Applications of Stochastic Calculus in Finance Chapter 3: Heath-Jarrow-Morton (HJM) methodology

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1 Dynamics of Forward Rates and Zero-coupon Bond Prices

Fix a filtered probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbf{P})$, which satisfies the usual conditions, and supports a d-dimensional Brownian motion $W = (W^1, \dots, W^d)^T$.

In forward rate models (HJM framework), we mainly model the dynamics of forward rates. The traded assets are the bank account and zero-coupon bonds with different maturities.

Assumption 1 *The forward rate follows SDE*

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW_t$$

which determines the zero-coupon bond price $P(t,T) = e^{-\int_t^T f(t,s)ds}$*. Moreover, the* $\mathbb{R}$*-valued drift* $\alpha(t,T)$ *and the* $\mathbb{R}^d$*-valued volatility* $\sigma(t,T) = (\sigma^1(t,T), \dots, \sigma^d(t,T))$ *are*

(1) both progressively measurable and smooth in T*; and*

(2) for any $T > 0$,

$$\int_0^T \int_0^T |\alpha(t,s)| dt ds < \infty; \quad \sup_{0\leq t\leq s\leq T} |\sigma(t,s)| < \infty.$$

Since $P(t,T) = e^{-\int_t^T f(t,u)du}$, we can derive the dynamics of $P(t,T)$ as follows.

Proposition 1. *For any* $T > 0$*, the zero-coupon bond price* $P(t,T)$ *follows*

$$P(t,T) = P(0,T) + \int_0^t P(s,T)(r_s + \alpha^*(s,T) + \frac{1}{2}|\sigma^*(s,T)|^2)ds + \int_0^t P(s,T)\sigma^*(s,T)dW_s$$

for $t \in [0,T]$*, where* $\sigma^*(s,T) = -\int_s^T \sigma(s,u)du$*, and* $\alpha^*(s,T) = -\int_s^T \alpha(s,u)du$.

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Proof. Note that

$$d(-\int_t^T f(t,s)ds) = f(t,t)dt - \int_t^T df(t,s)ds$$
$$= r_t dt - \int_t^T (\alpha(t,s)dt + \sigma(t,s)dW_t)ds.$$

Now the (stochastic) Fubini's theorem (see Filipovic [1] Chapter 6.5) implies that

$$d(-\int_t^T f(t,s)ds) = r_t dt - \int_t^T \alpha(t,s)dsdt - \int_t^T \sigma(t,s)dsdW_t$$
$$= r_t dt + \alpha^*(t,T)dt + \sigma^*(t,T)dW_t.$$

Applying Itô's formula to $e^{-\int_t^T f(t,s)ds}$ gives us

$$dP(t,T) = de^{-\int_t^T f(t,s)ds} = e^{-\int_t^T f(t,s)ds}d(-\int_t^T f(t,s)ds) + \frac{1}{2}e^{-\int_t^T f(t,s)ds}d\langle -\int_\cdot^T f(\cdot,s)ds\rangle_t$$
$$= P(t,T)(r_t dt + \alpha^*(t,T)dt + \sigma^*(t,T)dW_t) + \frac{1}{2}P(t,T)|\sigma^*(t,T)|^2 dt$$
$$= P(t,T)\left(r_t + \alpha^*(t,T) + \frac{1}{2}|\sigma^*(t,T)|^2\right)dt + P(t,T)\sigma^*(t,T)dW_t.$$

□

Proposition 2. *For any $T > 0$, the discounted zero-coupon bond price $P(t,T)/B_t$ follows*

$$\frac{P(t,T)}{B_t} = P(0,T) + \int_0^t \frac{P(s,T)}{B_s}(\alpha^*(s,T) + \frac{1}{2}|\sigma^*(s,T)|^2)ds + \int_0^t \frac{P(s,T)}{B_s}\sigma^*(s,T)dW_s$$

for $t \in [0,T]$.

Proof. Since $B_t = e^{\int_0^t r_s ds}$, apply Itô's formula to $P(t,T)e^{-\int_0^t r_s ds}$,

$$d\frac{P(t,T)}{B_t} = e^{-\int_0^t r_s ds}dP(t,T) - r_t e^{-\int_0^t r_s ds}P(t,T)dt$$
$$= \frac{P(t,T)}{B_t}(\alpha^*(t,T) + \frac{1}{2}|\sigma^*(t,T)|^2)dt + \frac{P(t,T)}{B_t}\sigma^*(t,T)dW_t. \quad \square$$

2 Absence of Arbitrage: HJM Drift Condition

Different from shot-rate models, where we have to exogenously specify the market price of risk Θ, we would endogenously determine the market price of risk in forward rate models, which is in line with the arbitrage theory in the Black-Scholes model.

Theorem 1. *(HJM drift condition)*
There exists an ELMM $\mathbf{Q}$ *iff the HJM drift condition is satisfied:*

$$\sigma^*(t,T)\Theta_t = \alpha^*(t,T) + \frac{1}{2}|\sigma^*(t,T)|^2$$

for any $T > 0$, $d\mathbf{P}\otimes dt$ *a.s., where* Θ_t *is called the market price of risk.*
In this case, the $\mathbf{Q}$*-dynamic of the forward rate* $f(t,T)$ *follows*

$$f(t,T) = f(0,T) - \int_0^t \sigma(s,T)(\sigma^*(s,T))^T ds + \int_0^t \sigma(s,T) dW_s^{\mathbf{Q}},$$

and the discounted T*-bond price is a* $\mathbf{Q}$*-local martingale and follows*

$$d\frac{P(t,T)}{B_t} = \frac{P(t,T)}{B_t}\sigma^*(t,T)dW_t^{\mathbf{Q}}.$$

Proof. From Proposition 2, we have

$$\begin{aligned} d\frac{P(t,T)}{B_t} &= \frac{P(t,T)}{B_t}\left[(\alpha^*(t,T) + \frac{1}{2}|\sigma^*(t,T)|^2)dt + \sigma^*(t,T)dW_t\right] \\ &= \frac{P(t,T)}{B_t}\sigma^*(t,T)\left(\Theta_t dt + dW_t\right), \end{aligned}$$

where Θ_t solves the market price of risk equation (HJM drift condition).

If $\mathbf{E}^{\mathbf{P}}[e^{\frac{1}{2}\int_0^T |\Theta_s|^2 ds}] < \infty$, then the stochastic exponential $\mathscr{E}(-\int_0^\cdot \Theta_s dW_s)$ is a martingale up to T by Novikov's condition. Define a new probability measure $\mathbf{Q}\sim\mathbf{P}$ by the Radon-Nikodym density:

$$\left.\frac{d\mathbf{Q}}{d\mathbf{P}}\right|_{\mathscr{F}_t} = \mathscr{E}(-\int_0^\cdot \Theta_s dW_s)_t.$$

By Girsanov's theorem, $W_t^{\mathbf{Q}} = W_t + \int_0^t \Theta_s ds$, $t\in[0,T]$, is a d-dimensional Brownian motion under $\mathbf{Q}$, and the discounted T-bond price $\frac{P(t,T)}{B_t}$ is a $\mathbf{Q}$-local martingale.

Next, we differentiate both sides of the HJM drift condition in T,

$$\partial_T\left(-\int_t^T \sigma(t,u)du\right)\Theta_t = \partial_T\left(-\int_t^T \alpha(t,u)du\right) + \frac{1}{2}\partial_T\left|\int_t^T \sigma(t,u)du\right|^2$$

which gives us

$$\begin{aligned} -\sigma(t,T)\Theta_t &= -\alpha(t,T) + \sigma(t,T)\left(\int_t^T \sigma(t,u)du\right)^T \\ &= -\alpha(t,T) - \sigma(t,T)(\sigma^*(t,T))^T. \end{aligned}$$

Therefore, by using the Girsanov's theorem again, we obtain the $\mathbf{Q}$-dynamic of the forward rate $f(t,T)$ as

$$\begin{aligned} df(t,T) &= \alpha(t,T)dt + \sigma(t,T)dW_t \\ &= -\sigma(t,T)(\sigma^*(t,T))^T dt + \sigma(t,T)(\Theta_t dt + dW_t) \\ &= -\sigma(t,T)(\sigma^*(t,T))^T dt + \sigma(t,T)dW_t^{\mathbf{Q}}. \quad \square \end{aligned}$$

We make some comments on the HJM drift condition. From first fundamental theorem of asset pricing,

if the HJM drift condition is satisfied, then there exists an ELMM, and therefore, the market is arbitrage-free.

Secondly, the HJM drift condition can be written component-wisely as

$$\sum_{j=1}^{d} \sigma^{*,j}(t,T)\Theta_t^j = \alpha^*(t,T) + \frac{1}{2}\sum_{j=1}^{d}\left(\sigma^{*,j}(t,T)\right)^2$$

or differentiating in T,

$$\sum_{j=1}^{d} \sigma^{i}(t,T)\Theta_t^i = \alpha(t,T) + \sum_{j=1}^{d}\sigma^{j}(t,T)\sigma^{*,j}(t,T).$$

for any $T > 0$. So the HJM drift condition represents infinitely many equation, one for each $T > 0$. For example, if in the market we are only given zero-coupon bonds with maturities $T_1, T_2, \ldots, T_n$ such that $(\sigma^j(t,T_i))_{1\le i\le n, 1\le j\le d}$ is a $n \times d$ matrix with rank d, then Θ_t is unique determined. From second fundamental theorem of asset pricing, the market is complete, so we can discuss about hedging (see Section 5).

Proposition 3. *Suppose that the HJM drift condition is satisfied. Then the ELMM* $\mathbf{Q}$ *is an EMM if either*

(1) the Novikov's condition $\mathbf{E}^{\mathbf{Q}}[e^{\frac{1}{2}\int_0^T |\sigma^*(s,T)|^2 ds}] < \infty$ *holds; or*

(2) the forward rate is nonnegative: $f(t,T) \ge 0$*.*

Proof. (1) Note that the discounted T-bond price is a stochastic exponential:

$$\frac{P(t,T)}{B_t} = P(0,T)\mathscr{E}\left(\int_0^{\cdot} \sigma^*(s,T)dW_s^{\mathbf{Q}}\right)_t.$$

(2) If $f(t,T) \ge 0$, then $P(t,T) = e^{-\int_t^T f(t,s)ds} \in [0,1]$, and

$$B_t = e^{\int_0^t r_s ds} = e^{\int_0^t f(s,s)ds} \in [1,\infty).$$

Hence, $\frac{P(t,T)}{B_t} \in [0,1]$. Since a bounded local martingale is a martingale, (2) is proved. $\square$

Hence, we see that the no-arbitrage assumption in short rate model is indeed satisfied if we impose the conditions in the above Proposition 3.

The summary of forward-rate models: the dynamics of the forward rate $f(t,T)$ are

$$
\begin{aligned}
df(t,T) &= \alpha(t,T)dt + \sigma(t,T)dW_t \quad \text{under } \mathbf{P} \\
&= -\sigma(t,T)(\sigma^*(t,T))^T dt + \sigma(t,T)dW_t^{\mathbf{Q}} \quad \text{under } \mathbf{Q}.
\end{aligned}
$$

The dynamics of the zero-coupon bond price $P(t,T)$ are

$$
\begin{aligned}
\frac{dP(t,T)}{P(t,T)} &= r_t dt + \sigma^*(t,T)dW_t^{\mathbf{Q}} \quad \text{under } \mathbf{Q} \\
&= (r_t + \sigma^*(t,T)\Theta_t)dt + \sigma^*(t,T)dW_t \quad \text{under } \mathbf{P}.
\end{aligned}
$$

Compare the above with the dynamics of the zero-coupon bond price in short-rate models.

3 The Corresponding Short-rate Dynamics

Given the dynamics of the forward rate $f(t,T)$, we can also recover the dynamics of the corresponding short rate by using $r_t = f(t,t)$.

Proposition 4. *Suppose that Assumption 1 is satisfied when $\alpha(t,T)$ and $\sigma(t,T)$ are replaced by $\partial_T\alpha(t,T)$ and $\partial_T\sigma(t,T)$ respectively. Moreover, $\int_0^T |\partial_u f(0,u)|du < \infty$. Then the short rate $r_t = f(t,t)$ follows*

$$
r_t = r_0 + \int_0^t \bar{\alpha}_u du + \int_0^t \sigma(u,u)dW_u
$$

where

$$
\bar{\alpha}_u = \alpha(u,u) + \partial_u f(0,u) + \int_0^u \partial_u\alpha(s,u)ds + \int_0^u \partial_u\sigma(s,u)dW_s.
$$

Proof. Recall that

$$
r_t = f(t,t) = f(0,t) + \int_0^t \alpha(s,t)ds + \int_0^t \sigma(s,t)dW_s.
$$

Note that

$$
f(0,t) = r_0 + \int_0^t \partial_u f(0,u)du.
$$

Moreover, by the (stochastic) Fubini's theorem (see Filipovic [1] Chapter 6.5),

$$\begin{aligned}\int_0^t \alpha(s,t)ds &= \int_0^t \alpha(s,s)ds + \int_0^t \int_s^t \partial_u \alpha(s,u)duds \\ &= \int_0^t \alpha(s,s)ds + \int_0^t \int_0^t \mathbf{1}_{\{u\geq s\}} \partial_u \alpha(s,u)duds \\ &= \int_0^t \alpha(s,s)ds + \int_0^t \int_0^t \mathbf{1}_{\{s\leq u\}} \partial_u \alpha(s,u)dsdu \\ &= \int_0^t \alpha(s,s)ds + \int_0^t \int_0^u \partial_u \alpha(s,u)dsdu,\end{aligned}$$

and

$$\begin{aligned}\int_0^t \sigma(s,t)dW_s &= \int_0^t \sigma(s,s)dW_s + \int_0^t \int_s^t \partial_u \sigma(s,u)dudW_s \\ &= \int_0^t \sigma(s,s)dW_s + \int_0^t \int_0^t \mathbf{1}_{\{u\geq s\}} \partial_u \sigma(s,u)dudW_s \\ &= \int_0^t \sigma(s,s)dW_s + \int_0^t \int_0^t \mathbf{1}_{\{s\leq u\}} \partial_u \alpha(s,u)dW_s du \\ &= \int_0^t \sigma(s,s)dW_s + \int_0^t \int_0^u \partial_u \sigma(s,u)dW_s du.\end{aligned}$$

The dynamic of the short rate r then follows by combining the above formulae. □

Under the ELMM $\mathbf{Q}$, the short rate r follows

$$r_t = f(t,t) = f(0,t) - \int_0^t \sigma(s,t)(\sigma^*(s,t))^T ds + \int_0^t \sigma(s,t)dW_s^{\mathbf{Q}}.$$

Therefore, the expectation of the future short rate $\mathbf{E}^{\mathbf{Q}}[r_t]$ does not equal to the current value $f(0,t)$ of the forward rate: $\mathbf{E}^{\mathbf{Q}}[r_t] \neq f(0,t)$. However, we shall prove later that $f(0,t)$ is equal to the expectation of r_t under the *forward measure*.

4 Example: Constant Volatility Forward Rate Model and the Ho-Lee Short Rate Model

The constant volatility forward rate model is given by

$$df(t,T) = \alpha(t,T)dt + \sigma dW_t$$

under the original physical probability measure $\mathbf{P}$, where $\alpha(t,T)$ is a *deterministic* function and $\sigma > 0$. By the HJM drift condition,

$$-\int_t^T \alpha(t,u)du + \frac{1}{2}\sigma^2(T-t)^2 = -\sigma(T-t)\Theta_t.$$

Differentiating against T yields

$$-\alpha(t,T)+\sigma^2(T-t) = -\sigma\Theta_t.$$

Under the ELMM $\mathbf{Q}$, the forward rate $f(t,T)$ follows

$$\begin{aligned} df(t,T) &= \alpha(t,T)dt + \sigma(dW_t^{\mathbf{Q}} - \Theta_t dt) \\ &= (\alpha(t,T) - \sigma\Theta_t)dt + \sigma dW_t^{\mathbf{Q}} \\ &= \sigma^2(T-t)dt + \sigma dW_t^{\mathbf{Q}} \end{aligned}$$

Next, we derive the corresponding short rate r_t. Note that

$$f(t,T) = f(0,T) + \int_0^t \sigma^2(T-s)ds + \sigma W_t^{\mathbf{Q}}.$$

Hence,

$$\begin{aligned} r_t = f(t,t) &= f(0,t) + \int_0^t \sigma^2(t-s)ds + \sigma W_t^{\mathbf{Q}} \\ &= f(0,t) + \frac{1}{2}\sigma^2 t^2 + \sigma W_t^{\mathbf{Q}}. \end{aligned}$$

In turn,

$$dr_t = (\partial_t f(0,t) + \sigma^2 t)dt + \sigma dW_t^{\mathbf{Q}}$$

which is the Ho-Lee model with the drift $b(t) = \partial_t f(0,t) + \sigma^2 t$.

We may also derive the short rate r_t using Proposition 4 directly. Recall

$$r_t = r_0 + \int_0^t \bar{\alpha}_u du + \int_0^t \sigma(u,u)dW_u^{\mathbf{Q}}$$

where

$$\bar{\alpha}_u = \alpha(u,u) + \partial_u f(0,u) + \int_0^u \partial_u \alpha(s,u)ds + \int_0^u \partial_u \sigma(s,u)dW_s^{\mathbf{Q}}.$$

Since $\alpha(u,s) = \sigma^2(u-s)$ and $\sigma(u,s) = \sigma$, it follows that

$$\bar{\alpha}_u = \partial_u f(0,u) + \sigma^2 u,$$

and

$$r_t = r_0 + \int_0^t (\partial_u f(0,u) + \sigma^2 u)du + \int_0^t \sigma dW_u^{\mathbf{Q}}.$$

5 Hedging in Bond Market

We discuss the hedging in bond market by using the bank account and the zero-coupon bonds, which is in line with the Black-Scholes model. In the following, we assume that the filtration $\{\mathscr{F}_t\}_{t\geq 0}$ is generated by the Brownian motion W, so W has the martingale representation property.

Suppose there are n zero-coupon bonds with maturities $T_1 \leq T_2 \leq \cdots \leq T_n$, and a contingent claim with maturity T_1 and payoff X which is $\mathscr{F}_{T_1}$-measurable. We shall construct a hedging strategy consisting of bank account B_t and zero-coupon bonds $P(t,T_i)$ for $1 \leq i \leq n$ to replicate X.

Introduce $\frac{X_t}{B_t} := \mathbf{E}^{\mathbf{Q}}[\frac{X}{B_T}|\mathscr{F}_t]$, which, by the martingale representation, follows:

$$d\frac{X_t}{B_t} = \sum_{j=1}^{d} h_t^j dW_t^{\mathbf{Q},j}$$

for some $h \in \mathscr{L}^2(\mathbb{R}^d)$.

On the other hand, the discounted value process $\frac{V_t}{B_t}$ follows

$$\begin{aligned} d\frac{V_t}{B_t} &= \sum_{i=1}^{n} \phi_t^i d\frac{P(t,T_i)}{B_t} \\ &= \sum_{i=1}^{n} \phi_t^i \frac{P(t,T_i)}{B_t} \sum_{j=1}^{d} \sigma^{*,j}(t,T_i) dW_t^{\mathbf{Q},j} \\ &= \sum_{i=1}^{n} \tilde{\phi}_t^i \sum_{j=1}^{d} \sigma^{*,j}(t,T_i) dW_t^{\mathbf{Q},j} \end{aligned}$$

where $\tilde{\phi}_t^i := \phi_t^i \frac{P(t,T_i)}{B_t}$.

In order to replicate X, we need to choose $\tilde{\phi}$ such that the hedging equation admits a solution:

$$h_t^j = \sum_{i=1}^{n} \tilde{\phi}_t^i \sigma^{*,j}(t,T_i)$$

or in matrix form

$$(\sigma^*(t,T))^T \tilde{\phi}_t^T = h_t^T.$$

Hence, we need to assume that the volatility matrix

$$\sigma^*(t,T) = \left\{\sigma^{*,j}(t,T_i)\right\}_{1\leq i\leq n, 1\leq j\leq d}$$

has rank d. Note that this rank d condition also guarantees the uniqueness of the market price of risk Θ_t.

6 The Musiela Parametrization

In practice, it is more natural to use *time to maturity*, rather than *time of maturity*. If we denote the time to maturity by x, then $x = T - t$, and the forward rate can be defined as

$$r(t,x) := f(t,t+x).$$

Recall that we have the standard HJM-type model under $\mathbf{Q}$:

$$df(t,T) = \sigma(t,T)\left(\int_t^T \sigma(t,s)ds\right)^T dt + \sigma(t,T)dW_t^{\mathbf{Q}}, \qquad (1)$$

we aim to find the dynamics for $r(t,x)$ under $\mathbf{Q}$. Note that

$$dr(t,x) = df(t,t+x) + \partial_T f(t,t+x)dt$$

we have derived the following Musiela's SPDE for $r(t,x)$:

$$\begin{aligned} dr(t,x) &= \left(\partial_T f(t,t+x) + \sigma(t,t+x)\left(\int_t^{t+x} \sigma(t,s)ds\right)^T\right)dt + \sigma(t,t+x)dW_t^{\mathbf{Q}} \\ &= \left(\partial_x r(t,x) + \sigma_0(t,x)\left(\int_0^x \sigma_0(t,s)ds\right)^T\right)dt + \sigma_0(t,x)dW_t^{\mathbf{Q}} \end{aligned}$$

with $\sigma_0(t,x) := \sigma(t,t+x)$.

7 Exercises

Exercise 1. (Vasciek model and corresponding forward rate)
The one dimensional Vasciek model is given by

$$dr_t = (a - br_t)dt + \sigma dW_t^{\mathbf{Q}}$$

under the EMM $\mathbf{Q}$, where $a, b, \sigma > 0$.

1. Explain why the corresponding short rate $(r_t)_{t\geq 0}$ provides an ATS, i.e. the price $P(t,T)$ of the corresponding zero-coupon bond price has the form:

$$P(t,T) = e^{-A(t)-B(t)r_t}$$

for some functions $A(t)$ and $B(t)$.
2. Write down the ODEs for $A(t)$ and $B(t)$, and solve the equations to obtain $A(t)$ and $B(t)$.
3. Recall that the forward rate $f(t,T)$ is defined as

$$f(t,T) = -\frac{\partial \ln P(t,T)}{\partial T}.$$

Prove the forward rate has $f(t,T)$ has the dynamic

$$df(t,T) = \frac{\sigma^2}{b}(e^{-b(T-t)} - e^{-2b(T-t)})dt + \sigma e^{-b(T-t)} dW_t^{\mathbf{Q}}.$$

Therefore the volatility $\sigma(t,T)$ of the forward rate has the form: $\sigma(t,T) = \sigma e^{-b(T-t)}$.

4. Show that the drift of the forward rate is nothing but $\sigma(t,T)\int_t^T \sigma(t,s)ds$.

Exercise 2. (Deterministic volatility forward rate model and corresponding short rate)

The deterministic volatility forward rate model is given by

$$df(t,T) = \alpha(t,T)dt + \sigma e^{-b(T-t)} dW_t$$

under the original physical probability measure **P**, where $\sigma > 0$ and $b > 0$.

1. Write down the corresponding HJM drift condition, and based on such a condition, prove that

$$-\alpha(t,T) + \frac{\sigma^2}{b}(e^{-b(T-t)} - e^{-2b(T-t)}) = -\sigma e^{-b(T-t)}\Theta_t,$$

where Θ_t is the market price of risk.

2. Prove that the forward rate has the following dynamic:

$$df(t,T) = \frac{\sigma^2}{b}(e^{-b(T-t)} - e^{-2b(T-t)})dt + \sigma e^{-b(T-t)} dW_t^{\mathbf{Q}}$$

under the ELMM **Q**.

3. By using $r_t = f(t,t)$, prove that the corresponding short rate has the dynamic:

$$dr_t = (a(t) - br_t)dt + \sigma dW_t^{\mathbf{Q}},$$

where $a(t)$ is given by

$$a(t) = \partial_t f(0,t) + bf(0,t) + \frac{\sigma^2}{2b} - \frac{\sigma^2}{2b}e^{-2bt}$$

which is nothing but the extended Vasicek model.
(Recall that an extended Vasicek model is given by

$$dr_t = (a(t) - br_t)dt + \sigma W_t^{\mathbf{Q}},$$

where $a(t)$ is some deterministic function, $b > 0$ and $\sigma > 0$.)

4. Using Proposition 4, derive the dynamics of the short rate and conclude that it is the same as the one proved in part (3).

References

1. Filipovic, Damir. *Term-Structure Models. A Graduate Course.* Springer, 2009.