EXAMINATION

7 October 2010 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   Enter all the candidate and examination details as requested on the front of your answer booklet.

2.   You must not start writing your answers in the booklet until instructed to do so by the supervisor.

3.   Mark allocations are shown in brackets.

4.   Attempt all 10 questions, beginning your answer to each question on a separate sheet.

5.   Candidates should show calculations where this is appropriate.

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

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1   A bond pays coupons in perpetuity on 1 June and 1 December each year. The annual coupon rate is 3.5% per annum. An investor purchases a quantity of this bond on 20 August 2009.

Calculate the price per £100 nominal to provide the investor with an effective rate of return per annum of 10%. [3]

2   A bond is redeemed at £110 per £100 nominal in exactly four years' time. It pays coupons of 4% per annum half-yearly in arrear and the next coupon is due in exactly six months' time. The current price is £110 per £100 nominal.

(i)   (a)   Calculate the gross rate of return per annum convertible half-yearly from the bond.

(b)   Calculate the gross effective rate of return per annum from the bond.
[2]

(ii)   Calculate the net effective rate of return per annum from the bond for an investor who pays income tax at 25%. [2]
[Total 4]

3   The annual rates of return from an asset are independently and identically distributed. The expected accumulation after 20 years of £1 invested in this asset is £2 and the standard deviation of the accumulation is £0.60.

(a)   Calculate the expected effective rate of return per annum from the asset, showing all the steps in your working.

(b)   Calculate the variance of the effective rate of return per annum.
[6]

4   A six-month forward contract was issued on 1 April 2009 on a share with a price of 700p at that date. It was known that a dividend of 20p per share would be paid on 1 May 2009. The one-month spot, risk-free rate of interest at the time of issue was 5% per annum effective and the forward rate of interest from 1 May to 30 September was 3% per annum effective.

(i)   Calculate the forward price at issue, assuming no arbitrage, explaining your working. [3]

It has been suggested that the forward price cannot be calculated without making a judgement about the expected price of the share when the forward contract matures.

(ii)   Explain why this statement is not correct. [2]

(iii)   Comment on whether the method used in part (i) would still be valid if it was not known with certainty that the dividend due on 1 May 2009 would be paid.
[1]
[Total 6]

5 (a)   Describe the characteristics of Eurobonds.
       (b)   Describe the characteristics of convertible bonds.

[6]

6 On 1 January 2001 the government of a particular country bought 200 million shares in a particular bank for a total price of £2,000 million. The shares paid no dividends for three years. On 30 June 2004 the shares paid dividends of 10 pence per share. On 31 December 2004, they paid dividends of 20 pence per share. Each year, until the end of 2009, the dividend payable every 30 June rose by 10% per annum compound and the dividend payable every 31 December rose by 10% per annum compound. On 1 January 2010, the shares were sold for their market price of £3,500 million.

   (i)   Calculate the net present value on 1 January 2001 of the government's investment in the bank at a rate of interest of 8% per annum effective.   [5]

   (ii)   Calculate the accumulated profit from the government's investment in the bank on the date the shares are sold using a rate of interest of 8% per annum effective.   [1]

[Total 6]

7 (i)   State the three conditions that are necessary for a fund to be immunised from small, uniform changes in the rate of interest.   [2]

   (ii)   A pension fund has liabilities of £10m to meet at the end of each of the next ten years. It is able to invest in two zero-coupon bonds with a term to redemption of three years and 12 years respectively. The rate of interest is 4% per annum effective.

   Calculate:

   (a)   the present value of the liabilities of the pension fund

   (b)   the duration of the liabilities of the pension fund

   (c)   the nominal amount that should be invested in the zero-coupon bonds to ensure that the present values and durations of the assets and liabilities is the same

[7]

   (iii)   One year later, just before the pension payment then due, the rate of interest is 5% per annum effective.

   (a)   Determine whether the duration of the assets and the liabilities are still equal.

   (b)   Comment on the practical usefulness of the theory of immunisation in the context of the above result.

[6]
[Total 15]

8   The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.05 + 0.001t & 0 \le t \le 20 \\ 0.05 & t > 20 \end{cases}$$

(i)   Derive and simplify as far as possible expressions for $v(t)$, where $v(t)$ is the present value of a unit sum of money due at time t.   [5]

(ii)   (a)   Calculate the present value of £100 due at the end of 25 years.

(b)   Calculate the rate of discount per annum convertible quarterly implied by the transaction in part (ii)(a).   [4]

(iii)   A continuous payment stream is received at rate $30e^{-0.015t}$ units per annum between $t = 20$ and $t = 25$. Calculate the accumulated value of the payment stream at time $t = 25$.   [4]

[Total 13]

9   The government of a particular country has just issued three bonds with terms to redemption of exactly one, two and three years respectively. Each bond is redeemed at par and pays coupons of 8% annually in arrear. The annual effective gross redemption yields from the one, two and three year bonds are 4%, 3% and 3% respectively.

(i)   Calculate the one-year, two-year and three-year spot rates of interest at the date of issue.   [8]

(ii)   Calculate all possible forward rates of interest from the above spot rates of interest.   [4]

An index of retail prices has a current value of 100.

(iii)   Calculate the expected level of the retail prices index in one year, two years' and three years' time if the expected real spot rates of interest are 2% per annum effective for all terms.   [5]

(iv)   Calculate the expected rate of inflation per annum in each of the next three years.   [2]

[Total 19]

10 On 1 April 2003 a company issued securities that paid no interest and that were to be redeemed for £70 after five years. The issue price of the securities was £64. The securities were traded in the market and the market prices at various different dates are shown in the table below.

Date	Market price of securities (£)
1 April 2003	64
1 April 2004	65
1 April 2005	60
1 April 2006	65
1 April 2007	68
1 April 2008	70

(i) Explain why the price of the securities might have fallen between 1 April 2004 and 1 April 2005. [1]

Two investors bought the securities at various dates. Investor X bought 100 securities on 1 April 2003 and 1,000 securities on 1 April 2005. Investor Y bought 100 securities every year on 1 April from 2003 to 2007 inclusive. Both investors held the securities until maturity.

(ii) Construct a table showing the nominal amount of the securities held and the market value of the holdings for X and Y on 1 April each year, just before any purchases of securities. [5]

(iii) (a) Calculate the effective money weighted rate of return per annum for X for the period from 1 April 2003 to 1 April 2008.

(b) Calculate the effective time weighted rate of return per annum for X for the period from 1 April 2003 to 1 April 2008.

[6]

(iv) (a) Determine whether the effective money weighted rate of return for Y is lower or higher than that for X for the period from 1 April 2003 to 1 April 2008.

(b) Determine the effective time weighted rate of return per annum for Y for the period from 1 April 2003 to 1 April 2008.

[7]

(v) Discuss the relationship between the different rates of return that have been calculated. [3]

[Total 22]

END OF PAPER