Seminar 3
Stochastic Volatility Models

Vega Institute

Problem 1 🧠

Prove Black-Scholes formula

a) for divivdent paying stock,
b) for futures-style options.

Problem 2 🍬

Let $V_t =$, $S_t^* = S_t/B_t$ and $V_t^* = V_t/B_t$. Prove that $dV_t^* = H_t \cdot dS_t^*$.

Problem 3 🧠

Prove that

a) $\dfrac{dP}{dQ} = 1 \Big/ \dfrac{dQ}{dP}$,

b) $\mathbb{E}^Q X = \mathbb{E}^P \left(\dfrac{dQ}{dP} X \right)$.

Problem 4 🍫

Let $P_t = P \mid \mathscr{F}_t$, $Q_t = Q \mid \mathscr{F}_t$ and $Z_t = \dfrac{dQ_t}{dP_t}$. Prove that if $Q \sim P$,

$$\mathbb{E}^Q(X \mid \mathscr{F}_t) = \frac{\mathbb{E}^P(Z_T X \mid \mathscr{F}_t)}{Z_t}.$$