EXAMINATION

25 September 2007 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   Enter all the candidate and examination details as requested on the front of your answer booklet.

2.   You must not start writing your answers in the booklet until instructed to do so by the supervisor.

3.   Mark allocations are shown in brackets.

4.   Attempt all 11 questions, beginning your answer to each question on a separate sheet.

5.   Candidates should show calculations where this is appropriate.

Graph paper is *not* required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.*

1 A 90-day government bill is purchased for £96 at the time of issue and is sold after 45 days to another investor for £97.90. The second investor holds the bill until maturity and receives £100.

Determine which investor receives the higher rate of return. [2]

2 An investor purchases a share for 769p at the beginning of the year. Halfway through the year he receives a dividend, net of tax, of 4p and immediately sells the share for 800p. Capital gains tax of 30% is paid on the difference between the sale and the purchase price.

Calculate the net annual effective rate of return the investor obtains on the investment. [4]

3 An insurance company offers a customer two payment options in respect of an invoice for £456. The first option involves 24 payments of £20 paid at the beginning of each month starting immediately. The second option involves 24 payments of £20.50 paid at the end of each month starting immediately. The customer is willing to accept a monthly payment schedule if the annual effective interest rate per annum he pays is less than 5%.

Determine which, if any, of the payment options the customer will accept. [4]

4 State the characteristics of an equity investment. [4]

5 A one-year forward contract is issued on 1 April 2007 on a share with a price of 900p at that date. Dividends of 50p per share are expected on 30 September 2007 and 31 March 2008. The 6-month and 12-month spot, risk-free rates of interest are 5% and 6% per annum effective respectively on 1 April 2007.

Calculate the forward price at issue, stating any assumptions. [4]

6 The annual effective forward rate applicable over the period t to $t + r$ is defined as $f_{t,r}$ where t and r are measured in years. $f_{0,1} = 4\%$, $f_{1,1} = 4.25\%$ $f_{2,1} = 4.5\%$, $f_{2,2} = 5\%$. Calculate the following:

(i) $f_{3,1}$ [1]

(ii)   All possible zero coupon (spot) yields that the above information allows you to calculate. [4]

(iii) The gross redemption yield of a four-year bond, redeemable at par, with a 3% coupon payable annually in arrears. [6]

(iv)   Explain why the gross redemption yield from the four-year bond is lower than the one-year forward rate up to time 4, $f_{3,1}$ [2]

[Total 13]

7 The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t) = \begin{cases} 0.04 + 0.01t & \text{for } 0 \le t \le 10 \\ 0.05 & \text{for } t > 10 \end{cases}$$

(i) Derive, and simplify as far as possible, expressions for $v(t)$ where $v(t)$ is the present value of a unit sum of money due at time t. [5]

(ii) (a) Calculate the present value of £1,000 due at the end of 15 years.

(b) Calculate the annual effective rate of discount implied by the transaction in (a). [4]

(iii) A continuous payment stream is received at a rate of $20e^{-0.01t}$ units per annum between $t = 10$ and $t = 15$. Calculate the present value of the payment stream.

[4]
[Total 13]

8 A pension fund makes the following investments (£m):

1 January 2004	1 July 2004	1 January 2005	1 January 2006
12.5	6.6	7.0	8.0

The rates of return earned on money invested in the fund were as follows:

1 January 2004 to 30 June 2004	1 July 2004 to 31 December 2004	1 January 2005 to 31 December 2005	1 January 2006 to 31 December 2006
5%	6%	6.5%	3%

You may assume that 1 January to 30 June and 1 July to 31 December are precise half year periods.

(i) Calculate the linked internal rate of return per annum over the three years from 1 January 2004 to 31 December 2006, using semi-annual sub-intervals. [3]

(ii) Calculate the time weighted rate of return per annum over the three years from 1 January 2004 to 31 December 2006. [3]

(iii) Calculate the money weighted rate of return per annum over the three years from 1 January 2004 to 31 December 2006. [4]

(iv) Explain the relationship between your answers to (i), (ii) and (iii) above. [2]

[Total 12]

PLEASE TURN OVER

9   The expected effective annual rate of return from a bank's investment portfolio is 6% and the standard deviation of annual effective returns is 8%. The annual effective returns are independent and $(1+i_t)$ is lognormally distributed, where i_t is the return in year t.

Deriving any necessary formulae:

(i)   calculate the expected value of an investment of £2 million after ten years. [6]

(ii)   calculate the probability that the accumulation of the investment will be less than 80% of the expected value. [3]

[Total 9]

10   A government is holding an inquiry into the provision of loans by banks to consumers at high rates of interest. The loans are typically of short duration and to high risk consumers. Repayments are collected in person by representatives of the bank making the loan. Campaigners on behalf of the consumers and campaigners on behalf of the banks granting the loans are disputing one particular type of loan. The initial loans are for £2,000. Repayments are made at an annual rate of £2,400 payable monthly in advance for two years.

The consumers' association case

The consumers' association asserts that, on this particular type of loan, consumers who make all their repayments pay interest at an annual effective rate of over 200%.

The banks' case

The banks state that, on the same loans, 40% of the consumers default on all their remaining payments after exactly 12 payments have been made. Furthermore half of the consumers who have not defaulted after 12 payments default on all their remaining payments after exactly 18 payments have been made. The banks also argue that it costs 30% of each monthly repayment to collect the payment. These costs are still incurred even if the payment is not made by the consumer. Furthermore, with inflation of 2.5% per annum, the banks therefore assert that the real rate of interest that the lender obtains on the loan is less than 1.463% per annum effective.

(i)   (a)   Calculate the flat rate of interest paid by the consumer on the loan described above.

        (b)   State why the flat rate of interest is not a good measure of the cost of borrowing to the consumer. [4]

(ii)   Determine, for each of the cases above, whether the assertion is correct. [10]

[Total 14]

11 A pension fund has liabilities to pay pensions each year for the next 60 years. The pensions paid will be £100m at the end of the first year, £105m at the end of the second year, £110.25m at the end of the third year and so on, increasing by 5% each year. The fund holds government bonds to meet its pension liabilities. The bonds mature in 20 years time and pay an annual coupon of 4% in arrears.

(i) Calculate the present value of the pension fund's liabilities at a rate of interest of 3% per annum effective. [4]

(ii) Calculate the nominal amount of the bond that the fund needs to hold so that the present value of the assets is equal to the present value of the liabilities. [3]

(iii) Calculate the duration of the liabilities. [6]

(iv) Calculate the duration of the assets. [4]

(v) Using your calculations in (iii) and (iv), estimate by how much more the value of the liabilities would increase than the value of the assets if there were a reduction in the rate of interest to 1.5% per annum effective. [4]

[Total 21]

END OF PAPER