Robust Optimal Planning of Power Distribution Networks Under Uncertain Conditions

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30 October, 2020

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Introduction

- Single-Stage Robust Optimization (decision variables $\mathbf{x}$, uncertain parameters $\mathbf{u}$)

$$\min_{\mathbf{x}} \max_{\mathbf{u}} f(\mathbf{x}, \mathbf{u})$$

- Using a preventive view, we wish to be safe against worst realization of the uncertain parameters
- For the worst uncertainty realization, we try to find minimum of the objective function
- Ultimate aim: **design a system that will operate with least costs, fulfilling all technical requirements, even in the case with most badly circumstances**

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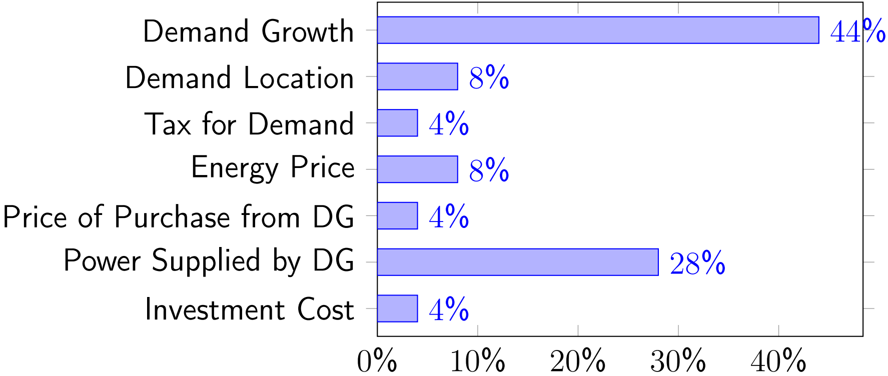
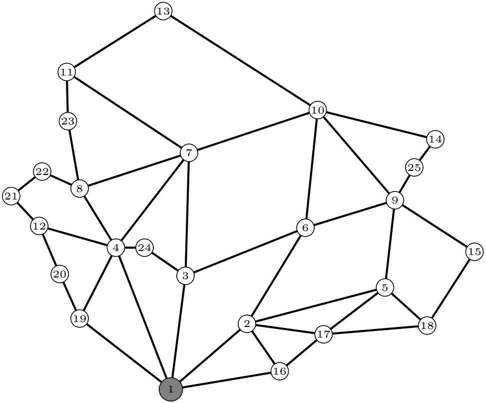
Uncertainty Topics in Research Papers



In 72% of the papers main topics are uncertainty in power demand and power output of DG (Distributed Generation – Renewables)

Source: V. Vahidinasab et al. "Overview of Electric Energy Distribution Networks Expansion Planning". In: *IEEE Access* 8 (2020), pp. 34750–34769. DOI: 10.1109/ACCESS.2020.2973455

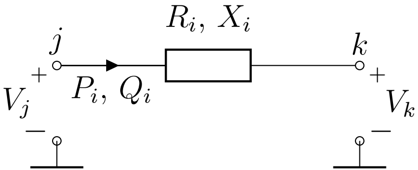
Problem Statement



- Exactly known lines routes of a meshed network and their parameters
- Uncertain data on load demand at each bus
- Find the least costs spanning tree that satisfy all voltage and current limits in worst possible case of uncertain data realization

Data and objective function from J. M. Nahman and D. M. Peric. "Optimal Planning of Radial Distribution Networks by Simulated Annealing Technique". In: *IEEE Transactions on Power Systems* 23.2 (May 2008), pp. 790–795. DOI: 10.1109/TPWRS.2008.920047

Line Model



$$V_j^2 - V_k^2 = 2(P_i R_i + Q_i X_i)$$

$$\Delta P_i = R_i \frac{P_i^2 + Q_i^2}{V_n^2}$$

M. E. Baran and F. F. Wu. "Network reconfiguration in distribution systems for loss reduction and load balancing". In: *IEEE Transactions on Power Delivery* 4.2 (Apr. 1989), pp. 1401–1407. DOI: 10.1109/61.25627

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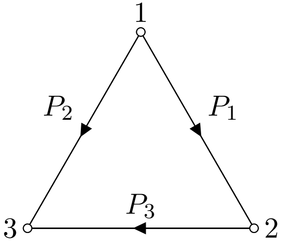
Network Representation

Incidence matrix (buses $\times$ lines)

$$A_{ij} = \begin{cases} 0 & \text{if line } j \text{ is not connected to bus } i, \\ 1 & \text{if line } j \text{ is directed away from bus } i, \\ -1 & \text{if line } j \text{ is directed toward bus } i. \end{cases}$$



$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

Power balance for all buses at once

$$\mathbf{A} \cdot \mathbf{P} = \begin{bmatrix} P_{\text{source1}} \\ -P_{\text{d2}} \\ -P_{\text{d3}} \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \mathbf{A}^T \cdot \begin{bmatrix} V_1^2 \\ V_2^2 \\ V_3^2 \end{bmatrix}$$

Difference of square of voltages at both ends of all branch at once

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Parameters and Variables

- $\mathcal{B}$ Set of buses
- $\mathcal{L}$ Set of lines
- $\mathcal{D}$ Set of load buses
- g Capital recovery rate
- c_i Cost of line i
- n_l Number of lines
- c_u Cost of undelivered energy
- α Load factor
- λ_i Failure rate of line i
- d_i Repair duration of line i
- P_i Active power flow in line i
- Q_i Reactive power flow in line i
- c_l Cost of losses
- β Loss factor, $\beta = 0.15\alpha + 0.85\alpha^2$
- R_i Resistance of line i
- X_i Reactance of line i
- V_n Nominal voltage
- V_s Voltage at source substation
- V_i Voltage at bus i
- $\underline{V}_i$ Minimum voltage at bus i
- W_i Square of the voltage at bus i
- $\underline{W}_i$ Square of the minimum voltage at bus i
- U_i Difference of square of voltages at both ends of branch i
- n_b Number of buses
- $P_{\mathrm{d}i}$ Active power demand at bus i
- $P_{\mathrm{d}i}^{\mathrm{ref}}$ Reference value of the active power demand at bus i
- $P_{\mathrm{d}i}^{\Delta}$ Deviation of the active power demand at bus i around its reference value
- $Q_{\mathrm{d}i}$ Reactive power demand at bus i
- $\overline{I}_i$ Maximum current in line i
- $\overline{S}_i$ Maximum power in line i ($\overline{S}_i = \sqrt{3}V_n\overline{I}_i$)

Deterministic Model

Objective = capital cost for the substation equipment and lines + costs for delivery interruptions + costs for active power losses

$$\min_{\mathbf{b},\mathbf{P},\mathbf{Q},\mathbf{V},\mathbf{U}} g\sum_{i\in\mathcal{L}} c_i b_i + c_u\alpha\sum_{i\in\mathcal{L}}\lambda_i d_i|P_i| + 8760c_l\beta\sum_{i\in\mathcal{L}} R_i\frac{P_i^2+Q_i^2}{V_n^2}$$

$$V_1 = V_s, \tag{1a}$$

$$V_i \geq \underline{V}_i, \quad i\in\mathcal{B} \tag{1b}$$

$$U_i = V_j^2 - V_k^2, \quad \text{line}\,(j\text{-}k),\ i\in\mathcal{L} \tag{1c}$$

$$b_i U_i = 2(P_i R_i + Q_i X_i), \quad i\in\mathcal{L} \tag{1d}$$

$$-b_i\overline{S}_i \leq P_i \leq b_i\overline{S}_i, \quad i\in\mathcal{L} \tag{1e}$$

$$-b_i\overline{S}_i \leq Q_i \leq b_i\overline{S}_i, \quad i\in\mathcal{L} \tag{1f}$$

$$P_i^2 + Q_i^2 \leq \overline{S}_i^2, \quad i\in\mathcal{L} \tag{1g}$$

$$\sum\nolimits_{j\in\mathcal{L}} A_{ij}P_j = -P_{\mathrm{d}i}, \quad i\in\mathcal{D} \tag{1h}$$

$$\sum\nolimits_{j\in\mathcal{L}} A_{ij}Q_j = -Q_{\mathrm{d}i}, \quad i\in\mathcal{D} \tag{1i}$$

$$\sum\nolimits_{i\in\mathcal{L}} b_i = n_b - 1, \tag{1j}$$

$$b_i \in \{0,1\}, \quad i\in\mathcal{L} \tag{1k}$$

Deterministic Model – Constraints

- (1a) and (1b) supply bus voltage and lower bound for all voltages
- (1c) difference of square of voltages at both ends of branch i, together with (1d) gives the branch power flow equation
- (1e) and (1f) constraints on active and reactive power flow (when $b_i = 0$, both P_i and Q_i are also 0)
- (1g) limits the combined power flow of P_i and Q_i to $\overline{S}_i$
- (1h) and (1i) bus load balance for active and reactive power
- (1j) together with the power balance constraints guarantee that the optimal network is radial

Exact Linearization

- Product of binary and continuous variables
  $F_i = b_i U_i \quad b_i \in \{0,1\} \qquad -V_n^2 \leq U_i \leq V_n^2$
  New variable $F_i\,(i \in \mathcal{L})$ and additional constraints

$$-V_n^2 b_i \leq F_i \leq V_n^2 b_i$$
$$U_i - V_n^2(1-b_i) \leq F_i \leq U_i + V_n^2(1-b_i)$$

- Absolute value of a continuous variable $H_i = |P_i|$
  New variable $H_i\,(i \in \mathcal{L})$ and additional constraints

$$H_i \geq -P_i$$
$$H_i \geq P_i$$

Deterministic Model (MIQCP)

$$\min_{\mathbf{b},\mathbf{P},\mathbf{Q},\mathbf{W},\mathbf{U},\mathbf{F},\mathbf{H}} \sum_{i\in\mathcal{L}} C_i b_i + \Lambda_i H_i + L_i(P_i^2 + Q_i^2)$$

$$b_i \in \{0,1\}, \quad i \in \mathcal{L} \tag{2a}$$
$$\sum\nolimits_{i\in\mathcal{L}} b_i = n_b - 1, \tag{2b}$$
$$W_1 = V_s^2, \tag{2c}$$
$$W_i \geq \underline{V}_i^2, \quad i \in \mathcal{B} \tag{2d}$$
$$U_i = \sum\nolimits_{j\in\mathcal{B}} A_{ji} W_j, \quad i \in \mathcal{L} \tag{2e}$$
$$F_i = 2(P_i R_i + Q_i X_i), \quad i \in \mathcal{L} \tag{2f}$$
$$-b_i \overline{S}_i \leq P_i \leq b_i \overline{S}_i, \quad i \in \mathcal{L} \tag{2g}$$
$$-b_i \overline{S}_i \leq Q_i \leq b_i \overline{S}_i, \quad i \in \mathcal{L} \tag{2h}$$
$$P_i^2 + Q_i^2 \leq \overline{S}_i^2, \quad i \in \mathcal{L} \tag{2i}$$
$$\sum\nolimits_{j\in\mathcal{L}} A_{ij} P_j = -P_{\mathrm{d}i}, \quad i \in \mathcal{D} \tag{2j}$$
$$\sum\nolimits_{j\in\mathcal{L}} A_{ij} Q_j = -Q_{\mathrm{d}i}, \quad i \in \mathcal{D} \tag{2k}$$
$$-V_n^2 b_i \leq F_i \leq V_n^2 b_i, \quad i \in \mathcal{L} \tag{2l}$$
$$U_i - V_n^2(1-b_i) \leq F_i \leq \leq U_i + V_n^2(1-b_i), \quad i \in \mathcal{L} \tag{2m}$$
$$H_i \geq -P_i, \quad i \in \mathcal{L} \tag{2n}$$
$$H_i \geq P_i, \quad i \in \mathcal{L}. \tag{2o}$$

Mixed Integer Quadratically Constraint Programing (MIQCP), solvable by CPLEX and GUROBI which are free for academic research.

Robust Optimization – References

- Two papers where the idea originated independently

  Bo Zeng and Long Zhao. "Solving two-stage robust optimization problems using a column-and-constraint generation method". In: *Operations Research Letters* 41.5 (2013), pp. 457–461. DOI: `https://doi.org/10.1016/j.orl.2013.05.003`

  D. Bertsimas et al. "Adaptive Robust Optimization for the Security Constrained Unit Commitment Problem". In: *IEEE Transactions on Power Systems* 28.1 (Feb. 2013), pp. 52–63. DOI: `10.1109/TPWRS.2012.2205021`

- A book on optimization under uncertainty

  Antonio J. Conejo et al. "Investment in Electricity, Generation and Transmission – Decision Making under Uncertainty". In: Springer International Publishing, 2016

- Excellent lectures (3 hours) explaining the core of the book

  Antonio J. Conejo: Adaptive Robust Optimization and its Applications to Power Systems. Department of Electrical Engineering, Technical University of Denmark (DTU), 13 June 2019

  `https://www.youtube.com/watch?v=Zk6y8joQLNQ`

Uncertainty Sets

- The load demand forecast is uncertain

$$\underline{P}_{\mathrm{d}i} = P_{\mathrm{d}i}^{\mathrm{ref}} - P_{\mathrm{d}i}^{\Delta}$$
$$\overline{P}_{\mathrm{d}i} = P_{\mathrm{d}i}^{\mathrm{ref}} + P_{\mathrm{d}i}^{\Delta}$$

- Limit the variability of uncertain variables using so-called uncertainty budget $\Gamma \in [0,1]$

$$\underline{P}_{\mathrm{d}i} \leq P_{\mathrm{d}i} \leq \overline{P}_{\mathrm{d}i}, \quad i \in \mathcal{D}, \tag{3a}$$
$$\sum_{i\in\mathcal{D}} \left| P_{\mathrm{d}i} - P_{\mathrm{d}i}^{\mathrm{ref}} \right| \leq \Gamma \cdot \sum_{i\in\mathcal{D}} P_{\mathrm{d}i}^{\Delta}, \tag{3b}$$
$$Q_{\mathrm{d}i} = Q_{\mathrm{d}i}^{\mathrm{ref}} / P_{\mathrm{d}i}^{\mathrm{ref}} \cdot P_{\mathrm{d}i}, \quad i \in \mathcal{D}. \tag{3c}$$

- Linearize $\left| P_{\mathrm{d}i} - P_{\mathrm{d}i}^{\mathrm{ref}} \right|$ replacing it with t_i and adding constraints

$$t_i \geq P_{\mathrm{d}i} - P_{\mathrm{d}i}^{\mathrm{ref}}, \quad i \in \mathcal{D} \tag{3d}$$
$$t_i \geq P_{\mathrm{d}i}^{\mathrm{ref}} - P_{\mathrm{d}i}, \quad i \in \mathcal{D}. \tag{3e}$$

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Two-level Optimization

Minimize costs under worst case uncertainty realization

$$\min_{\mathbf{b}} \max_{\mathcal{S}} \sum_{i \in \mathcal{L}} C_i b_i + \Lambda_i H_i + L_i(P_i^2 + Q_i^2)$$

$$\mathcal{S} = \{\mathbf{P}_{\mathrm{d}}, \mathbf{Q}_{\mathrm{d}}, \mathbf{t}, \mathbf{P}, \mathbf{Q}, \mathbf{W}, \mathbf{U}, \mathbf{F}, \mathbf{H}\}$$

- In the first level we search for an optimal network configuration controlling the vector $\mathbf{b}$ minimizing $\sum_{i \in \mathcal{L}} C_i b_i$
  - ▷ subject to constraints (2a) and (2b)
- In the second level we search for worst case uncertainty causing maximum operating costs $\sum_{i \in \mathcal{L}} \Lambda_i H_i + L_i(P_i^2 + Q_i^2)$ controlling variables from set $\mathcal{S}$ (main variables: $\mathbf{P}_{\mathrm{d}}$ and $\mathbf{Q}_{\mathrm{d}}$)
  - ▷ subject to constraints (2c) – (2o) and (3a) – (3c)

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Subproblem

Iteration count ν: $\mathbf{b} = \mathbf{b}^{(\nu-1)} \leftarrow$ fixed

$$\max_{\mathcal{S}} \sum_{i\in\mathcal{L}} \Lambda_i H_i + L_i(P_i^2 + Q_i^2)$$

$$W_1 = V_s^2,$$
$$W_i \ge \underline{V}_i^2, \quad i \in \mathcal{B}$$
$$U_i = \sum\nolimits_{j\in\mathcal{B}} A_{ji} W_j, \quad i \in \mathcal{L}$$
$$b_i^{(\nu-1)} \cdot U_i = 2(P_i R_i + Q_i X_i), \quad i \in \mathcal{L}$$
$$-b_i^{(\nu-1)} \overline{S}_i \le P_i \le b_i^{(\nu-1)} \overline{S}_i, \quad i \in \mathcal{L}$$
$$-b_i^{(\nu-1)} \overline{S}_i \le Q_i \le b_i^{(\nu-1)} \overline{S}_i, \quad i \in \mathcal{L}$$
$$P_i^2 + Q_i^2 \le \overline{S}_i^2, \quad i \in \mathcal{L},$$
$$\sum\nolimits_{j\in\mathcal{L}} A_{ij} P_j = -P_{\mathrm{d}i}, \quad i \in \mathcal{D}$$
$$\sum\nolimits_{j\in\mathcal{L}} A_{ij} Q_j = -Q_{\mathrm{d}i}, \quad i \in \mathcal{D}$$

$$P_i + M_i B_i \ge H_i, \quad i \in \mathcal{L}$$
$$-P_i + M_i(1 - B_i) \ge H_i, \quad i \in \mathcal{L}$$
$$H_i \ge -P_i, \quad i \in \mathcal{L}$$
$$H_i \ge P_i, \quad i \in \mathcal{L}.$$
$$\underline{P}_{\mathrm{d}i} \le P_{\mathrm{d}i} \le \overline{P}_{\mathrm{d}i}, \quad i \in \mathcal{D},$$
$$\sum\nolimits_{i\in\mathcal{D}} \left| P_{\mathrm{d}i} - P_{\mathrm{d}i}^{\mathrm{ref}} \right| \le \Gamma \cdot \sum\nolimits_{i\in\mathcal{D}} P_{\mathrm{d}i}^{\Delta},$$
$$Q_{\mathrm{d}i} = Q_{\mathrm{d}i}^{\mathrm{ref}} / P_{\mathrm{d}i}^{\mathrm{ref}} \cdot P_{\mathrm{d}i}, \quad i \in \mathcal{D},$$
$$t_i \ge P_{\mathrm{d}i} - P_{\mathrm{d}i}^{\mathrm{ref}} \quad i \in \mathcal{D}$$
$$t_i \ge P_{\mathrm{d}i}^{\mathrm{ref}} - P_{\mathrm{d}i}, \quad i \in \mathcal{D}$$

B_i is binary, $M_i = 2\overline{S}_i$, linearize $|P_i|$ when the problem is of maximization type, for details see:

`http://lpsolve.sourceforge.net/5.1/absolute.htm`

$$\mathrm{UB}_\nu = \sum\nolimits_{i\in\mathcal{L}} C_i b_i^{(\nu-1)} + \mathrm{OBJ}_\nu$$

Subproblem – Solvers Specifics

- The subproblem belongs to a MIQCP class, but now it is non-convex due to constraint $P_i^2 + Q_i^2 \leq \overline{S}_i^2$
- The above constraint should be replaced with piecewise linear approximation in case we are using CPLEX
- The piecewise linear approximation is easily done in Pyomo modeling language (Python), which has a special utility for the purpose
- GUROBI is capable of solving the problem in its original form

Master Problem (C&CG)

$$\min_{\mathbf{b},\eta,\mathbf{P}^{(k)},\mathbf{Q}^{(k)},\mathbf{W}^{(k)},\mathbf{U}^{(k)},\mathbf{F}^{(k)},k=1,\dots,\nu} \sum_{i\in\mathcal{L}} C_i b_i + \eta$$

$$b_i \in \{0,1\}, \quad i\in\mathcal{L}$$

$$\sum_{i\in\mathcal{L}} b_i = n_b - 1,$$

$$\eta \geq \sum_{i\in\mathcal{L}} \Lambda_i H_i^{(k)} +$$

$$+\sum_{i\in\mathcal{L}} L_i \left[{P_i^{(k)}}^2 + {Q_i^{(k)}}^2\right],$$

$$H_i^{(k)} \geq -P_i^{(k)}, \quad i\in\mathcal{L},$$

$$H_i^{(k)} \geq P_i^{(k)}, \quad i\in\mathcal{L},$$

$$W_1^{(k)} = V_s^2,$$

$$W_i^{(k)} \geq \underline{V}_i^2, \quad i\in\mathcal{B},$$

$$U_i^{(k)} = \sum_{j\in\mathcal{B}} A_{ji} W_j^{(k)}, \quad i\in\mathcal{L},$$

$$F_i^{(k)} = 2\left[P_i^{(k)} R_i + Q_i^{(k)} X_i\right], \quad i\in\mathcal{L},$$

$$-b_i\overline{S}_i \leq P_i^{(k)} \leq b_i\overline{S}_i, \quad i\in\mathcal{L}$$

$$-b_i\overline{S}_i \leq Q_i^{(k)} \leq b_i\overline{S}_i, \quad i\in\mathcal{L},$$

$${P_i^{(k)}}^2 + {Q_i^{(k)}}^2 \leq \overline{S}_i^2, \quad i\in\mathcal{L},$$

$$\sum_{j\in\mathcal{L}} A_{ij} P_j^{(k)} = -P_{\mathrm{d}i}^{(k)}, \quad i\in\mathcal{D},$$

$$\sum_{j\in\mathcal{L}} A_{ij} Q_j^{(k)} = -Q_{\mathrm{d}i}^{(k)}, \quad i\in\mathcal{D},$$

$$-V_n^2 b_i \leq F_i^{(k)} \leq V_n^2 b_i, \quad i\in\mathcal{L},$$

$$U_i^{(k)} - V_n^2(1-b_i) \leq F_i^{(k)} \leq U_i^{(k)} + V_n^2(1-b_i), \ i\in\mathcal{L}.$$

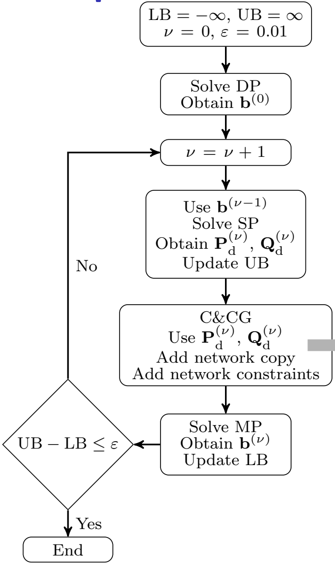
$$k = 1,2,\dots,\nu$$

problem expands in each iteration

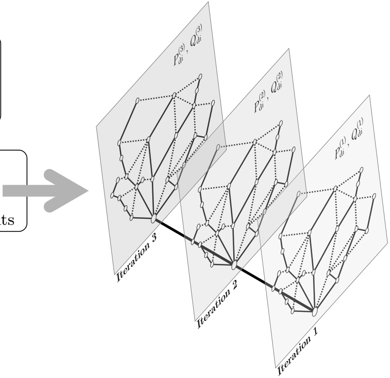
C&CG – Column and Constraint Generation

$$\mathrm{LB}_\nu = \sum_{i\in\mathcal{L}} C_i b_i^{(\nu)} + \eta^{(\nu)}$$

Robust Optimization – Block Diagram



DP	Deterministic Problem
SP	Subproblem
MP	Master Problem
C&CG	Column and Constraint Generation



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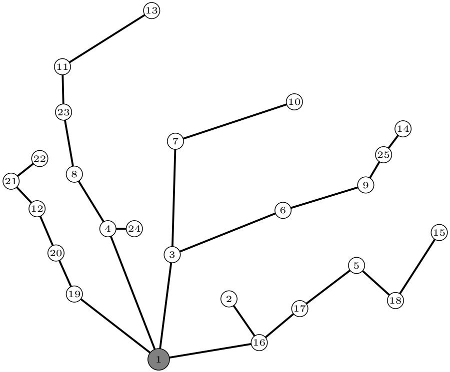
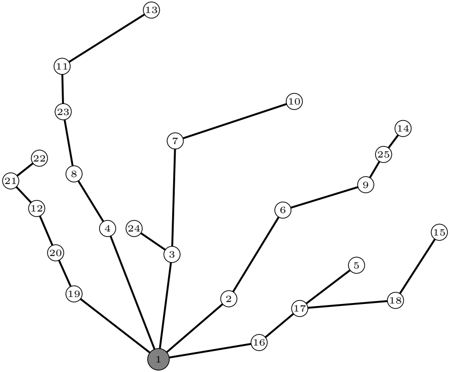
Optimal Network Configuration

We have used: $\underline{V}_i = 9.5$ kV, $P_{\mathrm{d}i}^{\Delta} = 50\% P_{\mathrm{d}i}^{\mathrm{ref}}$, $\Gamma = 1$ (huge uncertainty)



Deterministic Solution
4 lines from the supply bus
Objective: 68,934.98 $



Robust Solution
5 lines from the supply bus
Objective: 91,712.55 $
Increase: 33 %

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Possible Extensions to the Problem (1)

- Add uncertainty set for power generation from renewable energy sources

$$P_{\mathrm{g}i}^{\mathrm{ref}} - P_{\mathrm{g}i}^{\Delta} \leq P_{\mathrm{g}i} \leq P_{\mathrm{g}i}^{\mathrm{ref}} + P_{\mathrm{g}i}^{\Delta}, \quad i \in \mathcal{G}$$
$$\sum_{i\in\mathcal{G}} \left| P_{\mathrm{g}i} - P_{\mathrm{g}i}^{\mathrm{ref}} \right| \leq \Gamma_{\mathrm{g}} \cdot \sum_{i\in\mathcal{G}} P_{\mathrm{g}i}^{\Delta}$$

- Modify the power balance equation (red single term)

$$\sum_{j\in\mathcal{L}} A_{ij} P_j = -P_{\mathrm{d}i} + P_{\mathrm{g}i}, \quad i \in \mathcal{D}$$

- Add time snapshots (another dimension in all variables)
  - ▷ ${P_i}^t$, ${Q_i}^t$, ${P_{\mathrm{d}i}}^t$, ${Q_{\mathrm{d}i}}^t$, P_{gi}^t, etc.
  - ▷ Be careful with the uncertainty definitions going from snapshot $t-1$ to t since they are not independent

Possible Extensions to the Problem (2)

- Add control devices in the network: capacitor banks, electric vehicles, storage systems, etc.
- Adaptive Robust Optimization: once the uncertainty realizes it is possible to react and mitigate potential harmful effects on the system using the control devices in the network
- Three-level optimization (decision variables $\mathbf{x}$, uncertain parameters $\mathbf{u}$, system control variables $\mathbf{y}$ – control devices)

$$\min_{\mathbf{x}} \max_{\mathbf{u}} \min_{\mathbf{y}} f(\mathbf{x}, \mathbf{y}, \mathbf{u})$$

  Write the dual form for the third level (make it maximization) and merge it with the second level, solve it as a two-level optimization problem.
- Modify the power balance equation (red terms)

$$\sum_{j \in \mathcal{L}} A_{ij} P_j^t = -P_{\mathrm{d}i}^t + P_{\mathrm{g}i}^t + P_{\mathrm{veh}i}^t - P_{\mathrm{chg}i}^t + P_{\mathrm{dischg}i}^t, \quad i \in \mathcal{D}$$

Thank You for Your Attention!

Do you have any questions?

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