EXAMINATION

22 September 2008 (am)

Subject CT5 — Contingencies
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2.   *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3.   *Mark allocations are shown in brackets.*

4.   *Attempt all 14 questions, beginning your answer to each question on a separate sheet.*

5.   *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate (to the nearest integer) the lower quartile of the complete future lifetime of a person aged 25 exact who is subject to mortality according to ELT15 (Females). [3]

2 The profit signature of a 3-year assurance contract issued to a life aged 57 exact, with a premium payable at the start of each year of £500 is (−250, 150, 200).

Calculate the profit margin of the contract.

Basis:

Mortality   AM92 Ultimate
Lapses   None
Risk discount rate 12% per annum   [3]

3 In order to value the benefits in a final salary pension scheme as at 1 January 2008, a salary scale, s_x, has been defined so that $\dfrac{s_{x+t}}{s_x}$ is the ratio of a member's total earnings between ages $x + t$ and $x + t + 1$ to the member's total earnings between ages x and $x + 1$. Salary increases take place on 1 July every year. One member, whose date of birth is 1 April 1961, has an annual salary rate of £75,000 on the valuation date.

Write down an expression for the member's expected earnings during 2008. [3]

4 Write down an alternative expression for each of the following statements. Use notation as set out in the "International Actuarial Notation" section of the "Formulae and Tables for Examinations" where appropriate and express your answer as concisely as possible.

(i) Probability$[\text{maximum}\{T_x, T_y\} \le n]$ [1]

(ii) $E[g(K_x)]$ where $g(K_x) = v^{K_x+1}$ for $K_x < n$ and 0 for $K_x \ge n$ [1]

(iii) Probability$\{n < T_x \le m\}$ [1]

(iv) $\text{Limit}_{dt \to 0} \dfrac{1}{dt} \text{Probability}[\text{minimum}\{T_x, T_y\} \le t + dt \,|\, T_x > t, T_y > t]$ [1]

(v) $E[a_{\overline{\text{minimum}(n-1,\, K_x)}|} + 1]$ [1]

[Total 5]

5   (i)   Explain what is meant by $\ddot{s}_{x:\overline{n}|}$ [2]

    (ii)   Calculate $\ddot{s}_{50:\overline{20}|}$.   [3]

        Basis:
        Mortality: AM92 Ultimate
        Interest: 4% per annum

        [Total 5]

6   A select life aged 62 exact purchases a 3-year endowment assurance with sum assured £100,000. Premiums of £30,000 are payable annually in advance throughout the term of the policy or until earlier death. The death benefit is payable at the end of the policy year of death.

Calculate the expected value of the present value of the profit or loss to the office on the contract, using the following basis:

Interest   7.5% per annum
Expenses   Ignore
Mortality $q_{[x-t]+t} = \dfrac{1}{4-t} q_x$ for all x and for $t = 0$, 1 or 2.

                 $q_{62} = 0.018$, $q_{63} = 0.02$ and $q_{64} = 0.022$

        [6]

7   A certain population is subject to three modes of decrement: α, β and γ.

    (i)   Write down an expression for $(aq)_x^{\alpha}$ in terms of the single decrement table probabilities $q_x^{\alpha}, q_x^{\beta}$, and q_x^{γ}, assuming each of the three modes of decrement is uniformly distributed over the year of age x to $x + 1$ in the corresponding single decrement table.   [2]

    (ii)   Suppose now that in the single decrement table α, $_t p_x^{\alpha} = 1 - t^2 q_x^{\alpha}$ $(0 \leq t \leq 1)$, while decrements β and γ remain uniformly distributed. Derive a revised expression for $(aq)_x^{\alpha}$ in terms of the single decrement table probabilities $q_x^{\alpha}, q_x^{\beta}$, and q_x^{γ}.   [4]

        [Total 6]

8 A life insurance company sells 1,000 whole life annuities on 1 January 2007 to policyholders aged 65 exact. Each annuity is for £25,000 payable annually in arrear. 5 annuitants die during 2007.

The office holds reserves using the following basis:

Mortality   PFA92C20
Interest   4% per annum

(i) Calculate the profit or loss from mortality for this group for the year ending 31 December 2007. [4]

(ii) Explain why the mortality profit or loss has arisen. [2]
[Total 6]

9 A new member aged 35 exact, expecting to earn £40,000 in the next 12 months, has just joined a pension scheme. The scheme provides a pension on retirement for any reason of 1/60th of final pensionable salary for each year of service, with fractions counting proportionately. Final pensionable salary is defined as the average salary over the three years prior to retirement.

Members contribute a percentage of salary, the rate depending on age. Those under age 50 contribute 4% and those age 50 exact and over contribute 5%.

The employer contributes a constant multiple of members' contributions to meet exactly the expected cost of pension benefits.

Calculate the multiple needed to meet this new member's benefits.

All elements of the valuation basis are contained in the Example Pension Scheme Table in the Formulae and Tables for Examinations. [6]

10 Calculate the variance of the present value of benefits under an annuity payable to a life aged 35 exact. The annuity has payments of 1 per annum payable continuously for life.

Basis:

Mortality $\mu = 0.02$ throughout
Interest $\delta = 0.05$ [7]

11 A life insurance company has reviewed its mortality experience. For each age, it has pooled all the deaths and corresponding exposures from its entire portfolio over the previous ten years, and derived a single mortality table.

List three types of selection which might be likely to produce heterogeneity in this particular investigation. In each case, explain the nature of the heterogeneity and how it could be caused, and state how the heterogeneity could be reduced. [9]

12 A life insurance company is considering selling with-profit endowment policies with a term of twenty years and initial sum assured of £100,000. Death benefits are payable at the end of the policy year of death. Bonuses will vest at the end of each policy year.

The company is considering three different bonus structures:

   (1)   Simple reversionary bonuses of 4.5% per annum.

   (2)   Compound reversionary bonuses of 3.84615% per annum.

   (3)   Super compound bonuses where the original sum assured receives a bonus of 3% each year and all previous bonuses receive an additional bonus of 6% each year.

(i)   Calculate the amount payable at maturity under the three structures.   [4]

(ii)   Calculate the expected value of benefits under structure (2) for an individual aged 45 exact at the start, using the following basis:

   Interest   8% per annum
   Mortality   AM92 Select
   Expenses   ignore
                                                                    [4]

(iii)   Calculate the expected value of benefits, using the same policy and basis as in (ii) but reflecting the following changes:

   (a)   Bonuses vest at the start of each policy year (the death benefit is payable at the end of the policy year of death).

   (b)   The death benefit is payable immediately on death (bonuses vest at the end of each policy year).

   (c)   The death benefit is payable immediately on death, and bonuses vest continuously.   [3]
                                                                 [Total 11]

13 Two lives, a female aged 60 exact and a male aged 65 exact, purchase a policy with the following benefits:

(i) an annuity deferred ten years, with £20,000 payable annually in advance for as long as either of them is alive

(ii) a lump sum of £100,000 payable at the end of the policy year of the first death, should this occur during the deferred period

Level premiums are payable monthly in advance throughout the deferred period or until earlier payment of the death benefit.

Calculate the monthly premium.

Basis:

Mortality	Female	PFA92C20
	Male	PMA92C20

Interest   4% per annum

Expenses	Initial	£350
	Renewal	2.5% of each monthly premium excluding the first.

[14]

14 A life insurance company issues a decreasing term assurance policy to a life aged 55 exact. The death benefit, which is payable immediately on death, is £100,000 in the first policy year, £90,000 in the second year thereafter reducing by £10,000 each year until the benefit is £10,000 in the 10th year, with cover ceasing at age 65.

The policy is paid for by level annual premiums payable in advance for 10 years, ceasing on earlier death.

The life office uses the following basis for calculating premiums and reserves:

Basis:

Mortality   AM92 Select

Interest   4% per annum

Expenses	Initial	£300 plus 25% of the first premium
	Renewal	5% of all premiums excluding the first and $£50*(1.04)^t$ on each policy anniversary where t is the exact duration of the policy on the anniversary
	Claim	$£200*(1.04)^u$ where u is the exact duration of the policy at death, measured in years with fractions counting

(i)   Write down the gross premium future loss random variable at the start of the policy. Use P for the annual premium. [4]

(ii)   Calculate the premium, using the equivalence principle. [10]

(iii) Calculate the gross premium prospective reserve after 9 years. [2]

[Total 16]

END OF PAPER