EXAMINATION

4 April 2006 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1.   *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2.   *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3.   *Mark allocations are shown in brackets.*

4.   *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*

5.   *Candidates should show calculations where this is appropriate.*

Graph paper is *not* required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

1 An investment is discounted for 28 days at a simple rate of discount of 4.5% per annum. Calculate the annual effective rate of interest. [3]

2 An annuity certain with payments of £150 at the end of each quarter is to be replaced by an annuity with the same term and present value, but with payments at the beginning of each month instead.

Calculate the revised payments, assuming an annual force of interest of 10%. [3]

3 At time $t = 0$ the n-year spot rate of interest is equal to $(2.25 + 0.25n)$% per annum effective $(1 \leq n \leq 5)$.

    (a) Calculate the 2-year forward rate of interest from time $t = 3$ expressed as an annual effective rate of interest.

    (b) Calculate the 4-year par yield.

    (c) Without performing any further calculations, explain how you would expect the gross redemption yield of a 4-year bond paying annual coupons of 3.5% to compare with the par yield calculated in (b).

    [7]

4 An investor, who is liable to income tax at 20% but is not liable to capital gains tax, wishes to earn a net effective rate of return of 5% per annum. A bond bearing coupons payable half-yearly in arrear at a rate 6.25% per annum is available. The bond will be redeemed at par on a coupon date between 10 and 15 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

Calculate the maximum price that the investor is willing to pay for the bond. [5]

5 A share currently trades at £10 and will pay a dividend of 50p in one month's time. A six-month forward contract is available on the share for £9.70. Show that an investor can make a risk-free profit if the risk-free force of interest is 3% per annum. [4]

6 An actuarial student has created an interest rate model under which the annual effective rate of interest is assumed to be fixed over the whole of the next ten years. The annual effective rate is assumed to be 2%, 4% and 7% with probabilities 0.25, 0.55 and 0.2 respectively.

    (a) Calculate the expected accumulated value of an annuity of £800 per annum payable annually in advance over the next ten years.

    (b) Calculate the probability that the accumulated value will be greater than £10,000.

    [4]

7   A company has entered into an interest rate swap. Under the terms of the swap the company makes fixed annual payments equal to 6% of the principal of the swap. In return, the company receives annual interest payments on the principal based on the prevailing variable short-term interest rate which currently stands at 5.5% per annum.

    (a)   Describe briefly the risks faced by a counterparty to an interest rate swap.

    (b)   Explain which of the risks described in (a) are faced by the company.   [4]

8   An ordinary share pays annual dividends. A dividend of 25p per share has just been paid. Dividends are expected to grow by 2% next year and by 4% the following year. Thereafter, dividends are expected to grow at 6% per annum compound in perpetuity.

    (i)   State the main characteristics of ordinary shares.   [4]

    (ii)   Calculate the present value of the dividend stream described above at a rate of interest of 9% per annum effective from a holding of 100 ordinary shares.   [4]

    (iii)   An investor buys 100 shares in (ii) for £8.20 each. He holds them for two years and receives the dividends payable. He then sells them for £9 immediately after the second dividend is paid.

           Calculate the investor's real rate of return if the inflation index increases by 3% during the first year and by 3.5% during the second year assuming dividends grow as expected.   [4]
                                               [Total 12]

9   The force of interest $\delta(t)$ is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 & 0 < t \le 5 \\ 0.008t & 5 < t \le 10 \\ 0.005t + 0.0003t^2 & 10 < t \end{cases}$$

    (i)   Calculate the present value of a unit sum of money due at time $t = 12$.   [5]

    (ii)   Calculate the effective annual rate of interest over the 12 years.   [2]

    (iii)   Calculate the present value at time $t = 0$ of a continuous payment stream that is paid at the rate of $e^{-0.05t}$ per unit time between time $t = 2$ and time $t = 5$.   [3]
                                               [Total 10]

10 A piece of land is available for sale for £5,000,000. A property developer, who can lend and borrow money at a rate of 15% per annum, believes that she can build housing on the land and sell it for a profit. The total cost of development would be £7,000,000 which would be incurred continuously over the first two years after purchase of the land. The development would then be complete.

The developer has three possible project strategies. She believes that she can sell the completed housing:

- in three years time for £16,500,000
- in four years time for £18,000,000
- in five years time for £20,500,000

The developer also believes that she can obtain a rental income from the housing between the time that the development is completed and the time of sale. The rental income is payable quarterly in advance and is expected to be £500,000 in the first year of payment. Thereafter, the rental income is expected to increase by £50,000 per annum at the beginning of each year that the income is paid.

(i)   Determine the optimum strategy if this is based upon using net present value as the decision criterion. [9]

(ii) Determine which strategy would be optimal if the discounted payback period were to be used as the decision criterion. [2]

(iii) If the housing is sold in six years' time, the developer believes that she can obtain an internal rate of return on the project of 17.5% per annum. Calculate the sale price that the developer believes that she can receive. [6]

(iv) Suggest reasons why the developer may not achieve an internal rate of return of 17.5% per annum even if she sells the housing for the sale price calculated in (iii). [2]

[Total 19]

11 An actuarial student has taken out two loans.

Loan A: a five-year car loan for £10,000 repayable by equal monthly instalments of capital and interest in arrear with a flat rate of interest of 10.715% per annum.

Loan B: a five-year bank loan of £15,000 repayable by equal monthly instalments of capital and interest in arrear with an effective annual interest rate of 12% for the first two years and 10% thereafter.

The student has a monthly disposable income of £600 to pay the loan interest after all other living expenses have been paid.

Freeloans is a company which offer loans at a constant effective interest rate for all terms between three years and ten years. After two years, the student is approached by a representative of Freeloans who offers the student a 10-year loan on the capital outstanding which is repayable by equal monthly instalments of capital and interest in arrear. This new loan is used to pay off the original loans and will have repayments equal to half the original repayments.

(i) Calculate the final disposable income (surplus or deficit) each month after the loan payments have been made. [5]

(ii) Calculate the capital repaid in the first month of the third year assuming that the student carries on with the original arrangements. [5]

(iii) Estimate the capital repaid in the first month of the third year assuming that the student has taken out the new loan. [5]

(iv) Suggest, with reasons, a more appropriate strategy for the student. [2]
[Total 17]

12 A pension fund has liabilities of £3 million due in 3 years' time, £5 million due in 5 years' time, £9 million due in 9 years' time, and £11 million due in 11 years' time. The fund holds two investments, X and Y. Investment X provides income of £1 million payable at the end of each year for the next five years with no capital repayment. Investment Y is a zero coupon bond which pays a lump sum of £R at the end of n years (where n is not necessarily an integer). The interest rate is 8% per annum effective.

(i)   Investigate whether values of £R and n can be found which ensure that the fund is immunised against small changes in the interest rate.

You are given that $\sum_{t=1}^{5} t^2 v^t = 40.275$ at 8%.   [8]

(ii)   (a)   The interest rate immediately changes to 3% per annum effective. Calculate the revised present values of the assets and liabilities of the fund.

(b)   Explain your answer to (ii)(a).   [4]
[Total 12]

END OF PAPER