Swaption Pricing - Table of contents

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Swaptions are OTC interest rate trades. To be more specific, they are options on Interest Rate Swaps (IRS). Therefore, the **Interest rate swap** documentation is a prerequisite to this reading.
This document describes the Swaption product and explains how is it priced in MX.3.

1 - Product definition

To explain the Swaptions product, the following notations are used throughout the document:

- Let N be the Nominal (also called Notional) of the swaption (which is also the nominal of the underlying swap).
- Let τ_i be the year fraction of the i^{th} period of the ***floating*** leg. A year fraction is the time length (expressed in years) of a period.
- Let $\tilde{\tau}_i$ be the year fraction of the i^{th} period of the ***fixed*** leg. A year fraction is the time length (expressed in years) of a period.
- Let T_i be the start date of the $(i+1)^{th}$ period and the end date of the i^{th} period for the ***floating*** leg.
- Let $\tilde{T}_i$ be the start date of the $(i+1)^{th}$ period and the end date of the i^{th} period for the ***fixed*** leg.
- Let T be the maturity of the option. Of course the option must end before the start date of the first period of the underlying swap (which means $T <= T_0$).
- Let t be today.
- Let $DF(t,T_i)$ (respectively, $DF(t,\tilde{T}_i)$) be a discount factor between t and T_i (respectively $\tilde{T}_i$).
- Let K be the fixed rate of the option. K is also called Strike of the swaption and is also the fixed rate of the underlying swap.
- Let $F_i(T_{i-1},T_i)$ be the estimation (as of t) of the rate of the period defined by T_{i-1} and T_i . For the sake of brevity, we will often use $F_i(t) = F_i(T_{i-1},T_i)$. When the rate has been fixed, we will use the notation L_i .
- Let x be a double $(x)^+ = max(0,x)$.
- Let σ be a volatility. As MX.3 handles three different diffusion models, we will use three different notations:
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    - σ_{LN} for the log-normal model,
    - σ_{SLN} for the shifted log-normal model and
    - σ_N for the normal model.
- Let $W(.)$ be a standard Brownian motion.
- Let $\mathcal{N}(.)$ be the standard normal distribution function.

The computation of $F_i(t)$ is detailed in the documentation **Interest rate indices** .

1.1 - Description

A Swaption is an option on a IRS (interest rate swap): it gives the holder the right to enter into an IRS. This underlying product is explained in the **Interest rate swap** documentation. As a reminder, the NPV of a swap can be written as below:

- In the Linear convention :
-
    - $$NPV_{IRS}(t) = N \times \sum_{i=1}^{n} \tau_i \times F_i(t) \times DF(t,T_i) - N \times \sum_{i=1}^{m} \tilde{\tau}_i \times K \times DF(t,\tilde{T}_i)$$ for a payer swap (because the fixed rate is paid).

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  - $NPV_{IRS}(t) = N \times \sum_{i=1}^{n} \tau_i \times F_i(t) \times DF(t, T_i) - N \times \sum_{i=1}^{m} \tilde{\tau}_i \times K \times DF(t, \tilde{T}_i)$ for a payer swap (because the fixed rate is paid).
  - $NPV_{IRS}(t) = N \times \sum_{i=1}^{m} \tilde{\tau}_i \times K \times DF(t, \tilde{T}_i) - N \times \sum_{i=1}^{n} \tau_i \times F_i(t) \times DF(t, T_i)$ for a receiver swap (because the fixed rate is received).
- In the Yield convention :
  - $NPV_{IRS}(t) = N \times \sum_{i=1}^{n} (1 + F_i(t))^{\tau_i} \times DF(t, T_i) - N \times \sum_{i=1}^{m} (1 + K)^{\tilde{\tau}_i} \times DF(t, \tilde{T}_i)$ for a payer swap.
  - $NPV_{IRS}(t) = N \times \sum_{i=1}^{m} (1 + K)^{\tilde{\tau}_i} \times DF(t, \tilde{T}_i) - N \times \sum_{i=1}^{n} (1 + F_i(t))^{\tau_i} \times DF(t, T_i)$ for a receiver swap.
- In the Exponential convention :
  - $NPV_{IRS}(t) = N \times \sum_{i=1}^{n} e^{\tau_i \times F_i(t)} \times DF(t, T_i) - N \times \sum_{i=1}^{m} e^{\tilde{\tau}_i \times K} \times DF(t, \tilde{T}_i)$ for a payer swap.
  - $NPV_{IRS}(t) = N \times \sum_{i=1}^{m} e^{\tilde{\tau}_i \times K} \times DF(t, \tilde{T}_i) - N \times \sum_{i=1}^{n} e^{\tau_i \times F_i(t)} \times DF(t, T_i)$ for a receiver swap.

Since the three native models implemented (for the swaptions) in MX.3 apply only for the Linear convention, the following pricing formulas are restricted to the linear model.

Whenever the rate convention is different from the linear convention, MX.3 prices the option as if the convention is linear.

Now, defining the swap annuity as $A_S(t) = \sum_{i=1}^{n} \tilde{\tau}_i \times DF(t, \tilde{T}_i)$ and the par swap rate (also called swap rate) as $S(t) = \frac{\sum_{i=1}^{n} \tau_i \times F_i(t) \times DF(t, T_i)}{A_S(t)}$, the NPV of the underlying swap (in linear convention only) can be expressed as follows:

- $NPV_{IRS}(t) = N \times A_S(t) \times \Big(S(t) - K\Big)$ for a Payer swap.
- $NPV_{IRS}(t) = N \times A_S(t) \times \Big(K - S(t)\Big)$ for a Receiver swap.

Coming back to the Swaption, its payoff at maturity is given by:

- $NPV_{swaption}(T) = \Big(NPV_{IRS}(T)\Big)^+ = N \times A_S(T) \times \Big(S(T) - K\Big)^+$ for a payer swaption. Which means a Payer swaption is a call option.
- $NPV_{swaption}(T) = \Big(NPV_{IRS}(T)\Big)^+ = N \times A_S(T) \times \Big(K - S(T)\Big)^+$ for a receiver swaption. Which means a Receiver swaption is a put option.

To define a swaption in MX.3 a few steps only are required. From the e-Tradepad, the user simply needs to:

- $NPV_{swaption}(T) = \left(NPV_{IRS}(T)\right)^{+} = N \times A_S(T) \times \left(K - S(T)\right)^{+}$ for a receiver swaption. Which means a Receiver swaption is a put option.

To define a swaption in MX.3 a few steps only are required. From the e-Tradepad, the user simply needs to:

- Select a generator
- Input the Nominal
- Input the option Maturity
- Input the underlying swap maturity (the underlying swap starts at option maturity)
- Input the Strike

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1.2 - Pricing models

Since swaptions are options, we need a diffusion model to be able to price them.
Three vanilla diffusion models are implemented in MX.3: the following sections **1.2.1** , **1.2.2** and **1.2.3** describe these models and the pricing formulas.
In addition, more details about the log-normal and the shifted lognormal models can be found in the **Shifted log-normal model for interest Rate Derivatives** documentation.

1.2.1 - Log-normal model

The Log-normal model (also known as Black model) is the following diffusion model:
$dS(t) = \sigma_{LN} \times S(t) \times dW(t)$
Which leads to the following pricing formulas:

- For a payer swaption: $NPV(t) = N \times A_S(t) \times \left(S(t) \times \mathcal{N}(d_1) - K \times \mathcal{N}(d_2)\right)$
- For a receiver swaption: $NPV(t) = N \times A_S(t) \times \left(K \times \mathcal{N}(-d_1) - S(t) \times \mathcal{N}(-d_2)\right)$

where d_1 and d_2 are defined as follows:

- $d_1 = \dfrac{ln(S(t)/K)}{\sigma_{LN} \times \sqrt{T-t}} + \dfrac{1}{2} \times \sigma_{LN} \times \sqrt{T-t}$
- $d_2 = \dfrac{ln(S(t)/K)}{\sigma_{LN} \times \sqrt{T-t}} - \dfrac{1}{2} \times \sigma_{LN} \times \sqrt{T-t}$

The way the volatility is retrieved is explained in section **5** .

- $d_2 = \frac{ln(S(t)/K)}{\sigma_{LN} \times \sqrt{T-t}} - \frac{1}{2} \times \sigma_{LN} \times \sqrt{T-t}$

The way the volatility is retrieved is explained in section **5** .

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1.2.2 - Shifted log-normal model

The shifted log-normal model is an extension of the log-normal model. The log normal model prevents the rate from being negative. To release this constraint, the shifted log-normal model introduces a new parameter known as the shift (let α be this constant so that $\alpha > 0$) . Introducing this shift allows us to have $S(t) > -\alpha$. Then this model assumes the following diffusion model:
$dX(t) = \sigma_{SLN} \times X(t) \times dW(t)$, where $X(t) = S(t) + \alpha$.
Which leads to the following pricing formulas:

- For a payer swaption: $NPV(t) = N \times A_S(t) \times \Big((S(t)+\alpha) \times \mathcal{N}(d_1) - (K+\alpha) \times \mathcal{N}(d_2) \Big)$
- For a receiver swaption: $NPV(t) = N \times A_S(t) \times \Big((K+\alpha) \times \mathcal{N}(-d_1) - (S(t)+\alpha) \times \mathcal{N}(-d_2) \Big)$

where d_1 and d_2 are defined as follows:

- $d_1 = \frac{ln(\frac{S(t)+\alpha}{K+\alpha})}{\sigma_{LN} \times \sqrt{T-t}} + \frac{1}{2} \times \sigma_{LN} \times \sqrt{T-t}$
- $d_2 = \frac{ln(\frac{S(t)+\alpha}{K+\alpha})}{\sigma_{LN} \times \sqrt{T-t}} - \frac{1}{2} \times \sigma_{LN} \times \sqrt{T-t}$

The way the volatility is retrieved is explained in section **5**

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1.2.3 - Normal model

The normal model has also been introduced to release the constraint of the log-normal model (non-negatives rates). This model allows us to have $S(t) \in \mathbb{R}$ and assumes the following diffusion model:
$dS(t) = \sigma_N \times dW(t)$
Which leads to the following pricing formulas:

- For a payer swaption: $NPV(t) = N \times A_S(t) \times \sigma_N \times \sqrt{T-t} \times \Big(d_N \times \mathcal{N}(d_N) + \mathcal{N}'(d_N) \Big)$

$NPV(t) = N \times A_S(t) \times \sigma_N \times \sqrt{T-t} \times \Big(\mathcal{N}'(-d_N) - d_N \times \mathcal{N}(-d_N) \Big)$

- $d_2 = \frac{}{\sigma_{LN} \times \sqrt{T-t}} - \frac{1}{2} \times \sigma_{LN} \times \sqrt{T-t}$

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$dS(t) = \sigma_N \times dW(t)$
Which leads to the following pricing formulas:

- For a payer swaption: $NPV(t) = N \times A_S(t) \times \sigma_N \times \sqrt{T-t} \times \left(d_N \times \mathcal{N}(d_N) + \mathcal{N}'(d_N)\right)$
- For a receiver option: $NPV(t) = N \times A_S(t) \times \sigma_N \times \sqrt{T-t} \times \left(\mathcal{N}'(-d_N) - d_N \times \mathcal{N}(-d_N)\right)$

Where $d_N = \frac{S(t) - K}{\sigma_N \times \sqrt{T-t}}$
The way the volatility is retrieved is explained in section **5** .

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2 - Trade Ticket

In the application, the user enters and prices a new deal through the **Trade ticket** (located in the e-Tradepad). Figure **1** shows a Swaption trade ticket.
This section explains which information the user must input and which information and figures are given by the Trade ticket.

2.1 - Standard swaptions

This section describes the fields in the trade ticket that are used directly in the calculation of the trade NPV.

2.1.1 - Deal screen entry

The swaption is priced in MX.3 in the e-Tradepad: ***Pricing / e-Tradepad*** :