ISSN 1063-7729, Astronomy Reports, 2015, Vol. 59, No. 1, pp. 72–87. ©Pleiades Publishing, Ltd., 2015.
Original Russian Text ©A.A. Zlenko, 2015, published in Astronomicheskii Zhurnal, 2015, Vol. 92, No. 1, pp. 80–96.

A Celestial-Mechanical Model for the Tidal Evolution of the Earth–Moon System Treated as a Double Planet

A. A. Zlenko*

Abstract—A celestial-mechanical model for the motion of two viscoelastic spheres in the gravitational field of a massive point is considered, treating them as a double planet. The spheres move along quasicircular orbits in a single plane, with their rotational axes perpendicular to this plane. The deformation of the spheres is described using the classical theory of small deformations. A Kelvin–Voigt model is adopted for the viscous forces. A system of evoutionary equations is obtained and applied to analyze the joint translational–rotational tidal evolution of the Earth and Moon in the gravitational field of the Sun. This system has been numerically integrated several billion years into the past and into the future. The results are compared with the predictions of other theories, paleontological data, and astronomical observations.

DOI: 10.1134/S1063772915010096

1. INTRODUCTION The theory of tides originated with work by Newton and Laplace. The main achievements in this area were collected, systematized, and analyzed by Darwin [1], and further developed by MacDonald [2], who studied the evolution of the Earth–Moon system without including the influence of the Sun. Goldreich [3] used the method of MacDonald to investigate the oblateness of the Earth and the influence of solar tides, but neglecting the ellipticity of the lunar orbit, and correctly averaged the equations of motion using three time scales. The method of MacDonald was then used in various other studies. Beletskii [4] investigated the tidal evolution of the inclinations and rotations of celestial bodies. Webb [5] studied the evolution of the Earth–Moon system based on the ocean tides and compared his results with the model of Goldreich [3]. Krasinsky [6] combined the methods of MacDonald and Goldreich to reconstruct a dynamical history of the Earth–Moon system. Touma andWisdom [7] developed various models for tidal phenomena in detail. It was shown that the evolution of the Earth–Moon system based on the models of Darwin–Mignard and Darwin–Cowley–Goldreich is essentially equivalent to that predicted by the model of Goldreich. Efroimsky and Lainey [8] considered the effective dissipation function Q, which is proportional to the tidal frequency to the power α . They studied the tidal evolution of the Martian moon Phobos for α = 0.2, 0.3, 0.4. Note that α = 0 in the model of MacDonald and α = -1 in the model of Mignard [9, 10]. The main distinguishing property of the approach proposed by Ferraz-Mello et al. [11] is that, in contrast to many studies based on the theory of Darwin, different coefficients are introduced for the harmonics of the tidal wave, instead of one Love number. A critical analysis of the mathematical formulas in the above theories describing the tidal moments, slowing of planetary rotation, and the delay angle, as well as the accuracy and range of applicability of the theories and connections with rheological models, are considered by Efroimsky and Williams [12] and Efroimsky and Makarov [13]. Note that the qualitative conclusions derived for the simpler MacDonald theory essentially remain correct [12]. The subsequent development of tidal theories is concerned with the creation of rheological models. Churkin [14–16] established a generalized theory of the Love number and applied it to the rheological models of Guk, Maxwell, Voigt, and others. His theory for the rotation of the inelastic Earth was applied to a Voigt model for the Earth's interior, and numerical estimates of rheological corrections to the precession, nutation, and axial rotation of the Earth were obtained. Efroimsky [17] introduced complex Love numbers as a function of the tidal frequency to study tides in the case of a rotational–orbital resonance between a planet and one of its satellites. Vil'ke [18] developed a method for separating motions and averaging in systems with an infinite number of degrees of freedom, aimed at studying the motions of deformable bodies using a classical linear