On the Weakness of *Believe*

Abstract

There is a robust intuition that the attitude verb *believe* imparts some sort of weakness to the attitude holder regarding the prejacent proposition (cf. *I believe that Kim is on vacation* vs. *I know that Kim is on vacation*). This paper pits two alternative views on where in its semantics the weakness of *believe* resides, i.e., in its non-maximal modal force (Hawthorne et al. 2016; Rothschild 2020; cf. Moss 2019) or in its subjective modal flavor (cf. Lyons 1977; Kratzer 1981; Nuyts 2001; Papafragou 2006; Portner 2009). Based on an examination of three sets of data that tap directly into the modal strength of *believe* (i.e., occurrence in modal contexts, interaction with other epistemic expressions, and closure under conjunction), the emerging picture suggests that *believe* carries a strong modal force but conveys subjective (and thus 'weak') epistemic content.

Keywords: *believe*, modal strength, subjectivity, epistemic modality

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1 Introduction

The verb *believe* has played a central role in the development of formal semantics as it sets a benchmark for the analysis of other attitude predicates. This verb also underlies a number of widely studied phenomena, such as opacity, presupposition projection, neg-raising, the norm of assertion, and others. Given its theoretical significance, it is critical to understand its core interpretational properties. Ever since Hintikka (1969), it has become standard to analyze *believe* as involving universal quantification over possibilities. More specifically, a belief attribution is taken to state that the 'prejacent' (the clausal complement of *believe*) is true in all of the agent's doxastic alternatives. This is usually rendered as in (1), where $Dox_{x,w,t}$ stands for the set of x's doxastic alternatives in a world w and at a time t, i.e., the set of possibilities compatible with everything x believes in w and at t.

(1) $[\![\text{believe}]\!]^{w,t} = \lambda p \lambda x . \forall \langle w', t' \rangle \in Dox_{x,w,t} : p(\langle w', t' \rangle)$

Although very popular, this Hintikkan orthodoxy fails to address one key aspect of the semantics of *believe*, i.e., its felt modal weakness. There is a robust intuition that *believe* expresses some sort of 'weak' attitude towards the prejacent proposition, so that e.g. *I believe that Kim is on vacation* conveys a lower degree of certainty or commitment than does *I know that Kim is on vacation*. In spite of this intuition, the standard Hintikkan semantics makes no clear predictions as to whether the belief agent regards the prejacent as true, certain, likely, possible, or similar. The reason is that, according to (1), the strength of *believe* hinges on the kind of modal content that the set of doxastic alternatives *Dox* represents. Since *Dox* is defined as the set of possibilities compatible with everything the agent 'believes', the modal strength issue is merely pushed into the metalanguage. It is not derived from the semantics and needs to be independently stipulated.

Despite this omission, the structure of the Hintikkan semantics outlines quite clearly the two key factors that bear on the modal strength of *believe*: modal force and modal content. The modal force of *believe* is determined by the force of the quantifier, or more generally by the logical relation that holds between the set of doxastic alternatives and the prejacent proposition.[1] In turn, the modal content of *believe* is determined by the nature of the quantificational domain, i.e., what the set of doxastic alternatives stands for. As just mentioned, the Hintikkan semantics addresses only the force factor, stating that the prejacent is true across all doxastic alternatives, in line with the universal force of the quantifier. But this semantics fails to address the content factor, specifically how committing these doxastic alternatives are. Therefore, it fails to make predictions about the modal strength of *believe* more generally.

Surprisingly, the issue of the modal strength of *believe* has barely been touched upon in the literature, with a few notable exceptions (see Hawthorne et al. 2016; Moss 2019; Rothschild 2020). While work on graded modality has discussed the strength with which epistemic modals imply their prejacents (Karttunen 1972; Kratzer 1991; von Fintel and Gillies 2010, 2021; Lassiter 2016,

[1] That is, we can sensibly talk about 'modal force' in the absence of overt quantification. For example, the entry in (1) can be rephrased as $[\![\text{believe}]\!]^{w,t} = \lambda p \lambda x . Dox_{x,w,t} \subseteq p$, where the strong modal force is expressed by set-theoretic inclusion.

2017; Goodhue 2017; Giannakidou and Mari 2018; Del Pinal and Waldon 2019; and others), the focus has been primarily on modal auxiliaries and modal adjectives, which are impersonal and (in unembedded positions) adopt the perspective of the speaker. Since attitude verbs like *believe* project an external argument, the issue of modal strength here translates into the issue of the certainty or commitment level of the attitude holder regarding the prejacent proposition, and so it requires special attention.

The intuition that *believe* is 'weak' begs the question of where in its meaning this weakness resides. As discussed earlier, there are two analytical choices on which this weakness can be pinned: force or content. The former choice is made in Hawthorne et al. (2016) and Rothschild (2020) (see also Moss 2019), who attribute the weakness of *believe* to its non-maximal modal force. Roughly, the claim is that a belief attribution is true just when the agent's degree of certainty in the prejacent proposition is sufficiently high. I dub this view Weak Force (WF) and render it informally as shown in (2).

(2) WEAK FORCE (informal)

x believes p is true if and only if the degree of certainty that *x* assigns to *p* exceeds some vague contextual threshold

Alternatively, one could argue that *believe* owes its weakness not to its modal force (which is at a maximum) but to the kind of modal content it conveys. To the best of my knowledge, this view has not been explicitly defended previously. However, it is strongly suggested by a distinction drawn in prior literature between two flavors of epistemic modality, i.e., 'subjective' and 'objective' (Lyons 1977: ch.17; Kratzer 1981; Nuyts 2001; Papafragou 2006; Portner 2009: 4.2; see also Hamblin 1971; Krifka 2015; Geurts 2019). While this intuitive distinction has been fleshed out theoretically in various ways, I will broadly adopt Kratzer's (1981) characterization of the contrast in subjectivity as pertaining to what is publicly defendable. That is, while objective certainty entails public commitments and therefore demands strong evidence, subjective certainty merely describes the mental state of the agent and may be based on weak evidence. Assuming that *believe* lexicalizes this latter subjective flavor of epistemic modality, we may call this view Subjective Content (SC). It is stated informally in (3).

(3) SUBJECTIVE CONTENT (informal)

x believes p is true if and only if *x* assigns to *p* a maximal degree of subjective (in the intended sense) certainty

The main contribution of this paper is twofold. First, I will spell out WF and SC in sufficient detail, thus elucidating the background assumptions and the general predictions of these two views. Second, I will compare the predictions of WF and SC concerning three sets of empirical data: (i) the distribution of *believe* in modal contexts in which the agent's degree of certainty regarding the prejacent proposition is fixed by the prior context; (ii) the interaction of *believe* with other expressions of epistemic modality, such as *know*, *sure*, and *doubt*; (iii) the observation that *believe* is closed under conjunction introduction, i.e., the intuitively correct inference from *believe p* and

believe q to *believe (p and q)*. While most of these data turn out to be compatible with either view, the emerging picture suggests that SC makes more specific and arguably better predictions than WF overall.

The three sets of data listed above were chosen because they appear to reveal the most immediate effects of the modal strength of *believe*. That said, it is important to note that the finding that SC has an empirical edge over WF is suggestive rather than conclusive, as more empirical evidence could be adduced for or against either view. One potentially consequential piece of additional evidence involves neg-raising, a phenomenon whereby a matrix negation is interpreted as though it takes scope inside an embedded clause (Bartsch 1973; Horn 1989; Gajewski 2007; Romoli 2013; Homer 2015). Since *believe* is a classic neg-raising predicate, so that *x doesn't believe p* typically comes to mean *x believes not p*, one might wonder whether this property is characteristic of predicates of a particular strength. Indeed, Horn (1989) and Hawthorne et al. (2016) hypothesize that neg-raising may be licensed by 'weak' predicates (e.g., *want*, *like*, *advise*) but not by 'strong' predicates (e.g., *need*, *love*, *order*), and conclude from this that *believe* must carry a weak modal force. However, what this reasoning leaves open is whether the felt weakness of neg-raisers stems from their force or their content. In fact, a weakness in content seems to be the more likely option, as some neg-raisers (e.g., *feel*, *plan*, *advise*) presumably lack a force component entirely.[2] Given that the kind of weakness involved here is unclear, I will leave the detailed study of the link between neg-raising and modal strength for future research.

In order to get the comparison between WF and SC off the ground, I will assume a gradable semantics for *believe* within the standard degree-based approach to gradable predicates (Cresswell 1976; von Stechow 1984; Heim 1985; Kennedy and McNally 2005; Morzycki 2016; and others). According to this semantics, *believe* denotes a relation between individuals, propositions, and degrees, stating that the agent's degree of certainty in the prejacent proposition meets some threshold. This is formalized in (4), where the measure C is anchored to an agent (as well as worlds and times) and maps propositions to degrees of certainty.

(4) $[\![\text{believe}]\!]^{w,t} = \lambda p \lambda d \lambda x . C_{x,w,t}(p) \geq d$

The idea that *believe* is grammatically gradable is suggested by both WF and SC, as both views talk about degrees of certainty (of the relevant kind). This idea is further supported by the apparent ability of *believe* to interact with degree morphology, as seen in combinations like *partially believe*, *fully believe*, *believe more strongly than Jill*, and so on.[3]

We have been discussing the modal force and the modal content of *believe*. In the standard Hintikkan semantics in (1), these two components are encoded by the universal quantifier (which determines the logical relation between the quantificational domain and the prejacent proposition) and the quantificational domain itself (the set of the agent's doxastic alternatives), respectively.

[2] In addition, Homer (2015) argues that in British English epistemic *must* is a neg-raiser while epistemic *have to* is not, even though the former may feel stronger than the latter. See Horn (1989: 5.2) for more on crosslinguistic variation in neg-raising.

[3] For more discussion on the gradability of *believe*, see Bolinger (1972: ch.9) and Lassiter (2021). For a general discussion on graded modality, see Kratzer (1991), Portner (2009: ch.3), von Fintel and Gillies (2010; 2021), Yalcin (2010), Klecha (2014), Lassiter (2016; 2017), Santorio and Romoli (2017), and others.

But how are these two components encoded in the gradable semantics in (4)? In this latter case, the modal force of *believe* will depend on the default value of d, where a strong force corresponds to d being the maximum of the relevant scale ($d = 1$) and a non-strong force corresponds to d dropping below the maximum ($d < 1$). In turn, the modal content is encoded by C, which assigns degrees of certainty of some kind. In short, according to both (1) and (4), the modal strength of *believe* is a child of two parents: it depends on the force and the content of this verb. Thus, despite their differences, we can observe that at some level of generality the quantificational and the degree-based analyses of *believe* share the same fundamental components.

Before closing this introductory section, an important caveat is in order. It should be emphasized that the current paper analyzes the empirical properties of the English verb *believe* and not the philosophical notion of 'rational belief'. There exists a long-standing debate in the philosophical literature regarding when it is rational to believe a proposition and related questions concerning whether rational belief should be closed under conjunction introduction or under entailment more generally (Kyburg 1961; Hintikka 1962; Makinson 1965; Stalnaker 1984; Harman 1986; Foley 1992; Clarke 2013; Leitgeb 2014; Greco 2015; and others). Importantly, the notion of rational belief is a normative one, and therefore this notion need not be identical to the natural meaning of the verb *believe*, which is purely a matter of empirical discovery. Consequently, one should exert caution in drawing conclusions from assumed norms of rational belief to what the semantics of *believe* actually is.

The structure of the paper is as follows. Section 2 introduces the distinction between subjective and objective epistemic modality and juxtaposes two views on the weakness of *believe*, i.e., WF and SC. Section 3 presents three sets of empirical data on *believe*, which appear to favor SC over WF. Finally, Section 4 concludes the discussion and offers a brief outlook on subjectivity and epistemic modality more generally.

2 Two views on the weakness of *believe*

This section spells out two alternative accounts of *believe*, i.e., Weak Force (WF) and Subjective Content (SC). While both accounts entail that *believe* has a 'weak' semantics, they disagree on what this weakness is attributed to. WF posits that *believe* carries a weak modal force and assumes no special kind of epistemic flavor. By contrast, SC claims that *believe* carries a strong modal force but conveys a flavor which is subjective in the intended sense.

2.1 Weak Force

An unmodified use of *believe* is typically perceived as conveying some sort of 'weak' attitude regarding the prejacent proposition. For example, (5a) would generally imply a lower degree of certainty or commitment than does the variant with *know* in (5b).

(5) a. I believe that Kim is on vacation.
b. I know that Kim is on vacation.

One natural reaction to this contrast is to say that *believe*, unlike *know*, carries a non-maximal modal force. Hawthorne et al. (2016) and Rothschild (2020) follow this line of thought in the form of a threshold semantics (possibly considering the gradability of *believe*). More specifically, their claim is that the degree of certainty ascribed to the belief agent must exceed some vague contextual threshold. An informal version of this view was already presented in (2). Plugging in the gradable semantics for *believe* stated in (4), this view can be spelled out as in (6), where C is some generic measure of certainty shared among all epistemic modals and θ_{bel} is the relevant belief threshold. Notice that in the absence of overt degree morphology, the degree argument of *believe* is assumed to be filled by the null morpheme POS, an idea co-opted from the gradability literature (see Cresswell 1976 and much subsequent work).

(6) WEAK FORCE

$$([\![\text{POS}]\!]^{w,t}([\![\text{believe}]\!]^{w,t}(p)))(x) \quad \text{iff} \quad C_{x,w,t}(p) > \theta_{bel}$$

In essence, WF posits that *believe* operates similarly to *likely* in that the threshold value is taken somewhere from the middle of the scale (Yalcin 2010; Lassiter 2017). More abstractly, and extending beyond the modal domain, this view aligns *believe* with proportional quantifiers like *most* or *more than half* (Barwise and Cooper 1981; Hackl 2009; Solt 2016) in that it requires some sort of a 'majority' interpretation.

Hawthorne et al. (2016) actually add a second truth condition to (6), wherein the prejacent must be significantly more likely than any of its salient alternatives: $C_{x,w,t}(p) \gg C_{x,w,t}(q)$, for all salient alternatives q of p (see also Kahneman and Tversky 1982; Dorst 2019; Dorst and Mandelkern 2021; Holguín 2022). In the case of a binary choice between p and $\overline{p}$, this condition boils down to $C_{x,w,t}(p) \gg C_{x,w,t}(\overline{p})$, entailing that the likelihood of the prejacent significantly surpasses the midpoint of the scale. However, when several alternatives to the prejacent are salient in the context of utterance, the threshold may dip below the midpoint. While I will mainly set aside this second truth condition, it will become relevant in the discussion of the racehorse example at the end of Section 3.1.

Moss (2019) is a kindred account. Like WF, it assumes that *believe* conveys generic epistemic content. However, the proposed mechanism that derives the weakness of *believe* differs. While WF posits that the degree of certainty associated with *believe* must meet some lower contextual threshold, Moss suggests that *believe* conveys full certainty but allows for some amount of imprecision or 'loose speech' (Lasersohn 1999; Sauerland and Stateva 2011; Solt 2014; Klecha 2018). Notably, this entails that *believe* has the same strict content as e.g. *sure*, both requiring full certainty. Thus, to explain the observation that *believe* feels weaker than *sure* (cf. *I believe it's raining, but I'm not sure* vs. *#I'm sure it's raining, but I don't believe it*), Moss proposes that these two modals convey different loose contents, with *believe* generally interpreted to a lower standard of precision than *sure*. However, this necessitates lexical stipulations about how much imprecision each given modal expression can tolerate, and so this account ends up looking very similar to the threshold-based account of Hawthorne et al. (2016) and Rothschild (2020). That is, one can regard Moss (2019) as broadly favoring the WF view.

2.2 Subjective vs. objective epistemic modality

Carrying a weak force is not the only way for an epistemic modal to be lacking in strength. Another possibility is that it encodes non-subscribable modal content, i.e., modal content that does not make the relevant agent responsible for the prejacent proposition. Following this general line of thought, Lyons (1977: ch.17) points out that epistemic modality comes in two flavors, i.e. as 'subjective' or 'objective'. In (7), this is illustrated for epistemic *must*, which can exhibit either a subjective or an objective reading. I have preserved the original paraphrases of the two readings.[4]

(7) Alfred must be unmarried. (Lyons 1977: 791–792)

a. SUBJECTIVE: I confidently infer that Alfred is unmarried.

b. OBJECTIVE: In the light of what is known, it is necessarily the case that Alfred is unmarried.

According to these paraphrases, both readings of *must* involve an inference from some body of evidence, a fact that presumably stems from the strong force of *must* (cf. Stone 1994; Palmer 2001: 2.1.7; von Fintel and Gillies 2010; Mandelkern 2019; Waldon 2021). However, the quality of the evidence differs between the two readings. The subjective reading is based on less reliable evidence and merely voices an opinion, while the objective reading is based on knowledge and entails commitment to truth. For example, the subjective reading would arise if we learn that Alfred is dating someone, and from this we draw the somewhat risky conclusion that Alfred is unmarried. In turn, the objective reading would come about if we know that exactly one faculty member (whoever that might be) is not married and have already established that every faculty member, except for Alfred, is married. In this latter case, it becomes a matter of logical deduction to conclude that Alfred is unmarried.[5]

There have been two main attempts to render the subjective–objective distinction in more theoretical terms. Kratzer (1981), for one, proposes to expound this distinction not so much in terms of the available evidence but in terms of what we make of it. According to this proposal, the two readings share the same 'modal base' (the available and agreed-upon evidence), but they differ in 'ordering source' (the stereotypical assumptions one makes in order to draw inferences from the available evidence). While subjective modal claims involve risky stereotypical assumptions that may not be defendable, objective modal claims are based on established regularities and are defendable. An illustrative example is provided in (8) (original example in German; see Kratzer 1981: 307).

(8) *Lenz, who often has bad luck, is going to leave the Old World by boat today, on Friday 13th. On hearing about this, someone utters*:

a. Probably, the boat will sink.

b. It is probable that the boat will sink.

[4]Here the term 'reading' is used informally and is not intended to suggest that epistemic modals are lexically ambiguous. Further discussion on this matter can be found in the Conclusion.

[5]A similar contrast arises with weak-force epistemic auxiliaries, like *might* (cf. Lyons 1977: 797–798).

According to Kratzer, the claim in (8a) is subjective. It relies on superstitions about there being unlucky days and cannot be easily defended on objective grounds. Consequently, it necessitates a subjective background as an ordering source. By contrast, the claim in (8b) is objective. It is based on established facts about the boat, the technical equipment, or the weather, and can be defended on objective grounds. It thus requires an objective background as an ordering source.[6]

Nuyts (2001) and Papafragou (2006) put a different spin on the above distinction. For them, subjectivity hinges on the accessibility of the supporting evidence, specifically whether it is shared among speech participants. Subjective modality involves evidence that is only known to the speaker, while objective (or 'intersubjective') modality involves evidence that is shared among all speech participants. This distinction in accessibility does not seem to always cut the pie in the right way, though. For instance, in (7) the evidence is shared on both the 'dating' and the 'unmarried faculty member' scenarios, yet the intuitive distinction in subjectivity persists. This suggests that the Kratzer and the Nuyts–Papafragou accounts may be tracking two slightly different distinctions. In fact, Portner (2009: 4.2) demonstrates how these two accounts can coexist within a single formal model.

I will broadly adopt Kratzer's characterization of the contrast in subjectivity as being about what is publicly defendable. That is, I will view subjective modality as merely describing the mental state of the relevant agent. Since the contents of such state need not be backed by strong evidence, the agent is not held responsible for them. By contrast, objective modality is about contributing information to the public forum. It entails commitments for the agent and so it must be rooted in strong evidence. More formally, I will assume that subjective and objective certainty are semantically encoded by two different measure functions, which further specify the generic certainty measure C familiar from Section 1. Specifically, C^s maps propositions to degrees of subjective certainty in the intended sense. As its counterpart, I introduce the measure C^o, which maps propositions to degrees of objective certainty.

Notice that this usage of the terms 'subjective' and 'objective' is reminiscent but does not entirely align with philosophical parlance about probability (Hájek 2019 for an overview). In philosophy, Bayesians analyze probabilities in terms of betting behavior and view them as representing the subjective beliefs of individual agents, while frequentists locate probabilities in the world and view them as representing objective proportions. Notably, C^s and C^o are both subjective in this philosophical sense as they are anchored to an agent. Closer to home, there is a suggestive parallel within the former, Bayesian interpretation. That is, according to subjective Bayesians it is up to the agent which degrees of certainty to adopt as long as they are coherent, following the axioms of probability theory (e.g., de Finetti 1964; Savage 1972; Jeffrey 1990). By contrast, objective Bayesians require that degrees of certainty be additionally calibrated with the available evidence (e.g., Jaynes 1968; Rosenkrantz 1981; Williamson 2010). Despite the obvious similarities to the intended subjective–objective distinction, some differences remain. First, the concept of public commitments is essential to the intended distinction (see also Hamblin 1971; Krifka 2015; Geurts

[6]Presumably, this contrast has something to do with the fact that *probably* is a speaker-oriented adverb whereas *probable* is a modal adjective. Whatever the correct analysis, the important point is the felt contrast in subjectivity.

2019), but it does not seem to play a prominent role in Bayesian epistemology. Second, while subjective Bayesians disregard evidence entirely, it is questionable whether subjective modals like *believe* require no evidence whatsoever. Third, Bayesian epistemology primarily focuses on probabilities, whereas it remains unclear that epistemic expressions at large encode probability measures (e.g., *believe* is generally incompatible with proportional and percentage modifiers, like *two thirds* or *37 percent*). In sum, while intriguing parallels exist between the semantic subjective–objective distinction and various forms of Bayesian epistemology, one should avoid stretching these parallels too far.

I conclude this section by addressing two important points. The first point concerns the question of whether the two measures and corresponding flavors of certainty are related in some manner. In addressing this question, I suggest the following heuristic for how subjective and objective certainty are expected to stack up: we can view C^o as a more conservative variant of C^s, implying that speakers should support their public commitments with a sufficient degree of belief. That is, assuming that speakers are sincere, a given degree of commitment will be accompanied by an equal or greater degree of belief: $C^o_{x,w,t}(p) \leq C^s_{x,w,t}(p)$, for all sincere speakers x.[7] Conversely, a certain degree of belief does not necessitate any specific degree of commitment. For instance, an agent may give full credence to a proposition without being willing to publicly commit to it. In fact, expressions of subjective certainty in hedging sentences often serve to emphasize exactly this kind of opposition (see Section 3.2).

The second important point is that certain modal expressions may lexicalize a particular flavor of epistemic modality. Thus, mental state predicates like *believe* or *doubt* could reasonably be considered inherently subjective, conveying private opinions and inviting commitment hedging. By contrast, mental state predicates like *know* are inherently objective and commit the agent to the prejacent proposition. That said, most epistemic modals, e.g. *sure*, appear to be neutral and may convey either flavor of modality (see the Conclusion for a brief discussion).[8]

2.3 Subjective Content

In light of the distinction between subjective and objective epistemic modality drawn in the previous section, the intuition that *believe* denotes a 'weak' attitude may be fleshed out in terms of subjective content rather than a non-strong modal force. The view that *believe* is strong but subjective is formalized in (9) and spells out the informal statement in (3).

(9) SUBJECTIVE CONTENT

[7] This heuristic bears similarity to Geurts's (2019) Sincerity Principle, which roughly states that a commitment to p entails a lack of belief to *not* p. See also Lewis's (1980) Principal Principle, according to which subjective credences ought to match the objective chances.

[8] Nuyts (2001: 390–391) touches on both of these points, writing: "The mental state predicates systematically express subjectivity ... Because the mental state predicates are inherently subjective, they are frequently used as mitigating or hedging devices ... In such uses, it is usually quite obvious that speakers are absolutely certain about or convinced of what they are saying, but by using the mental state predicate they suggest that they are voicing a tentative and personal opinion which may be wrong, thus 'officially' leaving room for another opinion or for a reaction on the part of the hearer."

$$([\![\text{POS}]\!]^{w,t}([\![\text{believe}]\!]^{w,t}(p)))(x) \quad \text{iff} \quad C^{s}_{x,w,t}(p) = 1$$

Comparing (6) and (9), we notice that WF and SC differ in two respects only. First, they diverge in their choice of measure functions. WF ascribes to *believe* a generic measure of certainty shared with all epistemic modals with a scalar semantics. By contrast, SC posits that *believe* measures subjective certainty, thus conveying a weakness in content which contrasts with the objective certainty encoded by modals like *know*. Second, WF and SC diverge in modal force. While WF ascribes to *believe* a non-maximal force, requiring that the (generic) certainty associated with it exceeds some lower threshold value, SC does not weaken the force of *believe*. According to this latter view, the intuition that *believe* is weak is instead rooted in its subjectivity and the threshold value is kept at a maximum.

Overall, these two competing views pin the weakness of *believe* on two different components of its semantics: modal force vs. certainty type. In the following section, we will explore why SC appears to align more closely with the empirical evidence, demonstrating that *believe* exhibits the logical properties of a strong-force modal while plausibly conveying subjective content.

3 Data on *believe*

This section examines three sets of empirical data that turn out to largely favor SC over WF. The first two sets, which focus on the distribution of *believe* in modal contexts and its interaction with other epistemic expressions, indicate that *believe* fails to convey internal doubt but may signal a lack of commitment. This is consistent with *believe* encoding a subjective modal flavor. The third set of data, centered on the observation that *believe* is closed under conjunction, further suggests that this verb carries a strong modal force.

3.1 Distribution in modal contexts

I will begin by looking at examples in which the certainty level of the belief holder is fixed by the prior context. There are two main cases to consider here. In the first case, when the context assigns a maximal degree of certainty to an agent leaving no room for doubt in their mind, an attribution of belief to said agent is quite natural. This is shown in (10).

(10) CONTEXT: *We do not know whether the transfer student passed the midterm exam, but Jill has no doubt he did.*

Jill believes that the transfer student passed the midterm exam.

Conversely, it is hard to attribute genuine belief to an agent who doubts the prejacent proposition to some extent. This is illustrated in (11).

(11) CONTEXT: *Mueller finds it likely that the Russians hacked the election. But the evidence is inconclusive, so some doubt remains in his mind.*

? Mueller believes that the Russians hacked the election.

The degraded status of this utterance becomes even more evident when contrasted with the fully acceptable variant *Mueller believes that the Russians might have hacked the election*, where the prejacent expresses a mere possibility.[9]

Both of these data-points recommend SC over WF. According to the WF view, (10) and (11) should lead to the opposite judgments from what we actually observe. That is, if *believe* did not require full certainty, it would be puzzling why (10) is fully natural instead of being felt to be underinformative—especially in the presence of a stronger alternative that would fit the context, like *Jill is sure that the transfer student passed the midterm exam*. Here the *believe*-alternative should be on par with a statement like *Billy ate most of the cookies*, uttered in a context where it has already been established that Billy ate all of the cookies. The relative infelicity of (11) poses a problem for WF as well, given that a high (though non-maximal) degree of certainty should suffice to warrant belief. On this view, we would need to conjure up some independent issue with this utterance to account for its degraded status. In sum, WF appears to lack the explanatory power to accommodate examples as straightforward as (10) and (11).

On the other hand, SC accurately predicts the observed judgments in both cases. That is, (10) attributes no doubt to the agent and *believe* is licensed, whereas (11) attributes some amount of doubt to the agent and *believe* is not licensed. This is exactly as expected.

Data like these have implications that extend beyond their immediate scope as they bear on one kind of argument that has been put forward in support of WF. In particular, Hawthorne et al. (2016: 1400) seek to establish that the threshold for *believe* may drop below 50% when the prejacent is the most likely alternative among several contextually salient alternatives. Their main example (credited to Jeremy Goodman, p.c.) involves horse races and is cited in (12).

(12) CONTEXT: *In a three-horse race, the known statistical chances of winning are distributed as follows: horse A = 45%, horse B = 28%, horse C = 27%.*

I believe horse A will win.

The suggestion here is that *believe* cannot possibly require full credence because, in certain situations, the threshold value need not even reach 50%. At first blush, this appears to be compelling evidence for WF. However, this argument crucially conflates statistical chances with personal cre-

[9]Since the degraded status of (11) is less categorical, it is worth examining whether negating the utterance flips the judgment. However, there is no straightforward way to negate sentences with *believe* without triggering additional effects. For one, merely negating the target utterance in (11) invites the possibility of neg-raising, which would strengthen the reading to 'Mueller believes that the Russians didn't hack the election' and clash with the prior context. In order to suspend this stronger reading, we may add stress on *believe* (Gajewski 2007; Romoli 2013) and contrast it with *likely*, as shown in (i-a), creating a natural fit with the context. Another option is to use matrix negation by prefixing the sentence with, say, *It's not true that...* . While this results in two layers of embedding and may be somewhat stylistically dispreferred, (i-b) seems fine once again.

(i) CONTEXT: *Mueller finds it likely that the Russians hacked the election. But the evidence is inconclusive, so some doubt remains in his mind.*

a. Mueller doesn't BELIEVE that the Russians hacked the election. He only finds it LIKELY.

b. It's not true that Mueller believes that the Russians hacked the election. He only finds it likely.

In short, properly controlled negated variants of (11) indeed reverse the judgment, as expected.

dences. Yet, as already demonstrated by examples like (11) (see also (18) and (20) below), attributing belief is unnatural if the agent has any doubt in the prejacent proposition. Therefore, it is questionable whether in (12) the lower statistical estimates translate into a lack of internal certainty.

As additional evidence pointing into the same direction, notice that in the very same context one can equally well utter either of the variants in (13a). While holding such beliefs may not be entirely rational, the point is that such utterances would be felicitous.[10] Notice also that, as demonstrated in (13b), in this same context there can be disagreement about which horse will be the winner without a sense of confusion as to what the statistical chances are.

(13) a. I believe horse B / horse C will win.

b. Alfonso believes horse A will win, Benita believes horse B will win, and Camila believes horse C will win.

One plausible line of explanation is that, in all cases, the agent's subjective certainty is at a maximum and each specific choice is not solely based on statistical chances but also on things like personal preferences, gut feeling, or any other evidence that the belief agent deems relevant. This could be why sentences like these give the impression that the belief agents may not trust the numbers or may possess undisclosed information about the horses. Pursuing this line would allow the SC view to provide a systematic account for all the horse race data in (12)–(13). By contrast, the WF view has more work to do in order to justify why suboptimal alternatives, as in (13), can naturally represent personal beliefs. In a broader sense, the issue with WF is that if agents had the liberty to believe any proposition they wished, then there would be no interesting threshold semantics for *believe* to be had.

3.2 Interaction with other epistemic expressions

Arguably the most direct way of probing into the strength of *believe* involves examining its interaction with other epistemic expressions. There are several examples which juxtapose *believe* with another epistemic expression and appear to favor WF over SC on grounds of theoretical parsimony. That is, while such examples could be accommodated by either view, they might be taken to argue for WF as this view need not draw a distinction between different epistemic flavors in order to account for the data. This kind of general consideration notwithstanding, it turns out that in some cases SC makes more precise and empirically grounded predictions.

The first type of example I will discuss involves modal gradation sequences. One exemplar is based on the old dictum that knowledge is stronger than belief.[11] It can be illustrated by the natural gradation in (14).

(14) Scientists believe that there is water on Mars. In fact, they know it.

[10] Dorst and Mandelkern (2021: 4) make a similar point about *think* or, more generally, about making suboptimal guesses, stating: "To be clear, we are not claiming that people never have guesses like these. Our claim is normative: there is something peculiar—something irrational—about guesses like this."

[11] Gettier (1963) is a classic reference.

This example leaves the impression that the certainty level attributed to the scientists regarding the existence of water on Mars increades as the discourse advances from the first sentence to the second sentence. This observation suggests that *believe* encodes a lower degree of strength compared to *know*, although it does not necessarily pinpoint whether this difference is due to a contrast in modal force or a contrast in modal flavor.

While both WF and SC can explain the intuition of strengthening in (14), the nature of the explanation differs in each case. WF offers a straightforward explanation in terms of modal force. All this view has to say is that, since *believe* carries a weak force, it is only natural to strengthen it by using a modal with a strong force, like *know*. This explanation puts (14) on par with cases in which *likely* is strengthened to *certain* (cf. *It's likely he stole the money. In fact, it's certain*), where presumably these two modals merely differ in force.

In turn, SC states that *believe* lexicalizes a strong force, so it has to resort to a contrast in modal flavor when tackling (14). In order to make this more explicit, let us posit (15) as a minimal lexical entry for *know*. According to this entry, *know* differs from *believe* in at least two respects (cf. Percus 2006; Chemla 2008; Sauerland 2008; Schlenker 2012): it incorporates a factive component, presupposing that the prejacent proposition is true, and it lexicalizes objective epistemic certainty, encoded by C^o.[12]

(15) $[\![\text{know}]\!]^{w,t} = \lambda p \lambda x : p(\langle w,t \rangle) . C^o_{x,w,t}(p) = 1$

In the light of this semantics, the intuition of strengthening in (14) can be attributed to a shift from subjective to objective certainty, with everything these two concepts imply in terms of commitment level, supporting evidence, and so on.

We have observed that both WF and SC can account for the intuition of strengthening in modal gradation sequences with *believe* and *know*. But unlike WF, which operates with a single notion of certainty, SC relies on the additional assumption that *believe* and *know* differ in their epistemic flavor. This may be taken as a theoretical point in favor of WF, at least if there are no independent reasons to think that there is a subjective–objective opposition at play in natural language semantics. However, if such reasons do exist (as some of the remaining data suggests), then there is no parsimony argument to be made.

The second type of example I will discuss involves hedging sentences. Such sentences consist of a belief attribution accompanied with an explicit disavowal of full certainty (of the relevant kind). One example is cited in (16).[13]

(16) I believe it's raining, but I'm not sure it's raining. (Hawthorne et al. 2016: 1395)

Once more, both WF and SC can accommodate hedging sentences like these, albeit through distinct mechanisms and by making different additional predictions. Under WF, (16) attributes to

[12] Building on prior literature (Bolinger 1972: ch.9; Partee 2004; Stanley 2004; Wellwood 2019: ch.8; Lassiter 2021), I also tentatively assume that *know* is not grammatically gradable, i.e., it lacks a degree argument. Nothing important depends on this assumption, though.

[13] As noted in Hawthorne et al. (2016: 1400), some English speakers exhibit a slight preference for *think* over *believe* in such examples, presumably due to a register contrast. However, naturally occurring examples of this shape abound on the web. See also the experimental data reported at the end of the current section.

the speaker high but non-maximal certainty regarding the prejacent proposition. Importantly, there is a single notion of certainty involved here, one shared by *believe* and *sure*.

SC provides a very different explanation. According to this latter view, (16) establishes a contrast between maximal credence and lower commitment. This contrast can be made formally explicit by positing that *sure* is a strong gradable predicate that can convey an objective modal flavor. This is detailed in the preliminary entry in (17), where the default maximum standard associated with *sure* is supplied by the null morpheme POS.[14]

(17) a. $[\![\text{sure}]\!]^{w,t} = \lambda p \lambda d \lambda x . C^{o}_{x,w,t}(p) \geq d$ (preliminary)

b. $[\![\text{POS}]\!]^{w,t}([\![\text{sure}]\!]^{w,t}(p)) = \lambda x . C^{o}_{x,w,t}(p) = 1$

While each view takes the hedging data in stride, WF seems to win out on simplicity once again as it does not necessitate a differentiation between subjective and objective epistemic modality and corresponding measures of certainty. Nonetheless, independent evidence suggests that hedging sentences with *believe* do establish a contrast in modal flavor rather than a contrast in modal force. This evidence involves (i) expressions of doubt and (ii) manipulation of the agent, world and time parameters.

Starting with the former kind of evidence, notice that sentences containing *believe* cannot naturally be hedged with modal expressions that are inherently subjective, such as the dubitative verb *doubt*. This is exemplified in (18).

(18) # I believe Putin stole the election, but I doubt it.

Because WF operates with a single flavor of modality, it remains unclear why (16) and (18) should yield different judgments. To account for the observed contrast, this view would have to be supplemented with some independent mechanism. One simple thought is that (18) sounds contradictory simply because *doubt* conveys too much uncertainty for it to be compatible with *believe*. Indeed, in upward-entailing contexts *doubt p* typically implies not just that *p* is uncertain but that *p* is unlikely (Anand and Hacquard 2013). However, this strengthened meaning has been attributed to exhaustification arising from the fact that English *doubt* lacks a stronger scalemate whose meaning amounts to 'likely not' (Uegaki 2021). The key observation here is that the implication to unlikelihood melts away in downward-entailing contexts. That is, since *not doubt p* entails *certain p*, it follows that *doubt p* must entail *not certain p* rather than *likely not p*. But this makes (16) and (18) semantically equivalent, both amounting to *believe p but not certain p*, and so the intuitive difference in judgment is left unexplained.

One could still insist that, since in (18) *doubt* appears in an upward-entailing context, it is obligatorily exhaustified and so it clashes with *believe*. However, even if *doubt* must be strengthened and conveys unlikelihood, WF would still not rule out (18). The reason is that, according to this view, the threshold for *believe* can lie below the midpoint of the scale, making *believe* logically consistent with the exhaustified meaning of *doubt*. That is, (18) would be incorrectly predicted to

[14] In the concluding Section 4, I will suggest that *sure*—along with *might*, *must*, *possible*, *likely*, *certain*—actually exhibits epistemic indeterminacy, in the sense of being capable to convey either subjective or objective modality.

be acceptable in contexts where, say, the speaker considers the prejacent to be the most likely alternative among several other options but still deems this alternative unlikely overall. This prediction is at odds with the strong sense of unacceptability associated with (18).

By contrast, SC readily accounts for the unacceptability of (18). That is, if *believe* and *doubt* are assumed to be antonyms operating on a subjective scale, then (18) is correctly predicted to result in an epistemic contradiction, the reason being that this sentence ends up simultaneously attributing and denying full credence in the prejacent proposition. Formally, we can assume that *doubt* is a minimality predicate that lexicalizes the reverse scale to that of C^s, call it D^s.[15] This is spelled out in (19).

(19) a. $[\![\text{doubt}]\!]^{w,t} = \lambda p \lambda d \lambda x . D^s_{x,w,t}(p) \geq d$

b. $[\![\text{POS}]\!]^{w,t}([\![\text{doubt}]\!]^{w,t}(p)) = \lambda x . D^s_{x,w,t}(p) > 0$
$= \lambda x . C^s_{x,w,t}(p) < 1$

One important caveat regarding the theoretical implications of (18) is that not all linguistic expressions of doubt exhibit the same degree of incompatibility with *believe*. In order to probe the data more systematically, I tested online the minimally different hedging sentences in (20), which employ different lexical categories (the verb *doubt*, the adjective *doubtful*, and the noun *doubt*) and otherise introduce no further complexities. Three variants of each sentence were presented to 12 participants (with English as a first language and IP addresses located in the U.S.), recruited through the Prolific crowdsourcing platform. Participants were asked to rate each variant for naturalness on a seven-point Likert scale, where 1 corresponded to 'very unnatural' and 7 corresponded to 'very natural'. The obtained mean ratings are listed below.

(20) a. # I believe Putin stole the election, but I doubt that he did. (mean = 1.7)

b. ?? I believe Putin stole the election, but I'm doubtful that he did. (mean = 2.4)

c. ? I believe Putin stole the election, but I have some doubts that he did. (mean = 3.3)

As can be seen, all three versions received ratings towards the lower part of the scale, against a baseline mean of 4.7 for the respective control sentence with *not sure* in the hedge. I will not speculate on the reasons for this variation in acceptability, although it stands to reason that different expressions of doubt may engage with the agent's certainty in slightly different ways. When considering the theoretical implications of this data, the presence of some variability is a point in favor of WF, which allows for the belief threshold to vary. At the same time, the fact that none of the above examples is as natural as the variant with *not sure* suggests that there might be something inherently incompatible between *believe* and various expressions of doubt, as would be expected under SC.

Additional evidence supporting the analysis of hedging sentences with *believe* as contrasting modal flavor rather than modal force comes from manipulating the agent, world, and time parameters. Author et al. (2021) demonstrate experimentally that hedging sentences featuring combinations of third person/past tense/embedded features, while logically consistent, are generally rated

[15] That is, assuming that these are ratio scales, $D^s_{x,w,t}(p) > d$ ends up meaning the same as $C^s_{x,w,t}(p) < 1 - d$, for all degrees d between 0 and 1.

as somewhat less natural than canonical first person/present tense/main clause forms. For instance, participants found each of the sentences in (22) to be slightly less acceptable than the baseline form in (21).[16]

(21) I believe diplomacy is better than war, but I'm not sure.

(22) a. (?) Sean believes diplomacy is better than war, but he's not sure.
b. (?) One year ago I believed diplomacy was better than war, but I was not sure.
c. ? Suppose that I believe diplomacy is better than war, but I'm not sure.

Once again, in the absence of some independent mechanism, WF has little to say about what causes such subtle contrasts in judgment. This view draws a purely quantitative contrast between *believe* and *not sure*, rooted in a single epistemic measure and comparable to 'likely but not certain'. So we would expect such sequences to be equally natural, contrary to what we find.

SC, which draws a qualitative contrast between subjective and objective certainty, has a better shot at such data. That is, it has been independently argued that human communication primarily revolves around negotiating commitments rather than merely sharing beliefs or other mental states (Hamblin 1971; Krifka 2015; Geurts 2019). In light of this claim, we would expect that the subjective–objective contrast is most readily discerned in the presence of a salient speech context, since in such cases private beliefs and public commitments can be most sharply distinguished. This implies that the closer the parameters on the certainty measures align with the utterance context parameters, where hedging sentences with first person/present tense/main clause features constitute the canonical case, the more natural said contrast is expected to be. This is indeed what the data in (21)–(22) suggests.[17]

I close this section with a puzzle that seems to pose a challenge for both views. In Section 3.1, we established that an attribution of belief in the presence of doubt is degraded; the key example

[16]For the sake of simplicity, only minimally different sentences from the canonical form are listed here, and combinations of differing features are omitted.

[17]The observation that person/world/time parameters can influence the acceptability of 'epistemic contradictions' is not novel. Moore (1993) had already noted that while (i) sounds odd, sentences of this shape are not self-defeating. They become acceptable when embedded under modals or when rendered in the past tense or the third person, as shown in (ii)–(iv).

(i) # It's raining, but I don't believe it's raining.

(ii) Suppose that it's raining but I don't believe it.

(iii) I didn't believe it was raining, but as a matter of fact it was.

(iv) Jack doesn't believe it's raining, but it really is.

A detailed analysis of this paradigm is beyond the scope of this paper. The important thing to notice, though, is that only the infelicitous example in (i) combines a full commitment (stemming the unqualified assertion in the first part of the sentence) and lower credence (resulting from the negated belief attribution in the second part of the sentence) with the same person/world/time specifications. As discussed in Section 2.2, such a combination conveys a lack of sincerity and is expected to be pragmatically odd. (iii) does something similar, but here the belief is anchored in the past while the assertion is anchored in the present, mitigating any oddness. In (iv), the agents of the belief and the assertion differ, and (ii) entirely lacks an assertion of rain due to the embedded position of the first conjunct.

(11) is repeated in (23a). Nonetheless, notice that adding a hedge of a certain shape seems to obviate the problem, as shown in (23b).

(23) CONTEXT: *Mueller finds it likely that the Russians hacked the election. But the evidence is inconclusive, so some doubt remains in his mind.*

a. ? Mueller believes that the Russians hacked the election.

b. Mueller believes that the Russians hacked the election, but he is not fully certain.

Both views yield uniform and thus incorrect predictions regarding the contrast between (23a) and (23b), with WF predicting that both sentences should be fine and SC predicting that both sentences should be degraded. That is, as argued in Section 3.1, WF would be hard-pressed to justify the degraded status of (23a), given that the agent ascribes a high degree of certainty to the prejacent and so *believe* should be licensed. On the other hand, SC must explain why the belief attribution in (23b) seems fine despite the presence of doubt stipulated in the context. One possibility is that the presence of the approximator *fully* allows for some amount of imprecision (cf. Lasersohn 1999; Sauerland and Stateva 2011; Solt 2014; Klecha 2018) and acceptability is improved (although it is unclear that removing *fully* from the sentence significantly alters the judgment). The key point is that, given what we know so far, the apparent empirical contrast in (23) does not really let us choose between WF and SC.

To recap, the way *believe* interacts with other expressions of epistemic modality points at its subjective flavor. While some of the data can be folded under either WF or SC and the former view happens to be theoretically more parsimonious, certain empirical contrasts seem compatible only with the latter view. Furthermore, both views still face unresolved challenges.

3.3 Closure under conjunction

The force of a particular epistemic modal is expected to be reflected in the logical properties that it does or does not possess. One such property that can be employed as a diagnostic for modal force is conjunction closure. A modal M is said to be 'closed' under conjunction introduction just when M licenses the entailment pattern in (24).

(24) CONJUNCTION CLOSURE FOR MODALS

$M(p), M(q) \models M(p \textit{ and } q)$

How do epistemic modals behave with respect to this property? It is easy to observe that modals that uncontroversially carry a strong force are closed under conjunction. This is exemplified for *certain* in (25).

(25) It's certain that Sean is in Rome and it's certain that he is catholic.
$\models$ It's certain that Sean is in Rome and that he is catholic.

By contrast, modals that uncontroversially carry a non-strong force, like *probably* or *possible*, do not exhibit this property, as shown in (26)–(27).

(26) CONTEXT: *Each week Jack spends (in no particular order) 3 nights at the local pub and gets drunk, 2 nights at the same pub but stays sober, and 2 nights at home where he also gets drunk. On a given night, I say*:

a. Jack is probably at the pub. True (chance $= 5/7$)

b. Jack is probably drunk. True (chance $= 5/7$)

c. Jack is probably at the pub drunk. False (chance $= 3/7$)

(27) It's possible Jane is in Italy and it's also possible Jane is in France.
$\not\models$ It's possible Jane is in Italy and in France.

Crucially, *believe* aligns with strong-force modals in this respect (as also noted in Hawthorne et al. 2016 and Rothschild 2020). It licenses the entailment pattern in (24), as (28) and (29) demonstrate.

(28) Ron believes Mia is pretty and he also believes she is going to marry him.
$\models$ Ron believes that Mia is pretty and that she is going to marry him.

(29) ? John believes it will rain today and he believes it will rain tomorrow, but he doesn't quite believe it will rain today and tomorrow.

The problematic status of (29) is particularly revealing. It demonstrates that a conjunction of beliefs cannot lead to a reduced degree of certainty, even when such reduction is necessary for the sentence to be acceptable.[18]

Let us now rehearse the predictions that SC and WF make regarding the interaction of *believe* with conjunction. The finding that *believe* is closed under conjunction is good news for SC, the reason being that the closure property falls out directly from the assumption that *believe* carries a strong force. Here is an intuitive understanding of why this is the case. Suppose that *x believes p* and *x believes q* are both true. According to SC, it follows that $C^s_{x,w,t}(p) = 1$ and $C^s_{x,w,t}(q) = 1$ (in the relevant world w and at the relevant time t). These expressions say that, according to x, the entire certainty weight falls within p and the entire certainty weight also falls within q. In other words, all possibilities outside $p \cap q$ are discarded. We may conclude that $C^s_{x,w,t}(p \cap q) = 1$, or that *x believes (p and q)* is true as well.[19]

By contrast, the empirical observation that *believe* is closed under conjunction is consistent with WF but it does not follow from this view. The intuitive reason for this lies in the fact that if any amount of uncertainty is compatible with any given belief, the uncertainties associated with individual beliefs may compound when combined, potentially failing to surpass the contextual threshold. This implies that we could encounter a situation where *x believes p* and *x believes q* are

[18] This example is from Rothschild (2020: 1357), with *quite* being added here to block a neg-raised interpretation, which would lead to inconsistent beliefs and independently explain the infelicity.

[19] Assuming that the measure of *believe* is probabilistic, we can give the following rigorous proof of conjunction closure (world and time parameters are suppressed). Let $C^s_x(p) = 1$ and $C^s_x(q) = 1$, so *x believes p* and *x believes q* are both true. Since probabilities sum up to 1, we get $C^s_x(\overline{p}) = 0$ and $C^s_x(\overline{q}) = 0$, respectively. Now assume, for contradiction, that $C^s_x(p \cap q) < 1$. For the same reason as above, we get $C^s_x(\overline{p \cap q}) > 0$, i.e., $C^s_x(\overline{p} \cup \overline{q}) > 0$. Since probability measures are additive, the latter can be true only if $C^s_x(\overline{p}) > 0$ or $C^s_x(\overline{q}) > 0$, a contradiction. Our assumption is false, hence $C^s_x(p \cap q) = 1$, which means that *x believes (p and q)* must be true.

both true, but *x believes (p and q)* is false.[20] What this means is that WF does not guarantee that *believe* is closed under conjunction, contrary to intuition.

Before proceeding, I would like to emphasize two important points that will help us better understand the implications of conjunction closure for the semantics of *believe*. The first point is that the tight link between a strong modal force and conjunction closure is not reliant on the specific technical implementation, such as whether *believe* is given a threshold-based semantics or a standard quantificational semantics. To see why, let us recall the Hintikkan proposal stated in (1). This proposal encodes strong force via universal quantification over possibilities but still derives the closure property. This is so because, if all of x's doxastic alternatives are p-possibilities and all of x's doxastic alternatives are also q-possibilities, it necessarily follows that all of x's doxastic alternatives are $p \cap q$-possibilities. Despite this virtue, as discussed in Section 1, the Hintikkan semantics does not tell us whether the universal force of *believe* translates into full certainty, absent additional stipulations regarding the modal content of *believe*. What this suggests is that conjunction closure really is a diagnostic for modal force and is insensitive to the type of modal content involved.

The second important point is that a weak-force semantics for *believe* might still achieve conjunction closure if the selection of prejacents is suitably constrained. One such account is suggested in Holguín (2022: 19–21). Roughly, this account states that the prejacent of *believe* is an optimal disjunction of complete answers to the current Question Under Discussion (QUD) in the sense that, if a given complete answer is included, all more likely complete answers must be included as well.[21] This account is illustrated in (30) using the racehorse example from Section 3.1.

(30) CONTEXT: *In a three-horse race, the statistical chances of winning are distributed as follows: horse A = 45%, horse B = 28%, horse C = 27%.*

QUD: Which horse will win?

a. OPTIMAL ANSWERS: Horse A. / Horse A or B. / Horse A or B or C.

b. NON-OPTIMAL ANSWERS: Horse B. / Horse C. / Horse B or C. / Horse A or C.

It is indeed true that any two premises with prejacents drawn from the set of optimal answers will derive the closure property. But the issue is that, due to the optimality restriction, one of the premises will always asymmetrically entail the other, and so the conclusion ends up being logically equivalent to that stronger premise. For instance, while *I believe horse A will win* and *I believe horse A or B will win* jointly lead to the conclusion *I believe that horse A will win and that horse A or B will win*, this conclusion conveys the same information as the former and stronger premise,

[20] Here is a counterexample to the closure property within WF, fleshed out in probabilistic terms (once again, world and time parameters are suppressed). Consider a context in which the threshold for *believe* is 0.5, and let p and q be probabilistically independent. If $C_x(p) = 0.6$ and $C_x(q) = 0.7$, then *x believes p* and *x believes q* are true, as they both exceed the threshold of 0.5. However, *x believes (p and q)* turns out to be false, given that $C_x(p \cap q) = C_x(p) \times C_x(q) = 0.6 \times 0.7 = 0.42$ falls below the threshold of 0.5.

[21] While Holguín's account is centered on *think* (and what is rationally permitted to think more generally) rather than *believe*, for the sake of the argument I will assume that these two verbs are similar in all relevant respects. Additionally, the account does not actually require that the prejacent of *believe* be optimal in the above sense. It only assumes that, as a matter of some psychological regularity, people tend to form optimal beliefs like these.

given that $p \wedge (p \vee q)$ is logically equivalent to p. This shows that, on this account, we could never derive conjunction closure from belief attributions with independent prejacents, because at most one of these prejacents—i.e., the more likely one—could ever constitute an optimal answer to the QUD. And given the empirical observation in (13) that non-optimal answers can readily serve as prejacents of *believe*, conjunction closure is not derived in the general case.

I conclude this section with a brief note on conjunction closure and rational belief. There is a debate in the philosophical literature as to whether the beliefs of a rational agent should be closed under conjunction. While many authors agree that this should be so (e.g., Hintikka 1962; Stalnaker 1984; Harman 1986; Clarke 2013; Leitgeb 2014; Greco 2015; Yalcin 2016; Moss 2019; Holguín 2022), detractors point out that the closure property leads to the 'lottery paradox' (Kyburg 1961). A classic version of the lottery paradox goes as follows. Consider a fair lottery with 100 tickets and a single winner. It seems rational to believe the statement *Ticket 1 will not win*, as it has a solid 99% chance of being true. But the same goes for the statements *Ticket 2 will not win*, *Ticket 3 will not win*, and so on down the line up to *Ticket 100 will not win*. By the closure property, it should then be rational to believe the statement *No ticket will win*. But this contradicts the assumption that one ticket will win.

The presence of the lottery paradox may be taken as evidence in favor of WF. However, the important point is that the lottery paradox is about the norms of rational belief rather than the empirical properties of the verb *believe*. If rational belief is understood as reaching a sufficiently high but not necessarily maximal level of confidence, it is indeed reasonable to reject the closure property, or else we would be left with a paradox. However, if what is at issue is the semantics of the verb *believe*, the lottery paradox does not really arise for SC. The reason is that, if the belief agent harbors any doubt in some of the prejacent propositions, the corresponding premises will already be judged false (as seen in (11), (18) and (20)), rendering the conjunction closure inference vacuously true. To illustrate, consider the lottery scenario from the previous paragraph and assume that Jack's subjective credences exactly match the respective chances. In such a scenario, each premise in (31) will be deemed false because Jack's credence—coming in at 99%—will fall short of reaching full subjective certainty. And so, the inference cannot be falsified.

(31) Jack believes that ticket 1 will not win.
Jack believes that ticket 2 will not win.
...
Jack believes that ticket 100 will not win.
———————————————
Jack believes that no ticket will win.

Conversely, if Jack has absolutely no doubt that each individual ticket will lose, Jack must also have no doubt that no ticket will win. For instance, this could occur if a large lottery with a single winning ticket has been subdivided into several sub-lotteries. Since in a specific sub-lottery there may be no winning ticket, the sequence in (31) would sound entirely natural. In this scenario as well, SC does not lead to the lottery paradox.[22]

[22]It is worth noting that SC still derives the related 'preface paradox' (Makinson 1965). This paradox is about a

4 Conclusion and outlook

The idea that *believe* expresses universal quantification over possibilities hails from a long and venerable tradition in formal semantics. This paper challenged this dictum as non-explanatory, pointing out that it fails to account for the intuition that *believe* implies some sort of weakness regarding the prejacent proposition. To gain a deeper understanding of the source of this intuition, this paper compared two perspectives on the weakness associated with *believe*: WF, which posits that *believe* conveys standard epistemic content but has a weak force, and SC, which contends that *believe* carries a strong force but conveys content that is subjective in the intended sense. By examining the occurrence of *believe* in modal contexts, its interaction with other epistemic expressions, and its conjunction closure properties, we have arrived at the tentative conclusion that the latter view is more on the right track. That is, while much of the data can be accommodated by both views and WF turns out to be theoretically more parsimonious, SC seems to provide better empirical coverage overall.

Given that SC is a substantial contender for elucidating the semantic properties of *believe*, it becomes essential to explore its implications for the broader epistemic domain. A key assumption of this view is that epistemic modality comes in two varieties, i.e., subjective and objective, with certain modals potentially lexicalizing one of these flavors. But how deeply does the subjective–objective distinction cut into the epistemic domain? Answering this question goes far beyond the scope of this paper, whose modest goal is limited to understanding the sense in which *believe* expresses a weak attitude. The focus on *believe* notwithstanding, one may hypothesize that all epistemic modals are lexically encoded as subjective or objective, or else can be interpreted as conveying either of these flavors. Indeed, in some cases there is no real choice in how a modal is interpreted. That is, following the line of SC, we have assumed that *believe* and *doubt* are inherently subjective, while *know* is inherently objective. In other cases, there seems to be more flexibility. Take *sure* as an illustration. In the face of hedging sentences like (32a) and given the inherent subjectivity of *believe*, SC predicts that *sure* can be read objectively (Section 3.2). In addition, the (marginal) acceptability of (32b) and the inherent objectivity of *know* suggest that *sure* can be read subjectively as well.

(32) a. I believe she was at the party, but I'm not sure.
b. (?) I'm sure she was at the party, but I don't know it.

Building on Lyons (1977) and the discussion in Section 2.2, one could argue that the same holds for modals like *might*, *must*, *possible*, *likely*, and *certain*. How does the ambivalence of such modals come about? It hardly seems plausible that such modals are lexically ambiguous, in the sense of being listed twice in the lexicon—once as subjective and once as objective. Such an assumption would lead to multiplying the range of meanings without adding much explanatory value. Instead,

cautious scholar who asserts in the preface of her book that she believes there are errors within it, even though, upon inspecting the claims individually, she professes to believe each of them. The problem is that, since the preface paradox is about the author's own subjective credences, SC makes both the individual beliefs and the overall statement true. However, the issue here may simply be that such overall statements do not serve as proxies for the conjunction of all claims in the book and would typically include auxiliary assumptions that the author may not actually believe.

a more plausible scenario is that these modals are not inherently bound to subjectivity or objectivity and that the specific interpretation they receive is context dependent. If this hypothesis is on the right track, it leads to the basic typology of epistemic modals as shown in (33), with one example provided for each case.

(33) SUBJECTIVITY TYPOLOGY OF EPISTEMIC MODALS

a. SUBJECTIVE: $[\![\text{believe}]\!]^{w,t} = \lambda p \lambda d \lambda x . C^{s}_{x,w,t}(p) \geq d$

b. OBJECTIVE: $[\![\text{know}]\!]^{w,t} = \lambda p \lambda x : p(\langle w,t \rangle) . C^{o}_{x,w,t}(p) = 1$

c. SUBJECTIVE OR OBJECTIVE: $[\![\text{sure}]\!]^{w,t} = \lambda p \lambda d \lambda x . C^{i}_{x,w,t}(p) \geq d$, where $i \in \{s, o\}$

I leave the substantiation of these rather speculative remarks to future work.

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References

Anand, P. and V. Hacquard (2013). Epistemics and attitudes. Semantics & Pragmatics 6(8), 1–59.

Bartsch, R. (1973). 'Negative transportation' gibt es nicht. Linguistische Berichte 27, 1–7.

Barwise, J. and R. Cooper (1981). Generalized quantifiers and natural language. Linguistics and Philosophy 4, 159–219.

Bolinger, D. (1972). Degree Words. The Hague/Paris: Mouton.

Chemla, E. (2008). An epistemic step for anti-presuppositions. Journal of Semantics 25, 141–173.

Clarke, R. (2013). Belief is credence one (in context). Philosophers' Imprint 13(11), 1–18.

Cresswell, M. J. (1976). The semantics of degree. In B. Partee (Ed.), Montague Grammar, pp. 261–292. New York: Academic Press.

de Finetti, B. (1964). Foresight: Its logical laws, its subjective sources. In H. E. Kyburg and H. E. Smokler (Eds.), Studies in Subjective Probability, pp. 97–158. New York: Wiley.

Del Pinal, G. and B. Waldon (2019). Modals under epistemic tension: A defense of the restricted quantificational account of *must* and *might*. Natural Language Semantics 27, 135–188.

Dorst, K. (2019). Lockeans maximize expected accuracy. Mind 128(509), 175–211.

Dorst, K. and M. Mandelkern (2021). Good guesses. Philosophy and Phenomenological Research.

Foley, R. (1992). The epistemology of belief and the epistemology of degrees of belief. American Philosophical Quarterly 29(2), 111–124.

Gajewski, J. R. (2007). Neg-raising and polarity. Linguistics and Philosophy 30, 289–328.

Gettier, E. L. (1963). Is justified true belief knowledge? Analysis 23(6), 121–123.

Geurts, B. (2019). Communication as commitment sharing: Speech acts, implicatures, common ground. Theoretical Linguistics 1–30; 111–125(1-2), 45.

Giannakidou, A. and A. Mari (2018). A unified analysis of the future as epistemic modality: The view from Greek and Italian. Natural Language and Linguistic Theory 4(1), 55–69.

Goodhue, D. (2017). Must ϕ is felicitous only if ϕ is not known. Semantics & Pragmatics 10(14).

Greco, D. (2015). How i learned to stop worrying and love probability 1. Philosophical Perspectives 1, 179–201.

Hackl, M. (2009). On the grammar and processing of proportional quantifiers: *most* versus *more than half*. Natural Language Semantics 17, 63–98.

Hájek, A. (2019). Interpretations of probability. In Stanford Encyclopedia of Philosophy. https://plato.stanford.edu/entries/probability-interpret/.

Hamblin, C. L. (1971). Mathematical models of dialogue. Theoria 37(2), 130–155.

Harman, G. (1986). Change in View: Principles of Reasoning. Cambridge, MA: MIT Press.

Hawthorne, J., D. Rothschild, and L. Spectre (2016). Belief is weak. Philosophical Studies 173, 1393–1404.

Heim, I. (1985, May). Notes on comparatives and related matters. Technical report, University of Texas at Austin.

Hintikka, J. (1962). Knowledge and Belief. Ithaca: Cornell University Press.

Hintikka, J. (1969). Semantics for propositional attitudes. In J. W. D. et al. (Ed.), Philosophical Logic, pp. 21–45. Dordrecht: Reidel.

Holguín, B. (2022). Thinking, guessing, and believing. Philosophers' Imprint 22(6).

Homer, V. (2015). Neg-raising and positive polarity: The view from modals. Semantics & Pragmatics 8(4), 1–88.

Horn, L. R. (1989). A Natural History of Negation. Chicago: University of Chicago Press.

Jaynes, E. T. (1968). Prior probabilities. IEEE Transactions on Systems Science and Cybernetics 4(3), 227–241.

Jeffrey, R. (1990). The Logic of Decision. Chicago: The University of Chicago Press.

Kahneman, D. and A. Tversky (1982). Variants of uncertainty. Cognition 11, 143–157.

Karttunen, L. (1972). Possible and must. In J. Kimball (Ed.), Syntax and Semantics, pp. 1–20. New York: Academic Press.

Kennedy, C. and L. McNally (2005). Scale structure, degree modification, and the semantics of gradable predicates. Language 81(2), 345–381.

Klecha, P. (2014). Bridging the Divide: Scalarity and Modality. Ph.D. dissertation, University of Chicago.

Klecha, P. (2018). On unidirectionality in precisification. Linguistics and Philosophy 41(1), 87–124.

Kratzer, A. (1981). The notional category of modality. In H. J. Eikmeyer and H. Rieser (Eds.), Words, Worlds, and Contexts. New Approaches in Word Semantics, pp. 38–74. Berlin/New York: Walter de Gruyter.

Kratzer, A. (1991). Modality. In A. von Stechow and D. Wunderlich (Eds.), Semantik/Semantics: An International Handbook of Semantics, pp. 639–650. de Gruyter.

Krifka, M. (2015). Bias in commitment space semantics: Declarative questions, negated questions, and question tags. In Proceedings of Semantics and Linguistic Theory 25, pp. 328–345.

Kyburg, H. E. (1961). Probability and the Logic of Rational Belief. Middletown, CT: Wesleyan University Press.

Lasersohn, P. (1999). Pragmatic halos. Language 75(3), 522–551.

Lassiter, D. (2016). *Must*, knowledge, and (in)directness. Natural Language Semantics 24(2), 117–163.

Lassiter, D. (2017). Graded Modality: Qualitative and Quantitative Perspectives. Oxford: Oxford University Press.

Lassiter, D. (2021). Graded modality. In D. G. et al. (Ed.), The Wiley Blackwell Companion to Semantics, Volume 5. Wiley Blackwell.

Leitgeb, H. (2014). The stability theory of belief. The Philosophical Review 123(2), 173–204.

Lewis, D. K. (1980). A subjectivist's guide to objective chance. In R. C. Jeffrey (Ed.), Studies in Inductive Logic and Probability, Volume II, pp. 263–293. Berkeley: University of California Press.

Lyons, J. (1977). Semantics, vol. 2. Cambridge: Cambridge University Press.

Makinson, D. (1965). The paradox of the preface. Analysis 25(6), 205–207.

Mandelkern, M. (2019). What 'must' adds. Linguistics and Philosophy 42, 225–266.

Moore, G. E. (1993). Moore's paradox. In T. Baldwin (Ed.), G. E. Moore: Selected Writings, pp. 207–212. London: Routledge.

Morzycki, M. (2016). Modification. Cambridge, UK: Cambridge University Press.

Moss, S. (2019). Full belief and loose speech. The Philosophical Review 128(3), 255–291.

Nuyts, J. (2001). Subjectivity as an evidential dimension in epistemic modal expressions. Journal of Pragmatics 33, 383–400.

Palmer, F. R. (2001). Mood and Modality. Second Edition. Cambridge, UK: Cambridge University Press.

Papafragou, A. (2006). Epistemic modality and truth conditions. Lingua 116, 1688–1702.

Partee, B. (2004). Comments on Jason Stanley's 'On the linguistic basis for contextualism'. Philosophical Studies 119(1–2), 147–159.

Percus, O. (2006). Antipresuppositions. In A. Ueyama (Ed.), Theoretical and Empirical Studies of Reference and Anaphora: Toward the establishment of generative grammar as an empirical science, pp. 52–73. Report of the Grant-in-Aid for Scientific Research (B), Project No. 15320052, Japan Society for the Promotion of Science.

Portner, P. (2009). Modality. Oxford: Oxford University Press.

Romoli, J. (2013). A scalar implicature-based approach to neg-raising. Linguistics and Philosophy 36, 291–353.

Rosenkrantz, R. D. (1981). Foundations and Applications of Inductive Probability. Atascadero, CA: Ridgeview.

Rothschild, D. (2020). What it takes to believe. Philosophical Studies 177, 1345–1362.

Santorio, P. and J. Romoli (2017). Probability and implicatures: A unified account of the scalar effects of disjunction under modals. Semantics & Pragmatics 10(13).

Sauerland, U. (2008). Implicated presuppositions. In A. Steube (Ed.), The Discourse Potential of Underspecified Structures, pp. 581–600. Berlin: Mouton de Gruyter.

Sauerland, U. and P. Stateva (2011). Two types of vagueness. In N. Klinedinst (Ed.), Vagueness and Language Use, pp. 121–145. Palgrave.

Savage, L. J. (1972). The Foundations of Statistics. Second Edition. New York: Dover Publications.

Schlenker, P. (2012). *Maximize Presupposition* and Gricean reasoning. Natural Language Semantics 20, 391–429.

Solt, S. (2014). An alternative theory of imprecision. In Proceedings of Semantics and Linguistic Theory 24, pp. 514–533.

Solt, S. (2016). On measurement and quantification: the case of *most* and *more than half*. Language 92(1), 65–100.

Stalnaker, R. (1984). Inquiry. Cambridge, MA: MIT Press.

Stanley, J. (2004). On the linguistic basis of contextualism. Philosophical Studies 119, 119–146.

Stone, M. (1994). The reference argument of epistemic *must*. In First International Workshop in Computational Semantics, pp. 181–190.

Uegaki, W. (2021, December). The *doubt-whether* puzzle. Technical report, University of Edinburgh.

von Fintel, K. and A. Gillies (2010). *Must* ... stay ... strong! Natural Language Semantics 18(4), 351–383.

von Fintel, K. and A. Gillies (2021). Still going strong. Natural Language Semantics 29, 91–113.

von Stechow, A. (1984). Comparing semantic theories of comparison. Journal of Semantics 3, 1–77.

Waldon, B. (2021). Epistemic *must* and *might*: Evidence that argument is semantically encoded. In Proceedings of Chicago Linguistic Society 56(1), pp. 463–477.

Wellwood, A. (2019). The Meaning of *More*. Oxford: Oxford University Press.

Williamson, J. (2010). In Defence of Objective Bayesianism. Oxford: Oxford University Press.

Yalcin, S. (2010). Probability operators. Philosophy Compass 5(10), 916–937.

Yalcin, S. (2016). Belief as question-sensitive. Philosophy and Phenomenological Research, 1–25.