1)

2. To'liq differensial tenglamani yeching.

2.7

$$\left(\frac{x}{\sqrt{x^2+y^2}}+y\right)dx+\left(x+\frac{y}{\sqrt{x^2+y^2}}\right)dy=0$$

$$\left(\frac{y}{\sqrt{x^2+y^2}}+x\right)dy=\left(-\frac{x}{\sqrt{y^2+x^2}}-y\right)dx$$

$$\left(\frac{x\sqrt{y^2+x^2}+y}{\sqrt{y^2+x^2}}\right)dy=-\left(\frac{y\sqrt{y^2+x^2}+x}{\sqrt{y^2+x^2}}\right)dx$$

$$M(x,y)\,dy+N(x,y)\,dx=0$$

$$M(x,y)=\frac{x\sqrt{y^2+x^2}+y}{\sqrt{y^2+x^2}}$$

$$N(x,y)=\frac{y\sqrt{y^2+x^2}+x}{\sqrt{y^2+x^2}}$$

$M(x,y)'_x = N(x,y)'_y = 1 - \frac{xy}{(y^2+x^2)^{3/2}}$ 2)

$F(x,y): dF(x,y) = F'_y\,dy + F'_x\,dx$

$F(x,y) = \int N(x,y)\,dx = \int \frac{y\sqrt{y^2+x^2}+x}{\sqrt{y^2+x^2}}\,dx =$

$= \sqrt{x^2+y^2} + yx + C_y$

$(\sqrt{x^2+y^2} + yx)'_y = \frac{y}{\sqrt{x^2+y^2}} + x$

$C_y = \int M(x,y) - (\sqrt{x^2+y^2}+yx)\,dy =$

$= \int \frac{x\sqrt{y^2+x^2}+y}{\sqrt{x^2+y^2}} - \frac{y}{\sqrt{x^2+y^2}} - x\,dy = 0$

$F(x,y) = \sqrt{x^2+y^2} + yx + C_y = \sqrt{x^2+y^2} + yx$

Јавов: $\sqrt{x^2+y^2} + yx = C$

3) 3. Tenglamani integrallovchi ko'paytuvchi usulidan foydolanib yeching.

3.7 $(2y + xy^3)dx + (x + x^2y^2)dy = 0$

$(x^2y^2 + x)dy = (-xy^3 - 2y)dx$

$y = z^\lambda \quad x = z \quad y = z^\lambda$

$\lambda z^{\lambda-1}(z^{2\lambda+2} + z)dz = (-z^{3\lambda+1} - 2z^\lambda)dz$

$\lambda = 3\lambda + 1$

$y = \frac{1}{\sqrt{z}} \quad dy = -\frac{dz}{2z^{3/2}}$

$-\frac{x\,dz}{2z^{3/2}} - \frac{x^2 dz}{2z^{5/2}} = \left(-\frac{2}{\sqrt{z}} - \frac{x}{z^{3/2}}\right)dx$

$u = \frac{z}{x} \quad z = ux \quad dz = u\,dx + x\,du$

$\left(-\frac{1}{2u^{3/2}} - \frac{1}{2u^{5/2}}\right)u^{5/2}(u\,dx + x\,du) =$

$= 2\left(-\frac{2}{\sqrt{u}} - \frac{1}{u^{3/2}}\right) \cdot u^{5/2}dx \Rightarrow$

4)

$\Rightarrow (-ux - x)\,du = (-3u^2 - u)\,dx$

$(-u-1)\,x\,du = (-3u^2 - u)\,dx$

$\left(\frac{1}{u(3u+1)} + \frac{1}{3u+1}\right) du = \int \frac{1}{x}\,dx$

$\ln\left(\frac{u}{3u+1}\right) + \frac{\ln(3u+1)}{3} = \ln(x) + C$

$e^{f_1} = e^{f_2} \qquad e^{\ln(a)} = a$

$\frac{u}{\sqrt[3]{(3u+1)^2}}\; e^{C}x \qquad u = \frac{z}{x} \qquad x = -\frac{3}{y^2}$

$\frac{z}{x \cdot \sqrt[3]{\left(\frac{3z}{x}+1\right)^2}} = C_x \qquad z = \frac{1}{y^2}$

$\frac{1}{x\sqrt[3]{\left(\frac{3}{xy^2}+1\right)^2}\, y^2} = C_x$

<u>Javob</u> $\quad 1 = C(x^6y^6 + 6x^5y^4 + 9x^4y^2)$

6. Erkli o'zgaruvchili x ga nisbatan yechiladigan tenglamani integrallang.

6)

6.7 $x = y'(1 + y')$

$F(x, y, y') = 0 \quad x = f(y, y')$

$p = y' \quad p = y' = \frac{dy}{dx} \Rightarrow dx = \frac{dy}{p}$

$x = p(p+1)$

$dx = dp + 2p\,dp \quad dx = \frac{dy}{p}$

$dy = p(dp + 2p\,dp)$

$dy = \cancel{p(}\ (2p^2 + p)\,dp$

$M(y)\,dy = N(p)\,dp$

$\int 1\,dy = \int (2p^2 + p)\,dp$

$y = \frac{2p^3}{3} + \frac{p^2}{2} + c$

Javob: $x = p(p+1) \quad y = \frac{2p^3}{3} + \frac{p^2}{2} + c$

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7 Noma'lum funksiya y ga nisba-tan yechiladigan tenglamani integrallang.

7.7 $y\left(1+\frac{1}{y'^2}\right)^{3/2}=1.$

$$y\left(1+\frac{1}{(y')^2}\right)^{\frac{3}{2}}=1$$

$$\Rightarrow F(x, y, y')=0$$

$$y=f(x, y')$$

$$p=y'$$

$$p=y'=\frac{dy}{dx}$$

$$dy=p\,dx$$

$$\Rightarrow y=\frac{1}{\left(\frac{1}{p^2}+1\right)^{3/2}}$$

$dy =$

$\Rightarrow dy$

$\Rightarrow p$

$dx =$

$\Rightarrow$

$\Rightarrow$

$x =$

Javob:

$$dy = \frac{3p|p|\,dp}{(p^2+1)^{5/2}}$$

$$\Rightarrow dy = p\,dx$$

$$\Rightarrow p\,dx = \frac{3p|p|\,dp}{(p^2+1)^{5/2}}$$

$$dx = \frac{3|p|\,dp}{\sqrt{p^2+1}\,(p^4+2p^2+1)}$$

$$\Rightarrow M_{(x)}\,dx = N_{(p)}\,dp$$

$$\Rightarrow \int 1\,dx = \int \frac{3p}{\sqrt{p^2+1}\,(p^4+2p^2+1)}\,dp$$

$$x = C - \frac{|p|}{\sqrt{p^2+1}\,(p^3+p)}$$

Жавоб. $y = \frac{1}{\left(\frac{1}{p^2}+1\right)^{3/2}}$ $\quad x = C - \frac{|p|}{\sqrt{p^2+1}\,(p^3+p)}$

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8 Lagranj tenglamasini yeching

8.7

$$y = x(y')^2 + (y')^3$$

Formulalar

$$F(x, y, y') = 0$$
$$y = f(x, y')$$
$$p = y'$$
$$\Rightarrow p = y' = \frac{dy}{dx} \Rightarrow$$
$$dy = p\,dx$$

$$\Rightarrow \quad y = p^2 x + p^3$$
$$dy = p^2 dx + 2px\,dp + 3p^2 dp$$
$$dy = p\,dx$$
$$p\,dx = p^2 dx + 2px\,dp + 3p^2 dp$$
$$p\,dx = p^2 dx + 2px\,dp + 3p^2 dp \quad |:p$$
$$dx = p\,dx + 2x\,dp + 3p\,dp$$

$$dx = 2x\,dp + p(dx + 3dp)$$

$$(1-p)\,dx = (2x+3p)\,dp$$

$$dx = \frac{(2x+3p)\,dp}{1-p}$$

$$x' = \left(\frac{a_1 x + b_1 p + c_1}{ax + bp + c}\right) \quad \begin{cases} 1-p=0 \\ 2x+3=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} p=1 \\ x=-\frac{3}{2} \end{cases}$$

$$x = x_1 - \frac{3}{2} \qquad p = p_1 + 1$$

$$dp = dp_1$$

$$dx = dx_1$$

$$dx_1 = \left(-\frac{2x_1}{p_1} - 3\right) dp_1$$

$$M(kx, kp_1) = kM(x, p_1)$$

$$1 = \frac{-2x_1}{p_1} - 3 \rightarrow k^0$$

$$u = \frac{x_1}{p_1} \qquad x_1 = p_1 u$$

$$dx_1 = p_1 du + u dp_1$$

$$p_1 du + u dp_1 = (-2u-3) dp_1$$

$$p_1 du = (-3u-3) dp_1$$

$$-\frac{du}{3u+3} = \frac{dp_1}{p_1}$$

$$M_{(u)} du = N_{(p_1)} dp_1$$

$$\int -\frac{1}{3u+3} du = \int \frac{1}{p_1} dp_1$$

$$-\frac{\ln(u+1)}{3} = \ln(p_1) + c$$

$$\Rightarrow \frac{e^{f_1} = e^{f_2}}{e^{\ln a} = a} \Rightarrow$$

$$\Rightarrow \frac{1}{\sqrt[3]{u+1}} = e^c p_1 \qquad u = \frac{x_1}{p_1}$$

$$\frac{1}{\sqrt[3]{\frac{x_1}{p_1}+1}} = cp_1 \qquad x_1 = x + \frac{3}{2} \,/\, p_1 = p-1$$

$$\frac{1}{\sqrt[3]{\frac{x+\frac{3}{2}}{p-1}+1}} = c(p-1)$$

$$x = \frac{c}{p^2-2p+1} - p - \frac{1}{2}$$

$$x = \frac{c}{p^2-2p+1} - p - \frac{1}{2} \qquad y = p^2x + p^3$$

Javob:

$$y = p^2\left(\frac{c}{p^2-2p+1} - p - \frac{1}{2}\right) + p^3$$

$$x = \frac{c}{p^2-2p+1} - p - \frac{1}{2}.$$

9.7. $(2y'+x)y'' = 1$.

$(2y'+x)y''-1=0$

$y'^2+xy'-y-x=c$.

$y=px-x+p^2-c$.

$0=-dx+x\,dp+2p\,dp$

$du-2dp=u\,dp$

$du=(u+2)dp$.

$u=x+2p \quad (1)$

$x=c_1e^p-2p-2$

$y=px-x+p^2-c$.

Відпов:

$y=p(c_1e^p-2p-2)-c_1e^p+p^2+2p-c+2$

$x=c_1e^p-2p-2$

11 Tenglamani yeching

11.7 $y'' = a e^{y}$

$F(y, y', \ldots y^{(n)}) = 0$

$y' = u \quad y'' = u \cdot u' \quad y' = u(y)$

$u u' = a \cdot e^{y}$

$u u' = a e^{y} \Rightarrow u'(y) = \frac{du}{dy}$

$\frac{u\,du}{dy} = a e^{y} \;/ \cdot dy$

$u\,du = a \cdot e^{y} dy$

$u\,du = a \cdot e^{y} dy$

$M(u)\,du = N(y)\,dy$

$\int u\,du = \int (a e^{y} dy)$

$\frac{u^2}{2} = a e^{y} + c$

$u^2 = 2(ae^y + c) \qquad u = y'$

$(y')^2 = 2(ae^y + c)$

$\Rightarrow \quad v = e^y$

$v' = e^y y' \quad y' = \frac{v'}{e^y} \Rightarrow$

$\Rightarrow (y')^2 = 2(ae^y + c)$

$$\frac{(v')^2}{v^2} = 2(a \cdot v + c)$$

$F(x, v, v') = 0$

$$\frac{(v')^2}{v^2} = 2(a \cdot v + c) \Rightarrow \begin{cases} v' = -\sqrt{2}v\sqrt{a \cdot v + c} \\ v' = \sqrt{2}v\sqrt{a \cdot v + c} \end{cases}$$

I $\quad v' = -\sqrt{2}v\sqrt{a \cdot v + c}$

$v'(x) = \frac{dv}{dx} \qquad \frac{dv}{dx} = -\sqrt{2}v\sqrt{av + c}$

$dv = -\sqrt{2}v\sqrt{av + c} \cdot dx$

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$$\frac{dv}{v\sqrt{a\cdot v+c}}=-\sqrt{2}\,dx$$

$$M_{(v)}dv=N_{(x)}\,dx$$

$$\int\frac{1}{v\sqrt{av+c}}\,dv=\int-\sqrt{2}\,dx$$

$$\frac{\ln\left(\frac{\sqrt{av+c}-\sqrt{c}}{\sqrt{av+c}+\sqrt{c}}\right)}{\sqrt{c}}=c_1-\sqrt{2}x$$

$$\frac{\ln(\sqrt{ae^y+c}-\sqrt{c})}{\sqrt{c}}-\frac{\ln(\sqrt{ae^y+c}+\sqrt{c})}{\sqrt{c}}=$$

$$=c_1-\sqrt{2}x \qquad v=e^y$$

$$\frac{\ln(\sqrt{ae^y+c}-\sqrt{c})}{\sqrt{c}}-\frac{\ln(\sqrt{ae^y+c}+\sqrt{c})}{\sqrt{c}}=$$

$$=c_1-2x \qquad v'=\sqrt{2}\,v\sqrt{av+c}\Rightarrow$$

$$v'(x) = \frac{dv}{dx} \qquad \frac{dv}{dx} = \sqrt{2}\, v \sqrt{av+c}$$

$$= c_1 - \sqrt{2}x$$

II $\quad v' = \sqrt{2}v\sqrt{av+c}$

$$v'(x) = \frac{dv}{dx} \qquad \frac{dv}{dx} = \sqrt{2}v\sqrt{av+c} \,/\cdot dx$$

$$dv = \sqrt{2}\, v \sqrt{a \cdot v + c}\, dx$$

$$\frac{dv}{v = \sqrt{av+c}} = \sqrt{2}\, dx$$

$$M\, v(dv) = N_{(x)}\, dx$$

$$\int \frac{1}{v\sqrt{av+c}}\, dv = \int \sqrt{2}\, dx$$

$$\frac{\ln\left(\frac{\sqrt{av+c} - \sqrt{c}}{\sqrt{av+c} + \sqrt{c}}\right)}{\sqrt{c}} = \sqrt{2}x + c_2$$

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$$\frac{\ln\left(\sqrt{ae^{y}+c}-\sqrt{c}\right)}{\sqrt{c}}-\frac{\ln\left(\sqrt{ae^{y}+c}+\sqrt{c}\right)}{\sqrt{c}}=$$

$$=\sqrt{2}x+C_2.$$

Javob:

$$\frac{\ln\left(\sqrt{ae^{y}+c}-\sqrt{c}\right)}{\sqrt{c}}-\frac{\ln\left(\sqrt{ae^{y}+c}+\sqrt{c}\right)}{\sqrt{c}}=C_1-\sqrt{2c}$$

$$\frac{\ln\left(\sqrt{ae^{y}+c}-\sqrt{c}\right)}{\sqrt{c}}-\frac{\ln\left(\sqrt{ae^{y}+c}+\sqrt{c}\right)}{\sqrt{c}}=\sqrt{2}x+C_2$$