

Задачи на производные

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Правила дифференцирования и таблица производных

ToDo

DER-1

$$y = \arctan \frac{\operatorname{tg} x}{\sqrt{2}}$$

Решение:

$$\begin{aligned} y' &= (\operatorname{tg} x)' \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{1 + \frac{\operatorname{tg}^2 x}{2}} = \left[\operatorname{tg}^2 x = \frac{1}{\cos^2 x} - 1 \right] \\ &= \frac{\sqrt{2}}{\cos^2 x \cdot \left(2 + \frac{1}{\cos^2 x} - 1 \right)} = \frac{\sqrt{2}}{\cos^2 x + 1} \end{aligned}$$

Ответ:

$$\frac{\sqrt{2}}{\cos^2 x + 1}$$

DER-2

$$y = \left(\frac{1}{3} \right)^{\arcsin x^2}$$

Решение:

$$\begin{aligned} y' &= \left(\frac{1}{3} \right)^{\arcsin x^2} \cdot \ln \frac{1}{3} \cdot (\arcsin x^2)' = \\ &= \left(\frac{1}{3} \right)^{\arcsin x^2} \cdot \ln \frac{1}{3} \cdot \frac{2x}{\sqrt{1-x^4}} \end{aligned}$$

Ответ:

$$\left(\frac{1}{3} \right)^{\arcsin x^2} \cdot \frac{2x \cdot \ln \frac{1}{3}}{\sqrt{1-x^4}}$$

DER-3

$$y = \frac{2+x^2}{\sqrt{1+x^2}}$$

Решение:

$$\begin{aligned} y' &= \frac{2x\sqrt{1+x^2} - (2+x^2)\frac{x}{\sqrt{1+x^2}}}{1+x^2} = \\ &= \frac{2x+2x^3 - (2x+x^3)}{(1+x^2)^{\frac{3}{2}}} = \frac{x^3}{(1+x^2)^{\frac{3}{2}}} \end{aligned}$$

Ответ:

$$\frac{x^3}{(1+x^2)^{\frac{3}{2}}}$$

DER-4

$$y = (x+1)^{\frac{1}{x}}$$

Решение:

$$\begin{aligned} \ln y &= \ln((x+1)^{\frac{1}{x}}) = \frac{1}{x} \ln(x+1) \\ (\ln y)' &= \frac{y'}{y} = -\frac{1}{x^2} \ln(x+1) + \frac{1}{x(x+1)} \\ y' &= \frac{x - (x+1) \ln(x+1)}{x^2} \cdot (x+1)^{\frac{1-x}{x}} \end{aligned}$$

Ответ:

$$\frac{x - (x+1) \ln(x+1)}{x^2} \cdot (x+1)^{\frac{1-x}{x}}$$

DER-5

$$y = \frac{\sin(2x) + 1}{\sin(x) - \cos(x)}$$

Решение:

$$\begin{aligned}
y' &= \frac{(\sin(2x) + 1)'(\sin(x) - \cos(x)) - (\sin(2x) + 1)(\sin(x) - \cos(x))'}{(\sin(x) - \cos(x))^2} = \\
&= \frac{2 \cos(2x)(\sin(x) - \cos(x)) - (\sin(2x) + 1)(\cos(x) + \sin(x))}{\sin^2(x) - 2 \sin(x) \cos(x) + \cos^2(x)} = \\
&= \frac{2(2 \cos^2(x) - 1)(\sin(x) - \cos(x)) - (2 \sin(x) \cos(x) + 1)(\cos(x) + \sin(x))}{1 - \sin(2x)} = \\
&= \frac{4 \cos^2(x) \sin(x) - 4 \cos^3(x) - 2 \sin(x) + 2 \cos(x) - 2 \sin(x) \cos^2(x) - 2 \sin^2(x) \cos(x) - \cos(x) - \sin(x)}{1 - \sin(2x)} = \\
&= \frac{2 \cos^2(x) \sin(x) - 4 \cos^3(x) - 3 \sin(x) + \cos(x) - 2 \sin^2(x) \cos(x)}{1 - \sin(2x)} = \\
&= \frac{2 \cos^2(x) \sin(x) - 4 \cos(x)(1 - \sin^2(x)) - 3 \sin(x) + \cos(x) - 2 \sin^2(x) \cos(x)}{1 - \sin(2x)} = \\
&= \frac{2 \cos^2(x) \sin(x) - 3 \cos(x) - 3 \sin(x) + 2 \sin^2(x) \cos(x)}{1 - \sin(2x)} = \\
&= \frac{(\cos(x) + \sin(x))(\sin(2x) - 3)}{1 - \sin(2x)}
\end{aligned}$$

Ответ:

$$\frac{(\cos(x) + \sin(x))(\sin(2x) - 3)}{1 - \sin(2x)}.$$

DER-6

$$y = e^{-x} \frac{x - 2}{(1 - x)^2}$$

Решение:

$$\begin{aligned}
y' &= \frac{(e^{-x}(x - 2))'(1 - x)^2 - (e^{-x}(x - 2))(2(1 - x))(-1)}{(1 - x)^4} = \\
&= \frac{(-e^{-x}(x - 2) + e^{-x})(1 - x) + 2e^{-x}(x - 2)}{(1 - x)^3} = \frac{(-xe^{-x} + 3e^{-x})(1 - x) + 2e^{-x}(x - 2)}{(1 - x)^3} = \\
&= \frac{x^2e^{-x} - 2xe^{-x} - e^{-x}}{(1 - x)^3} = (e^{-x}) \frac{x^2 - 2x - 1}{(1 - x)^3}.
\end{aligned}$$

Ответ: $(e^{-x}) \frac{x^2 - 2x - 1}{(1 - x)^3}.$

DER-7

$$y = e^{2x}(3 \cos(3x) - 2 \sin(3x))$$

Решение

$$\begin{aligned}(e^{2x}(3 \cos(3x) - 2 \sin(3x)))' &= 2e^{2x}(3 \cos(3x) - 2 \sin(3x)) + e^{2x}(3 \cos(3x) - 2 \sin(3x))' = \\ &= 2e^{2x}(3 \cos(3x) - 2 \sin(3x)) + e^{2x}(-9 \sin(3x) - 6 \cos(3x)) = -13 \sin(3x)e^{2x}\end{aligned}$$

Ответ: $-13 \sin(3x)e^{2x}$.

DER-8

$$y = (e^x + e^{-x})^{\cos 2x}.$$

Решение.

$$\begin{aligned}y' &= \left(e^{\cos 2x \ln(e^x + e^{-x})} \right)' = e^{\cos 2x \ln(e^x + e^{-x})} \cdot (\cos 2x \ln(e^x + e^{-x}))' = \\ &= (e^x + e^{-x})^{\cos 2x} \cdot ((\cos 2x)' \ln e^x + e^{-x} + \cos 2x \cdot (\ln(e^x + e^{-x})))' = \\ &= (e^x + e^{-x})^{\cos 2x} \cdot \left(-2 \sin 2x \ln(e^x + e^{-x}) + \cos 2x \frac{e^x - e^{-x}}{e^x + e^{-x}} \right).\end{aligned}$$

Ответ: $(e^x + e^{-x})^{\cos 2x} \cdot \left(-2 \sin 2x \ln(e^x + e^{-x}) + \cos 2x \frac{e^x - e^{-x}}{e^x + e^{-x}} \right)$.

DER-10

$$y = 2^{\operatorname{arccctg} \sqrt{x^2+1}}$$

Решение

$$\begin{aligned}y' &= \left(2^{\operatorname{arccctg} \sqrt{x^2+1}} \right)' = 2^{\operatorname{arccctg} \sqrt{x^2+1}} \ln 2 \left(\operatorname{arccctg} \sqrt{x^2+1} \right)' = \\ &= -\frac{2^{\operatorname{arccctg} \sqrt{x^2+1}} \ln 2 \cdot (\sqrt{x^2+1})'}{(\sqrt{x^2+1})^2 + 1} = -\frac{\ln 2 \cdot 2^{\operatorname{arccctg} \sqrt{x^2+1}} \left(\frac{1}{2} \right) \cdot 2x}{(x^2 + x + 1) (\sqrt{x^2+1})} = \\ &= 2^{\operatorname{arccctg} \sqrt{1+x^2}} \ln 2 \cdot \frac{-x}{(2+x^2)\sqrt{1+x^2}}\end{aligned}$$

Ответ: $2^{\operatorname{arccctg} \sqrt{1+x^2}} \ln 2 \cdot \frac{-x}{(2+x^2)\sqrt{1+x^2}}$

DER-11

$$y = x \ln (x + \sqrt{x^2 + 1})$$

Решение

$$\begin{aligned} y' &= \left(x \ln (x + \sqrt{x^2 + 1}) \right)' = \\ &= x' \ln (x + \sqrt{x^2 + 1}) + x \left(\ln (x + \sqrt{x^2 + 1}) \right)' = \\ &= \ln (x + \sqrt{x^2 + 1}) + \frac{x}{x + \sqrt{x^2 + 1}} \cdot \left(x + \sqrt{x^2 + 1} \right)' = \\ &= \ln (x + \sqrt{x^2 + 1}) + \frac{x}{x + \sqrt{x^2 + 1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x \right) = \\ &= \ln (x + \sqrt{x^2 + 1}) + \frac{x}{x + \sqrt{x^2 + 1}} \cdot \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}} = \\ &= \ln (x + \sqrt{x^2 + 1}) + \frac{x}{\sqrt{x^2 + 1}} \end{aligned}$$

Ответ: $\ln (x + \sqrt{x^2 + 1}) + \frac{x}{\sqrt{x^2 + 1}}$

DER-12

$$y = \arcsin \frac{x + 2}{2x + 2}, \quad x > 0$$

Решение

$$\begin{aligned} y' &= \frac{1}{\sqrt{1 - \frac{(x+2)^2}{(2x+2)^2}}} \cdot \frac{2x + 2 - 2(x + 2)}{(2x + 2)^2} = \\ &= \frac{2x + 2}{\sqrt{(2x + 2)^2 - (x + 2)^2}} \cdot \frac{-2}{(2x + 2)^2} = -\frac{1}{(x + 1)\sqrt{3x^2 + 4x}} \end{aligned}$$

Ответ: $-\frac{1}{(x+1)\sqrt{3x^2+4x}}$

DER-13

$$y = \ln \operatorname{tg} x + \frac{1}{2} \operatorname{ctg} 2x$$

Решение

$$\begin{aligned} y' &= \frac{1}{\operatorname{tg} x} \frac{1}{\cos^2 x} + \frac{1}{2} \left(-\frac{2}{\sin^2 2x} \right) = \frac{\cos x}{\sin x} \frac{1}{\cos^2 x} - \frac{1}{\sin^2 2x} = \\ &= \frac{1}{\sin x \cos x} - \frac{1}{\sin^2 x} = \frac{2}{\sin 2x} - \frac{1}{\sin^2 2x} \end{aligned}$$

Ответ: $\frac{2}{\sin 2x} - \frac{1}{\sin^2 2x}$

DER-14

$$y = \log_2 \frac{\cos x + x \sin x}{\sin x - x \cos x}$$

Решение

$$y' = \frac{1}{\ln 2} \left(\ln \frac{\cos x + x \sin x}{\sin x - x \cos x} \right)' =$$

$$\frac{1}{\ln 2} \frac{\cos x + x \sin x}{\sin x - x \cos x} \frac{(-\sin x + \sin x + x \cos x)(\sin x - x \cos x) - (\cos x + x \sin x)(\cos x - \cos x + x \sin x)}{(\sin x - x \cos x)^2} =$$

$$\frac{1}{\ln 2} \frac{\cos x + x \sin x}{\sin x - x \cos x} \frac{x \cos x (\sin x - x \cos x) - (\cos x + x \sin x) x \sin x}{(\sin x - x \cos x)^2} =$$

$$\frac{1}{\ln 2} \frac{\cos x + x \sin x}{\sin x - x \cos x} \frac{-x^2 \cos^2 x - x^2 \sin^2 x}{(\sin x - x \cos x)^2} =$$

$$\frac{1}{\ln 2} \frac{-x^2}{(\cos x + x \sin x)(\sin x - x \cos x)} =$$

$$\frac{-x^2}{\ln 2} \frac{1}{\cos x \sin x - x \cos^2 x + x \sin^2 x - x^2 \sin x \cos x} =$$

$$\frac{-x^2}{\ln 2} \frac{2}{(1 - x^2) \sin 2x - 2x \cos 2x}$$

Ответ: $\frac{-x^2}{\ln 2} \frac{2}{(1 - x^2) \sin 2x - 2x \cos 2x}$

DER-15

$$y = x (\cos(2 \ln x) + 2 \sin(2 \ln x)) .$$

$$y' = \cos(2 \ln x) + 2 \sin(2 \ln x) + x \left(-\sin(2 \ln x) \frac{2}{x} + 2 \cos(2 \ln x) \frac{2}{x} \right) = 5 \cos(2 \ln x).$$

Ответ: $5 \cos(2 \ln x).$

DER-16

$$y = x \arccos x - \sqrt{1 - x^2}$$

Решение

$$y' = \arccos x - \frac{x}{\sqrt{1 - x^2}} - \frac{-2x}{2\sqrt{1 - x^2}} = \arccos x$$

Ответ: $\arccos x.$

DER-17

$$y = x^2 \sqrt[3]{x^2 + 4x + 1}$$

Решение

$$y' = 2x \sqrt[3]{x^2 + 4x + 1} + \frac{x^2(2x + 4)}{3 \sqrt[3]{(x^2 + 4x + 1)^2}} = \frac{6x(x^2 + 4x + 1) + 2x^3 + 4x^2}{3 \sqrt[3]{(x^2 + 4x + 1)^2}} = \frac{8x^3 + 28x^2 + 6x}{3 \sqrt[3]{(x^2 + 4x + 1)^2}}$$

Ответ: $\frac{8x^3 + 28x^2 + 6x}{3 \sqrt[3]{(x^2 + 4x + 1)^2}}$.

DER-18

$$y = \operatorname{arctg} x + \frac{1}{3} \operatorname{arctg} x^3$$

Решение

$$\begin{aligned} y' &= \frac{1}{1+x^2} + \frac{1}{3} \frac{3x^2}{1+x^6} = \frac{1}{1+x^2} + \frac{x^2}{1+x^6} = \frac{1+x^6+x^2(1+x^2)}{(1+x^2)(1+x^6)} = \\ &= \frac{1+x^2+x^4+x^6}{(1+x^2)(1+x^6)} = \frac{(1+x^2)+x^4(1+x^2)}{(1+x^2)(1+x^6)} = \frac{1+x^4}{1+x^6} \end{aligned}$$

Ответ: $\frac{1+x^4}{1+x^6}$.

DER-19

$$y = \frac{\sin x}{\cos^3 x}$$

Решение

$$\begin{aligned} y' &= \frac{\cos x \cdot \cos^3 x - \sin x \cdot (-3 \cos^2 x \sin x)}{\cos^6 x} = \frac{\cos^4 x + 3 \sin^2 x \cos^2 x}{\cos^6 x} = \\ &= \frac{\cos^2 x + 3 \sin^2 x}{\cos^4 x} = \frac{1 + 2 \sin^2 x}{\cos^4 x} = \frac{1 + (1 - \cos 2x)}{\cos^4 x} = \frac{2 - \cos 2x}{\cos^4 x} \end{aligned}$$

Ответ: $\frac{2 - \cos 2x}{\cos^4 x}$.

DER-20

$$y = \arccos \frac{1-x^3}{1+x^3}$$

Решение

$$\begin{aligned} y' &= -\frac{1}{\sqrt{1 - \left(\frac{1-x^3}{1+x^3}\right)^2}} \cdot \frac{-3x^2(1+x^3) - 3x^2(1-x^3)}{(1+x^3)^2} = -\frac{1+x^3}{\sqrt{(1+x^3)^2 - (1-x^3)^2}} \cdot \frac{-3x^2 - 3x^5 - 3x^2 + 3x^5}{(1+x^3)^2} \\ &= \frac{1}{\sqrt{4x^3}} \frac{6x^2}{1+x^3} = \frac{3\sqrt{x}}{1+x^3} \end{aligned}$$

Ответ: $\frac{3\sqrt{x}}{1+x^3}$.