INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2018

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
December 2018

A. General comments on the aims *of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

B. General comments on *student performance in this diet of the examination*

This exam was done well by those students who had prepared thoroughly. There were a large number of students however who were unprepared.

Most questions were very straightforward. Those of particular challenge were Questions 5, 7, 10 (b) and 12 (b).

The Examiners hope the comments below will assist in future studies.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1

(a) $_{12}q_{[54]} = \dfrac{\left(l_{[54]} - l_{66}\right)}{l_{[54]}} = \dfrac{\left(9585.6916 - 8695.6199\right)}{9585.6916} = 0.09285$ [1]

(b) $\ddot{a}_{65}^{(6)} = \ddot{a}_{65} - \dfrac{5}{12} = 12.276 - \dfrac{5}{12} = 11.859$ [1]

(c)

$$\bar{s}_{43:\overline{10|}} = (1.04)^{10} \times \frac{l_{43}}{l_{53}} \times \bar{a}_{43:\overline{10|}} = (1.04)^{10} \times \frac{l_{43}}{l_{53}} \times \bar{a}_{43} - \bar{a}_{53}$$

$$= (1.04)^{10} \times \frac{9826.2060}{9630.0522} \times \left(19.319 - 0.5\right) - \left(16.524 - 0.5\right) = 12.400$$

[2]

[Total 4]

Generally well done. The main issue was remembering the definition of $\bar{s}_{43:\overline{10|}}$ in (c)

Q2

(a)

$$\bar{Z} = \begin{cases} \bar{a}_{\overline{T_y|}} - \bar{a}_{\overline{T_x|}} & \text{if } T_y > T_x \\ 0 & \text{otherwise} \end{cases}$$

Alternatives

(1) $\bar{a}_{\overline{T_y|}} - \bar{a}_{\overline{\min\left(T_x, T_y\right)|}}$

(2) $\bar{a}_{\overline{T_y|}} - \bar{a}_{\overline{T_{x:y}|}}$

(3) $v^{T_x}\, \bar{a}_{\overline{T_y - T_x|}}$

[2]

(b)

$$E(\bar{Z}) = \bar{a}_y - \bar{a}_{xy} = \frac{1-\bar{A}_y}{\delta} - \frac{1-\bar{A}_{xy}}{\delta} = \frac{\bar{A}_{xy}-\bar{A}_y}{\delta}$$ [2]

[Total 4]

Surprisingly many students struggled with this and tried to answer with non-random variable functions. This gained no marks. A mark was also lost if the 0 otherwise" was missing from a random variable answer.

Q3

Value of benefits:

$$= 30000 \times (\ddot{a}_{\overline{5}|}^{(12)} + v^5 \times \frac{l_{70}}{l_{65}} \times \ddot{a}_{70}^{(12)}) \qquad [1]$$

$$= 30000 \times (\frac{1-v^5}{d^{(12)}} + v^5 \times \frac{l_{70}}{l_{65}} \times (\ddot{a}_{70} - \frac{11}{24})) \qquad [2]$$

$$= 30000 \times (\frac{(1-0.82193)}{0.039157} + v^5 \times \frac{9392.621}{9703.708}(12.934 - \frac{11}{24})) \qquad [1]$$

$$= 434{,}189$$

A very straightforward question of its type. Generally answered well. Main error was incorrect application of the monthly adjustment.

Q4

(i) A group of lives is selected from a larger group in a non-random way with regard to their mortality. [1]

For example, in the underwriting of applicants for life insurance policies, the lives selected for standard premium rates have lower mortality rates than the population from which they were selected. [1]

Some of this effect on mortality will be temporary, i.e. the longer the time since the selection date, the less will be the difference in mortality of the select group compared to the equivalent non-selected group at the same age. [1]

Thus, the effect of the initial selection tends to 'wear off' over time. This is temporary initial selection. [1]

[Maximum 3 marks]

[Credit any sensible example]

(ii) The initial underwriting process is based on a proposal form and may sometimes include a medical questionnaire and a medical examination. The absence of medical questions will allow more impaired lives to be included within the initial group of lives. [1]

The impact of temporary selection will be reduced in two ways:

- The period of selection will be shorter [1/2]
- The select mortality rates will be closer to the ultimate rates [1/2]

[Total 5]

Most students got some of the valid points above, but many had omissions. Credit was given by examiners for any acceptable points or examples not contained in the solution above.

Q5

In the first instance the following relationship is used:

$p_x = e^{-\int_0^1 \mu_{x+t}dt}$ which is equivalent to $e^{-\mu}$ for constant μ

$\Rightarrow \mu = -\ln(p_x)$

Thus the following values can be obtained:

$\mu_x = -\ln(p_x) = -\ln(0.99) = 0.01005$ and $\mu_{x+1} = -\ln(p_{x+1}) = -\ln(0.975) = 0.02532$

[2]

EPV of benefits:

$$100,000 \times \int_0^1 (0.01005 \times e^{-(0.01005+.05)t})dt + 150,000 \times e^{-0.06005} \times \int_0^1 (0.02532 \times e^{-(0.02532+.05)t})dt$$

$$= 1005 \times \left(\frac{1-e^{-0.06005}}{0.06005}\right) + 3798 \times e^{-0.06005} \times \left(\frac{1-e^{-0.07532}}{0.07532}\right)$$

$$= 1005 \times 0.970567 + 3798 \times 0.941717 \times 0.963268$$

$$= 975.42 + 3445.26$$

$$= 4421 \text{ rounded}$$

[2 marks each of first 2 lines; 1 mark for result]

Overall this question was poorly answered. Many students failed to appreciate the simple link $\mu = -\ln(p_x)$ and tried all sorts of approximations. A small amount of credit was given for close approximations but to score well required the above solution.

Q6

(i)

(a) The directly standardised mortality rate is calculated as

$$\frac{\sum_x {}^sE^c_{x,t}\, m_{x,t}}{\sum_x {}^sE^c_{x,t}}$$

[2]

(b)

The indirectly standardised mortality rate is calculated as

$$\frac{\sum_x {}^sE^c_{x,t}\ {}^sm_{x,t}}{\sum_x {}^sE^c_{x,t}} \Bigg/ \frac{\sum_x E^c_{x,t}\ {}^sm_{x,t}}{\sum_x E^c_{x,t}\ m_{x,t}}$$

$$= \begin{array}{c}\text{Crude mortality rate}\\ \text{for standard population}\end{array} \Bigg/ \frac{\text{Expected deaths in population}}{\text{Actual deaths in population}}$$

[2]

(ii)

Directly standardised

$$\frac{(140{,}000 \text{ x } 125 / 10{,}236 + 156{,}000 \text{ x } 156 / 11{,}256 + 168{,}000 \text{ x } 166 / 10{,}633)}{464{,}000}$$

=0.013997

[2]

Indirectly standardised

Crude mortality rate for standard population
(140,000 x 0.00169 + 156,000 x 0.00220 + 168,000 x 0.00277) / 464,000 = 0.002253
Expected deaths in population
(10,236 x 0.00169 + 11,256 x 0.00220 + 10,633 x 0.00277) = 71.5
Actual deaths in population = 447
Indirectly standardised rate = 0.002253 / (71.5 / 447) = 0.014079 [2]

[Total 8]

Part (i) was knowledge based and part (ii) a straightforward application. The question was generally well done by fully prepared students.

Q7

Derive the constant force of mortality for 70 to 71 using

$$\mu = -\log(p_{70}) = -\log\left(\frac{9112.449}{9238.134}\right) = 0.013698 \qquad [1]$$

$$_{0.25}p_{70.75} = e^{-0.25\times 0.013698} = 0.996581 \qquad [1/2]$$

$$\bar{A}_{70.75} = \int_0^{0.25} v^t \times {}_tp_{70.75} \times \mu dt + v^{0.25} \times {}_{0.25}p_{70.75} \times \bar{A}_{71} \qquad [2]$$

$$\bar{A}_{71} \approx 1.04^{0.5} \times A_{71} = 1.04^{0.5} \times (1 - d\ddot{a}_{71}) = 1.04^{0.5} \times \left(1 - \frac{0.04}{1.04} \times 11.136\right) = 0.583014 \qquad [2]$$

$$\int_0^{0.25} v^t \times {}_tp_{70.75} \times \mu dt = 0.013698 \int_0^{0.25} e^{-\log(1.04)t} \times e^{-0.013698t} dt$$

$$= 0.013698 \times \left[\frac{-e^{-(\log(1.04)+0.013698)t}}{\log(1.04)+0.013698}\right]_0^{0.25} \qquad [2]$$

$$= 0.003402$$

Therefore $\bar{A}_{70.75} = 0.003402 + v^{0.25} \times 0.996581 \times 0.583014 = 0.578754$ [1/2]

[Total 8]

Students generally struggled with this question and failed to spot the same link as in Q5. Because of the low impact on the final answer from the first quarter many students successfully got an answer close to the solution using other approximations. In this case this was given a fair proportionate credit.

Q8

(i)

- Allocated premiums are invested in the fund(s) chosen by the policyholder which purchases a number of units within the fund(s)
- Each investment fund is divided into units, which are priced regularly (usually daily)
- Policyholder receives the value of the units allocated to their own policy
- Benefits are directly linked to the value of the underlying investments
- Unallocated premiums are directed to the company's non-unit fund
- Bid/offer spread is used to help cover expenses and contribute towards profit
- Charges are made from the unit account periodically to cover expenses and benefits (i.e. fund management charge) and may be varied after notice of change given.
- Unit-linked contracts may offer guaranteed benefits (e.g. minimum death benefit)

- Unit-linked contracts are generally endowment assurance and whole of life contracts

0.5 mark for each feature [max 4]

(ii) To calculate the expected reserves at the end of each year we have (utilising the end of year cashflow figures):

$p_{53} = 0.996461 \quad p_{52} = 0.996848 \quad p_{51} = 0.997191$

$${}_3V = \frac{1075.23}{1.025} = 1{,}049.00$$

$${}_2V \times 1.025 - p_{53} \times {}_3V = 355.10 \Rightarrow {}_2V = 1{,}366.23$$

$${}_1V \times 1.025 - p_{52} \times {}_2V = 401.56 \Rightarrow {}_1V = 1{,}720.47$$

[3]

The revised cash flow for year 1 will become:

$1{,}798.01 - p_{51} \times 1{,}720.47 = 82.37$ [1]

Revised profit vector becomes (82.37, 0, 0, 0) and
Net present value of profits = 82.37/(1.045) = 78.82 [1]

[Total 9]

In (i) very few students got the maximum number of points requested. There were no issues with (ii) except for many students got arithmetical errors whilst appreciating the techniques required.

Q9

(i) The dependent rates of decrement are calculated for each policy year using:-

$$(aq)_x^j = \frac{\mu^j}{\mu^d + \mu^r + \mu^s}\left[1 - e^{-(\mu^d + \mu^r + \mu^s)}\right]$$

where d denotes mortality, r retirement and s surrender

$\Rightarrow$ for policy years 1 and 2

$$(aq)^d = \frac{\mu^d}{\mu^d + \mu^r + \mu^s}\left[1 - e^{-(\mu^d + \mu^r + \mu^s)}\right] = \frac{0.015}{0.085}\left[1 - e^{-(0.085)}\right] = 0.01438$$

$$(aq)^r = \frac{\mu^r}{\mu^d + \mu^r + \mu^s}\left[1 - e^{-(\mu^d + \mu^r + \mu^s)}\right] = \frac{0.02}{0.085}\left[1 - e^{-(0.085)}\right] = 0.019174$$

$$(aq)^s = \frac{\mu^s}{\mu^d + \mu^r + \mu^s}\left[1 - e^{-(\mu^d + \mu^r + \mu^s)}\right] = \frac{0.05}{0.085}\left[1 - e^{-(0.085)}\right] = 0.047934$$

$\Rightarrow$ for policy year 3

$$(aq)^d = \frac{\mu^d}{\mu^d + \mu^r}\left[1 - e^{-(\mu^d + \mu^r)}\right] = \frac{0.015}{0.035}\left[1 - e^{-(0.035)}\right] = 0.014741$$

$$(aq)^r = \frac{\mu^r}{\mu^d + \mu^r}\left[1 - e^{-(\mu^d + \mu^r)}\right] = \frac{0.02}{0.035}\left[1 - e^{-(0.035)}\right] = 0.019654$$

which gives the following multiple decrement table:

Year t	μ^d	μ^s	μ^r	$(aq)^d$	$(aq)^s$	$(aq)^r$	(ap)	$_{t-1}(ap)$
1	.015	.05	.02	.014380	.047934	.019174	.918512	1.0
2	.015	.05	.02	.014380	.047934	.019174	.918512	.918512
3	.015	0	.02	.014741	0	.019654	.956505	.843665

[3]

(ii) Cash flows:

Year t	Premium P	Expenses E	Interest on P-E	Death Claim	Surrender Claim	Redundancy Claim	Maturity Claim	Profit Vector
1	12500.00	312.50	304.69	545.96	200.18	254.78	0	11491.27
2	12500.00	312.50	304.69	545.96	400.37	509.56	0	11036.30
3	12500.00	312.50	304.69	559.64	0	783.49	37658.61	-26509.55

[4]

Note: allowance for ½ year interest roll up is included in death, surrender and redundancy costs

Year t	Profit Vector	Cum probability of survival	Profit signature	Discount factor	NPV of Profit signature
1	11491.27	1	11491.27	.961538	11049.30
2	11036.30	.918512	10136.98	.924556	9372.21
3	-26509.55	.843665	-22365.18	.888996	-19882.56

=> Total NPV of profit = 538.94 [1½]

NPV of premium = 12,500 x (1 + .918512x .961538 + .843665 x .924556)
= 33,290.01 [1]

Therefore, profit margin = 538.94/33,290.01 = 1.62% [½]

Generally well attempted although often with arithmetical errors. Credit was given for understanding the process even though accuracy was not always attained.

Q10

(a) Let P be the annual premium for the policy. Then (functions at 6%):-

EPV of premiums:

$P\ddot{a}_{[35]} = 15.993P$

[0.5]

EPV of benefits:

$85,000A_{[35]}$

[1]

EPV of expenses:

$0.75P + 350 + (85 + 0.025P)a_{[35]}$

[2]

Equation of value gives:-

$P\ddot{a}_{[35]} = 85,000A_{[35]} + 350 + 0.75P + (0.025P + 85)a_{[35]}$

$P \times 15.993 = 85,000 \times 0.09475 + 350 + 0.75P + (0.025P + 85) \times 14.993$

$\Rightarrow P = \dfrac{9,678.155}{14.868175} = 650.93$

[0.5]

(b) Let $P^{/}$ be the required minimum office premium. Then the insurer's loss random variable for this policy is given by (where K and T denote the curtate and complete future lifetime of a policyholder):-

$L = 85,000v^{K_{[35]}+1} + 350 + 0.75P^{/} + (0.025P^{/} + 85)a_{\overline{K_{[35]}|}} - P^{/}\ddot{a}_{\overline{K_{[35]}+1|}}$ [2]

We need to find a value of t such that

$P(L > 0) = P(T < t) = 0.05 \Rightarrow P(T \geq t) = 0.95$ [0.5]

Using AM92 Select, we require:-

$\dfrac{l_{[35]+t}}{l_{[35]}} \geq 0.95 \Rightarrow l_{[35]+t} \geq 0.95 l_{[35]} = 0.95 \times 9892.9151 = 9398.269$ [1]

As $l_{58} = 9413.8004$ *and* $l_{59} = 9354.004$ then t lies between 23 and 24 so $K_{[35]} = 23$.

[0.5]

We therefore need the minimum premium such that

$L = 0 = 85,000v^{24} + 350 + 0.75P^{/} + (0.025P^{/} + 85)a_{\overline{23|}} - P^{/}\ddot{a}_{\overline{24|}}$

$\Rightarrow 0 = 85,000 \times 0.24698 + 350 + 0.75P^{/} + (0.025P^{/} + 85) \times 12.3034 - 13.3034P^{/}$

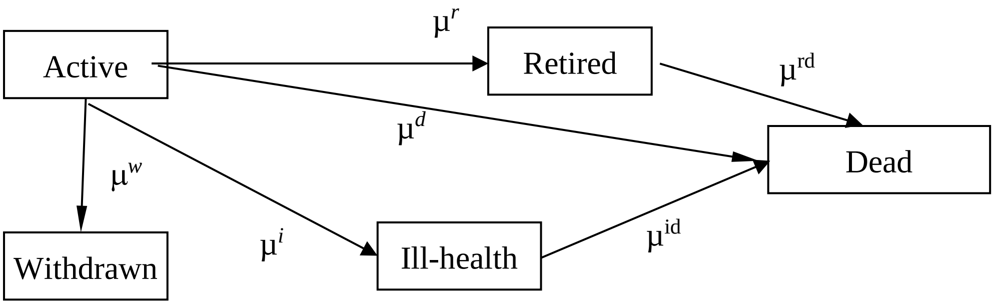
$\Rightarrow P^{/} = \dfrac{22,389.089}{12.2458} = 1828.31$

[2]

No issues with part (i). Most students struggled with part (ii) generally failing to appreciate how to approach the problem.

Q11

(i)



Ill health means ill health retirement

[1/2 for each box, 1/2 for each labelled arrow, extra ½ if complete, total 6]

(ii)

$$\text{Value} = 30,000 \times \frac{s_{50}}{s_{49}} \times \frac{1}{80} \times \frac{\left[\left(10\left({}^{z}M_{50}^{ra} + {}^{z}M_{50}^{ia}\right) + \left({}^{z}\bar{R}_{50}^{ra} + {}^{z}\bar{R}_{50}^{ia}\right)\right]}{{}^{s}D_{50}}$$

$$= 30,000 \times \frac{9.165}{9.031} \times \frac{1}{80} \times \frac{\left[10(128026 + 45392) + (1604000 + 363963)\right]}{16460}$$

$$= 85,596$$

[3 marks for formula, 1 for calculation]

Very straightforward question done well. Main omissions were parts of the diagrams and the salary factor in (ii).

Q12

(a) Let P be the monthly premium for version A of the contract. Then equation of value (at 4% p.a. interest) is: -

$$12P\ddot{a}^{(12)}_{[35]:\overline{30}|} = \frac{100,000}{1.04}\times 1.04^{0.5}\times A^{@0\%}_{[35]} + 225\times 1.04^{0.5}\times A^{@4\%}_{[35]} + 275 + 0.4\times 12P$$

[½] [1] [½] [½] [½]

$$+0.025\times 12P(\ddot{a}^{(12)}_{[35]:\overline{30}|} - \frac{1}{12}) + 0.025\times 12P(\ddot{a}^{(12)}_{[35]:\overline{30}|} - \frac{1}{12})$$

[½]

$$12P(.95\ddot{a}^{(12)}_{[35]:\overline{30}|}) - 4.75P = \frac{100,000}{1.04}\times 1.04^{0.5}\times A^{@0\%}_{[35]} + 225\times 1.04^{0.5}\times A^{@4\%}_{[35]} + 275$$

where $\ddot{a}^{(12)}_{[35]:\overline{30}|} = \ddot{a}_{[35]:\overline{30}|} - \frac{11}{24}\left(1 - v^{30}\ {}_{30}p_{[35]}\right) = 17.631 - \frac{11}{24}\left(1 - 0.30832\times\frac{8821.2612}{9892.9151}\right)$

$= 17.631 - \frac{11}{24}\left(1 - 0.27492\right) = 17.2987$ [½]

and $A^{@0\%}_{[35]} = 1$ [½]

$$\Rightarrow 12P\ (.95\times 17.2987) - 4.75P = \frac{100,000}{(1.04)^{0.5}} + 225\times 1.04^{0.5}\times 0.19207 + 275$$

$$192.4552P = 98,058.0676 + 44.0716 + 275$$

$\Rightarrow P^{/} = \frac{98,377.1392}{192.4552} = £511.17$ [½]

[Total 5]

(b) Let b be the simple bonus per annum for version B of the contract that can be supported by a monthly premium P = 511.17. Then equation of value (at 4% p.a. interest) is:

$$12P(.95\ddot{a}^{(12)}_{[35]:\overline{30}|}) - 4.75P = (100,000(1-b) + 225)\bar{A}_{[35]} + 100,000b(I\bar{A})_{[35]} + 275$$

[1½] [1] [½] [1] [½]

$$\Rightarrow 192.4552P = (100,000(1-b) + 225)\times 1.04^{0.5}\times 0.19207 + 100,000b\times 1.04^{0.5}\times 7.47005 + 275$$

$$98,377.32 = 19,587.374(1-b) + 44.0716 + 761,798.61b + 275$$

$\Rightarrow b = \frac{78,470.88}{742,211.24} = .1057 \Rightarrow 10.6\%$ [½]

[Total 5]

> *Part (a) was generally done well although some students struggled to include all the required parameters in the equation of value.*
>
> *Part (b) proved challenging for many despite the fact that it was in reality a quite simple formula.*

Q13

(a) Annual net premium for the decreasing term assurance is given by:

$$P = \frac{520{,}000 A^{1}_{40:\overline{25|}} - 20{,}000 (IA)^{1}_{40:\overline{25|}}}{\ddot{a}_{40:\overline{25|}}}$$

$$\text{where } A^{1}_{40:\overline{25|}} = A_{40:\overline{25|}} - v^{25}\,{}_{25}p_{40}$$

$$= 0.38907 - 0.37512 \times \frac{8821.2612}{9856.2863} = 0.38907 - 0.33573 = 0.05334$$

$$\text{and } (IA)^{1}_{40:\overline{25|}} = (IA)_{40} - v^{25}\,{}_{25}p_{40}\left[25A_{65} + (IA)_{65}\right]$$

$$= 7.95699 - 0.33573 \times \left[25 \times 0.52786 + 7.89442\right] = 0.87612$$

$$P = \frac{520{,}000 \times 0.05334 - 20{,}000 \times 0.87612}{15.884} = 643.06$$

2½ marks for premium formula, ½ mark for evaluating the first assurance function, 1 mark for evaluating the second assurance function and ½ mark for correct answer for *P* [4.5]

(b) Reserve at the <u>end</u> of the 17th policy year given by:

$${}_{17}V = 180{,}000 A^{1}_{57:\overline{8|}} - 20{,}000 (IA)^{1}_{57:\overline{8|}} - P\ \ddot{a}_{57:\overline{8|}}$$

$$\text{where } A^{1}_{57:\overline{8|}} = A_{57:\overline{8|}} - v^{8}\,{}_{8}p_{57}$$

$$= 0.73701 - 0.73069 \times \frac{8821.2612}{9467.2906} = 0.73701 - 0.68083 = 0.05618$$

$$\text{and } (IA)^{1}_{57:\overline{8|}} = (IA)_{57} - v^{8}\,{}_{8}p_{57}\left[8A_{65} + (IA)_{65}\right]$$

$$= 8.52268 - 0.68083 \times \left[8 \times 0.52786 + 7.89442\right] = 0.27286$$

$${}_{17}V = 180{,}000 \times 0.05618 - 20{,}000 \times 0.27286 - 643.06 \times 6.838$$

$$= 10{,}112.40 - 5{,}457.20 - 4{,}397.24 = 257.96$$

2½ marks for reserve formula, ½ mark for evaluating the first each assurance function, 1 mark for evaluating the second assurance function and ½ mark for correct answer for *V* [4]

Therefore, sum at risk per policy in the 17th policy year is:

DSAR = 180,000 – 257.96 = 179,742.04

[1]

Mortality profit = EDS – ADS

$$EDS = 1527 \times q_{56} \times 179,742.04 = 1527 \times 0.005025 \times 179,742.04 = 1,379,192.13$$

$$ADS = 9 \times 179,742.04 = 1,617,678.36$$

[1]

i.e. mortality profit = - 238,486.23 (i.e. a loss) [0.5]

Generally well done by students who had prepared. The main error was establishing the correct duration and sum assured for the reserve in (b).

END OF EXAMINERS' REPORT