Seminar 4
Stochastic Volatility Models

Vega Institute

Problem 1 🧠

Let $X_t = \left((B_t^1)^2 + \ldots + (B_t^n)^2\right)^{1/2}$, B_t^i are independent Brownian motions. Prove that

$$dX_t = \frac{n-1}{2X_t}dt + d\tilde{B}_t.$$

Problem 2 💅

Find recurrent formula for $\beta_k(t) = \mathbb{E}[B_t^k]$.

Problem 3 💅

Solve SDE by applying you rich mathematical experience:

$$\begin{pmatrix} dX_1 \\ dX_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} dt + \begin{pmatrix} X_2 \\ X_1 \end{pmatrix} dB_t.$$

Problem 4 🧠 🪶

Solve SDEs:

a) $dX_t = X_t dt + dB_t$,
b) $dX_t = (\mu - X_t)dt + \sigma dB_t$,
c) $dX_t = rdt + \alpha X_t dB_t$,
d) $dX_t = f(t, X_t)dt + c(t)X_t dB_t$.

Problem 5 🧠

Find integrating factor and solve SDE:

$$dX_t = X_t^\gamma dt + \alpha X_t dB_t, \quad X_0 = x > 0.$$

When does the solution explode?