Seminar 7
Stochastic Volatility Models

Vega Institute

Problem 1

Prove that for any arbitrage free model:
1. $C(T,K)$ is non-increasing function of K,
2. $C(T,K)$ is convex function of K.

Problem 2

Prove that

$$\mathbb{E}S_T^{1+p} = \mathbb{E}\int_0^\infty (p+1)pK^{p-1}(S_T-K)^+\,dK.$$

Problem 3

Prove that Lee formula does not contradict SVI model given $\big(1+|\rho|\big)b \le 2$.

Problem 4

Let $c(k) = e^{\alpha k}C_T(k)$ for some $k > 0$. Let $\psi(u)$ be the Fourier transform:

$$\psi(u) = \int_{\mathbb{R}} e^{iuk}c(k)\,dk.$$

Prove that

$$C_T(k) = \frac{e^{-\alpha k}}{\pi}\int_0^\infty e^{-iuk}\psi(u)\,du.$$

Problem 5

Prove inverse DFT formula:

$$X_m = \sum_{n=0}^{N-1} e^{-\frac{2\pi i}{N}nm}x_n,$$

$$x_n = \frac{1}{N}\sum_{m=0}^{N-1} e^{\frac{2\pi i}{N}nm}X_m.$$

Problem 6

Prove its periodicity:

$$X_{m+N} = X_m.$$

Problem 7

Prove the Plancherel theorem:

$$\sum_{n=0}^{N-1} |x_n|^2 = \sum_{m=0}^{N-1} |X_m|^2$$

Problem 8

Let F be the CDF of target non-singular distribution. How can we sample from it, if we can sample from uniform distribution?