INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2016 (with mark allocations)

Subject CT1 – Financial Mathematics Core Technical

Introduction

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

F Layton
Chair of the Board of Examiners
June 2016

A. General comments on the *aims of this subject and how it is marked*

1. CT1 provides a grounding in financial mathematics and its simple applications. It introduces compound interest, the time value of money and discounted cashflow techniques which are fundamental building blocks for most actuarial work.

2. Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may be penalised where excessive rounding has been used or where insufficient working is shown.

B. General comments on *student performance in this diet of the examination*

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made the question was generally answered well by most candidates.

2. Performance was of a similar standard to most recent diets with the weaker performance in September 2015 being an exception to the general standard. As in previous diets, the non-numerical questions were often answered poorly by marginal candidates.

C. Pass Mark

The Pass Mark for this exam was 60%.

Solutions

Q1 Convertible Securities

- Generally unsecured loan stocks.
- Can be converted into ordinary shares of the issuing company.
- Pay interest/coupons until conversion.
- The date of conversion might be a single date or, at the option of the holder, one of a series of specified dates.
- Risk characteristics of convertible vary as the final date for convertibility approaches (behaviour will tend towards the security into which it is likely to convert)
- Generally less volatility than in the underlying share price before conversion.
- Combine lower risk of debt securities with the potential for gains from equity investment.
- Security and marketability depend upon issuer.
- Generally provide higher income than ordinary shares and lower income than conventional loan stock or preference shares.
- Option to convert will have time value which is reflected in price of the security.

[½ mark for each valid point]

[MAX 3]

Despite being a bookwork question, this was answered poorly. This performance is consistent with questions in previous years on the same area of the syllabus.

Q2 (i) PV of asset proceeds is:

$$V_A(0.08) = 5.5088v_{8\%}^{5} + 13.7969v_{8\%}^{20} = 6.7093$$

PV of liability outgo is:

$$V_L(0.08) = 6v_{8\%}^{8} + 11v_{8\%}^{15} = 6.7093 = V_A(0.08)$$

Hence, condition (1) for immunisation is satisfied. [2]

Also, DMT of asset proceeds is:

$$\tau_A(0.08) = \frac{5\times 5.5088v_{8\%}^{5} + 20\times 13.7969v_{8\%}^{20}}{6.7093} = 11.618$$

And, DMT of liability outgo is:

$$\tau_L(0.08) = \frac{8\times 6v_{8\%}^{8} + 15\times 11v_{8\%}^{15}}{6.7093} = 11.618 = \tau_A(0.08)$$

Hence, condition (2) for immunisation is also satisfied. [3]

(ii) Yes, the insurance company is immunised.

As the asset proceeds are received at times 5 and 20, whereas the liability outgo is paid at times 8 and 15, the spread of the asset proceeds around the DMT is greater than the spread of the liability outgo around the same DMT. [2]

[TOTAL 7]

Part (i) was generally answered well; however, candidates must include sufficient factors and workings to demonstrate that the respective asset and liability values are the same for each of the two conditions. Part (ii) was often answered poorly. To get full marks for this part, candidates were required to make reference to the actual data in the question rather than just repeating the theory (e.g. stating the actual figures for the spread of the assets and the liabilities around the DMT).

Q3 (i) Issue price (per £100 nominal) = $\frac{4}{1+i_i}+\frac{4}{(1+i_2)^2}+\frac{4}{(1+i_3)^3}+\frac{105}{(1+i_3)^3}$ [1½]

where i_t is the t-year zero coupon rate at time $t = 0$ and we have that:

$$\left(1+i_{t-1}\right)^{t-1} * (1+f_{t-1,1}) = (1+i_t)^t$$

where $f_{t-1,1}$ is the one-year forward rate at time $t - 1$

we have $1 + i_1 = 1.04$ (i_1 is given)

$$(1+i_2)^2 = (1+i_1)(1+f_{1,1})$$

$$= 1.04*1.05$$

$$(1+i_3)^3 = (1+i_2)^2(1+f_{2,1})$$

$$= 1.04*1.05*1.06$$ [1½]

$$\Rightarrow \text{Issue Price} = \frac{4}{1.04}+\frac{4}{1.04*1.05}+\frac{4+105}{1.04*1.05*1.06}$$

$$= £101.68$$ [1]

(ii) Let y_{c_3} be the 3 year par yield (%). Then y_{c_3} is given by

$$100 = y_{c_3}\left(\frac{1}{1+i_1}+\frac{1}{(1+i_2)^2}+\frac{1}{(1+i_3)^3}\right)+\frac{100}{(1+i_3)^3} \qquad [1½]$$

$$= y_{c_3}\left(\frac{1}{1.04}+\frac{1}{1.04*1.05}+\frac{1}{1.04*1.05*1.06}\right)+\frac{100}{1.04*1.05*1.06}$$

$$= y_{c_3}*2.741205336+86.39159583$$

$$\Rightarrow y_{c_3} = 4.9644\% \qquad [1½]$$

[TOTAL 7]

Generally well answered.

Q4 (i) $\left(1+\frac{i^{(4)}}{4}\right)^4 = 1.05 \Rightarrow i^{(4)} = 0.049089$

$$\frac{D}{R}(1-t_1) = \frac{0.07}{1.08}\times 0.75 = 0.04861$$

$$\Rightarrow i^{(4)} > (1-t_1)g$$

$\Rightarrow$ Capital gain on contract and we assume loan is redeemed as late as possible (i.e. after 20 years) to obtain minimum yield. [2]

Let price of stock = P

$$P = 0.07\times 100000\times 0.75\times a_{\overline{20|}}^{(4)} + (108000-0.40(108000-P))v^{20} \text{ at } 5\%$$

$$\Rightarrow P = \frac{5250\, a_{\overline{20|}}^{(4)} + 64800\, v^{20}}{1-0.40\, v^{20}}$$

$$= \frac{5250\times 1.018559\times 12.4622+64800\times 0.37689}{1-0.40\times 0.37689}$$

$$= £107,228.63 \qquad [3]$$

(above uses factors from Formulae and Tables Book – exact answer is £107,228.67)

(ii) As the redemption date is at the option of the borrower, it is outside the investor's control when the stock will be redeemed. Hence the investor must assume a worst case scenario in pricing the loan. [2]

[TOTAL 7]

> Part (i) was answered well. The reasoning of marginal candidates in part (ii) was often unclear. The key point is that the date of redemption is out of the control of the investor.

Q5 (i) Loan = $950\, a_{\overline{15|}} + 250(Ia)_{\overline{15|}}$ at 6%

$= 950 \times 9.7122 + 250 \times 67.2668$

$= £26{,}043.29$

[3]

(ii) Capital outstanding after 9 payments:

$3200\, a_{\overline{6|}} + 250(Ia)_{\overline{6|}} = 3200 \times 4.9173 + 250 \times 16.3767 = £19{,}829.54$

[2]

(iii) Capital outstanding after 14 payments $= 4700v$ at 6%
$= £4{,}433.96$
$=$ Capital in final payment

$\Rightarrow$ Interest in final payment $= 4700 - 4433.96$
$= £266.04$

[2]

(above uses factors from Formulae and Tables Book – exact answers are £26,043.34 for (i) and £19,829.61 for (ii))

[TOTAL 7]

> The best answered question on the paper.

Q6 (i) $$pv = 10{,}000 \times \exp\left[-\int_7^{10} (0.01t - 0.04)\,dt\right] \times \exp\left[-\int_5^{7} (0.10 - 0.01t)\,dt\right]$$ [1]

$$= 10{,}000 \exp \times \left(-\left[\frac{0.01t^2}{2} - 0.04t\right]_7^{10}\right) \times \exp\left(-\left[0.10t - \frac{0.01t^2}{2}\right]_5^7\right)$$

$$= 10{,}000 \times \exp\left(-\left[\frac{0.01*51}{2} - 0.04 \times 3\right]\right) \times \exp\left(-\left[0.10*2 - \frac{0.01*24}{2}\right]\right)$$

$= 10{,}000\exp(-0.255 + 0.12 - 0.20 + 0.12)$
$= 10{,}000\exp(-0.215)$

$= £8{,}065.41$ [4]

(ii) Required discount rate p.a. convertible monthly is given by

$$10{,}000\left(1 - \frac{d^{(12)}}{12}\right)^{12\times5} = 8{,}065.41$$

$d^{(12)} = 4.2923\%$ p.a. convertible monthly.

[2]
[TOTAL 7]

Generally well answered.

Q7 Forward price of the contract is:

$$K_0 = (S_0 - I)e^{\delta T} = (8.70 - I)e^{0.07}$$ [1]

where I is the present value of the income expected during the contract

$$\Rightarrow I = 1.10 \times e^{-0.07\times 8/12} = 1.049846$$ [1]

$$\Rightarrow K_0 = (8.70 - 1.049846) \times e^{0.07} = 8.204853$$ [½]

Forward price of contract set up at time r (where r = 5 months) is

$$K_r = (S_r - I_r)e^{\delta(T-r)}$$ [1]

where I_r is the value at time r of the income expected during the contract

$$= 1.10 \times e^{-0.065 \times 3/12} = 1.082269 \qquad [1]$$

$$\Rightarrow K_r = (9.90 - 1.082269)e^{0.065 \times 7/12} = 9.158489 \qquad [½]$$

Value of original forward contract

$$= (K_r - K_0)e^{-\delta(T-r)}$$

$$= (9.158489 - 8.204853)e^{-0.065 \times \frac{7}{12}}$$

$$= 0.918154$$

$$= £0.92 \qquad [2]$$

[TOTAL 7]

> Although this question was answered better than similar questions in past diets, the workings shown by marginal candidates were often unclear.

Q8 (i) Work in £000's

Let total accumulation at 1/6/20 be X, and i_y = investment return for the year starting from 1 June 2016 + y

$$E(X) = E\left[3(1+i_1)(1+i_2)(1+i_3) + 3(1+i_2)(1+i_3) + 3(1+i_3) + 3 + 110\right] \qquad [1½]$$

Due to independence:

$$E(X) = 3\left[E(1+i_1)E(1+i_2)E(1+i_3) + E(1+i_2)E(1+i_3) + E(1+i_3)\right] + 113$$

[1]

$$= 3\left[(1+E[i_1])(1+E[i_2])(1+E[i_3]) + (1+E[i_2])(1+E[i_3]) + (1+E[i_3])\right] + 113$$

[½]

where $E(i_y)$ $= 0.55 \times 6\% + 0.45 \times 5.5\%$

$= 5.775\%$ [1]

(ii) $E(X) = 3\,\ddot{s}_{\overline{3}|}^{5.775\%} + 113$

$$= 3\left(\frac{1.05775^3 - 1}{0.05775/1.05775}\right) + 113$$

$= 123.080$ (= £123,080 for £100,000 nominal)

[2]

[TOTAL 6]

> Whilst the calculations were often correct, relatively few candidates followed the instructions to derive the required formula for these calculations. For full marks, such derivation was required including identifying where the independence assumption is used.

Q9 (i) Cash Flows:

Issue Price:	Jan 14	$-0.97 \times 100{,}000 = -£97{,}000$
Interest Payments:	July 14	$0.03 \times 100{,}000 \times \frac{122.3}{120.0} = £3{,}057.50$
	Jan 15	$0.03 \times 100{,}000 \times \frac{124.9}{120.0} = £3{,}122.50$
	July 15	$0.03 \times 100{,}000 \times \frac{127.2}{120.0} = £3{,}180.00$
	Jan 16	$0.03 \times 100{,}000 \times \frac{129.1}{120.0} = £3{,}227.50$
Capital redeemed:	Jan 16	$100{,}000 \times \frac{129.1}{120.0} = £107{,}583.33$

[3]

(ii) Express all amounts in "January 2014 money", and we get:

$$97000 = 3057.50 \times \frac{122.3}{124.9} v^{\frac{1}{2}} + 3122.50 \times \frac{122.3}{127.2} v$$

$$+ 3180.00 \times \frac{122.3}{129.1} v^{1\frac{1}{2}} + \frac{122.3}{131.8} \times (107583.33 + 3227.50) v^2$$ [2]

$$\Rightarrow 97000 = 2993.85 v^{\frac{1}{2}} + 3002.22 v + 3012.50 v^{1\frac{1}{2}} + 102823.71 v^2$$ [1]

Try 7%, RHS = 98232.04
8%, RHS = 96499.48

$$i = 0.07 + \left(\frac{98232.04 - 97000}{98232.04 - 96499.48}\right) \times 0.01 \qquad [2]$$

= 7.7% p.a. effective real yield (exact answer is 7.708%).

[TOTAL 8]

> This question seemed to strongly differentiate between stronger and weaker candidates. Common errors from the latter included not correctly allowing for the time lag in part (i) or not uplifting the nominal cashflows for inflation at all.

Q10 (i) TWRR is i such that:

$$(1+i)^3 = \frac{12,700}{12,000} \times \frac{13,000}{12,700 + 2,600} \times \frac{14,100}{13,000 - 3,700} \times \frac{17,200}{14,100 + 1,800}$$

= 1.474830 ⇒ i = 13.8% [2 for formula, 1 for solution]

(ii) If the MWRR achieved by the fund were 13.8% p.a., then fund value at 31 December 2015 would be (in £000's):

$$12000 \times (1.138)^3 + 2600 \times (1.138)^{2\frac{1}{2}} - 3700 \times (1.138)^{1\frac{1}{2}} + 1800 \times (1.138)^{\frac{1}{2}}$$

=18,706 which is greater than 17,200. This means that the MWRR must be less than 13.8% p.a. [2]

(iii) The MWRR is lower because the fund performed badly immediately after receiving the large positive cash flow in July 2013 and also performed well immediately after the large negative cash flow in July 2014. [2]

(iv) The TWRR is not influenced by the amount and timing of the cash flows (which are generally considered to be outside of the control of the fund manager) and, thus, better reflects the manager's performance over the period. [2]

[TOTAL 9]

> Parts (i) and (ii) were answered well. In part (ii), it is not necessary to calculate the MWRR.
>
> As in previous diets, candidates had difficulty explaining the relative values for the MWRR and TWRR. For full marks in part (iii), candidates needed to make reference to the actual data in the question. In part (iv), the key point is that

> the amount and timing of the cash flows are generally considered to be outside of the control of the fund manager.

Q11 (i) 10,000 shares give a total dividend on the next payment date of £650.

Then, working in half-year periods, we have:

$$V = 650\times\left(v_{6\%} + 1.02v_{6\%}^2 + 1.02^2 v_{6\%}^3 + \ldots\right) \quad [2]$$

$$= 650v_{6\%}\times\left(1 + 1.02v_{6\%} + \left(1.02v_{6\%}\right)^2 + \ldots\right)$$

$$= 650v_{6\%}\times\left(\frac{1}{1-1.02v_{6\%}}\right)$$

$$= £16{,}250 \quad [2]$$

(ii) (a) We now have

$$v = 650v_{6\%}\times\left(\frac{1}{1-1.025v_{6\%}}\right) = £18{,}571.43 \quad [1]$$

(b) The higher rate of dividend growth means that expected future dividend income is increased and, thus, the investor is prepared to pay a higher price to purchase the shares. [1]

(iii) (a) The rumoured change in legislation might be thought of as increasing the uncertainty of the future growth prospects for the company (without necessarily either increasing or decreasing them).

Thus it is appropriate that the investor requires a higher return to compensate for this greater uncertainty. [1]

(b) We now have:

$$v = 650v_{7\%}\times\left(\frac{1}{1-1.02v_{7\%}}\right) = £13{,}000 \quad [1]$$

(c) The higher risk (as reflected by the higher effective rate of return required) means that the investor is now prepared to pay a lower maximum price to purchase the shares. [1]

(iv) (a) Lower inflation is likely to lead to lower (nominal) profits and, thus, lower (nominal) dividend payments.

Also, as many investors are more concerned with real returns (i.e. in excess of inflation), it is appropriate to reduce the effective rate of return to reflect the lower expected inflation. [2]

(b) We now have:

$$v = 650v_{5\%} \times \left(\frac{1}{1 - 1.01v_{5\%}} \right) = £16,250 \qquad [1]$$

(c) In this case, the maximum price that the investor is prepared to pay is unchanged. Lower expected inflation leads to lower nominal dividend payments, which are then discounted at a lower nominal interest rate. Thus, the price is unaffected (i.e. equities are a real asset).

[2]

[TOTAL 14]

> The calculations in this question were relatively simple and generally done well. The explanatory parts of the questions were answered better than expected.

Q12 (i)

$$(\bar{I}\bar{a})_{\overline{n}|} = \int_0^n te^{-\delta t} dt = \left[t \times \frac{e^{-\delta t}}{-\delta} \right]_0^n - \int_0^n \frac{e^{-\delta t}}{-\delta} dt$$

$$= -\frac{n.e^{-\delta n}}{\delta} + \frac{1}{\delta} \int_0^n e^{-\delta t} \, dt$$

$$= -\frac{n.e^{-\delta n}}{\delta} + \frac{1}{\delta} \left[-\frac{e^{-\delta t}}{\delta} \right]_o^n$$

$$= -\frac{n.e^{-\delta n}}{\delta} + \frac{1}{\delta} \left[\frac{1 - e^{-\delta n}}{\delta} \right] = \frac{\bar{a}_{\overline{n}|} - nv^n}{\delta}$$

[4]

(ii) Project lasts for 33 years as follows:

Time take to reopen mine = 1 year

Time taken for net revenue to go from zero to \$3,600,000 is 12 years

$$\left(\text{from } \frac{3,600,000}{300,000} = 12 \right)$$

Time taken for net revenue to decline to \$600,000 is 20 years

$$\left(\text{from } \frac{3,600,000-600,000}{150,000} = 20\right)$$

[2]

(iii) PV of reopening costs and additional costs $= 700,000\,\bar{a}_{\overline{1|}} + 200,000\,\bar{a}_{\overline{33|}}$ at 25%

where

$$\bar{a}_{\overline{1|}}^{25\%} = \frac{i}{\delta} a_{\overline{1|}} = 1.120355 \times 0.8 = 0.896284$$

$$\bar{a}_{\overline{33|}}^{25\%} = \frac{i}{\delta} \,.\, a_{\overline{33|}} = 1.120355 \times 3.9975 = 4.478619$$

$\Rightarrow$ PV = 627,399 + 895,724 = 1,523,123 [3]

PV of net revenue

$$v\,.\,300,000\left(\bar{Ia}\right)_{\overline{12|}} + v^{13}\left\{3,600,000\,\bar{a}_{\overline{20|}} - 150,000\left(\bar{Ia}\right)_{\overline{20|}}\right\}$$ [2]

where $\bar{a}_{\overline{12|}} = \frac{i}{\delta} . a_{\overline{12|}} = 1.120355 \times 3.7251 = 4.173434$ [1]

$$\left(\bar{Ia}\right)_{\overline{12|}} = \frac{\bar{a}_{\overline{12|}} - 12v^{12}}{\delta} = \frac{4.173434 - 12 \times 0.06872}{0.223144} = 15.0073$$ [1]

$$\bar{a}_{\overline{20|}} = \frac{i}{\delta} . a_{\overline{20|}} = 1.120355 \times 3.9539 = 4.429772$$ [1]

$$\left(\bar{Ia}\right)_{\overline{20|}} = \frac{4.429772 - 20v^{20}}{\delta} = 18.818324$$ [1]

$\Rightarrow$ PV of net revenue

$$= \frac{300,000}{1.25} \times 15.0073 + 0.05498\{3,600,000 \times 4.429772 - 150,000 \times 18.818324\}$$ [1]

= 3,601,752 + 721,581 = 4,323,333 [1]

$\Rightarrow$ Price to obtain IRR of 25% p.a. is:

4,323,333 – 1,523,123 = \$2,800,210. [1]

(above uses factors from Formulae and Tables Book – exact answer is 4,323,319 – 1,523,115 = \$2,800,204)

[TOTAL 18]

The proof in part (i) was answered very poorly. Also, since the result is given, candidates must provide enough steps in deriving the result to convince the examiners that they haven't just jumped to the result. In part (iii), the workings of many marginal candidates were very unclear. The examiners recommend that candidates set out their working clearly e.g. by calculating each component of the costs and benefits separately. This enables examiners to give full credit for correct working even if errors are made in the calculations.

END OF EXAMINERS' REPORT