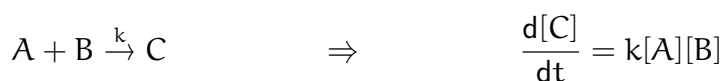


## Part 1 Introduction to Dynamical Systems

1. Consider an nonautonomous system  $m\ddot{x} + b\dot{x} + kx = F \cos t$ . Re-write it as a first-order system of higher dimensionality.
2. Solve the logistic equation  $\dot{N} = rN(1 - N/K)$ ,  $N(0) = N_0$ .
3. Find fixed points and classify their stability (by different methods):
  - (a)  $\dot{x} = 4x^2 - 16$ , by graphical analysis or linear stability analysis;
  - (b)  $\dot{x} = 1 - x^{14}$ , by graphical analysis or linear stability analysis;
  - (c)  $\dot{x} = 1 + \frac{1}{2}\cos x$ , by graphical analysis or linear stability analysis;
  - (d)  $\dot{x} = 1 - 2\cos x$ , by linear stability analysis;
  - (e)  $\dot{x} = e^x - \cos x$ , by graphical analysis;
  - (f)  $\dot{N} = rN(1 - N/K)$ , by linear stability analysis;
- 4.

The Law of Mass Action: the rate of the chemical reaction is proportional to the product of the concentrations of the molecular species involved in the reaction



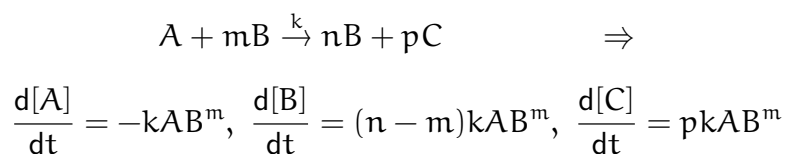
$k$  is called the *rate constant*.

The main principle for constructing dynamic equations:

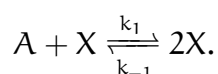
rate of change = inflow rate - outflow rate



The Law of Mass Action for the more general reaction:



Consider the following reaction of autocatalysis



Assuming the concentration  $[A] = a$  is constant, write down an ODE for this reaction and explain the system's behaviour.

5. Consider the following reactions



Assuming the concentrations  $[A] = a$ ,  $[B] = b$  and  $[C] = c$  are constant, write down an ODE for this model and classify the stability of fixed points as a parameter  $p = k_1 a - k_2 b$  varies.

6. Construct the bifurcation diagrams (a diagram of values of fixed points as a function of a parameter) for the following systems:

(a)  $\dot{x} = \mu - |x|$

(b)  $\dot{x} = \mu - x^2 + 4x^4$

(c)  $\dot{x} = r + x - x^3$

7. Sketch the phase portrait of each of the following linear systems:

(a)  $\dot{x} = -3x, \quad \dot{y} = -2y$

(b)  $\dot{x} = -x + y, \quad \dot{y} = -x - y$

(c)  $\dot{x} = 3y, \quad \dot{y} = -3x$