EXAMINATION

23 September 2008 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   Enter all the candidate and examination details as requested on the front of your answer booklet.

2.   You must not start writing your answers in the booklet until instructed to do so by the supervisor.

3.   Mark allocations are shown in brackets.

4.   Attempt all 12 questions, beginning your answer to each question on a separate sheet.

5.   Candidates should show calculations where this is appropriate.

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

1 A 91-day government bill is purchased for £95 at the time of issue and is redeemed at the maturity date for £100. Over the 91 days, an index of consumer prices rises from 220 to 222.

Calculate the effective real rate of return per annum. [3]

2 (i) State the strengths and weaknesses of using the money-weighted rate of return as opposed to the time-weighted rate of return as a measure of an investment manager's skill. [3]

(ii) An investor had savings totalling £41,000 in an account on 1 January 2006. He invested a further £12,000 in this account on 1 August 2006. The total value of the account was £45,000 on 31 July 2006 and was £72,000 on 31 December 2007.

Assuming that the investor made no further deposits or withdrawals in relation to this account, calculate the annual effective time-weighted rate of return for the period 1 January 2006 to 31 December 2007. [2]

[Total 5]

3 (i) A forward contract with a settlement date at time T is issued based on an underlying asset with a current market price of B.

The annualised risk-free force of interest applying over the term of the forward contract is δ and the underlying asset pays no income. Show that the theoretical forward price is given by $K = Be^{\delta T}$, assuming no arbitrage. [3]

(ii) An asset has a current market price of 200p, and will pay an income of 10p in exactly three months' time.

Calculate the price of a forward contract to be settled in exactly six months, assuming a risk-free rate of interest of 8% per annum convertible quarterly. [3]

[Total 6]

4 Describe the characteristics of commercial property (i.e. commercial real estate) as an investment. [5]

5 A bank offers two repayment alternatives for a loan that is to be repaid over ten years. The first requires the borrower to pay £1,200 per annum quarterly in advance and the second requires the borrower to make payments at an annual rate of £1,260 every second year in arrears.

Determine which terms would provide the best deal for the borrower at a rate of interest of 4% per annum effective. [5]

6 A pension fund holds an asset with current value £1 million. The investment return on the asset in a given year is independent of returns in all other years. The annual investment return in the next year will be 7% with probability 0.5 and 3% with probability 0.5. In the second and subsequent years, annual investment returns will be 2%, 4% or 6% with probability 0.3, 0.4 and 0.3, respectively.

   (i)   Calculate the expected accumulated value of the asset after 10 years, showing all steps in your calculations. [3]

   (ii) Calculate the standard deviation of the accumulated value of the asset after 10 years, showing all steps in your calculations. [4]

   (iii) Without doing any further calculations explain how the mean and variance of the accumulation would be affected if the returns in years 2 to 10 were 1%, 4%, or 7%, with probability 0.3, 0.4 and 0.3 respectively. [2]
   [Total 9]

7 The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t) = \begin{cases} 0.05 + 0.02t & \text{for } 0 \le t \le 5 \\ 0.15 & \text{for } t > 5 \end{cases}$$

   (i)   Calculate the present value of £1,000 due at the end of 12 years. [5]

   (ii) Calculate the annual effective rate of discount implied by the transaction in (i). [2]
   [Total 7]

8 A tax advisor is assisting a client in choosing between three types of investment. The client pays tax at 40% on income and 40% on capital gains.

   Investment A requires the investment of £1m and provides an income of £0.1m per year in arrears for ten years. Income tax is deducted at source. At the end of the ten years, the investment of £1m is returned.

   In Investment B, the initial sum of £1m accumulates at the rate of 10% per annum compound for ten years. At the end of the ten years, the accumulated value of the investment is returned to the investor after deduction of capital gains tax.

   Investment C is identical to Investment B except that the initial sum is deemed, for tax purposes, to have increased in line with the index of consumer prices between the date of the investment and the end of the ten-year period. The index of consumer prices is expected to increase by 4% per annum compound over the period.

   (i)   Calculate the net rate of return expected from each of the investments. [7]

   (ii) Explain why the expected rate of return is higher for Investment C than for Investment B and is higher for Investment B than for Investment A. [3]
   [Total 10]

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9 Three bonds, paying annual coupons in arrears of 6%, are redeemable at £105 per £100 nominal and reach their redemption dates in exactly one, two and three years' time respectively. The price of each of the bonds is £103 per £100 nominal.

(i) Calculate the gross redemption yield of the three-year bond. [3]

(ii) Calculate to three decimal places all possible spot rates, implied by the information given, as annual effective rates of interest. [4]

(iii) Calculate to three decimal places all possible forward rates, implied by the information given, as annual effective rates of interest. [4]
[Total 11]

10 An insurance company is considering two possible investment options.

The first investment option involves setting up a branch in a foreign country. This will involve an immediate outlay of £0.25m, followed by investments of £0.1m at the end of one year, £0.2m at the end of two years, £0.3m at the end of three years and so on until a final investment is made of £1m in ten years' time. The investment will provide annual payments of £0.5m for twenty years with the first payment at the end of the eighth year. There will be an additional incoming cash flow of £5m at the end of the 27th year.

The second investment option involves the purchase of 1 million shares in a bank at a price of £4.20 per share. The shares are expected to provide a dividend of 21p per share in exactly one year, 22.05p per share in two years and so on, increasing by 5% per annum compound. The shares are expected to be sold at the end of ten years, just after a dividend has been paid, for £5.64 per share.

(i) Determine which of the options has the higher net present value at a rate of interest of 7% per annum effective. [9]

(ii) Without doing any further calculations, determine which option has the higher discounted mean term at a rate of interest of 7% per annum effective. [2]
[Total 11]

11   A company has a liability of £400,000 due in ten years' time.

The company has exactly enough funds to cover the liability on the basis of an effective interest rate of 8% per annum. This is also the interest rate on which current market prices are calculated and the interest rate earned on cash.

The company wishes to hold 10% of its funds in cash, and to invest the balance in the following securities:

- a zero-coupon bond redeemable at par in twelve years' time

- a fixed-interest stock which is redeemable at 110% in sixteen years' time bearing interest at 8% per annum payable annually in arrear

(i)   Calculate the nominal amounts of the zero-coupon bond and the fixed-interest stock which should be purchased to satisfy Redington's first two conditions for immunisation.   [10]

(ii)   Calculate the amount which should be invested in each of the assets mentioned in (i).   [2]

(iii)   Explain whether the company would be immunised against small changes in the rate of interest if the quantities of stock in part (i) are purchased.   [2]
[Total 14]

12 An individual takes out a 25-year bank loan of £300,000 to purchase a house.

The individual agrees to pay only the interest payments, monthly in arrear, for the first 15 years whereupon he repays half of the capital as a lump sum. He then pays only the interest for the remaining 10 years, quarterly in arrear, and repays the other half of the capital as a lump sum at the end of the term.

(i) Calculate the total amount of interest paid by the individual, assuming an effective rate of interest of 8½% p.a. [5]

(ii) The individual believes that he can earn a nominal rate of interest convertible half-yearly of 9% p.a. from a separate savings account.

Calculate the level contribution he must make monthly in advance to the savings account in order to repay half the capital after 15 years. [4]

(iii) The individual made the monthly contributions calculated in (ii) to the savings account. However, over the first 15 years, the effective rate of return earned on the savings account was 10% per annum.

The individual used the proceeds at that time to repay as much of the loan as possible and then decided to repay the remainder of the loan by level instalments of interest and capital. After the first 15 years, the effective rate of interest changed to 7% per annum.

Calculate the level payment he must make, payable monthly in arrear, to repay the loan over the final 10 years of the loan. [5]

[Total 14]

END OF PAPER