CLOSED-FORM FORMULA PRICING

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Linear IRD – The building blocks

Zero coupon bonds:

- The basic building block is the ZC bond, its price is tractable under HW.
- Denoting x_t^j the value of x at (t,j), the ZC price is:

$$P(t,T,x_t^j) = \frac{P^M(0,T)}{P^M(0,t)} \exp\left(A(t,T) - B(t,T)x_t^j\right)$$

- where:

$$B(t,T) = \frac{1-e^{-a(T-t)}}{a}$$

$$A(t,T) = -\frac{1}{2}\theta(t)B(t,T)^2$$

  - With $P^M(0,u) = e^{-R^M(0,u)\times u}$ is read from the market's curve.

LIBOR estimation:

- The value at (t,j) of LIBOR index with start date s, end date e and tenor δ can be deduced from the zero coupon prices:

$$\frac{P\left(t,s,x_t^j\right)}{P\left(t,e,x_t^j\right)} = 1+\delta\times L\left(t,s,e,x_t^j\right) \Leftrightarrow L\left(t,s,e,x_t^j\right) = \frac{1}{\delta}\left(\frac{P\left(t,s,x_t^j\right)}{P\left(t,e,x_t^j\right)} - 1\right)$$

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Linear IRD – Interest rates swaps

- These two building blocks give us the price of the IRS (e.g. received):

$$MV(t) = MV_{\text{fixed leg}}(t) - MV_{\text{floating leg}}(t)$$

- The fixed leg price is given by the keyword `IRFixedStream(amounts, payment_dates)`:

$$MV_{\text{fixed leg}}(t) = \sum_{k=1}^{\text{fixed flows}} \underbrace{N \times DCF_k \times SR}_{\text{amounts}} \times P\left(t, T_k^p\right)$$

- While the floating leg price is given by `IRLiborStream(amounts, payment_dates, libors_info, forward_curve)`:

$$MV_{\text{floating leg}}(t) = \sum_{k=1}^{\text{floating flows}} \underbrace{N \times DCF_k}_{\text{amounts}} \times L(t, T_{k-1}, T_k) \times P\left(t, T_k^p\right)$$

- For a vanilla IRS, two different curves can be involved:
  - $L(t, T_{k-1}, T_k)$ are computed using the forward estimation curve;
  - $P(t, T_k)$ are computed using the discount curve.

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Optional IRD – Options on ZC bond

Options on ZC bond:

- In HW framework, The ZC bond price is lognormal;
- As a result, the ZC bond option can be calculated analytically using Black formula using two inputs:
  - The forward ZC bond price:

$$F(t,T_1,T_2) = \frac{P(t,T_2)}{P(t,T_1)}$$

  - The forward ZC bond variance is given by:

$$\Sigma(t,T_1,T_2)^2 = B(T_1,T_2)^2 \int_t^{T_1} e^{-2a(T_1-u)}\sigma^2(u)du$$

- The ZC bond option with strike K is then:

$$ZCBondOption = P(t,T_1) * BlackForward(F(t,T_1,T_2),\Sigma(t,T_1,T_2),K)$$

  - Where:

$$BlackForward(F,\Sigma,K) = \eta FN(\eta d_+) - \eta KN(\eta d_-)$$

  - N is the standard cumulative gaussian, η is a cofficient equal to 1 for a call and -1 for a put, and:

$$d_\pm = \frac{\ln\left(\frac{F}{K}\right)}{\Sigma} \pm \frac{\Sigma}{2}$$

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Optional IRD – Cap / Floors

Cap / Floors are strips of ZC bond call / put options:

- A cap / floor is a strip of caplets / floorlets.
- The value of each caplet, with fixing date T_f, payment date T_p, nominal N and strike K is:

$$MV(t) = N \times \delta \times \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{T_p} r(s)ds}\left(L(T_f, T_p) - K\right)^+ | \mathcal{F}_t\right]$$

  - Remark: this is the vanilla case where the payment date and the libor end date are the same.

- Switching to the T_f forward measure :

$$MV(t) = N \times \delta \times P(t, T_f) \times \mathbb{E}^{T_f}\left[P(T_f, T_p)\left(L(T_f, T_p) - K\right)^+ | \mathcal{F}_t\right]$$

Where $P(t, T)$ is the zero coupon using the discount curve.

- Replacing the LIBOR by its definition: $L(T_f, T_p) = \frac{1}{\delta}\left(\frac{1}{P^L(T_f, T_p)} - 1\right)$ in the payoff leads to:

$$P(T_f, T_p)\left(L(T_f, T_p) - K\right)^+ = \frac{1}{\delta}\left(\frac{P(T_f, T_p)}{P^L(T_f, T_p)} - P(T_f, T_p) - \delta K P(T_f, T_p)\right)^+ = \frac{(1 + \delta K)}{\delta}\frac{P(0, T_p)P^L(0, T_f)}{P(0, T_f)P^L(0, T_p)}\left(\frac{1}{1 + \delta K} - P^L(T_f, T_p)\right)^+$$

Where $P^D(t, T)$ is the zero coupon using the estimation curve (Libor curve).

- The caplet / floorlet is equivalent to $(1 + \delta K)\frac{P(0, T_p)P^L(0, T_f)}{P(0, T_f)P^L(0, T_p)}$ ZC bond put / call options, for which we have an analytic formula under HW:

$$MV(t) = N(1 + \delta K)\frac{P(0, T_p)P^L(0, T_f)}{P(0, T_f)P^L(0, T_p)}P(t, T_f)BlackForwardPut\left(F^L(t, T_f, T_p), \frac{1}{1 + \delta K}, \Sigma(t, T_f, T_p)^2\right)$$

Where $F^L(t, T_1, T_2)$ is the forward zero coupon (using Libor curve) starting at T_1 and maturing at T_2 seen from date t.

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