DER-13. Найти производную функции

$$y = \ln tg x + \frac{1}{2} ctg 2x$$

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$$y' = (\ln tg x)' + \frac{1}{2}(ctg 2x)' = \frac{1}{tg x}\frac{1}{\cos^2 x} + \frac{1}{2}\left(-\frac{1}{\sin^2 2x}\right)\cdot 2 =$$

$$= \frac{\cos x}{\sin x}\frac{1}{\cos^2 x} - \frac{1}{\sin^2 2x} = \frac{1}{\sin x \cos x} - \frac{1}{4\sin^2 x\cos^2 x} =$$

$$= \frac{4\sin x\cos x - 1}{4\sin^2 x\cos^2 x} = \frac{2\sin(2x) - 1}{\sin^2 2x}$$

Ответ: $\frac{4\sin x\cos x - 1}{4\sin^2 x\cos^2 x}$, или $\frac{2\sin(2x)-1}{\sin^2(2x)}$.

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DER-5 Найти производную функции

$$y = e^{-x}\frac{x-2}{(1-x)^2}$$

$$y' = (e^{-x})'\frac{x-2}{(1-x)^2} + e^{-x}\left(\frac{x-2}{(1-x)^2}\right)' = -e^{-x}\frac{x-2}{(1-x)^2} +$$

$$+ e^{-x}\frac{1\cdot(1-x)^2 + 2(1-x)(x-2)}{(1-x)^4} = e^{-x}\frac{2-x}{(1-x)^2} +$$

$$+ e^{-x}\frac{1}{(1-x)^2} + 2e^{-x}\frac{(x-2)}{(1-x)^3} = e^{-x}\left(\frac{3-x}{(1-x)^2} + 2\frac{x-2}{(1-x)^3}\right) =$$

$$= e^{-x}\left(\frac{\overset{3-4x+x^2}{(3-x)(1-x)} + 2x-4}{(1-x)^3}\right) = e^{-x}\frac{x^2-2x-1}{(1-x)^3}$$

Ответ: $y'(x) = e^{-x}\frac{x^2-2x-1}{(1-x)^3}$.

ПРОИЗВОДНАЯ

№1

$$y = x^2 \sqrt[3]{x^2+4x+1}$$

$$y' = 2x\sqrt[3]{x^2+4x+1} + \frac{x^2(2x+4)}{3\sqrt[3]{(x^2+4x+1)^2}}$$

$$y' = \frac{6x(x^2+4x+1) + x^2(2x+4)}{3\sqrt[3]{(x^2+4x+1)^2}}$$

$$y' = \frac{8x^3+28x^2+6x}{3\sqrt[3]{(x^2+4x+1)^2}}$$

№2

$$y = \frac{2+x^2}{\sqrt{1+x^2}}$$

$$y' = \frac{2x\sqrt{1+x^2} - \frac{x(2+x^2)}{\sqrt{1+x^2}}}{1+x^2}$$

$$y' = \frac{x^3}{(1+x^2)\sqrt{1+x^2}}$$

№3

$$y = \frac{\sin x}{\cos^3 x}$$

$$y' = \frac{\cos^4 x + 3\sin^2 x \cos^2 x}{\cos^6 x}$$

$$y' = \frac{\cos^2 x + 3\sin^2 x}{\cos^4 x}$$

$$y' = \frac{1+2\sin^2 x}{\cos^4 x}$$

№4

$$y = e^{-x}\frac{x-2}{(1-x)^2}$$

$$y' = e^{-x}\frac{(1-x)^2 + 2(1-x)(x-2)}{(1-x)^4} - e^{-x}\frac{x-2}{(1-x)^2}$$

$$y' = e^{-x}\,\frac{1-x+2(x-2)-(x-2)(1-x)}{(1-x)^3}$$

$$y' = e^{-x}\,\frac{+x^2-2x-1}{(1-x)^3}$$

№5

$$y = 3^{\sin^2\frac{x}{2}}$$

$$y = e^{\sin^2\frac{x}{2}\cdot\ln 3}$$

$$y' = e^{\sin^2\frac{x}{2}\ln 3}\cdot 2\cdot\frac{1}{2}\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}\cdot\ln 3$$

$$y' = 3^{\sin^2\frac{x}{2}}\cdot\frac{\ln 3}{2}\sin x$$

№6

$$y = x\ln(x+\sqrt{x^2+1})$$

$$y' = \ln(x+\sqrt{x^2+1}) + \frac{x}{x+\sqrt{x^2+1}}\left(1+\frac{x}{\sqrt{x^2+1}}\right)$$

$$y' = \ln(x+\sqrt{x^2+1}) + \frac{x}{\sqrt{x^2+1}}$$

№7

$$y = \arcsin\frac{x+2}{2x+2}$$

$$y' = \left(\frac{x+2}{2x+2}\right)'\cdot\frac{+1}{\sqrt{1-\left(\frac{x+2}{2x+2}\right)^2}}$$

$$y' = \frac{1}{2}\cdot\frac{-1}{(x+1)^2}\cdot\frac{(2x+2)}{\sqrt{3x^2+4x}}$$

$$y' = \frac{-1}{(x+1)\sqrt{3x^2+4x}}$$

№8

$$y = (e^x+e^{-x})^{\cos 2x}$$

$$y^{\cancel{\prime}} = e^{\cos 2x \cdot \ln(e^{x} + e^{-x})}$$

$$y' = (e^{x} + e^{-x})^{\cos 2x} \left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \cos 2x - 2 \sin 2x \ln(e^{x} + e^{-x}) \right)$$

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DER-13 Найти производную функции

$y = 2^{\operatorname{arcctg}\sqrt{x^2+1}}$

$(f(g(x)))' = f'(g(x))\,g'(x)$

$(a^x)' = a^x \log a$

$(\operatorname{arctg}(x))' = \dfrac{1}{x^2+1}$

Реш-е $y' = \left(2^{\operatorname{arctg}\sqrt{x^2+1}}\right)' =$

$= 2^{\operatorname{arctg}\sqrt{x^2+1}} \log 2 \cdot \left(\operatorname{arctg}\sqrt{x^2+1}\right)' =$

$= \dfrac{2^{\operatorname{arctg}\sqrt{x^2+1}} \cdot \log 2 \cdot \left(\sqrt{x^2+1}\right)'}{\left(\sqrt{x^2+1}\right)^2+1} = \dfrac{\log 2 \cdot 2^{\operatorname{arctg}\sqrt{x^2+1}} \left(\dfrac{1}{\cancel{2}}\right) \cdot \cancel{2}x}{\left(x^2+1+1\right)\left(\sqrt{x^2+1}\right)} =$

$= \boxed{\dfrac{x \cdot \log 2 \cdot 2^{\operatorname{arctg}\sqrt{x^2+1}}}{\left(x^2+2\right)\left(\sqrt{x^2+1}\right)}}$

Производная

① $y = x^2 \cdot \sqrt[3]{x^2+4x+1}$

$$y' = 2x \cdot \sqrt[3]{x^2+4x+1} + x^2 \cdot \frac{1}{3} \cdot (x^2+4x+1)^{-2/3} \cdot (2x+4) =$$

$$= 2x\sqrt[3]{(x^2+4x+1)} + \frac{2}{3}x^2(x+2)\frac{1}{\sqrt[3]{(x^2+4x+1)^2}} =$$

$$= \frac{2x(x^2+4x+1) + \frac{2}{3}x^2(x+2)}{\sqrt[3]{(x^2+4x+1)^2}} = \frac{2x\left(x^2+4x+1+\frac{x^2}{3}+\frac{2\cdot 2}{3}\right)}{\sqrt[3]{(x^2+4x+1)^2}} =$$

$$= \frac{\frac{2}{3}x(4x^2+14x+3)}{\sqrt[3]{(x+4x+1)^2}}$$

② $y = \log_2 \frac{\cos x + x\sin x}{\sin x - x\cos x}$ //wolfram: $\log_2[\ldots]$ //$(\log_2 x)' = \frac{1}{x\ln 2}$

$$y' = \frac{\sin x - x\cos x}{\cos x + x\sin x} \cdot \frac{1}{\ln 2} \cdot \frac{(\cancel{-\sin x} + \cancel{\sin x} + x\cos x)(\sin x - x\cos x) - (\cos x + x\sin x)(\cancel{\cos x} - \cancel{\cos x} + x\sin x)}{(\sin x - x\cos x)^2} =$$

$$= \frac{1}{\ln 2} \cdot \frac{x\cos x(\sin x - x\cos x) - x\sin x(\cos x + x\sin x)}{(\cos x + x\sin x)(\sin x - x\cos x)} = \frac{-x^2\cos^2 x - x^2\sin^2 x}{\ln 2(\cos x + x\sin x)(\sin x - x\cos x)} =$$

$$= \frac{-x^2}{\ln 2} \frac{(\cos^2 x + \sin^2 x)}{(\cos x + x\sin x)(\sin x - x\cos x)} = \frac{-x^2}{\ln 2(\cos x + x\sin x)(\sin x - x\cos x)}$$

③ $y = \frac{2+x^2}{\sqrt{1+x^2}}$

$$y' = \frac{2x\sqrt{1+x^2} - (2+x^2) \cdot \frac{1}{\cancel{2}} \cdot (1+x^2)^{-\frac{1}{2}} \cdot \cancel{2}x}{(1+x^2)} = \frac{2x\sqrt{1+x^2} - \frac{2+x^2}{\sqrt{1+x^2}}x}{(1+x^2)} = \frac{2x(1+x^2) - (2+x^2)x}{\sqrt{1+x^2}(1+x^2)} =$$

$$= \frac{2x + 2x^3 - 2x - x^3}{\sqrt{1+x^2}(1+x^2)} = \frac{x^3}{(1+x^2)^{3/2}}$$

④ $y = e^{-x}\frac{x-2}{(1-x)^2}$

$$y' = (e^{-x})' \cdot \frac{x-2}{(1-x)^2} + e^{-x} \cdot \frac{(x-2)'(1-x)^2 - (x-2)((1-x)^2)'}{(1-x)^4} = -e^{-x} \cdot \frac{(x-2)}{(1-x)^2} + e^{-x} \cdot \frac{(1-x)^2 - (x-2)(-2(1-x))}{(1-x)^4}$$

$$= -e^{-x} \cdot \frac{x-2}{(1-x)^2} + e^{-x}\frac{(1-x)+2(x-2)}{(1-x)^3} = e^{-x} \cdot \frac{1-x+2x-4-(x-2)(1-x)}{(1-x)^3} =$$

$$= e^{-x}\frac{1-x+\cancel{2x}-4-x+2+x^2-\cancel{2x}}{(1-x)^3} = e^{-x} \cdot \frac{x^2-2x-1}{(1-x)^3}$$

//$(tg x)' = \frac{1}{\cos^2 x}$

//$(ctg x)' = -\frac{1}{\sin^2 x}$

⑤ $y = \ln tg x + \frac{1}{2} ctg 2x$

$$y' = \frac{1}{tg x} \cdot \frac{1}{\cos^2 x} + \frac{1}{2} \cdot 2 \cdot \left(-\frac{1}{\sin^2(2x)}\right) =$$

$$= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{1}{\sin x \cdot \cos x} - \frac{1}{\sin^2(2x)} =$$

$$= \csc(x) \cdot \sec(x) - \csc^2(2x)$$

$\csc(x) \cdot \sec(x) - \csc^2(2x) = \frac{1}{\sin x} \cdot \frac{1}{\cos x} - \frac{1}{\sin^2(2x)}$

⑥ $y = x\arccos x - \sqrt{1-x^2}$ //$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$$y' = \arccos x + \frac{-x}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) =$$

$$= \arccos x - \frac{x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} = \arccos x$$

(7) $y = \operatorname{arctg} x + \frac{1}{3}\operatorname{arctg} x^3$ $\qquad$ // $(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$$y' = \frac{1}{1+x^2} + \frac{1}{3}\cdot 3x^2 \cdot \frac{1}{1+x^6} =$$

$$= \frac{1}{1+x^2} + \frac{x^2}{1+x^6} = \frac{1+x^6+x^2+x^4}{(1+x^2)(1+x^6)} = \frac{(1+x^2)+x^4(x^2+1)}{(1+x^2)(1+x^6)} = \frac{1+x^4}{1+x^6}$$

(8) $y = e^{2x}\cdot(3\cos 3x - 2\sin 3x)$

$$y' = 2e^{2x}(3\cos 3x - 2\sin 3x) + e^{2x}(-9\sin 3x - 6\cos 3x) =$$

$$= e^{2x}(\cancel{6\cos 3x} - 4\sin 3x - 9\sin 3x - \cancel{6\cos 3x}) =$$

$$= -13e^{2x}\cdot\sin 3x$$

$(a^x)' = a^x \ln a$

(9) $y = 3^{\sin^2\frac{x}{2}}$

$$y' = 3^{\sin^2\frac{x}{2}}\cdot\ln 3\cdot\cancel{2}\sin\frac{x}{2}\cdot\cos\frac{x}{2}\cdot\frac{1}{\cancel{2}} = \frac{\ln 3}{2}\cdot 3^{\sin^2\frac{x}{2}}\sin x$$

(10) $y = x\ln(x + \sqrt{x^2+1})$

$$y' = \ln(x+\sqrt{x^2+1}) + x\cdot\frac{1}{x+\sqrt{x^2+1}}\cdot\left(1 + \frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot\cancel{2}x\right) =$$

$$= \ln(x+\sqrt{x^2+1}) + \frac{x}{\cancel{x+\sqrt{x^2+1}}}\cdot\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}} =$$

$$= \ln(x+\sqrt{x^2+1}) + \frac{x}{\sqrt{x^2+1}}$$

// $(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

(11) $y = 2^{\operatorname{arcctg}\sqrt{x^2+1}}$

$$y' = \ln 2\cdot\left(-\frac{1}{1+(x^2+1)}\right)\frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot\cancel{2}x\cdot 2^{\operatorname{arcctg}\sqrt{1+x^2}} = -\ln 2\cdot\frac{x}{\sqrt{x^2+1}\cdot(x^2+2)}\cdot 2^{\operatorname{arcctg}\sqrt{1+x^2}}$$

(12) $y = \frac{\sin x}{\cos^3 x}$

$$y' = \frac{\cos x\cdot\cos^3 x - \sin x\cdot 3\cos^2 x\cdot(-\sin x)}{\cos^6 x} = \frac{\cos^2 x + 3\sin^2 x}{\cos^4 x} = \frac{1+2\sin^2 x}{\cos^4 x}$$

(13). $y = \arcsin\frac{x+2}{2x+2}$

$$y' = \frac{1}{\sqrt{1-\left(\frac{x+2}{2x+2}\right)^2}}\cdot\frac{1(2x+2)-(x+2)\cdot 2}{(2x+2)^2} \underset{x>0}{=} \frac{(2x+2)}{\sqrt{(2x+2)^2-(x+2)^2}}\cdot\frac{\cancel{2x}+2-\cancel{2x}-4}{(2x+2)^2} =$$

$$= \frac{-2}{(2x+2)\sqrt{(2x+2-x-2)(2x+2+x+2)}} = \frac{-1}{(x+1)\sqrt{(x\cdot(3x+4))}}$$

(14) $y = (1+x)^{\frac{1}{x}}$ $\qquad$ $(1+x)^{\frac{1}{x}} = e^{\ln(1+x)^{\frac{1}{x}}} = e^{\frac{1}{x}\ln(1+x)}$

$$y' = e^{\frac{1}{x}\ln(1+x)}\cdot\left[-\frac{1}{x^2}\ln(1+x) + \frac{1}{x}\cdot\frac{1}{1+x}\right] = e^{\ln(1+x)^{1/x}}\cdot\left[-\frac{\ln(1+x)}{x^2} + \frac{1}{x(1+x)}\right] =$$

$$= (1+x)^{\frac{1}{x}}\cdot\frac{1}{x^2}\cdot\left[\frac{x}{1+x} - \ln(1+x)\right] = \frac{(1+x)^{\frac{1}{x}-1}}{x^2}\cdot\left(x - (1+x)\ln(1+x)\right)$$