INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

12 April 2016 (am)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 List the characteristics of convertible bonds. [3]

2 An insurance company has liabilities of £6 million due in exactly 8 years' time and a further £11 million due in exactly 15 years' time.

The assets held by the insurance company consist of:

- a 5-year zero-coupon bond of nominal amount £5.5088 million; and
- a 20-year zero-coupon bond of nominal amount £13.7969 million.

The current rate of interest is 8% per annum effective at all durations.

(i) Show that the first two conditions of Redington's theory for immunisation against small changes in the rate of interest are satisfied. [5]

(ii) Explain, without doing any further calculations, whether the insurance company will be immunised against small changes in the rate of interest. [2]

[Total 7]

3 At time $t = 0$, the one-year zero-coupon yield is 4% per annum effective and the one-year forward rate per annum effective at time t ($t = 1, 2, \ldots$) is given by:

$$f_{t,1} = (4 + t)\%.$$

(i) Determine the issue price per £100 nominal of a three-year 4% coupon bond issued at time $t = 0$, paying coupons annually in arrear and redeemable at 105%. [4]

(ii) Determine the three-year par yield at time $t = 0$. [3]

[Total 7]

4 A loan of nominal amount £100,000 is to be issued bearing coupons payable quarterly in arrear at a rate of 7% per annum. Capital is to be redeemed at £108 per £100 nominal on a coupon date between 15 and 20 years inclusive after the date of issue. The date of redemption is at the option of the borrower.

An investor who is liable to income tax at 25% and capital gains tax at 40% wishes to purchase the entire loan at the date of issue.

(i) Determine the price which the investor should pay to ensure a net effective yield of at least 5% per annum. [5]

(ii) Explain the significance of the redemption date being at the option of the borrower in relation to your calculation in part (i). [2]

[Total 7]

5 A loan is to be repaid by a series of instalments payable annually in arrear for 15 years. The first instalment is £1,200 and payments increase thereafter by £250 per annum.

Repayments are calculated using a rate of interest of 6% per annum effective.

Determine:

(i) the amount of the loan. [3]

(ii) the capital outstanding immediately after the 9th instalment has been made. [2]

(iii) the capital and interest components of the final instalment. [2]

[Total 7]

6 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.06 & 0 \le t \le 4 \\ 0.10 - 0.01t & 4 < t \le 7 \\ 0.01t - 0.04 & 7 < t \end{cases}.$$

(i) Calculate, showing all working, the value at time $t = 5$ of £10,000 due for payment at time $t = 10$. [5]

(ii) Calculate the constant rate of discount per annum convertible monthly which leads to the same result as in part (i). [2]

[Total 7]

7 A one-year forward contract on a share was agreed on 1 September 2015 when the share price was £8.70 and the risk-free force of interest was 7% per annum. The stock was expected to pay a dividend of £1.10 eight months after the date of issue.

The price of the share was £9.90 on 1 February 2016 and the risk-free force of interest was 6.5% per annum. The dividend expectation was unchanged.

Calculate, showing all working, the value of the contract to the holder of the long forward position on 1 February 2016. [7]

8 An individual is planning to purchase £100,000 nominal of a bond on 1 June 2016 which will be redeemable at 110% on 1 June 2020. The bond will pay coupons of 3% per annum at the end of each year.

The individual wishes to invest the coupon payments on deposit until the bond is redeemed. It is assumed that, in any year, there is a 55% probability that the rate of interest will be 6% per annum effective and a 45% probability that it will be 5.5% per annum effective. It is also assumed that the rate of interest in any one year is independent of that in any other year.

(i) Derive the necessary formula to determine the mean value of the total accumulated investment on 1 June 2020. [4]

(ii) Calculate the mean value of the total accumulated investment on 1 June 2020. [2]

[Total 6]

9 In January 2014, the government of a country issued an index-linked bond with a term of two years. Coupons were payable half-yearly in arrear, and the annual nominal coupon rate was 6%. The redemption value, before indexing, was £100 per £100 nominal. Interest and capital payments were indexed by reference to the value of an inflation index with a time lag of six months.

A tax-exempt investor purchased £100,000 nominal at issue and held it to redemption. The issue price was £97 per £100 nominal.

The inflation index was as follows:

Date	*Inflation Index*
July 2013	120.0
January 2014	122.3
July 2014	124.9
January 2015	127.2
July 2015	129.1
January 2016	131.8

(i) Set out a schedule of the investor's cashflows, showing the amount and month of each cashflow. [3]

(ii) Determine the annual effective real yield obtained by the investor to the nearest 0.1% per annum. [5]

[Total 8]

10 The following table gives information concerning a fund held by an investment manager:

Year	*2012*	*2013*	*2014*	*2015*
Value of fund at 30 June	–	12,700,000	13,000,000	14,100,000
Net cash flow received on 1 July	–	2,600,000	–3,700,000	1,800,000
Value of fund at 31 December	12,000,000	13,500,000	12,900,000	17,200,000

(i) Calculate, to the nearest 0.1% and showing all working, the annual effective time-weighted rate of return (TWRR) achieved by the fund during the period from 31 December 2012 to 31 December 2015. [3]

(ii) Show that the annual effective money-weighted rate of return (MWRR) achieved by the fund over the same period is less than the answer obtained in part (i) above. [2]

(iii) Explain why you would expect the outcome described in part (ii) for this fund. [2]

(iv) Explain which of the two measures referred to in parts (i) and (ii) is a better indicator of the investment manager's performance over the period. [2]

[Total 9]

11 An investor is considering the purchase of 10,000 ordinary shares in Enterprise plc.

Dividends from the shares are payable half-yearly in arrear. The next dividend is due in exactly six months and is expected to be 6.5 pence per share.

The required rate of return is 6% per half-year effective and an estimated rate of future dividend growth is 2% per half-year.

(i) Calculate, showing all working, the maximum price that the investor should pay for the shares. [4]

As a result of a recently announced expansion plan, the investor increases the estimated rate of future dividend growth to 2.5% per half-year.

(ii) (a) Calculate, showing all working, the maximum price the investor should now pay for the shares.

(b) Explain the difference between your answers to part (i) and part (ii)(a). [2]

It is rumoured that new legislation may affect the operation of Enterprise plc.

As a result, the investor decides to increase her required rate of return to 7% per half-year effective. The estimated dividend growth rate remains at 2% per half-year

(iii) (a) Explain why it might be appropriate for the investor to increase her required rate of return.

(b) Calculate the maximum price that the investor should now pay for the shares.

(c) Explain the difference between your answers to part (i) and part (iii)(b). [3]

In the prevailing economic circumstances, investors are expecting lower inflation in the wider economy.

As a result, the investor decides to reduce both the assumed rate of dividend growth and her required rate of return to 1% and 5% per half-year effective respectively.

(iv) (a) Explain why it is appropriate for the investor to reduce both the future dividend growth rate and the required rate of return in this case.

(b) Calculate the maximum price that the investor should now pay for the shares.

(c) Explain the difference between your answers to part (i) and part (iv)(b). [5]

[Total 14]

12 (i) Show that:

$$\left(\bar{I}\bar{a}\right)_{\overline{n}|} = \frac{\bar{a}_{\overline{n}|} - nv^n}{\delta}.$$ [4]

A company is considering the purchase of a gold mine which has recently ceased production.

The company forecasts that:

- the cost of re-opening the mine will be \$900,000, which will be incurred continuously over the first twelve months.
- additional costs are expected to be constant throughout the term of the project at \$200,000 per annum, excluding the first year. These are also incurred continuously.
- after the first twelve months, the rate of revenue will grow continuously and linearly from zero per annum to \$3,600,000 per annum at a constant rate of \$300,000 per annum.
- when the rate of revenue reaches \$3,600,000 per annum it will then decline continuously and linearly at a constant rate of \$150,000 per annum until it reaches \$600,000 per annum.
- when the rate of revenue declines to \$600,000 per annum production will stop and the mine will have zero value.

(ii) Determine the overall term of the project. [2]

(iii) Calculate, showing all working, the price that the company should pay in order to earn an internal rate of return (IRR) of 25% per annum effective. [12]

[Total 18]

END OF PAPER