Home assignment 3

12.11.2022

Problem 1

We can trade a stock and a call option on the stock and use the bank account with interest rate $r = 20\%$.

1. The initial price of the stock is $S_0 = \$100$. At $t = 1$ the stock pays the dividend $D = \$6$ and then its price S_1 takes one of the values: \$90, \$110, and \$130.
2. The call has strike $K^C = \$110$ and is traded at initial price $C_0 = \$5$. The delivery takes place at $t = 1$ after the dividend payment.

(a) Is the model arbitrage-free? Is it complete?

(b) Compute all AFPs for the put option with strike $K^P = \$102$ and the delivery taking place at $t = 1$ after the dividend payment. Can we replicate the put? If yes, then find the numbers Δ_0 and Δ_0^C of stocks and calls in the replicating strategy.

Problem 2

We can use the bank account with interest rate $r = 25\%$ and trade the forward on a stock at the forward price $F = 4$. The forward expires at $t = 1$, when the price of the stock S_1 takes one of the values: \$2, \$5, or \$8. We can not trade the stock.

(a) Is the model arbitrage-free? Is it complete?

(b) Compute AFPs for the put option on the stock with strike $K = 6$ and maturity $t = 1$.

Problem 3

We can trade a forward contract on a stock and a put option on forward and use a money market account with interest rate $r = \frac{1}{4}$.

1. The forward expires at $N = 2$. The initial forward price is $F_0 = 4$. At $t = 1$, the forward price F_1 takes one of the values: 2, 4, or 9.
2. The put expires at $t = 1$ and gives its holder the right to enter into the forward agreement to sell the stock at N for the price $K = 4$. Note that, after the exercise, there is no payment at $t = 1$. The initial price of the put is $P_0 = \frac{4}{5}$.

(a) For a risk-neutral probability measure $\widetilde{\mathbb{P}}$, compute

$$\widetilde{p}_1 = \widetilde{\mathbb{P}}(F_1 = 2), \quad \widetilde{p}_2 = \widetilde{\mathbb{P}}(F_1 = 4), \quad \widetilde{p}_3 = \widetilde{\mathbb{P}}(F_1 = 9).$$

(b) For the digital option paying $V_1 = 1_{\{F_1=2\}}$ at $t = 1$, compute the AFP V_0 and the numbers Δ_0^F and Δ_0^P of forwards and puts in the replicating strategy.

Problem 4

The bank account pays the interest rate $r = 20\%$. We can trade the forward contract and the put option on the same stock and with the same maturity $t = 1$. The forward price $F = \$4$. The put option has strike $K = \$4$ and is traded at the price $P_0 = \$0.5$. The stock price S_1 at $t = 1$ takes one of the values: \$1, \$4, or \$8. We can not trade the stock.

(a) Is the model arbitrage-free? Is it complete?

(b) Compute all RNPs $\widetilde{\mathbb{P}}$.

(c) Compute all AFPs of the put option with strike $K^P = \$2$ and maturity $t = 1$.

(d) Compute all AFPs of the call option with strike $K^C = \$5$ and maturity $t = 1$.