INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

18 April 2017 (pm)

Subject CT1 – Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes of planning and reading time before the start of this examination. You may make separate notes or write on the exam paper but not in your answer booklet. Calculators are not to be used during the reading time. You will then have three hours to complete the paper.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 10 questions, beginning your answer to each question on a new page.*

6. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Calculate the nominal rate of discount per annum convertible monthly which is equivalent to:

(i) an effective rate of interest of 1% per quarter. [2]

(ii) a force of interest of 5% per annum. [2]

(iii) a nominal rate of discount of 4% per annum convertible every three months. [2]

[Total 6]

2 A bank offers two repayment alternatives for a loan that is to be repaid over sixteen years:

Option 1: the borrower pays £7,800 per annum quarterly in arrear.

Option 2: the borrower makes payments at an annual rate of £8,200 every second year in arrear.

Determine which option would provide the better deal for the borrower at a rate of interest of 5% per annum effective. [5]

3 A one-year forward contract on a stock was agreed on 1 March 2017 when the stock price was £78 and the risk-free force of interest was 14% per annum. The stock was expected to pay dividends of £3.20 on 1 June and 1 December 2017.

On 1 April 2017, the price of the stock was £80 and the risk-free force of interest was 11% per annum. The dividend expectations were unchanged.

Determine the value of the contract to the holder of the long forward position on 1 April 2017. [7]

4 An investor borrows money from a bank in order to invest in a business venture. The initial loan is £500,000, with further loans of £250,000 made in 6 months' time and £250,000 made in 12 months' time.

The business venture will provide the investor with an income of £2 million in exactly 10 years' time and £3 million in exactly 15 years' time.

The bank offers a force of interest, $\delta(t)$, as a function of time t (measured in years) which is given by:

$$\delta(t) = \begin{cases} 0.04 & \text{for } 0 \le t \le 2 \\ 0.02 + kt & \text{for } t > 2 \end{cases}$$

(i) Derive expressions for $v(t)$ which cover all values of t. [5]

(ii) Determine the minimum value of k that would ensure that the discounted payback period is exactly 10 years. [4]

[Total 9]

5 An investment fund has liabilities of £11 million due in 7 years' time and £8.084 million in 11 years' time.

The manager of the fund will meet the liabilities by investing in zero-coupon bonds. The manager is able to buy zero-coupon bonds for whatever term is required and there are adequate funds at the manager's disposal.

(i) Explain whether it is possible for the manager to immunise the fund against small changes in the rate of interest by purchasing a single zero-coupon bond. [2]

The manager decides to purchase two zero-coupon bonds, one paying £15.363 million in 7.5 years' time and the other paying £3.787 million in 14.25 years' time. The current interest rate is 5.5% per annum effective.

(ii) Determine whether the investment fund satisfies the necessary conditions to be immunised against small changes in the rate of interest. [7]

[Total 9]

6 Exactly three months ago an investor purchased an office building for £5.8 million with the intention of renting it out. In three months' time the investor will spend £850,000 on necessary refurbishments and improvements.

A tenant has agreed to lease the building in six months' time for 35 years. The tenant will pay an initial rent of £1.250 million per annum payable monthly in arrear. The rent will be increased at five-yearly intervals at a rate of 4.2% per annum compound. It has further been agreed that at the end of the lease period the tenant will buy the building from the investor for £11.5 million.

The investor pays income tax at a rate of 35% and is expecting a net effective rate of return of 8% per annum.

Calculate, showing all workings, the net present value of the project to the investor at the time of purchase. [11]

7 A fixed interest bond was issued on 1 January 2017 with a term of 20 years and is redeemable at 105%. The security pays a coupon of 4% per annum, payable half-yearly in arrear.

An investor is liable to income tax at the rate of 30% and capital gains tax at the rate of 40%. Income tax and capital gains tax are both collected on 1 June each year in relation to gross payments made during the previous 12 months.

The investor bought £10,000 nominal of the stock at an issue price of £9,800.

(i) Show that the net redemption yield obtained by the investor will be between 3% and 4% per annum effective. [7]

The inflation rate over the term of the bond is assumed to be 2% per annum.

(ii) Calculate the net effective annual real redemption yield that would be obtained by the investor. [3]

(iii) Explain, without doing any further calculations, how your answers to parts (i) and (ii) would alter if the tax were collected on 1 April instead of 1 June each year. [2]

[Total 12]

8 Two investment funds A and B are administered by different managers. The initial values of the two funds on 1 January 2015 were £1.5 million and £2.3 million, respectively. The funds received additional net cash flows at the beginning of 2015 and 2016, as follows:

	Fund Net Cash Flows	
	1 January 2015	*1 January 2016*
Fund A	£300,000	£1,700,000
Fund B	£2,000,000	£200,000

The fund managers achieved the following annual returns during 2015 and 2016:

	Fund Annual Returns	
	2015	*2016*
Fund A	42%	3%
Fund B	36%	2%

(i) Calculate the annual effective time weighted rate of return for each fund for the period 1 January 2015 to 31 December 2016. [3]

(ii) Calculate the annual effective money weighted rate of return per annum for each fund for the period 1 January 2015 to 31 December 2016. [8]

(iii) Comment on your answers to parts (i) and (ii) by explaining which of the two measures is the better indicator of the comparative performance of the managers for the given two-year period. [3]

[Total 14]

9 Let f_t denote the one-year effective forward rate of interest over the year from time t to $(t + 1)$. Let i_t be the t-year effective spot rate over the period 0 to t.

The annual effective gross redemption yield from an n-year bond which pays coupons of 5% annually in arrear is given by:

$$g_n = 0.07 + 0.001n \qquad \text{for } n = 1, 2 \text{ and } 3$$

Each bond is redeemed at par and is exactly one year from the next coupon payment. It is assumed that no arbitrage takes place.

(i) Calculate i_1, i_2 and i_3 as percentages to three decimal places. [7]

(ii) Calculate f_0, f_1 and f_2 as percentages to three decimal places. [4]

(iii) Explain why the one-year forward rates increase more quickly with term than the spot rates. [2]

[Total 13]

10 An individual aged exactly 65 intends to retire in five years' time and receive an annuity-certain. The annuity will be payable monthly in advance and will cease after 20 years. The annuity will increase at each anniversary of the commencement of payment at the rate of 3% per annum.

The individual would like the initial level of annuity to be £20,000 per annum. The price of the annuity will be the present value of the payments on the date it commences using an interest rate of 7% per annum effective.

(i) Calculate the price of the annuity. [4]

In order to purchase the annuity described in part (i), the individual invests £200,000 on his 65th birthday in a particular fund.

The investment return on the fund in any given year is independent of returns in all other years and the annual return is:

- 4% with a probability of 60%.
- 7% with a probability of 40%.

(ii) Calculate, showing all workings, the expected accumulation of the investment at the time of retirement. [3]

(iii) Calculate, showing all workings, the standard deviation of the investment at the time of retirement. [4]

(iv) Determine the probability that the individual will have sufficient funds to purchase the annuity. [3]

[Total 14]

END OF PAPER