CREDIT SUISSE

Lecture 4: Stock Process Modeling

Assignment

1) Write a short .fsx script implementing simulation of single stock prices as values of Geometric Brownian Motion with given drift r and volatility σ (assume both are constant).
   - Input parameters should be as follows:
     `count <N>; steps <n>; price <`S_0`>; drift <`r`>; vol <`σ`>; years <t>; seed <seed>`
     Where `N` is number of paths, `n` is number of generated prices in time horizon `[0, t]` (`t` in years; assume every year has 365 days), S_0 is an initial price of the stock and `seed` should correspond to your pseudo-random numbers generator. Assume equal time offsets, and stock prices being continuous – trading 24h a day on weekends and holidays.
   - Your application should output a CSV file called `output.txt` with one row for each path and the following columns for each row – Final Stock Price, Realized Volatility.
   - Use Box-Muller transform based Gaussian random numbers generator. Check that the numbers are normally distributed.
2) Test your implementation:
   - Check what happens when the volatility is set to 0. Why?
   - Generate N (N>=1000) paths of n time steps (n>=250) starting from the same stock price value, using the same drift and volatility and ending on the same date (in one year's time, for simplicity: t=1). In a spreadsheet, plot the histogram of the final stock prices and compare these with log normal density (S_0 being the initial stock price, r and σ being GBM parameters):

$$\frac{1}{x\sqrt{2\pi}\sigma} e^{-\frac{\left(\ln x - \ln S_0 - r + \frac{\sigma^2}{2}\right)^2}{2\sigma^2}}$$

   - For each path calculate the realized volatility, check if the average matches input volatility.
3) Take some real-life stock daily prices' series, calculate log-returns (in a spreadsheet) and plot the histogram. Plot appropriate normal distribution density function on the same chart (estimate mean and std deviation matching historical returns).
4) Extend the Pricer app for taking into account the time value of money (i.e. discounting) when computing the value of Payment. Assume constant continuously compounded interest rates with intensity r, which should be passed to the app via Market data section.

Deliverables

- Code files with an implementation of 1) and 4).
- Short report (in .pdf, .doc or .docx format) containing summary of the exercises 2) and 3), including charts