

Autoencoders - an Introduction

What they are and what you can do with them

Umberto Michelucci^{1,2}

¹ Presenting author, umberto.michelucci@toelt.ai

² Research and Development, TOELT, Advanced AI Lab

March 13, 2023



Table of Contents



- 1 Introduction
- 2 Structure of an autoencoder
- 3 Classification with latent features
- 4 Data Compression
- 5 Anomaly Detection

Autoencoders

Umberto
Michelucci

Introduction

Structure of an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Introduction



You can find a complete discussion at [1]
<https://arxiv.org/pdf/2201.03898.pdf>

GitHub:

<https://github.com/toelt-llc/ETH-ZURICH-GDSC-WORKSHOPS-2022>



Normally in Machine learning we use the following notation for **supervised learning**:

\mathbf{x}_i indicates the input observations (e.g. images)

y_i indicate the expected value or label.

Now suppose we have only unlabelled observations. We have only a training dataset S_T with M observations:

$$S_T = \{\mathbf{x}_i \mid i = 1, \dots, M\} \quad \text{with } \mathbf{x}_i \in \mathbb{R}^n$$



Notation

Normally in Machine learning we use the following notation for **supervised learning**:

\mathbf{x}_i indicates the input observations (e.g. images)

y_i indicate the expected value or label.

Autoencoders

Umberto
Michelucci

Introduction

Structure of an autoencoder

Classification with latent features

Data Compression

Anomaly Detection

Notation

Normally in Machine learning we use the following notation for **supervised learning**:

\mathbf{x}_i indicates the input observations (e.g. images)

y_i indicate the expected value or label.

Now suppose we have only unlabelled observations. We have only a training dataset S_T with M observations:



Normally in Machine learning we use the following notation for **supervised learning**:

\mathbf{x}_i indicates the input observations (e.g. images)

y_i indicate the expected value or label.

Now suppose we have only unlabelled observations. We have only a training dataset S_T with M observations:

$$S_T = \{\mathbf{x}_i \mid i = 1, \dots, M\} \quad \text{with } \mathbf{x}_i \in \mathbb{R}^n$$

History

Autoencoders were first introduced¹ by Rumelhart, Hinton, and Williams [2] in 1986 with the goal of learning to reconstruct the input observations \mathbf{x}_i with the lowest error possible.

¹https://web.stanford.edu/class/psych209a/ReadingsByDate/02_06/PDPVolIChapter8.pdf



Autoencoders were first introduced¹ by Rumelhart, Hinton, and Williams [2] in 1986 with the goal of learning to reconstruct the input observations x_i with the lowest error possible.

If you have problems imagining what that means, think of having a dataset made of images. An autoencoder would be an algorithm that can give as output an image that is as similar as possible to the input one.

¹https://web.stanford.edu/class/psych209a/ReadingsByDate/02_06/PDPVol1Chapter8.pdf

Definition



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Definition

An autoencoder is a type of algorithm with the primary purpose of learning an "informative" representation of the data that can be used for different applications [3] by learning to reconstruct a set of input observations well enough.

What does it mean **well enough**?

What does it mean **informative**?

Definition



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Definition

An autoencoder is a type of algorithm with the primary purpose of learning an "informative" representation of the data that can be used for different applications [3] by learning to reconstruct a set of input observations well enough.

What does it mean **well enough**?

What does it mean **informative**?

Definition



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Definition

An autoencoder is a type of algorithm with the primary purpose of learning an "informative" representation of the data that can be used for different applications [3] by learning to reconstruct a set of input observations well enough.

What does it mean **well enough**?

What does it mean **informative**?



The *informative* aspect is important

Building an algorithm that reconstruct perfectly the input is very easy...

It is enough to use the identify function $I(x) = x$

This is not useful at all (and dumb)

Autoencoders

Umberto
Michelucci

Introduction

Structure of an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



The *informative* aspect is important

Building an algorithm that reconstruct perfectly the input is very easy...

It is enough to use the identify function $I(x) = x$

This is not useful at all (and dumb)

Autoencoders

Umberto
Michelucci

Introduction

Structure of an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



The *informative* aspect is important

Building an algorithm that reconstruct perfectly the input is very easy...

It is enough to use the identify function $I(x) = x$

This is not useful at all (and dumb)

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



The *informative* representation

For example, an informative representation of hand-written digits could be the number of lines required to write each number or the angle of each line and how they connect.

Learning how to write numbers certainly does not require to learn the gray values of each pixel in the input image.

We humans do not certainly learn to write by filling pixels with gray values.

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



The *informative* representation

For example, an informative representation of hand-written digits could be the number of lines required to write each number or the angle of each line and how they connect.

Learning how to write numbers certainly does not require to learn the gray values of each pixel in the input image.

We humans do not certainly learn to write by filling pixels with gray values.

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



The *informative* representation

For example, an informative representation of hand-written digits could be the number of lines required to write each number or the angle of each line and how they connect.

Learning how to write numbers certainly does not require to learn the gray values of each pixel in the input image.

We humans do not certainly learn to write by filling pixels with gray values.

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Structure of an autoencoder

Structure of an autoencoder



The structure of an autoencoder looks typically like this

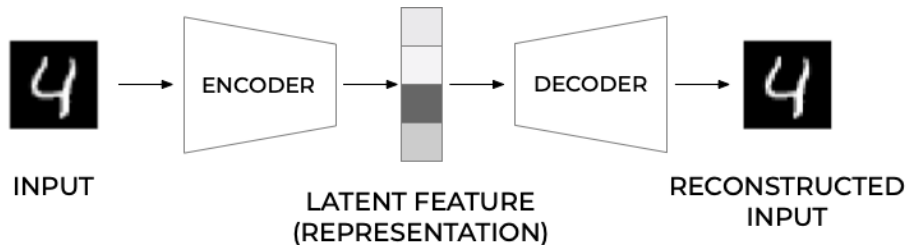


Figure: The typical structure of an autoencoder.

Autoencoders

Umberto Michelucci

Introduction

Structure of an autoencoder

Classification with latent features

Data Compression

Anomaly Detection

Elements of an autoencoder



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

The main elements of an autoencoder are the following:

- 1 **Encoder:** generally speaking is a function $\mathbf{h}_i = g(\mathbf{x}_i)$ that depends on some parameters;
- 2 **Latent Features:** the \mathbf{h}_i are normally just an array of numbers (in 1 or 2 dimension depending on the function $g()$);
- 3 **Decoder:** a second function that has as output the reconstructed image $\tilde{\mathbf{x}}_i = f(\mathbf{x}_i) = f(g(\mathbf{x}_i))$;
- 4 The functions $f()$ and $g()$ are typically neural networks.

Elements of an autoencoder



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

The main elements of an autoencoder are the following:

- 1 **Encoder:** generally speaking is a function $\mathbf{h}_i = g(\mathbf{x}_i)$ that depends on some parameters;
- 2 **Latent Features:** the \mathbf{h}_i are normally just an array of numbers (in 1 or 2 dimension depending on the function $g()$);
- 3 **Decoder:** a second function that has as output the reconstructed image $\tilde{\mathbf{x}}_i = f(\mathbf{x}_i) = f(g(\mathbf{x}_i))$;
- 4 The functions $f()$ and $g()$ are typically neural networks.

Elements of an autoencoder



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

The main elements of an autoencoder are the following:

- 1 **Encoder:** generally speaking is a function $\mathbf{h}_i = g(\mathbf{x}_i)$ that depends on some parameters;
- 2 **Latent Features:** the \mathbf{h}_i are normally just an array of numbers (in 1 or 2 dimension depending on the function $g()$);
- 3 **Decoder:** a second function that has as output the reconstructed image $\tilde{\mathbf{x}}_i = f(\mathbf{x}_i) = f(g(\mathbf{x}_i))$;
- 4 The functions $f()$ and $g()$ are typically neural networks.

Elements of an autoencoder



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

The main elements of an autoencoder are the following:

- 1 **Encoder:** generally speaking is a function $\mathbf{h}_i = g(\mathbf{x}_i)$ that depends on some parameters;
- 2 **Latent Features:** the \mathbf{h}_i are normally just an array of numbers (in 1 or 2 dimension depending on the function $g()$);
- 3 **Decoder:** a second function that has as output the reconstructed image $\tilde{\mathbf{x}}_i = f(\mathbf{x}_i) = f(g(\mathbf{x}_i))$;
- 4 The functions $f()$ and $g()$ are typically neural networks.

Another take on the *informative* aspect



An **informative representation** means to obtain a latent representation \mathbf{h}_i that is useful for other purposes.

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Bottleneck in autoencoders



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

- 1 To enforce \mathbf{h}_i to be useful, we impose that it must be of lower dimension than \mathbf{x}_i ;
- 2 The fact that the dimension of \mathbf{h}_i is lower than that of \mathbf{x}_i is called a **bottleneck**.

Bottleneck in autoencoders



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

- 1 To enforce \mathbf{h}_i to be useful, we impose that it must be of lower dimension than \mathbf{x}_i ;
- 2 The fact that the dimension of \mathbf{h}_i is lower than that of \mathbf{x}_i is called a **bottleneck**.



Example of bottleneck

A diagram of a bottleneck with Feed Forward Neural Networks is the following

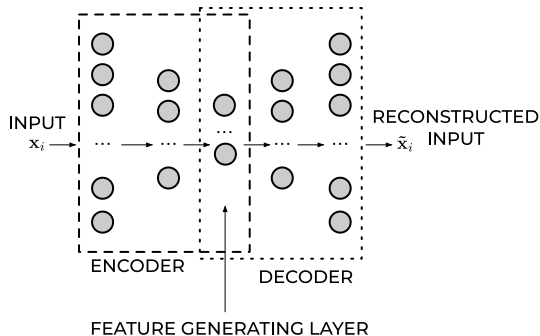


Figure: A typical architecture of a Feed-Forward Autoencoder. The number of neurons in the layers at first goes down as we move through the network until it reaches the middle and then starts to grow again until the last layer has the same number of neurons as the input dimensions.

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



Information

The encoder can reduce the number of dimensions of the input observation (n) and create a learned representation (\mathbf{h}_i) of the input that has a smaller dimension $q < n$. This learned representation is enough for the decoder to reconstruct the input accurately (if the autoencoder training was successful as intended).



Activation Function of the Output Layer I

The most used ones are ReLU and sigmoid.

$$\text{ReLU}(x) = \max(0, x) \quad (1)$$

and

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad (2)$$

Warning

ReLU activation function for the output layer is well suited for cases when the input observations \mathbf{x}_i assume a wide range of positive real values.

Warning

sigmoid activation function for the output layer is well suited for cases when the input observations \mathbf{x}_i assume a range of values in $[0, 1]$

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Loss Functions



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

For autoencoders one can use both typical loss functions:

- 1 MSE
- 2 Cross-entropy

Warning

An essential prerequisite for using the binary cross-entropy loss function is that the inputs must be normalized between 0 and 1 and that the activation function for the last layer must be a sigmoid or softmax function.

Reconstruction Error



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

The typical reconstruction error (RE) is a metric that gives an indication of how good (or bad) the autoencoder was able to reconstruct the input observation.

$$\text{RE} = \text{MSE} = \frac{1}{M} \sum_{i=1}^M |\mathbf{x}_i - \tilde{\mathbf{x}}_i|^2 \quad (3)$$

Reconstruction Error - an example

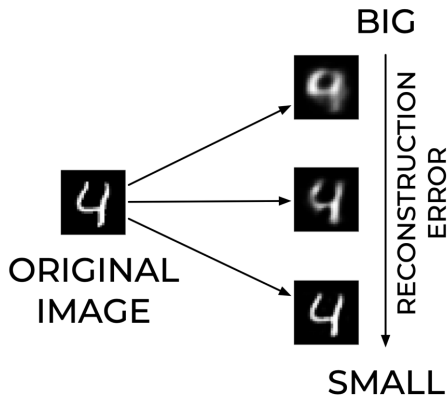


Figure: An example of big and small reconstruction error when an autoencoder tries to reconstruct an image.



Classification with latent features

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

An example



Dataset: MNIST (handwritten digits, 28x28 gray levels, 60000 training images, 10000 test images)

Model: With kNN with $k = 7$ on MNIST (60000 images, $\mathbf{x}_i \in \mathbb{R}^{784}$) takes ca. 16.6 minutes (ca. 1000 sec.) with an accuracy of 96.4%.

Model with latent features: if we consider an FFA with 8 neurons in the middle layer and again train a kNN algorithm on the latent features $g(\mathbf{x}_i) \in \mathbb{R}^8$ we get an accuracy of 89% in 1.1 sec

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

An example



Dataset: MNIST (handwritten digits, 28x28 gray levels, 60000 training images, 10000 test images)

Model: With kNN with $k = 7$ on MNIST (60000 images, $\mathbf{x}_i \in \mathbb{R}^{784}$) takes ca. 16.6 minutes (ca. 1000 sec.) with an accuracy of 96.4%.

Model with latent features: if we consider an FFA with 8 neurons in the middle layer and again train a kNN algorithm on the latent features $g(\mathbf{x}_i) \in \mathbb{R}^8$ we get an accuracy of 89% in 1.1 sec

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

An example



Dataset: MNIST (handwritten digits, 28x28 gray levels, 60000 training images, 10000 test images)

Model: With kNN with $k = 7$ on MNIST (60000 images, $\mathbf{x}_i \in \mathbb{R}^{784}$) takes ca. 16.6 minutes (ca. 1000 sec.) with an accuracy of 96.4%.

Model with latent features: if we consider an FFA with 8 neurons in the middle layer and again train a kNN algorithm on the latent features $g(\mathbf{x}_i) \in \mathbb{R}^8$ we get an accuracy of 89% in 1.1 sec

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

An example



Autoencoders

Umberto
Michelucci

Introduction

Structure of an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Input Data	Accuracy	Running Time
Original data $\mathbf{x}_i \in \mathbb{R}^{784}$	96.4%	1000 sec. \approx 16.6 min.
Latent Features $\mathbf{g}(\mathbf{x}_i) \in \mathbb{R}^8$	89%	1.1 sec.

Table: the different in accuracy and running time when applying the kNN algorithm to the original 784 features or the 8 latent features for the MNIST dataset.



An example

Input Data	Accuracy	Running Time
Original data $\mathbf{x}_i \in \mathbb{R}^{784}$	85.4%	1040 sec. \approx 16.6 min.
Latent Features $\mathbf{enc}(\mathbf{x}_i) \in \mathbb{R}^8$	79.9%	1.2 sec.
Latent Features $\mathbf{enc}(\mathbf{x}_i) \in \mathbb{R}^{16}$	83.6%	3.0 sec.

Table: the difference in accuracy and running time when applying the kNN algorithm to the original 784 features with a FFA with 8 neurons and with a FFA with 16 neurons for the Fashion MNIST dataset.

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



Data Compression

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



Data Compression

You can do also data compression by using instead of the original dataset the latent features (16 neurons).

```
np.save('temp_orig', mnist_x_test)
```

```
! ls -al temp_orig*
```

```
-rw-r--r--  1 umberto  staff  31360128 May  5 13:45 temp_orig.npy
```

and with the latent features

```
np.save('temp_encoded', encoded_imgs)
```

```
! ls -al temp_encoded*
```

```
-rw-r--r--  1 umberto  staff   320128 May  5 13:45 temp_encoded.npy
```

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection



Anomaly Detection

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Anomaly Detection



- 1 We consider an autoencoder with only three layers with 784 neurons in the first, 64 in the latent feature generation layer, and again 784 neurons in the output layer;
- 2 We will train it with the MNIST dataset and in particular with the 60000 training portion of it;
- 3 Let us choose an image of a shoe from this dataset and add it to the testing portion of the MNIST dataset (that now will have 10001 images).

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Anomaly Detection



- 1 We consider an autoencoder with only three layers with 784 neurons in the first, 64 in the latent feature generation layer, and again 784 neurons in the output layer;
- 2 We will train it with the MNIST dataset and in particular with the 60000 training portion of it;
- 3 Let us choose an image of a shoe from this dataset and add it to the testing portion of the MNIST dataset (that now will have 10001 images).

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

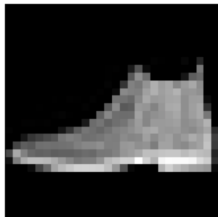
Data
Compression

Anomaly
Detection

Anomaly Detection



- 1 We consider an autoencoder with only three layers with 784 neurons in the first, 64 in the latent feature generation layer, and again 784 neurons in the output layer;
- 2 We will train it with the MNIST dataset and in particular with the 60000 training portion of it;
- 3 Let us choose an image of a shoe from this dataset and add it to the testing portion of the MNIST dataset (that now will have 10001 images).



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Anomaly Detection - how the autoencoder reconstruct the shoe

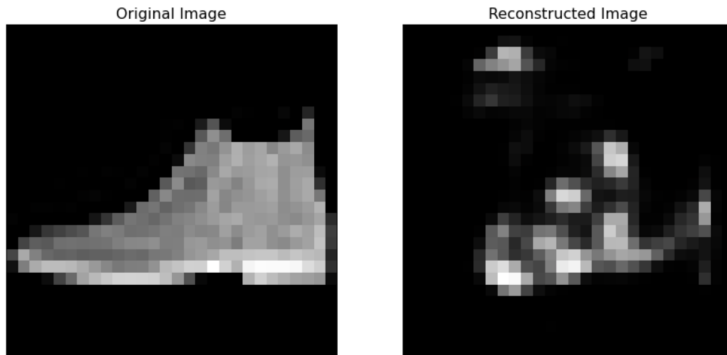


Figure: The shoe and the autoencoder's reconstruction trained on the 60000 hand-written images of the MNIST dataset. This image has the biggest RE in the entire 10001 test dataset we built with a value of 0.062.

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Anomaly Detection - Algorithm



- 1 One train an autoencoder on the entire dataset (or if possible, on a portion of the dataset known not to have any outlier)
- 2 For each observation (or input) of the portion of the dataset known to have the wanted outliers one calculates the RE
- 3 One sorts the observations by the RE.
- 4 One classifies the observations with the highest RE as outliers. Note that how many observations are outliers will depend on the problem at hand and require an analysis of the results and usually lot of knowledge of the data and the problem

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Anomaly Detection - Algorithm



- 1 One train an autoencoder on the entire dataset (or if possible, on a portion of the dataset known not to have any outlier)
- 2 For each observation (or input) of the portion of the dataset known to have the wanted outliers one calculates the RE
- 3 One sorts the observations by the RE.
- 4 One classifies the observations with the highest RE as outliers. Note that how many observations are outliers will depend on the problem at hand and require an analysis of the results and usually lot of knowledge of the data and the problem

Autoencoders

Umberto
Michelucci

Introduction

Structure of an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Anomaly Detection - Algorithm



- 1 One train an autoencoder on the entire dataset (or if possible, on a portion of the dataset known not to have any outlier)
- 2 For each observation (or input) of the portion of the dataset known to have the wanted outliers one calculates the RE
- 3 One sorts the observations by the RE.
- 4 One classifies the observations with the highest RE as outliers. Note that how many observations are outliers will depend on the problem at hand and require an analysis of the results and usually lot of knowledge of the data and the problem

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Anomaly Detection - Algorithm



- 1 One train an autoencoder on the entire dataset (or if possible, on a portion of the dataset known not to have any outlier)
- 2 For each observation (or input) of the portion of the dataset known to have the wanted outliers one calculates the RE
- 3 One sorts the observations by the RE.
- 4 One classifies the observations with the highest RE as outliers. Note that how many observations are outliers will depend on the problem at hand and require an analysis of the results and usually lot of knowledge of the data and the problem

Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Assumption in anomaly detection



Autoencoders

Umberto
Michelucci

Introduction

Structure of
an
autoencoder

Classification
with latent
features

Data
Compression

Anomaly
Detection

Warning

If one train the autoencoder on the entire dataset at disposal, there is an essential assumption: the outliers are a negligible part of the dataset and their presence will not influence (or will influence in an insignificant way) how the autoencoder learns to reconstruct the observations.

Umberto
Michelucci

Structure of an autoencoder

Classification with latent features

Data Compression

Anomaly Detection

- U. Michelucci, “An introduction to autoencoders,” *arXiv preprint arXiv:2201.03898*, 2022.
- D. E. Rumelhart, G. E. Hinton, and R. J. Williams, “Learning internal representations by error propagation, parallel distributed processing, vol. 1,” *Foundations. MIT Press, Cambridge*, 1986.
- D. Bank, N. Koenigstein, and R. Giryes, “Autoencoders,” *arXiv preprint arXiv:2003.05991*, 2020.