№ 1.1

Построить для $y'(x)=f(x)$ разност. схему с наив. пор. аппрокс.

$$\frac{y_k-y_{k-2}}{2h}=a_1 f_k+a_0 f_{k-1}+a_{-1}f_{k-2}$$

$$\frac{y(x_k)-y(x_k-2h)}{2h}=a_1 f(x_k)+a_0 f(x_k-h)-a_{-1}f(x_k-2h)$$

$$y(x_k-2h)=y(x_k)-2hy'(x_k)+2h^2y''(x_k)+\frac{4}{3}h^3y'''(x_k)+O(h^4)$$

Т.о.

$$\frac{y_k-y(x_k-2h)}{2h}=y'(x_k)-2hy''(x_k)+\frac{2}{3}h^2y'''(x_k)+O(h^3)$$

$$f(x_k-h)=f(x_k)-hf'(x_k)+\frac{h^2}{2}f''(x_k)+O(h^3)$$

$\cancel{y'(x_k)=}$

$$\frac{y_k-y(x_k-2h)}{2h}-a_1 f(x_k)-a_0 f(x_k-h)-a_{-1}f(x_k-2h)=$$

$$=y'(x_k)-hy''(x_k)+\frac{2}{3}h^2y'''(x_k)+O(h^4)-a_1 f(x_k)-a_0 f(x_k-h)-a_{-1}f(x_k-2h)$$

$$f(x_k)-y'(x_k)=a_1 f(x_k)+a_0 f(x_k-h)+a_{-1}f(x_k-2h)$$

$p=4$

Т.о. $a_0+a_1+a_{-1}=1$ $\quad a_1-a_{-1}=0$

$-a_0-2a_{-1}=-1$ $\quad a_0=\frac{2}{3} \Rightarrow a_1=a_{-1}=\frac{1}{6}$

$\frac{1}{2}a_0+2a_{-1}=\frac{2}{3}$

Т.о. $$\frac{y_k-y_{k-2}}{2h}=\frac{1}{6}f_k+\frac{2}{3}f_{k-1}+\frac{1}{6}f_{k-2}$$

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№1.2.

Исследовать устойч.

$$\theta \cdot \frac{y_{k+1}-y_k}{h} + (1-\theta)\frac{y_k - y_{k-1}}{h} = f_k \quad , \theta \in [0,1]$$

Л-уст.: $y_k - \mu^k \Rightarrow \frac{\theta(\mu^{k+1}-\mu^k)}{h} + \frac{(1-\theta)(\mu^k - \mu^{k-1})}{h} = 0$

$\theta\mu^2 - \theta\mu + \mu - \theta\mu + \theta - 1 = 0$

$\theta\mu^2 + (1-2\theta)\mu + \theta - 1 = 0$

$\theta = 0 \Rightarrow \mu = 1$

$\theta \neq 0 \Rightarrow \mu_{1,2} = \frac{2\theta - 1 \pm 1}{2\theta} \Rightarrow$

$\Rightarrow \mu_1 = 1, \mu_2 = \frac{\theta - 1}{\theta}$

$\left|\frac{\theta-1}{\theta}\right| \le 1 \Rightarrow -1 \le \frac{\theta - 1}{\theta} \le 1 \Rightarrow 0 \le \frac{1}{\theta} \le 2 \Rightarrow$

$\Rightarrow \theta \ge \frac{1}{2}$

(+)

Ответ: $\theta = \{0\} \cup [\frac{1}{2}; 1]$

№1.5

Постр. аппрокс. на решении 2-ого пор. по т. $x_0 = 0$, $x_1 = h$

$u'(0) - u(0) = 0$ для $\nexists$ $u'' - 2u = \sin x - 1$

$u(h) = u(0) + h^2 u(0) + \frac{h^2}{2} u''(0) + O(h^3)$

$u'(0) = \frac{u(h) - u(0)}{h} - \frac{h}{2} u''(0) + O(h^2)$

для $u'' - 2u = \sin x - 1$: $u''(0) - 2u(0) = -1 \Rightarrow u''(0) = 2u(0) - 1 + O(h^2)$

$u'(0) = \frac{u(h) - u(0)}{h} - \frac{h}{2}(2u(0) - 1) + O(h^2) \Rightarrow u'(0) - u(0) = \frac{u(h) - u(0)}{h} - \frac{h}{2}(2u(0) - 1) - u(0) + O(h^2)$

$u(0) = u_0$; $u(h) = u_1$ $\Rightarrow \frac{u_1 - u_0}{h} - u_0(h+1) + \frac{h}{2} = 0$ аппрокс. $u'(0) - u(0)$ с точн. $O(h^2)$

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№1.3.

$y' = y,\ y(0) = 1$

$\frac{y_{k+1} - y_k}{h} = \frac{y_{k+1} + y_k}{2}$, $y_0 = 1,\ k \geq 0$

В разложении ошибки $y(x_N) - y_N = C_1 h + C_2 h^2 + \ldots$ найти пост. C_1 для $x_N = Nh = 1$

$\frac{y_{k+1} - y_k}{h} = \frac{y_{k+1} + y_k}{2} \Rightarrow y_{k+1} \cdot 2 - y_k \cdot 2 = y_{k+1} h + y_k \cdot h \Rightarrow$

$\Rightarrow y_k = y_{k-1}\left(\frac{2+h}{2-h}\right) \Rightarrow y_k = \left(\frac{2+h}{2-h}\right)^k y_0$

$y(x_N) - y_N = \cancel{y(x_N)}\, e - \left(\frac{2+h}{2-h}\right)^N = e - e^{N \ln\left(\frac{2+h}{2-h}\right)} =$

$= e - e^{N(\ln(2+h) - \ln(2-h))} = e - e^{N\left(\frac{h}{2} - \frac{h^2}{8} + \frac{h^3}{24} + \frac{h}{2} + \frac{h^2}{8} + \frac{h^3}{24} + O(h^4)\right)} =$

$= e - e^{\frac{1}{h}\left(h + \frac{h^3}{12} + O(h^4)\right)} = e - e^{1 + \frac{h^2}{12} + O(h^3)} = e\left(1 - \left(1 + \frac{h^2}{12} + O(h^3)\right)\right) = -\frac{eh^2}{12} \Rightarrow$

$\Rightarrow \underline{C_1 = 0}$

№ 1.9.

$y' + 5y = \sin 2x$, $y(0) = 2$

$$\frac{y_{k+1} - y_k}{h} + 5\,\frac{y_{k+1} - y_k}{2} = \frac{\sin(2h(k+1)) + \sin(2hk)}{2} = \frac{f_{k+1} + f_k}{2}, \quad y_0 = 2$$

$$\frac{y(x_k + h) - y(x_k)}{h} + 5\,\frac{y(x_k + h) + y(x_k)}{2} = \frac{f_{k+1} + f_k}{2}$$

$$y(x_k + h) = y(x_k + \tfrac{h}{2}) + \tfrac{h}{2}\, y'(x_k + \tfrac{h}{2}) + \tfrac{h^2}{8}\, y''(x_k + \tfrac{h}{2})$$

$$y(x_k) = y(x_k + \tfrac{h}{2}) - \tfrac{h}{2}\, y'(x_k + \tfrac{h}{2}) + \tfrac{h^2}{8}\, y''(x_k + \tfrac{h}{2})$$

$$y'(x_k + \tfrac{h}{2}) + 5y(x_k + \tfrac{h}{2}) + O(h^2) = y'(x_k + \tfrac{h}{2}) + 5y(x_k + \tfrac{h}{2})$$

Т.о. $\text{пот}\left|\frac{y_{k+1} - y_k}{h} + 5\frac{y_{k-1} - y_k}{2} - \frac{f_{k+1} + f_k}{2}\right| = O(h^2)$

<u>L-уст.</u>: $\mu = 1$ - хар. уравн. $\Rightarrow \|\mu\| \le 1$

тогда $\frac{y_{k+1} - y_k}{h} + \frac{5}{2}\,\frac{y_{k+1} + y_k}{2} = \frac{\sin(2kh) + \sin(2h(k+1))}{2}$, $y(0) = 2$

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имеет сх-ть 2-ого порядка