Канон. вид ортог. оператора

$$\frac{1}{7}\begin{pmatrix} 6-\lambda & -2 & 3 \\ 2 & -3-\lambda & -6 \\ 3 & 6 & -2-\lambda \end{pmatrix}$$

$(6-\lambda)(-3-\lambda)(-2-\lambda) + 3\cdot2\cdot6 + 3\cdot2\cdot6 + 9(\lambda+3) + 4(-2-\lambda) +$
$+ 36(6-\lambda) = (6-\lambda)(3+\lambda)(2+\lambda) + 72 + 9\lambda + 27 - 8 - 4\lambda +$
$+ 216 - 36\lambda = \cancel{6\lambda^2 + 12\lambda} +$
$= (18 + 3\lambda - \lambda^2)(2+\lambda) + 307 - 31\lambda =$
$= 36 + 18\lambda + 6\lambda + 3\lambda^2 - 2\lambda^2 - \lambda^3 + 307 - 31\lambda =$
$= \cancel{\lambda^3 + 4\lambda^2 - 7\lambda + 343}$
$= -\lambda^3 + \lambda^2 - 7\lambda + 343 = -(\lambda-7)(\lambda^2+6\lambda+49)$

$$\begin{array}{l|l} \lambda^3 - \lambda^2 + 7\lambda - 343 & \lambda - 7 \\ \hline -\ \lambda^3 - 7\lambda^2 & \lambda^2 + 6\lambda + 49 \\ \quad 6\lambda^2 + 7\lambda - 343 & \\ \quad -\ 6\lambda^2 - 42\lambda & \\ \qquad 49\lambda - 343 & \\ \qquad -\ 49\lambda - 343 & \\ \qquad\qquad 0 & \end{array}$$

$D = 36 - 196 = -160$

$\lambda = \dfrac{-6 \pm \sqrt{-160}}{2} =$

$= -3 \pm 2\sqrt{10}\, i$

$\lambda = 7$

$\lambda = -3 \pm 2\sqrt{10}\, i$

$9 + 40 = 7^2$

$\cancel{\begin{matrix} 7 & 0 & 0 \\ 0 & \cos\left(\frac{3}{7}\right) & \\ 0 & & \end{matrix}}$

$$\cancel{\#}\begin{pmatrix} \cancel{1} & 0 & 0 \\ 0 & -\frac{3}{7} & -\frac{2\sqrt{10}}{7} \\ 0 & \frac{2\sqrt{10}}{7} & -\frac{3}{7} \end{pmatrix}$$

$$a_0 + \frac{a_1}{1+x} + \frac{a_2}{(1+x)^2} + \dots + \frac{a_n}{(1+x)^n} = 0$$

$$a_0(1+x)^n + a_1(1+x)^{n-1} + \dots + a_n = 0$$

$\downarrow$ дифф

$$na_0(1+x)^{n-1} + (n-1)a_1(1-x)^{n-1} + \dots + a_{n-1} = 0$$

x^k - базис $\Rightarrow$ лин. независимы

$$ax^5 + bx^4 + cx^3 + dx^2 + ex + f \to (d-a)x^2 + (e-b)x + (f-c)$$

$$\begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} \begin{pmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ -1 & 0\ 0\ 1\ 0 & 0 \\ 0 & -1\ 0\ 0\ 1 & 0 \\ 0 & 0\ -1\ 0\ 0 & 1 \end{pmatrix}$$

Все инвариантные прямые

$Ax + By + C = 0$ - все прямые в прообразе

Если прямая ~~нерско~~ инвариантна, то

$k(A\tilde{x} + B\tilde{y} + C) = 0$

$\tilde{x} = 3x - 2y + 5$

$\tilde{y} = 2x - y + 5$

$A(3x - 2y + 5) + B(2x - y + 5) + C = 0$

$x\underbrace{(3A + 2B)}_{kA} + y\underbrace{(-2A - B)}_{kB} + \underbrace{(5A + 5B + C)}_{kC} = 0$

$\begin{cases} 3A + 2B = kA \\ -2A - B = kB \end{cases}$

$\begin{cases} (3-k)A + 2B = 0 \\ 2A + (k+1)B = 0 \end{cases}$

Если ур-я лин. нез. то ∃ единств. реш (0,0), чего быть не может =>

$\begin{vmatrix} 3-k & 2 \\ 2 & k+1 \end{vmatrix} = 0 \Rightarrow (3-k)(k+1) - 4 = 0$

$3 + 3k - k^2 - k - 4 = 0$

$k^2 - 2k + 1 = 0$

$k = 1$

$\begin{cases} 3A + 2B = A \\ 2A + B = -B \\ 5A + 5B + C = C \end{cases} \Rightarrow A = -B$

$x - y + C = 0$ - все инвар. прямые

$\vec{l} = \{-1; 2; 0; 1\}$

$$\begin{cases} 3x+z=7 \\ 2x-y+v=0 \end{cases} \to \begin{cases} 3x+z=0 \\ 2x-y+v=0 \end{cases}$$

$$\begin{pmatrix} 3 & 0 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 3 & 0 & 1 & 0 \\ 0 & -1 & -\frac{2}{3} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & \frac{2}{3} & 1 \end{pmatrix}$$

$$\begin{cases} x = -\frac{1}{3}z \\ y = -\frac{2}{3}z - v \end{cases}$$

$(0,1)$

$(1,0)$

$$\cancel{\begin{cases} 0 \ldots \\ 1 \ldots -\frac{2}{3}z - v \end{cases} \ldots = 0}$$

$$\begin{cases} x = 0 \\ y = -1 \end{cases}$$

$$\begin{cases} x = -\frac{1}{3} \\ y = -\frac{2}{3} \end{cases}$$

$(0; -1; 0, 1)$
$(-\frac{1}{3}; -\frac{2}{3}, 1, 0)$ — базис пр-ва

$\sim$ $(0; -1, 0, 1) = v_1$

$(-1, -2, 3, 0) = v_2$

$\cancel{p = (a, b, c, d)} = \alpha v_1 + \beta v_2$ — проекция

$\cancel{-a+2b-d}$ $\quad l - p \perp v_1, \quad l - p \perp v_2$

$\cancel{l - p = (-1-a; 2-b, -c; 1-d)}$

$\cancel{b-2+1-d}$

Ищем p: $\begin{cases} \alpha v_1 + \beta v_2 = p \\ l - p \perp v_2 \\ l - p \perp v_1 \end{cases}$

$p = (-\beta; -\alpha - 2\beta; 3\beta; \alpha)$

$l - p = (-1-\beta, 2-\alpha-2\beta; -3\beta, 1-\alpha)$

$\begin{cases} -2 + \alpha + 2\beta + 1 + \alpha = 0 \\ 1 + \beta, -4 + 2\alpha + 4\beta - 9\beta = 0 \end{cases}$

$\begin{cases} 2\alpha + 2\beta = 1 \\ 2\alpha - 4\beta = 3 \end{cases}$ $\quad 6\beta = -2 \quad \beta = -\frac{1}{3}$

$\alpha = \frac{3+4\beta}{2} = \frac{3}{2} - \frac{4}{6} = \frac{9}{6} - \frac{4}{6} = \frac{5}{6}$

$p = \left(\frac{1}{3}; -\frac{5}{6} + \frac{4}{6}; -1; \frac{5}{6}\right)$

$p = \left(\frac{1}{3}; -\frac{1}{6}; -1; \frac{5}{6}\right) \qquad l = (-1; 2; 0, 1)$

$\cos\alpha = \frac{(a, b)}{|a| \cdot |b|}$

$\cancel{\cos\alpha = \sqrt{\frac{1}{9} + \frac{5}{36}}}$

$\cos(l, p) = \frac{\left|-\frac{1}{3} - \frac{1}{3} + \frac{5}{6}\right|}{\sqrt{\frac{1}{9} + \frac{1}{36} + 1 + \frac{25}{36}}\sqrt{1+4+1}} =$

$= \frac{\left|\frac{5}{6} - \frac{4}{6}\right|}{\sqrt{\frac{66}{36}}\sqrt{6}} = \frac{\frac{1}{6}}{\sqrt{11}} = \frac{1}{6\sqrt{11}}$

N3

$$(I)\begin{cases} x = \cancel{t} - s + r \\ y = -t - s + r \\ z = 3 + 3t - s - r \\ v = 2t + s - r \end{cases} \qquad (II)\begin{cases} 3x + z = 7 \\ 2x - y + v = 0 \end{cases}$$

(t, s, r)

$e_1 = (0, -1, 3, 2)$

$e_2 = (-1, -1, -1, 1)$

$e_3 = (1; 1, -1, -1)$

$\vec{n} = (a, b, c, d)$

$$\begin{cases} -b + 3c + 2d = 0 \\ -a \ \ldots \end{cases}$$

$$\begin{pmatrix} 0 & -1 & 3 & 2 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 0 & -2 & 0 \\ 0 & 1 & -3 & -2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$d = 1 \Rightarrow \begin{cases} a = -1 \\ b = 2 \\ c = 0 \\ d = 1 \end{cases}$$

$\vec{n} = (-1, 2, 0, 1)$ – нормаль к (I)

Найдем базис (II)

$$\begin{pmatrix} 3 & 0 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & \frac{1}{3} & 0 \\ 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \sim$$

$$\sim\begin{pmatrix}1&0&1/3&0\\0&-\frac{1}{2}&-\frac{1}{3}&-\frac{1}{2}\end{pmatrix}\sim\begin{pmatrix}3&0&1&0\\0&-3&-2&-3\end{pmatrix}\sim\begin{pmatrix}3&0&1&0\\0&3&2&3\end{pmatrix}$$

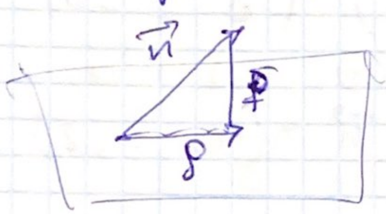
$$\begin{cases}3x=-z\\3y=-2z-3v\end{cases}\qquad (0,1)\to\begin{cases}x=0\\y=-1\end{cases}$$

$$(1;0)\to\begin{cases}x=-1/3\\y=-2/3\end{cases}$$

$e_1=(0,-1;0,1)$

$e_2=(-1,-2;3;0)$ – базис (II)

найдем проекцию $\vec{n}$ на (II)



$n-p\perp(\mathrm{II})$

$$\begin{cases}p=\alpha e_1+\beta e_2\\n-p\perp e_1\\n-p\perp e_2\end{cases}$$

$\vec{p}=(-\beta;-\alpha-2\beta;3\beta;\alpha)$

$n-p=(-1+\beta,\ 2+\alpha+2\beta,\ -3\beta,\ 1-\alpha)$

$-\cancel{2+\alpha+2\beta}$ $\begin{cases}-2-\alpha-2\beta+1-\alpha=0\\1-\beta-4-2\alpha-4\beta-9\beta=0\end{cases}$

$$\begin{cases}-2\alpha-2\beta=1\\-2\alpha-14\beta=3\end{cases}\to\quad 12\beta=-2\Rightarrow\beta=-\frac{1}{6}$$

$$\alpha=\frac{2\beta-1}{2}=\beta-\frac{1}{2}=-\frac{1}{6}-\frac{3}{6}=-\frac{4}{6}$$

$$\vec{p}=\left(\frac{1}{6};\frac{4}{6}+\frac{2}{6};-\frac{3}{6};-\frac{4}{6}\right)=$$

$$=\left(\frac{1}{6};1;-\frac{1}{2};-\frac{2}{3}\right)=(6;6;-3;-4)$$

$$\cos(\vec{p},\vec{n})=\frac{|-6+12-4|}{\sqrt{1+4+1}\sqrt{36+36+9+16}}=a\Rightarrow 90-\arccos a$$

Расстояния

№4

$$\begin{cases} x = -2 \\ y = 1+2t \\ z = 4-3t \end{cases} \qquad \frac{x}{1} = \frac{y-1}{-2} = \frac{z+4}{6}$$

$$\vec{l_1} = \{0, 2, -3\} \qquad \vec{l_2} = \{1; -2, 6\}$$

$$\vec{n} = \{a, b, c\}$$

$$\begin{cases} (\vec{n}, \vec{l_1}) = 0 \\ (\vec{n}, \vec{l_2}) = 0 \end{cases} \qquad \begin{cases} 2b - 3c = 0 \\ a - 2b + 6c = 0 \end{cases}$$

$$\begin{pmatrix} 1 & -2 & 6 \\ 0 & 2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & -3 \end{pmatrix}$$

$a = -6$

$b = 3$ $\qquad \vec{n} = (-6; 3; 2)$

$c = 2$

$$Ax + By + Cz + D = 0$$

$$-6x + 3y + 2z + D = 0$$

подставим точку на l_1

$$12 + 3 + 8 + D = 0 \Rightarrow D = -23$$

плоск: $-6x + 3y + 2z - 23 = 0$

$$\rho(l_2, \text{плоск}) = \rho((1, -1, 2), \text{плоск}) =$$

$$= \frac{|-6 \cdot 1 - 3 + 2 \cdot 2 - 23|}{\sqrt{36 + 9 + 4}} = \frac{28}{7} = 4$$

N5

$$\alpha: \begin{cases} -x+y+z+v=3 \\ -3y+2z-4v=4 \end{cases} \qquad \beta: (1;3;-3;-1) + \langle(1,0,1,1)\rangle$$

$$\begin{pmatrix} -1 & 1 & 1 & 1 \\ 0 & -3 & 2 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 1 & -\frac{2}{3} & \frac{4}{3} \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & -\frac{5}{3} & \frac{1}{3} \\ 0 & 1 & -\frac{2}{3} & \frac{4}{3} \end{pmatrix} \qquad \begin{aligned} x &= -\frac{5}{3}z + \frac{1}{3}v \\ y &= -\frac{2}{3}z + \frac{4}{3}v \end{aligned}$$

$$e_1^\alpha = (-5, -2, 3, 0)$$
$$e_2^\alpha = (1, 4, 0, 3)$$
$$e_1^\beta = (1, 0, 1, 1)$$

$$\vec{n} = (a \ b \ c \ d)$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 4 & 0 & 3 \\ 5 & 2 & -3 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 4 & -1 & 2 \\ 0 & 2 & -8 & -5 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1/4 & 1/2 \\ 0 & 0 & -\frac{15}{2} & -6 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1/4 & 1/2 \\ 0 & 0 & 1 & \frac{12}{15} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 4 & -4 & 2 \\ 0 & 0 & 1 & \frac{12}{15} \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & \frac{3}{15} \\ 0 & 4 & 0 & \frac{42}{15} \\ 0 & 0 & 1 & \frac{12}{15} \end{pmatrix} \qquad \begin{aligned} \vec{n} &= (3; -10{,}5; -12; 15) \\ &= (6, -21; -24; 30) \end{aligned}$$

$6x - 21y - 24z + 30v + C = 0$

$\cancel{C = 5}$ подставили точку из α:

$\cancel{(0, 0, 4, -1)}$

$(-4, 0, 0, -1)$

$\Downarrow$

$C = 10$

$6x - 21y - 24z + 30v + 10 = 0$

до точки $(1, 3, -3; -1)$

$\cancel{\rho = \arccos}$ $\rho = \dfrac{|6 - 21 \cdot 3 + 24 \cdot 3 - 30 + 10|}{\sqrt{36 + 21^2 + 24^2 + 30^2}} = 5$

N6 $V_1 = \langle \{1, -1, 1, 0\}, \{-2, -1, 1, -1\},$

$\cancel{\{-2, -1, 1, -1\}}, \{1, 0, 1, 2\} \rangle$

$V_2 = \langle \{0, 1, 1, 1\} \rangle$

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ -2 & -1 & 1 & -1 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & \cdot \end{pmatrix}$$

$\to$ они линейно независимы

$\vec{x} = \{4, 1, 0, 1\}$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 4 \\ -2 & -1 & 1 & -1 & 1 \\ 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 4 \\ -1 & -1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & 2 & 1 & 1 \end{array}\right)$$

$x_1 = 1,5$

$x_2 = -1,625$

$x_3 = -0,75$

$x_4 = 0,875$

$$\vec{x} = \underset{\uparrow\; V_2}{(0;\ 0,85;\ 0,875;\ 0,875)} + \underset{\uparrow\; V_1}{(x_1 e_1 + x_2 e_2 + x_3 e_3)}$$

N7 $$\left(\begin{array}{ccc|ccc} e_1 & e_2 & e_3 & \tilde{e}_1 & \tilde{e}_2 & \tilde{e}_3 \\ | & | & | & | & | & | \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & & 0 & \\ & 1 & & \\ 0 & & 1 & \underbrace{\ \ldots\ }_{C} \end{array}\right)$$

$A_{\tilde{e}} = C^{-1} A C$