

1)

- a) False. We don't have enough information to conclude that **T1=Number**, meaning we can't conclude that the expression **(g a)** is even valid and hence can't conclude the entire expression's return-value type.
- b) False. Don't have enough information to conclude that **T1=T2**, and hence can't conclude that the listed return types of **f** match.
- c) False. **(lambda () (f x))** is of type **[empty -> typeof((f x))]**, which is equal to type **[empty -> T2]**, which according to the definitions so far in the course – can't be equal to **T2** (recursion is disallowed in type equalities.)
- d) False. Type of **x** is unknown.

2)

a) Renaming of bound variables:

((lambda (f x1) (f 1 x1)) + #t) turns to **((lambda (f x) (f 1 x)) + #t)**

Assignment of type variables:

((lambda (f x) (f 1 x)) + #t)	T0
(lambda (f x) (f 1 x))	T1
+	T+
#t	T#t
(f 1 x)	T2
f	Tf
1	Tnum1
x	Tx

Construction of type equations:

(subexpressions)

((lambda (f x) (f 1 x)) + #t)	T1 = [T+ * T#t -> T0]
(lambda (f x) (f 1 x))	T1 = [Tf*Tx -> T2]
(f 1 x)	Tf = [Tnum1*Tx -> T2]

(primitives)

+	T+ = [Number*Number -> Number]
#t	T#t = Boolean
1	Tnum1 = Number

Solving of equations:

1	T1 = [T+ * T#t -> T0]	{ }
2	T1 = [Tf*T _x -> T2]	
3	Tf = [Tnum1*T _x -> T2]	
4	T+ = [Number*Number -> Number]	
5	T#t = Boolean	
6	Tnum1 = Number	

1		{ T1 := [T+ * T#t -> T0] }
2	T1 = [Tf*T _x -> T2]	
3	Tf = [Tnum1*T _x -> T2]	
4	T+ = [Number*Number -> Number]	
5	T#t = Boolean	
6	Tnum1 = Number	

2		{ T1 = [T+ * T#t -> T0], T1 = [Tf*T _x -> T2] }
3	Tf = [Tnum1*T _x -> T2]	
4	T+ = [Number*Number -> Number]	
5	T#t = Boolean	
6	Tnum1 = Number	
7	T+ = Tf	
8	T#t = T_x	
9	T0 = T2	

3		{ T1 = [T+ * T#t -> T0], T1 = [[Tnum1*T _x -> T2] * T _x -> T2],
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		Tf = [Tnum1*Tx -> T2] }
4	T+ = [Number*Number -> Number]	
5	T#t = Boolean	
6	Tnum1 = Number	
7	T+ = Tf	
8	T#t = Tx	
9	T0 = T2	

4		{ T1 = [[Number*Number -> Number]*T#t -> T0], T1 = [[Tnum1*Tx -> T2]*Tx -> T2], Tf = [Tnum1*Tx -> T2], T+ = [Number*Number -> Number] }
5	T#t = Boolean	
6	Tnum1 = Number	
7	T+ = Tf	
8	T#t = Tx	
9	T0 = T2	

5		{ T1 = [[Number*Number -> Number]* Boolean -> T0] , T1 = [[Tnum1*Tx -> T2]*Tx -> T2], Tf = [Tnum1*Tx -> T2], T+ = [Number*Number -> Number], T#t = Boolean }
6	Tnum1 = Number	
7	T+ = Tf	
8	T#t = Tx	
9	T0 = T2	

6		{ T1 = [[Number*Number -> Number]* Boolean -> T0], T1 = [[Number *Tx -> T2]*Tx -> T2], Tf = [Number *Tx -> T2], T+ = [Number*Number -> Number], T#t = Boolean,
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		Tnum1 = Number }
7	T+ = Tf	
8	T#t = Tx	
9	T0 = T2	

7		{ T1 = [[Number*Number -> Number]* Boolean -> T0], T1 = [[Number*Tx -> T2]*Tx -> T2], Tf = [Number*Tx -> T2], T+ = [Number*Number -> Number], T#t = Boolean, Tnum1 = Number }
8	T#t = Tx	
9	T0 = T2	
10	Tx=Number	
11	T2=Number	

8		{ T1 = [[Number*Number -> Number]* Boolean -> T0], T1 = [[Number* Boolean -> T2]* Boolean -> T2], Tf = [Number* Boolean -> T2], T+ = [Number*Number -> Number], T#t = Boolean, Tnum1 = Number, T+ = Tf, Boolean = Tx }
9	T0 = T2	
10	Tx=Number	
11	T2=Number	

9		{ T1 = [[Number*Number -> Number]* Boolean -> T0], T1 = [[Number* Boolean -> T2]* Boolean -> T0],
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		<pre> Tf = [Number* Boolean -> T2], T+ = [Number*Number -> Number], T#t = Boolean, Tnum1 = Number, T+ = Tf, Boolean = Tx, T0 = T2 } </pre>
10	Tx=Number	
11	T2=Number	

10	Boolean=Number ((ERROR))	<pre> { T1 = [[Number*Number -> Number]* Boolean -> T2], T1 = [[Number* Boolean -> T2]* Boolean -> T2], Tf = [Number* Boolean -> T2], T+ = [Number*Number -> Number], T#t = Boolean, Tnum1 = Number, T+ = Tf, Boolean = Tx, T0 = T2 } </pre>
11	T2=Number	

b) Renaming of bound variables:

`((lambda (f x) (f1 x1 1)) + *)` turns into `((lambda (f x) (f x 1)) + *)`

Assignment of type variables:

1	<code>((lambda (f x) (f x 1)) + *)</code>	T0
2	<code>(lambda (f x) (f x 1))</code>	T1
3	<code>+</code>	T+

4	*	Tm
5	(f x 1)	T2
6	f	Tf
7	x	Tx
8	1	Tnum1

Construction of type equations:

(primitives)

+	T+ = [Number*Number -> Number]
*	Tm = [Number*Number -> Number]
1	Tnum1 = Number

(subexpressions)

((lambda (f x) (f x 1)) + *)	T1 = [T+ * Tm -> T0]
(lambda (f x) (f x 1))	T1 = [Tf * Tx -> T2]
(f x 1)	Tf = [Tx * Tnum1 -> T2]

Solving of equations:

1	Tf = [Tx * Tnum1 -> T2] => Tf = [Tx * Tnum1 -> T2]	{ }
2	T1 = [Tf * Tx -> T2]	
3	T1 = [T+ * Tm -> T0]	
4	Tnum1 = Number	
5	Tm = [Number*Number -> Number]	
6	T+ = [Number*Number -> Number]	

2	T1 = [Tf * Tx -> T2] => T1 = [[Tx * Tnum1->T2] *Tx -> T2]	{ Tf = [Tx * Tnum1 -> T2] }
3	T1 = [T+ * Tm -> T0]	
4	Tnum1 = Number	
5	Tm = [Number*Number -> Number]	

6	$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
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3	$T1 = [T+ * Tm \rightarrow T0]$ \Rightarrow $[[Tx * Tnum1 \rightarrow T2] * Tx \rightarrow T2] = [T+ * Tm \rightarrow T0]$	<pre>{ Tf = [Tx * Tnum1 -> T2], T1 = [[Tx * Tnum1->T2]*Tx -> T2] }</pre>
4	$Tnum1 = \text{Number}$	
5	$Tm = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
6	$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	

4	$Tnum1 = \text{Number}$ \Rightarrow $Tnum1 = \text{Number}$	<pre>{ Tf = [Tx * Tnum1 -> T2], T1 = [[Tx * Tnum1->T2]*Tx -> T2] }</pre>
5	$Tm = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
6	$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
7	$[Tx * Tnum1 \rightarrow T2] = T+$	
8	$Tx = Tm$	
9	$T2 = T0$	

5	$Tm = [\text{Number} * \text{Number} \rightarrow \text{Number}]$ \Rightarrow $Tm = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	<pre>{ Tf = [Tx * Number -> T2], T1 = [[Tx * Number - >T2]*Tx -> T2], Tnum1 = Number }</pre>
6	$T+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	
7	$[Tx * Tnum1 \rightarrow T2] = T+$	
8	$Tx = Tm$	
9	$T2 = T0$	

6	$T_+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$ \Rightarrow $T_+ = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	<pre>{ Tf = [Tx * Number -> T2], T1 = [[Tx * Number - >T2]*Tx -> T2], Tnum1 = Number, Tm = [Number*Number -> Number] }</pre>
7	$[Tx * Tnum1 \rightarrow T2] = T_+$	
8	$Tx = Tm$	
9	$T2 = T0$	

7	$[Tx * Tnum1 \rightarrow T2] = T_+$ \Rightarrow $[Tx * \text{Number} \rightarrow T2] =$ $[\text{Number} * \text{Number} \rightarrow \text{Number}]$	<pre>{ Tf = [Tx * Number -> T2], T1 = [[Tx * Number - >T2]*Tx -> T2], Tnum1 = Number, Tm = [Number*Number -> Number], T+ = [Number*Number -> Number] }</pre>
8	$Tx = Tm$	
9	$T2 = T0$	

8	$Tx = Tm$ \Rightarrow $Tx = [\text{Number} * \text{Number} \rightarrow \text{Number}]$	<pre>{ Tf = [Tx * Number -> T2], T1 = [[Tx * Number - >T2]*Tx -> T2], Tnum1 = Number, Tm = [Number*Number -> Number], T+ = [Number*Number -> Number] }</pre>
9	$T2 = T0$	
10	$Tx = \text{Number}$	
11	$T2 = \text{Number}$	

9	$T2 = T0$ \Rightarrow $T2 = T0$	<pre> { Tf = [Tx * Number -> T2], T1 = [[Tx * Number - >T2]*Tx -> T2], Tnum1 = Number, Tm = [Number*Number -> Number], T+ = [Number*Number -> Number], Tx = [Number*Number -> Number] } </pre>
10	$Tx = \text{Number}$	
11	$T2 = \text{Number}$	

10	$Tx = \text{Number}$ \Rightarrow $\text{[Number*Number -> Number] = Number}$ [[ERROR]]	<pre> { Tf = [Tx * Number -> T0], T1 = [[Tx * Number - >T2]*Tx -> T0], Tnum1 = Number, Tm = [Number*Number -> Number], T+ = [Number*Number -> Number], Tx = [Number*Number -> Number] } </pre>
11	$T2 = \text{Number}$	