INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

23 September 2013 (pm)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

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1 The rate of interest is 4.5% per annum effective.

(i) Calculate:

    (a)   the annual effective rate of discount.
    (b)   the nominal rate of discount per annum convertible monthly.
    (c)   the nominal rate of interest per annum convertible quarterly.
    (d)   the effective rate of interest over a five year period.

[5]

(ii) Explain why your answer to part (i)(b) is higher than your answer to part (i)(a). [2]

[Total 7]

2 A nine-month forward contract is issued on 1 March 2012 on a share with a price of £1.80 at that date. Dividends of 10p per share are expected on 1 September 2012.

Calculate the forward price at issue assuming a risk-free rate of interest of 4% per annum effective and no arbitrage. [3]

3 A fixed-interest security pays coupons of 4% per annum, half-yearly in arrear and will be redeemed at par in exactly ten years.

(i) Calculate the price per £100 nominal to provide a gross redemption yield of 3% per annum convertible half-yearly. [2]

(ii) Calculate the price, 91 days later, to provide a net redemption yield of 3% per annum convertible half-yearly if income tax is payable at 25%. [2]

[Total 4]

4 Describe the characteristics of the cash flows that are paid and received in respect of:

(i) an index-linked security. [2]
(ii) an equity. [3]

[Total 5]

5　An investor is considering the purchase of two government bonds, issued by two countries A and B respectively, both denominated in euro.

Both bonds provide a capital repayment of €100 together with a final coupon payment of €6 in exactly one year. The investor believes that he will receive both payments from the bond issued by Country A with certainty. He believes that there are four possible outcomes for the bond from Country B, shown in the table below.

Outcome	Probability
No coupon or capital payment	0.1
Capital payment received, but no coupon payment received	0.2
50% of capital payment received, but no coupon payment received	0.3
Both coupon and capital payments received in full	0.4

The price of the bond issued by Country A is €101.

(i)　Calculate the price of the bond issued by Country B to give the same expected return as that for the bond issued by Country A. [3]

(ii)　Calculate the gross redemption yield from the bond issued by Country B assuming that the price is as calculated in part (i). [1]

(iii)　Explain why the investor might require a higher expected return from the bond issued by Country B than from the bond issued by Country A. [2]

[Total 6]

6　A pension fund is considering investing in a major infrastructure project. The fund has been asked to make an investment of £2m for a 1% share in revenues from building a road. No other costs will be incurred by the pension fund. The following revenues are expected to arise from the project:

In the first year, 40,000 vehicles a day will use the road, each paying a toll of £1.

In the second year, 50,000 vehicles a day will use the road, each paying a toll of £1.10.

In the third year, both the number of vehicles using the road and the level of tolls will rise by 1% from their level in the second year. They will both continue to rise by 1% per annum compound until the end of the 20th year.

At the end of the 20th year, it is assumed that the road has no value as it will have to be completely rebuilt.

You should assume that all revenue is received continuously throughout the year and that there are 365 days in all years.

Calculate the net present value of the investment in the road at a rate of interest of 8% per annum effective. [10]

7   An insurance company has just written contracts that require it to make payments to policyholders of £10 million in five years' time. The total premiums paid by policyholders at the outset of the contracts amounted to £7.85 million. The insurance company is to invest the premiums in assets that have an uncertain return. The return from these assets in year t, i_t, has a mean value of 5.5% per annum effective and a standard deviation of 4% per annum effective. $(1+i_t)$ is independently and lognormally distributed.

   (i)   Calculate the mean and standard deviation of the accumulation of the premiums over the five-year period. You should derive all necessary formulae. [Note: You are not required to derive the formulae for the mean and variance of a lognormal distribution.]   [9]

   A director of the insurance company is concerned about the possibility of a considerable loss from the investment strategy suggested in part (i). He therefore suggests investing in fixed-interest securities with a guaranteed return of 4 per cent per annum effective.

   (ii) Explain the arguments for and against the director's suggestion.   [3]

   [Total 12]

8   Mrs Jones invests a sum of money for her retirement which is expected to be in 20 years' time. The money is invested in a zero coupon bond which provides a return of 5% per annum effective. At retirement, the individual requires sufficient money to purchase an annuity certain of £10,000 per annum for 25 years. The annuity will be paid monthly in arrear and the purchase price will be calculated at a rate of interest of 4% per annum convertible half-yearly.

   (i)   Calculate the sum of money the individual needs to invest at the beginning of the 20-year period.   [5]

   The index of retail prices has a value of 143 at the beginning of the 20-year period and 340 at the end of the 20-year period.

   (ii) Calculate the annual effective real return the individual would obtain from the zero coupon bond.   [2]

   The government introduces a capital gains tax on zero coupon bonds of 25 per cent of the nominal capital gain.

   (iii) Calculate the net annual effective real return to the investor over the 20-year period before the annuity commences.   [3]

   (iv) Explain why the investor has achieved a negative real rate of return despite capital gains tax only being a tax on the profits from an investment.   [2]

   [Total 12]

9 A bank makes a loan to be repaid by instalments paid annually in arrear. The first instalment is £400, the second is £380 with the payments reducing by £20 per annum until the end of the 15th year, after which there are no further repayments. The rate of interest charged is 4% per annum effective.

(i) Calculate the amount of the loan. [3]

(ii) Calculate the capital and interest components of the first payment. [2]

At the beginning of the ninth year, the borrower can no longer make the scheduled repayments. The bank agrees to reduce the capital by 50 per cent of the loan outstanding after the eighth repayment. The bank requires that the remaining capital is repaid by a 10-year annuity paid annually in arrear, increasing by £2 per annum. The bank changes the rate of interest to 8% per annum effective.

(iii) Calculate the first repayment under the revised loan. [5]
[Total 10]

10 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = 0.05 + 0.002t$$

Calculate the accumulated value of a unit sum of money:

(i)   (a)   accumulated from time $t = 0$ to time $t = 7$.
       (b)   accumulated from time $t = 0$ to time $t = 6$.
       (c)   accumulated from time $t = 6$ to time $t = 7$.

[5]

(ii) Calculate, using your results from part (i) or otherwise:

     (a)   the seven-year spot rate of interest per annum from time $t = 0$ to time $t = 7$.

     (b)   the six-year spot rate of interest per annum from time $t = 0$ to time $t = 6$.

     (c) $f_{6,1}$ where $f_{6,1}$ is the one-year forward rate of interest per annum from time $t = 6$. [3]

(iii) Explain why your answer to part (ii)(c) is higher than your answer to part (ii)(a). [2]

(iv) Calculate the present value of an annuity that is paid continuously at a rate of $30e^{-0.01t+0.001t^2}$ units per annum from $t = 3$ to $t = 10$. [5]
[Total 15]

11   A pension fund has liabilities to meet annuities payable in arrear for 40 years at a rate of £10 million per annum.

The fund is invested in two fixed-interest securities. The first security pays annual coupons of 5% and is redeemed at par in exactly ten years' time. The second security pays annual coupons of 10% and is redeemed at par in exactly five years' time. The present value of the assets in the pension fund is equal to the present value of the liabilities of the fund and exactly half the assets are invested in each security. All assets and liabilities are valued at a rate of interest of 4% per annum effective.

(i)   Calculate the present value of the liabilities of the fund.   [1]

(ii)   Calculate the nominal amount held of each security purchased by the pension fund.   [6]

(iii)   Calculate the duration of the liabilities of the pension fund.   [3]

(iv)   Calculate the duration of the assets of the pension fund.   [4]

(v)   Without further calculations, explain whether the pension fund will make a profit or loss if interest rates fall uniformly by 1.5% per annum effective.   [2]

[Total 16]

END OF PAPER