Basics of MC, ex3

p.7. Inversion method:

$$f(x) = \frac{10}{\pi 2} \cdot \frac{4}{x^2+4}$$

- CDF for interval $[-10, 10]$:

$$F(x) = \int_{-10}^{x} f(t)\,dt = \frac{10}{\pi}\left[\operatorname{atan}\left(\frac{x}{2}\right) + \operatorname{atan}(5)\right]$$

- Since $F(10) \neq 1$, we divide $f(t)$ by $F(10)$ to get the normalized $F(x)$:

$$F(x) = \int_{-10}^{x} \frac{f(t)}{\frac{20}{\pi}\cdot \operatorname{atan}(5)}\,dt = \frac{\operatorname{atan}\left(\frac{x}{2}\right) + \operatorname{atan}(5)}{2\operatorname{atan}(5)}$$

- Solve for x to get the formula for the RN's:

$$u = \frac{\operatorname{atan}\left(\frac{x}{2}\right) + \operatorname{atan}(5)}{2\operatorname{atan}(5)}$$

$$\Rightarrow \boxed{x = -2\tan\left[\operatorname{atan}(5) - 2\cdot u\cdot \operatorname{atan}(5)\right]}$$