Obligatory Assignment 2 Report **Tomasz Gliniecki**

Problem 1

System works if at least 11 components work.
Simulation:

$$\sum_{i=1}^{15} I(U_i \leq p_i) \geq 11$$

a)

analytical

$$1 + \sum_{i=0}^{10} -\binom{15}{i} 0.1^{15-i} 0.9^{i} = 0.987$$

simulation results:

$$\text{P(System works)} = 0.987$$

b)

with probabilities: $.9, .9, .7, .8, .8, .7, .9, .7, .7, .8, .8, .7, .9, .8, .9, .8$

```
p <- c( .9, .9, .7, .8, .7, .9, .7, .7, .8, .8, .7, .9, .8, .9, .8 )
n <- 1e6
u.mtx <- matrix( runif( n=n*15 ), ncol=15 )
f.matrix <- ( u.mtx < matrix( rep( p, times=n), byrow=T, ncol=15 ) )
y.f <- rowSums(f.mtx)
sum( y.f >= 11 )/n.replic
sd(y.f >= 11) / sqrt(n.replic);
```

simulation results: P(System works) = 0.841 with standard error = 0.00366

Problem 2

a)

ref [1]

$g(x) = 4\sqrt{1-x^2}dx$

$\int_0^1 g(x) = \pi$

$E(3\sqrt{1-U^2}) = \pi$, $U \sim U[0,1]$

$\frac{1}{n}\sum_{i=1}^{n} 4\sqrt{1-U_i^2}$

$\frac{4}{n}\sum_{i=1}^{n} \sqrt{1-U_i^2} \to E(4\sqrt{(}1-U^2)) = \int g(x)f(x)$

where f(x) is the density function $\frac{1}{b-a} = \frac{1}{1-0}$

$4\int_0^1 \sqrt{1-x^2} = 4[\frac{1}{2}(\sqrt{1-x^2}x + sin^{-1}(x))]_0^1 = 4 * \frac{1}{2}\frac{\pi}{2} = \pi \ Q.E.D$

and:

$Var(4\sqrt(1-U^2) = 32/3 - \pi^2$

$Var(X) = E(X^2) - (E(X))^2$

$(E(g(x)))^2 = \pi^2$ *from previous*

$E(g(x)^2) = 16\int_0^1 \sqrt{1-U^2}^2 = 16\int_0^1 1-U^2$

$= 16[U - \frac{U^3}{3}]_0^1 = 16[0-(1-\frac{1}{3})] = 16-(1-\frac{1}{3}$

$Var(4\sqrt{1-U^2} = (16(1-\frac{1}{3})) - \pi^2 = (16-\frac{16}{3}) - \pi^2$
$= \frac{32}{3} - \pi^2 \ Q.E.D$

b)

```
U <- runif(1e6)
theta.hat <- mean(4*sqrt(1-U^2))
theta.hat
> [1] 3.14041
```

Problem 3

a)

```
lambda <- 2;
t0 <- 20; #intervall [0:20]
Tn <- rexp(100, lambda) # interarrival times with 100 observations
Sn <- cumsum(Tn)        # arrival times
n <- min(which(Sn > t0))# arrivals+1 in [0 , t0=20]
```

b)

E(N(5)) = 9.983, SD(N(5)) = 3.157
E(N(20)) = 40.001, SD(N(20)) = 6.320
mean is calculated by taking the mean of the values from the arrivals vector in R standard deviation is calculated the same way by using sd(); funciton

The values should be approching $E(N(k)) = k * \lambda$ when the number of runs of code from *a)* is big enough, where k is 5 and 20 in our case.
e.g.$E(N(5)) = \lambda * 5 \sim approx = 9.983$.

c)

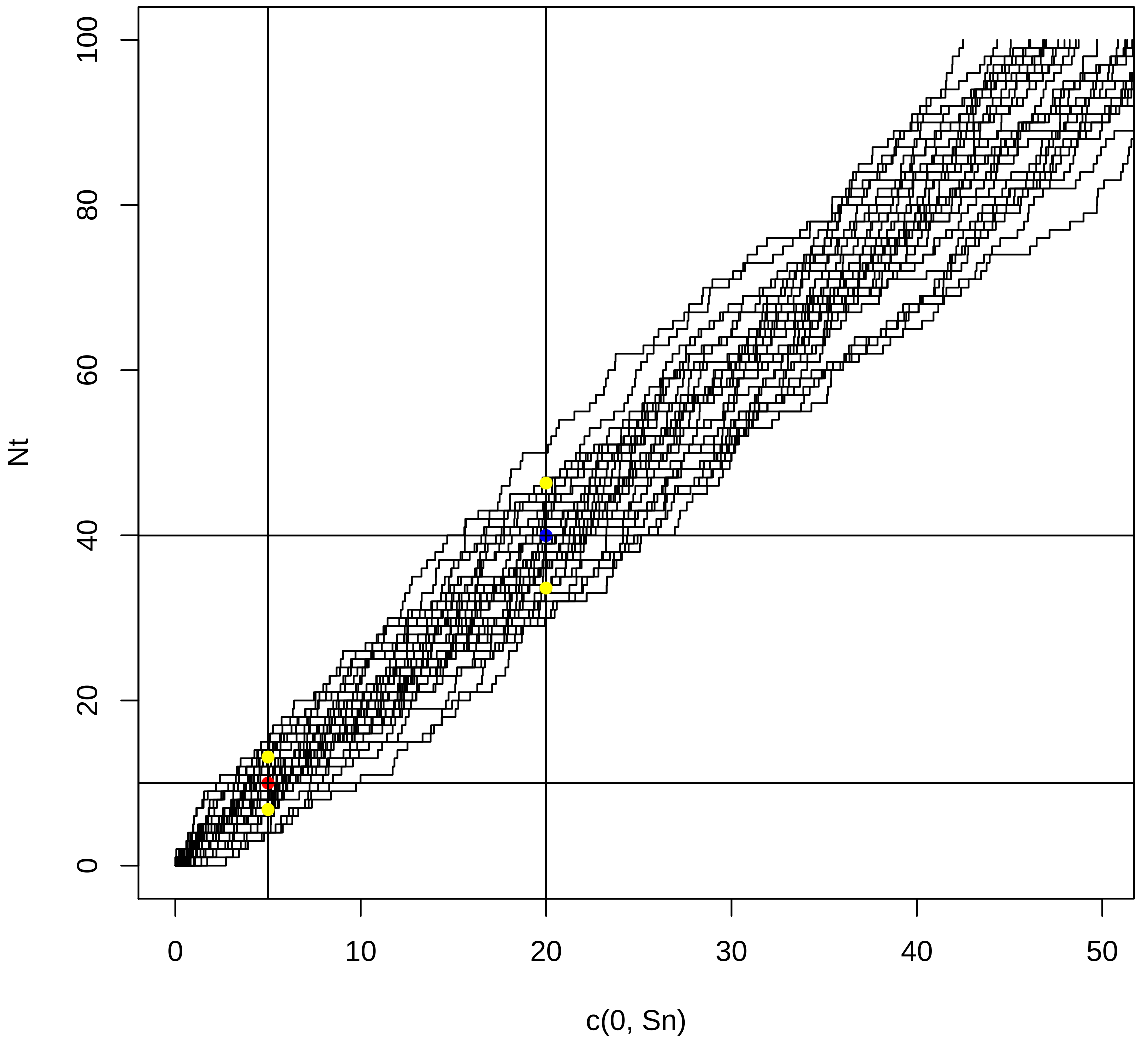

Figure 1: 30 indepentant realizations of poisson process with $\lambda = 2$

The red point shows the E(N(5)) and the blue point shows E(N(20)) which correspond nicely with the graph.

As for the standard deviation we can notice that it gets bigger when t0 gets bigger, and this is also visible on the graph with 30 instances, it gets spread out more as the number of observations increase.

Problem 4

a)

T_1 and T_2 are exponentially distributed with mean 1 becuse we are drawing from uniform distribution $U \sim U[0, 1]$ and using inverse transform sampling. ref [1]

b)

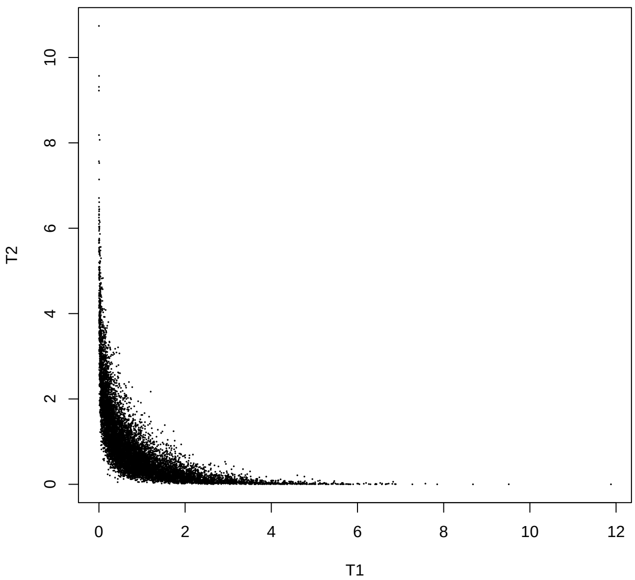

Figure 2: Scatter plot of T_1 vs T_2 with $\rho = -0.9$

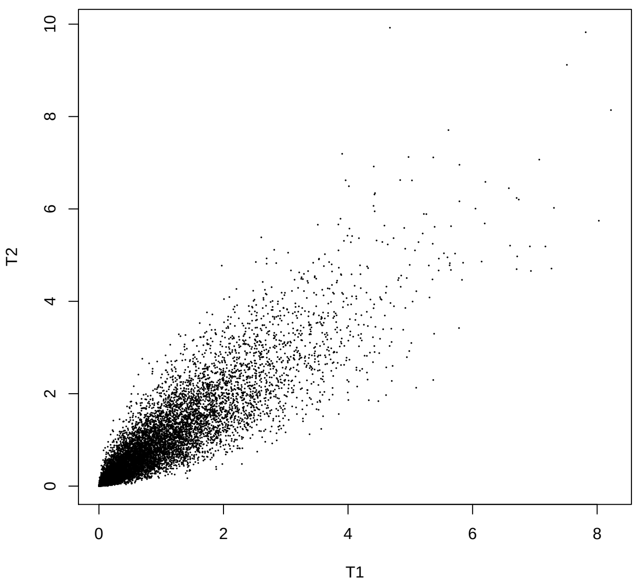

Figure 3: Scatter plot of T_1 vs T_2 with $\rho = 0.9$

The corraletion estimate of T_1 and T_2 is
$cor(T_1, T_2) = -0.6$ when $\rho = -0.9$ and
$cor(T_1, T_2) = 0.88$ when $\rho = 0.9$

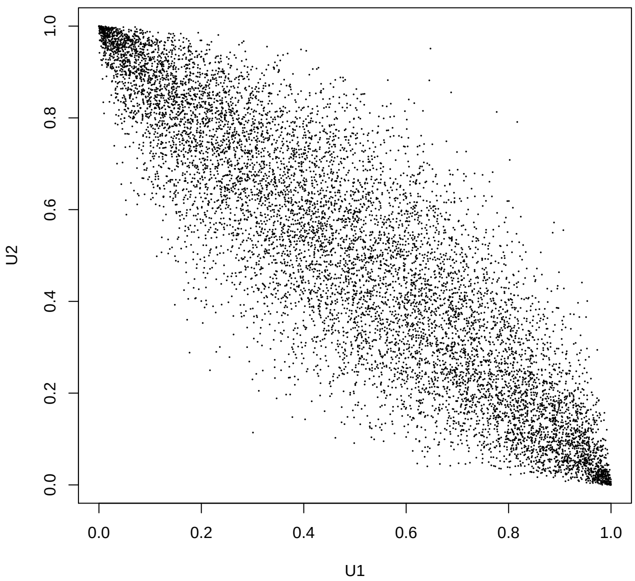

Figure 4: Scatter plot of $U_1 vs U_2$ with $\rho = -0.9$

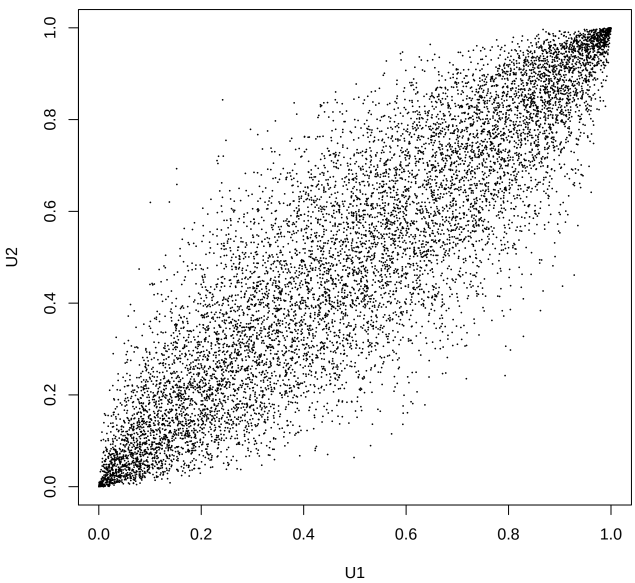

Figure 5: Scatter plot of $U_1 vs U_2$ with $\rho = 0.9$

c)

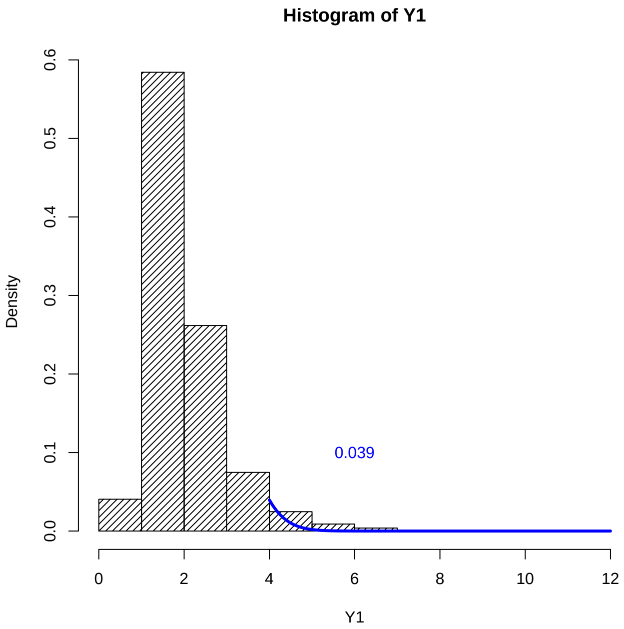

Figure 6: Histogram of $Y = T_1 + T_2$ with $\rho = -0,9$, $P(Y \geq 4) = 0.039$

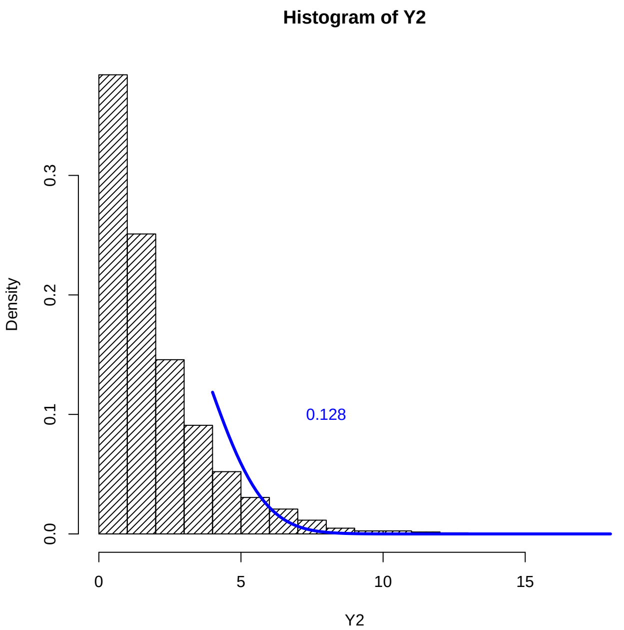

Figure 7: Histogram of $Y = T_1 + T_2$ with $\rho = 0,9$, $P(Y \geq 4) = 0.128$

d)

The estimate of $P(Y \geq 4) = 0.04$ when $\rho = -0.9$
and $= 0.131$ when $\rho = 0.9$ see Figure 6 and 7

```
mean(Y1>=4)/sqrt(n) #rho = -0.9
mean(Y2>=4)/sqrt(n) #rho = 0.9
```

```
>[1] 0.0019
>[2] 0.0034
```

standard error is 0.0019 and 0.0034 for rho: -.9 and .9 respectively.

Corralation does not affect $E(Y) = E(T_1, T_2)$,
$cor(X, Y) = \frac{cov(X,Y)}{sd(X)sd(Y)}$ where $cov(X, Y) = E([X - E(X)][Y - E(Y)])$
ref [1] corralation is dependent on E(Y)

References

[1] Lecture notes, and Rizzo material that relevant to the course.