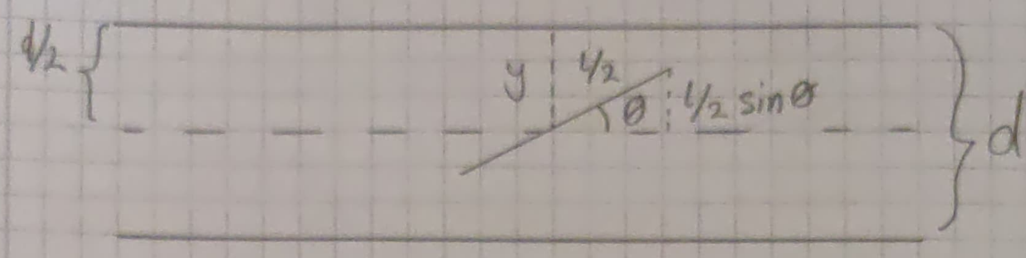
Basics of MC ex1

P01. Buffon's Needle:



We use y to tell the distance of the needles middle part from the closest line and θ to tell the angle of the needle with a line parallel to the edges.

From this we get that $0 \le y \le d/2$ and $0 \le \theta \le \pi/2$

All needle tosses should give random uniforman values for both y and θ. With this information we can get the probability density functions for both variables:

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{for } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

$$\hookrightarrow f(y) = \begin{cases} \frac{2}{d}, & \text{for } 0 \le y \le d/2 \\ 0, & \text{otherwise} \end{cases} \text{ and } f(\theta) = \begin{cases} \frac{2}{\pi}, & \text{for } 0 \le \theta \le \pi/2 \\ 0, & \text{otherwise} \end{cases}$$

Since the variables are independent, we can combine ~~them~~ their probabilities with multiplication.

$$f(y,\theta) = \begin{cases} \frac{4}{d\pi}, & y \in [0, \frac{d}{2}] \text{ and } \theta \in [0, \frac{\pi}{2}] \\ 0, & \text{otherwise} \end{cases}$$

Condition for the needle to cross a line is that $\frac{l}{2} \sin\theta > y$

PDF gives us probability by integrating it with appropriate limits.

Probability for the needle to cross the line:

$$P = \int_0^{\pi/2} \int_0^{l/2 \sin\theta} 4/d\pi \, dy \, d\theta = \int_0^{\pi/2} \frac{2l}{d\pi} \sin\theta = \frac{2l}{d\pi}$$