INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

25 September 2017 (pm)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 11 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 (i) Calculate the time in days for £6,000 to accumulate to £7,600 at:

(a) a simple rate of interest of 3% per annum.

(b) a compound rate of interest of 3% per annum effective.

(c) a force of interest of 3% per annum. [6]

Note: You should assume there are 365 days in a year.

(ii) Calculate the effective rate of interest per half year which is equivalent to a force of interest of 3% per annum. [1]

[Total 7]

2 Describe how cash flows are exchanged in an interest rate swap. [2]

3 An investor is considering two investments. One is a 91-day deposit which pays a compound rate of interest of 3% per annum effective. The second is a government bill.

Calculate the annual simple rate of discount from the government bill if both investments are to provide the same effective rate of return. [3]

4 A one-year forward contract was issued on 1 April 2016 on a share with a price of \$4.00 at that date. Dividends of \$0.10 per share were expected on 30 September 2016 and 31 March 2017. The 6-month and 12-month risk-free spot rates of interest were 5% and 6% per annum effective respectively on 1 April 2016.

Calculate the forward price at issue, stating any assumptions. [4]

5 An individual invests £100 in an asset. The expected accumulation of this asset after 20 years is £200 and the standard deviation of the accumulation after 20 years is £50.

(i) Calculate the expected effective rate of return per annum. [1]

(ii) Calculate the standard deviation of the effective rate of return per annum. [4]

[Total 5]

6 An investor has a choice of two 15-year savings plans, A and B, issued by a company. In both plans, the investor pays contributions of \$100 at the start of each month and the contributions accumulate at an effective rate of interest of 4% per annum before any allowance is made for expenses.

In plan A, the company charges for expenses by deducting 1% from the annual effective rate of return.

In plan B, the company charges for expenses by deducting \$15 from each of the first year's monthly contributions before they are invested. In addition it deducts 0.3% from the annual effective rate of return.

Calculate the percentage by which the accumulated amount in Plan B is greater than the accumulated amount in Plan A, at the end of the 15 years. [6]

7 Two investors, A and B, value corporate bonds using different models.

- Investor A uses the average gross redemption yield from all government securities with the addition of a risk premium of 1% per annum effective.
- Investor B uses the spot rates of interest derived from the government bond yield curve also with the addition of a risk premium of 1% per annum effective to value each payment.

The investors are valuing a particular corporate bond which has half-yearly coupon payments paid at a rate of 5% per annum and a term to redemption of exactly two years. The bond is redeemed at 110% and tax is payable on coupons only at a rate of 20%.

The average gross redemption yield from all government securities is 3% per annum effective.

(i) Calculate the price that investor A would pay for the corporate bond. [3]

Over time t, the spot rate of interest from the yield curve of government securities, y_t is given by $y_t = 0.015t$ per annum effective for $t \leq 2$.

(ii) Calculate the price that investor B would pay for the corporate bond. [3]

(iii) Calculate the forward rate of interest from government securities from $t = 1$ to $t = 2$. [2]

(iv) Giving two reasons, explain why the spot yield curve might rise with term to redemption. [3]

[Total 11]

8 A loan is to be repaid by an increasing annuity. The first payment will be £100 and the payments will increase by £50 per annum. Payments will be made annually in arrear for ten years. The repayments are calculated using a rate of interest of 5% per annum effective.

(i) Calculate the amount of the loan. [2]

(ii) Calculate:

(a) the interest component of the sixth instalment.

(b) the capital component of the sixth instalment. [4]

Immediately after the sixth instalment, the borrower asks to repay the remaining loan using level annual instalments. The lender agrees, but changes the interest rate at the time of the alteration to 6% per annum effective.

(iii) Calculate the revised instalment. [3]

[Total 9]

9 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.09 - 0.003t & 0 \le t \le 10 \\ 0.06 & t > 10 \end{cases}$$

(i) Calculate the corresponding constant effective annual rate of interest for the period from $t = 0$ to $t = 10$. [4]

(ii) Express the rate of interest in part (i) as a nominal rate of discount per annum convertible half-yearly. [1]

(iii) Calculate the accumulation at time $t = 15$ of £1,500 invested at time $t = 5$. [3]

(iv) Calculate the corresponding constant effective annual rate of discount for the period $t = 5$ to $t = 15$. [1]

(v) Calculate the present value at time t = 0 of a continuous payment stream payable at a rate of $10e^{0.01t}$ from time $t = 11$ to time $t = 15$. [6]

[Total 15]

10 An insurance company has liabilities of £100 million due in 10 years' time and £200 million due in 20 years' time.

The company's assets consist of a zero-coupon bond and a level annuity paid annually in arrear. The zero coupon bond will pay £144.054 million in 15 years' time. The current interest rate is 3% per annum effective at all terms to redemption.

Redington's first two conditions for immunisation against small changes in the rate of interest have been satisfied for this insurance company.

(i) (a) Calculate the present value of the liabilities.

(b) Calculate the discounted mean term of the liabilities. [4]

(ii) Show that the term of the annuity is 41 years. [6]

(iii) Determine the annual rate of payment of the annuity. [1]

(iv) State Redington's third condition for immunisation, explaining whether you think it is fulfilled. [2]

The insurance company decides to sell the zero-coupon bond it holds and invest the proceeds in another zero-coupon bond with a shorter term to maturity.

(v) Explain the risks of implementing this decision. [2]

[Total 15]

11 A university offers its students three financing options for a degree course that lasts exactly three years.

Option A

Fees are paid during the term of the course monthly in advance. The fees are £10,000 per annum in the first year and rise by 5% on the first and second anniversaries of the start of the course.

Option B

The university makes a loan to the students which is repaid in instalments after the end of the course. The instalments are determined as follows:

- No payments are made until three years after the end of the course.
- Over the following 15 years, students pay the university £1,300 per year, quarterly in advance.
- After 15 years of payments, the quarterly instalments are increased to £1,500 per year, quarterly in advance.

- After a further 15 years of payments, the quarterly instalments are increased to £1,800 per year, quarterly in advance, for a further 15-year period after which there are no more payments.

Option C

- Students pay to the university 3% of all their future earnings from work, with the payments made annually in arrear.

A particular student wishes to attend the university. He expects to leave university at the end of the three-year course and immediately obtain employment. The student expects that his earnings will rise by 3% per annum compound at the end of each year for 10 years and then he will take a five-year career break.

After the career break, he expects to restart work on the salary he was earning when the career break started. He then expects to receive salary increases of 1% per annum compound at the end of each year until retiring 45 years after graduating.

The student wishes to take the financing option with the lowest net present value at a rate of interest of 3% per annum effective.

(i) Calculate the present value of the payments due under option A. [4]

(ii) Calculate the present value of the payments due under option B. [5]

(iii) Calculate the initial level of salary that will lead the payments under option C to have the lowest present value of the three options. [8]

(iv) Comment on whether the student should use the same interest rate to evaluate all three options. [2]

The university is concerned that this scheme exposes it to considerable financial risk.

(v) Explain three risks which the university faces. [4]

[Total 23]

END OF PAPER