INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

30 September 2015 (pm)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all nine questions, beginning your answer to each question on a new page.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 An investor wishes to obtain a rate of interest of 3% per annum effective from a 91-day treasury bill.

Calculate:

(a) the price that the investor must pay per £100 nominal.

(b) the annual simple rate of discount from the treasury bill.

[3]

2 The nominal rate of discount per annum convertible monthly is 5.5%.

(i) Calculate, giving all your answers as a percentage to three decimal places:

(a) the equivalent force of interest.

(b) the equivalent effective rate of interest per annum.

(c) the equivalent nominal rate of interest per annum convertible monthly.

[3]

(ii) Explain why the nominal rate of interest per annum convertible monthly calculated in part (i)(c) is less than the equivalent annual effective rate of interest calculated in part (i)(b) [1]

(iii) Calculate, as a percentage to three decimal places, the effective annual rate of discount offered by an investment that pays £159 in eight years' time in return for £100 invested now. [1]

(iv) Calculate, as a percentage to three decimal places, the effective annual rate of interest from an investment that pays 12% interest at the end of each two-year period. [1]

[Total 6]

3 An insurance company has sold a pension product to an individual. Under the arrangement, the individual is to receive an immediate annuity of £500 per year annually in arrear for 12 years. The insurance company has invested the premium it has received in a fixed-interest bond that pays coupons annually in arrear at the rate of 5% per annum and which is redeemable at par in exactly eight years.

(i) Calculate the duration of the annuity at an interest rate of 4% per annum effective. [2]

(ii) Calculate the duration of the bond at an interest rate of 4% per annum effective. [3]

(iii) State with reasons whether the insurance company will make a profit or a loss if there is a small increase in interest rates at all terms. [2]

[Total 7]

4 A nine-month forward contract was issued on 1 October 2015 on a share with a price at that date of £10. Dividends of 50 pence per share are expected on 1 November 2015 and 1 May 2016. The risk-free force of interest is 5% per annum.

(i) Calculate the forward price at issue, stating any further assumptions made and showing all workings. [4]

(ii) Explain why the expected price of the share nine months after issue does not have to be taken into account when pricing the forward. [2]

[Total 6]

5 An individual can obtain a force of interest per annum at time t, measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \le t \le 3 \\ 0.005 & t > 3 \end{cases}$$

(i) Determine the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of £5,000 per annum from time $t = 3$ to time $t = 6$. [5]

(ii) Determine the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]

(iii) Determine the amount an individual would accumulate from the investment of £300 from time $t = 0$ to time $t = 50$. [2]

[Total 10]

6 Three bonds, each paying annual coupons in arrear of 3% and redeemable at £100 per £100 nominal, reach their redemption dates in exactly one, two and three years' time, respectively.

The price of each bond is £101 per £100 nominal.

(i) Determine the gross redemption yield of the three-year bond. [3]

(ii) Calculate the one-year, two-year and three-year spot rates of interest implied by the information given. [5]

(iii) Calculate the one-year forward rate starting from the end of the second year, $f_{2,1}$. [2]

The pattern of spot rates is upward sloping throughout the yield curve.

(iv) Explain, with reference to the various theories of the yield curve, why the yield curve might be upward sloping. [4]

[Total 14]

7 A special type of loan is to be issued by a company. The loan is made up of 100,000 bonds, each of nominal value €100. Coupons will be paid semi-annually in arrear at a rate of 4% per annum. The bonds are to be issued on 1 October 2015 at a price of €100 per €100 nominal. Income tax will be paid by the bond holders at a rate of 25% on all coupon payments.

Exactly half the bonds will be redeemed after ten years at €100 per €100 nominal. The bonds that are redeemed will be determined by lot (i.e. the bonds will be numbered and half the numbered bonds will be chosen randomly for redemption). Coupon payments on the remaining bonds will be increased to 7% per annum and these bonds will be redeemed 20 years after issue at €130 per €100 nominal.

An individual buys a single bond.

Calculate, as an effective rate of return per annum:

(i) the maximum rate of return the individual can obtain from the bond. [5]

(ii) the minimum rate of return the individual can obtain from the bond. [2]

(iii) the expected rate of return the individual will obtain from the bond [2]

An investor is considering buying the whole loan.

(iv) Show that the rate of return that the investor will obtain is greater than the expected rate of return that the above individual who buys a single bond will receive. [5]

[Total 14]

8 (i) State the characteristics of an equity. [4]

An investor was considering investing in the shares of a particular company on 1 August 2014. The investor assumed that the next dividend would be payable in exactly one year and would be equal to 6 pence per share.

Thereafter, dividends will grow at a constant rate of 1% per annum and are assumed to be paid in perpetuity. All dividends will be taxed at a rate of 20%. The investor requires a net rate of return from the shares of 6% per annum effective.

(ii) Derive and simplify as far as possible a general formula which will allow you to determine the value of a share for different values of:

- the next expected dividend.
- the dividend growth rate.
- the required rate of return.
- the tax rate.

(iii) Calculate the value of one share to the investor. [5]

The company announces some news that makes the shares more risky.

(iv) Explain what would happen to the value of the share, using the formula derived in part (ii). [2]

The investor bought 1,000 shares on 1 August 2014 for the price calculated in part (iii). He received the dividend of 6 pence on 1 August 2015 and paid the tax due on the dividend. The investor then sold the share immediately for 120 pence. Capital gains tax was charged on all gains of at a rate of 25%. On 1 August 2014, the index of retail prices was 123. On 1 August 2015, the index of retail prices was 126.

(v) Determine the net real return earned by the investor. [3]

[Total 14]

9 A student has inherited £1m and is considering investing the money in two projects, A and B.

Project A requires the investment of the whole sum in properties that are to be let out to tenants. The details are:

- The student expects to receive an income from rents at an annual rate of £60,000 a year for four years after an initial period of one year in which no income will be received.
- Rents are expected to rise thereafter at the start of each year at a rate of 0.5% per annum.
- The income will be received monthly in advance.
- The project involves costs of £10,000 per annum in the first year, rising at a constant rate of 0.5% per annum.
- The costs will be incurred at the beginning of each year.
- At the end of 20 years, the student expects to be able to sell the properties for £2m after which there will be no further revenue or costs.

Project B involves the investment of the whole sum in an investment fund.

- The fund is expected to pay an income of £60,000 per annum annually in advance and return the whole invested sum at the end of 20 years.

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(i) (a) Calculate the payback period for project B.

(b) Show, by general reasoning or otherwise, that the payback period from project A is longer than that from project B. [5]

(ii) (a) Define the discounted payback period.

(b) Determine the discounted payback period from project B at a rate of interest of 1% per annum effective.

(c) Show, by general reasoning or otherwise, that the discounted payback period from project A is longer than that from project B. [5]

(iii) Determine the internal rate of return from project B expressed as an annual effective return. [3]

(iv) Show that the internal rate of return from project A is higher than that from project B. [10]

(v) Discuss which project is the better project given your answers to parts (i)–(iv) above. [3]

[Total 26]

END OF PAPER