Quantum Optics

lecture course

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1 Secondary quantization

1.1 Introduction

Second quantization starts with an expansion of a scalar or vector field (or wave functions) in a basis consisting of a complete set of functions. These expansion functions depend on the coordinates of a single particle. The expansion coefficients have been promoted from ordinary numbers to operators, creation and annihilation operators. A creation operator creates a particle in the corresponding basis function and an annihilation operator annihilates a particle in this function.

1.2 System for A

Let us write Maxwell equations in vacuum without any charge in the system (in CGS units):

$$\begin{cases} \operatorname{rot}\mathbf{E} = -\dfrac{1}{c}\dfrac{\partial \mathbf{H}}{\partial t}, & \text{(1a)} \\ \operatorname{rot}\mathbf{H} = \dfrac{1}{c}\dfrac{\partial \mathbf{E}}{\partial t}, & \text{(1b)} \\ \operatorname{div}\mathbf{E} = 0, & \text{(1c)} \\ \operatorname{div}\mathbf{H} = 0. & \text{(1d)} \end{cases}$$

It's more convenient to work with potentials but not the fields itself. If we know $\mathbf{A}$ и φ, we know the field

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\varphi, \tag{2}$$

$$\mathbf{H} = \operatorname{rot}\mathbf{A}. \tag{3}$$

It's easy to construct two (generally speaking infinitely many) different potential which can lead to the same EM fields. So we can put one arbitrary condition for $\mathbf{A}$ and φ. Let us use Lorentz gauge:

$$\operatorname{div}\mathbf{A} = 0. \tag{4}$$

Let us obtain an equation for $\mathbf{A}$. Substitution of field to the (1b) gives us

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{1}{c}\nabla\frac{\partial\varphi}{\partial t} \tag{5}$$

and since

$$\operatorname{rot}\operatorname{rot}\mathbf{A} = \underbrace{\operatorname{grad}\operatorname{div}\mathbf{A}}_{\hookrightarrow=0} - \operatorname{div}\operatorname{grad}\mathbf{A} = -\Delta\mathbf{A} \tag{6}$$

then

$$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = \frac{1}{c}\nabla\frac{\partial\varphi}{\partial t}. \tag{7}$$

If we do $\nabla\cdot$(2) then we get

$$\underbrace{\operatorname{div}\mathbf{E}}_{\hookrightarrow=0} = -\underbrace{\frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\mathbf{A}}_{\hookrightarrow=0} - \Delta\varphi \quad \to \quad \Delta\varphi = 0 \quad \to \quad \nabla\varphi = 0. \tag{8}$$

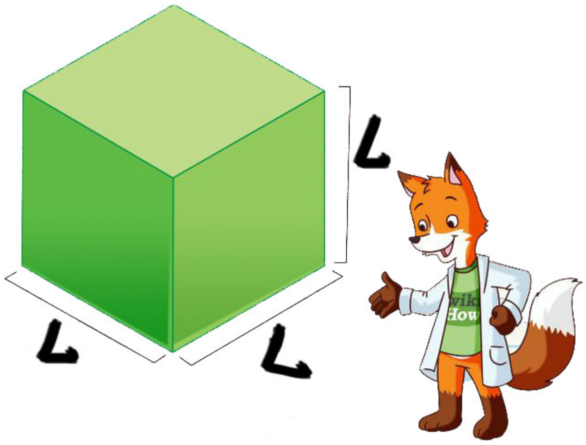

Рис. 1: Formulation of the problem

So we have system for $\mathbf{A}$:

$$\begin{cases} \Delta \mathbf{A} - \dfrac{1}{c^2}\dfrac{\partial^2 \mathbf{A}}{\partial t^2} = 0, \\ \\ \operatorname{div} \mathbf{A} = 0. \end{cases} \tag{9}$$

Remark: in deriving this system we put $\rho = 0$ and $\mathbf{j} = 0$.

1.3 Formulation of the problem

Let us consider a cube with length of the edge L (fig 1). Boundary conditions are periodic with the period L. System is considered to be conservative.