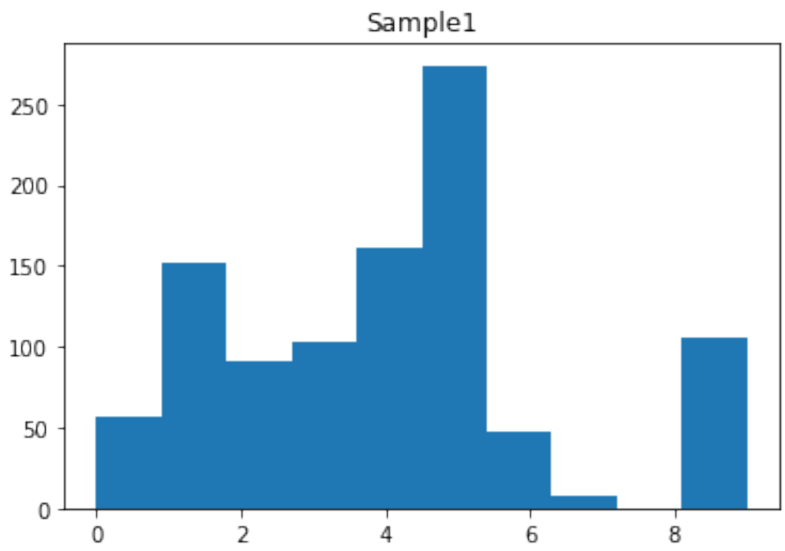
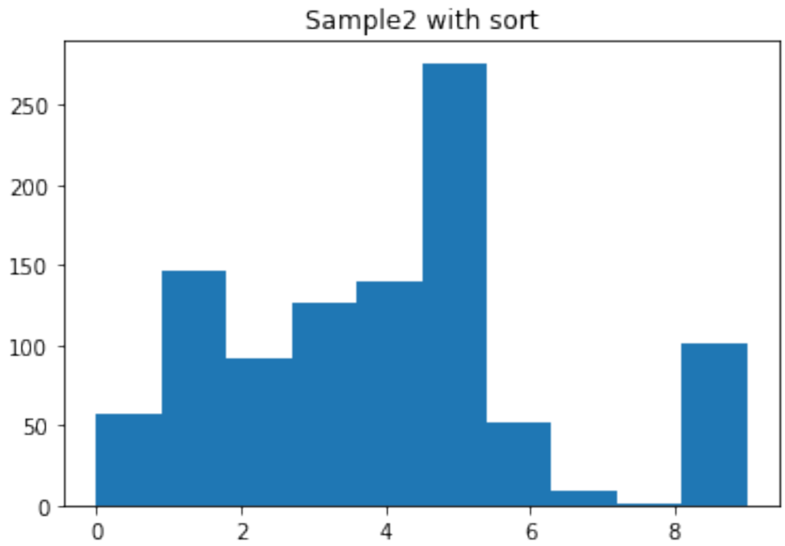
In [1]:
```
#task1: generate a sample from 0...9 with weights
#0.12, 0.3, 0.167, 0.24, 0.31, 0.54, 0.111, 0.02, 0.001, 0.2

import numpy as np
import matplotlib.pyplot as plt
import time

from operator import itemgetter
%matplotlib inline
N=1000

def generate_from0_to9(weights,N):
    mas_of_randoms=np.random.random(N)
    otv=np.zeros(N)
    for i in range(N):
        s=mas_of_weights[0]
        j=0
        while(s<mas_of_randoms[i]):
            s=s+mas_of_weights[j+1]
            j=j+1
        otv[i]=j
    return otv

def generate_from0_to9_with_sort(weights,N):

    dict_of_data={mas_of_weights[i]:i for i in range(10)}
    list_keys = list(dict_of_data.keys())
    list_keys.sort(reverse=True)
    new_chisla=np.zeros(10)
    for i in range(len(list_keys)):
        new_chisla[i]=dict_of_data[list_keys[i]]

    mas_of_randoms=np.random.random(N)
    otv=np.zeros(N)
    for i in range(N):
        s=list_keys[0]
        j=0
        while(s<mas_of_randoms[i]):
            s=s+list_keys[j+1]
            j=j+1
        otv[i]=new_chisla[j]
    return otv





mas_of_weights=np.array([0.12, 0.3, 0.167, 0.24, 0.31, 0.54, 0.111, 0.
mas_of_weights=mas_of_weights/mas_of_weights.sum()

%time data1=generate_from0_to9(mas_of_weights,N)
%time data2=generate_from0_to9_with_sort(mas_of_weights,N)
```

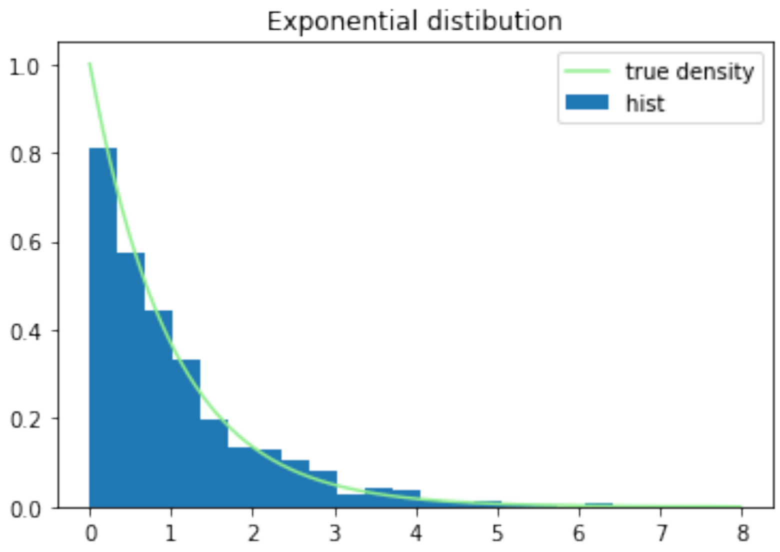
```
plt.hist(data1)
plt.title("Sample1")
plt.show()

plt.hist(data2)
plt.title("Sample2 with sort")
plt.show()


#we see that with sorting is faster
```

```
CPU times: user 2.54 ms, sys: 167 µs, total: 2.71 ms
Wall time: 3.06 ms
CPU times: user 1.65 ms, sys: 110 µs, total: 1.76 ms
Wall time: 1.7 ms
```



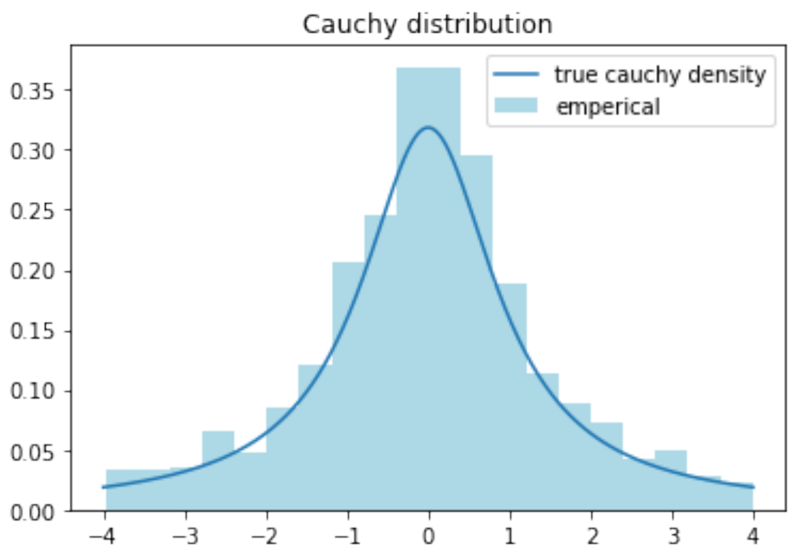


In [2]:
```
#task2: build a sample from Exp(lamda)
#distribution function F: y=1-exp{-lamda*x}
#F^(-1): x=-ln(1-y)/lamda

import numpy as np
import matplotlib.pyplot as plt
lamda=1
N=1000 #amount of sample
K=300 #amount of borders
Left_border=0
Right_border=8


mas_of_randoms = [np.random.rand() for i in range(N)]
observed_distribution=np.zeros(N)
for i in range(0,N):
    observed_distribution[i]=np.log(1-mas_of_randoms[i])/(-lamda)


true_density=np.zeros(N)
tmp=np.arange(0,8,0.01)
true_density=lamda*np.exp(-lamda*tmp)


plt.plot(tmp,true_density,label='true density',color='lightgreen')
plt.hist(observed_distribution,density=True,bins=20,label="hist ")
plt.title("Exponential distibution")
plt.legend()
plt.show()
```

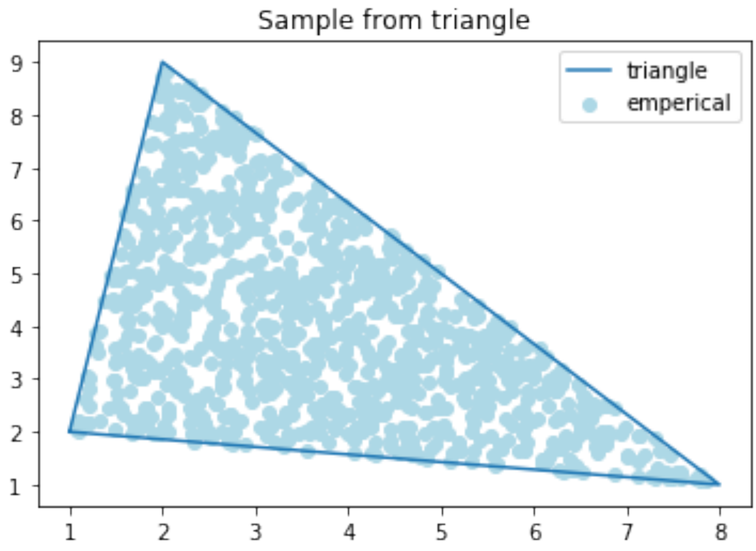


In [3]:
```
#task3: cauchy distribution
```

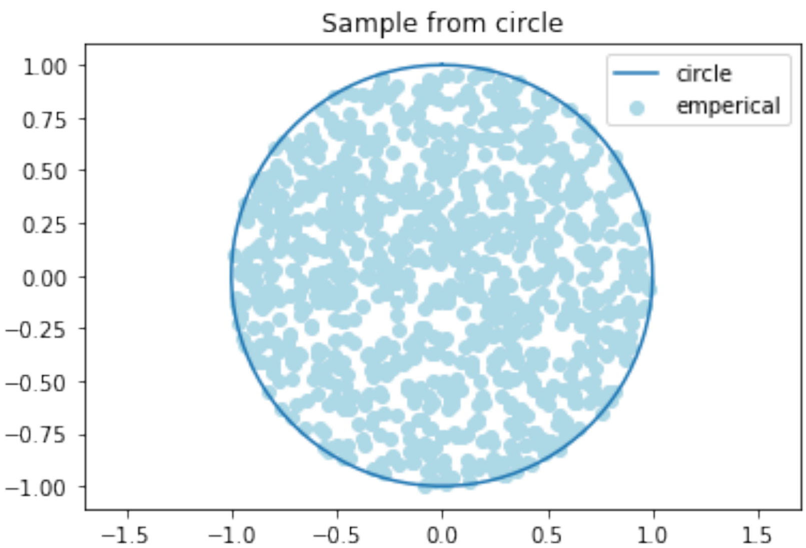
```
import numpy as np
import matplotlib.pyplot as plt
from scipy import stats
cauchy_distrib = stats.cauchy()
N=1000
C=4
def h_density(x):
    return 1/8

def f_density(x):
    return cauchy_distrib.pdf(x)
plotnost_x = np.linspace(-4, 4, N)
plotnost_y=np.zeros(N)
for i in range(N):
    plotnost_y[i]=cauchy_distrib.pdf(plotnost_x[i])


fitting_x=np.zeros(N)
step=0
while(step<N):
    z=8*np.random.rand()-4 #uniform on [-4,4]
    u=np.random.rand()
    if (u<=f_density(z)/(C*h_density(z))):
        fitting_x[step]=z
        step=step+1


plt.hist(fitting_x,density=True,bins=20,label="emperical",color='light
plt.plot(plotnost_x,plotnost_y,label="true cauchy density")
plt.title("Cauchy distribution")
plt.legend()
plt.show()
```



In [11]:
```
#task4: sample from triangle
import numpy as np
import matplotlib.pyplot as plt
import math

triangle_x=[1,2,8,1]
triangle_y=[2,9,1,2]
N=1000

xrand = np.zeros(N)
yrand =np.zeros(N)
step=0

while(step<N):
    x=np.random.rand()
    y=np.random.rand()
    if y<=1-x:
        xrand[step]=x*5*np.sqrt(2)
        yrand[step]=y*5*np.sqrt(2)
    else:
        xrand[step]=(1-y)*5*np.sqrt(2) # reflecting relative to y=1-x
        yrand[step]=(1-x)*5*np.sqrt(2)

    step=step+1

phi=np.arctan(-1/7)
mas_of_x=xrand*math.cos(phi)-yrand*math.sin(phi)+1
mas_of_y=xrand*math.sin(phi)+yrand*math.cos(phi)+2

plt.scatter(mas_of_x,mas_of_y,label="emperical",color='lightblue')
plt.plot(triangle_x,triangle_y,label="triangle")
plt.title("Sample from triangle")
plt.legend()
plt.show()
```



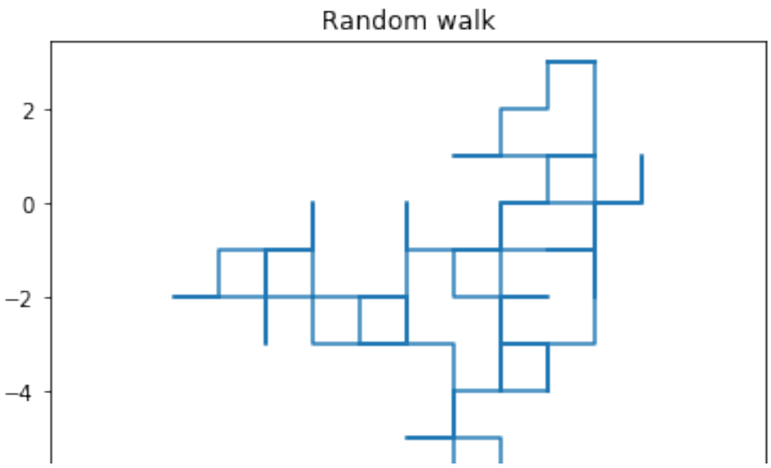
In [12]:
```python
#task5: sample from a circle
import numpy as np
from matplotlib.patches import Circle
import math
N=1000

t = np.linspace(0, 2 * np.pi, 100)

mas_of_phi=[2*np.pi*np.random.rand() for i in range(N)]
mas_of_r=[np.sqrt(np.random.rand()) for i in range(N)]

mas_of_x=np.zeros(N)
mas_of_y=np.zeros(N)
for i in range(N):
    mas_of_x[i]=mas_of_r[i]*math.cos(mas_of_phi[i])
    mas_of_y[i]=mas_of_r[i]*math.sin(mas_of_phi[i])

plt.scatter(mas_of_x,mas_of_y,label="emperical",color='lightblue')
plt.plot(np.sin(t), np.cos(t),label='circle')
plt.axis('equal')
plt.title("Sample from circle")
plt.legend()
plt.show()
```

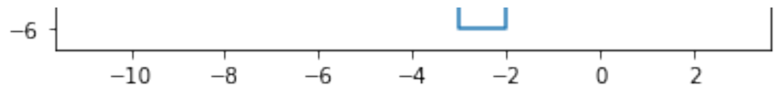


In [27]:
```python
#task6: random walk
import numpy as np
from matplotlib.patches import Circle
import math

N=100

a_walk=np.ones(N)
b_walk=np.ones(N)
a_where_minus=np.random.choice(np.arange(N),N//2,replace=False)
```

```
a_where_minus=np.random.choice(np.arange(N),N//2,replace=False)
b_where_minus=np.random.choice(np.arange(N),N//2,replace=False)
a_walk[a_where_minus]=-1
b_walk[b_where_minus]=-1

x_walk=(a_walk+b_walk)/2
y_walk=(a_walk-b_walk)/2

print("a_walk:", a_walk[0:16])
print("b_walk:",b_walk[0:16])
print("x_walk:",x_walk[0:16])
print("y_walk:",y_walk[0:16])

mas_of_x=np.zeros(N+1)
mas_of_y=np.zeros(N+1)
mas_of_x[0]=0
mas_of_y[0]=0
for i in range(1,N):
    mas_of_x[i]=mas_of_x[i-1]+x_walk[i-1]
    mas_of_y[i]=mas_of_y[i-1]+y_walk[i-1]

print("mas_of_x:",mas_of_x[0:15])
print("mas_of_y:",mas_of_y[0:15])

plt.plot(mas_of_x,mas_of_y)
plt.axis('equal')
plt.title("Random walk")
plt.show()
```

```
a_walk: [-1. -1.  1.  1.  1. -1.  1. -1.  1.  1.  1. -1.  1. -1. -1.
 -1.]
b_walk: [ 1.  1. -1. -1. -1. -1.  1. -1.  1. -1. -1. -1.  1. -1.  1.
 -1.]
x_walk: [ 0.  0.  0.  0.  0. -1.  1. -1.  1.  0.  0. -1.  1. -1.  0.
 -1.]
y_walk: [-1. -1.  1.  1.  1.  0.  0.  0.  0.  1.  1.  0.  0.  0. -1.
 0.]
mas_of_x: [ 0.  0.  0.  0.  0.  0. -1.  0. -1.  0.  0.  0. -1.  0. -
1.]
mas_of_y: [ 0. -1. -2. -1.  0.  1.  1.  1.  1.  1.  2.  3.  3.  3.
 3.]
```

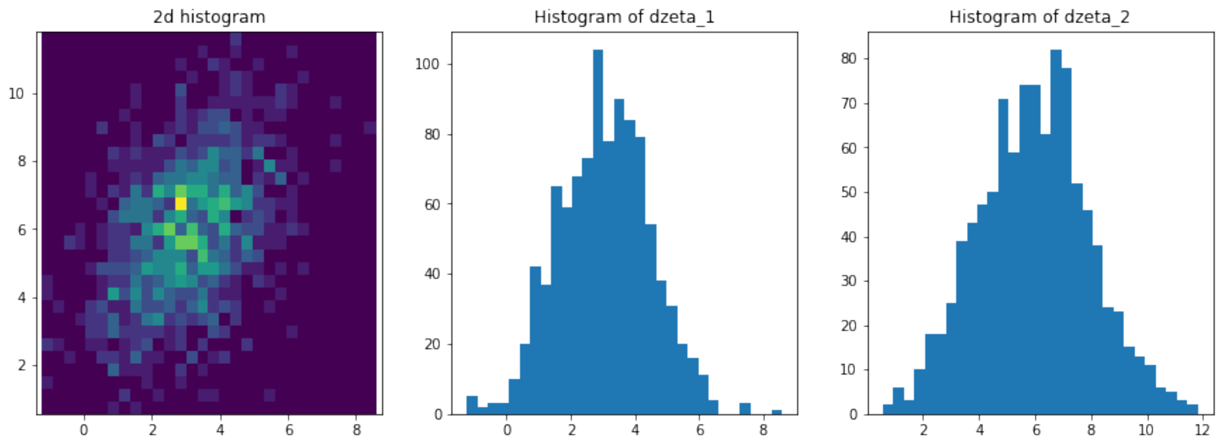




Мы хотим нормальный гауссовский вектор со средним (3,6) и ковариационной матрицей ((2,1),(1,4)); при замене eta=A*ksi (где ksi - независимые стандартные нормальные), матрица ковариации меняется как* AA^t, для получения этого разложения применяем разложение Холецкого: np.linalg.cholesky(a)

In [33]:
```
#Box-Muller number2
import numpy as np
import matplotlib.pyplot as plt

N=1000

def give_indep_stand_norm():
    x=-1+2*np.random.rand()
    y=-1+2*np.random.rand()
    while((x*x + y*y >1) or (x*x + y*y ==0)):
        x=-1+2*np.random.rand()
        y=-1+2*np.random.rand()

    s= x*x + y*y
    a = x*np.sqrt(-2 * np.log(s)/s)
    b = y*np.sqrt(-2 * np.log(s)/s)
    return a,b

needed_variance=np.array([2,1,1,4]).reshape(2,2)
A=np.linalg.cholesky(needed_variance)

#vect_ksi=np.array([give_indep_stand_norm() for i in range(N)]) #vect
vect_ksi=np.transpose(np.array([give_indep_stand_norm() for i in range

#vect_eta=np.transpose(vect_ksi@A +np.array([3,6]))
vect_eta=A@vect_ksi +  np.array([3,6]).reshape(2,1)

#print(A)
#print(vect_ksi)
#print(vect_eta)

dzeta_1=vect_eta[0]
dzeta_2=vect_eta[1]


fig, ax = plt.subplots(1, 3, figsize=(15, 5))
ax[0].hist2d(dzeta_1, dzeta_2, bins=30)
ax[0].axis('equal')
ax[0].set_title("2d histogram")
ax[1].hist(dzeta_1, bins=30)
ax[1].set_title("Histogram of dzeta_1")
```

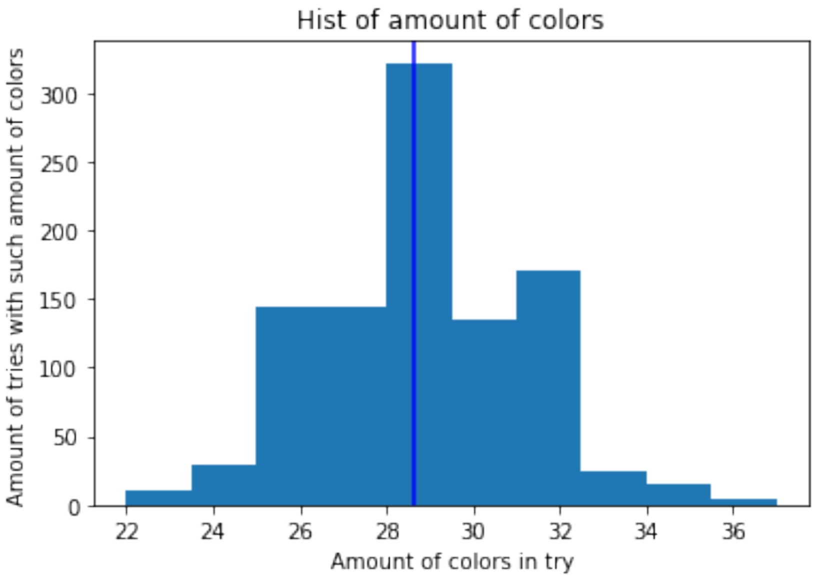
```
ax[2].hist(dzeta_2, bins=30)
ax[2].set_title("Histogram of dzeta_2")

plt.show()
```



In [40]:
```
#task8: Partitions of a set
import numpy as np
import matplotlib.pyplot as plt
import math
N=100
tries=1000

def give_Bell_numbers(N):
    mas=np.zeros(N+1)
    mas[0]=1
    for n in range(1,N+1):
        s=0
        for k in range (0,n):
            s=s+mas[k]* math.factorial(n-1)/( math.factorial(k)* math.
        mas[n]=s
    return mas
def find_amount_of_colors(mas_of_weights,mas_of_values):
    K=np.random.choice(mas_of_values, None, True, mas_of_weights[1:])
    mas_of_col=np.zeros(N)
    for i in range(N):
        mas_of_col[i]=int(1+K*np.random.rand())
    return len(np.unique(mas_of_col))


Bell_numbers=np.zeros(N+1)
Bell_numbers= give_Bell_numbers(N)
#print(Bell_numbers)

mas_of_weights=np.zeros(N+1)
s=0
for k in range(1,N+1):
```

```
    mas_of_weights[k]=math.pow(k,N)/((math.e)*(math.factorial(k))*(Bel
    s=s+mas_of_weights[k]
mas_of_weights=mas_of_weights/s
mas_of_values=1+np.arange(N)

mas_of_tries=np.zeros(tries)
for i in range(tries):
    mas_of_tries[i]=find_amount_of_colors(mas_of_weights,mas_of_values

print("mean amount of colors=",mas_of_tries.mean())
plt.hist(mas_of_tries)
plt.axvline(mas_of_tries.mean(),c='b')
plt.title("Hist of amount of colors")
plt.xlabel("Amount of colors in try")
plt.ylabel("Amount of tries with such amount of colors")
plt.show()
```

```
mean amount of colors= 28.613
```



In [ ]: