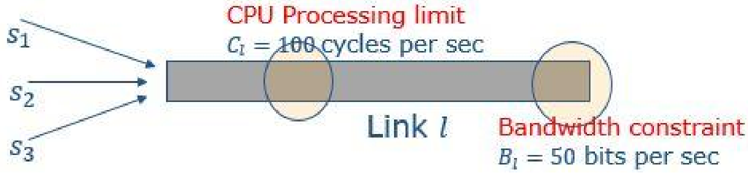
0.2.6 Lecture 6 - 140402 - Flow Control

Paper: *Song Chong et al., "Flow Control with Processing Constraint," IEEE Communications Letters, 2005*

Problem:

Figure 1: Single Link Model



$$\begin{aligned} \underset{r_s(t)}{\text{maximize}} \quad & \sum_{s=1}^{3} log(r_s) \\ \text{subject to} \quad & \sum w_s r_s \leq C_l; \\ & \sum r_s \leq B_l; \\ & r_s \geq 0. \end{aligned}$$

Questions:

- Is it a convex optimization problem? Why?
- Find the optimal solution r_s^* for each source
- Show the algorithm to find optimal solution. Is it distributed and why?

Answer 6:

- Objective function is summation of logarithm and it is concave function, maximize of concave is similar to minimize of convex function. The feasible set is intersection of convex sets: half-spaces and also a convex set. So this problem is a convex optimization problem.
- Find the optimal solution r^* for each source. Lagrange Dual function of the primal problem is

$$g(r_s, \lambda_C, \lambda_B) = sup\{\sum_{s=1}^{3} log(r_s) + \lambda_C(C_l - \sum w_s r_s) + \lambda_B(B_l - \sum r_s)\} \tag{43}$$

where λ_C, λ_B are price parameters for constraints of CPU Processing and Bandwidth Limitation, respectively.

$$g(r_s, \lambda_C, \lambda_B) = sup\{\sum_{s=1}^{3}[log(r_s) - r_s(\lambda_C w_s + \lambda_B)] + [\lambda_C C_l + \lambda_B B_l]\} \quad (44)$$

Solve the above equation by taking gradient respect to r_s

$$\frac{\partial L}{\partial r_s} = \frac{1}{r_s^*} - \lambda_C w_s - \lambda_B = 0 \rightarrow r_s^* = \frac{1}{\lambda_C w_s + \lambda_B} \quad (45)$$

Substitute r_s^*, we obtain the dual problem

$$\begin{aligned} \underset{\lambda_C, \lambda_B}{\text{minimize}} \quad & \sum_{s=1}^{3} log(r_s^*) - 3 + [\lambda_C C_l + \lambda_B B_l] \\ \text{subject to} \quad & \lambda_C, \lambda_B \geq 0. \end{aligned}$$

The dual problem is an unconstrained convex optimization problem which can be solved by gradient method.

$$\begin{aligned} \frac{\partial g}{\partial \lambda_C} &= C_l - \sum_{s=1}^{3} r_s^* w_s \\ \frac{\partial g}{\partial \lambda_B} &= B_l - \sum_{s=1}^{3} r_s^*. \end{aligned} \quad (46)$$

- Algorithm: firstly, initialize link prices for CPU processing and Bandwidth: $\lambda_C(0)$ and $\lambda_B(0)$

  Link algorithm: update link prices

  $\lambda_C(t+1) = \lambda_C(t) + \Delta(C_l - \sum_{s=1}^{3} r_s(t) w_s)$

  $\lambda_B(t+1) = \lambda_B(t) + \Delta(B_l - \sum_{s=1}^{3} w_s)$

  Source algorithm: update source sending rates

  $r_s(t+1) = \frac{1}{\lambda_C(t+1) w_s + \lambda_B(t+1)}$

This is a distributed algorithm because it uses local information at link or source, without any global knowledge. More specificly, to update sending rates, we only need feedback current price from link; to update link prices, we need know the current sender rate.