ODE 1 $y'' + y = 4\cos x + (x^2+1)e^x$

хар ур-е: $\lambda^2 + 1 = 0$

$\lambda = \pm i$

$y_0 = x(A\cos x + B\sin x) + (Cx^2 + Dx + E)e^x$

$y = C_1\cos x + C_2\sin x + x(A\cos x + B\sin x) + (Cx^2 + Dx + E)e^x$

ODE 2 $y''' - y'' - 6y' = e^{3x} - \sin 3x$

$\lambda^3 - \lambda^2 - 6\lambda = 0$

$\lambda = 0$

$\lambda = 3$

$\lambda = -2$

$y_0 = Axe^{3x} + B\cos 3x + C\sin 3x$

$y = C_1e^{3x} + C_2e^{-2x} + C_3 + B\cos 3x + C\sin 3x + Axe^{3x}$

ODE 3 $y''' + y' = -4\sin x + e^{2x}\sin 4x$

$\lambda^3 + \lambda = 0$

$\lambda = 0$

$\lambda = i$

$\lambda = -i$

$y = C_1 + C_2\cos x + C_3\sin x + x(A\cos x + B\sin x) +$

$+ e^{2x}(C\cos 4x + D\sin 4x)$

ODE 4 $y^4 - 3y' = x + e^{3x}\sin x$

$\lambda^2 - 3\lambda \Rightarrow \lambda = 0, \lambda = 3$

$y = C_1 + C_2e^{3x} + x(Ax + B) + e^{3x}(C\cos x + D\sin x)$

ODE5 $y'' + 4y = x\sin 2x - x^2$

$\lambda^2 + 4 = 0$

$\lambda = \pm 2i$

$y = C_1\cos 2x + C_2 \sin 2x + x\left((Ax+B)\cos 2x + (Cx+D)\sin 2x\right) +$
$+ Ex^2 + Dx + F$

ODE6 $y''' - 4y' + 3y = xe^x + \cos 2x$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda = 1$

$\lambda = 3$

$y = C_1 e^x + C_2 e^{3x} + xe^x(Ax+B) + C\cos 2x + D\sin 2x$

ODE7 $y'' + 2y' + 5y = 2xe^{-x} - x^2\cos x$

$\lambda^2 + 2\lambda + 5 = 0$

$\lambda = -1 \pm 2i$

$y = C_1 e^{-x}\cos 2x + C_2 e^{-x}\sin 2x + (A+Bx)e^{-x} +$
$+ \left((Cx^2+Dx+E)\cos x + (Fx^2+Gx+H)\sin x\right)$

ODE8 $y'' + 2y' + 2y = e^{-x}\cos x + x^3 - 2x^2 + 10$

$\lambda^2 + 2\lambda + 2 = 0$

$\lambda = -1 \pm i$

$y = C_1 e^{-x}\cos x + C_2 e^{-x}\sin x + xe^{-x}(A\cos x + B\sin x) +$
$+ Cx^3 + Dx^2 + Ex + D$

ODE9 $y''' - 3y' + 2y = \cos 2x + x^3 e^{2x}$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda = 0$ $\lambda = 1$ $y = C_1 e^x + C_2 e^{2x} + A\cos 2x + B\sin 2x +$

$\lambda = 1$ $\lambda = 2$ $+ xe^{2x}(Cx^3 + Dx^2 + Ex + F)$

~~$y = C_1 + C_2 e^x + A\cos 2x + B\sin 2x + e^{2x}(Ax^3 + Ex^2 + Fx + G)$~~

ODE 10 $y''' + 4y'' = x - 1 + \cos 4x$

$\lambda^3 + 4\lambda = 0$

$\lambda_1 = \lambda_2 = 0$

$\lambda_3 = -4$

$y = C_1 + C_2 x + C_3 e^{-4x} + x^2(Ax+B) + C\cos 4x + D\sin 4x$

ODE 11 $y'' - 3y' = x + \cos 2x$

$\lambda^2 - 3\lambda = 0$

$\lambda = 0, \ \lambda = 3$

$y = C_1 + C_2 e^{3x} + x(Ax+B) + D\cos 2x + E\sin 2x$

ODE 12 $y'' - 8y' + 20y = 5x e^{4x}\sin 2x - 2x^2$

$\lambda^2 - 8\lambda + 20 = 0$

$\lambda = 4 \pm 2i$

$y = C_1 e^{4x}\cos 2x + C_2 e^{4x}\sin 2x + x e^{4x}((Ax+B)\cos 2x + (Cx+D)\sin 2x) + Ex^2 + Fx + G$

ODE 13 $y'' + 3y' - 4y = e^{-4x} + x e^{-x}\sin 2x$

$\lambda^2 + 3\lambda - 4 = 0$

$\lambda = 1$

$\lambda = -4$

$y = C_1 e^{x} + C_2 e^{-4x} + Axe^{-4x} + e^{-x}((Bx+C)\sin 2x + (Dx+E)\cos 2x)$

ODE 14 $y'' + y = x\sin x + e^x \cos 2x$

$\lambda^2 + 1 = 0$

$\lambda = \pm i$

$y = C_1 \cos x + C_2 \sin x + x((Ax+B)\sin x + (Cx+D)\cos x) + e^x(E\cos 2x + F\sin 2x)$

ODE15 $y'' + 4y = 2\sin 2x - 3\cos 2x + 1$

$\lambda^2 + 4 = 0$

$\lambda = \pm 2i$

$y = C_1 \cos 2x + C_2 \sin 2x + x(A\cos 2x + B\sin 2x) + C$

ODE16 $y'' + y = \sin x - 2e^{-x}$

$\lambda^2 + 1 = 0$

$\lambda = \pm i$

$y = C_1 \cos x + C_2 \sin x + x(A\sin x + B\cos x) + Ce^{-x}$

ODE17 $y'' + 9y = 2x\sin 3x + xe^{3x}$

$\lambda^2 + 9 = 0$

$\lambda = \pm 3i$

$y = C_1 \cos 3x + C_2 \sin 3x + x((Ax+B)\cos 3x + (Cx+D)\sin 3x) +$
$+ (Ex+F)e^{3x}$

ODE18 $y'' + 6y' + 10y = 3xe^{-3x} - 2e^{-3x}\cos x$

$\lambda^2 + 6\lambda + 10 = 0$

$\lambda = -3 \pm i$

$y = e^{-3x}(C_1 \cos x + C_2 \sin x) + (Ax+B)e^{-3x} +$
$+ xe^{-3x}(C\cos x + D\sin x)$

ODE19 $y'' - 9y = 3e^{3x} - \cos x$

$\lambda^2 - 9 = 0$

$\lambda = \pm 3$

$y = Ae^{3x} + Be^{-3x} + Axe^{3x} + B\cos x + D\sin x$

ODE20 $y'' - 2y' + 5y = e^x \cos 2x - x^2$

$\lambda^2 - 2\lambda + 5 = 0 \quad ; \quad \lambda = 1 \pm 2i$

$y = C_1 e^x \cos 2x + C_2 e^x \sin 2x + xe^x(A\cos 2x + B\sin 2x) + Cx^2 + Dx + E$