

## ST909: Applications of Stochastic Calculus in Finance (Spring Term 2024)

Lecturer: Gechun LIANG

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Office Hours: Tue 2:30-3:30pm and Wed 3.30-4.30pm.

Office Number: 2.07, MS Building.

Lecture Times and Place: Week 2-10

Tue 12-2pm in 1007 WBS for lectures;

Wed 1-3pm in 1015 WBS for lectures/seminars.

All the lectures and seminars will be available on the Moodle/MS Teams channel for ST909.

Lecture Notes and Assignments: Lecture notes are posted on the Moodle. Handwritten notes and lecture slides will also be posted after each lecture. There are no assignments. However, students are encouraged to rewrite lecture notes themselves in the minimal number of pages that is sufficient to remember the essential content of the course.

Aims: To give a thorough understanding of how stochastic calculus is used in continuous time finance. To develop an in-depth understanding of models used for various asset classes.

### Course Outline

#### 1. Interest rate models

- Short-rate models (Affine term structure, Vasicek model, Hull-White model, Ho-Lee model, CIR model)
- Heath-Jarrow-Morton (HJM) methodology (Forward rate model, HJM drift condition, hedging in bond market)
- LIBOR market models (Change of numeraire technique,  $T$ -Forward Measure, Black-Scholes Model with random interest rates, LIBOR model, Black's formula for interest rate caps)
- Main references: (1) Liang's KCL lecture notes (2) Bjork's book Chapters 15, 19-23 (3) Filipovic's book Chapters 1, 5-7, 11.

#### 2. Credit risk models

- Stochastic calculus for single jump processes (Cadlag functions with bounded variation, change of variables formula, stochastic exponential, Girsanov's theorem)
- Reduced form credit risk models (Filtration switching formula, risk-neutral valuation of defaultable cash flows, credit default swaps(CDS))
- Contagion models (Basket CDS, correlated intensity model, change of probability measure method, total hazard rate construction)
- Main references: (1) Liang's Wawick lecture notes (2) Filipovic's book Chapter 12 (3) Jarrow's book Chapter 7

#### 3. Volatility models

- Local volatility models (CEV model, implied volatility, Dupire's formula)
- Stochastic volatility models (Hull-White model, Heston's model, SABR model)

- More volatility models (Stochastic local volatility models, Gyongy's projection, uncertainty volatility, rough volatility)
- Main references: (1) Hobson's Warwick lecture notes (2) Alos' book Chapter 2 (3) Musiela and Rutkowski's book Chapter 7

Further Reading: The lecture notes are prepared mainly based on the following resources.

1. E. Alos: Malliavin Calculus in Finance Theory and Practice, Chapman & Hall/CRC. Available in Warwick library.
2. T. Bjork: Arbitrage Theory in Continuous time, Oxford. Available in Warwick library.
3. D. Filipovic: Term-Structure Models. A Graduate Course. Springer. Available in Warwick library
4. D. Hobson: Lecture notes on Volatility Models, Warwick. Available on personal website.
5. R. Jarrow: Continuous-Time Asset Pricing Theory, Springer. Available in Warwick library.
6. G. Liang: Lectures notes on Interest Rates and Credit Risk, KCL. Available on personal website.
7. G. Liang: Lecture notes on Systemic Risk and Contagion Risk, Warwick. Available on personal website.
8. M. Musiela and M. Rutkowski: Martingale Methods in Financial Modeling, Springer. Available in Warwick library.

Examinations:

- 2 one-hour class tests (each 10%): Weeks 8 and 10
- 2-hour examination in April/May (80%).