

Exercises for the Lecture AI - Knowledge Acquisition and Representation

Probability Theory

1) *The Cookie paradox*

Three cookies, Almond (A), Blueberry (B) and Chocolate Chunk (C), are locked in their cookie jar. It is common knowledge that every day, one of them will be eaten by the cookie monster and the others are spared (for now). Only the guard of the cookie monster knows which one will be eaten as he feeds him one cookie per day to keep him friendly. Cookie A asks the guard a favor: Please take a message to one of my friends B and C to let him know that he will be spared in the morning. The guard agrees, and comes back later and tells A that he gave the spare message to B. What are A's chances of being eaten the next day, given this information? (Answer this *mathematically*, not by energetic waving of hands!)

2) *Three stoves*

Since the three plates of a stove can get differently hot, the likelihood that an otherwise equally prepared Sauce bearnaise succeeds differ.

On plates 1, 2, and 3, it curdles with probability 5%, 4% and 2%, respectively. The likelihood of a place being used by a cook be 25% for plate 1, 35% for plate 2 and the remainder for plate 3.

How likely is it that a failed Sauce Bearnaise has been prepared on place 1/2/3?

3) *Rational beliefs*

- (a) Would it be rational for an agent to maintain the three beliefs

$$P(A) = 0.4, P(B) = 0.3 \text{ and } P(A \vee B) = 0.5?$$

If this is the case, which probability range is rationally to be assumed for $P(A \wedge B)$?

- (b) What if $P(A \vee B) = 0.8$?

- (c) Proof, that if $P(B|A) > P(B)$ holds, then $P(A|\neg B) < P(A)$ also holds.

4) *Poker Game*

Consider the domain of dealing five-card poker hands from a standard deck of 52 cards, under the assumption that the dealer is fair.

- (a) How many atomic events are there in the joint probability distribution (i.e., how many five-card hands are there)?
- (b) What is the probability of each atomic event?
- (c) What is the probability of being dealt a royal straight flush (the ace, king, queen, jack and ten of the same suit)?
- (d) What is the probability of four of a kind?