$dS_t^i = S_t^i \mu^i dt + S_t^i \sigma^i dW_t^i \quad i=1,2, \quad dW_t^1 dW_t^2 = 0$

$V_T = \mathbb{1}(S_T^1 > S_T^2)$

1 путь

р.н.м.: $dS_t^i = S_t^i \sigma^i dW_t^i \quad \mathbb{P}$

$V_T = \mathbb{1}\left(S_T^1 > S_T^2\right) = \mathbb{1}\left(\frac{S_T^1}{S_T^2} > 1\right)$

$V_0^? = \mathbb{E}^{\mathbb{P}}\left[\mathbb{1}(S_T^1 > S_T^2)\right]$

$\tilde{\sigma} = \sqrt{(\sigma^1)^2 + (\sigma^2)^2}$

рынок: $S_t^1, S_t^2, \dots S_t^n$ — рисковые

B_t — безрисковый

март. мера:

$\forall \; i = \overline{1,n} \quad \frac{S_t^i}{B_t}$ — мартингал

2 путь

B_t — безрисковый актив

$V_t = g(X_t)$

$V_0 = \mathbb{E}^{\mathbb{Q}}\left[\frac{g(X_T)}{B_T} \Big| \mathcal{F}_0\right]$

$dB_t = B_t r dt, \quad B_0 = 1$

$V_0 = e^{-rT} \mathbb{E}^{\mathbb{Q}}\left[g(X_T) | \mathcal{F}_0\right]$

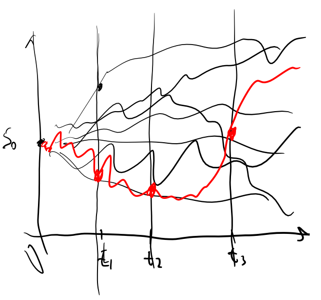
S_t^2 — Numeraire $\quad \frac{S_t^1}{S_t^2}$ — мартингал | получили меру $\mathbb{Q}$

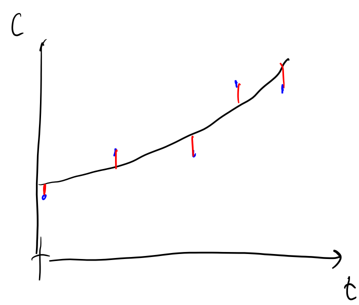
$B_t \equiv 1 \quad \frac{B_t}{S_t^2}$ — мартингал | $\frac{1}{S_t^2}$ — март. — S_t^2

$V_T = \mathbb{1}\left(\frac{S_T^1}{S_T^2} > 1\right)$

$V_0^{S^2} = \mathbb{E}^{\mathbb{Q}}\left[\frac{\mathbb{1}(S_T^1/S_T^2 > 1)}{S_T^2}\right]$

$V_0^{\$} = V_0^{S^2} \cdot S_0^2 = S_0^2 \, \mathbb{E}^{\mathbb{Q}}\left[\frac{\mathbb{1}(S_T^1/S_T^2 > 1)}{S_T^2}\right]$





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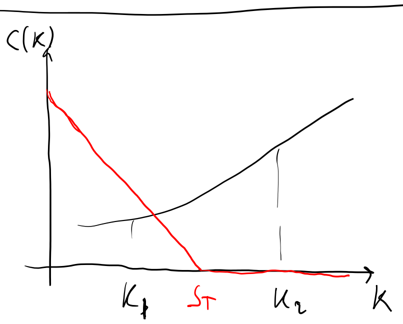
① 1)



$C(T,K)$

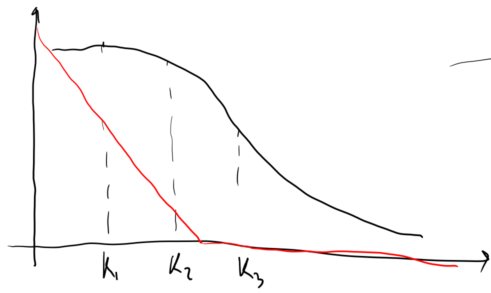
$\exists$ в t нарушение

$C(t,K_1) < C(t,K_2), \quad K_1 < K_2$

1) купить 1 продать 2

2) в конце $C(T,K_2) = (S_T - K_2)^+ \overset{п.н.}{\le} (S_T - K_1)^+$ | арбитраж

2) $C(K)$ выпукла



$\exists K_2 = \frac{1}{2}(K_1 + K_3)$

1) портфель: $C(t,K_1) - 2C(t,K_2) + C(t,K_3) \overset{\text{н.ч.}}{<} 0$ | арбитраж

2) в конце: $C(T,K_1) - 2C(T,K_2) + C(T,K_3) \overset{\text{н.ч.}}{\geq} 0$

$PnL \overset{\text{н.ч.}}{>} 0 \Rightarrow$ арбитраж

$C(T,K) = (S_T - K)^+$

⑧ $F_\xi(x) = \mathbb{P}(\xi \leq x)$ ACHTUNG!

$F_\xi(\xi) = \mathbb{P}(\xi \leq \xi) = 1$

$F_\xi(x) = \mathbb{P}(\{\omega : \xi(\omega) \leq x\})$

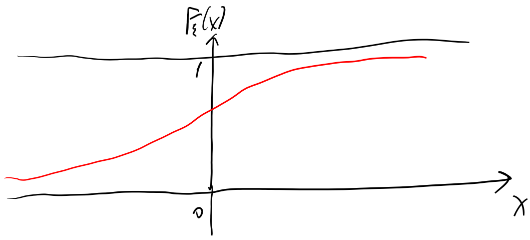
$\eta : F_\xi(\xi)$

$\mathbb{P}(\eta \leq y) = \mathbb{P}(F_\xi(\xi) \leq y) = \mathbb{P}(\xi \leq F_\xi^{-1}(y)) = F_\xi(F_\xi^{-1}(y)) = y, \quad y \in [0,1]$

$\mathbb{P}(\eta \leq y) = y, \ y \in [0,1] \Rightarrow \eta \sim U[0,1]$.

$\eta = F_\xi(\xi) \sim U \Rightarrow F_\xi^{-1}(\eta) \overset{d}{=} \xi$

1)

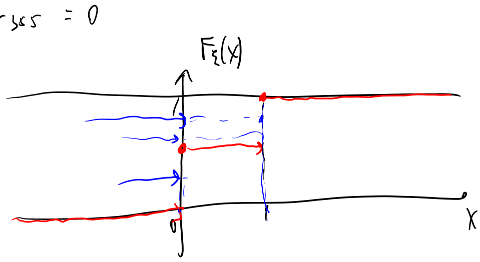


всё OK

$\exists \ \forall x \in \mathbb{R} \quad F_\xi(x) \in (0,1)$

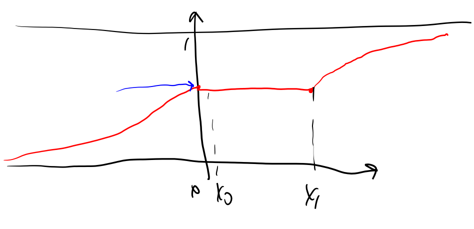
$\eta_{3 \varepsilon 5} = 0$

2)



все OK

3)



любое из x_0, x_1 не важно, т.к. вер. 0.

⑥ $X_m = \sum_{n=0}^{N-1} e^{-\frac{2\pi i}{N} nm} x_n$

$X_{m+N} = \sum_{n=0}^{N-1} e^{-\frac{2\pi i}{N} n(m+N)} x_n = \sum_{n=0}^{N-1} e^{-\frac{2\pi i}{N} nm} \cdot \underbrace{e^{-\frac{2\pi i}{N} nN}}_{= e^{-2\pi i n} = 1} x_n = \sum_{n=0}^{N-1} e^{-\frac{2\pi i}{N} nm} x_n = X_m$