2- Shaxsiy uy topshiqlari

1 Bernulli tenglamasining berilgan shartni qanoatlantiruvchi yechimini toping

1. 7. $2y' + y\cos x = y^{-1}\cos x(1+\sin x),\ y(0)=1.$

$$y(x)\cos(x) + 2\frac{d}{dx}y(x) =$$

$$= \frac{(\sin x + 1)\cos(x)}{y(x)}$$

Javob: $y(x) = -\sqrt{c_1 e^{-\sin(x)} + \sin(x)}$

$$y(x) = \sqrt{c_1 e^{-\sin(x)} + \sin(x)}.$$

1) 2. Tо'liq differentsial tenglamani yeching.

2.7

$$\left(\frac{x}{\sqrt{x^2+y^2}}+y\right)dx+\left(x+\frac{y}{\sqrt{x^2+y^2}}\right)dy=0$$

$$\left(\frac{y}{\sqrt{x^2+y^2}}+x\right)dy=\left(-\frac{x}{\sqrt{y^2+x^2}}-y\right)dx$$

$$\left(\frac{x\sqrt{y^2+x^2}+y}{\sqrt{y^2+x^2}}\right)dy=-\left(\frac{y\sqrt{y^2+x^2}+x}{\sqrt{y^2+x^2}}\right)dx$$

$$M(x,y)\,dy+N(x,y)\,dx=0$$

$$M(x,y)=\frac{x\sqrt{y^2+x^2}+y}{\sqrt{y^2+x^2}}$$

$$N(x,y)=\frac{y\sqrt{y^2+x^2}+x}{\sqrt{y^2+x^2}}$$

$M(x,y)'_x = N(x,y)'_y = 1 - \dfrac{xy}{(y^2+x^2)^{3/2}}$ 2)

$F(x,y) : dF(x,y) = F'_y\,dy + F'_x\,dx$

$F(x,y) = \int N(x,y)\,dx = \int \dfrac{y\sqrt{y^2+x^2}+x}{\sqrt{y^2+x^2}}\,dx =$

$= \sqrt{x^2+y^2} + yx + C_y$

$(\sqrt{x^2+y^2} + yx)'_y = \dfrac{y}{\sqrt{x^2+y^2}} + x$

$C_y = \int M(x,y) - (\sqrt{x^2+y^2}+yx)\,dy =$

$= \int \dfrac{x\sqrt{y^2+x^2}+y}{\sqrt{x^2+y^2}} - \dfrac{y}{\sqrt{x^2+y^2}} - x\,dy = 0$

$F(x,y) = \sqrt{x^2+y^2} + yx + C_y = \sqrt{x^2+y^2} + yx$

Jawob: $\sqrt{x^2+y^2} + yx = C$

3) 3. Tenglamani integrallovchi ko'paytuvchi usulidan foydalanib yeching.

3.7 $(2y + xy^3)dx + (x + x^2y^2)dy = 0$

$(x^2y^2 + x)dy = (-xy^3 - 2y)dx$

$y = z^{\lambda}$ $\quad x = z$ $\quad y = z^{\lambda}$

$\lambda z^{\lambda-1}(z^{2\lambda+2} + z)dz = (-z^{3\lambda+1} - 2z^{\lambda})dz$

$\lambda = 3\lambda + 1$

$y = \frac{1}{\sqrt{z}}$ $\quad dy = -\frac{dz}{2z^{3/2}}$

$-\frac{x\,dz}{2z^{3/2}} - \frac{x^2\,dz}{2z^{5/2}} = \left(-\frac{2}{\sqrt{z}} - \frac{x}{z^{3/2}}\right)dx$

$u = \frac{z}{x}$ $\quad z = ux$ $\quad dz = u\,dx + x\,du$

$\left(-\frac{1}{2u^{3/2}} - \frac{1}{2u^{5/2}}\right)u^{5/2}(u\,dx + x\,du) =$

$= 2\left(-\frac{2}{\sqrt{u}} - \frac{1}{u^{3/2}}\right) \cdot u^{5/2}dx \Rightarrow$

$\Rightarrow \quad (-4x - x)\, du = (-3u^2 - u)\, dx \qquad 4)$

$(-u-1)\, x\, du = (-3u^2 - u)\, dx$

$$\left(\frac{1}{u(3u+1)} + \frac{1}{3u+1}\right) du = \int \frac{1}{x}\, dx$$

$$\ln\left(\frac{u}{3u+1}\right) + \frac{\ln(3u+1)}{3} = \ln(x) + C$$

$$e^{f_1} = e^{f_2} \qquad e^{\ln(a)} = a$$

$$\frac{u}{\sqrt[3]{(3u+1)^2}} \, e^{c} x \qquad u = \frac{z}{x} \qquad z = -\frac{3}{y^2}$$

$$\frac{z}{x \cdot \sqrt[3]{\left(\frac{3z}{x}+1\right)^2}} = c_x \qquad z = \frac{1}{y^2}$$

$$\frac{1}{x\sqrt[3]{\left(\frac{3}{xy^2}+1\right)^2}\, y^2} = c_x$$

Jaвoв $\quad 1 = c(x^6 y^6 + 6x^5 y^4 + 9x^4 y^2)$

4 Berilgan tenglamani Eyler usulida yeching.

$$y' = e^{-x} - y^2 \qquad y(0) = 0$$

$$\frac{d}{dx} y(x) = e - y^2(x) \qquad y(0) = 0$$

$$y(x) = \frac{\left(-e^{2c + e^{\frac{1}{2}}} - e^{2x e^{\frac{1}{2}}}\right) e^{\frac{1}{2}}}{e^{2c + e^{\frac{1}{2}}} - e^{2x} e^{\frac{1}{2}}}$$

$$0 = \frac{\left(-e^{2c + e^{\frac{1}{2}}} - 1\right) e^{1/2}}{e^2 c + e^{\frac{1}{2}} - 1} \qquad c_1 = -\frac{\pi}{2e^{\frac{1}{2}}}$$

$$y(x) = \frac{\left(1 - e^{2x e^{\frac{1}{2}}}\right) e^{\frac{1}{2}}}{-e^{2x e^{1/2}} - 1} \qquad c_1 = \frac{\pi}{2e^{1/2}}$$

$$y(x) = \frac{\left(1 - e^{2x e^{1/2}}\right) e^{1/2}}{-e^{2x e^{1/2}} - 1} \qquad \int \frac{1}{y^2 - 1} dy = \int (-1) dx$$

$$\frac{\log\left((y) - e^{1/2}\right)}{2e^{1/2}} - \frac{\log\left(y + e^{\frac{1}{2}}\right)}{2e^{1/2}} = \cos h^{t} - x$$

Javob

$$y(x) = \frac{\left(-e^{2c_1 e^{\frac{1}{2}}} - e^{2x e^{\frac{1}{2}}}\right) e^{\frac{1}{2}}}{e^{2c_1 e^{1/2}} - e^{2x e^{1/2}}}$$

5 Hosilasiga nisbatan yechiladigan tenglamani integrallang.

<u>5.7</u> $x^2y'^2 - 2xyy' = x^2y^2 - x^4$

$$\frac{x^3 y(x)}{\frac{d}{dx} y(x)} = -x^4 + x^2y^2(x)$$

$$y(x) = -\sqrt{x^2 - \sqrt{c_1 + x^4}}$$

$$y(x) = \sqrt{x^2 - \sqrt{c_1 + x^4}}$$

$$y(x) = -\sqrt{x^2 + \sqrt{c_1 + x^4}}$$

<u>Javob</u> $y(x) = \sqrt{x^2 + \sqrt{c_1 + x^4}}$

6. Erkli o'zgaruvchili x ga nisbatan yechiladigan tenglamani integrallang. 6)

6.7 $x = y'(1+y')$

$F(x, y, y') = 0 \qquad x = f(y, y')$

$p = y' \qquad p = y' = \frac{dy}{dx} \Rightarrow dx = \frac{dy}{p}$

$x = p(p+1)$

$dx = dp + 2p\,dp \qquad dx = \frac{dy}{p}$

$dy = p(dp + 2p\,dp)$

$dy = \cancel{p(}\ (2p^2 + p)\,dp$

$M(y)\,dy = N(p)\,dp$

$\int 1\,dy = \int (2p^2 + p)\,dp$

$y = \frac{2p^3}{3} + \frac{p^2}{2} + c$

Javob: $x = p(p+1) \quad y = \frac{2p^3}{3} + \frac{p^2}{2} + c$

7 Noma'lum funksiya y ga nisbatan yechiladigan tenglamani integrallang.

7.7 $y\left(1+\frac{1}{y'^2}\right)^{3/2}=1$.

$$y\left(1+\frac{1}{(y')^2}\right)^{\frac{3}{2}}=1.$$

$\Rightarrow$ $F(x, y, y')=0$

$y=f(x, y')$

$p=y'$

$p=y'=\frac{dy}{dx}$

$dy=p\,dx$

$$\Rightarrow\ y=\frac{1}{\left(\frac{1}{p^2}+1\right)^{3/2}}$$

$$dy = \frac{3p|p|\,dp}{(p^2+1)^{5/2}}$$

$$\Rightarrow dy = p\,dx$$

$$\Rightarrow p\,dx = \frac{3p|p|\,dp}{(p^2+1)^{5/2}}$$

$$dx = \frac{3|p|\,dp}{\sqrt{p^2+1}\,(p^4+2p^2+1)}$$

$$\Rightarrow M_{(x)}\,dx = N_{(p)}\,dp$$

$$\Rightarrow \int 1\,dx = \int \frac{3p}{\sqrt{p^2+1}\,(p^4+2p^2+1)}\,dp$$

$$x = C - \frac{|p|}{\sqrt{p^2+1}\,(p^3+p)}$$

Jaδoв:

$$y = \frac{1}{\left(\frac{1}{p^2}+1\right)^{3/2}} \quad x = C - \frac{|p|}{\sqrt{p^2+1}\,(p^3+p)}$$

8 Lagranj tenglamasini yeching

8.7

$$y = x(y')^2 + (y')^3$$

Formulalar

$$F(x, y, y') = 0$$

$$y = f(x, y')$$

$$p = y'$$

$$\Rightarrow p = y' = \frac{dy}{dx} \Rightarrow$$

$$dy = p\,dx$$

$$\Rightarrow \quad y = p^2 x + p^3$$

$$dy = p^2 dx + 2px\,dp + 3p^2 dp$$

$$dy = p\,dx$$

$$p\,dx = p^2 dx + 2px\,dp + 3p^2 dp$$

$$p\,dx = p^2 dx + 2px\,dp + 3p^2 dp \quad |:p$$

$$dx = p\,dx + 2x\,dp + 3p\,dp$$

$$dx = 2x\,dp + p(dx + 3dp)$$

$$(1-p)\,dx = (2x+3p)\,dp$$

$$dx = \frac{(2x+3p)\,dp}{1-p}$$

$$x' = \left(\frac{a_1x + b_1p + c_1}{ax + bp + c}\right) \qquad \begin{cases} 1-p=0 \\ 2x+3=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} p = 1 \\ x = -\frac{3}{2} \end{cases}$$

$$x = x_1 - \frac{3}{2} \qquad p = p_1 + 1$$

$$dp = dp_1$$

$$dx = dx_1$$

$$dx_1 = \left(-\frac{2x_1}{p_1} - 3\right)dp_1$$

$$M(kx, kp_1) = kM(x, p_1)$$

$$1 = \frac{-2x_1}{p_1} - 3 \to k^0$$

$u = \frac{x_1}{p_1}$ $\qquad x_1 = p_1 u$

$dx_1 = p_1 du + u dp_1$

$p_1 du + u dp_1 = (-2u - 3) dp_1$

$p_1 du = (-3u - 3) dp_1$

$$-\frac{du}{3u+3} = \frac{dp_1}{p_1}$$

$$M_{(u)} du = N_{(p_1)} dp_1$$

$$\int -\frac{1}{3u+3} du = \int \frac{1}{p_1} dp_1$$

$$-\frac{\ln(u+1)}{3} = \ln(p_1) + c$$

$$\Rightarrow \quad e^{f_1} = e^{f_2} \quad \Rightarrow$$

$$e^{\ln a} = a$$

$$\Rightarrow \quad \frac{1}{\sqrt[3]{u+1}} = e^c p_1 \qquad u = \frac{x_1}{p_1}$$

$$\frac{1}{\sqrt[3]{\frac{x_1}{p_1}}+1}=cp_1 \qquad x_1=x+\frac{3}{2} \;/\; p_1=p-1$$

$$\frac{1}{\sqrt[3]{\frac{x+\frac{3}{2}}{p-1}}+1}=c(p-1)$$

$$x=\frac{c}{p^2-2p+1}-p-\frac{1}{2}$$

$$x=\frac{c}{p^2-2p+1}-p-\frac{1}{2} \qquad y=p^2x+p^3$$

Javob:

$$y=p^2\left(\frac{c}{p^2-2p+1}-p-\frac{1}{2}\right)+p^3$$

$$x=\frac{c}{p^2-2p+1}-p-\frac{1}{2}.$$

9 Tartib pasaytirish mumkin bo'lgan yuqori tartibli differensial

9.7 $xy'' = y' + x\sin\left(\frac{y'}{x}\right) \Rightarrow$

$$\Rightarrow x\frac{d^2}{dx^2}y(x) = x\sin\left(\frac{\frac{d}{dx}y(x)}{x}\right) + \frac{d}{dx}y(x)$$

Javob:

$$x\frac{d^2}{dx^2}y(x) = x\sin\left(\frac{\frac{d}{dx}y(x)}{x}\right) + \frac{d}{dx}y(x).$$

10 Tenglamani yeching

10.7 $yy'' - 3y'^2 + 3yy' - y^2 = 0$

$$y^2(x) + 4y(x)\frac{d}{dx}y(x) - 3\left(\frac{d}{dx}y(x)\right)^2 = 0$$

Javob:

$$y(x) = c_1 e^{x/3} \qquad y(x) = c_1 e^{x}$$

11 Tenglamani yeching

11.7 $y'' = a e^{y}$

$F(y, y', \dots y^{(n)}) = 0$

$y' = u \quad y'' = u \cdot u' \quad y' = u(y)$

$uu' = a \cdot e^{y}$

$uu' = a e^{y} \Rightarrow u'(y) = \frac{du}{dy}$

$\frac{u\,du}{dy} = a e^{y} \quad / \cdot dy$

$u\,du = a \cdot e^{y} dy$

$u\,du = a \cdot e^{y} dy$

$M(u)\,du = N(y)\,dy$

$\int u\,du = \int (a e^{y} dy)$

$\frac{u^2}{2} = a e^{y} + c$

$u^2 = 2(ae^y + c) \qquad u = y'$

$(y')^2 = 2(ae^y + c)$

$\Rightarrow \quad v = e^y$

$v' = e^y y' \quad y' = \frac{v'}{e^y} \Rightarrow$

$\Rightarrow (y')^2 = 2(ae^y + c)$

$$\frac{(v')^2}{v^2} = 2(a \cdot v + c)$$

$F(x, v, v') = 0$

$$\frac{(v')^2}{v^2} = 2(a \cdot v + c) \Rightarrow \begin{cases} v' = -\sqrt{2}v\sqrt{a \cdot v + c} \\ v' = \sqrt{2}v\sqrt{a \cdot v + c} \end{cases}$$

I $\quad v' = -\sqrt{2}\, v\sqrt{a \cdot v + c}$

$$v'(x) = \frac{dv}{dx} \qquad \frac{dv}{dx} = -\sqrt{2}\, v\sqrt{av + c}$$

$dv = -\sqrt{2}\, v\sqrt{av + c} \cdot dx$

$$\frac{dv}{v\sqrt{a\cdot v+c}} = -\sqrt{2}\,dx$$

$$M_{(v)}\,dv = N_{(x)}\,dx$$

$$\int \frac{1}{v\sqrt{av+c}}\,dv = \int -\sqrt{2}\,dx$$

$$\frac{\ln\left(\frac{\sqrt{av+c}-\sqrt{c}}{\sqrt{av+c}+\sqrt{c}}\right)}{\sqrt{c}} = c_1 - \sqrt{2}x$$

$$\frac{\ln\left(\sqrt{ae^y+c}-\sqrt{c}\right)}{\sqrt{c}} - \frac{\ln\left(\sqrt{ae^y+c}+\sqrt{c}\right)}{\sqrt{c}} =$$

$$= c_1 - \sqrt{2}x \qquad v = e^y$$

$$\frac{\ln\left(\sqrt{ae^y+c}-\sqrt{c}\right)}{\sqrt{c}} - \frac{\ln\left(\sqrt{ae^y+c}+\sqrt{c}\right)}{\sqrt{c}} =$$

$$= c_1 - 2x \qquad v' = \sqrt{2}\,v\sqrt{av+c} \Rightarrow$$

$$\upsilon'(x) = \frac{d\upsilon}{dx} \qquad \frac{d\upsilon}{dx} = \sqrt{2}\,\upsilon\sqrt{a\upsilon+c} =$$

$$= c_1 - \sqrt{2}x$$

II $\quad \upsilon' = \sqrt{2}\upsilon\sqrt{a\upsilon+c}$

$$\upsilon'(x) = \frac{d\upsilon}{dx} \qquad \frac{d\upsilon}{dx} = \sqrt{2}\upsilon\sqrt{a\upsilon+c} \,/\cdot dx$$

$$d\upsilon = \sqrt{2}\,\upsilon\sqrt{a\cdot\upsilon+c}\,dx$$

$$\frac{d\upsilon}{\upsilon\sqrt{a\upsilon+c}} = \sqrt{2}\,dx$$

$$M\,\upsilon(d\upsilon) = N(x)\,dx$$

$$\int \frac{1}{\upsilon\sqrt{a\upsilon+c}}\,d\upsilon = \int\sqrt{2}\,dx$$

$$\frac{\ln\left(\frac{\sqrt{a\upsilon+c}-\sqrt{c}}{\sqrt{a\upsilon+c}+\sqrt{c}}\right)}{\sqrt{c}} = \sqrt{2}x + c_2$$

$$\frac{\ln\left(\sqrt{ae^{y}+c}-\sqrt{c}\right)}{\sqrt{c}}-\frac{\ln\left(\sqrt{ae^{y}+c}+\sqrt{c}\right)}{\sqrt{c}}=$$

$$=\sqrt{2}x+c_2$$

Javob:

$$\frac{\ln\left(\sqrt{ae^{y}+c}-\sqrt{c}\right)}{\sqrt{c}}-\frac{\ln\left(\sqrt{ae^{y}+c}+\sqrt{c}\right)}{\sqrt{c}}=c_1-\sqrt{2}x$$

$$\frac{\ln\left(\sqrt{ae^{y}+c}-\sqrt{c}\right)}{\sqrt{c}}-\frac{\ln\left(\sqrt{ae^{y}+c}+\sqrt{c}\right)}{\sqrt{c}}=\sqrt{2}x+c_2$$