付録 4.B　積分など

m，n を整数とする．このとき以下が成り立つ．

$$\sin(m\pi) = 0,\ \ \cos(m\pi) = \begin{cases} 1, & m：偶数, \\ -1, & m：奇数. \end{cases}$$

$$\cos^2(mx) = \frac{1+\cos(2mx)}{2}. \qquad \sin^2(mx) = \frac{1-\cos(2mx)}{2}.$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\Big[\cos\Big\{(m+n)x\Big\} + \cos\Big\{(m-n)x\Big\}\Big].$$

$$\sin(mx)\sin(nx) = -\frac{1}{2}\Big[\cos\Big\{(m+n)x\Big\} - \cos\Big\{(m-n)x\Big\}\Big].$$

$$\sin(mx)\cos(nx) = \frac{1}{2}\Big[\sin\Big\{(m+n)x\Big\} + \sin\Big\{(m-n)x\Big\}\Big].$$

$$\int_{-\pi}^{\pi}\cos(mx)\,dx = \left[\frac{1}{m}\sin(mx)\right]_{-\pi}^{\pi} = \frac{1}{m}(\sin(m\pi) - \sin(-m\pi)) = 0\ \ (m \neq 0).$$

$$\int_{-\pi}^{\pi}\sin(mx)\,dx = \left[-\frac{1}{m}\cos(mx)\right]_{-\pi}^{\pi} = -\frac{1}{m}(\cos(m\pi) - \cos(-m\pi)) = 0\ \ (m \neq 0).$$

$$\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,dx = \int_{-\pi}^{\pi}\frac{1}{2}\Big[\cos\Big\{(m+n)x\Big\} + \cos\Big\{(m-n)x\Big\}\Big]dx = 0\ \ (m \neq n).$$

$$\int_{-\pi}^{\pi}\cos(mx)\sin(nx)\,dx = \int_{-\pi}^{\pi}\frac{1}{2}\Big[\sin\Big\{(m+n)x\Big\} + \sin\Big\{(m-n)x\Big\}\Big]dx = 0.$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax} + C \quad （C\ は積分定数）.$$

$$\int e^{jk\Omega_0 x}dx = \frac{1}{jk\Omega_0}e^{jk\Omega_0 x} + C = -\frac{j}{k\Omega_0}e^{jk\Omega_0 x} + C \quad （C\ は積分定数）.$$

部分積分：

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx.$$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C \quad （C\ は積分定数）.$$

$$\int x\sin x\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C \quad （C\ は積分定数）.$$