Chapter 7 Solutions

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Ex 1. Note that

$$V_T^i = V_0^i \exp\{(r - \tfrac{1}{2}\sigma_i^2)T + \sigma_i W_T^i\}$$

$$= V_0^i \exp\{(r - \tfrac{1}{2}\sigma_i^2)T + \sigma_i \sqrt{T}\, Z^i\} \quad \text{where } Z^i = W_T^i/\sqrt{T}.$$

Use the tower property of conditional expectation and factor decomposition

$Z^i = \rho_i \xi + \sqrt{1-\rho_i^2}\,\xi^i$, we have

$$Q(V_T^1 < K_1 \cdots V_T^m < K_m) = Q(Z^1 < -d_1 \cdots Z^m < -d_m)$$

$$= Q\left(\xi^i < \frac{-d_i - \rho_i \xi}{\sqrt{1-\rho_i^2}},\ 1 \le i \le m\right)$$

$$= E\left[Q\left(\xi^i < \frac{-d_i - \rho_i \xi}{\sqrt{1-\rho_i^2}},\ 1 \le i \le m \,\middle|\, \xi\right)\right]$$

$$= E\left[\prod_{i=1}^{m} Q\left(\xi^i < \frac{-d_i - \rho_i \xi}{\sqrt{1-\rho_i^2}}\,\middle|\, \xi\right)\right]$$

$$= E\left[\prod_{i=1}^{m} \Phi\left(\frac{-d_i - \rho_i \xi}{\sqrt{1-\rho_i^2}}\right)\right]$$

$$= \int \prod_{i=1}^{m} \Phi\left(\frac{-d_i - \rho_i x}{\sqrt{1-\rho_i^2}}\right) \phi(x)\, dx$$

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