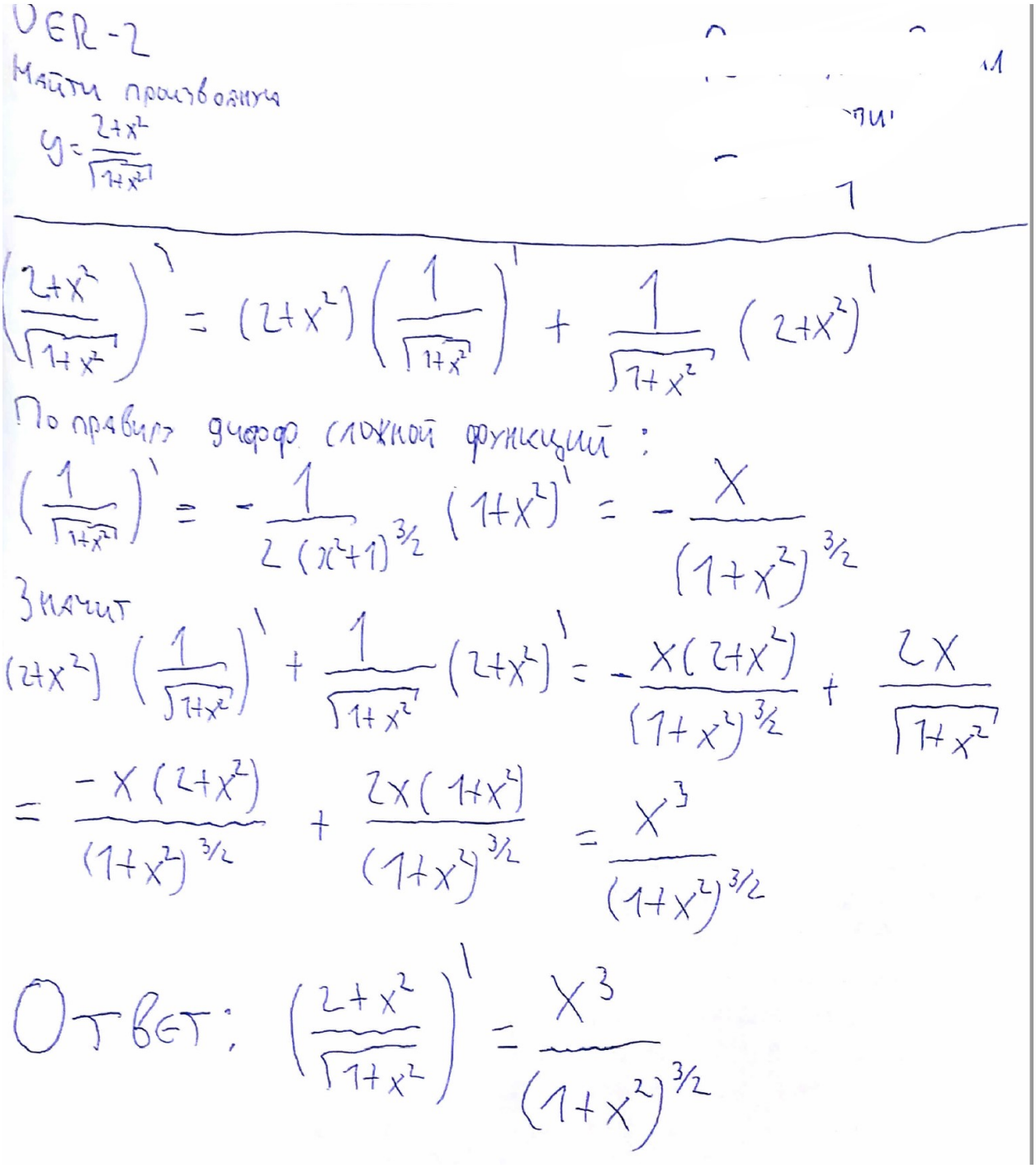
6 Подсказки и решения

6.1 Производные

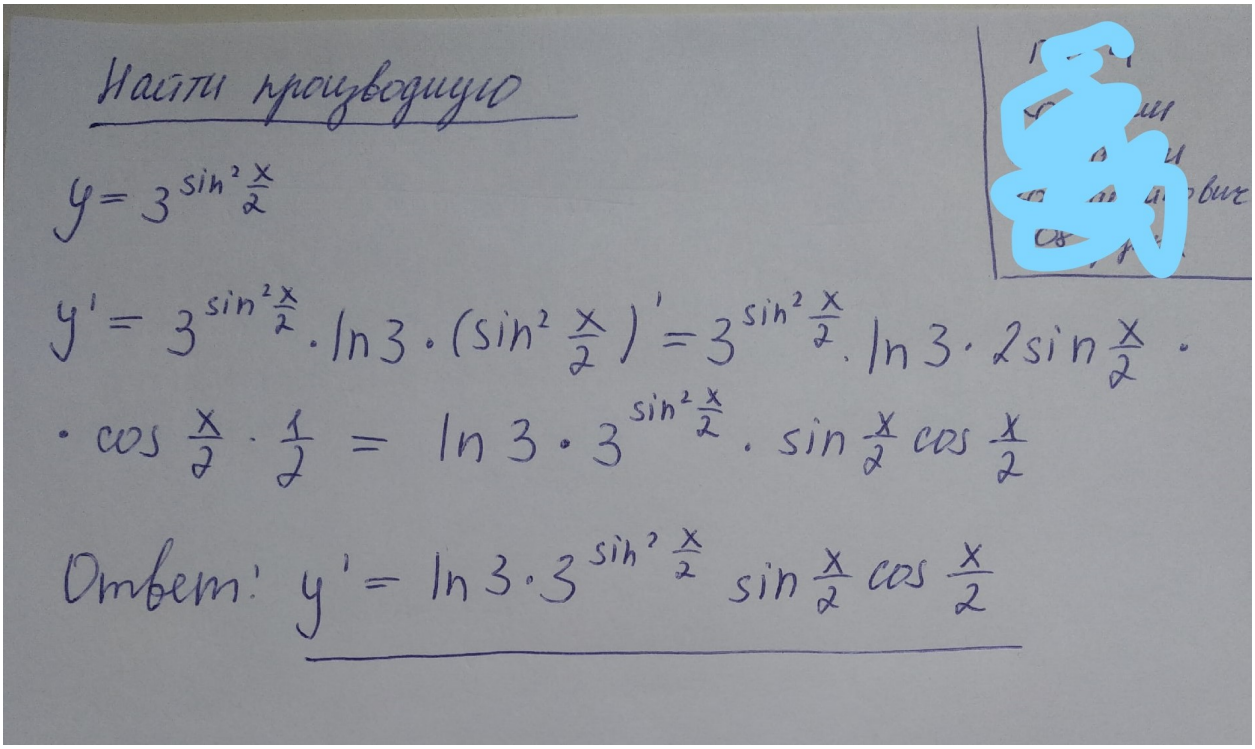
6.1.1 Таблица производных

$C' = 0$	$(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$
$x' = 1$	$(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$
$(x^n)' = n \cdot x^{n-1}$	$(\operatorname{arctg} x)' = \dfrac{1}{1+x^2}$
$(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$	$(\operatorname{arcctg} x)' = -\dfrac{1}{1+x^2}$
$(e^x)' = e^x$	$(\operatorname{sh} x)' = \operatorname{ch} x$
$(a^x)' = a^x \ln a$	$(\operatorname{ch} x)' = \operatorname{sh} x$
$(\ln x)' = \dfrac{1}{x}$	$(\operatorname{th} x)' = \dfrac{1}{\operatorname{ch}^2 x}$
$(\log_a x)' = \dfrac{1}{x \ln a}$	$(\operatorname{cth} x)' = -\dfrac{1}{\operatorname{sh}^2 x}$
$(\sin x)' = \cos x$	$(\operatorname{arcsh} x)' = \dfrac{1}{\sqrt{x^2+1}}$
$(\cos x)' = -\sin x$	$(\operatorname{arcch} x)' = \dfrac{1}{\sqrt{x^2-1}}$
$(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$	$(\operatorname{arcth} x)' = \dfrac{1}{1-x^2}$
$(\operatorname{ctg} x)' = -\dfrac{1}{\sin^2 x}$	$(\operatorname{arccth} x)' = \dfrac{1}{1-x^2}$

6.1.2 DER-2


DER-2
Найти производную
$y = \frac{2+x^2}{\sqrt{1+x^2}}$
$\left(\frac{2+x^2}{\sqrt{1+x^2}}\right)' = (2+x^2)\left(\frac{1}{\sqrt{1+x^2}}\right)' + \frac{1}{\sqrt{1+x^2}}(2+x^2)'$
По правилу дифф сложной функции:
$\left(\frac{1}{\sqrt{1+x^2}}\right)' = -\frac{1}{2(x^2+1)^{3/2}}(1+x^2)' = -\frac{x}{(1+x^2)^{3/2}}$
Значит
$(2+x^2)\left(\frac{1}{\sqrt{1+x^2}}\right)' + \frac{1}{\sqrt{1+x^2}}(2+x^2)' = -\frac{x(2+x^2)}{(1+x^2)^{3/2}} + \frac{2x}{\sqrt{1+x^2}}$
$= \frac{-x(2+x^2)}{(1+x^2)^{3/2}} + \frac{2x(1+x^2)}{(1+x^2)^{3/2}} = \frac{x^3}{(1+x^2)^{3/2}}$
Ответ: $\left(\frac{2+x^2}{\sqrt{1+x^2}}\right)' = \frac{x^3}{(1+x^2)^{3/2}}$

6.1.3 DER-7



Найти производную

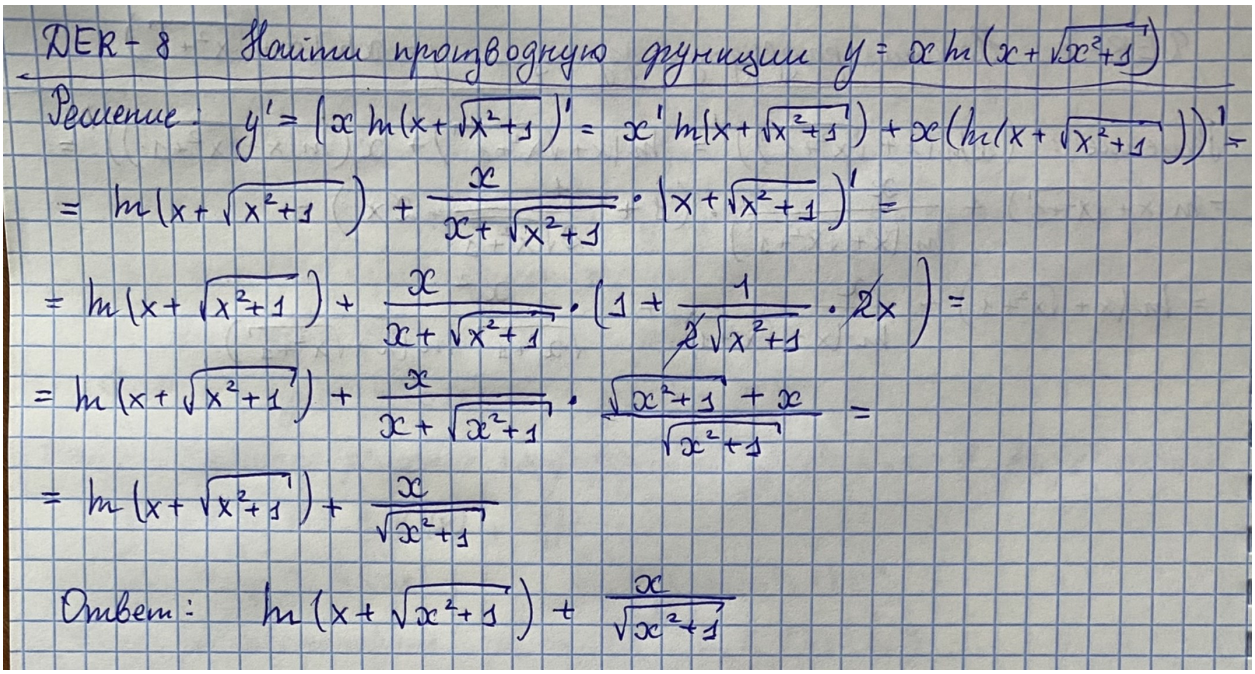
$y = 3^{\sin^2 \frac{x}{2}}$

$y' = 3^{\sin^2 \frac{x}{2}} \cdot \ln 3 \cdot (\sin^2 \frac{x}{2})' = 3^{\sin^2 \frac{x}{2}} \cdot \ln 3 \cdot 2 \sin \frac{x}{2} \cdot$

$\cdot \cos \frac{x}{2} \cdot \frac{1}{2} = \ln 3 \cdot 3^{\sin^2 \frac{x}{2}} \cdot \sin \frac{x}{2} \cos \frac{x}{2}$

Ответ: $y' = \ln 3 \cdot 3^{\sin^2 \frac{x}{2}} \sin \frac{x}{2} \cos \frac{x}{2}$

6.1.4 DER-8



DER-8 Найти производную функции $y = x \ln(x + \sqrt{x^2+1})$

Решение: $y' = (x \ln(x+\sqrt{x^2+1}))' = x' \ln(x+\sqrt{x^2+1}) + x(\ln(x+\sqrt{x^2+1}))' =$

$= \ln(x+\sqrt{x^2+1}) + \frac{x}{x+\sqrt{x^2+1}} \cdot (x+\sqrt{x^2+1})' =$

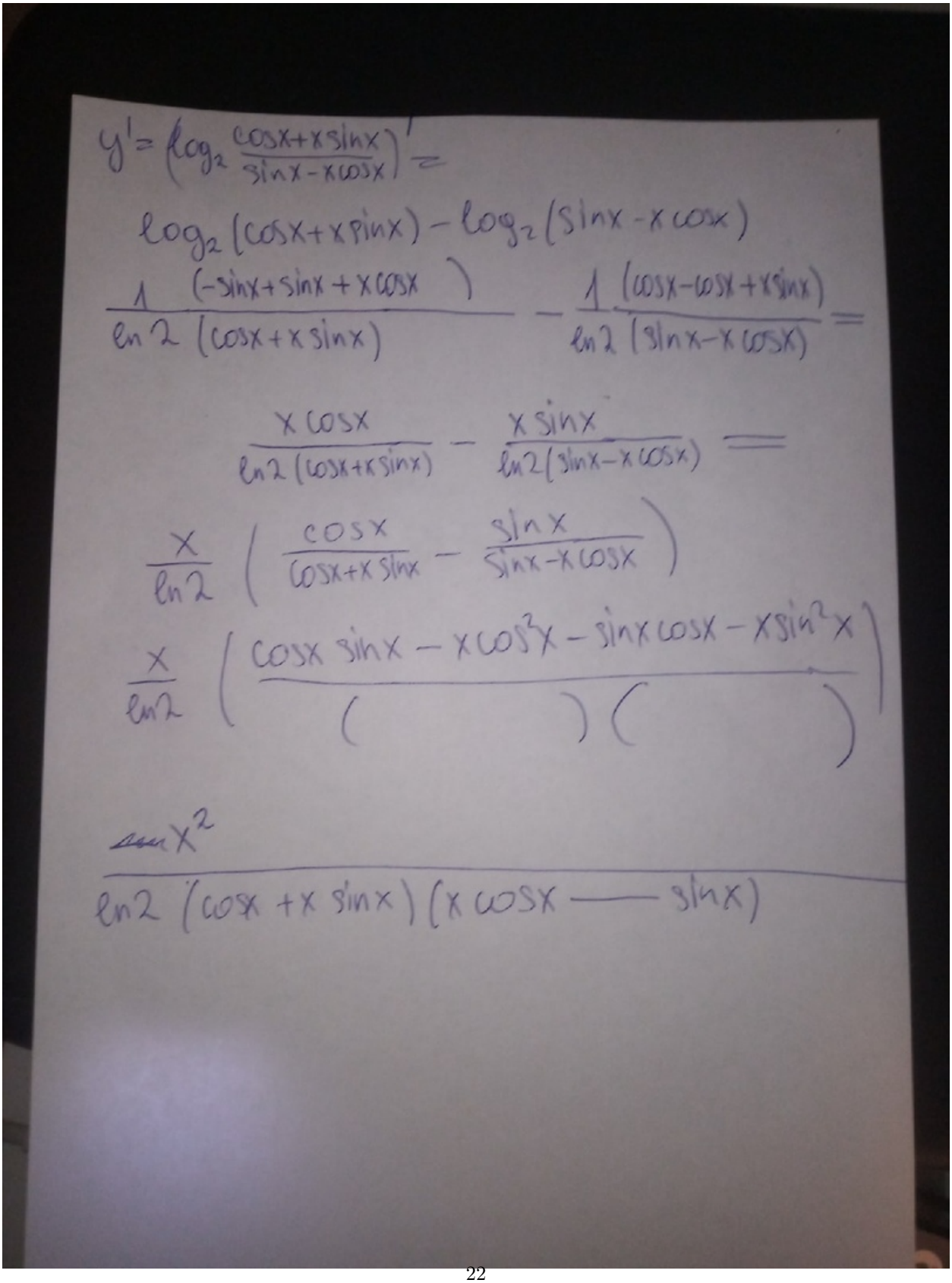
$= \ln(x+\sqrt{x^2+1}) + \frac{x}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2+1}} \cdot 2x\right) =$

$= \ln(x+\sqrt{x^2+1}) + \frac{x}{x+\sqrt{x^2+1}} \cdot \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}} =$

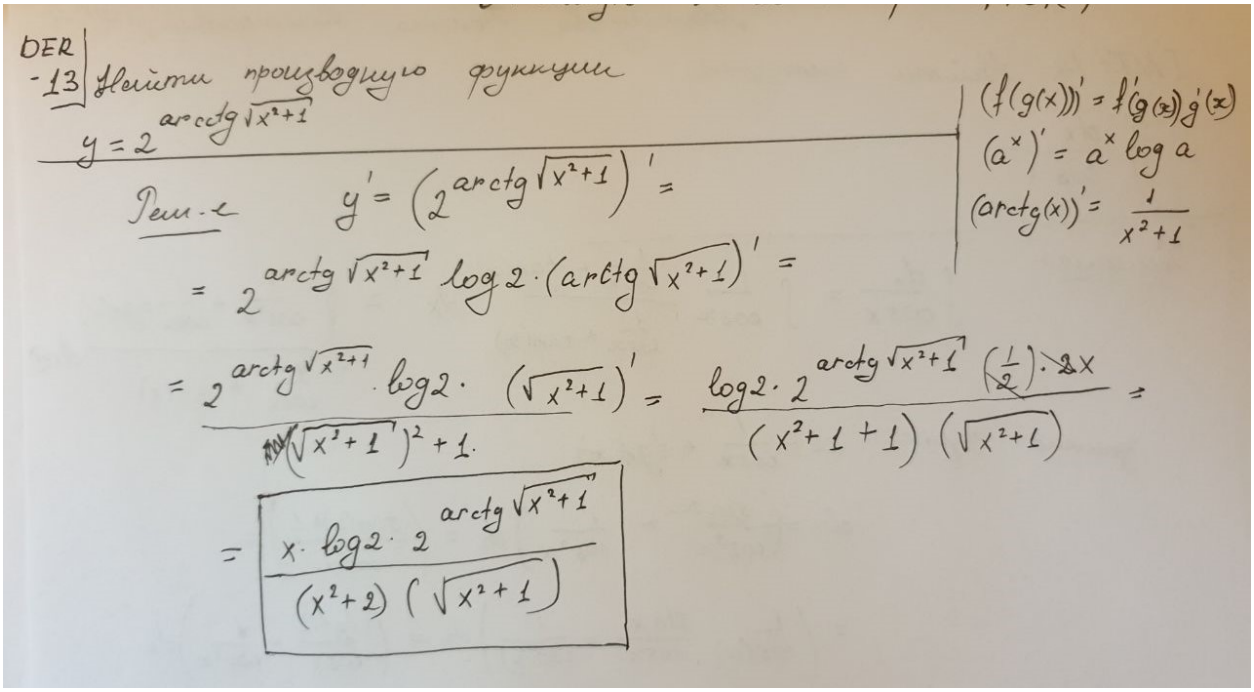
$= \ln(x+\sqrt{x^2+1}) + \frac{x}{\sqrt{x^2+1}}$

Ответ: $\ln(x+\sqrt{x^2+1}) + \frac{x}{\sqrt{x^2+1}}$

6.1.5 DER-10


$y' = \left(\log_2 \frac{\cos x + x \sin x}{\sin x - x \cos x}\right)' =$
$\log_2(\cos x + x \sin x) - \log_2(\sin x - x \cos x)$
$\frac{1}{\ln 2}\frac{(-\sin x + \sin x + x\cos x)}{(\cos x + x \sin x)} - \frac{1}{\ln 2}\frac{(\cos x - \cos x + x \sin x)}{(\sin x - x \cos x)} =$
$\frac{x \cos x}{\ln 2 (\cos x + x \sin x)} - \frac{x \sin x}{\ln 2 (\sin x - x \cos x)} =$
$\frac{x}{\ln 2}\left(\frac{\cos x}{\cos x + x \sin x} - \frac{\sin x}{\sin x - x \cos x}\right)$
$\frac{x}{\ln 2}\left(\frac{\cos x \sin x - x\cos^2 x - \sin x \cos x - x \sin^2 x}{(\quad)(\quad)}\right)$
x^2
$\ln 2 (\cos x + x \sin x)(x \cos x - \sin x)$

6.1.6 DER-13



6.2 Интегралы

6.2.1 Таблица интегралов

$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad \int \frac{dx}{x} = \ln|x|$$

$$\int e^x dx = e^x \qquad \int a^x dx = \frac{a^x}{\ln a}$$

$$\int \sin x\, dx = -\cos x \qquad \int \cos x\, dx = \sin x$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x \qquad \int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} \qquad \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$