INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

29 April 2014 (am)

Subject CT5 – Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 13 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

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1   Define Class Selection giving two distinct examples.   [2]

2   Calculate:

(a) $\ddot{a}^{(4)}_{25:\overline{20}|}$

(b) $(I\bar{A})^{1}_{25:\overline{20}|}$

Basis: Mortality   AM92
        Interest   4% per annum
    [4]

3   For a particular species of animal the mortality and rate of interest are shown according to the Basis below.

Calculate $A_{3:\overline{5}|}$.

Basis: Mortality $l_x = l_0 e^{-0.15x}$
        Interest   5% per annum
    [4]

4   Outline with examples the advantages of "cash flow" or "discounted emerging costs" techniques for product pricing, compared with the use of traditional commutation functions.   [4]

5   A pension scheme provides a pension of 1/40 of final pensionable salary for each year of service, limited to a maximum of 2/3 of final pensionable salary, upon retirement at age 65. No other retirement age is allowed.

Final pensionable salary is defined as average annual salary over the 3 years immediately preceding retirement.

A member is now aged exactly 45 and has 16 years of past service. He earned £40,000 in the previous 12 months.

Calculate the expected present value now of this member's expected total pension on retirement.

Basis:

PEN Tables in the Formulae and Tables for Actuarial Examinations – Interest 4% per annum
    [4]

6 (a) Describe the Method of Constant Force of Mortality.

(b) Calculate $_{2.75}q_{85.5}$ using the method given in (a) above.

Basis: Mortality ELT15 (Males) [5]

7 A Joint Life Annuity is issued to a male aged 65 exact and a female aged 62 exact. The annuity is payable quarterly in arrear the first payment commencing 3 months after issue.

The annuity has the following conditions:

- £10,000 per annum whilst both lives survive.

- If the male life predeceases the female life the annuity reduces to £7,500 per annum payable for the remainder of her lifetime.

- If the female life predeceases the male life the annuity reduces to £6,000 per annum payable for the remainder of his lifetime.

Calculate the expected present value of the annuity.

Basis: Mortality PMA92C20 (male) and PFA92C20 (female)
      Interest 4% per annum
      Expenses Ignore

[6]

8 A double decrement table is to be constructed from two single decrement tables. The modes of decrement are α and β. The basis for each of the single decrement tables is shown below:

Basis:

In the table for single decrement α: $l^{\alpha}_{x+t} = l^{\alpha}_x - t^3 d^{\alpha}_x$ for $0 \le t \le 1$

In the table for single decrement β: $l^{\beta}_{x+t} = l^{\beta}_x - t^5 d^{\beta}_x$ for $0 \le t \le 1$

The l function represents the number of lives and the d function the number of decrements in the appropriate table.

(i) Show that

$$_t p^{\beta}_x \mu^{\beta}_{x+t} = 5t^4 q^{\beta}_x \text{ for } 0 \le t \le 1$$

[3]

(ii) Hence or otherwise show that

$$(aq)^{\beta}_x = q^{\beta}_x \left(1 - \frac{5}{8} q^{\alpha}_x\right)$$

[5]

[Total 8]

9 Calculate the expected present value and the variance of $\bar{A}^1_{x:\overline{20|}}$ given the basis below.

Basis: Mortality $\mu_x = 0.03$ for all x
         Force of interest $\delta = 5\%$ per annum throughout

[8]

10 (i) Define the following terms in words without giving any formulae:

        (a)   crude mortality rate
        (b)   directly standardised mortality rate
        (c)   indirectly standardised mortality rate
        (d)   area comparability factor

[4]

The following table gives a summary of mortality for a particular occupational group compared to the whole population.

| | Occupational Group | | Whole Population | |
Age Group	Number	Deaths	Number	Deaths
20–34	20,000	67	1,000,000	3,500
35–49	15,000	92	1,500,000	7,800
50–64	11,000	125	700,000	8,000
TOTAL	46,000	284	3,200,000	19,300

(ii) Calculate the crude mortality rate, the directly standardised mortality rate and the indirectly standardised mortality rate for the occupational group.   [5]

[Total 9]

11 On 1 January 2008, a life insurance company issued a number of without profit endowment policies maturing at age 60 to lives then aged 40 exact. The sum assured is payable at the end of year of death or on survival to the end of the term and level premiums are payable annually in advance throughout the term of the contract.

Premiums and reserves on each policy are both calculated on the following basis:

Mortality	AM92 Select
Interest	4% per annum
Initial commission	60% of the first premium
Renewal commission	6% of each annual premium excluding the first

(i) Calculate the annual office premium per £1,000 sum assured for each policy.

[2]

(ii) Calculate the gross premium prospective reserve per £1,000 sum assured for each policy in force at 31 December 2012.   [2]

(iii)   Calculate the profit or loss to the company in 2013 in respect of these policies given the following information:

- The total sums assured in force on 1 January 2013 were £15,500,000

- The company incurred expenses relating to these policies of £76,500 on 1 January 2013 (including renewal commission).

- The total sums assured paid on 31 December 2013 in respect of deaths during 2013 were £295,000.

- The total sums assured surrendered during 2013 were £625,000. The surrender value on each policy (which was paid on 31 December 2013) was calculated as 85% of the gross premium prospective reserve applicable at the date of payment of the surrender value.

- The company earned interest of 3.5% per annum on its assets during 2013.

[10]
[Total 14]

12 A life assurance company issues a policy which provides a three-year temporary annuity of £15,000 per annum payable annually in arrear to a male life aged 65 exact. The single premium payable at outset on the policy is £42,000.

The company uses the following basis to profit test the policy:

Mortality	PMA92C20
Interest earned on cash flow and reserves	5% per annum
Initial commission	1% of the single premium
Initial expenses	£350
Renewal expenses	£55 per annuity payment which is assumed to increase by 3% per annum from inception of the policy
Risk discount rate	7% per annum

In addition, the company establishes reserves on the policy at the beginning and end of each policy year where:

$$_tV = 15,000 \times (3 - t) \text{ for } t = 1 \text{ and } 2$$
$$_tV = 0 \text{ otherwise}$$

(i) Calculate the net present value of the expected profits on the policy:

   (a) allowing for reserves
   (b) ignoring reserves

   [10]

(ii) Briefly comment on the reason for the difference in the two values calculated in part (i). [2]

(iii) Describe briefly how the net present value calculated in part (i)(a) and part (i)(b) would change if a risk discount rate of 4% per annum had been used (instead of 7% per annum) and state the reasons for the difference. [3]

[Total 15]

13 On 1 January 2003, an insurance company issued a 35 year with profit endowment assurance policy to a life aged 30 exact for a sum assured of £60,000. The sum assured (together with any bonuses attaching) is payable at maturity or immediately on death, if earlier. Level premiums are payable annually in advance throughout the policy term or until earlier death. Compound reversionary bonuses vest at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

The company calculates the premium on the following basis:

Mortality	AM92 Ultimate
Interest	6% per annum
Initial expenses	£250 plus 60% of the first year's premium, incurred at outset
Renewal expenses	2.5% of the second and each subsequent year's premium, incurred at the beginning of the respective policy years
Bonuses:	1.92308% per annum

(i) Show that the annual premium is approximately £1,146. [8]

(ii) Express, in stochastic form, the gross future loss random variable for this policy at duration t, where t is an integer and $0 < t < 35$.

You should use T_x, K_x or both, together with the elements of the premium basis that are relevant.

Assume bonuses declared follow the assumptions stated in the premium basis. [3]

On 31 December 2012, and just after the 10th bonus has been declared, the life wishes to surrender the policy. The insurance company calculates a surrender value equal to the gross prospective policy reserve, using the premium basis above.

(iii) Calculate the surrender value payable by the insurance company given that the total actual past bonus additions to the policy have followed the assumptions stated in the premium basis (including the bonus just vested).
. [6]
[Total 17]

END OF PAPER