EXAMINATION

5 April 2006 (pm)

Subject CT5 — Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2.   *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3.   *Mark allocations are shown in brackets.*

4.   *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

5.   *Candidates should show calculations where this is appropriate.*

Graph paper is *not* required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.*

1   It is possible to model the mortality of current active members of a pension scheme using the following three-state continuous-time Markov model, with age-dependent forces of transition ρ_x, μ_x and v_x:
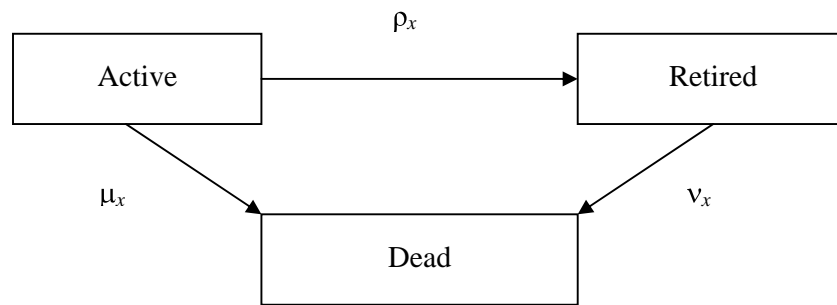
A pension scheme provides a benefit of £10,000 payable on death regardless of whether death occurs before or after retirement. Give an expression to value this benefit for an active life currently aged x. [2]

2   (i)   In the context of with-profit policies, describe the super compound method of adding bonuses. [2]

(ii)   Suggest a reason why a life insurance company might use the super compound method of adding bonuses as opposed to the compound method. [1]
[Total 3]

3   Using the PMA92C20 table for both lives calculate:

(a) $\mu_{65:60}$

(b) $_5p_{65:60}$

(c) $_2q^1_{65:65}$

[4]

4   State the main difference between an overhead expense and a direct expense incurred in writing a life insurance policy and give an example of each. [4]

5   A life office issues term assurance policies to 500 lives all aged 30 exact with a term of 25 years. The benefit of £10,000 is payable at the end of the year of death of any of the lives into a special fund. Calculate the expected share of this fund for each survivor after 25 years.

Basis:

Mortality   AM92 Select
Interest   4% per annum

[4]

6   A life office has issued for a number of years whole-life regular premium policies to a group of lives through direct advertising. Assured lives are only required to complete an application form with no further evidence of health. Outline the forms of selection that the insurer should expect to find in the mortality experience of the lives.

[5]

7   (i)   Show that:

$$\frac{\partial}{\partial t} \, _s p_{x+t} = \, _s p_{x+t}(\mu_{x+t} - \mu_{x+t+s})$$

[2]

        (ii)   Prove Thiele's differential equation for a whole-life assurance issued to a life aged x to be as follows:

$$\frac{\partial}{\partial t} \, _t \overline{V}_x = -(1 - \, _t \overline{V}_x)\mu_{x+t} + \delta \, _t \overline{V}_x + \overline{P}_x$$

[4]

[Total 6]

8   (i)   Calculate the expected present value of an annuity-due of 1 per annum payable annually in advance until the death of the last survivor of two lives using the following basis:

        First life: male aged 70, mortality table PMA92C20
        Second life: female aged 67, mortality table PFA92C20
        Rate of interest: 4% per annum

[2]

        (ii)   Give an expression for the variance of the annuity-due in terms of annuity functions.

[5]

[Total 7]

9 (i) Express fully in words:

$$\bar{a}_{\overline{xy:n|}}$$ [3]

(ii) Express $\bar{a}_{\overline{xy:n|}}$ as the expected value of random variables and hence show that

$$\bar{a}_{\overline{xy:n|}} = \frac{1 - \bar{A}_{\overline{xy:n|}}}{\delta}$$ [4]

[Total 7]

10 A 20-year special endowment assurance policy is issued to a group of lives aged 45 exact. Each policy provides a sum assured of £10,000 payable at the end of the year of death or £20,000 payable if the life survives until the maturity date. Premiums on the policy are payable annually in advance for 15 years or until earlier death.

You are given the following information:

Number of deaths during the 13th policy year   4
Number of policies in force at the end of the 13th policy year   195

(i) Calculate the profit or loss arising from mortality in the 13th policy year. [7]

(ii) Comment on your results. [2]

Basis:

Mortality   AM92 Ultimate
Interest   4% per annum
Expenses   none

[Total 9]

11 An employer wishes to introduce a lump-sum retirement benefit payable immediately on retirement at 65 or earlier other than on the grounds of ill-health. The amount of the benefit is £1,000 for each year of an employee's service, with proportionate parts of a year counting.

(i) Give a formula to value this benefit for an employee currently aged x with n years of past service, defining all terms used. [5]

(ii) Using the Pension Scheme Tables from the Actuarial Formulae and Tables, calculate the value for an employee currently aged 30 exact with exactly 10 years past service. [2]

(iii) Calculate the level annual contribution payable continuously throughout this employee's service to fund the future retirement benefit. [3]

[Total 10]

12 (i) Define the following terms without giving detailed formulae:

      (a)   Crude Mortality Rate
      (b)   Directly Standardised Mortality Rate
      (c)   Indirectly Standardised Mortality Rate

[3]

(ii) The data in the following table are taken from data published by the Office of National Statistics in 2001.

	England and Wales		**Tyne and Wear**	
	Population	*Number of births*	*Population*	*Number of births*
Under 25	3,149,000	153,000	71,000	4,000
25–35	3,769,000	339,000	74,000	6,000
35+	3,927,000	103,000	82,000	1,000

      (a)   Using the population for England and Wales as the standard population calculate crude birth rates and the directly and indirectly standardised birth rates for Tyne and Wear.

      (b)   State an advantage of using the Indirectly Standardised Birth Rate and comment briefly on the answers you have obtained.

[8]
[Total 11]

13 A life aged 35 exact purchases a 30-year with-profit endowment assurance policy. Level premiums are payable monthly in advance throughout the duration of the contract. The sum assured of £250,000 plus declared reversionary bonuses are payable at maturity or at the end of the year of death if earlier.

(i)   Show that the monthly premium is £647.47 if the life insurance company assumes that future simple reversionary bonuses will be declared at the rate of 2% per annum and vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

Basis:

mortality   AM92 Select
interest   4% per annum
initial expenses   £250 plus 50% of the gross annual premium
renewal expenses   3% of the second and subsequent monthly premiums
claims expenses   £300 on death; £150 on maturity

[7]

(ii) At age 60 exact, immediately before the premium then due, the life wishes to surrender the policy. The life insurance company calculates a surrender value equal to the gross retrospective policy value, assuming the same basis as in (i) above.

Calculate the surrender value using the retrospective policy value at the end of the 25th policy year immediately before the premium then due and just after the declared bonus has increased the sum assured plus reversionary bonuses to £375,000. Assume that the life insurance company has declared a simple bonus throughout the duration of the policy consistent with the bonus loading assumption used to derive the premium in (i) above. [6]

(iii) State with a reason whether the surrender value would have been larger, the same or smaller than in (ii) above if the office had used the prospective gross premium policy value, on the same basis. [1]

[Total 14]

14 A life insurance company issues a 3-year unit linked endowment policy to a life aged 45 exact under which level premiums are payable yearly in advance. In the 1st year, 35% of the premium is allocated to units and 105% in the 2nd and 3rd years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.5% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death and surrender benefits are paid.

If the policyholder dies during the term of the policy, the death benefit of the bid value of the units is payable at the end of the year of death. The policyholder may surrender the policy only at the end of each year. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit.

The company uses the following assumptions in its profit test of this contract:

Rate of growth on assets in the unit fund	5% per annum
Rate of interest on non-unit fund cash flows	4% per annum
Independent rates of mortality	AM92 Ultimate
Independent rates of withdrawal	5% per annum
Initial expenses	£250
Renewal expenses	£50 per annum on the 2nd and 3rd premium dates
Initial commission	20% of 1st premium
Renewal commission	2.5% of the 2nd and 3rd years' premiums

The company sets premiums so that the net present value of the profit on the policy is 15% of the annual premium.

(i) Using a risk discount rate of 8% per annum, calculate the premium for the policy on the assumption that the company does not zeroise future expected negative cash flows. [12]

(ii) Explain why the company might need to zeroise future expected negative cash flows on the policy. [2]

[Total 14]

END OF PAPER