Algorithm 0.1: $(First\ Order\ Sweeping\ Method\ for\ AQM\ )$

Choose initial control trajectory : $u_0, \ldots, u_N, \delta u_0, \ldots, \delta u_N, u_{+0} = u_0, \ldots, u_{+N} = u_N$, $\mathcal{J}_{-} = \infty, \gamma$, *Iter* $= 1, \triangle_0$, TOL, *MaxIter*, *and* $\rho \in (0,1)$.

Do:

Initialisation: $x_{+0}, \mathcal{J}_{-}$ *are given*

for $i = 0, 1, \ldots, N-1$

Forward Sweep:

Evaluate:

if *Iter* > 1 : $u_{+i} = u_i + \triangle \delta u_i$

$x_{+i+1} = f_i(x_{+i}, u_{+i})$

$\mathcal{J}_{+} = \phi(x_{+N})$

if $\mathcal{J}_{+} < \mathcal{J}_{-}$: $\mathcal{J}_{-} = \mathcal{J}_{+}$, $\triangle = \triangle_0$

else: $\triangle = \frac{\triangle}{\rho}$, *goto* **Forward Sweep**

Adjoint Initialisation $\bar{x}_N = \phi^{\top}_{x_+}(x_{+N}), \gamma_N = 0$

for $i = N-1, N-2, \ldots, 0$

Backward sweep: $x_{+i} = x_i,\ u_{+i} = u_i,$

Evaluate:

$\bar{x}_i = f^{\top}_{x,i} \bar{x}_{i+1},\ \bar{u}_i = -H_{x,i}(x_i, u_i, \bar{x}_i)$

$\gamma_i = \parallel \bar{u}_i \parallel^2,\ \delta u_i = -\bar{u}_i$

$\gamma = \gamma_N$

if $\parallel \gamma \parallel <$ TOL: *stop*

Iter $=$ *Iter* $+ 1$

while: $Iter \leq MaxIter$