over depth of the accumulated radiation. To illustrate such an effect, consider a horizontal surface where incident radiation (intensity I_0) makes an angle θ with the horizontal. In penetrating to a depth z, the radiation travels a distance $z/\sin\theta$ through the material, and is attenuated by a factor of $\exp(-z/\sin\theta)$. Integrating over all depth gives

$$\int_0^\infty I_0 \mathrm{e}^{-\frac{z}{\sin\theta}} \mathrm{d}z = I_0 \sin\theta \tag{1}$$

The factor of $\sin\theta$ on the right hand side of this relation is the foreshortening factor mentioned above. Thus, foreshortening is built into and is a result of this model, and will not be applied as an additional factor.

3. Geometric shielding by thick objects

An object on the surface of a flat, level landform that has an unobstructed view of the sky in all directions will receive the maximum flux of cosmic irradiation. For a cosmic ray intensity given by $I(\theta,\phi)$, where θ is the inclination angle measured up from the horizontal and ϕ is the azimuthal angle of incoming radiation as viewed by the irradiated material, then the total flux, F, that an object would receive from this unshielded exposure is given by

$$F = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I(\theta,\phi) \cos\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \tag{2}$$

The form of $I(\theta,\phi)$ generally used (Heidbreder et al., 1971; Lal, 1958) is:

$$I(\theta,\phi) = I_0 \sin^m(\theta) \tag{3}$$

for $\theta \geq 0$, and $I(\theta,\phi) = 0$ for $\theta < 0$, where I_0 is the maximum intensity and m is an experimentally determined constant. Currently, a value of 2.3 is assumed for m in most studies. The maximum (unshielded) flux is then given by

$$F_{\mathrm{Max}} = \frac{2\pi I_0}{m+1}. \tag{4}$$

Should a fraction of the sky be obscured by an object thick enough (i.e., a few meters) to block out essentially all of the spallation-producing cosmic rays, the decrease in flux can be determined either by placing appropriate limits on the integrals in (2) or, equivalently, by calculating the flux from the shielded region of sky and removing it from the total given in (4).

For a 'rectangular' obstruction that blocks incident cosmic rays from the ground up to a constant inclination angle θ_0, and that extends through an azimuth of $\Delta\phi$, the missing flux, δF, is given by

$$\delta F = \frac{I_0 \Delta\phi}{m+1} \sin^{m+1}(\theta_0). \tag{5}$$

For a set of n obstructions, each with a corresponding θ_i and $\Delta\phi_i$, a rate of production shielding factor, S, can be calculated as the ratio of the remaining flux to the maximum flux:

$$S = 1 - \frac{1}{360°} \sum_{i=1}^{n} \Delta\phi_i \sin^{m+1}\theta_i. \tag{6}$$

The response of this shielding factor for single obstructions under variations in θ_i and $\Delta\phi_i$ can be seen in Fig. 1.

In addition to changing the surface rate of production, geometric shielding will also affect the effective attenuation length of the cosmic radiation that is incident on a surface. Performing an average, weighted according to the intensity distribution given in (3), of the effective attenuation lengths for radiation incident at different zenith angles, the following relation is obtained for n rectangular obstructions extending from the horizontal up to angles of θ_i and subtending azimuthal angles of $\Delta\phi_i$:

$$\Lambda^* = \Lambda \frac{1 - \sum_{i=1}^{n} \frac{\Delta\phi_i}{360°} \sin^{m+2}\theta_i}{1 - \sum_{i=1}^{n} \frac{\Delta\phi_i}{360°} \sin^{m+1}\theta_i} \tag{7}$$

where Λ^* is the effective attenuation length with obstructions and Λ is the unshielded attenuation length. Fig. 2 shows the magnitude of this effect for a single rectangular obstruction.

A similar procedure can be followed for triangular obstructions, although the resulting integral does not have a closed form, and must therefore be solved numerically. For the purpose of determining a scaling factor for rate of production, it is possible to model a triangular obstruction with a rectangular

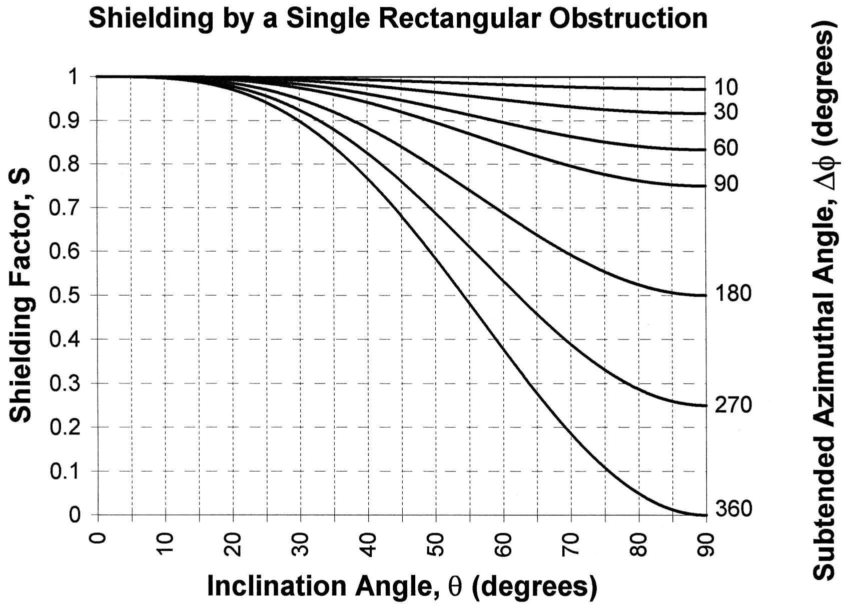

Fig. 1. Shielding factor that results from a single, 'rectangular', cosmic-ray-blocking obstruction that subtends an azimuthal angle $\Delta\phi$ through a constant zenith angle θ measured up from the horizontal. Plot based on $m = 2.3$.

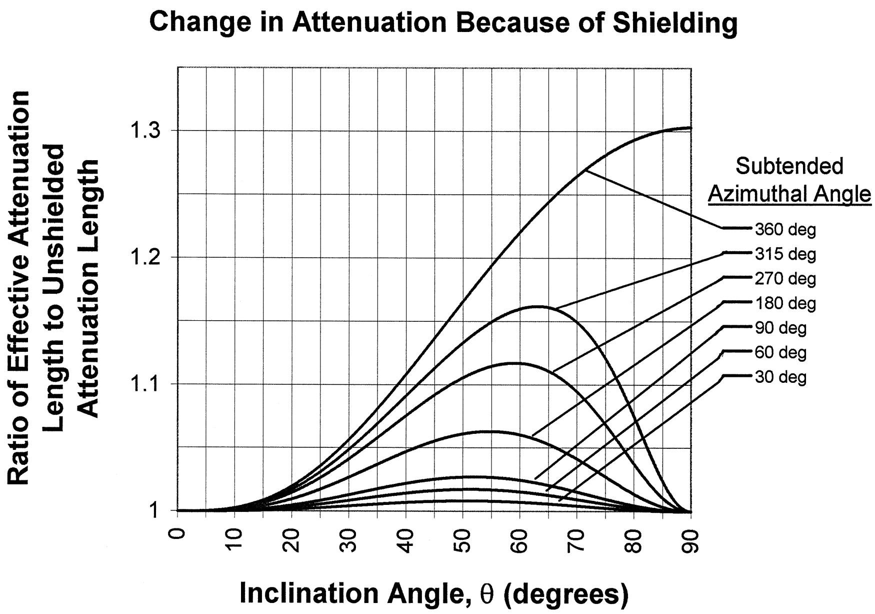

Fig. 2. Change in effective attenuation length that results from a single, 'rectangular', cosmic-ray-blocking obstruction that subtends an azimuthal angle $\Delta\phi$ through a constant zenith angle θ measured up from the horizontal. Plot based on $m = 2.3$. Vertical axis represents the ratio of the attenuation length adjusted for the obstruction to the attenuation length appropriate for an unshielded exposure.

equivalent, which can then be used in (6). Because the intensity of cosmic rays varies with inclination angle, the equivalent 'rectangular' inclination angle for modeling a triangular obstruction is not exactly half of the maximum angle (although this is a reasonable first estimate). The polynomial

$$\theta_R = 0.62\,\theta_T - (6.5 \times 10^{-4})\,\theta_T^2 \tag{8}$$

produces a rectangular inclination angle, θ_R, that provides the same shielding effect when applied over the same azimuthal range as the base of a triangular obstruction having a maximum inclination angle of θ_T (all angles measured in degrees). This relation is accurate over a 90 degree range in zenith angle to within two degrees. For zenith angles less than 50 degrees, $\theta_R = 0.62\ \theta_T$ gives an approximation which is accurate to better than 0.5 degrees. This polynomial was fitted for $m = 2.3$, but less than a six degree variation in θ_R occurs over a range of m values from 2 to 3.

A scaling factor for the attenuation length for triangular obstructions is given in Fig. 3. These curves can be modeled accurately for a maximum zenith angle (θ_T) less than 50 degrees by the following formula:

$$\frac{\Lambda^*}{\Lambda} = 1 + \left[0.08\frac{1}{10e^{-0.09(\theta_T - 20°)} + 1} - 0.002\right] \times \frac{\Delta\phi}{360°} \qquad (\theta_T < 50°) \tag{9}$$

where $\Delta\phi$ is the azimuthal angle subtended by the base of the obstruction. Fig. 3 must be used for zenith angles larger than 50°.

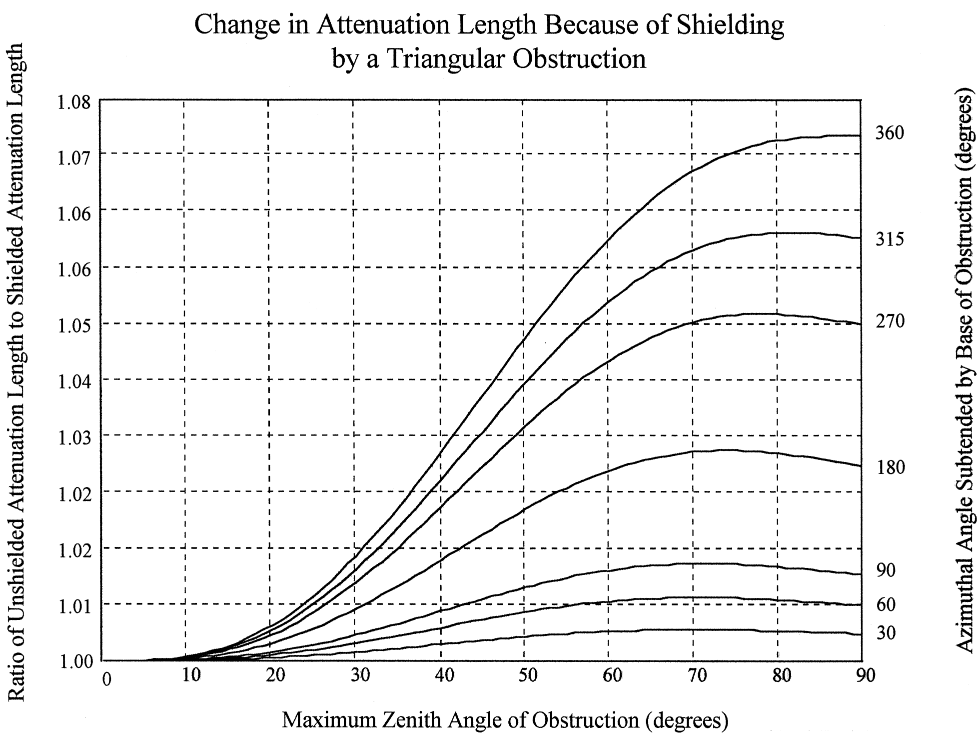

Fig. 3. Change in effective attenuation length that results from a single, 'triangular', cosmic-ray-blocking obstruction that subtends an azimuthal angle $\Delta\phi$ at its base and reaches a maximum zenith angle θ_T measured up from the horizontal. Plot based on $m = 2.3$. Vertical axis represents the ratio of the attenuation length adjusted for the obstruction to the attenuation length appropriate for an unshielded exposure.

4. Shielding by surface slope as a function of depth

If a sampling surface is sloped, the effect of geometric shielding on rate of production and attenuation length cannot be calculated in a straightforward manner as the extent of the shielding varies in a complex way with azimuthal angle.

The following model is based on the assumption of a uniform slope with a dip angle (maximum inclination) given by α (see Fig. 4). Defining $\phi = 0$ in the direction of maximum slope angle, the directional inclination (slope angle in the direction ϕ), γ, is given by the relation

$$\tan\gamma = \cos\phi \tan\alpha \tag{10}$$

The flux of collimated, unidirectional, spallation-inducing cosmic rays is attenuated exponentially as it travels through matter (Lal, 1991). The intensity of that flux, after traversing a distance d of material, is given by:

$$I(d) = I(0)\,e^{-d/\lambda}, \tag{11}$$

where $I(0)$ is the unattenuated flux, and λ is the attenuation length, which is dependent on particle energy and type. λ is the attenuation length for particle flux, not that for the rate of production for cosmogenic-nuclides, although the two can be related as described below.

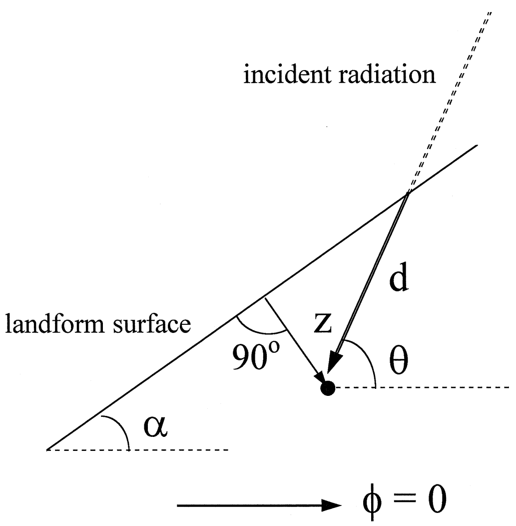

Fig. 4. Sketch of the relevant angles for scaling of rates of production because of a sloped surface. Sketch shows cross-section with maximum slope, α. z is measured perpendicularly down from the slope. For azimuthal angles other than $\phi = 0$, α would be replaced by γ and z would not be measured perpendicularly to the different, shallower slope.

For a target located a distance z perpendicularly below the surface of a slope, the flux penetration distance d depends on the cosmic ray inclination angle θ and on γ:

$$d = \frac{z\cos(\gamma)}{\cos(\alpha)\sin(\theta-\gamma)} \qquad (\theta > \gamma) \tag{12}$$

Eq. (12) is only appropriate for values of θ that are greater than γ; values of θ less than γ will not contribute to the cosmic ray flux at the target.

The total flux seen by a target at depth z on a slope of dip angle α for cosmic rays incident from all directions (θ, ϕ) is given by

$$F = \int_{\phi=0}^{2\pi}\int_{\theta=\gamma}^{\frac{\pi}{2}} I(\theta,\phi)\,e^{-d/\lambda}\cos(\theta)\,d\theta\,d\phi \tag{13}$$

Using (12), along with the geometrical relation

$$\sin(\theta \pm |\gamma|) = \frac{\sin(\theta)}{\sqrt{1+\tan^2\alpha\cos^2\phi}} \pm \frac{\cos(\theta)}{\sqrt{1+\dfrac{1}{\tan^2\alpha\cos^2\phi}}} \tag{14}$$

it is possible to solve (13) numerically for a given distribution of cosmic rays, $I(\theta,\phi)$, and specified dip angle, α. Because the rate of production is proportional to the flux of cosmic rays, such a result can be used to define a scaling factor,

$$S(z,\alpha) = \frac{F}{F_{max}}, \tag{15}$$

that will scale the rate of production to account for the effects of shielding. $S(z,\alpha)$ varies from 0 to 1, and scales the rate of production that would occur at the surface of a flat, horizontal surface to obtain the rate of nuclide production at a depth of z in a slope of dip angle α.

To relate the attenuation length λ for collimated, unidirectional radiation to the attenuation length Λ for the decrease in rate of production with depth, Eq.

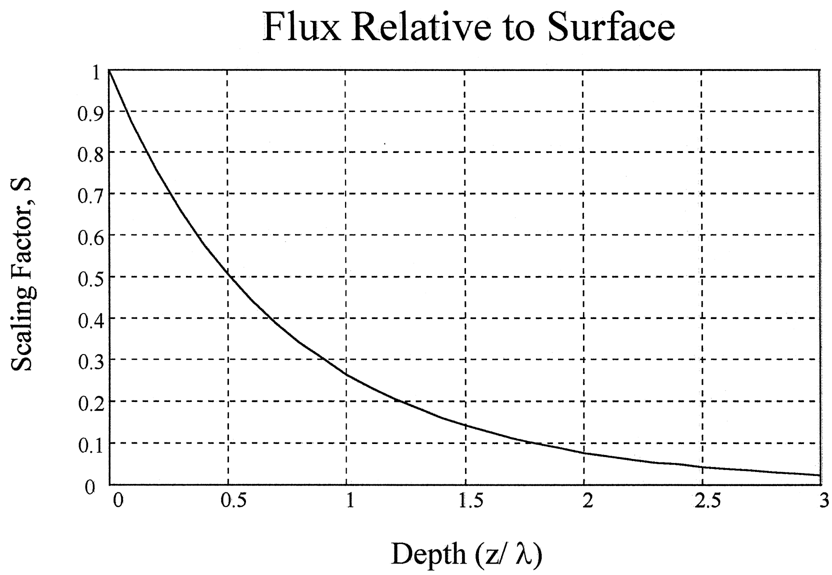

Fig. 5. Attenuation of cosmic-ray flux (and thereby rate of production) with depth for a uniform, horizontal sampling surface. Depth is measure in fractions of the attenuation length appropriate for collimated radiation passing through a homogeneous material of uniform density.

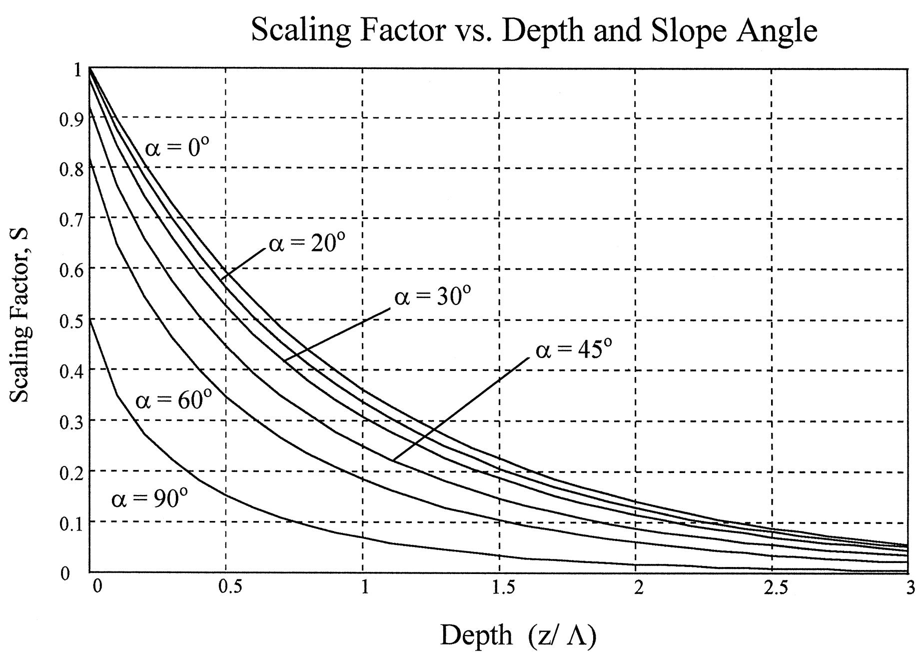

Fig. 6. Scaling factor for the rate of production as a function of sampling depth for various surface dip angles (with $m = 2.3$). Depth is measured in fractions of the attenuation length normally used to model decrease in rate of production with depth as a simple exponential.

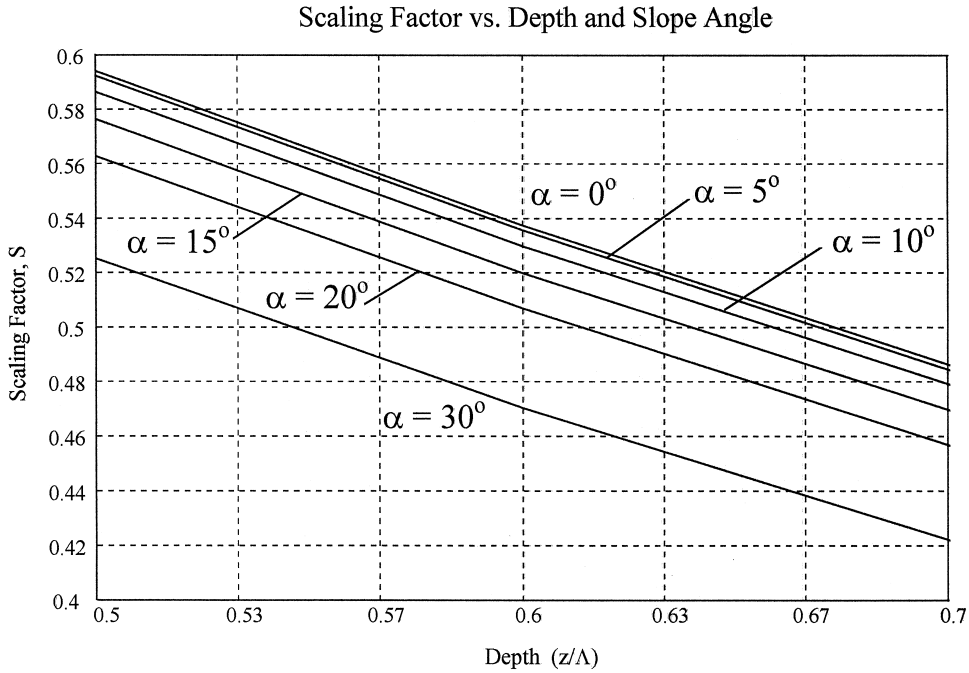

Fig. 7. An enlarged region from Fig. 6, illustrating the magnitude of the slope effects for shallow slopes.

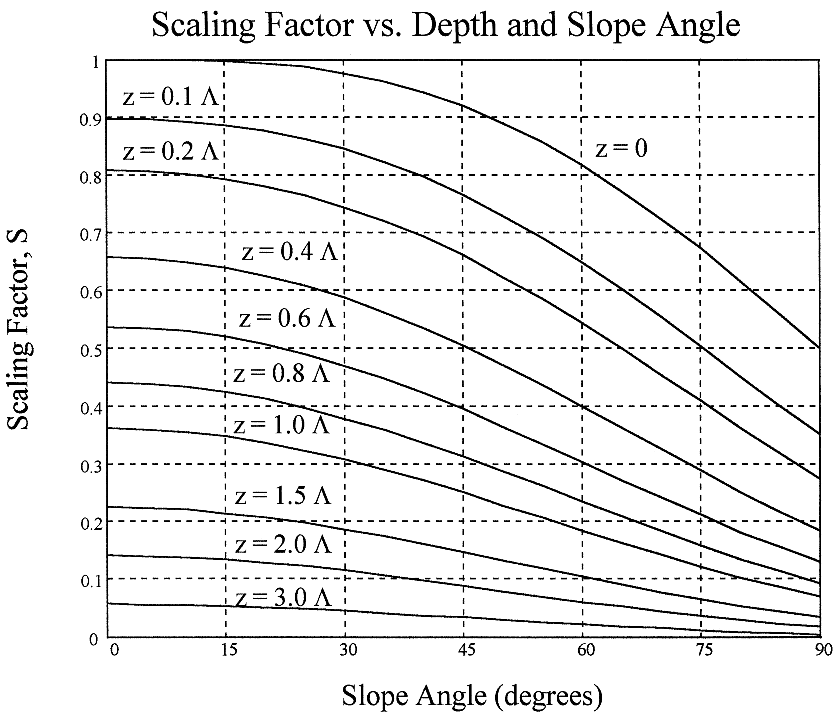

Fig. 8. Scaling factor for rate of production (with $m = 2.3$) as a function of dip angle for various sampling depths (measured in fractions of the attenuation length normally used to model decrease in rate of production with depth as a simple exponential).

(13) was solved for an intensity distribution as specified by (3) for a uniform, horizontal surface ($\alpha = 0$). This solution is given exactly by the formula

$$S(z,0) = (m+1) \cdot \left(\frac{z}{\lambda}\right)^{m+1} \Gamma\left(-m-1, \frac{z}{\lambda}\right), \tag{16}$$

where Γ represents the incomplete gamma function. Eq. (16) is shown graphically in Fig. 5 for $m = 2.3$.

Because the incomplete gamma function can be unwieldy, it is desirable to model this function as a simple, exponential decay of the form

$$S(z,0) \cong \mathrm{e}^{-\frac{z}{\Lambda}} \tag{17}$$

Using a least squares fit of (16) to (17), it is found that $\lambda = 1.3\ \Lambda$ reproduces the standard exponential decay very closely. By replacing λ with $1.3\ \Lambda$ in Eq. (13), results can be obtained in terms of the standard attenuation length that is currently used to describe decreases in production rate with depth.

Eq. (13) can again be solved numerically (using the distribution in (3) with $m = 2.3$) to obtain the scaling factor as a function of dip angle and depth (Figs. 6–8). These curves can be approximated by:

$$S(z,\alpha) = (1 - 3.6 \times 10^{-6} \alpha^{2.64}) \mathrm{e}^{-\left(\frac{z}{\Lambda}\right)\left(1 + \frac{\alpha^2}{5000}\right)} \tag{18}$$

Eq. (18) matches the numerical solutions of (13) closely for slope angles of 25° or less, with discrepancies in S of less than 0.02, and less well for slope angles of 30° or more (discrepancies in S ranging from 0.025 at 30° to 0.043 at 70°).

The effective shielding factors from obstructions, as presented in Section 3, and sloped surfaces, as presented in this section, are not, in general, independent of each other. Only for surface samples with obstructions that are downhill can these scaling effects be applied independently to the same sampling site. Otherwise, the effective shielding factor must be determined by solving a variation of Eqs. (2) and (13) with integration limits appropriate to the situation.

For a large, vertical slope, such as a cliff face, the surface rate of production is half that for a horizontal surface with no obstructions. This rate of production is valid extending from approximately two attenuation lengths from the top to the bottom, where it matches with the 50% decrease from geometric shielding. Near the top, the increase in production from rays that pass through the top of the cliff can be estimated by adding half of the production for a horizontal surface to the production from the vertical face. Determination and definition of the attenuation length, however, are not straightforward.

When sampling a vertical surface, a considerably larger reduction occurs in the rate of production for a thick sample. For example, for a rock having a density of 3 $\mathrm{g/cm^3}$, the rate of production drops by 10% in 6 cm for a horizontal surface versus approximately 2 cm for a vertical surface.

5. Summary

Equations have been developed to describe the effects of geometric shielding for thick rectangular and triangular obstructions. In addition, a theoretical model for the attenuation of the cosmic ray flux has been developed to predict nuclide rates of production at depth in uniformly sloped surfaces. Scaling factors have been plotted by solving this theoretical model numerically, and an Eq. (18) is presented which closely models these graphs.

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