# INSTITUTE AND FACULTY OF ACTUARIES

# EXAMINATION

27 September 2016 (am)

## Subject CT1 – Financial Mathematics
## Core Technical

*Time allowed: Three hours*

***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 12 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

***Graph paper is NOT required for this paper.***

*AT THE END OF THE EXAMINATION*

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

*In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

**1** The nominal rate of interest per annum convertible quarterly is 5%.

Calculate, giving all the answers as a percentage to three decimal places:

(i) the equivalent annual force of interest. [1]

(ii) the equivalent effective rate of interest per annum. [1]

(iii) the equivalent nominal rate of discount per annum convertible monthly. [2]

[Total 4]

**2** The nominal rate of interest per annum convertible quarterly is 2%.

Calculate the present value of a payment stream paid at a rate of €100 per annum, monthly in advance for 12 years. [4]

**3** Describe the characteristics of a repayment loan (or repayment mortgage). [3]

**4** The following table shows the cashflows paid into a fund on three different dates, together with the value of the fund on each date immediately before the cash flow takes place. There were no other cashflows except on the dates shown.

| | *1 January 2014* | *1 January 2015* | *1 January 2016* |
|---|---|---|---|
| Value of fund (£m) | 112 | *X* | 160 |
| Cash flow (£m) | 23 | 43 | 32 |

During 2014, the rate of return on the fund was 10% per annum effective.

(i) Calculate *X*. [1]

(ii) Calculate, showing all workings, the annual effective time weighted rate of return on the fund over the two-year period from 1 January 2014 to 1 January 2016. [3]

[Total 4]

**5** A zero-coupon bond was issued on 1 January 1975 with a redemption date of 1 January 2015. An investor bought the bond to provide a yield to maturity of 5% per annum convertible half yearly. On a particular date the borrower defaulted, repaying 80% of the capital to all bondholders. The investor obtained a rate of return until the date of default which was equivalent to a force of interest of 4.8% per annum.

Determine the date on which the borrower defaulted. [5]

**6** At the beginning of 2015 a 182–day commercial bill, redeemable at £100, was purchased for £96 at the time of issue and later sold to a second investor for £97.50. The initial purchaser obtained a simple rate of interest of 3.5% per annum before selling the bill.

(i) Calculate the annual simple rate of return which the initial purchaser would have received if they had held the bill to maturity. [2]

(ii) Calculate the length of time in days for which the initial purchaser held the bill. [2]

The second investor held the bill to maturity.

(iii) Calculate the annual effective rate of return achieved by the second investor. [2]

[Total 6]

**7** A nine-month forward contract was issued on 1 April 2015 on a share with a price of £1.10 at that date. Dividends of £0.10 per share were expected on 1 July 2015, 1 October 2015 and 1 January 2016.

(i) Calculate, showing all workings, the forward price assuming a risk-free rate of interest of 8% per annum convertible half-yearly and no arbitrage. [4]

(ii) Explain why you do not need to use the expected price of the share at the time the forward matures in the calculation of the forward price. [2]

[Total 6]

**8** Three bonds, each paying annual coupons in arrear of 4% and redeemable at par, reach their redemption dates in exactly one, two and three years' time, respectively. The price of each bond is £96 per £100 nominal.

(i) Calculate the gross redemption yield of the three-year bond. [3]

(ii) Calculate, showing all workings, the one-year and two-year spot rates of interest implied by the information given. [3]

(iii) Calculate the forward rate of interest applicable over the second year. [2]

(iv) Explain whether the three-year spot rate will be higher than or lower than the three-year gross redemption yield. [2]

[Total 10]

**9** An insurance company has just written single premium contracts that require it to make payments to policyholders of £10,000,000 in five years' time. The total single premiums paid by policyholders amounted to £8,000,000.

The insurance company is to invest the premiums in assets that have an uncertain return. The return from these assets in year $t$, $i_t$, is independent of the returns in all previous years with a mean value of 5.5% per annum effective and a standard deviation of 4% per annum effective. $(1 + i_t)$ is lognormally distributed.

(i) Calculate, deriving all necessary formulae, the mean and standard deviation of the accumulation of the premiums over the five-year period. [9]

A director of the company is concerned about the possibility of a considerable loss from the investment in the assets suggested in part (i). Instead, the director suggests investing in fixed interest securities with a guaranteed return of 4% per annum effective.

(ii) Set out the arguments for and against the director's position. [3]

[Total 12]

**10** A particular charity invests its assets in a fund on which it has a target rate of return of 8% per annum effective. From time-to-time, the charity also invests in projects that help achieve its charitable objectives whilst providing a rate of return. Projects that are accepted by the charity must fulfil each of the following criteria:

1. a minimum annual effective internal rate of return of 2% less than the target return on the investment fund.

2. a payback period of no more than ten years.

3. a positive cash flow during the fifth year or earlier.

The charity is considering investing in a social enterprise project that involves providing loans to farmers in low-income countries to help them develop better resilience against poor weather conditions. The details are as follows:

- The project involves making loans of £1m at the start of each year for three years, the first loan being made at the beginning of 2017.

- The loans will be paid back from the extra income obtained by the farmers from the beginning of 2020.

- The repayments in each year will be through level monthly instalments paid in advance with the rate of payment of the instalments increasing by 1% per year for 10 years after which the payments stop.

- The annual rate of repayment in 2020 will be £495,000.

- The charity will also incur costs at the end of each of the years in which income is received of £50,000 per annum.

(i) Explain why, in general, the payback period is not an appropriate decision criterion for an investment project. [2]

(ii) Determine which of the three criteria used by the charity are met in this case. [12]

[Total 14]

**11** The government of a heavily indebted country has a range of bonds currently in issue. These include bonds with nominal amounts outstanding of £4bn and £5bn with terms to redemption of exactly three years and ten years respectively from the current time. Both bonds pay annual coupons in arrear of 4%. The government is negotiating a restructuring of its debt portfolio and proposes to transform the three and ten year bonds into perpetuities paying an annual coupon of 5% in arrear. The yield curve is currently flat with gross redemption yields at 6% per annum effective.

(i) Calculate, showing all workings, the duration of the current portfolio of three-year and ten-year bonds. [7]

(ii) Calculate, showing all workings, the duration of the proposed portfolio of bonds. [4]

The government's objective is that the present value of the proposed portfolio of bonds will be 80% of the present value of the current portfolio of three-year and ten-year bonds.

(iii) Determine the nominal amount of the new bonds that the government will have to issue to achieve the objective. [2]

[Total 13]

**12** The force of interest, $\delta(t)$, is a function of time and at any time $t$ (measured in years) is given by:

$$\delta(t) = \begin{cases} 0.03 & \text{for } 0 \le t \le 10 \\ at & \text{for } 10 < t \le 20 \\ bt & \text{for } t > 20 \end{cases}$$

where $a$ and $b$ are constants.

The present value of £100 due at time 20 is 50.

(i) Calculate $a$. [5]

The present value of £100 due at time 28 is 40.

(ii) Calculate $b$. [4]

(iii) Calculate the equivalent annual effective rate of discount from time 0 to time 28. [2]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per annum between $t = 3$ and $t = 7$.

(iv) (a) Calculate, showing all workings, the present value of the payment stream.

(b) Determine the level continuous payment stream per annum from time $t = 3$ to time $t = 7$ that would provide the same present value as the answer in part (iv)(a) above. [8]

[Total 19]

## END OF PAPER