Adjusted Bachelier Model for Caps/Floors on RFR

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1. Introduction

This document is used to summarize the evaluation formulas of adapted Bachelier model for Cap/Floor on RFR.

1.1. Background

IBOR rates (forward looking term rates) will be ceased as reference in financial transactions and replaced by nearly risk-free compounded indices (RFR). Compounded RFR rates are “backward looking” and are set in arrears. The compounded indices will be partially fixed all along the accrual period and will only be fully known at the end of the calculation period. For vanilla interest rate options such as European swaption and Cap/Floor, the handling transition from IBOR to Compounded RFR is different.

European Swaption: European swaption’s maturity is before the first compounding period of the underlying swap. So, the evaluation is the same as IBOR-based European swaption. The swap rate can still be estimated with annuity measure and there would be no adaptation required for its evaluation model.

Cap/Floor: IBOR Cap/Floor’s payoff is known in advance but Cap/Floor on RFR payoff would not be known until the end of the period. Therefore, the pricing model adaptations are required to diffuse till the accrual end date. When entering compounding period, the compounded RFR rate is being partially fixed, and the volatility should decrease also.

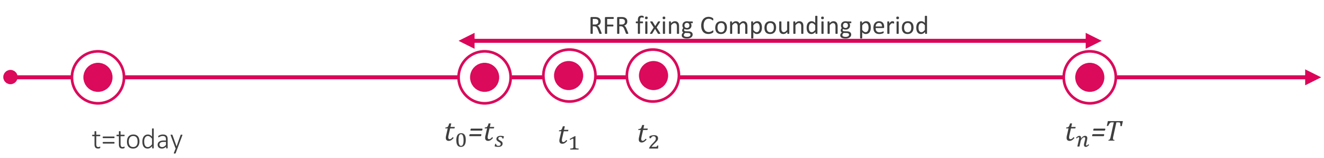
This document summarizes the evaluation methodology for Caps/Floors on RFR. It focuses on two types of setups:

- IBOR proxy volatility: Due to liquidity, some clients may want to use IBOR volatilities to evaluate Caps/Floors on RFR. This is a transition model which relies on the assumption that there is a linear relationship between Compounded RFR rate and IBOR rate.
- RFR volatility: We would expect the market liquidity progressively shift to Caps/Floors on RFR rate and RFR volatility market quotes would be available. In addition, different calculation frequency (for example, 1M, 3M, 6M etc.) involves different period of volatility decay, we would need dedicated volatility surface for each tenor like IBOR.

1.2. Notation and Definition

To explain the product and evaluation formula, some notations used in the document are listed as below:

- The Compounded rate from t_s to t_n is computed as $R(t_0, t_n) = \frac{1}{\tau}\left[\prod_{i=0}^{n-1}\left(1 + \tau_i F(t_i,\ t_{i+1})\right) - 1\right]$
    - F: OIS rate
    - τ: The day count fraction for the compound RFR rate
    - τ_i: The day count fraction for OIS rate
    - $t_s = t_0$: Caplet compounding period start date
    - $T = t_n$: Caplet compounding period end date



- A caplet or floorlet is an option on floating index and its payoff is described as below:
    - N: The notional of the caplet or floorlet
    - K: The strike of the caplet or floorlet
    - τ: The day count fraction for the accrual period
    - R: The floating index where the caplet or floorlet is referenced with
    - $DF_{t,T}$: The discount factor between t and T
    - The Caplet Payoff is $N \times \tau \times (R - K)^+$ and Floorlet Payoff is $N \times \tau \times (K - R)^+$

Note: *The payoffs for different strategies (Collar, Strangle, Cap Spread, Floor Spread) are listed in Appendix.*

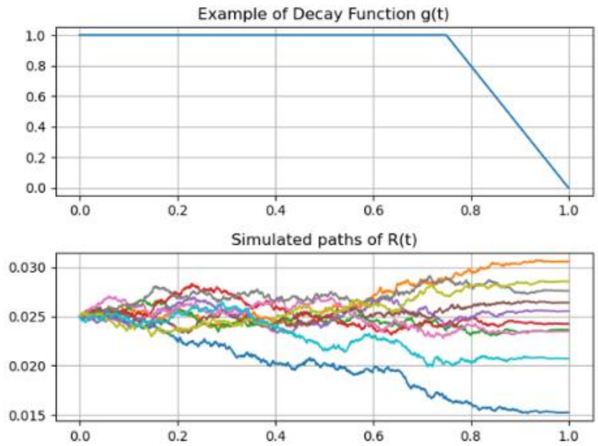
2. Adjusted Bachelier Model with Proxy Volatility

2.1. Forward RFR Diffusion with Decay Function

Like IBOR rates, Compounded RFR rate can be modelled as martingale under the T-Forward measure. Yet, compounded RFR rate should keep diffusing till the accrual end date with variance decreasing monotonically to 0. Following Mercurio's article (Section 3), we can approximate the Compounded RFR rate by a simple normal diffusion:

$$\boldsymbol{dR(t) = \sigma g(t) dW_t} \textbf{ where } \boldsymbol{g(t) = \min\{\frac{(T-t)_+}{T-t_s}, 1\}}$$

- $g(t)$ is chosen as a piecewise decay function which is worth 1 up util the first fixing date then decreases linearly till the last fixing date.



$t_s = 9m \; and \; T = 1y, R_0 = 2.5\% \; and \; \sigma = 10\%$

Figure 1: g(t) function and simulation path where t_s=9m and T=1y.

- $R(T)$ is normally distributed with mean $R(0)$ and variance $\boldsymbol{\Sigma_{RFR}^2(T)T = \sigma^2 \int_0^T g(t)^2 dt}$.

2.2. Bachelier Model for IBOR and adaptation for RFR Rate

The classical Bachelier model assumes the IBOR rate diffusion as $dF = \sigma_N dW_t$. The pricing formulas for caplet and floorlet on IBOR are as below:

- $Bachelier_{Caplet}(F, K, \sigma_N, T_{expiry}) = DF_{0,T} \times \left(N \times \tau \times \sigma_N \sqrt{T_{expiry}} \left(dN(d) + N'(d)\right)\right)$
- $Bachelier_{Floorlet}(F, K, \sigma_N, T_{expiry}) = DF_{0,T} \times \left(N \times \tau \times \sigma_N \sqrt{T_{expiry}} \left(N'(-d) - dN(-d)\right)\right)$

where $d = \frac{F-K}{\sigma_N \sqrt{T_{expiry}}}$, σ_N is constant normal volatility and T_{expiry} is the *option expiry date*.

Similarly, to extend the classical Bachelier model with RFR diffusion with decay function, the pricing formulas for caplet and floorlet on Compounded RFR are as below:

- $Bachelier_{Caplet}(F, K, \Sigma_{RFR}, T) = DF_{0,T} \times \left(N \times \tau \times \Sigma_{RFR} \sqrt{T} \left(dN(d) + N'(d)\right)\right)$
- $Bachelier_{Floorlet}(F, K, \Sigma_{RFR}, T) = DF_{0,T} \times \left(N \times \tau \times \Sigma_{RFR} \sqrt{T} \left(N'(-d) - dN(-d)\right)\right)$

where $d = \frac{F-K}{\Sigma_{RFR}\sqrt{T}}$, $\Sigma_{RFR} = \sqrt{\frac{\boldsymbol{\sigma^2 \int_0^T g(t)^2 dt}}{T}}$ and T is *the end date of the accrual period*.

2.3. Choices of σ and Normal Vol Conversion

The fallback rate will be compounded RFR rate in the IBOR accrual period plus a spread. The spread is computed on the discontinuation announcement dates as historical mean of the spread between IBOR and RFR. So, due to the fallback mechanism, there is an almost linear dependency between IBOR rates and RFR rates. We can deduce RFR diffusion volatility σ from normal IBOR volatilities. More precisely, given the caplet on RFR, we will use its moneyness $k = F - K$ to extract the Bachelier normal volatility on IBOR of the same moneyness, $\sigma = \Sigma^2_{Libor}(k,\ t_s)$. Then, we can derive the variance depending on the condition of t_s as below:

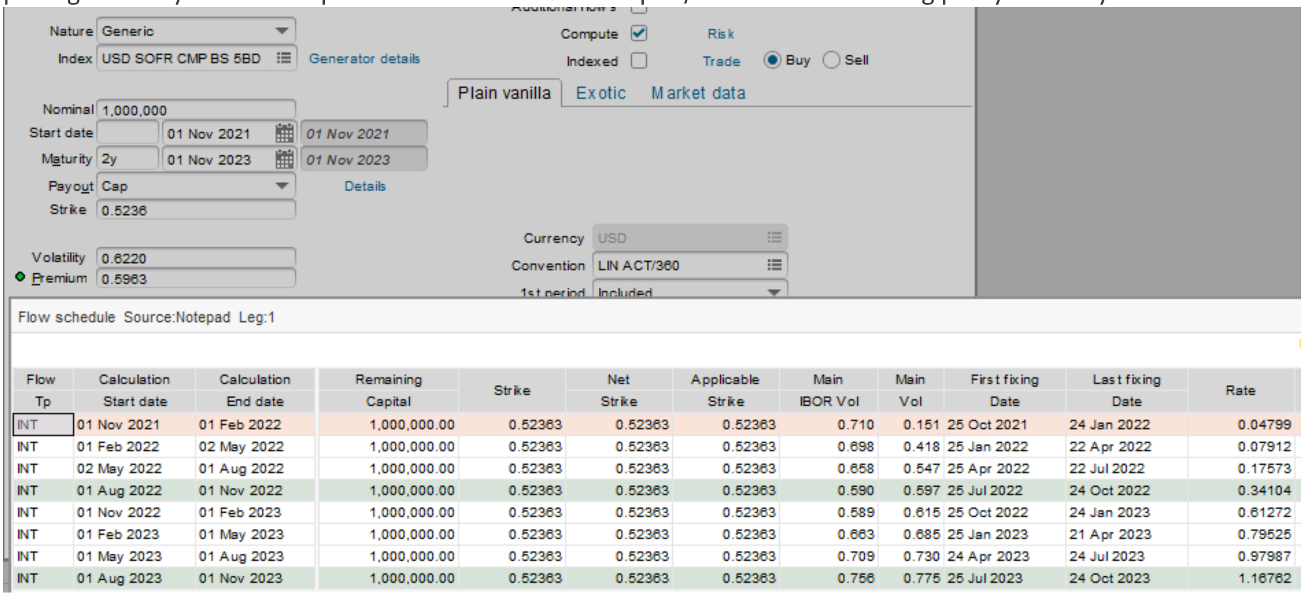
- $t_s > 0 \rightarrow \Sigma^2_{RFR}(T)T = \sigma^2 \int_0^T g(t)^2 dt = \sigma^2 t_s + \sigma^2 \frac{\frac{1}{3}(T-t)^3}{(T-t_s)^2} = \sigma^2(t_s + \frac{T-t_s}{3})$
- $t_s < 0 \rightarrow \Sigma^2_{RFR}(T)T = \sigma^2 \int_0^T g(t)^2 dt = \frac{\sigma^2}{3}\frac{T^3}{(T-t_s)^2}$

The IBOR volatility used in the above pricing formula is interpolated *linearly or V2T* after the first fixing date. Here, we define t_{min} as the first IBOR maturity available where flat extrapolation is adopted before t_{min}. Typically, if Par volatility is input, the first IBOR maturity would be 9m. So, if compounded RFR rate's first fixing is before t_{min}, its corresponding IBOR volatility is at t_{min}. In addition, we adopt a simple step, $\Sigma^{Approx}_{RFR}(t_{vol}) = \Sigma_{RFR}(T)\sqrt{\frac{T}{t_{vol}}}$ to convert the normal on IBOR into normal on RFR. t_{vol} is the pillar where Libor volatility is interpolated or extrapolated at. Similar adjustment can also be found in Piterbarg's article (Section 3.2). Here, we further define the condition of $t_s$and t_{min}.

- $t_s > t_{min}$, $\Sigma^{Approx}_{RFR} = \Sigma_{Libor}(k, t_s)\sqrt{1 + \frac{T-t_s}{3t_s}} \rightarrow Bachelier\left(F, K, \Sigma^{Approx}_{RFR}, t_s\right)$
- $0 < t_s \leq t_{min}$, $\Sigma^{Approx}_{RFR} = \Sigma_{Libor}(k, t_{min})\sqrt{\frac{t_s}{t_{min}} + \frac{T-t_s}{3t_{min}}} \rightarrow Bachelier\left(F, K, \Sigma^{Approx}_{RFR}, t_{min}\right)$
- $t_s < 0$, $\Sigma^{Approx}_{RFR}(t_{min}) = \Sigma_{Libor}(k, t_{min})\sqrt{\frac{T^3}{3(T-t_s)^2 t_{min}}} \rightarrow Bachelier\left(F, K, \Sigma^{Approx}_{RFR}, t_{min}\right)$

2.4. Computation Example Cap/Floor with Ibor Proxy Volatility in MX.3

To elaborate the proxy volatility interpolation and volatility conversion process, a detailed computation example is provided as below. The system date is *18 Nov 2021* and USD SOFR CMP BS 5BD (observation shift by 5 business days) with *3-month (Proxy to USD LIBOR 3M)* computation frequency is chosen. There are two steps to derive the pricing volatility used in adapted Bachelier Model for Caplet/Floorlet on RFR using proxy volatility.



Flow Tp	Calculation Start date	Calculation End date	Remaining Capital	Strike	Net Strike	Applicable Strike	Main IBOR Vol	Main Vol	First fixing Date	Last fixing Date	Rate
INT	01 Nov 2021	01 Feb 2022	1,000,000.00	0.52363	0.52363	0.52363	0.710	0.151	25 Oct 2021	24 Jan 2022	0.04799
INT	01 Feb 2022	02 May 2022	1,000,000.00	0.52363	0.52363	0.52363	0.698	0.418	25 Jan 2022	22 Apr 2022	0.07912
INT	02 May 2022	01 Aug 2022	1,000,000.00	0.52363	0.52363	0.52363	0.658	0.547	25 Apr 2022	22 Jul 2022	0.17573
INT	01 Aug 2022	01 Nov 2022	1,000,000.00	0.52363	0.52363	0.52363	0.590	0.597	25 Jul 2022	24 Oct 2022	0.34104
INT	01 Nov 2022	01 Feb 2023	1,000,000.00	0.52363	0.52363	0.52363	0.589	0.615	25 Oct 2022	24 Jan 2023	0.61272
INT	01 Feb 2023	01 May 2023	1,000,000.00	0.52363	0.52363	0.52363	0.663	0.685	25 Jan 2023	21 Apr 2023	0.79525
INT	01 May 2023	01 Aug 2023	1,000,000.00	0.52363	0.52363	0.52363	0.709	0.730	24 Apr 2023	24 Jul 2023	0.97987
INT	01 Aug 2023	01 Nov 2023	1,000,000.00	0.52363	0.52363	0.52363	0.756	0.775	25 Jul 2023	24 Oct 2023	1.16762

Figure 2: Cap Pricing Screen on USD SOFR CMP BS 5BD.

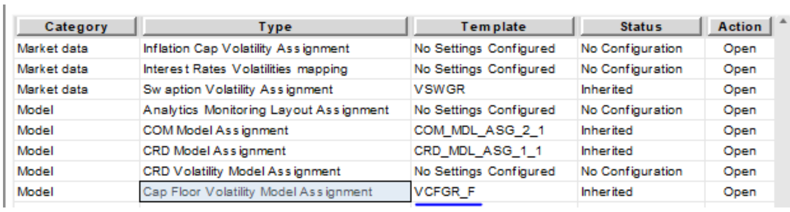
- **Computation of σ from IBOR volatility**: The moneyness scale, k is defined as $F - K$. Based on the estimated Compound RFR rate and the strike, we can first derive the RFR caplet/floorlet moneyness, k. Before applying the pricing formula, the key computation dates should be retrieved first as below.
  - t_{min} is the IBOR maturity for flat extrapolation. When we go to *Market Data/Rates/Caps and floors volatilities*, open the corresponding IBOR volatility group. If client maintains Par volatility in MX.3, we can switch the View from *Par volatility* to *Forward Volatility*. t_{min} should be the last caplet's fixing date of the first quoted pillar. For USD LIBOR 3M, it would be *9m (18 Aug 2022)* rather than *1Y (22 Nov 2022)*. We can observe that before t_{min}, the volatility is extrapolated constantly as shown in Figure 3: Forward-Forward Caplet Volatility View.


Forward Volatilities
USD LIBOR 3M Forward volatilities

Maturities	0.00	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00
18 Nov 2021	0.39	0.47	0.68	0.88	1.06	1.23	1.40	1.56	1.71	1.86	2.01	2.15	2.29	2.43	2.57
18 Feb 2022	0.39	0.47	0.68	0.88	1.06	1.23	1.40	1.56	1.71	1.86	2.01	2.15	2.29	2.43	2.57
19 May 2022	0.39	0.47	0.68	0.88	1.06	1.23	1.40	1.56	1.71	1.86	2.01	2.15	2.29	2.43	2.57
18 Aug 2022	0.39	0.47	0.68	0.88	1.06	1.23	1.40	1.56	1.71	1.86	2.01	2.15	2.29	2.43	2.57
18 Nov 2022	0.50	0.55	0.70	0.88	1.05	1.22	1.38	1.53	1.68	1.82	1.96	2.10	2.24	2.37	2.50

Figure 3: Forward-Forward Caplet Volatility View.

- The Ibor volatility interpolation setting is available under General Settings/Models/Cap Floor Volatility Model Assignment. The Smile is commonly interpolated Linearly in Strike Scale, and Time is either Linear or V2T.



Category	Type	Template	Status	Action
Market data	Inflation Cap Volatility Assignment	No Settings Configured	No Configuration	Open
Market data	Interest Rates Volatilities mapping	No Settings Configured	No Configuration	Open
Market data	Swaption Volatility Assignment	VSWGR	Inherited	Open
Model	Analytics Monitoring Layout Assignment	No Settings Configured	No Configuration	Open
Model	COM Model Assignment	COM_MDL_ASG_2_1	Inherited	Open
Model	CRD Model Assignment	CRD_MDL_ASG_1_1	Inherited	Open
Model	CRD Volatility Model Assignment	No Settings Configured	No Configuration	Open
Model	Cap Floor Volatility Model Assignment	VCFGR_F	Inherited	Open

Figure 4: Cap Floor Volatility Model Assignment.

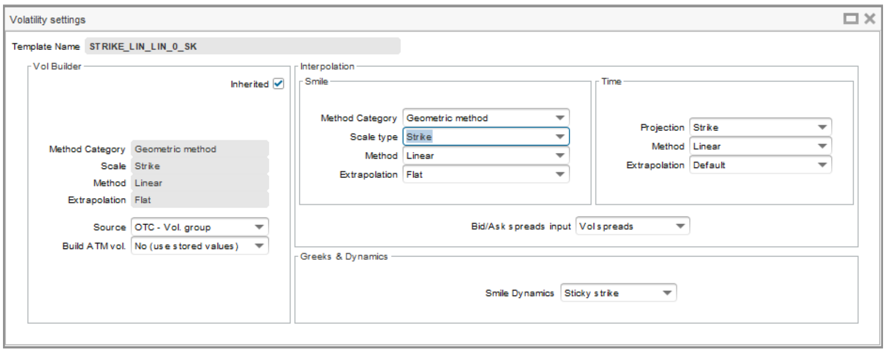


Figure 5: Volatility model setting template for Cap Floor Volatility Model Assignment.

- The first period (from *1 Nov 2021* to *1 Feb 2022*) in the sample cap/floor's moneyness is $k = 0.04799 - 0.52363 = -0.47564$ for compound RFR rate. Then, a USD LIBOR 3M caplet with t_{min} maturity is (18 Aug 2022) and same moneyness ($k = 0.60041 - 1.07605 = -0.47564$) is priced and the proxy volatility (0.710%) is matching.

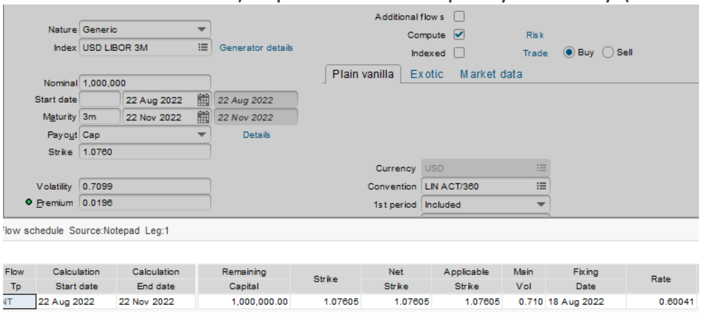


Figure 6: Cap Pricing Screen on USD LIBOR 3M with the same moneyness scale on First IBOR maturity date, 18 Aug 2022.

- The Last period (from *1 Aug 2023* to *1 Nov 2023*) moneyness is $k = 1.16762 - 0.52363 = 0.643$ for compound RFR rate. Then, a USD LIBOR 3M caplet with Fixing date = First Fixing Date (*25 Jul 2023*) and same moneyness ($k = 1.35828 - 0.71428 = 0.643$) is priced and the proxy volatility (0.756%) is matching.

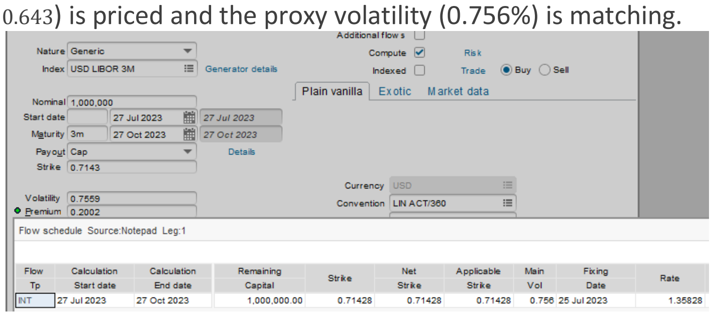
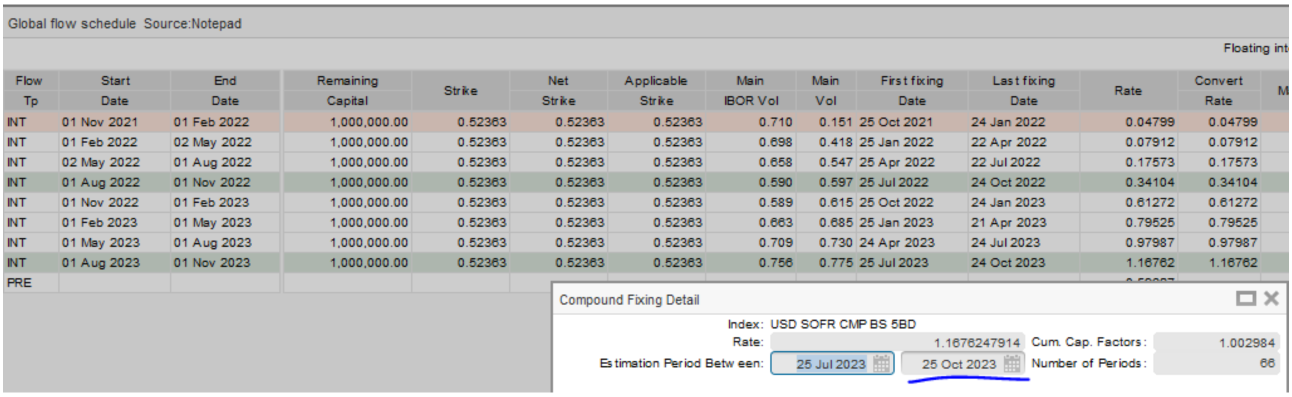


Figure 7: Cap Pricing Screen on USD LIBOR 3M with the same moneyness scale on Fixing date, 25 Jul 2023.

Note: *The assumption is that RFR proxy volatility is interpolated based on moneyness scale. So, the moneyness scale is first derived for RFR Caplet/Floorlet and then the corresponding Ibor volatility is interpolated accordingly.*

- **Vol Conversion**: Once σ is derived based on the same moneyness scale and fixing date on each caplet/floorlet period, system would carry out volatility conversion step. This volatility conversion step is a technical step which does not change the variance.
  - t_s and T are the compounded RFR rate *estimation start and end date* for each period. For t_s, it should be the same date as First Fixing Date in the flow screen. However, for T, it might not be the same as End Date nor Last fixing date. So, we would suggest to *right click→Rate Details* on Rate to check the exact estimation start and end date.



Global flow schedule Source:Notepad

Floating int

Flow Tp	Start Date	End Date	Remaining Capital	Strike	Net Strike	Applicable Strike	Main IBOR Vol	Main Vol	First fixing Date	Last fixing Date	Rate	Convert Rate
INT	01 Nov 2021	01 Feb 2022	1,000,000.00	0.52363	0.52363	0.52363	0.710	0.151	25 Oct 2021	24 Jan 2022	0.04799	0.04799
INT	01 Feb 2022	02 May 2022	1,000,000.00	0.52363	0.52363	0.52363	0.698	0.418	25 Jan 2022	22 Apr 2022	0.07912	0.07912
INT	02 May 2022	01 Aug 2022	1,000,000.00	0.52363	0.52363	0.52363	0.658	0.547	25 Apr 2022	22 Jul 2022	0.17573	0.17573
INT	01 Aug 2022	01 Nov 2022	1,000,000.00	0.52363	0.52363	0.52363	0.590	0.597	25 Jul 2022	24 Oct 2022	0.34104	0.34104
INT	01 Nov 2022	01 Feb 2023	1,000,000.00	0.52363	0.52363	0.52363	0.589	0.615	25 Oct 2022	24 Jan 2023	0.61272	0.61272
INT	01 Feb 2023	01 May 2023	1,000,000.00	0.52363	0.52363	0.52363	0.663	0.685	25 Jan 2023	21 Apr 2023	0.79525	0.79525
INT	01 May 2023	01 Aug 2023	1,000,000.00	0.52363	0.52363	0.52363	0.709	0.730	24 Apr 2023	24 Jul 2023	0.97987	0.97987
INT	01 Aug 2023	01 Nov 2023	1,000,000.00	0.52363	0.52363	0.52363	0.756	0.775	25 Jul 2023	24 Oct 2023	1.16762	1.16762
PRE												

Compound Fixing Detail

Index: USD SOFR CMP BS 5BD
Rate: 1.1676247914 Cum. Cap. Factors: 1.002984
Estimation Period Between: 25 Jul 2023 25 Oct 2023 Number of Periods: 66

Figure 8: Compounding Fixing Detail in Flow Screen to check estimation start and end ate.

If t_s, t_{min} and T are derived, we can use the formula listed in section 2.3 to derive the Main Vol in the Flow details in Figure 2. Here, t_{min} is 18 Aug 2022 and with the value of *0.747945*.

- The first period volatility is derived by $\Sigma_{Libor}(k, t_{min})\sqrt{\frac{T^3}{3(T-t_s)^2 t_{min}}}$ where $t_s < 0$.
- The second and third period volatiles are computed as $\Sigma_{Libor}(k, t_{min})\sqrt{\frac{t_s}{t_{min}} + \frac{T-t_s}{3t_{min}}}$ where $0 < t_s \le t_{min}$.
- The remaining periods' volatiles are $\Sigma_{Libor}(k, t_s)\sqrt{1 + \frac{T-t_s}{3t_s}}$ where $t_s > t_{min}$.

Estimation Start	Estimation End	Ibor Vol	Ts	T	Main Vol (Excel)	Main Vol (MX)	Diff
25-10-21	25-01-22	0.70990654	(0.06575342)	0.18630137	0.151194073	0.151194073	0.000000%
25-01-22	25-04-22	0.697624477	0.18630137	0.43287671	0.417978001	0.417978001	0.000000%
25-04-22	25-07-22	0.658395979	0.43287671	0.68219178	0.546851544	0.546851544	0.000000%
25-07-22	25-10-22	0.589626885	0.68219178	0.93424658	0.596782813	0.596782813	0.000000%
25-10-22	25-01-23	0.589407835	0.93424658	1.18630137	0.61534053	0.61534053	0.000000%
25-01-23	24-04-23	0.662979811	1.18630137	1.43013699	0.685315352	0.685315352	0.000000%
24-04-23	25-07-23	0.70920041	1.43013699	1.68219178	0.729735312	0.729735312	0.000000%
25-07-23	25-10-23	0.7559389	1.68219178	1.93424658	0.774586844	0.774586844	0.000000%

- **Apply Bachelier Pricing Formula**: Then, we can use Σ_{RFR}^{Approx} and its corresponding tenor to apply Bachelier pricing formula to derive the annualized coupon rate accordingly.
  - $t_s > t_{min}$, $\Sigma_{RFR}^{Approx} = \Sigma_{Libor}(k, t_s)\sqrt{1 + \frac{T-t_s}{3t_s}}$ *and* $\check{t} = t_s$
  - $0 < t_s \le t_{min}$, $\Sigma_{RFR}^{Approx} = \Sigma_{Libor}(k, t_{min})\sqrt{\frac{t_s}{t_{min}} + \frac{T-t_s}{3t_{min}}}$ *and* $\check{t} = t_{min}$
  - $t_s < 0$, $\Sigma_{RFR}^{Approx}(t_{min}) = \Sigma_{Libor}(k, t_{min})\sqrt{\frac{T^3}{3(T-t_s)^2 t_{min}}}$ *and* $\check{t} = t_{min}$

$$\rightarrow \boldsymbol{Coupon\ Rate = Bachelier\left(F, K, \Sigma_{RFR}^{Approx}, \check{t}\right)}$$

- **Drive the Flow and Sensitivity in flow details:** Then, we can use the formula below to derive the analytical sensitivity in the flow detail screen. More details regarding the definition and formulas for flow screen sensitivities are available in *docid: 973, section 3.2.1*.

Floating interest payments USD

Rate	Margin	Rate Factor	Payment Date	Annualized coupon	Flows	Cur	Delta	Delta Csh	Gamma	Gamma Csh	Vega(%)	Vega Csh	Theta	Theta Pct
0.04799	0.0000	1.0000	01 Feb 2022	0.475642	1,215.53	USD	-99.9862	-25.5520	0.0041	0.0010	0.0000	0.2514	-0.0003	-0.0000
0.07912	0.0000	1.0000	02 May 2022	0.463588	1,158.97	USD	-89.0593	-22.2648	0.5182	0.1295	0.0242	242.4056	-0.3100	-0.0000
0.17573	0.0000	1.0000	01 Aug 2022	0.411492	1,040.16	USD	-76.9019	-19.4391	0.6436	0.1627	0.0552	551.6508	-0.6664	-0.0000
0.34104	0.0000	1.0000	01 Nov 2022	0.309951	792.10	USD	-63.8247	-16.3108	0.7261	0.1856	0.0836	835.5742	-0.9053	-0.0000
0.61272	0.0000	1.0000	01 Feb 2023	0.195387	499.32	USD	-44.0462	-11.2562	0.6633	0.1695	0.1012	1,011.9848	-0.8792	-0.0000
0.79525	0.0000	1.0000	01 May 2023	0.181471	448.64	USD	-35.7967	-8.8497	0.5002	0.1237	0.1031	1,031.1794	-0.7956	-0.0000
0.97987	0.0000	1.0000	01 Aug 2023	0.166551	425.63	USD	-30.0554	-7.6808	0.3988	0.1019	0.1082	1,082.3843	-0.7434	-0.0000
1.16762	0.0000	1.0000	01 Nov 2023	0.158430	404.88	USD	-26.0753	-6.6637	0.3234	0.0826	0.1087	1,087.4887	-0.6792	-0.0000

Figure 9: Flow Screen to check annualized coupon, flow, and sensitivities.

- The annualized coupon is derived based on Bachelier formula as $\Sigma_{RFR}\sqrt{T}\left(dN(d)+N'(d)\right)$ and $\Sigma_{RFR}\sqrt{T}\left(N'(-d)-dN(-d)\right)$ for caplet and floorlet respectively.
- Delta Cash: $\tau \times\ N(d)$ and $\tau \times (N(d)-1)$ for caplet and floorlet respectively.
- Gamma Cash: $\frac{\tau \times N'(d)}{\sigma\sqrt{\check{t}}}$ for both caplet and floorlet.
- Theta: $-Notional \times \tau \times \frac{\sigma}{2\sqrt{t_s}} \times \frac{N'(d)}{365}$
- Vega Cash: $\frac{\partial NPV}{\Sigma_{Libor}} = DF_{0,T} \times \tau\ \sqrt{t_s}\ \times N'(d) \times \frac{\partial \Sigma_{RFR}^{Approx}}{\partial \Sigma_{Libor}}$ with $\frac{\partial \Sigma_{RFR}^{Approx}}{\partial \Sigma_{Libor}} = \begin{cases} \sqrt{\frac{T^3}{3(T-t_s)^2 t_{min}}}, t_s < 0 \\ \sqrt{\frac{t_s}{t_{min}} + \frac{T-t_s}{3t_{min}}},\ \ 0 < t_s \leq t_{min} \\ \sqrt{1 + \frac{T-t_s}{3t_s}},\ \ \ t_s > t_{min} \end{cases}$

- **Compute Market Value and Sensitivity in Simulation**: The DV01 (zero) is first computed analytically and then DV01 (par) is derived from Jacobian Matrix, $\frac{\partial Z}{\partial R}$. The DV01 can be validated in simulation by shifting the market rate by 1bp from last pillar with backward cumulative method. The pricing is based on forward-forward volatility and if client inputs Par volatility, we cannot directly shift up calibrated forward-forward volatility. So, Vega should be validated through analytical formula. For Vega, there is an additional adjustment, $\frac{\partial \Sigma_{RFR}^{Approx}}{\partial \Sigma_{Libor}}$ applied to derive Ibor volatility sensitivity.

We would support the standard sensitivity output field in the path below. More details regarding the sensitivities are available in docid: 973, section 3.2.1.

- DV01 (Zero): *Risk Engine.Results.Outputs.Interest rates.Delta.Zero.Value*
- DV01 (Par): *Risk Engine.Results.Outputs.Interest rates.Delta.Par.Value*
- Normal Vega: *Risk Engine.Results.Outputs.Interest rates.Vega.Normal.Value*

There are a few settings which can impact the sensitivity results of cap/floor in simulation viewers:

- *General settings/Rates/Rates general settings/Cap/floors sensitivity = Analytical*: The setting controls how sensitivities are computed, and it is recommended to set as Analytical.
- *General settings/Shared/Curves/Rate/Rate propagation=Keep market quotes constant:* Keep the market quotes constant by recomputing calibrated zero coupon rates.
- *General settings/Shared/Curves/Rate/Rate/basis propagation and computations=Keep market spread constant/Impact sensitivities:* Both scenarios and sensitivities rely on keeping market quote constant.
- *General settings/Shared/Models/Binary option smile drift/Call/puts spread and Strike gap=0.01*: The setting controls the pricing to digital caps and floors.
- *General settings/Shared/Sensitivities/Vega breakdown uses=Volatility surface pillars*: The Vega sensitivities are projected to the volatility surface pillars. When it is set to “Sensitivities dates”, it would project the Vega corresponding to the Caplet/Floorlet pricing dates.

3. Adjusted Bachelier Model with RFR Volatility

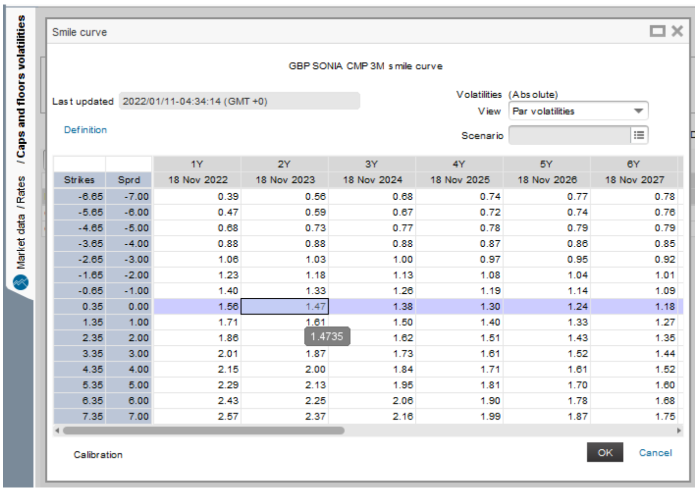
3.1. Market Convention for RFR Cap/Floor Volatility

As the liquidity of RFR options is growing, RFR cap/floor volatility quotes are now available in some markets. The quoted volatilities are volatilities on compounded RFR indices rather than the daily RFR rate. The market convention is to quote RFR caplet volatility in the last fixing date convention. For example, ICAP quotes GBP SONIA Caps/Floors volatilities and we can replicate the premiums by plugging in the Bachelier formula the quoted volatility with adjustment to use end date as expiry (as opposed to the start date for Ibor Cap/Floor).

- $Bachelier_{Caplet}(F, K, \Sigma_{RFR}, T) = DF_{0,T} \times \left(N \times \tau \times \Sigma_{RFR}\sqrt{T}\,\left(dN(d) + N'(d)\right)\right)$
- $Bachelier_{Floorlet}(F, K, \Sigma_{RFR}, T) = DF_{0,T} \times \left(N \times \tau \times \Sigma_{RFR}\sqrt{T}\,\left(N'(-d) - dN(-d)\right)\right)$

where $d = \frac{F-K}{\Sigma_{RFR}\sqrt{T}}$ and T is the *last fixing date* of the compounded RFR rate.

Consequently, the volatility in the MX.3 should be interpolated and displayed in last fixing date convention when RFR cap/floor volatility is provided in Caps and floors volatilities. The maturities in Forward Volatilities correspond to the last fixing date. Like Ibor Cap/Floor, Murex can bootstrap forward-forward caplet volatility from the quoted par RFR volatilities. After the first quoted pillar, we would continue using classical V2T interpolation based on last fixing date accordingly.



Market data / Rates / Caps and floors volatilities

Smile curve

GBP SONIA CMP 3M smile curve

Last updated 2022/01/11-04:34:14 (GMT +0)

Volatilities (Absolute)

View Par volatilities

Definition

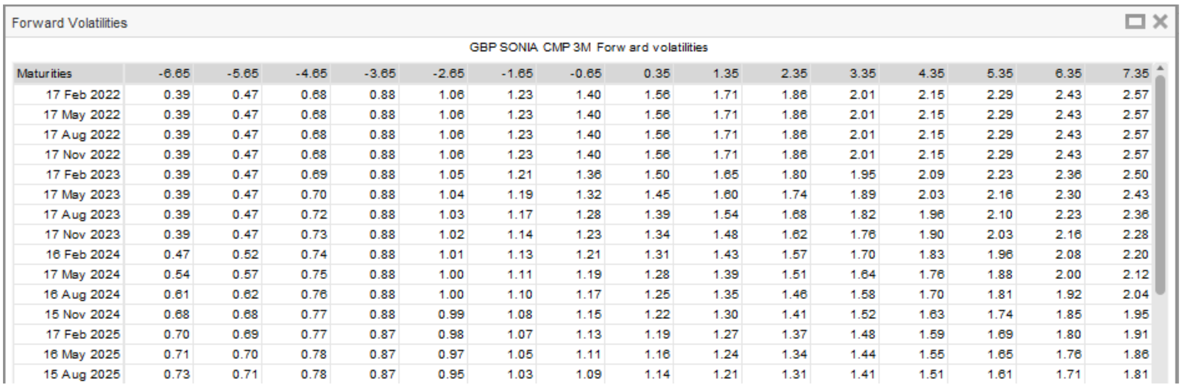
Scenario

		1Y	2Y	3Y	4Y	5Y	6Y
Strikes	Sprd	18 Nov 2022	18 Nov 2023	18 Nov 2024	18 Nov 2025	18 Nov 2026	18 Nov 2027
-6.65	-7.00	0.39	0.56	0.68	0.74	0.77	0.78
-5.65	-6.00	0.47	0.59	0.67	0.72	0.74	0.76
-4.65	-5.00	0.68	0.73	0.77	0.78	0.79	0.79
-3.65	-4.00	0.88	0.88	0.88	0.87	0.86	0.85
-2.65	-3.00	1.06	1.03	1.00	0.97	0.95	0.92
-1.65	-2.00	1.23	1.18	1.13	1.08	1.04	1.01
-0.65	-1.00	1.40	1.33	1.26	1.19	1.14	1.09
0.35	0.00	1.56	1.47	1.38	1.30	1.24	1.18
1.35	1.00	1.71	1.61	1.50	1.40	1.33	1.27
2.35	2.00	1.86	1.4735	1.62	1.51	1.43	1.35
3.35	3.00	2.01	1.87	1.73	1.61	1.52	1.44
4.35	4.00	2.15	2.00	1.84	1.71	1.61	1.52
5.35	5.00	2.29	2.13	1.95	1.81	1.70	1.60
6.35	6.00	2.43	2.25	2.06	1.90	1.78	1.68
7.35	7.00	2.57	2.37	2.16	1.99	1.87	1.75

Calibration

OK Cancel

Figure 10: GBP Sonia CMP 3M Par volatility in Caps/Floors Volatilities.



Forward Volatilities

GBP SONIA CMP 3M Forward volatilities

Maturities	-6.65	-5.65	-4.65	-3.65	-2.65	-1.65	-0.65	0.35	1.35	2.35	3.35	4.35	5.35	6.35	7.35
17 Feb 2022	0.39	0.47	0.68	0.88	1.06	1.23	1.40	1.56	1.71	1.86	2.01	2.15	2.29	2.43	2.57
17 May 2022	0.39	0.47	0.68	0.88	1.06	1.23	1.40	1.56	1.71	1.86	2.01	2.15	2.29	2.43	2.57
17 Aug 2022	0.39	0.47	0.68	0.88	1.06	1.23	1.40	1.56	1.71	1.86	2.01	2.15	2.29	2.43	2.57
17 Nov 2022	0.39	0.47	0.68	0.88	1.06	1.23	1.40	1.56	1.71	1.86	2.01	2.15	2.29	2.43	2.57
17 Feb 2023	0.39	0.47	0.69	0.88	1.05	1.21	1.36	1.50	1.65	1.80	1.95	2.09	2.23	2.36	2.50
17 May 2023	0.39	0.47	0.70	0.88	1.04	1.19	1.32	1.45	1.60	1.74	1.89	2.03	2.16	2.30	2.43
17 Aug 2023	0.39	0.47	0.72	0.88	1.03	1.17	1.28	1.39	1.54	1.68	1.82	1.96	2.10	2.23	2.36
17 Nov 2023	0.39	0.47	0.73	0.88	1.02	1.14	1.23	1.34	1.48	1.62	1.76	1.90	2.03	2.16	2.28
16 Feb 2024	0.47	0.52	0.74	0.88	1.01	1.13	1.21	1.31	1.43	1.57	1.70	1.83	1.96	2.08	2.20
17 May 2024	0.54	0.57	0.75	0.88	1.00	1.11	1.19	1.28	1.39	1.51	1.64	1.76	1.88	2.00	2.12
16 Aug 2024	0.61	0.62	0.76	0.88	1.00	1.10	1.17	1.25	1.35	1.46	1.58	1.70	1.81	1.92	2.04
15 Nov 2024	0.68	0.68	0.77	0.88	0.99	1.08	1.15	1.22	1.30	1.41	1.52	1.63	1.74	1.85	1.95
17 Feb 2025	0.70	0.69	0.77	0.87	0.98	1.07	1.13	1.19	1.27	1.37	1.48	1.59	1.69	1.80	1.91
16 May 2025	0.71	0.70	0.78	0.87	0.97	1.05	1.11	1.16	1.24	1.34	1.44	1.55	1.65	1.76	1.86
15 Aug 2025	0.73	0.71	0.78	0.87	0.95	1.03	1.09	1.14	1.21	1.31	1.41	1.51	1.61	1.71	1.81

Figure 11: GBP Sonia CMP 3M caplet volatility in Caps/Floors Volatilities with Default flat extrapolation.

Forward Volatilities

GBP SONIA CMP 3M Forward volatilities

Maturities	-6.65	-5.65	-4.65	-3.65	-2.65	-1.65	-0.65	0.35	1.35	2.35	3.35	4.35	5.35	6.35	7.35
17 Feb 2022	0.25	0.30	0.43	0.55	0.67	0.80	0.96	1.08	1.13	1.20	1.29	1.37	1.45	1.54	1.62
17 May 2022	0.35	0.42	0.61	0.79	0.96	1.14	1.37	1.55	1.62	1.72	1.84	1.96	2.08	2.19	2.31
17 Aug 2022	0.38	0.46	0.66	0.85	1.03	1.23	1.47	1.67	1.75	1.86	1.98	2.11	2.24	2.37	2.49
17 Nov 2022	0.39	0.47	0.68	0.88	1.07	1.27	1.53	1.73	1.81	1.92	2.05	2.19	2.32	2.45	2.58
17 Feb 2023	0.39	0.47	0.70	0.88	1.05	1.22	1.41	1.57	1.68	1.81	1.95	2.08	2.21	2.35	2.48
17 May 2023	0.39	0.47	0.71	0.88	1.04	1.19	1.32	1.46	1.59	1.73	1.87	2.01	2.14	2.28	2.40
17 Aug 2023	0.39	0.47	0.72	0.88	1.03	1.16	1.26	1.37	1.52	1.67	1.82	1.96	2.09	2.22	2.35
17 Nov 2023	0.39	0.47	0.73	0.88	1.02	1.14	1.20	1.30	1.47	1.63	1.78	1.92	2.05	2.18	2.31
16 Feb 2024	0.51	0.55	0.74	0.88	1.01	1.12	1.19	1.28	1.42	1.56	1.70	1.83	1.96	2.08	2.20
17 May 2024	0.58	0.60	0.75	0.88	1.00	1.11	1.19	1.27	1.38	1.51	1.63	1.76	1.88	1.99	2.11
16 Aug 2024	0.64	0.64	0.76	0.88	1.00	1.10	1.18	1.25	1.35	1.46	1.58	1.69	1.81	1.92	2.03
15 Nov 2024	0.68	0.68	0.77	0.88	0.99	1.09	1.17	1.24	1.32	1.42	1.53	1.64	1.75	1.86	1.97
17 Feb 2025	0.70	0.69	0.77	0.87	0.98	1.07	1.14	1.20	1.28	1.38	1.48	1.59	1.70	1.80	1.91
16 May 2025	0.72	0.70	0.78	0.87	0.96	1.05	1.11	1.16	1.24	1.34	1.44	1.55	1.65	1.76	1.86
15 Aug 2025	0.73	0.71	0.78	0.87	0.95	1.03	1.08	1.13	1.21	1.31	1.41	1.51	1.61	1.71	1.81

Figure 12: GBP Sonia COMP 3M caplet volatility in Caps/Floors Volatilities with RFR Decay extrapolation.

3.2. Adaptation of the RFR volatility before the First Quoted Pillar

By default, a cap/floor volatility surface is constructed in such a way that flat forward volatilities prior to the first quoted pillar are assumed regardless of the underlying of the volatility group as shown in Figure 11: GBP Sonia CMP 3M caplet volatility in Caps/Floors Volatilities with Default flat extrapolation. However, in the cases where the underlying of a Cap/floor volatility group is an RFR compounded index with daily compounding, there is an option of changing time extrapolation method for the maturities prior to the first pillar. The new method is called RFR Decay time extrapolation. For RFR compounded index, it would make more sense to use RFR Decay extrapolation method before the first pillar. The reason is that a flat volatility would overprice the short maturity options. RFR Decay time extrapolation would make sure the pricing of the partially fixed compounded index to be priced properly with decreasing variance.

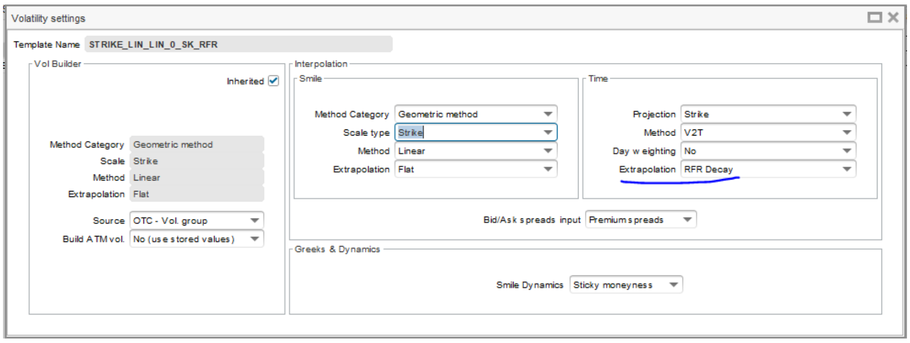


Figure 13: Extrapolation Method for RFR rate in Volatility Model Setting.

Following RFR diffusion, $dR = \sigma g(t)dW$ with decay function, $g(t) = \min\{\frac{(t_e-t)_+}{t_e-t_s}, 1\}$, we would assume that the constant diffusion volatility is σ before the first pillar. Suppose the first quoted RFR caplet volatility is $\sigma_{RFR,0}$ and its first and last fixing dates are $t_{s,0}$ ($t_{s,0} > 0$) and $t_{e,0}$ respectively.

$$\sigma_{RFR,0}^2 t_{e,0} = \sigma^2 \int_0^{t_{e,0}} g(t)^2 dt = \sigma^2 \times \left(t_{s,0} + \frac{1}{3} \times \frac{(t_{e,0} - t_{s,0})^3}{\tau^2}\right) \rightarrow \sigma = \sigma_{RFR,0} \sqrt{\frac{t_{e,0}}{t_{s,0} + \frac{1}{3} \times \frac{(t_{e,0} - t_{s,0})^3}{\tau^2}}}$$

To simplify the notation, we use $f(t_s, t_e) = t_s^+ + \frac{1}{3} \times \frac{(t_e - t_s^+)^3}{\tau^2}$ *where* $t_s^+ = \max(t_s, 0)$. τ can be approximated by first and last fixing date as $t_e - t_s + 1$. Moreover, τ is derived for each period and shared in both $f(t_s, t_e)$ and $f(t_{s,0}, t_{e,0})$. Following the convention of using last fixing date as expiry in Bachelier formula, we can derive the Σ_{RFR} as below.

$$\Sigma_{RFR}^2(T)T = \sigma_{RFR,0}^2 \times \frac{t_{e,0}}{T} \times \frac{f(t_s, t_e)}{f(t_{s,0}, t_{e,0})} \rightarrow \Sigma_{RFR}(T) = \sigma_{RFR,0} \sqrt{\frac{t_{e,0}}{T} \times \frac{f(t_s, T)}{f(t_{s,0}, t_{e,0})}} \quad when\ T < t_{e,0}$$

Therefore, pricing of the caplet before the first quoted pillar, we would need to solve caplet volatility of $\sigma_{RFR,0}$ to match corresponding cap premiums based on the Par RFR volatility. Then, we can derive pricing volatility as $\Sigma_{RFR}(T) = \sigma_{RFR,0} \sqrt{\frac{t_{e,0}}{T} \times \frac{f(t_s,T)}{f(t_{s,0},t_{e,0})}}$ accordingly.

Note: $\sigma_{RFR,0}$ is solved from par volatility based on bootstrapping process rather than par volatility itself.

3.3. Bootstrapping from Par Volatility to Forward-Forward Volatility

When input RFR volatility is Par volatility, forward-forward caplet volatility used for pricing and risk are retrieved via bootstrapping process. More details can be found in *docid: 828, section 3.3*.

3.3.1. Consistency Check Before Bootstrapping

Before the bootstrapping process, consistency checks are performed:

- The input volatility should always be a positive value and we choose 0 as the lower bound.
- A non-arbitrable condition is that the variance does not decrease over time.

If any of the consistency checks failed, the bootstrapping process would remove the instrument and continue until the consistency checks are passed to proceed with subsequent quotes.

3.3.2. Bootstrapping Optimization Method and Stopping Criteria

The bootstrapping process utilizes Newton-Raphson method with Par volatility as first guess and the precision of 10^{-4} on premium prices with nominal of 1.0.

3.3.3. Bootstrapping Process for RFR Cap/Floor

For RFR Cap/Floor bootstrapping, based on the 1Y GBP Sonia CMP 3M Par Cap/Floor volatility, system would first solve 3m caplet forward volatility $\sigma_{RFR,0} = 1.73$ whose start date is 9m and end date is 1y. With $\sigma_{RFR,0}$, system then derive corresponding Bachelier pricing volatility $\Sigma_{RFR}(T)$ and try to match premiums with Par Cap/Floor volatility as shown in the excel sample computation below.

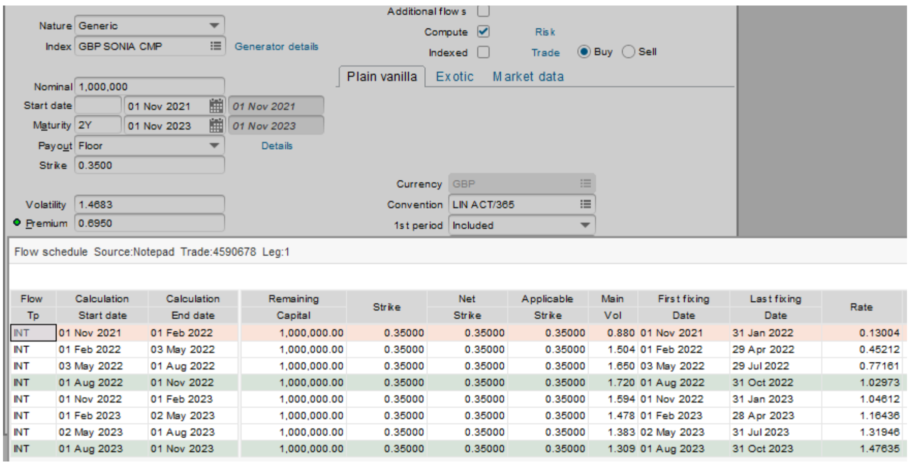
Start	End	Last fixing date	T	Forward	Strike	Volatility	DF	d	Coupon	NPV
18-11-21	18-02-22	17-02-22	0.249315068	0.179	0.35	1.5599	0.999554198	-0.21955	0.232687	0.232583
18-02-22	18-05-22	17-05-22	0.493150685	0.506198581	0.35	1.5599	0.998321977	0.14259	0.51955	0.518678
18-05-22	18-08-22	17-08-22	0.745205479	0.825260762	0.35	1.5599	0.996249669	0.352937	0.807957	0.804927
18-08-22	18-11-22	17-11-22	0.997260274	1.071266056	0.35	1.5599	0.993566862	0.463014	1.04754	1.040801

Start	End	First fixing date	ts	Last fixing date	T	Forward	Strike	DF	f(ts,te)	Volatility	d	Coupon	NPV
18-11-21	18-02-22	18-11-21	0	17-02-22	0.249315	0.179	0.35	0.999554	0.081308	1.08228	-0.31643	0.140792	0.140729
18-02-22	18-05-22	18-02-22	0.252054795	17-05-22	0.493151	0.5061986	0.35	0.998322	0.330624	1.55176	0.143339	0.517292	0.516424
18-05-22	18-08-22	18-05-22	0.495890411	17-08-22	0.745205	0.8252608	0.35	0.99625	0.577199	1.66790	0.330083	0.843048	0.839886
18-08-22	18-11-22	18-08-22	0.747945205	17-11-22	0.99726	1.0712661	0.35	0.993567	0.829253	1.728167	0.417932	1.108395	1.101265

After the first quoted pillar forward-forward volatility is derived based on RFR decay method, system starts the bootstrapping process for subsequent pillars. System would only solve the last caplet's forward-forward volatility and use the corresponding time interpolation method (typically V2T or Linearly) to derive the forward-forward volatilities in-between. So, there is only one volatility to solve per market quoted par volatility. The last pillar's forward-forward volatility can be accessed by change the view to Calibrated volatilities.

3.4. Computation Example of Cap/Floor with RFR Volatility in MX.3

To elaborate the RFR volatility extrapolation and interpolation process, a detailed computation example is provided as below. The system date is *18 Nov 2021* and GBP SONIA CMP with *3-month* computation frequency is chosen. There are two cases to derive the pricing volatility used in adapted Bachelier Model for Caplet/Floorlet on RFR volatility.



Flow Tp	Calculation Start date	Calculation End date	Remaining Capital	Strike	Net Strike	Applicable Strike	Main Vol	First fixing Date	Last fixing Date	Rate
INT	01 Nov 2021	01 Feb 2022	1,000,000.00	0.35000	0.35000	0.35000	0.880	01 Nov 2021	31 Jan 2022	0.13004
INT	01 Feb 2022	03 May 2022	1,000,000.00	0.35000	0.35000	0.35000	1.504	01 Feb 2022	29 Apr 2022	0.45212
INT	03 May 2022	01 Aug 2022	1,000,000.00	0.35000	0.35000	0.35000	1.650	03 May 2022	29 Jul 2022	0.77161
INT	01 Aug 2022	01 Nov 2022	1,000,000.00	0.35000	0.35000	0.35000	1.720	01 Aug 2022	31 Oct 2022	1.02973
INT	01 Nov 2022	01 Feb 2023	1,000,000.00	0.35000	0.35000	0.35000	1.594	01 Nov 2022	31 Jan 2023	1.04612
INT	01 Feb 2023	02 May 2023	1,000,000.00	0.35000	0.35000	0.35000	1.478	01 Feb 2023	28 Apr 2023	1.16436
INT	02 May 2023	01 Aug 2023	1,000,000.00	0.35000	0.35000	0.35000	1.383	02 May 2023	31 Jul 2023	1.31946
INT	01 Aug 2023	01 Nov 2023	1,000,000.00	0.35000	0.35000	0.35000	1.309	01 Aug 2023	31 Oct 2023	1.47635

Figure 14: Cap Pricing Screen on GBP SONIA CMP.

- **Extrapolation of volatility before first quote pillar with RFR Decay**: If the last fixing date of caplet is before $t_{e,0}$, RFR decay extrapolation method would be used to extrapolate pricing volatility in Bachelier model. The extrapolated pricing volatility is derived as $\Sigma_{RFR}(T) = \sigma_{RFR,0}\sqrt{\frac{t_{e,0}}{T} \times \frac{f(t_s,T)}{f(t_{s,0},t_{e,0})}}$ where $f(t_s,t_e) = t_s^+ + \frac{1}{3} \times \frac{(t_e - t_s^+)^3}{\tau^2}$ and $t_s^+ = \max(t_s, 0)$.
  - $t_{s,0}$ is the last caplet of the first quoted cap/floor's calculation start date (*18 Aug 2022*).
  - $t_{e,0}$ is the last caplet of the first quoted cap/floor's calculation end date (*17 Nov 2022*).

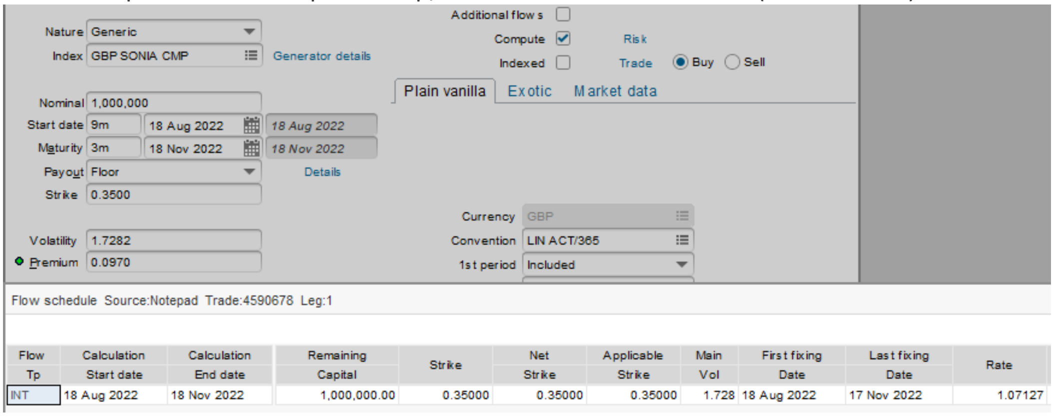


Figure 15: Last Caplet of the first quoted cap/floor's start, end, and last fixing date.

  - $\sigma_{RFR,0}$ is the forward-forward caplet volatility (1.728) derived from the bootstrapping process.
  - τ is the day count fraction which is based on calculation start (*18 Aug 2022*) and end date (*18 Nov 2022*).

For the caplet whose start date is 1 Aug 2022, end date 1 Nov 2022, and last fixing date is 31 Oct 2022, $f(t_s,t_e) = 0.782678$ and $f(t_{s,0},t_{e,0}) = 0.829253$. $\Sigma_{RFR}(T) = \sigma_{RFR,0}\sqrt{\frac{t_{e,0}}{T} \times \frac{f(t_s,T)}{f(t_{s,0},t_{e,0})}} = 1.728\sqrt{\frac{0.993567}{0.950685} \times \frac{0.782678}{0.829253}} = 1.7197.$

- **Interpolation of volatility after first quote pillar**: The market convention is to quote RFR caplet volatility in the last fixing date convention. Consequently, the volatility in the MX.3 should be interpolated in last fixing date convention. This setting can be enabled at *Static settings/Pricing/Risk-free rate cap volatility interpolation at* to update the value to "*Last fixing date*".

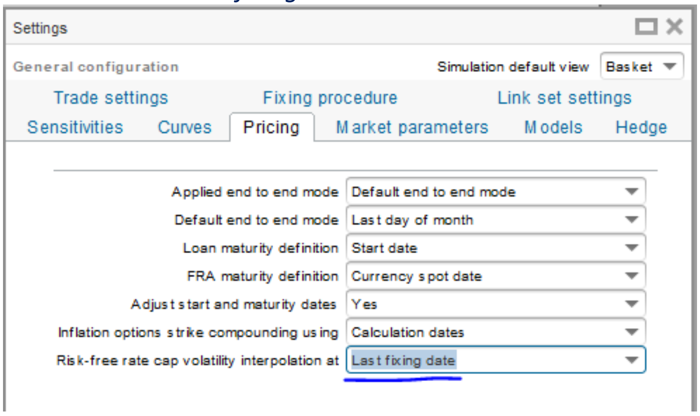


Figure 16: RFR cap interpolation based on Last fixing date setting.

  - The interpolation follows the volatility setting template in Cap/floor volatility model assignment. Typically, the smile interpolation is set to linear in strike scale and time interpolation is set to V2T or linearly. System first linearly interpolates in strike scale to derive volatility of strike on quoted pillar, $\sigma(T,K)$ and then system would use V2T or Linear method to derive the pricing volatility on last fixing date.

$$\boldsymbol{\Sigma_{RFR}(T,K)} = \begin{cases} \sigma(T_{pre},K) + \dfrac{\sigma(T_{post},K) - \sigma(T_{pre},K)}{T_{post} - T_{pre}} \times (T - T_{pre}), & \textit{time interpolation in Linear} \\ \sqrt{\dfrac{\sigma(T_{pre},K)^2 T_{pre} + \dfrac{\sigma(T_{post},K)^2 T_{post} - \sigma(T_{pre},K)^2 T_{pre}}{T_{post} - T_{pre}} \times (T - T_{pre})}{T}}, & \textit{time interpolation in V2T} \end{cases}$$

- **Apply Bachelier Pricing Formula**: Then, we can use $\Sigma_{RFR}(T)$ and its corresponding last fixing date, T to apply Bachelier pricing formula to derive the annualized coupon rate accordingly.

$$\rightarrow \boldsymbol{Coupon\ Rate = Bachelier(F, K, \Sigma_{RFR}(T), T)}$$

- **Drive the Flow and Sensitivity in flow details:** Then, we can use the formula below to derive the analytical sensitivity in the flow detail screen. More details regarding the definition and formulas for flow screen sensitivities are available in *docid: 973, section 3.2.1*.

Floating interest payments GBP

Rate	Margin	Rate Factor	Payment Date	Annualized coupon	Flows	Cur	Delta	Delta Csh	Gamma	Gamma Csh	Vega(%)	Vega Csh	Theta	Theta Pct
0.13004	0.0000	1.0000	01 Feb 2022	0.291822	735.55	GBP	-71.0578	-17.9105	0.8630	0.2175	0.0388	388.0066	-2.3080	-0.0000
0.45212	0.0000	1.0000	03 May 2022	0.350679	874.30	GBP	-45.9405	-11.4537	0.3962	0.0988	0.0658	658.2542	-3.0594	-0.0000
0.77161	0.0000	1.0000	01 Aug 2022	0.362704	894.34	GBP	-37.9425	-9.3557	0.2771	0.0683	0.0779	778.6880	-2.5470	-0.0000
1.02973	0.0000	1.0000	01 Nov 2022	0.383244	965.99	GBP	-34.2588	-8.6351	0.2192	0.0552	0.0898	897.7644	-2.2377	-0.0000
1.04612	0.0000	1.0000	01 Feb 2023	0.404055	1,018.44	GBP	-34.5269	-8.7027	0.2108	0.0531	0.1010	1,010.0845	-1.8499	-0.0000
1.16436	0.0000	1.0000	02 May 2023	0.374007	922.21	GBP	-32.3144	-7.9679	0.2024	0.0499	0.1051	1,050.7820	-1.4934	-0.0000
1.31946	0.0000	1.0000	01 Aug 2023	0.335809	837.22	GBP	-29.5306	-7.3624	0.1915	0.0478	0.1105	1,105.3036	-1.2508	-0.0000
1.47635	0.0000	1.0000	01 Nov 2023	0.300571	757.60	GBP	-26.8981	-6.7798	0.1805	0.0455	0.1141	1,140.5884	-1.0683	-0.0000

Figure 17: Flow Screen to check annualized coupon, flow, and sensitivities.

  - The annualized coupon is derived based on Bachelier formula as $\Sigma_{RFR}\sqrt{T}\left(dN(d) + N'(d)\right)$ and $\Sigma_{RFR}\sqrt{T}\left(N'(-d) - dN(-d)\right)$ for caplet and floorlet respectively.
  - Delta Cash: $\tau \times N(d)$ and $\tau \times (N(d) - 1)$ for caplet and floorlet respectively.
  - Gamma Cash: $\frac{\tau \times N'(d)}{\sigma\sqrt{t}}$ for both caplet and floorlet.
  - Theta: $-Notional \times \tau \times \frac{\sigma}{2\sqrt{t_s}} \times \frac{N'(d)}{365}$
  - Vega Cash: $\frac{\partial NPV}{\Sigma_{RFR}} = DF_{0,T} \times \tau \times \sqrt{t_s} \times N'(d)$
- **Compute Market Value and Sensitivity in Simulation**: The DV01 (zero) is first computed analytically and then DV01 (par) is derived from Jacobian Matrix, $\frac{\partial Z}{\partial R}$. The DV01 can be derived in simulation by shifting the market rate by 1bp from last pillar with backward cumulative method. The pricing is based on forward-forward volatility and if client inputs Par volatility, we cannot directly shift up calibrated forward-forward volatility. So, Vega should be validated through analytical formula.

We would support the standard sensitivity output field in the path below. More details regarding the sensitivities are available in docid: 973, section 3.2.1.

  - DV01 (Zero): *Risk Engine.Results.Outputs.Interest rates.Delta.Zero.Value*
  - DV01 (Par): *Risk Engine.Results.Outputs.Interest rates.Delta.Par.Value*
  - Normal Vega: *Risk Engine.Results.Outputs.Interest rates.Vega.Normal.Value*

There are a few settings which can impact the sensitivity results of cap/floor in simulation viewers:

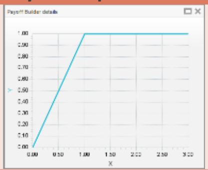
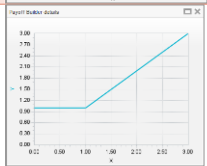
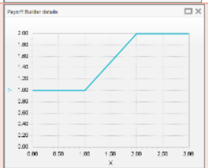
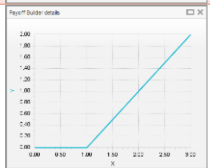
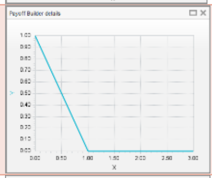
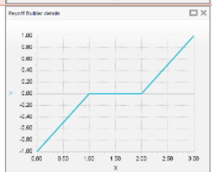
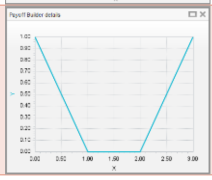
  - *General settings/Rates/Rates general settings/Cap/floors sensitivity = Analytical*: The setting controls how sensitivities are computed, and it is recommended to set as Analytical.
  - *General settings/Shared/Curves/Rate/Rate propagation=Keep market quotes constant:* Keep the market quotes constant by recomputing calibrated zero coupon rates.
  - *General settings/Shared/Curves/Rate/Rate/basis propagation and computations=Keep market spread constant/Impact sensitivities:* Both scenarios and sensitivities rely on keeping market quote constant.
  - *General settings/Shared/Models/Binary option smile drift/Call/puts spread and Strike gap=0.01*: The setting controls the pricing to digital caps and floors.
  - *General settings/Shared/Sensitivities/Vega breakdown uses=Volatility surface pillars*: The Vega sensitivities are projected to the volatility surface pillars. When it is set to “Sensitivities dates”, it would project the Vega corresponding to the Caplet/Floorlet pricing dates.

Appendix

Embedded Options

The embedded options features make it possible to add a cap or floor feature to any floating leg. This feature can be selected by modifying the embedded option field or by opening exotic window and setting the embedded option flag to the desired case. The rate factor and margin can be applied to the floating index. Its handling can be summarized as below:

- $Caplet(K) = Max(RateFactor \times Rate + Margin - K, 0) =$
$$\begin{cases} RateFactor \times Max\left(Rate - \frac{K-Margin}{Rate\ Factor}, 0\right) = RateFactor \times Caplet(\frac{K-Margin}{Rate\ Factor}), & RateFactor > 0 \\ -RateFactor \times Max\left(\frac{K-Margin}{Rate\ Factor} - Rate, 0\right) = -RateFactor \times Floorlet(\frac{K-Margin}{Rate\ Factor}), & RateFactor < 0 \end{cases}$$
- $Floorlet(K) = Max(K - RateFactor \times Rate - Margin, 0) =$
$$\begin{cases} RateFactor \times Max\left(\frac{K-Margin}{Rate\ Factor} - Rate, 0\right) = RateFactor \times Floorlet(\frac{K-Margin}{Rate\ Factor}), & RateFactor > 0 \\ -RateFactor \times Max\left(Rate - \frac{K-Margin}{Rate\ Factor}, 0\right) = -RateFactor \times Caplet(\frac{K-Margin}{Rate\ Factor}), & RateFactor < 0 \end{cases}$$
- $\frac{K-Margin}{Rate\ Factor}$ is applicable strike used to interpolate/extrapolate the pricing volatility.

Option Type	Payoff	Option Breakdown	Payoff Graph
Capped Rate	Payoff = Min (Rate, Strike)	**Strike - Max (Strike - Rate, 0) = Strike - Floorlet**	
Floored Rate	Payoff = Max (Rate, Strike)	**Strike + Max (Rate – Strike, 0) = Strike + Caplet**	
Collared Rate	If Rate > Strike2, Payoff = Strike 2 If Rate < Strike1, Payoff = Strike1 Else, Payoff = Rate	**Strike1 + Caplet (Strike1) – Caplet (Strike2)**	
Cap	Payoff = Max (Rate – Strike, 0)	**Caplet (Strike)**	
Floor	Payoff = Max (Strike – Rate, 0)	**Floorlet (Strike)**	
Collar	If Rate > Strike2, Payoff = Rate - Strike 2 If Rate < Strike1, Payoff = Rate - Strike1 Else, Payoff = 0	**-Floorlet (Strike1) + Caplet (Strike2)**	
Strangle	If Rate > Strike2, Payoff = Rate - Strike 2 If Rate < Strike1, Payoff = Strike1 - Rate Else, Payoff = 0	**Floorlet (Strike1) + Caplet (Strike2)**	

Current Index Estimation Mode

Due to the settlement process, market practitioners require a short delay between the last fixing of the compounded rate observation and actual coupon payment. There are a few flavors to derive daily compounded rate:

- Payment delay: The interest rate payments are delayed by X days after the end of an interest period.
- Lockout: The RFR is no longer updated but frozen for X days prior to the end of the accrual period.
- Observation Shift: both RFR rate as well as day weight shift back X days to earlier observation period.
- Lookback: Shift RFR rate back X days but with day weight remaining against the interest period.

Although SONIA adopts lookback method, it seems that SOFR leans toward observation shift convention. The difference between Observation shift and lookback is due to day weighting in interest periods or observation period. With 5 business days shift, the observation shift and lookback are different when there is a holiday in the period.

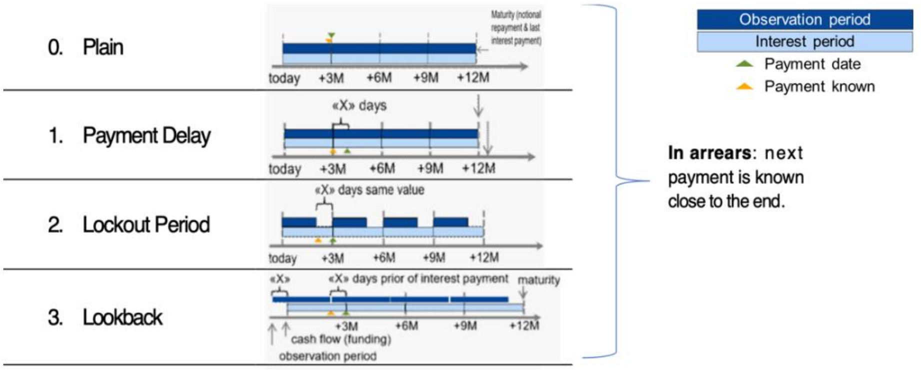


Figure 18: Compound RFR index flavors.

There are two modes, underlying and current index mode in MX.3 to estimate compounded index. The setting is available under Index definition under *Estimation mode*.

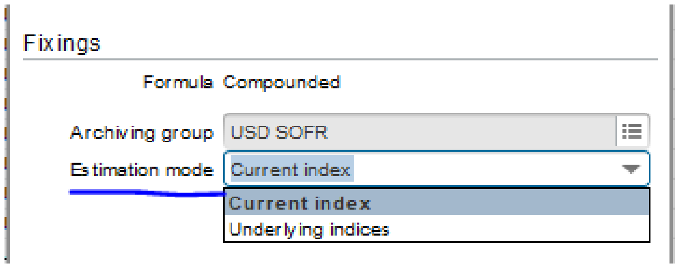


Figure 19: Compound index estimation mode setting.

With underlying index mode, the overnight rate, $F(t_i, t_{i+1})$ is estimated by discount factor from t_i to t_{i+1} as $\frac{1}{\tau_i} \times (\frac{DF_{0,t_i}}{DF_{0,t_{i+1}}} - 1)$. The compounded rate is then computed as $R(t_0, t_n) = \frac{1}{\tau}\left[\prod_{i=0}^{n-1}\left(1 + \tau_i F(t_i,\ t_{i+1})\right) - 1\right]$.

With current index mode, the calculation is simpler, and it allows better performance. It approximates the compounded index period from t_s to T by $\frac{1}{\tau}(\frac{DF_{0,t_s}}{DF_{0,T}} - 1)$. The approximation period (t_s and T) can be accessed by right click to view compound fixing details' estimation period. The rational is that the compounded rate can be approximated as $\frac{1}{\tau}\left[\prod_{i=0}^{n-1}\left(1 + \tau_i F(t_i,\ t_{i+1})\right) - 1\right] = \frac{1}{\tau}\left[\left(\frac{DF_0}{DF_1}\right) * \left(\frac{DF_1}{DF_2}\right) * \cdots * \left(\frac{DF_{t_{n-1}}}{DF_{t_n}}\right) - 1\right] = \frac{1}{\tau}(\frac{DF_{t_s}}{DF_{t_n}} - 1)$.

There are two limitations in current index estimation mode:

- When fixing is on Friday, it should cover the weight of 3 days practically when computing compound RFR rate. This day weight practice on Friday is not considered in current index mode. Practically, this difference is very small in terms of compound RFR rate estimation.
- When the day weight and observation period are not aligned, there could be difference in terms of fixing rate and its corresponding day weight. This limitation is mainly on the lookback flavor.