**1)**

1. False. We don’t have enough information to conclude that **T1=Number**, meaning we can’t conclude that the expression **(g a)** is even valid and hence can’t conclude the entire expression’s return-value type.
2. False. Don’t have enough information to conclude that **T1=T2** , and hence can’t conclude that the listed return types of **f** match.
3. False. **(lambda () (f x))** is of type **[empty -> typeof((f x))]**, which is equal to type **[empty -> T2]**, which according to the definitions so far in the course – can’t be equal to **T2** (recursion is disallowed in type equalities.)
4. False. Type of **x** is unknown.

**2)**

1. **Renaming of bound variables:**

**((lambda (f x1) (f 1 x1)) + #t)** turns to **((lambda (f x) (f 1 x)) + #t)**

**Assignment of type variables:**

|  |  |
| --- | --- |
| ((lambda (f x) (f 1 x)) + #t) | T0 |
| (lambda (f x) (f 1 x)) | T1 |
| + | T+ |
| #t | T#t |
| (f 1 x) | T2 |
| f | Tf |
| 1 | Tnum1 |
| x | Tx |

**Construction of type equations:**

(subexpressions)

|  |  |
| --- | --- |
| ((lambda (f x) (f 1 x)) + #t) | T1 = [T+ \* T#t -> T0] |
| (lambda (f x) (f 1 x)) | T1 = [Tf\*Tx -> T2] |
| (f 1 x) | Tf = [Tnum1\*Tx -> T2] |

(primitives)

|  |  |
| --- | --- |
| + | T+ = [Number\*Number -> Number] |
| #t | T#t = Boolean |
| 1 | Tnum1 = Number |

**Solving of equations:**

|  |  |  |
| --- | --- | --- |
| 1 | T1 = [T+ \* T#t -> T0] | **{ }** |
| 2 | T1 = [Tf\*Tx -> T2] |  |
| 3 | Tf = [Tnum1\*Tx -> T2] |  |
| 4 | T+ = [Number\*Number -> Number] |  |
| 5 | T#t = Boolean |  |
| 6 | Tnum1 = Number |  |

|  |  |  |
| --- | --- | --- |
| 1 |  | **{ T1 := [T+ \* T#t -> T0] }** |
| 2 | T1 = [Tf\*Tx -> T2] |  |
| 3 | Tf = [Tnum1\*Tx -> T2] |  |
| 4 | T+ = [Number\*Number -> Number] |  |
| 5 | T#t = Boolean |  |
| 6 | Tnum1 = Number |  |

|  |  |  |
| --- | --- | --- |
| 2 |  | **{**  T1 = [T+ \* T#t -> T0],  **T1 = [Tf\*Tx -> T2]**  **}** |
| 3 | Tf = [Tnum1\*Tx -> T2] |  |
| 4 | T+ = [Number\*Number -> Number] |  |
| 5 | T#t = Boolean |  |
| 6 | Tnum1 = Number |  |
| 7 | **T+ = Tf** |  |
| 8 | **T#t = Tx** |  |
| 9 | **T0 = T2** |  |

|  |  |  |
| --- | --- | --- |
| 3 |  | **{** T1 = [T+ \* T#t -> T0],  T1 = [**[Tnum1\*Tx -> T2]**\*Tx -> T2],  **Tf = [Tnum1\*Tx -> T2] }** |
| 4 | T+ = [Number\*Number -> Number] |  |
| 5 | T#t = Boolean |  |
| 6 | Tnum1 = Number |  |
| 7 | T+ = Tf |  |
| 8 | T#t = Tx |  |
| 9 | T0 = T2 |  |

|  |  |  |
| --- | --- | --- |
| 4 |  | **{**  T1 = [**[Number\*Number -> Number]**\*T#t -> T0],  T1 = [[Tnum1\*Tx -> T2]\*Tx -> T2],  Tf = [Tnum1\*Tx -> T2],  **T+ = [Number\*Number -> Number]**  **}** |
| 5 | T#t = Boolean |  |
| 6 | Tnum1 = Number |  |
| 7 | T+ = Tf |  |
| 8 | T#t = Tx |  |
| 9 | T0 = T2 |  |

|  |  |  |
| --- | --- | --- |
| 5 |  | **{**  T1 = [[Number\*Number -> Number]\* **Boolean** -> T0],  T1 = [[Tnum1\*Tx -> T2]\*Tx -> T2],  Tf = [Tnum1\*Tx -> T2],  T+ = [Number\*Number -> Number],  **T#t = Boolean**  **}** |
| 6 | Tnum1 = Number |  |
| 7 | T+ = Tf |  |
| 8 | T#t = Tx |  |
| 9 | T0 = T2 |  |

|  |  |  |
| --- | --- | --- |
| 6 |  | **{**  T1 = [[Number\*Number -> Number]\* Boolean -> T0],  T1 = [[**Number**\*Tx -> T2]\*Tx -> T2],  Tf = [**Number**\*Tx -> T2],  T+ = [Number\*Number -> Number],  T#t = Boolean,  **Tnum1 = Number**  **}** |
| 7 | T+ = Tf |  |
| 8 | T#t = Tx |  |
| 9 | T0 = T2 |  |

|  |  |  |
| --- | --- | --- |
| 7 |  | **{**  T1 = [[Number\*Number -> Number]\* Boolean -> T0],  T1 = [[Number\*Tx -> T2]\*Tx -> T2],  Tf = [Number\*Tx -> T2],  T+ = [Number\*Number -> Number],  T#t = Boolean,  Tnum1 = Number  **}** |
| 8 | T#t = Tx |  |
| 9 | T0 = T2 |  |
| 10 | **Tx=Number** |  |
| 11 | **T2=Number** |  |

|  |  |  |
| --- | --- | --- |
| 8 |  | **{**  T1 = [[Number\*Number -> Number]\* Boolean -> T0],  T1 = [[Number\* **Boolean** -> T2]\* **Boolean** -> T2],  Tf = [Number\* **Boolean** -> T2],  T+ = [Number\*Number -> Number],  T#t = Boolean,  Tnum1 = Number,  T+ = Tf,  **Boolean** **= Tx**  **}** |
| 9 | T0 = T2 |  |
| 10 | Tx=Number |  |
| 11 | T2=Number |  |
|  |  |  |

|  |  |  |
| --- | --- | --- |
| 9 |  | **{**  T1 = [[Number\*Number -> Number]\* Boolean -> T0],  T1 = [[Number\* **Boolean** -> T2]\* **Boolean** -> **T0**],  Tf = [Number\* **Boolean** -> T2],  T+ = [Number\*Number -> Number],  T#t = Boolean,  Tnum1 = Number,  T+ = Tf,  Boolean = Tx,  **T0 = T2**  **}** |
| 10 | Tx=Number |  |
| 11 | T2=Number |  |
|  |  |  |
|  |  |  |

|  |  |  |
| --- | --- | --- |
| 10 | **Boolean=Number**  **((ERROR))** | **{**  T1 = [[Number\*Number -> Number]\* Boolean -> **T2**],  T1 = [[Number\* **Boolean** -> T2]\* **Boolean** -> T2],  Tf = [Number\* **Boolean** -> T2],  T+ = [Number\*Number -> Number],  T#t = Boolean,  Tnum1 = Number,  T+ = Tf,  Boolean = Tx,  T0 = T2  **}** |
| 11 | T2=Number |  |
|  |  |  |
|  |  |  |
|  |  |  |

1. **Renaming of bound variables:**

**((lambda (f x) (f1 x1 1)) + \*)** turns into **((lambda (f x) (f x 1)) + \*)**

**Assignment of type variables:**

|  |  |  |
| --- | --- | --- |
| 1 | ((lambda (f x) (f x 1)) + \*) | T0 |
| 2 | (lambda (f x) (f x 1)) | T1 |
| 3 | + | T+ |
| 4 | \* | Tm |
| 5 | (f x 1) | T2 |
| 6 | f | Tf |
| 7 | x | Tx |
| 8 | 1 | Tnum1 |

**Construction of type equations:**

(primitives)

|  |  |
| --- | --- |
| + | T+ = [Number\*Number -> Number] |
| \* | Tm = [Number\*Number -> Number] |
| 1 | Tnum1 = Number |

(subexpressions)

|  |  |
| --- | --- |
| ((lambda (f x) (f x 1)) + \*) | T1 = [T+ \* Tm -> T0] |
| (lambda (f x) (f x 1)) | T1 = [Tf \* Tx -> T2] |
| (f x 1) | Tf = [Tx \* Tnum1 -> T2] |

**Solving of equations:**

|  |  |  |
| --- | --- | --- |
| 1 | Tf = [Tx \* Tnum1 -> T2]  **=>**  **Tf = [Tx \* Tnum1 -> T2]** | **{ }** |
| 2 | T1 = [Tf \* Tx -> T2] |  |
| 3 | T1 = [T+ \* Tm -> T0] |  |
| 4 | Tnum1 = Number |  |
| 5 | Tm = [Number\*Number -> Number] |  |
| 6 | T+ = [Number\*Number -> Number] |  |

|  |  |  |
| --- | --- | --- |
| 2 | T1 = [Tf \* Tx -> T2]  **=>**  T1 = [**[Tx \* Tnum1->T2]**\*Tx -> T2] | **{**  **Tf = [Tx \* Tnum1 -> T2]**  **}** |
| 3 | T1 = [T+ \* Tm -> T0] |  |
| 4 | Tnum1 = Number |  |
| 5 | Tm = [Number\*Number -> Number] |  |
| 6 | T+ = [Number\*Number -> Number] |  |

|  |  |  |
| --- | --- | --- |
| 3 | T1 = [T+ \* Tm -> T0]  **=>**  **[[Tx \* Tnum1->T2]\*Tx -> T2]** = [T+ \* Tm -> T0] | **{**  Tf = [Tx \* Tnum1 -> T2],  **T1 = [[Tx \* Tnum1->T2]\*Tx -> T2]**  **}** |
| 4 | Tnum1 = Number |  |
| 5 | Tm = [Number\*Number -> Number] |  |
| 6 | T+ = [Number\*Number -> Number] |  |

|  |  |  |
| --- | --- | --- |
| 4 | Tnum1 = Number  **=>**  Tnum1 = Number | **{**  Tf = [Tx \* Tnum1 -> T2],  T1 = [[Tx \* Tnum1->T2]\*Tx -> T2]  **}** |
| 5 | Tm = [Number\*Number -> Number] |  |
| 6 | T+ = [Number\*Number -> Number] |  |
| 7 | **[Tx \* Tnum1 -> T2] = T+** |  |
| 8 | **Tx = Tm** |  |
| 9 | **T2 = T0** |  |

|  |  |  |
| --- | --- | --- |
| 5 | Tm = [Number\*Number -> Number]  **=>**  Tm = [Number\*Number -> Number] | **{**  Tf = [Tx \* **Number** -> T2],  T1 = [[Tx \* **Number** ->T2]\*Tx -> T2],  **Tnum1 = Number**  **}** |
| 6 | T+ = [Number\*Number -> Number] |  |
| 7 | [Tx \* Tnum1 -> T2] = T+ |  |
| 8 | Tx = Tm |  |
| 9 | T2 = T0 |  |

|  |  |  |
| --- | --- | --- |
| 6 | T+ = [Number\*Number -> Number]  **=>**  T+ = [Number\*Number -> Number] | **{**  Tf = [Tx \* Number -> T2],  T1 = [[Tx \* Number ->T2]\*Tx -> T2],  Tnum1 = Number,  **Tm = [Number\*Number -> Number]**  **}** |
| 7 | [Tx \* Tnum1 -> T2] = T+ |  |
| 8 | Tx = Tm |  |
| 9 | T2 = T0 |  |

|  |  |  |
| --- | --- | --- |
| 7 | [Tx \* Tnum1 -> T2] = T+  **=>**  [Tx \* **Number** -> T2] = **[Number\*Number -> Number]** | **{**  Tf = [Tx \* Number -> T2],  T1 = [[Tx \* Number ->T2]\*Tx -> T2],  Tnum1 = Number,  Tm = [Number\*Number -> Number],  **T+ = [Number\*Number -> Number]**  **}** |
| 8 | Tx = Tm |  |
| 9 | T2 = T0 |  |

|  |  |  |
| --- | --- | --- |
| 8 | Tx = Tm  **=>**  Tx = **[Number\*Number -> Number]** | **{**  Tf = [Tx \* Number -> T2],  T1 = [[Tx \* Number ->T2]\*Tx -> T2],  Tnum1 = Number,  Tm = [Number\*Number -> Number],  T+ = [Number\*Number -> Number]  **}** |
| 9 | T2 = T0 |  |
| 10 | **Tx = Number** |  |
| 11 | **T2 = Number** |  |

|  |  |  |
| --- | --- | --- |
| 9 | T2 = T0  **=>**  T2 = T0 | **{**  Tf = [Tx \* Number -> T2],  T1 = [[Tx \* Number ->T2]\*Tx -> T2],  Tnum1 = Number,  Tm = [Number\*Number -> Number],  T+ = [Number\*Number -> Number],  **Tx = [Number\*Number -> Number]**  **}** |
| 10 | Tx = Number |  |
| 11 | T2 = Number |  |

|  |  |  |
| --- | --- | --- |
| 10 | Tx = Number  **=>**  **[Number\*Number -> Number] = Number**  **[[ERROR]]** | **{**  Tf = [Tx \* Number -> **T0**],  T1 = [[Tx \* Number ->T2]\*Tx -> **T0**],  Tnum1 = Number,  Tm = [Number\*Number -> Number],  T+ = [Number\*Number -> Number],  Tx = [Number\*Number -> Number]  **}** |
| 11 | T2 = Number |  |