

Построить для $y'(x) = f(x)$ разност. схему с наиб. порядком аппрокс.

$$\frac{y_k - y_{k-2}}{2h} = a_1 f_k + a_0 f_{k-1} + a_{-1} f_{k-2}$$

$$\frac{y(x_k) - y(x_{k-2}))}{2h} = a_1 f(x_k) + a_0 f(x_{k-1}) + a_{-1} f(x_{k-2})$$

$$y(x_{k-2}) = y(x_k) - 2L y'(x_k) + 2L^2 y''(x_k) - \frac{4}{3} L^3 y'''(x_k) + O(L^4)$$

П.О.

$$\frac{y_k - y_{k-2}}{2h} = y'(x_k) - 2L y''(x_k) + \frac{2}{3} L^2 y'''(x_k) + O(L^3)$$

$$f(x_{k-1}) = f(x_k) - L f'(x_k) + \frac{L^2}{2} f''(x_k) + O(L^3)$$

$$y'(x_k) =$$

$$\frac{y_k - y_{k-2}}{2h} = a_1 f(x_k) + a_0 f(x_{k-1}) + a_{-1} f(x_{k-2}) =$$

$$= y'(x_k) - L y''(x_k) + \frac{2}{3} L^2 y'''(x_k) + O(L^4) - a_1 f(x_k) - a_0 f(x_{k-1}) - a_{-1} f(x_{k-2})$$

$$f(x_k) - y'(x_k) = a_1 f(x_k) + a_0 f(x_{k-1}) + a_{-1} f(x_{k-2})$$

$p=4$

П.О.

$$a_0 + a_1 + a_{-1} = 1$$

$$-a_0 - 2a_{-1} = -1$$

$$\frac{1}{2} a_0 + 2a_{-1} = \frac{2}{3}$$

$$a_1 - a_{-1} = 0$$

$$a_0 = \frac{2}{3} \Rightarrow a_1 = a_{-1} = \frac{1}{6}$$

$$\text{П.О. } \frac{y_k - y_{k-2}}{2h} = \frac{1}{6} f_k + \frac{2}{3} f_{k-1} + \frac{1}{6} f_{k-2}$$

1.2.

Исследовать устойчивость.

$$\theta \cdot \frac{y_{k+1} - y_k}{h} + (1-\theta) \frac{y_k - y_{k-1}}{h} = f_k, \quad \theta \in [0, 1]$$

$$1\text{-уст.}: y_k - \mu^k \Rightarrow \frac{\theta(\mu^{k+1} - \mu^k)}{h} + \frac{(1-\theta)(\mu^k - \mu^{k-1})}{h} = 0$$

$$\theta \mu^2 - \theta \mu + \mu - \theta \mu + \theta - 1 = 0$$

$$\theta \mu^2 + (1-2\theta)\mu + \theta - 1 = 0$$

$$\theta = 0 \Rightarrow \mu = 1$$

$$\theta \neq 0 \Rightarrow \mu_{1,2} = \frac{2\theta - 1 \pm 1}{2\theta} \Rightarrow$$

$$\Rightarrow \mu_1 = 1, \mu_2 = \frac{\theta - 1}{\theta}$$



$$\left| \frac{\theta - 1}{\theta} \right| \leq 1 \Rightarrow -1 \leq \frac{\theta - 1}{\theta} \leq 1 \Rightarrow 0 \leq \frac{1}{\theta} \leq 2 \Rightarrow \theta \geq \frac{1}{2}$$

Ответ: $\theta = 10 \cup [\frac{1}{2}; 1]$

1.5

Получ. аппрокс. на решении 2-ого зап. по м. $x_0 = 0, x_1 = h$

$$u'(0) - u(0) = 0 \text{ где } u'' - 2u = \sin x - 1$$

$$u(h) = u(0) + h^2 u''(0) + \frac{h^2}{2} u''(0) + O(h^3)$$

$$u'(0) = \frac{u(h) - u(0)}{h} = \frac{h}{2} u''(0) + O(h^2)$$

$$\text{где } u'' - 2u = \sin x - 1; \quad u'(0) - 2u(0) = -1 \Rightarrow u''(0) = 2u(0) - 1 + O(h^2)$$

$$u'(0) = \frac{u(h) - u(0)}{h} = \frac{h}{2} (2u(0) - 1) + O(h^2) \Rightarrow u'(0) - u(0) = \frac{u(h) - u(0)}{h} - \frac{h}{2} (2u(0) - 1) + O(h^2)$$

$$u(0) = u_0; \quad u(h) = u_1 \Rightarrow \frac{u_1 - u_0}{h} - \frac{h}{2} (2u_0 - 1) + \frac{h}{2} = 0 \text{ аппрокс. } u'(0) - u(0) \text{ (маш. } O(h^2))$$



№1.3.

$$y' = y, y(0) = 1$$

$$\frac{y_{k+1} - y_k}{h} = \frac{y_{k+1} + y_k}{2}, y_0 = 1, k \geq 0$$

В разложении ошибки $y(x_{k+1}) - y_k = C_1 h + C_2 h^2 + \dots$ найти погр. C_1
 где $x_k = kh = 1$

$$\frac{y_{k+1} - y_k}{h} = \frac{y_{k+1} + y_k}{2} \Rightarrow y_{k+1} \cdot 2 - y_k \cdot 2 = y_{k+1} \cdot h + y_k \cdot h \Rightarrow$$

$$\Rightarrow y_k = y_{k-1} \left(\frac{2+h}{2-h} \right) \Rightarrow y_k = \left(\frac{2+h}{2-h} \right)^k y_0$$

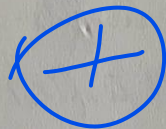
$$y(x_{k+1}) - y_k = y(x_k) e - \left(\frac{2+h}{2-h} \right)^k = e - e^{\ln \left(\frac{2+h}{2-h} \right)^k} =$$

$$= e - e^{k(\ln(2+h) - \ln(2-h))} = e - e^{k \left(\frac{1}{2} - \frac{h^2}{8} + \frac{h^3}{24} + \frac{h}{2} + \frac{h^3}{8} + \frac{h^3}{24} + O(h^4) \right)} =$$

$$= e - e^{\frac{1}{2}k + \frac{k^3}{12} + O(k^4)} = e - e^{1 + \frac{h^2}{12} + O(h^3)}$$

$$= e - e = e - e^{1 + \frac{h^2}{12} + O(h^3)} = e \left(1 - \left(1 + \frac{h^2}{12} + O(h^3) \right) \right) = -\frac{eh^2}{12} \Rightarrow$$

$$\Rightarrow \underline{C_1 = 0}$$



Н.У.

$$y' + 5y = \sin 2x, y(0) = 2$$

$$\frac{y_{k+1} - y_k}{h} + 5 \frac{y_{k+1} + y_k}{2} = \frac{\sin(2L(k+1)) + \sin(2Lk)}{2} = \frac{b_{k+1} + b_k}{2}, y_0 = 2$$

$$\frac{y(x_k + h) - y(x_k)}{h} + 5 \frac{y(x_k + h) + y(x_k)}{2} = \frac{b_{k+1} + b_k}{2}$$

$$y(x_k + h) = y(x_k + \frac{h}{2}) + \frac{h}{2} y'(x_k + \frac{h}{2}) + \frac{h^2}{8} y''(x_k + \frac{h}{2})$$

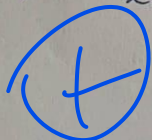
$$y(x_k) = y(x_k + \frac{h}{2}) - \frac{h}{2} y'(x_k + \frac{h}{2}) + \frac{h^2}{8} y''(x_k + \frac{h}{2})$$

$$y'(x_k + \frac{h}{2}) + 5y(x_k + \frac{h}{2}) + O(h^2) = y'(x_k - \frac{h}{2}) + 5y(x_k + \frac{h}{2})$$

$$\text{т.о. } \text{ном} \left(\frac{y_{k+1} - y_k}{h} + 5 \frac{y_{k+1} + y_k}{2} - \frac{b_{k+1} + b_k}{2} \right) = O(h^2)$$

L-уст.: $\mu = 1$ - неуст. способ. $\text{т.о. } \|\mu\| \leq 1$

тогда $\frac{y_{k+1} - y_k}{h} + \frac{5}{2} \frac{y_{k+1} + y_k}{2} = \frac{\sin(2Lk) + \sin(2L(k+1))}{2}, y(0) = 2$



илим м. то 2-оо нөпсүм