# Exercises for "Financial Derivatives with C++"

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## Contents

The issue time for all options coincides with the initial time. The maturities, barrier, and exercise times are strictly greater than the initial time.

## Straddle option

$K$ : the strike.

$T$ : the maturity.

The straddle option is the sum of the standard European put and call options with the same strike and maturity. The payoff at $T$ is given by

$$V(T) = |S(T) - K|,$$

where $S(T)$ is the price of the stock at $T$.

# Corridor option

$N$ : the notional amount.

$L$ : the lower barrier.

$U$ : the upper barrier.

$(t_m)_{m=1,\ldots,M}$ : the barrier times.

Last barrier time $t_M$ is the maturity of the option. At maturity, the option pays the product of the notional on the percentage of the barrier times when the price of the stock is less than the upper barrier and greater than the lower barrier:

$$X(t_M) = N \frac{1}{M} \sum_{m=1}^{M} 1_{\{L < S(t_m) < U\}}.$$

Here, $S(t)$ is the price of the stock at time $t$.

# American butterfly

$K$ : the middle strike.

$W$ : the size of wing $(W < K)$.

$(t_m)_{m=1,\ldots,M}$ : the exercise times.

A holder of the option can exercise it at any time $t_m$. In this case, he receives the following payments:

1. a long position in the call with strike $K - W$,

2. a long position in the call with strike $K + W$,

3. 2 short positions in the call with strike $K$.

If not exercised, the option expires worthless.

## Down-and-rebate option

$N$ : the notional.

$L$ : the lower barrier.

$(t_m)_{m=1,...,M}$ : the barrier times.

The option pays notional at first barrier time $t_m$ when spot price $S(t_m)$ stays below lower barrier $L$. Otherwise, the option expires worthless.

## Up-and-in American put

$U$ : the upper barrier.

$(t_j)_{j=1,...,J}$ : the barrier times.

$K$ : the strike.

$(u_n)_{n=1,...,N}$ : the exercise times $(u_1 > t_1,\ u_N > t_J)$.

The option becomes American put option (with the given set of exercise times) after the first barrier time $u_n$ when the stock price is above upper barrier $U$. If $u_n = t_j$, that is, the barrier time is also an exercise time, then the American put can still be exercised at $t_j$. If the price of the stock stays below $U$ for all barrier times, then the option expires worthless.

## Cancellable currency swap

Parameters of underlying swap:

$N^{dom}$ : the notional in domestic currency (USD).

$N^{for}$ : the notional in foreign currency (EUR).

$R^{dom}$ : the fixed rate in the swap (USD).

$\delta L^{for}$ : the spread for foreign float (EUR rate).

$\delta t$ : the interval of time between payments given as year fraction.

$M$ : the total number of payments.

**Brief description.** At initial time we pay notional $N^{for}$ in EUR and receive notional $N^{dom}$ in USD. Later, at payment times we pay fixed rate $R^{dom}$ in USD and receive float rate $L^{for}$ plus spread $\delta L^{for}$ in EUR. At maturity we also pay notional $N^{dom}$ in USD and receive notional $N^{for}$ in EUR. At payment times we have the right to terminate the swap. In this case, the last transaction takes place at the next payment time and consists of interest payments and notionals (the same payments as at maturity).

We denote by $(t_m)_{m=1,\dots,M}$ the payment times of the swap:

$$t_m = t_0 + m\delta t, \quad m = 1, \dots, M.$$

1. At initial time $t_0$ we pay $N^{for}$ EUR and receive $N^{dom}$ USD.

2. At payment time $t_{m+1}$,

   (a) We receive float interest plus spread in EUR:

   $$N^{for}(L^{for}(t_m, t_m + \delta t) + \delta L^{for})\delta t \quad (\text{EUR})$$

   where $L^{for}(t_m, t_m + \delta t)$ is the the float (LIBOR) rate for EUR at $t_m$ for the period of $\delta t$ years.

   (b) We pay fixed interest in USD:

   $$N^{dom}R^{dom}\delta t.$$

   Then,

   (a) If the swap has been terminated at $t_m$ or, if current time $t_{m+1}$ is the maturity, we pay notional $N^{dom}$ USD and receive notional $N^{for}$ EUR. There are no payments after that.

   (b) Otherwise, we have the right to terminate the swap. In this case, the payments at $t_{m+2}$ will still take place.

In the single asset model in `cfl`, the forward curve defines the forward exchange rate. The exchange rate is the number of units of domestic currency needed to buy one unit of foreign currency.