Home assignment 2

9.11.2022

Submission deadline: 11 November, 23:59
Send your solutions to info@vega-institute.org

Problem 1
We can borrow and lend money at year n and up to year $n+1$ at interest rate r_n. A *float note* has notional $A = \$1000$ and maturity $N = 20$. It pays coupon yearly and notional A at maturity. The coupon paid at time $n+1$ is computed at time n and is given by Ar_n. Compute V_0, the arbitrage-free price of the note at its issue time 0.

Problem 2
An *interest rate swap* is a financial contract between A and B with the parameters:

$R:$ the swap rate,

$m:$ the number of payments per year,

$n:$ the total number of payments.

There is no cost of entering the swap for A and B. The payments take place at times

$$t_k = \frac{k}{m}, \quad k = 1, \dots, n,$$

given as year fractions. At time t_k,
a) A pays to B the *fixed interest* R/m;
b) B pays to A the *float interest* $L(t_{k-1}, t_k)/m$, where $L(s,t)$ is the interest rate computed at time s for maturity t.

We assume that the payments occur $m = 2$ times per year for $n = 4$ periods, that we can trade the discount factors at time 0 for every maturity t_k, and that we have an access to the bank account that pays the interest rate $L(t_k, t_{k+1})$ between t_k and t_{k+1}. The discount factors have the values:

$$d(0.5) = 0.95, \quad d(1) = 0.9, \quad d(1.5) = 0.85, \quad d(2) = 0.8.$$

Compute the arbitrage-free value for the swap rate R.

Problem 3
We can trade a futures contract that expires at time N and borrow/lend money from a bank account at the constant single-period interest rate r. The futures price at n is denoted by G_n. We recall that a long position in the contract taken at time m yields zero payment at m and payments $G_n - G_{n-1}$ at subsequent times $n = m+1, \dots, N$. At maturity, the futures price G_N coincides with the price of the stock.

Construct the trading strategy (from the futures and the bank account) that allows you to receive exactly one stock at maturity N. For this strategy, compute the total wealth X_n and the number of futures Δ_n at time n in terms of G_n and r.

Problem 4
The N-period currency swap with the foreign notional A, the domestic notional B, and the foreign fixed *swap rate* q generates the following cash flow:
a) At initial time 0 we pay A in foreign currency and receive B in domestic currency.
b) At every time $0 < n < N$ we pay Br_{n-1} in domestic currency, where r_{n-1} is the domestic rate between $n-1$ and n, and receive Aq in foreign currency.
c) At maturity N we pay $B(1 + r_{N-1})$ in domestic currency and receive $A(1+q)$ in foreign currency.

At every time n we can borrow and lend at the domestic rate r_n; the rate r_n is stochastic, that is, unknown to us before n. At time 0 we can trade the domestic discount factor $D(0,n)$ and the forward exchange rate $F(0,n)$ for every delivery time $n = 0, 1, \ldots, N$. In particular, we can buy/sell foreign currency at the spot rate $S_0 = F(0,0)$.

At time 0 the foreign and domestic payments have identical values: $AS_0 = B$, and the swap rate q is set to make the value of the swap to be 0. Compute q if $N = 3$, $S_0 = 100$, and

$$
\begin{aligned}
&D(0,1) = 0.9, \quad D(0,2) = 0.8, \quad D(0,3) = 0.7 \\
&F(0,1) = 110, \quad F(0,2) = 120, \quad F(0,3) = 130.
\end{aligned}
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