EXAMINATION

30 September 2009 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   Enter all the candidate and examination details as requested on the front of your answer booklet.

2.   You must not start writing your answers in the booklet until instructed to do so by the supervisor.

3.   Mark allocations are shown in brackets.

4.   Attempt all 10 questions, beginning your answer to each question on a separate sheet.

5.   Candidates should show calculations where this is appropriate.

Graph paper is *not* required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

1   A 182-day government bill, redeemable at £100, was purchased for £96 at the time of issue and was later sold to another investor for £97.89. The rate of return received by the initial purchaser was 5% per annum effective.

(a)   Calculate the length of time in days for which the initial purchaser held the bill.

(b)   Calculate the annual simple rate of return achieved by the second investor.

[4]

2   List the characteristics of an equity investment.   [4]

3   An investor bought a number of shares at 78 pence each on 31 December 2005. She received dividends on her holding on 31 December 2006, 2007 and 2008. The rate of dividend per share is given in the table below:

Date	Rate of dividend per share	Retail price index
31.12.2005	- - - - - -	147.7
31.12.2006	4.1 pence	153.4
31.12.2007	4.6 pence	158.6
31.12.2008	5.1 pence	165.1

On 31 December 2008, she sold her shares at a price of 93 pence per share.

Calculate, using the retail price index values shown in the table, the effective annual real rate of return achieved by the investor   [7]

4   A fixed-interest security has just been issued. The security pays half-yearly coupons of 5% per annum in arrear and is redeemable at par 20 years after issue.

(i)   Calculate the price to provide an investor with a net redemption yield of 6% per annum effective. The investor pays tax at a rate of 20% on income and is not subject to capital gains tax.   [3]

(ii)   Determine the annual effective gross redemption yield of this security assuming the price calculated in (i) is paid.   [5]

(iii)   Determine the real annual effective gross redemption yield of this security if the rate of inflation is constant over the twenty years at 3% per annum.   [2]

[Total 10]

5   The force of interest $\delta(t)$ at time t is $a + bt^2$ where a and b are constants. An amount of £100 invested at time $t = 0$ accumulates to £130 at time $t = 5$ and £200 at time $t = 10$.

   (i)   Calculate the values of a and b. [6]

   (ii)   Calculate the constant rate of interest per annum convertible monthly that would give rise to the same accumulation from time $t = 0$ to time $t = 5$. [2]

   (iii)   Calculate the constant force of interest that would give rise to the same accumulation from time $t = 5$ to time $t = 10$. [2]

   [Total 10]

6   (i)   Distinguish between a future and an option. [2]

   An investor wishes to purchase a one year forward contract on a risk-free bond which has a current market price of £97 per £100 nominal. The bond will pay coupons at a rate of 7% per annum half yearly. The next coupon payment is due in exactly six months and the following coupon payment is due just before the forward contract matures. The six-month risk-free spot interest rate is 5% per annum effective and the 12-month risk-free spot interest rate is 6% per annum effective.

   (ii)   Stating all necessary assumptions:

   (a)   Calculate the forward price of the bond.

   (b)   Calculate the six-month forward rate for an investment made in six months' time.

   (c)   Calculate the purchase price of a risk-free bond with exactly one year to maturity which is redeemed at par and which pays coupons of 4% per annum half-yearly in arrears.

   (d)   Calculate the gross redemption yield from the bond in (c).

   (e)   Comment on why your answer in (d) is close to the one-year spot rate.

   [10]
   [Total 12]

7 A member of a pensions savings scheme invests £1,200 per annum in monthly instalments, in advance, for 20 years from his 25th birthday. From the age of 45, the member increases his investment to £2,400 per annum. At each birthday thereafter the annual rate of investment is further increased by £100 per annum. The investments continue to be made monthly in advance for 20 years until the individual's 65th birthday.

(i) Calculate the accumulation of the investment at the age of 65 using a rate of interest of 6% per annum effective. [6]

At the age of 65, the scheme member uses his accumulated investment to purchase an annuity with a term of 20 years to be paid half-yearly in arrear. At this time the interest rate is 5% per annum convertible half-yearly.

(ii) Calculate the annual rate of payment of the annuity. [3]

(iii) Calculate the discounted mean term of the annuity, in years, at the time of purchase. [3]

[Total 12]

8 A bank offers a customer two different repayment options on a loan of £50,000 as follows:

Option 1 – level instalments of capital and interest are paid annually in arrear over a period of 20 years.

Option 2 – over the 20-year term the customer pays only interest on the loan, annually in arrear at a rate of 5.5% per annum with the whole of the capital amount payable at the end of the term. The customer will take out a separate savings policy which involves making monthly payments in advance such that the proceeds will be sufficient to repay the loan at the end of its term. The payments into the savings policy accumulate at a rate of interest of 4% per annum effective.

(i) Determine the effective rate of interest per annum that would be paid by the customer on the loan under Option 1, given that the level annual instalment on this loan is £4,012.13. [3]

(ii) Determine the annual effective rate of interest paid by a customer under Option 2. [7]

[Total 10]

9 A life insurance company is issuing a single premium policy which will pay out £20,000 in twenty years time. The interest rate the company will earn on the invested funds over the first ten years of the policy will be 4% per annum with a probability of 0.3 and 6% per annum with a probability of 0.7. Over the second ten years the interest rate earned will be 5% per annum with probability 0.5 and 6% per annum with probability 0.5.

(i) Calculate the premium that the company would charge if it calculates the premium using the expected annual rate of interest in each ten year period. [2]

(ii) Calculate the expected profit to the company if the premium is calculated as in (i). The rate of interest in the second ten year period is independent of that in the first ten year period. [3]

(iii) Explain why, despite the company using the expected rate of interest to calculate the premium, there is a positive expected profit. [2]

(iv) By considering each possible outcome in (ii):

    (a) Find the range of possible profits.
    (b) Calculate the standard deviation of the profit to the company. [7]

[Total 14]

10 A group of experts is analysing options to try to avert problems caused by climate change. They agree on the following expected costs and benefits of climate change over the next 50 years, starting from the current time. All figures are given in 2009 dollars.

Costs of climate change:

- Serious events will occur once every three years, in arrear, each giving rise to costs of $30bn, incurred immediately on the date of the event.

- Communities affected by climate change will incur costs of $20bn per annum incurred continuously, increasing at a continuous rate of 1% per annum.

- Other costs, assumed to be $40bn per annum, will be incurred annually in arrear.

Benefits arising from climate change:

- Benefits from higher crop yields and lower heating costs are assumed to be $10bn per annum, incurred annually in arrear.

The experts are considering whether to recommend investment in a carbon storing technology which, it is believed, will reduce all the costs and benefits listed above to zero. The technology requires a one-off investment immediately of $440bn. Costs are then assumed to be $50bn per annum incurred annually in arrear for 50 years.

The experts do not agree about the appropriate rate of interest at which to evaluate the options available. One group believes that the net present value of using the carbon storage technology should be evaluated at a real rate of return of 4% per annum effective. A second group believe that it should be evaluated at a real rate of return of 1% per annum effective.

(i)   Define what is meant by the discounted payback period of an investment and indicate its main disadvantage as an investment decision criterion.   [3]

(ii) Explain why the project must have a discounted payback period when the interest rate is 1.5% and the internal rate of return is higher than 1.5%.   [2]

(iii) Calculate the net present value of the carbon storing technology at a real rate of interest of 1% per annum effective.   [5]

(iv) Calculate the net present value of the carbon storing technology at a real rate of interest of 4% per annum effective.   [5]
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(v)   Comment on whether the investment in the carbon storing technology should go ahead.   [2]
          [Total 17]

END OF PAPER