INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

22 September 2014 (am)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 10 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

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1   Describe how cash flows are exchanged in an interest rate swap.   [2]

2   A life insurance company is issuing a single premium policy which will pay out £200,000 in 20 years' time. The interest rate the company will earn on the invested fund throughout the 20 years will be 4% per annum effective with probability 0.25 or 7% per annum effective with probability 0.75. The insurance company uses the expected annual interest rate to determine the premium.

   (i)   Calculate the premium.   [2]

   (ii)   Calculate the expected profit made by the insurance company at the end of the policy.   [2]

   [Total 4]

3   A 91-day treasury bill is bought for £98.83 and is redeemed at £100.

   (i)   Calculate the annual effective rate of interest from the bill.   [3]

   (ii)   Calculate the annual equivalent simple rate of interest.   [2]

   [Total 5]

4   A fund had a value of £2.0 million on 1 January 2013. On 1 May 2013, £2.5 million was invested. Immediately before this investment, the value of the fund was £2.1 million. At the close of business on 31 December 2013, the value of the fund was £4.2 million.

   (i)   Calculate the annual effective time-weighted rate of return for 2013.   [2]

   (ii)   Calculate the annual effective money-weighted rate of return for 2013.   [3]

   (iii)   Comment on your answers to parts (i) and (ii).   [2]

   [Total 7]

5   Calculate, at a rate of interest of 5% per annum effective:

   (i) $a^{(12)}_{\overline{5}|}$ [1]

   (ii) $_{4|}a_{\overline{15}|}$ [1]

   (iii) $(I\bar{a})_{\overline{10}|}$ [1]

   (iv) $(\overline{I\bar{a}})_{\overline{10}|}$ [1]

(v) the present value of an annuity that is paid annually in advance for 10 years with a payment of 12 in the first year, 11 in the second year and thereafter reducing by 1 each year. [2]

[Total 6]

6 A Eurobond has been issued by a company that pays annual coupons of 5% per annum annually in arrear and is redeemable at par in exactly 10 years' time.

(i) Calculate the purchase price of the bond at issue at a rate of interest of 4% per annum effective assuming that tax is paid on the coupon payments at a rate of 20%. [2]

(ii) Calculate the discounted mean term of the bond at a rate of interest of 4% per annum effective, ignoring tax. [3]

(iii) (a) Explain why the discounted mean term of the gross payments from the bond is lower than the discounted mean term of the net payments.

(b) State two factors other than the size of the coupon payments that would affect the discounted mean term of the bond. [3]

(iv) Calculate the price of the bond three months after issue at a rate of interest of 4% per annum effective assuming tax is paid on the coupon payments at a rate of 20%. [1]

[Total 9]

7 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & \text{for } 0 < t \le 10 \\ 0.003t & \text{for } 10 < t \le 20 \\ 0.0001t^2 & \text{for } t > 20 \end{cases}$$

(i) Calculate the present value of a unit sum of money due at time $t = 28$. [7]

(ii) (a) Calculate the equivalent constant force of interest from $t = 0$ to $t = 28$.

(b) Calculate the equivalent annual effective rate of discount from $t = 0$ to $t = 28$. [3]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per unit time between $t = 3$ and $t = 7$.

(iii) Calculate the present value of the payment stream. [4]

[Total 14]

8 (i) Explain what is meant by the following theories of the shape of the yield curve:

   (a) market segmentation theory
   (b) liquidity preference theory [4]

Short-term, one-year annual effective interest rates are currently 6%; they are expected to be 5% in one year's time; 4% in two years' time and 3% in three years' time.

   (ii) Calculate the gross redemption yields from one-year, two-year, three–year and four-year zero coupon bonds using the above expected interest rates. [4]

The price of a coupon-paying bond is calculated by discounting individual payments from the bond at the zero–coupon yields in part (ii).

   (iii) Calculate the gross redemption yield of a bond that pays a coupon of 4% per annum annually in arrear and is redeemed at 110% in exactly four years. [5]

   (iv) Explain why the gross redemption yield of a bond that pays a coupon of 8% per annum annually in arrear and is redeemed at par would be greater than that calculated in part (iii). [2]

The government introduces regulations that require banks to hold more government bonds with very short terms to redemption.

   (v) Explain, with reference to market segmentation theory, the likely effect of this regulation on the pattern of spot rates calculated in part (ii). [2]
   [Total 17]

9 A government issued a number of index-linked bonds on 1 June 2012 which were redeemed on 1 June 2014. Each bond had a nominal coupon of 2% per annum, payable half yearly in arrear and a nominal redemption price of 100%. The actual coupon and redemption payments were indexed according to the increase in the retail price index between three months before the issue date and three months before the relevant payment dates. No adjustment is made to allow for the actual date of calculation of the price index within the month or the precise coupon payment date within the month.

The values of the retail price index in the relevant months were:

Date	Retail Price Index
March 2012	112
June 2012	113
September 2012	116
December 2012	117
March 2013	117
June 2013	118
September 2013	120
December 2013	121
March 2014	121
June 2014	122

An investor purchased £3.5m nominal of the bond at the issue date and held it until it was redeemed. The investor was subject to tax on coupon payments at a rate of 25%.

(i)   Calculate the incoming net cash flows the investor received.   [5]

(ii)   Express the cash flows in terms of 1 June 2012 prices.   [4]

(iii)   Calculate the purchase price of the bond per £100 nominal if the real net redemption yield achieved by the investor was 1.5% per annum effective.   [3]

When the investor purchased the security, he expected the retail price index to rise much more slowly than it did in practice.

(iv)   Explain whether the investor's expected net real rate of return at purchase would have been greater than 1.5% per annum effective.   [2]

In September 2012, the government indicated that it might change the price index to which payments were linked to one which tends to rise more slowly than the retail price index.

(v)   Explain the likely impact of such a change on the market price of index-linked bonds.   [2]

[Total 16]

10 A student is considering whether to attend university or enter a profession immediately upon leaving school. If he enters the profession immediately, his salary is expected to be as follows.

   Year 1: £15,000
   Year 2: £18,000
   Year 3: £20,000

In each subsequent year the expected salary would rise by 1% per annum compound. The salary is assumed to be received monthly in arrear for 40 years.

If he attends university, the fees and other costs will be £15,000 per annum for three years, paid annually in advance. After attending university, the student's potential earnings will rise. Immediately after leaving university, he expects to earn £22,000 in the first year, £25,000 in the second year and £28,000 in the third year. Thereafter, his salary is expected to rise each year by 1.5% per annum compound. The salary would be paid monthly in arrear for 37 years.

(i) Calculate the present value of the student's earnings if he enters the profession immediately at a rate of interest of 7% per annum effective. [7]

(ii) Calculate the net present value of the decision to attend university at a rate of interest of 7% per annum effective and hence determine whether attending university would be a more attractive option. [9]

(iii) Explain why attending university would be relatively more attractive at lower interest rates. [2]

The student wishes to consider the effect of taxation on earnings.

(iv) Determine the rate of income tax above which the option of attending university would be less attractive financially than that of entering the profession immediately. [2]

[Total 20]

END OF PAPER