INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

20 April 2015 (pm)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 12 questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

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1 Explain why the running yield from property investments tends to be greater than that from equity investments. [3]

2 Calculate the time in days for £3,000 to accumulate to £3,800 at:

(a) a simple rate of interest of 4% per annum.
(b) a compound rate of interest of 4% per annum effective.

[4]

3 A 182-day treasury bill, redeemable at \$100, was purchased for \$96.50 at the time of issue and later sold to another investor for \$98 who held the bill to maturity. The rate of return received by the initial purchaser was 4% per annum effective.

(i) Calculate the length of time in days for which the initial purchaser held the bill. [2]

(ii) Calculate the annual simple rate of return achieved by the second investor. [2]

(iii) Calculate the annual effective rate of return achieved by the second investor. [2]

[Total 6]

4 (i) Describe what is meant by the "no arbitrage" assumption in financial mathematics. [2]

A 9-month forward contract is issued on 1 April 2015 on a stock with a price of £6 per share on that date. Dividends are assumed to be received continuously and the dividend yield is 3.5% per annum.

(ii) Calculate the theoretical forward price per share of the contract, assuming no arbitrage and a risk-free force of interest of 9% per annum. [2]

The actual forward price per share of the contract is £6.30 and the risk-free force of interest is as in part (ii).

(iii) Outline how an investor could make an arbitrage profit. [2]

[Total 6]

5 An investor pays £120 per annum into a savings account for 12 years. In the first four years, the payments are made annually in advance. In the second four years, the payments are made quarterly in advance. In the final four years, the payments are made monthly in advance.

The investor achieves a yield of 6% per annum convertible half-yearly on the investment.

Calculate the accumulated amount in the savings account at the end of 12 years. [7]

6 An ordinary share pays annual dividends. The next dividend is expected to be 6p per share and is due in exactly six months' time. It is expected that subsequent dividends will grow at a rate of 6% per annum compound and that inflation will be 4% per annum. The price of the share is 175p and dividends are expected to continue in perpetuity.

Calculate the expected effective real rate of return per annum for an investor who purchases the share. [6]

7 In a particular country, insurance companies are required by regulation to value their liabilities using spot rates of interest derived from the government bond yield curve.

Over time t (measured in years), the spot rate of interest is equal to:

$$i = 0.02t \text{ for } t \leq 5$$

An insurance company in this country has a group of annuity policies which involve making payments of £1m per annum for four years and £2m per annum in the fifth year. All payments are assumed to be paid halfway through the year.

(i) Calculate the value of the insurance company's liabilities. [3]

(ii) Outline two reasons why the spot yield curve might rise with term to redemption. [3]

(iii) Calculate the forward rate of interest from time $t = 3.5$ to time $t = 4.5$. [2]
[Total 8]

8 A fixed-interest security, redeemable at par in 10 years, pays annual coupons of 9% in arrear and has just been issued at a price to give an investor who does not pay tax a rate of return of 7% per annum effective.

(i) Calculate the price of the security at issue. [2]

(ii) Calculate the discounted mean term (duration) of the security at issue. [3]

(iii) Explain how your answer to part (ii) would differ if the annual coupons on the security were 3% instead of 9%. [2]

(iv) (a) Calculate the effective duration (volatility) of the security at the time of issue.

(b) Explain the usefulness of effective duration for an investor who expects to sell the security over the next few months. [3]
[Total 10]

9 A property development company has just purchased a retail outlet for \$4,000,000. A further \$900,000 will be spent refurbishing the outlet in six months' time.

An agreement has been made with a prospective tenant who will occupy the outlet beginning one year after the purchase date. The tenant will pay rent to the owner for five years and will then immediately purchase the outlet from the property development company for \$6,800,000. The initial rent will be \$360,000 per annum and this will be increased by the same percentage compound rate at the beginning of each successive year. The rental income is received quarterly in advance.

Calculate the compound percentage increase in the annual rent required to earn the company an internal rate of return of 12% per annum effective. [9]

10 The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 4 \\ 0.12 - 0.01t & \text{for } 4 < t \le 9 \\ 0.05 & \text{for } t > 9 \end{cases}$$

(i) Determine the discount factor, $v(t)$, that applies at time t for all $t \ge 0$. [5]

(ii) Calculate the present value at $t = 0$ of a payment stream, paid continuously from $t = 10$ to $t = 12$, under which the rate of payment at time t is $100e^{0.03t}$. [4]

(iii) Calculate the present value of an annuity of £1,000 paid at the end of each year for the first three years. [3]

[Total 12]

11 On 1 January 2016, a student plans to take out a five-year bank loan for £30,000 that will be repayable by instalments at the end of each month. Under this repayment schedule, the instalment at the end of January 2016 will be X, the instalment at the end of February 2016 will be $2X$ and so on, until the final instalment at the end of December 2020 will be $60X$. The bank charges a rate of interest of 15% per annum convertible monthly.

(i) Prove that $(Ia)_{\overline{n}|} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$. [3]

(ii) Show that $X = £26.62$. [4]

The student is concerned that she will not be able to afford the later repayments and so she suggests a revised repayment schedule. The student would borrow £30,000 on 1 January 2016 as before. She would now repay the loan by 60 level monthly instalments of $36X = £958.32$ but the first repayment would not be made until the end of January 2019 and hence the final instalment is paid at the end of December 2023.

(iii) Calculate the APR on the revised loan schedule and hence determine whether you believe the bank should accept the student's suggestion. [5]

(iv) Explain the difference in the total repayments made under the two arrangements. [2]

[Total 14]

12 In any year, the yield on investments with an insurance company has mean j and standard deviation s and is independent of the yields in all previous years.

(i) Derive formulae for the mean and variance of the accumulated value after n years of a single investment of 1 at time 0 with the insurance company. [5]

Each year the value of $(1+i_t)$, where i_t is the rate of interest earned in the t^{th} year, is lognormally distributed. The rate of interest has a mean value of 0.04 and standard deviation of 0.12 in all years.

(ii) (a) Calculate the parameters μ and σ^2 for the lognormal distribution of $(1+i_t)$.

(b) Calculate the probability that an investor receives a rate of return between 6% and 8% in any year.

[8]

(iii) Explain whether your answer to part (ii) (b) looks reasonable. [2]

[Total 15]

END OF PAPER