Mechatronics2

Positive Position Feedback (PPF)

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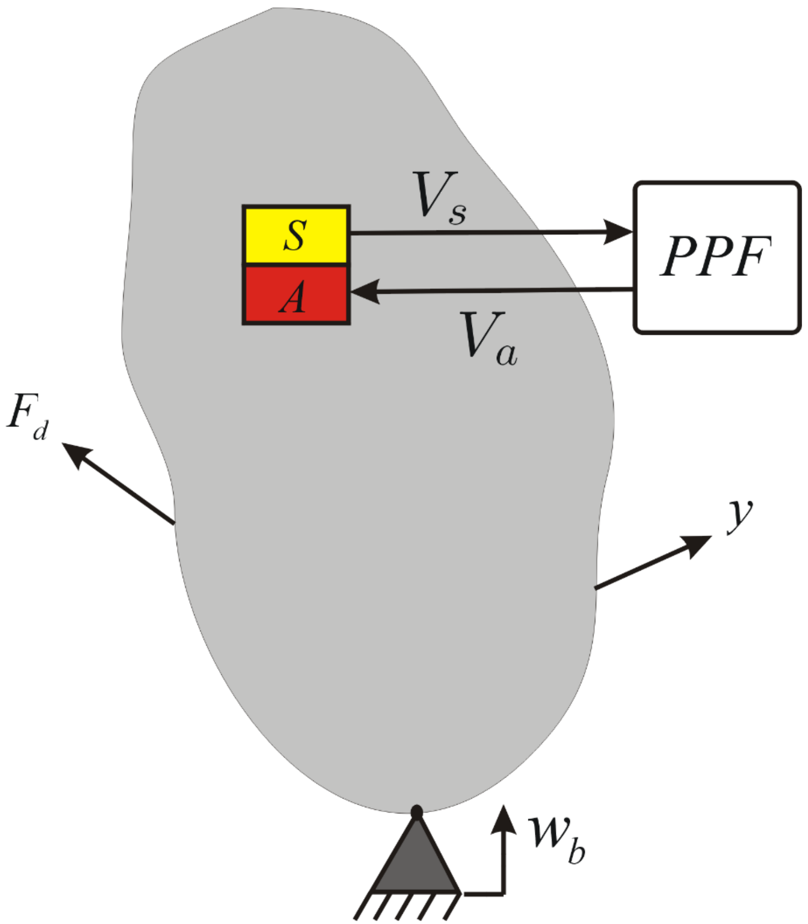
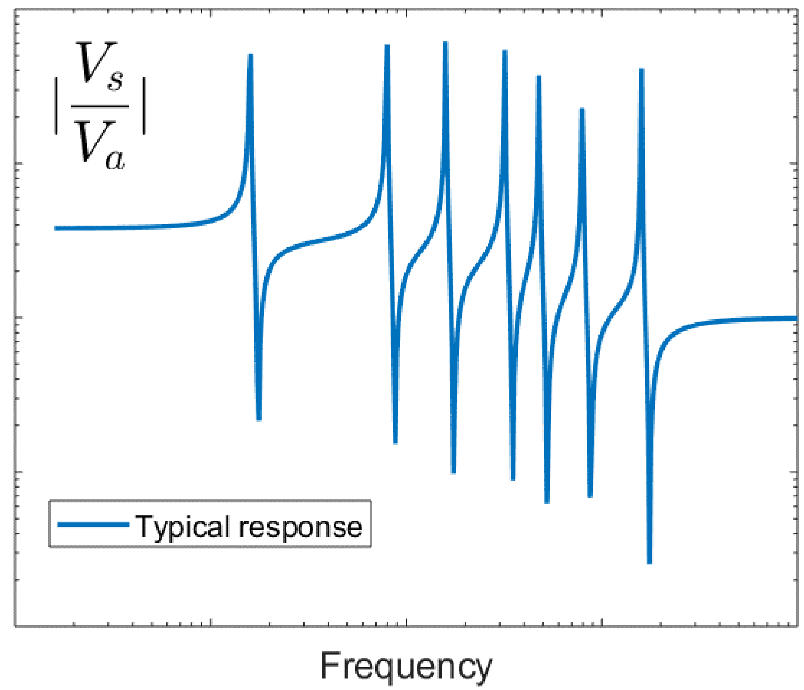
Content

1. Theory
2. Exercise

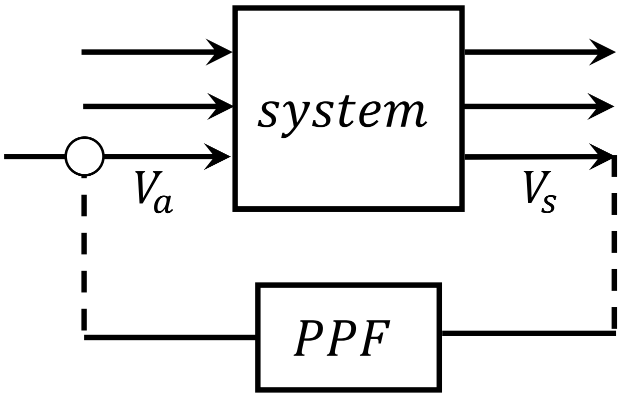
1. Theory

Primary system



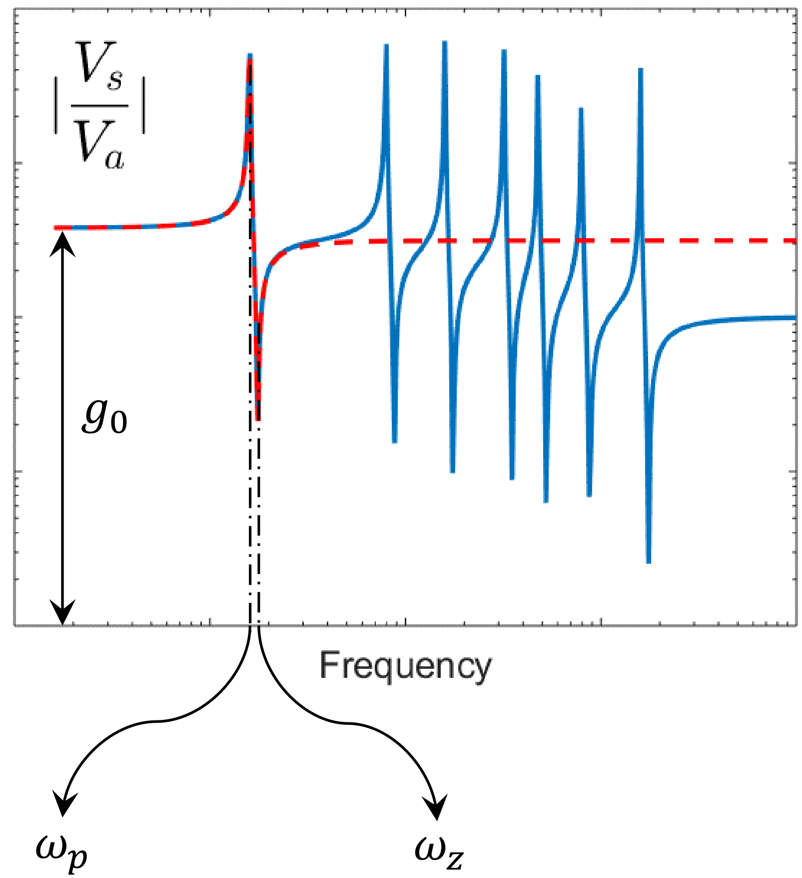


Primary system

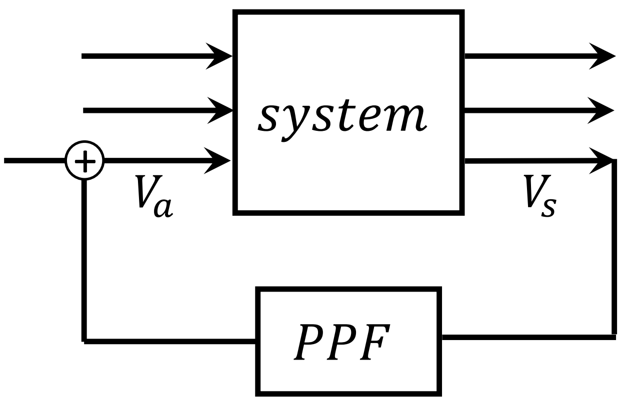


$$G = \frac{V_s}{V_a} = g_0 \frac{\frac{s^2}{\omega_z^2} + 1}{\frac{s^2}{\omega_p^2} + 1}$$

$$PPF = \frac{g_f \omega_f^2}{s^2 + 2\xi_f \omega_f s + \omega_f^2}$$



Feedback analysis



$$G = \frac{V_s}{V_a} = g_0 \frac{\frac{s^2}{\omega_z^2} + 1}{\frac{s^2}{\omega_p^2} + 1}$$

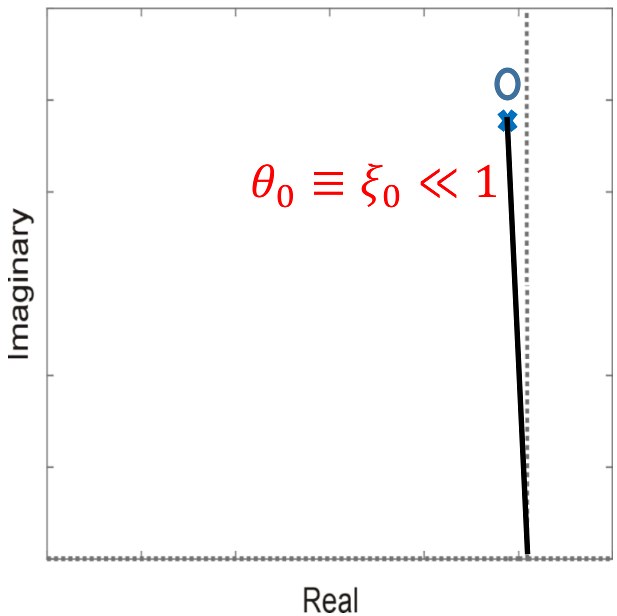
$$PPF = \frac{g_f \omega_f^2}{s^2 + 2\xi_f \omega_f s + \omega_f^2}$$

$$Loop\ Gain: \quad -G \times PPF$$

$$Sensitivity: \quad \frac{1}{1 - G \times PPF}$$

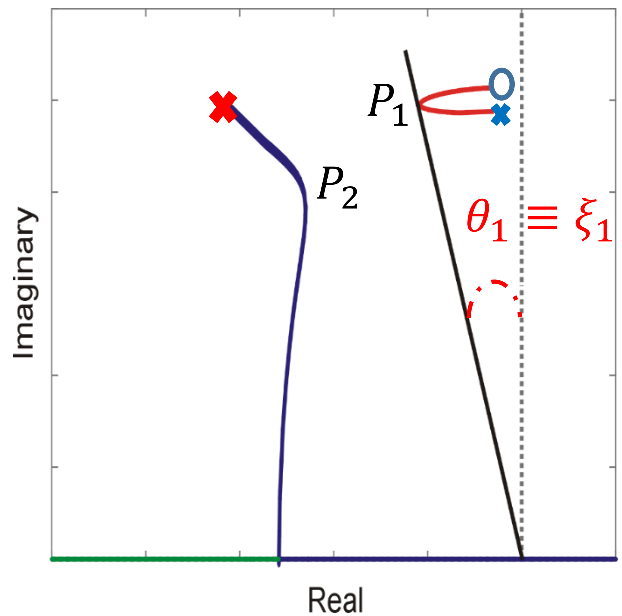
$$Closed\ loop: \quad \frac{G}{1 - G \times PPF}$$

How does PPF work?



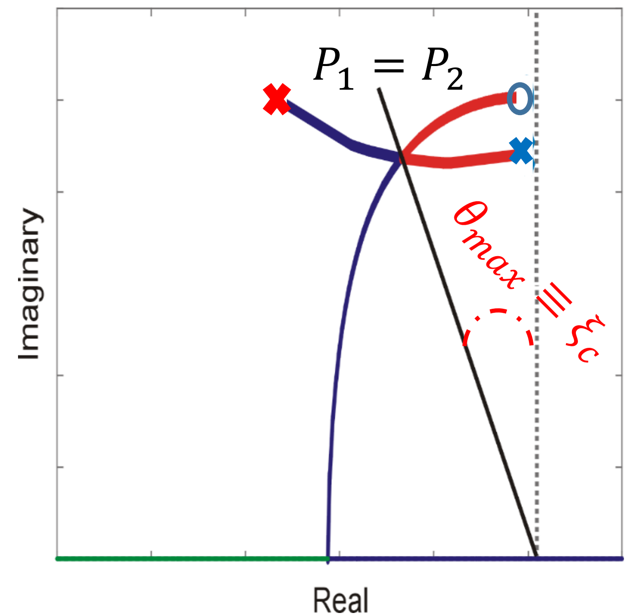
No Control

2 ***<u>different</u>*** poles in C.L.



PPF compensator

2 ***<u>merged</u>*** poles in C.L.

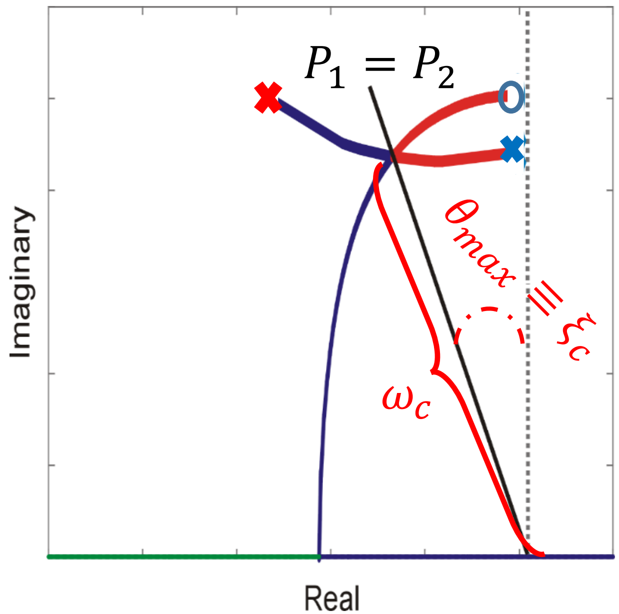


Optimal PPF control

Mathematical model

$$\text{Characteristic Eq. from general case}$$
$$\rightarrow \left(s^2 + 2\zeta_f\omega_f s + \omega_f^2\right)\left(s^2 + \omega_p^2\right) - g_0 g_f\left(s^2 + \omega_z^2\right)$$
$$= s^4 + 2\zeta_f\omega_f s^3 + \left(\omega_p^2 + \omega_f^2 - g_0 g_f\right)s^2$$
$$+ 2\zeta_f\omega_f\omega_p^2 s + \left(\omega_f^2\omega_p^2 - g_0 g_f\omega_z^2\right)$$

$$\text{Characteristic Eq. from merged poles}$$
$$\rightarrow (s^2 + 2\zeta_c\omega_c s + \omega_c^2)^2$$
$$= s^4 + 4\zeta_c\omega_c s^3 + (4\zeta_c^2\omega_c^2 + 2\omega_c^2)s^2 + 4\zeta_c\omega_c^3 s + \omega_c^4$$



Nondimensional variables $\longrightarrow$ $\alpha = \dfrac{\omega_f}{\omega_p}$, $\beta = \dfrac{g_f}{g0}$, $\gamma = \dfrac{\omega_z}{\omega_p}$

Mathematical Model

$$I\ and\ II \begin{cases} Eq.a: \quad 4\zeta_c\omega_c = 2\zeta_f\alpha\omega_p \\ Eq.b: \quad (4\zeta_c^2+2)\omega_c^2 = (\alpha^2+1)\omega_p^2 - \dfrac{\beta\alpha^2}{\gamma^2}g_0^2\omega_p^2 \\ Eq.c: \quad 4\zeta_c\omega_c^3 = 2\zeta_f\alpha\omega_p^3 \\ Eq.d: \quad \omega_c^4 = \alpha^2\omega_p^4 - \beta\alpha^2 g_0^2\omega_p^4 \end{cases}$$

$$\rightarrow \begin{cases} \dfrac{\omega_c}{\omega_p} = \dfrac{\alpha\zeta_p}{2\zeta_c} \\ \left(\dfrac{\omega_c}{\omega_p}\right)^3 = \dfrac{\alpha\zeta_p}{2\zeta_c} \end{cases}$$

$$\boxed{\rightarrow \omega_c = \omega_p}$$

The resonance frequency of the closed-loop system is the same as the resonance frequency of the primary system.

Mathematical model

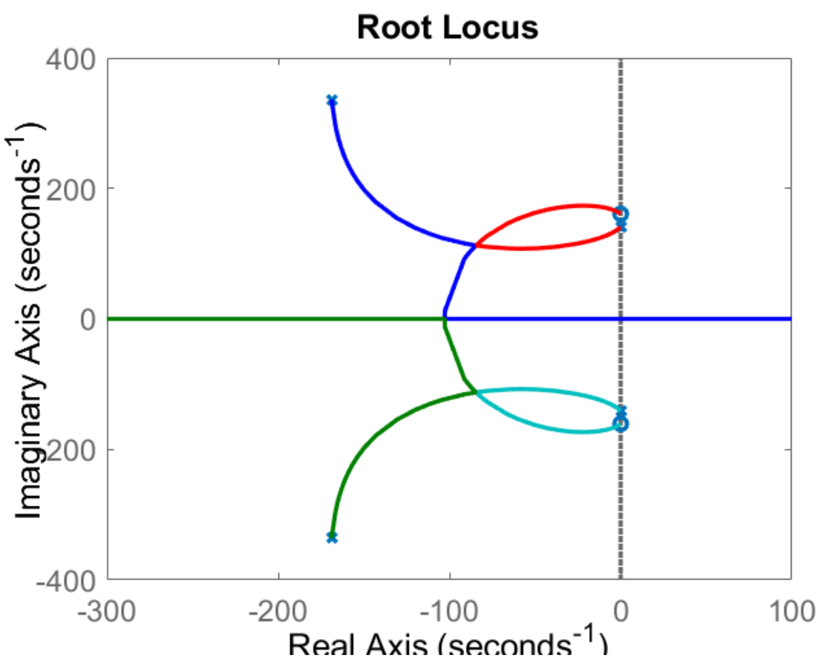
$$Simplified \begin{cases} Eq.a: \quad 4\zeta_c = 2\zeta_f \alpha \\[2ex] Eq.b: \quad 4\zeta_c^2 + 1 = \alpha^2 - \dfrac{\beta \alpha^2}{\gamma^2} g_0^2 \\[2ex] Eq.d: \quad 1 = \alpha^2 - \beta \alpha^2 g_0^2 \end{cases}$$

4 Unknowns
3 Equations

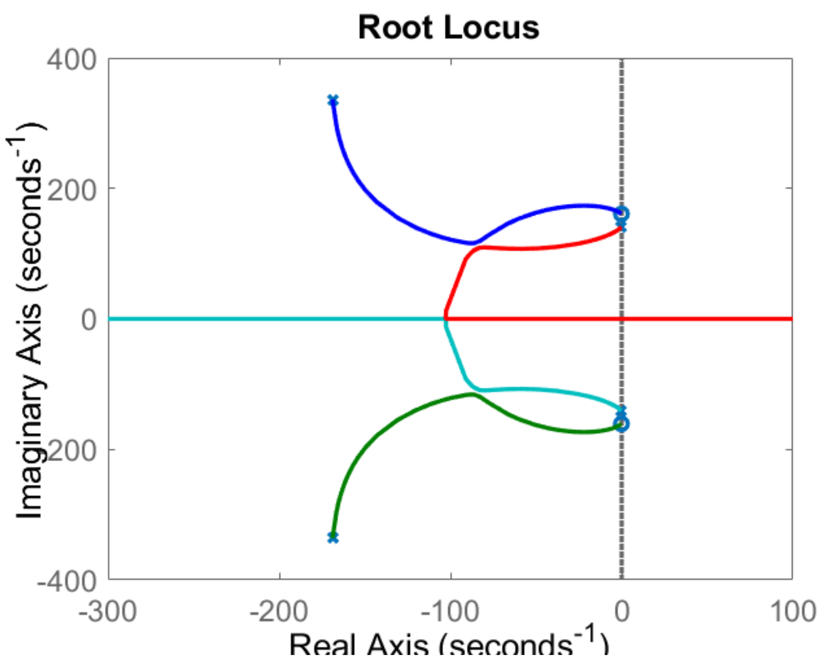
Designing a PPF control which realizes equal poles for each value of the target damping

$$\alpha = \sqrt{\frac{4\gamma^2 \xi_c^2}{\gamma^2 - 1} + 1} \qquad \xi_f = 2\xi_c \sqrt{\frac{\gamma^2 - 1}{4\gamma^2 \xi_c^2 + \gamma^2 - 1}} \qquad \beta = \frac{1}{g_0^2} \frac{4\gamma^2 \xi_c^2}{4\gamma^2 \xi_c^2 + \gamma^2 - 1}$$

lightly damped structure



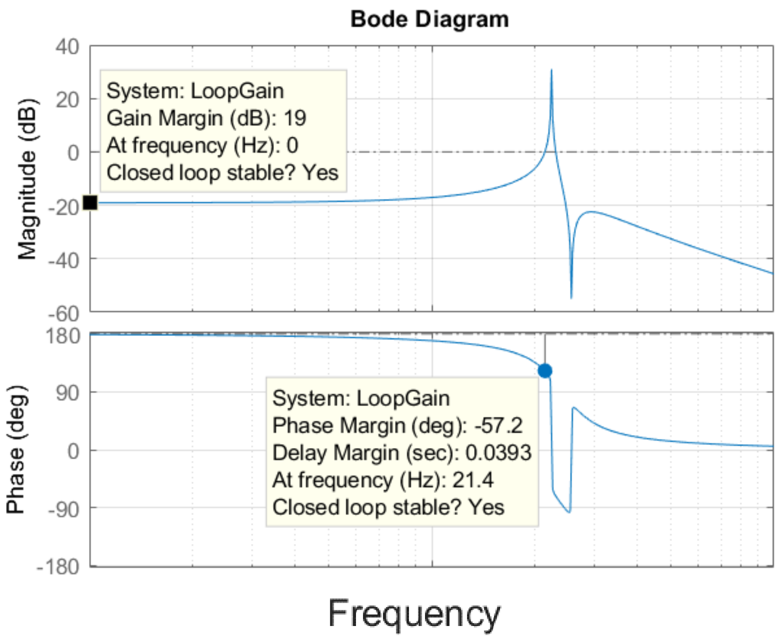
Undamped primary system



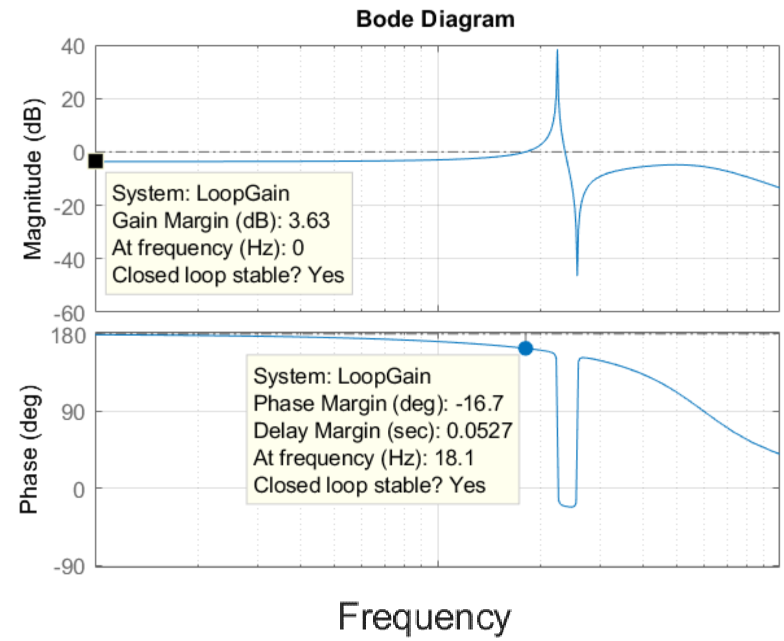
Lightly damped primary system

Stability analysis

Low value of the target damping = 0.1

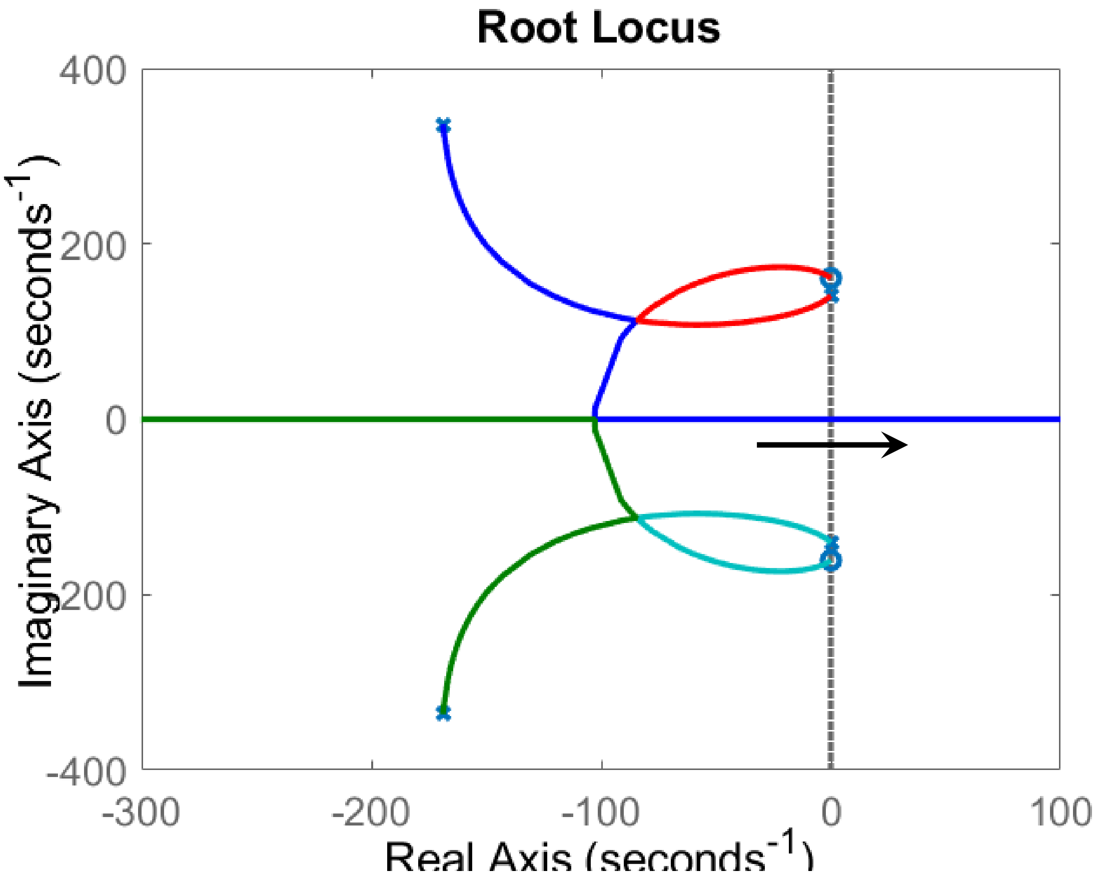


High value of the target damping = 0.6



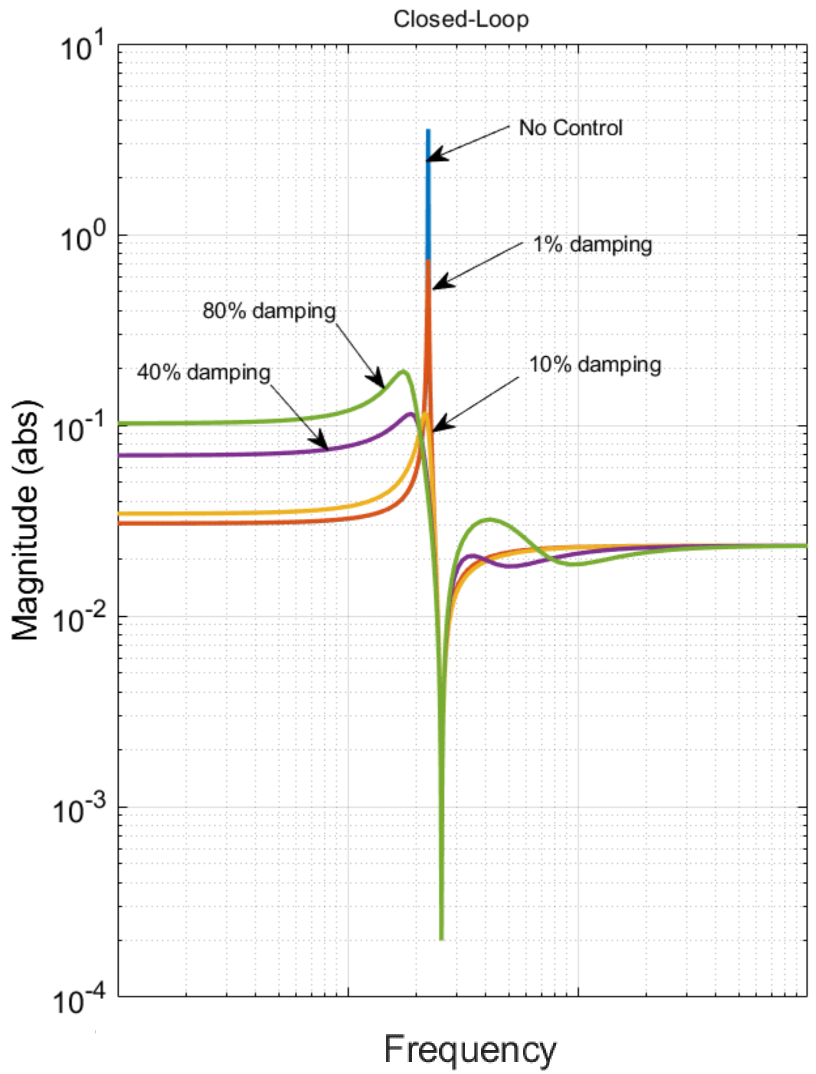
- The closed-loop system is stable because the phase is always bounded between 180 and -180.
- One trade-off in the system is that the higher value of the target damping, the lower value of the phase margin and gain margin are.

Stability analysis



★ By increasing the value of the feedback gain, one pole of the closed-loop system crosses the imaginary axis and makes the system unstable.

Optimal PPF



The other trade-off in the system is that the more damping is added to the system, the more amplification of the static response the closed-loop system has.

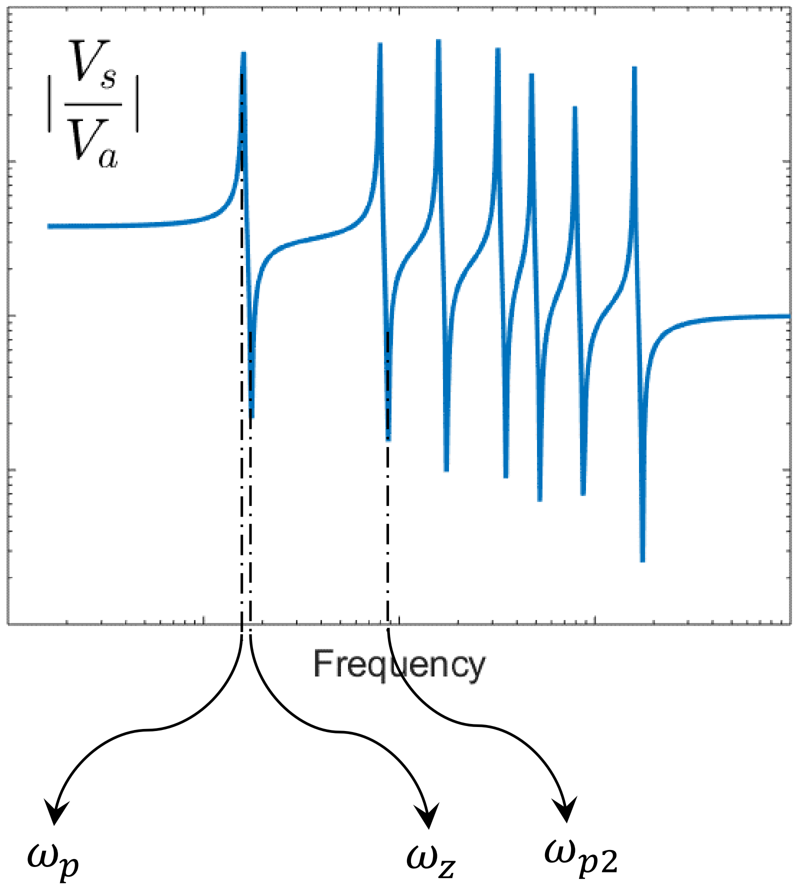
Limitation

$$\alpha = \sqrt{\frac{4\gamma^2\xi_c^2}{\gamma^2 - 1} + 1}$$

★

The more damping is added to the system, the farther the pole of the PPF is placed.

$$\boxed{\omega_f = \alpha\omega_p < \omega_{p2}}$$



Conclusions

- Maximum damping happens when the two loops in Root-Locus are intersecting
- For each value of closed-loop damping, there is one controller which causes merged poles of closed-loop and subsequently maximum damping
- More damping requires higher control frequency! This results in some problems
  - The amplification at low frequency
  - Coupling with the next modes (for continuous structure) The control frequency should be tuned far enough from the next resonance frequency
- Although the formula has been developed for undamped primary system, they can be used for lightly damped structure as well

2. Exercise

How the code works

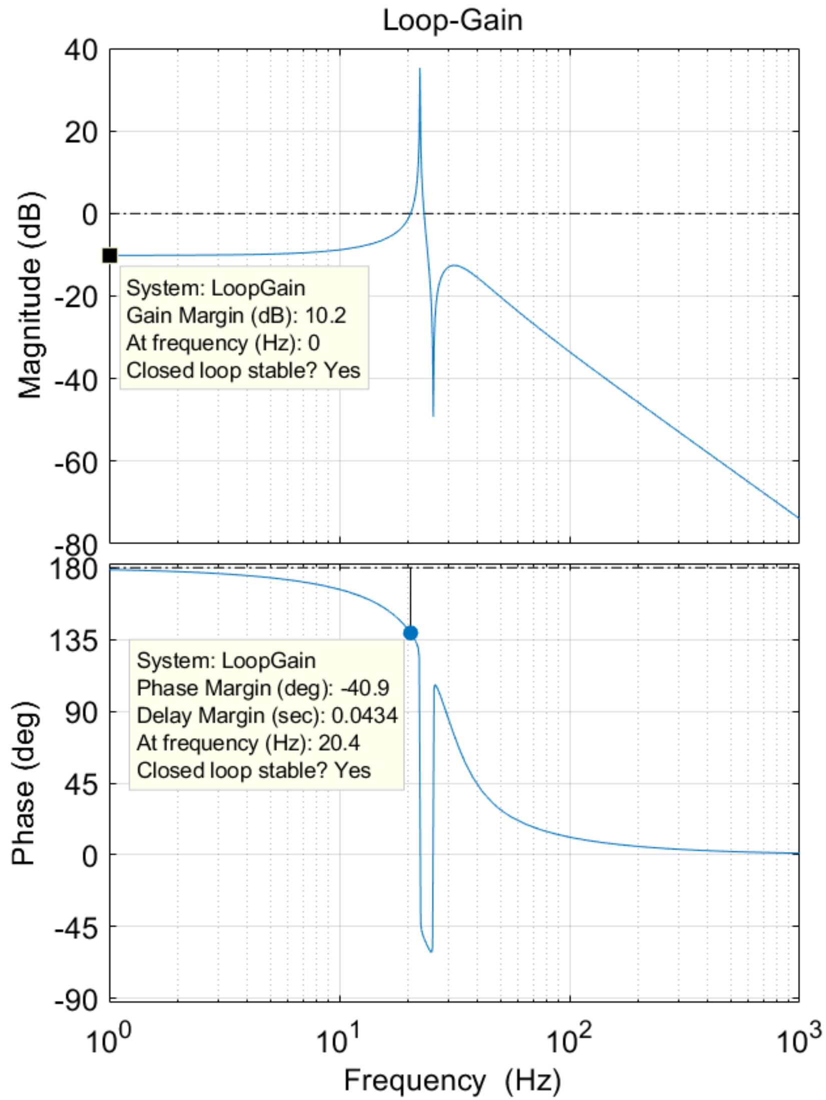
```
clear all
clc

global Wp Wz g0 G

s = tf('s');

Wp = 22.4*2*pi;
Wz = 25.6*2*pi;
g0 = 0.0305;

G = g0*(s^2/Wz^2+2*0.001/Wz*s+1)/(s^2/Wp^2+2*0.001/Wp*s+1);

PPF = MAXDPPF(0.2)
```

Frequency of the pole of the primary system

Frequency of the zero of the primary system

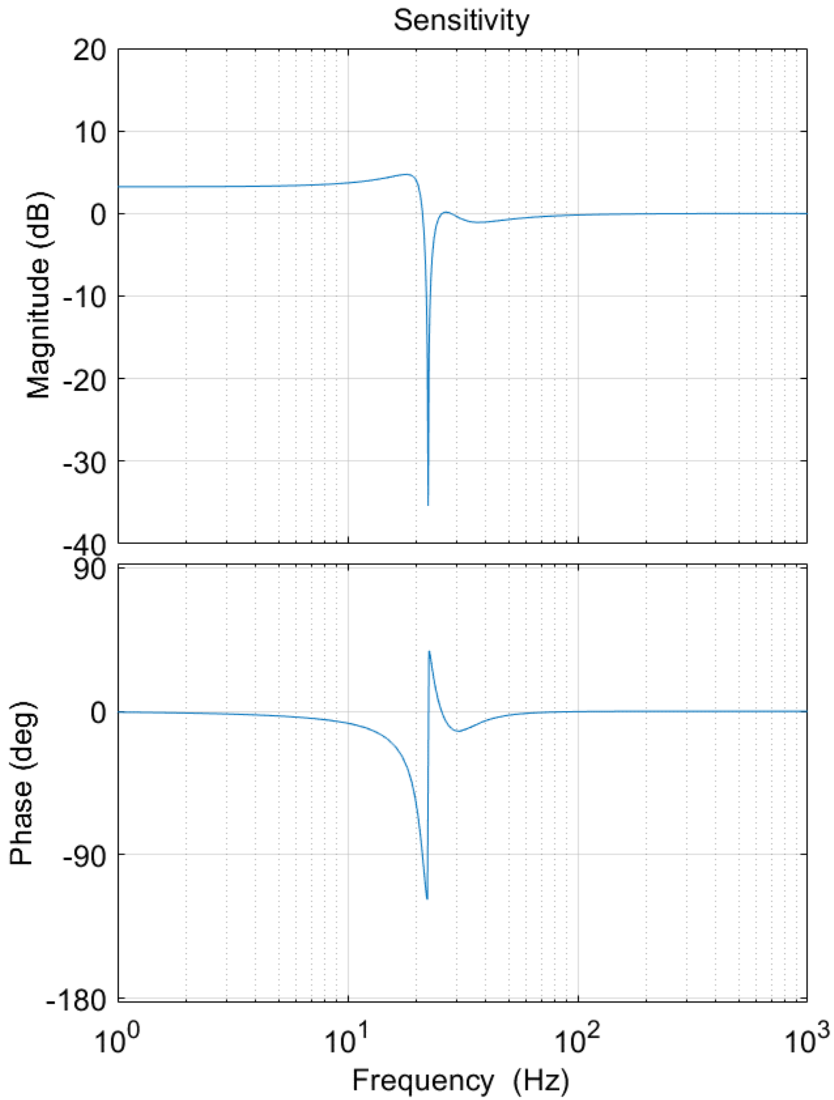
Constant gain of the primary system

Loop-Gain

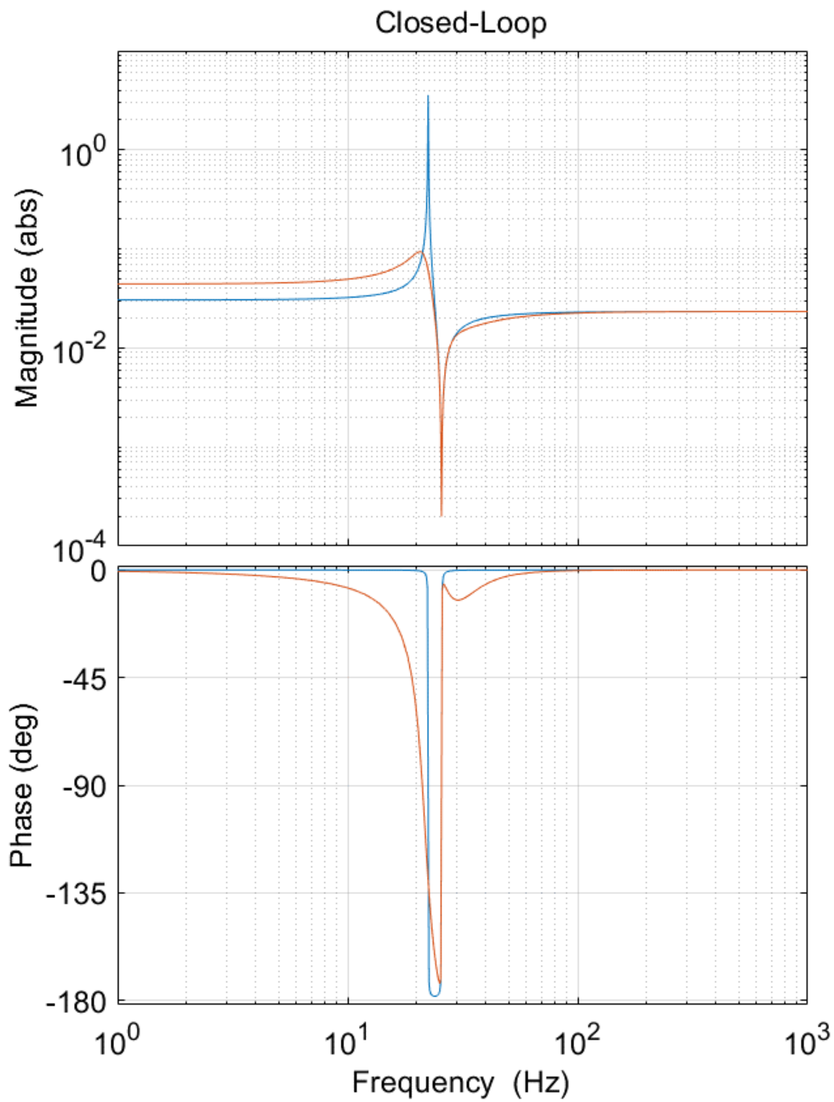


Loop-Gain
Magnitude (dB)
40
20
0
-20
-40
-60
-80
System: LoopGain
Gain Margin (dB): 10.2
At frequency (Hz): 0
Closed loop stable? Yes
Phase (deg)
180
135
90
45
0
-45
-90
System: LoopGain
Phase Margin (deg): -40.9
Delay Margin (sec): 0.0434
At frequency (Hz): 20.4
Closed loop stable? Yes
10^0
10^1
10^2
10^3
Frequency (Hz)

Sensitivity



Closed-Loop



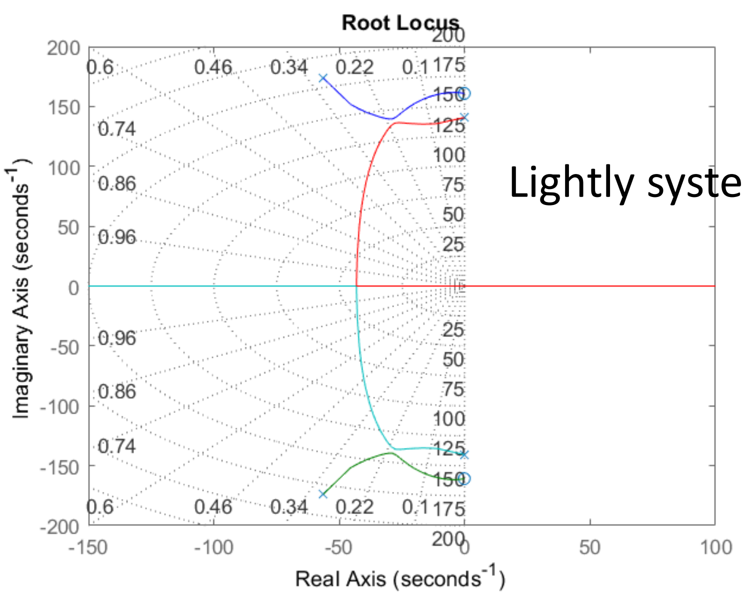
Closed-Loop
Magnitude (abs)
10^0
10^{-2}
10^{-4}
Phase (deg)
0
-45
-90
-135
-180
10^0
10^1
10^2
10^3
Frequency (Hz)

Checking the compatibility

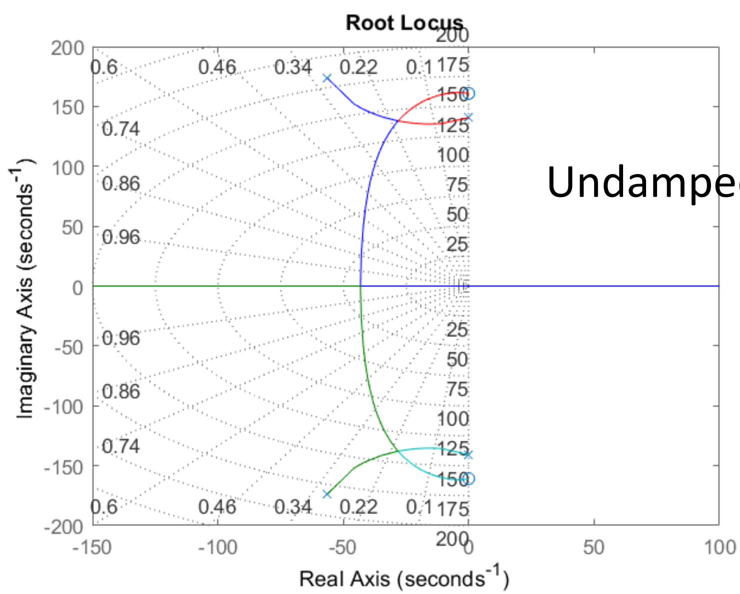
figure; rlocus(-G*PPF)

H = g0*(s^2/Wz^2+1)/(s^2/Wp^2+1); % Remove the damping ratio

figure; rlocus(-H*PPF)



Lightly system



Undamped system