

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

28 September 2018 (pm)

Subject CT1 – Financial Mathematics Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 10 questions, beginning your answer to each question on a new page.*
6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** An investor is considering two investments. One investment is a 91-day bond issued by a bank which pays a rate of interest of 4% per annum effective. The second is a 91-day treasury bill which pays out €100.
- (i) Calculate the price of the treasury bill and the annual simple rate of discount from the treasury bill if both investments are to provide the same effective rate of return. [3]
 - (ii) Suggest one factor, other than the rate of return, which might determine which investment is chosen. [1]
- [Total 4]
- 2** The effective rate of discount per annum is 5%.
- Calculate:
- (i) the equivalent force of interest; [1]
 - (ii) the equivalent rate of interest per annum convertible monthly; [2]
 - (iii) the equivalent rate of discount per annum convertible monthly. [1]
- [Total 4]
- 3** An investment fund is valued at £60m on 1 January 2016 and at £70m on 1 January 2017. Immediately after the valuation on 1 January 2017, £100m is paid into the fund. On 1 July 2018, the value of the fund is £300m.
- (i) Calculate the effective time-weighted rate of return per annum over the whole period. [3]
 - (ii) Explain why the money-weighted rate of return per annum would be higher than the time-weighted rate of return per annum. [2]
- [Total 5]

4 A company issues a loan stock which pays coupons at a rate of 6% per annum half-yearly in arrears. The stock is to be redeemed at 103% after 25 years.

- (i) (a) Calculate the price per £100 nominal at issue which would provide a gross redemption yield of 3% per annum convertible half yearly.
- (b) Calculate the price per £100 nominal three months after issue which would provide a gross redemption yield of 3% per annum convertible half-yearly.

[3]

An investor, who is liable to income tax at 30% and capital gains tax at 40%, bought the stock at issue at a price which gave him a net redemption yield of 10% per annum effective.

- (ii) Calculate the price the investor paid.

[4]

[Total 7]

5 (i) Explain the main difference:

- (a) between options and futures;
- (b) between call options and put options.

[4]

A 12-month forward contract is issued on 1 March 2017 on a share with a price of £1.10 at that date. Dividends of £0.10 per share are expected on 1 June, 1 September and 1 December 2017.

- (ii) Calculate the forward price at issue assuming a risk-free rate of interest of 5% per annum convertible half-yearly and no arbitrage.

[4]

[Total 8]

6 In a particular investment fund, i_t is the effective rate of return in the t^{th} year. Let S_n be the accumulation of £1 invested over a period of n years.

Assume the mean of i_t is 0.08, the standard deviation of i_t is 0.07 and that $1 + i_t$ is independently and lognormally distributed.

- (i) Determine the distribution of S_{10} .

[5]

An investor is considering investing £6,000 in the fund for 10 years.

- (ii) Determine the amount of the accumulated value after 10 years such that there is a 97.5% probability of the investor actually achieving an amount greater than this.

[3]

[Total 8]

- 7 The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & 0 \leq t \leq 10 \\ 0.003t & t > 10 \end{cases}$$

- (i) Calculate the present value of a unit sum of money due at time $t = 20$. [4]
 - (ii) Calculate the equivalent constant force of interest from $t = 0$ to $t = 20$. [2]
 - (iii) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $e^{-0.06t}$ from time $t = 4$ to time $t = 8$. [4]
- [Total 10]

- 8 Two countries have recently signed a free-trade treaty and an insurance company in one of the countries is considering establishing a subsidiary in the other. The country in which the investment will take place currently has a small insurance market, but it is expected to grow slowly over the next ten years and then rapidly thereafter.

The company expects to make investments of £15m in each of the next five years to establish the subsidiary. These costs are assumed to be incurred at the end of each year.

The subsidiary will start business immediately. Upon starting business, the following costs and revenues are expected.

- Costs at a rate of £3m per year will be incurred continuously throughout the first 30 years of the subsidiary's life.
- Revenues of £3.1m per year will be received continuously throughout the first 10 years of the subsidiary's life.
- In the 11th year, revenues will be received continuously at a rate of £3.2m. The rate at which revenues will be received is then expected to increase at a rate of 5% per annum from the end of the 11th year to the end of the 30th year with increases occurring at the end of each year from the end of the 11th year.

At the end of the 30th year, the company assumes that it will sell the subsidiary.

- (i) (a) Define the term "payback period".
(b) State two reasons why the payback period is a poor decision-making criterion in the above circumstances. [4]
- (ii) Calculate the amount for which the company will have to sell the subsidiary at the end of 30 years so that the project breaks even at a rate of interest of 6% per annum effective. [9]

Some directors are concerned that the project is too risky.

- (iii) Suggest two ways in which risk could be taken into account when appraising the project. [2]
- [Total 15]

- 9 (i) Describe the cash flows which are paid and received in respect of an index-linked security. [2]

An investor bought £1m nominal of an index-linked bond on 31 December 2015 for £100 per £100 nominal. Nominal coupon payments of 1% were received on 30 June and 31 December each year. The bond was sold for £101 per £100 nominal on 31 December 2017 immediately after the coupon due on that date had been received.

The coupon payments from the bond were linked to the retail prices index (RPI) with a three-month lag with cash payments being rounded to the nearest pound. RPI inflation was 2% per annum effective from three months before the bond was issued until three months before it was sold.

Assume that all months are of equal length.

- (ii) Calculate the cash payments received by the investor from the index-linked bond. [3]
- (iii) Calculate, to the nearest 0.1%, the effective rate of return per annum obtained from the bond over the holding period (before allowing for inflation). [5]

The real rate of return obtained from the bond over the holding period was 1% per annum convertible half-yearly.

- (iv) Calculate the rate of inflation in the three months to 31 December 2017, expressing your answer as an annual effective rate. [7]
- [Total 17]

- 10** (i) Describe the characteristics of a repayment mortgage. [3]

A bank has just granted a loan of \$10,000 to a business to be repaid in ten equal instalments, annually in arrears. The rate of interest is 4% per annum effective.

- (ii) (a) Calculate the amount of the annual repayment.
(b) Calculate the duration (discounted mean term) of the repayments. [5]

The bank wishes to immunise itself from changes in interest rates in relation to this particular asset. For this purpose, the bank has issued two zero-coupon bonds. The first bond is of nominal amount \$5,000 and has a term to redemption of two years.

- (iii) Determine the nominal amount of the second zero-coupon bond and its term to redemption such that the present value and durations of the assets and liabilities are equal. [6]

Immediately upon the loan being granted, the bank agrees to a request to change the terms of the loan. The loan is now to be repaid monthly in arrears over 25 years and the rate of interest remains unchanged.

- (iv) (a) Calculate the revised monthly instalment.
(b) Explain, without further calculation, the main risk to the bank of a change in interest rates.
(c) Determine the interest and capital portions of the 121st repayment under this new arrangement. [8]
[Total 22]

END OF PAPER