INT1

$$\int \frac{dx}{(x^2+1)(x^2-3)} = \frac{1}{4}\int \frac{dx}{x^2-3} - \frac{1}{4}\int \frac{dx}{x^2+1} =$$

$$= \frac{1}{4}\cdot\frac{1}{2\sqrt{3}} \ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| - \frac{1}{4}\operatorname{arctg} x + C$$

INT2

$$\int \frac{x-4}{\sqrt{x^2-2}}\,dx = \frac{1}{2}\int \frac{dx^2}{\sqrt{x^2-2}} - 4\int \frac{dx}{\sqrt{x^2-2}} = \sqrt{x^2-2} -$$

$$- 4\ln\left| x+\sqrt{x^2-2}\right| + C$$

INT3

$$\int \frac{dx}{x^4-1} = \frac{1}{2}\int \frac{dx}{x^2-1} - \frac{1}{2}\int \frac{dx}{x^2+1} =$$

$$= \frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\operatorname{arctg} x + C$$

INT4

$$\int \frac{e^x+e^{2x}}{1-e^x}\,dx = \int \frac{e^x\,de^x}{1-e^x} + \int \frac{de^x}{1-e^x} = -\int \frac{1-e^x}{1-e^x}\,de^x + 2\int \frac{de^x}{1-e^x} =$$

$$= e^{-x} - 2\ln|e^x-1| + C$$

INT5

$$\int (5^x-2^x)^2\,dx = \int 25^x dx - 2\int 10^x dx + \int 4^x dx =$$

$$= \frac{25^x}{\ln 25} - \frac{2\cdot 10^x}{\ln 10} + \frac{4^x}{\ln 4}$$

INT6

$$\int \frac{dx}{\cos x} = \int \frac{d\sin x}{1-\sin^2 x} = -\frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$$

INT7

$$\int x(x-2)^5\,dx = \int (x-2)^6\,d(x-2) + 2\int (x-2)^5\,d(x-2) =$$
$$= \frac{(x-2)^7}{7} + \frac{2(x-2)^6}{6} + C$$

INT8

$$\int x\sqrt{1-2x}\,dx = -\frac{1}{2}\int x\sqrt{1-2x}\,d(1-2x) = \left| t = 1-2x \right| =$$
$$= -\frac{1}{4}\int (1-t)\sqrt{t}\,dt = -\frac{1}{4}\int \sqrt{t}\,dt + \frac{1}{4}\int t^{3/2}\,dt =$$
$$= C \cancel{+} -\frac{1}{2}\cdot\frac{t^{3/2}}{3} + \frac{1}{4}\cdot\frac{2}{5}t^{5/2} = \frac{(1-2x)^{5/2}}{10} - \frac{(1-2x)^{3/2}}{6} + C$$

INT9

$$\int \frac{2x-7}{\sqrt{3x+1}}\,dx = \underbrace{\frac{2}{3}\int \frac{x}{\sqrt{3x+1}}\,d(3x+1)}_{} - \frac{7}{3}\int \frac{d(3x+1)}{\sqrt{3x+1}} \;(=)$$

$$\left| \begin{aligned} t &= 3x+1 \\ x &= \frac{t-1}{3} \end{aligned} \right| \qquad -\frac{7}{3}\cdot 2\sqrt{3x+1}$$

$$\frac{2}{9}\int \frac{t-1}{\sqrt{t}}\,dt = \frac{2}{9}\int \sqrt{t}\,dt - \frac{2}{9}\int \frac{dt}{\sqrt{t}} =$$
$$= \frac{2}{9}\cdot\frac{2}{3}t^{3/2} - \frac{2}{9}\cdot 2\sqrt{t}$$

$$(=)\ \frac{4}{27}(3x+1)^{3/2} - \frac{4}{9}\sqrt{3x+1} - \frac{14}{3}\sqrt{3x+1} + C =$$
$$= \frac{4}{27}(3x+1)^{3/2} - \frac{46}{9}\sqrt{3x+1} + C$$

INT10

$$\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{dx}{\sqrt{1+x^2}} =$$
$$= \arcsin x + \ln\left|x + \sqrt{1+x^2}\right| + C$$

INT11

$$\int \frac{dx}{x(\ln^2 x+2)} = \int \frac{d\ln x}{\ln^2 x+2} = \frac{1}{\sqrt{2}}\,\mathrm{arctg}\left(\frac{\ln x}{\sqrt{2}}\right)+C$$

INT12

$$\int \ln^2 x\,dx = x\ln^2 x - \int 2\ln x\cdot\frac{1}{x}\cdot x\,dx =$$
$$= x\ln^2 x - 2x\ln x + 2\int x\cdot\frac{1}{x}\,dx = x\ln^2 x - 2x\ln x + 2x + C$$

INT13

$$\int\frac{dx}{1+\cos x} = \int\frac{dx}{1+2\cos^2\frac{x}{2}-1} = \frac{1}{2}\int\frac{dx}{\cos^2\frac{x}{2}} = \int\frac{d\frac{x}{2}}{\cos^2\frac{x}{2}} = \mathrm{tg}\frac{x}{2}+C$$

INT14

$$\int \mathrm{tg}^2 x\,dx = -\int\frac{\sin x}{\cos^2 x}\,d\cos x = \int \sin x\, d\frac{1}{\cos x} =$$
$$= \frac{\sin x}{\cos x} - \int\frac{d\sin x}{\cos x} = \mathrm{tg}x - x + C$$

INT15

$$\int\frac{dx}{\sin^2 x(1+\mathrm{tg}x)} = -\int\frac{d(\mathrm{ctg}x)}{1+\mathrm{tg}x} = -\int\frac{\mathrm{ctg}x\,d(\mathrm{ctg}x)}{\mathrm{ctg}x+1} =$$
$$= -\int\frac{t\,dt}{t+1} = -\int\frac{t+1}{t+1}\,dt + \int\frac{dt}{t+1} = -t+\ln|t+1|+C$$
$$= -\mathrm{ctg}x + \ln|\mathrm{ctg}x+1| + C$$

INT16

$$\int\frac{x^2}{1-x^2}\,dx = -\int\frac{1-x^2}{1-x^2}\,dx + \int\frac{dx}{1-x^2} = -x+\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|+C$$

INT17

$$\cancel{\int x\sin x\,dx = x^2\sin x - \int(\sin x + x\cos x)\,dx =}$$
$$\cancel{= x^2\sin x + \cos x - x^2\cos x + \int(\cos x - x\sin x)\,dx}$$
$$\cancel{2I = x^2\sin x + \cos x - x^2\cos x + \sin x}$$

$$\int x\sin x\,dx = -\int x\,d\cos x = -x\cos x + \int\cos x\,dx =$$
$$= -x\cos x + \sin x + C$$

INT18

$$I=\int \sqrt{2-x^2}\,dx = x\sqrt{2-x^2} - \int x\,d\sqrt{2-x^2} =$$

$$= x\sqrt{2-x^2} + 2\cdot\frac{1}{2}\int \frac{x^2}{\sqrt{2-x^2}}\,dx =$$

$$= x\sqrt{2-x^2} - \int \frac{2-x^2+2}{\sqrt{2-x^2}}\,dx = x\sqrt{2-x^2} -$$

$$- \int \sqrt{2-x^2}\,dx - 2\int \frac{dx}{\sqrt{2-x^2}} = x\sqrt{2-x^2} - I - 2\arcsin\frac{x}{\sqrt{2}} + c$$

$$\Rightarrow I = \frac{x\sqrt{2-x^2}}{2} - \arcsin\frac{x}{\sqrt{2}} + C$$

INT19

$$\int x\,ctg^2x\,dx = \int x\,\frac{1-\sin^2x}{\sin^2x}\,dx = \int \frac{x}{\sin^2x}\,dx - \int x\,dx$$

$$= -\frac{x^2}{2} - \int x\,d(ctg x) = -\frac{x^2}{2} - x\,ctg x + \int ctg x\,dx =$$

$$= -\frac{x^2}{2} - x\,ctg x + \int \frac{\cos x}{\sin x}\,dx = -\frac{x^2}{2} - x\,ctg x + \int \frac{d\sin x}{\sin x} =$$

$$= -\frac{x^2}{2} - x\,ctg x + \ln|\sin x| + c$$

INT20

$$\int arctg x\,dx = x\,arctg x - \int x\,d\,arctg x =$$

$$= x\,arctg x - \int \frac{x}{1+x^2}\,dx = x\,arctg x - \frac{1}{2}\int \frac{d(1+x^2)}{1+x^2} =$$

$$= x\,arctg x - \frac{1}{2}\ln|1+x^2| + c$$