INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2018

Subject CT5 – Contingencies
Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer
Chair of the Board of Examiners
December 2018

A. General comments on the aims *of this subject and how it is marked*

1. The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

2. CT5 introduces the fundamental building blocks of all life insurance and pensions actuarial work.

B. General comments on *student performance in this diet of the examination*

This alternative paper was sat by only 16 students from centres that could not sit the main examination because of severe adverse weather conditions and were forced to delay their attempt by a few days.

Only a small number of these students passed the examination and the remainder were all significantly well below the pass mark required. Unfortunately a large proportion of these gained very few marks at all.

Hence it is difficult to give a complete commentary with such a small sample set.

However of the group examined it appeared that a large proportion were poorly prepared for a very standard paper. Questions which gave particular difficulty were 2, 6, 8, 10, 12 and 13.

It is hoped the detail below will give assistance to students preparing for future examinations.

C. Pass Mark

The Pass Mark for this exam was 60.

Solutions

Q1

(a) $_{10|5}q_{49} = \dfrac{(l_{59} - l_{64})}{l_{49}} = \dfrac{(9354.0040 - 8934.8771)}{9733.8865} = 0.043059$ [1]

(b) $e_{[43]:\overline{25|}} = e_{[43]} - \dfrac{l_{68}}{l_{[43]}} \times e_{68} = 36.189 - \dfrac{8404.4916}{9823.5994} \times 14.418 = 23.854$ [1]

(c) $A_{40:\overline{15|}}^{\;\;\;1} = v^{15} \times \dfrac{l_{55}}{l_{40}} = 0.55527 \times \dfrac{9557.8179}{9856.2863} = 0.538455$ [1]

[Total 3]

Very straightforward question which was satisfactorily done by students who had prepared well.

Q2

For the first policy year:- $({}_0V + P - I)(1+i) = (1+f)S \times q_x + {}_1V \times p_x$

[2]

For subsequent policy years:- $({}_tV + P - eP)(1+i) = (1+f)S \times q_{x+t} + {}_{t+1}V \times p_{x+t}$

[2]

Standard knowledge based question which was poorly done. Most students could not recognise the difference between first and subsequent years' results and struggled to complete all the factors within the formulae.

Q3

$$\bar{A}_{\overline{40:50}} = \bar{A}_{40} + \bar{A}_{50} - \bar{A}_{40:50} = \int_0^\infty v^t\, {}_tp_{40}\mu_{40+t}dt + \int_0^\infty v^t\, {}_tp_{50}\mu_{50+t}dt - \int_0^\infty v^t\, {}_tp_{40:50}\left(\mu_{40+t} + \mu_{50+t}\right)dt$$

$$= 0.02 \times \int_0^\infty e^{-(0.02+\ln(1.04))t}dt + 0.03 \times \int_0^\infty e^{-(0.03+\ln(1.04))t}dt - 0.05 \times \int_0^\infty e^{-(0.05+\ln(1.04))t}dt$$

$$= \frac{0.02}{0.02+\ln(1.04)} + \frac{0.03}{0.03+\ln(1.04)} - \frac{0.05}{0.05+\ln(1.04)}$$

$$= 0.210708$$

$$A_{\overline{40:50}} = \frac{\bar{A}_{\overline{40:50}}}{(1.04)^{0.5}} = 0.210708 \times 0.98058 = 0.2066$$

[Total 4]

Well prepared students completed this question without issues

Q4

Value of benefits
Let premium = P

$$100,000\bar{A}_{65:65} + 0.5P \times v^{10} \times \frac{l_{75:75}}{l_{65:65}} - 50,000 \times v^{10} \times \frac{l_{75:75}}{l_{65:65}} \times \bar{A}_{75:75}$$

$$= 100,000 \times 0.55077 + 0.5P \times 0.67556 \times \frac{8405.160}{9647.797} \times \frac{8784.955}{9703.708}$$

$$- 50,000 \times 0.67556 \times \frac{8405.160}{9647.797} \times \frac{8784.955}{9703.708} \times 0.71861$$

$$= 55,077 + 0.2664P - 19,145$$

$$= 35,932 + 0.2664P$$

[3]

Where

$\bar{A}_{65:65} = (1.04)^{0.5} \times \left(1 - d\ddot{a}_{65:65}\right) = (1.04)^{0.5} \times \left(1 - 0.038462 \times 11.958\right) = 0.55077$ [0.5]

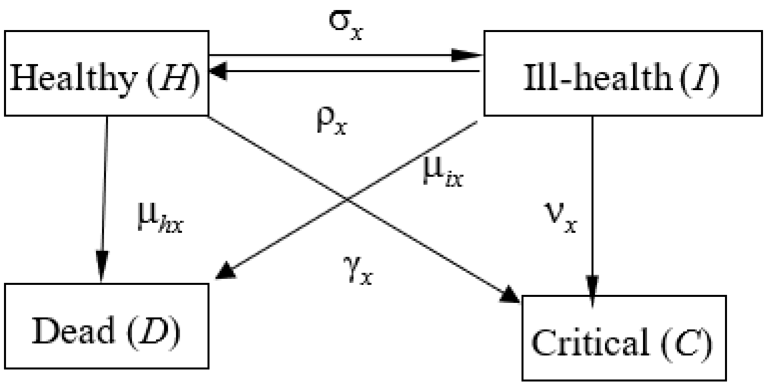
$\bar{A}_{75:75} = (1.04)^{0.5} \times \left(1 - d\ddot{a}_{75:75}\right) = (1.04)^{0.5} \times \left(1 - 0.038462 \times 7.679\right) = 0.71861$ [0.5]

Hence P= 35,932 / (1-0.2664) = 48,980 [1]

[Total 5]

Very straightforward question completed satisfactorily by well prepared students.

Q5



[0.5 for each box, 0.5 for each arrow]

Generally done well.

Q6

(i)

- active members of the scheme, i.e. those not in receipt of any benefits and for whom contributions are being paid
- pensioners who retired at the normal retirement age and are now receiving a pension
- pensioners who retired under the ill-health retirement rules of the scheme and are now receiving a pension benefit
- deferred pensioners, i.e. those for whom no contributions are currently being paid and who are entitled to a pension at some future date
- pensioners who retired under the early (before normal pension age) retirement rules of the scheme and are now receiving a pension benefit

[1 mark each bullet]

(ii)

- The mortality of those who retired early (but in good health) or at normal retirement age is likely to be lower than that of ill-health retirement pensioners. This is an example of class selection.
- The mortality of ill-health retirement pensioners is likely to depend on duration since retirement for a few years following the date of retirement, and subsequently only on age attained. This is an example of temporary initial selection.
- Underwriting at the date of joining a scheme tends to be very limited, e.g. actively at work, and so there tends to be only very slight temporary initial selection.
- Different sections of a large scheme, e.g. works and staff, may exhibit different levels of mortality. This is an example of class selection.
- Among the active members of the scheme ill-health retirement acts as a selective decrement, resulting in lighter mortality among the remaining active members. This is sometimes termed the "healthy worker" or the "active lives mortality" effect.

- Withdrawal from a scheme is associated with voluntary or compulsory termination of employment (changing jobs or redundancy). If voluntary resignation is the cause this tends to select these with lighter mortality (and ill-health retirement) rates. If redundancy is the cause withdrawal rates tend to vary markedly over time as economic conditions vary. This is an example of time selection.

[1 mark for each of two examples]

[Total 7]

Question generally done poorly. Students did not answer the question in the specifics of a pension fund but tried to generalise.

Q7

	$q_{[x]}$	$q_{[x-1]+1}$	$q_{[x-2]+2}$
45	**0.001201**		
46		**0.001557**	
47			**0.001802**

[0.5]

Therefore, using a cash flow approach: -

yr	prem.	expense	interest	death claim	mat claim	profit vector	cumulative prob of survival	discount factor	net present value
1	5000.00	165.00	145.05	60.05	0.00	4920.00	1.000000	.96154	4730.77
2	5000.00	125.00	146.25	116.78	0.00	4904.47	0.998799	.92456	4529.02
3	5000.00	125.00	146.25	180.20	14972.97	10131.92	0.997244	.88900	-8982.45
	[0.5]	[0.5]	[1]	[1.5]	[0.5]	[0.5]	[0.5]	[0.5]	[0.5]

Total net present value of profit = 277.35 [0.5]

Question completed satisfactorily by well prepared students. Those not well prepared supplied very poor answers not showing how to approach the question.

Q8

The reserves for the 2 death claim policies at 31st December 2017 were:-

$$ {}_{15}V + {}_{21}V = 25,000\left(1-\frac{\ddot{a}_{58}}{\ddot{a}_{43}}\right)+10,000\left(1-\frac{\ddot{a}_{58}}{\ddot{a}_{37}}\right) $$

$$ = 25,000\left(1-\frac{14.847}{19.319}\right)+10,000\left(1-\frac{14.847}{20.625}\right) = 5,787.05+2,801.45 = 8,588.50 $$

[3]

Total death strain at risk (*DSAR*) at 31st December 2017:

$DSAR = 1,567,000-(576,000+8,588.50) = 982,411.50$ [1.5]

Expected death strain (*EDS*) = $q_{57} \times DSAR = 0.005650 \times 982,411.50 = 5,550.62$

[1.5]

Actual death strain (*ADS*) = $(35,000-8,588.50) = 26,411.50$ [0.5]

Mortality profit = *EDS* – *ADS* = 5,550.62 – 26,411.50 = -20,860.88 i.e. a loss [0.5]

A bulk standard CT5 question done poorly.

Q9

(i)

	Country		Region A		Region B	
Age	Population	Number of deaths	Population	Number of deaths	Population	Number of deaths
0-19	800,000	450	54,000	30	75,000	30
20-39	650,000	626	42,000	45	60,000	65
40-59	567,000	1,201	40,000	65	45,000	65
60+	400,000	9,563	52,000	1,200	32,000	653
Total	**2,417,000**	**11,840**	**188,000**	**1,340**	**212,000**	**813**

	Region A		Region B	
Age	Expected deaths based on standard population	Expected deaths based on standard mortality	Expected deaths based on standard population	Expected deaths based on standard mortality
0-19	444	30	320	42
20-39	696	40	704	58
40-59	921	85	819	95
60+	9,231	1,243	8,163	765
Total	11,292	1,398	10,006	960

a)
Region A = 1,340 / 188,000 = 0.007128

Region B = 813 / 212,000 = 0.003835
b)
Region A = 11,292 / 2,417,000 = 0.004672
Region B = 10,006 / 2,417,000= 0.004140
c)
Region A = 1,340 / 1,398 = 0.958512
Region B = 813 / 960 = 0.846875

[1 for each, Total 6]

(ii) The crude rate for Region A is exaggerated by the shape of the population which has more lives at the older ages.

The standardised rate shows this – Region A does have higher mortality but not by much.
The mortality ratio shows the same position as the standardised rates

[1 for each, max of 2]
[Total 8]

Another standard CT5 question. Part (i) was done reasonably well but few students gave the proper explanations for part (ii)

Q10

First construct the multiple decrement table from the independent rates.

Age	q_x^d	q_x^i	μ_x^d	μ_x^i	$(aq)_x^d$	$(aq)_x^i$	$(ap)_x$
63	0.004251	0.05	0.00426	0.051293	0.004144	0.049894	0.945962
64	0.005073	0.05	0.005086	0.051293	0.004945	0.049874	0.945181

[0.5 for each of five final columns]

To value the benefits, we need to make an assumption on the timing of the payment. We will assume that deaths and ill-health retirements occur on average in the middle of the year as an approximation to the payments throughout the year. [0.5]

Value of retirement at 65 benefit:

$$= 10{,}000v^2 \times (ap)_{63} \times (ap)_{64} \times \ddot{a}_{65}$$
$$= 10{,}000v^2 \times 0.945962 \times 0.945181 \times 13.666 = 112{,}970$$

[2]

Value of ill-health retirement benefit

$5000\ddot{a}_{\overline{10|}} \times (v^{0.5} \times 0.049894 + v^{1.5} \times 0.945962 \times 0.049874) = 3940$

Where $\ddot{a}_{\overline{10|}} = 1 + 7.4353 = 8.4353$

[2]

Value of death benefit

$50{,}000 \times \left(v^{0.5} \times 0.004144 + v^{1.5} \times 0.945962 \times 0.004945\right) = 424$ [1½]

Total value=112,970+3,940+424=117,334 [½]

[Total 9]

This was a more challenging higher skills question on the paper. Well prepared students completed it with little issue but the remainder did not demonstrate they knew where to begin.

Q11

Let annuity income be A
Value of annuity given by

$A\left(\ddot{a}^{(12)}_{\overline{65:60}} - \ddot{a}^{(12)}_{65:60}\right) = A(17.177 - 12.224) = 4.953A$ [2]

Where

$\ddot{a}^{(12)}_{65:60} = \ddot{a}_{65:60} - \frac{11}{24} = 12.682 - \frac{11}{24} = 12.224$ [1]

$\ddot{a}^{(12)}_{\overline{65:60}} = \ddot{a}^{(12)}_{65} + \ddot{a}^{(12)}_{60} - \ddot{a}^{(12)}_{65:60} = 13.666 - \frac{11}{24} + 16.652 - \frac{11}{24} - 12.224 = 17.177$ [1]

Value of funeral benefit

$2,000\bar{A}_{\overline{65:60}} = 2,000 \times \left(1 - \delta\bar{a}_{\overline{65:60}}\right) = 2,000 \times \left(1 - \ln(1.04) \times 17.136\right) = 656$ [1]

Where

$\bar{a}_{\overline{65:60}} = \bar{a}_{65} + \bar{a}_{60} - \bar{a}_{65:60} = (13.666 - 0.5) + (16.652 - 0.5) - (12.682 - 0.5) = 17.136$ [2]

Equation of value

10,000=656+4.953A [1]

Hence

A=1,887 [1]

A further standard CT5 question. Done well by students who were prepared.

Q12

Annual premium	£3500.00	Allocation % (1st yr)	97.5%
Risk discount rate	5.0%	Allocation % (2nd yr)	97.5%
Interest on investments (1st yr)	4.0%	B/O spread	5.0%
Interest on investments (2nd yr)	3.5%	Management charge	1.0%
Interest on non-unit funds (1st and 2nd yrs)	2.5%	Policy Fee	60
Death benefit (% of bid value of units)	125%		

		% premium

Initial expense	235	5.0%
Renewal expense	75	2.5%
Death claim expense	100	
Maturity claim expense	60	

Mortality table:

x	t	$q^d_{[x]+t-1}$	$q^s_{[x]+t-1}$	$(aq)^d_{[x]+t-1}$	$(aq)^s_{[x]+t-1}$	$(ap)_{[x]+t-1}$	${}_{t-1}(ap)_{[x]}$
55	1	0.003358	0.05000	0.003358	0.04983	0.946810	1.000000
56	2	0.004903	0.00000	0.004903	0.00000	0.995097	0.946810

[1]

Unit fund (per policy at start of year)

	yr 1	*yr 2*
value of units at start of year	0.000	3276.059
alloc	3412.500	3412.500
B/O	170.625	170.625
policy fee	60.000	60.000
interest	127.275	226.028
management charge	33.092	66.840
value of units at year end	3276.059	6617.122

[2.5]

Non unit fund cash flows (per policy at start of year)

	yr 1	*yr 2*
unallocated premium + pol fee	147.500	147.500
b/o spread	170.625	170.625
expenses	410.000	162.500
interest	-2.297	3.891
man charge	33.092	66.840

[2]

(i)

a) if p/h dies in the 1st year of contract, non-unit cash flows at end of the year are:-

$$yr1 = \left(147.5 + 170.625 - 410.0 - 2.297 + 33.092 - 0.25 \times 3276.059 - 100\right) = -980.095$$ [1]

b) if p/h surrenders in the 1st year of contract, non-unit cash flows at end of the year are:-

$$yr1 = \left(147.5 + 170.625 - 410.0 - 2.297 + 33.092 + 750\right) = 688.920$$ [1]

c) if p/h dies in the 2nd year of contract, non-unit cash flows at end of the year are:-

$$yr1 = \left(147.5 + 170.625 - 410.0 - 2.297 + 33.092\right) = -61.08$$ [1]

d) if p/h survives to the end of the contract, non-unit cash flows at end of the year are:-

$yr\,1 = -61.08$ (derived in c) above) [0.5]

(ii)

a) if p/h dies in the 2nd year of contract, non-unit cash flows at end of the year are:-

$$yr\,2 = \left(147.5 + 170.625 - 162.5 + 3.891 + 66.84 - 0.25 \times 6617.122 - 100\right) = -1527.925$$ [1]

b) if p/h survives to the end of the contract, non-unit cash flows at end of the year are:-

$$yr\,2 = \left(147.5 + 170.625 - 162.5 + 3.891 + 66.84 - 60\right) = 166.356$$ [1]

(iii)

if p/h dies in the 1st year of contract, expected present value of cash flow is given by:-

$$-980.095 \times v \times \left(aq\right)^{d}_{[55]} = -933.424 \times 0.003358 = -3.134$$ [1]

if p/h surrenders in the 1st year of contract, expected present value of cash flow is given by:-

$$688.920 \times v \times \left(aq\right)^{s}_{[55]} = 656.114 \times 0.049832 = 32.695$$ [1]

if p/h dies in the 2nd year of contract, expected present value of cash flows is given by:-

$$\left[-61.08 \times v - 1527.925 \times v^2\right] \times \left(ap\right)_{[55]} \times \left(aq\right)^{d}_{[55]+1} = \left[-58.171 - 1385.873\right] \times 0.946810 \times 0.004903 = -6.704$$

[1.5]

if p/h survives to the end of the contract, expected present value of cash flows is given by:-

$$\left[-61.08 \times v + 166.356 \times v^2\right] \times {}_2\left(ap\right)_{[55]} = \left[-58.171 + 150.89\right] \times 0.946810 \times 0.995097 = 87.357$$

[1]

=> expected present value of profit = -3.134 + 32.695 – 6.704 + 87.357 =110.214 [0.5]

> *Part (i) was reasonably attempted but students showed few ideas about how to progress to part (ii). Part (ii) needed to consolidate the work within (i) and (ii) but few students were able to progress this.*

Q13

(i) Let P be the gross annual premium for the policy. Gross future loss random variable =

$$250,000v^{K_{[62]}+1}+315+55a_{\overline{K_{[62]}}|}-P(0.975\ddot{a}_{\overline{K_{[62]}+1)}|}-0.1) \quad if \quad K_{[62]}<3$$ [3]

$$315+55a_{\overline{2}|}-P(0.975\ddot{a}_{\overline{3)}|}-0.1) \quad if \quad K_{[62]}\geq 3$$ [2]

(ii) If E (Gross future loss random variable) = 0

$$\Rightarrow 250,000A^{1}_{[62]:\overline{3}|}+315+55\left[\ddot{a}_{[62]:\overline{3}|}-1\right]=P\left[0.975\ddot{a}_{[62]:\overline{3}|}-0.1\right]$$

$$where\ \mathrm{A}^{1}_{[62]:\overline{3}|}=\mathrm{A}_{[62]:\overline{3}|}-v^{3}\,{}_{3}p_{[62]}=0.88990-0.889\times\frac{8821.2612}{9097.7405}=0.88990-0.86198=0.02792$$

$$and\ \ \ddot{a}_{[62]:\overline{3}|}=2.863$$

$$\Rightarrow 250,000\times 0.02792+315+55\times 1.863=2.69143P$$

$$\Rightarrow P=\frac{6980.0+315+102.465}{2.69143}=2,748.53$$

3 marks for premium formula, ½ mark for evaluating the assurance function, and ½ mark for correct answer for P

Mortality table:

x	t	$q_{[x]+t-1}$	$p_{[x]+t-1}$	${}_{t-1}p_{[x]}$
62	1	0.007164	0.992836	1.000000
63	2	0.010815	0.989185	0.992836
64	3	0.012716	0.987284	0.982098

[1]

Cash flows (per policy at start of year) assuming annual premium is denoted by P:

Year	1	2	3
Premium	P	P	P
Expenses	0.125P + 315	0.025P + 55	0.025P + 55
Interest	0.035P – 12.60	0.039P –2.20	0.039P –2.20
Claim	1791.00	2703.75	3179.00
Profit vector	0.910P-2118.60	1.014P –2760.95	1.014P –3236.20
Cumulative probability of survival	1.000000	0.992836	0.982098

Profit signature	0.910*P*-2118.60	1.00674*P*-2741.170	0.99585*P*-3178.266
Discount factor	0.96154	0.92456	0.88900
NPV of profit	0.875*P*-2037.119	0.93079*P*-2534.377	0.88531*P*-2825.478

½ mark for each correct line (excluding lines 1,7 & 9) in table => [3.5]

Therefore:

$$\sum_{1}^{3} NPV = 0 = 2.6911P - 7396.974 \Rightarrow P = \frac{7396.974}{2.6911} = 2,748.68$$ [1]

which is consistent with the premium calculated in (ii) above (allowing for rounding)

(iv) If reserves are set up in the above cash flows, then profit is deferred. However, as the earned interest rate is equal to the risk discount rate, there is no change to the *NPV* or premium

[1.5]

Part (i) was very poorly done as students always struggle with random variable questions. Part (ii) being a standard approach was better done. The best prepared students tackled (iii) and (iv) in a satisfactory manner but the remainder showed little knowledge in this area.

END OF EXAMINERS' REPORT