Problem 1

System works if at least 11 components work.

Simulation:

$$\sum_{15}^{i=1} I(U_i \leq p_i) \geq 11$$

a) simulation results: P(A) = 0.987

b) simulation results: P(A) = 0.841 with standard error = 0.00366

Problem 2

a)

$$g(x) = \int_0^1 4\sqrt{1-x^2}dx = \pi$$
$$E(3\sqrt{1-U^2}) = \pi,\ U \sim U[0,1]$$
$$\frac{1}{n}\sum_{i=1}^{n} 4\sqrt{1-U_i^2}$$
$$\frac{4}{n}\sum_{i=1}^{n} \sqrt{1-U_i^2} \rightarrow E(4\sqrt{(1-U^2)}) = \int g(x)f(x)$$
where f(x) is the density function $\frac{1}{b-a} = \frac{1}{1-0}$
$$4\int_0^1 \sqrt{1-x^2} = 4[\frac{1}{2}(\sqrt{1-x^2}x + sin^{-1}(x))]_0^1 = 4 * \frac{1}{2}\frac{\pi}{2} = \pi$$

b)

```
U <- runif(1e6)
theta.hat <- mean(4*sqrt(1-U^2))
theta.hat
> [1] 3.14041
```

Problem 4

a)

T_1 and T_2 are exponentially distributed becuse we are drawing from uniform distribution and correcting it with $-ln(U_n)$ to make it exponential.

b)

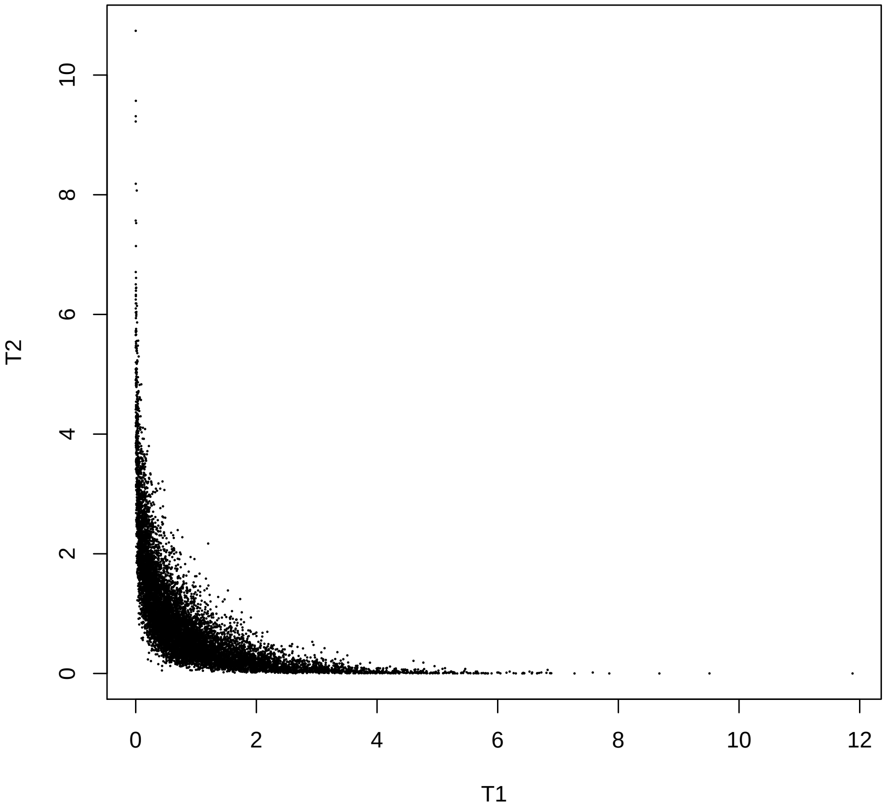

Figure 1: Scatter plot of T_1 vs T_2 with $\rho = -0.9$

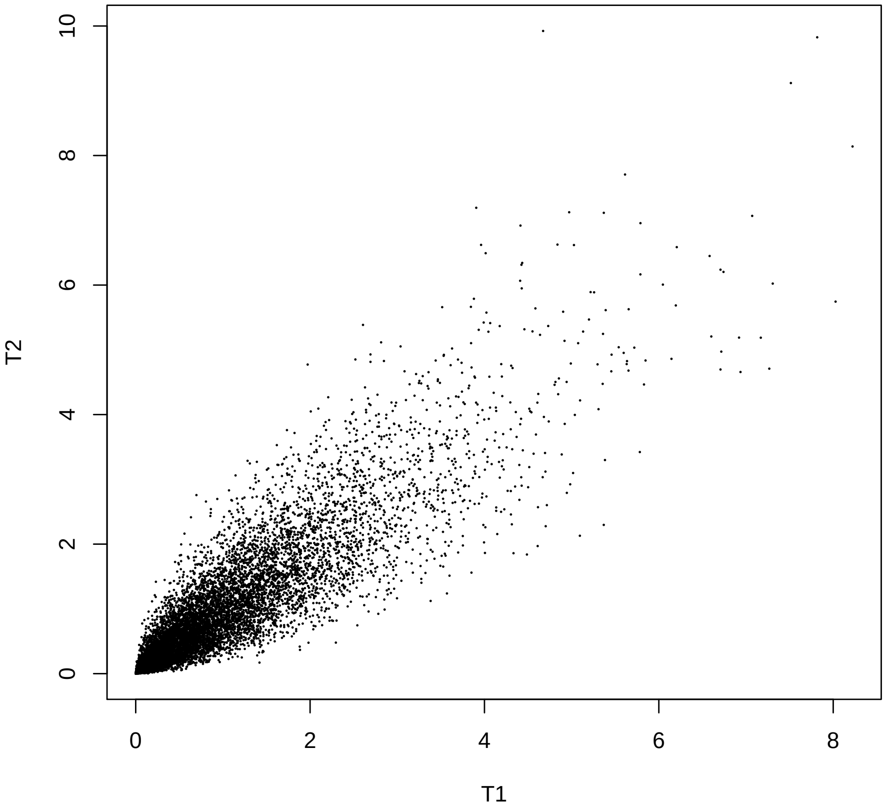

Figure 2: Scatter plot of T_1 vs T_2 with $\rho = 0.9$

The corraletion estimate of T_1 and T_2 is $cor(T_1, T_2) = -0.6$ when $\rho = -0.9$ and $cor(T_1, T_2) = 0.88$ when $\rho = 0.9$

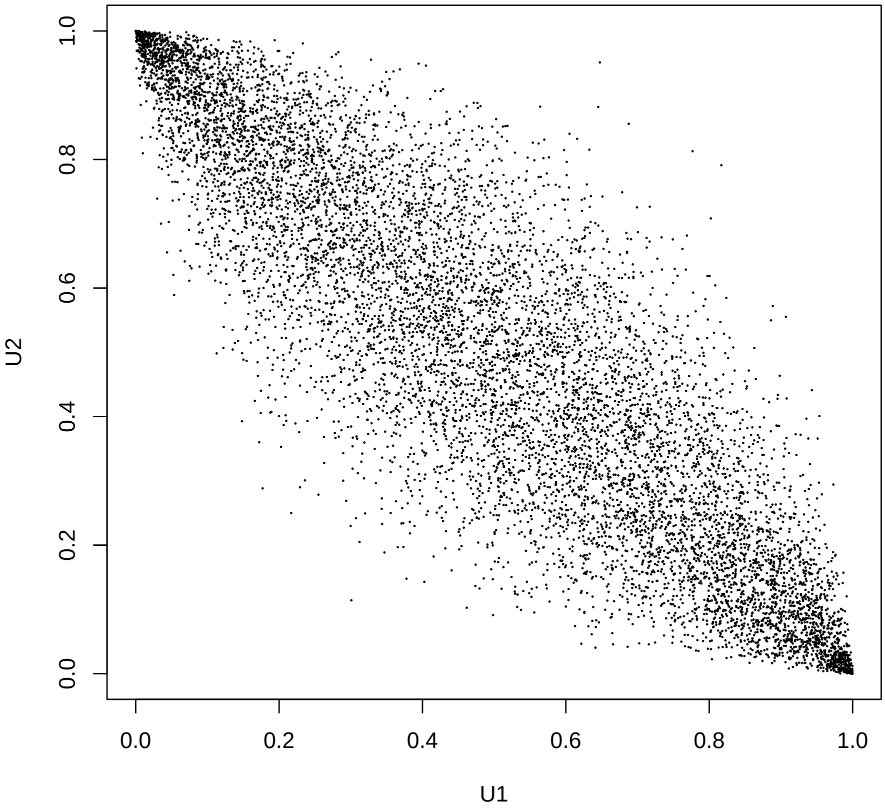

Figure 3: Scatter plot of $U_1 vs U_2$ with $\rho = -0.9$

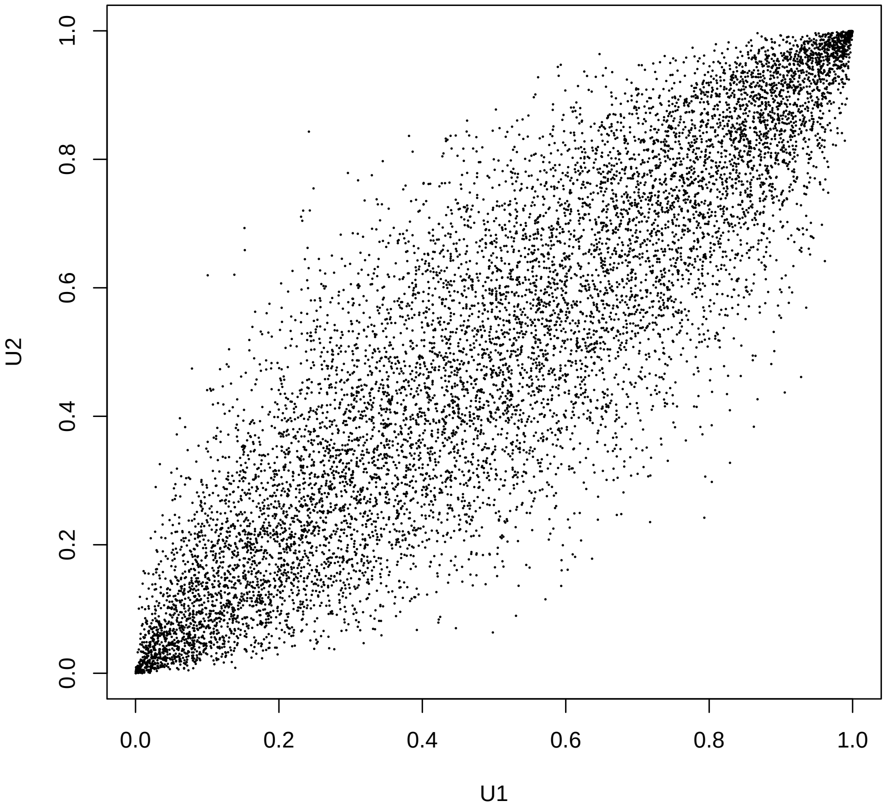

Figure 4: Scatter plot of $U_1 vs U_2$ with $\rho = 0.9$

0.1 c)

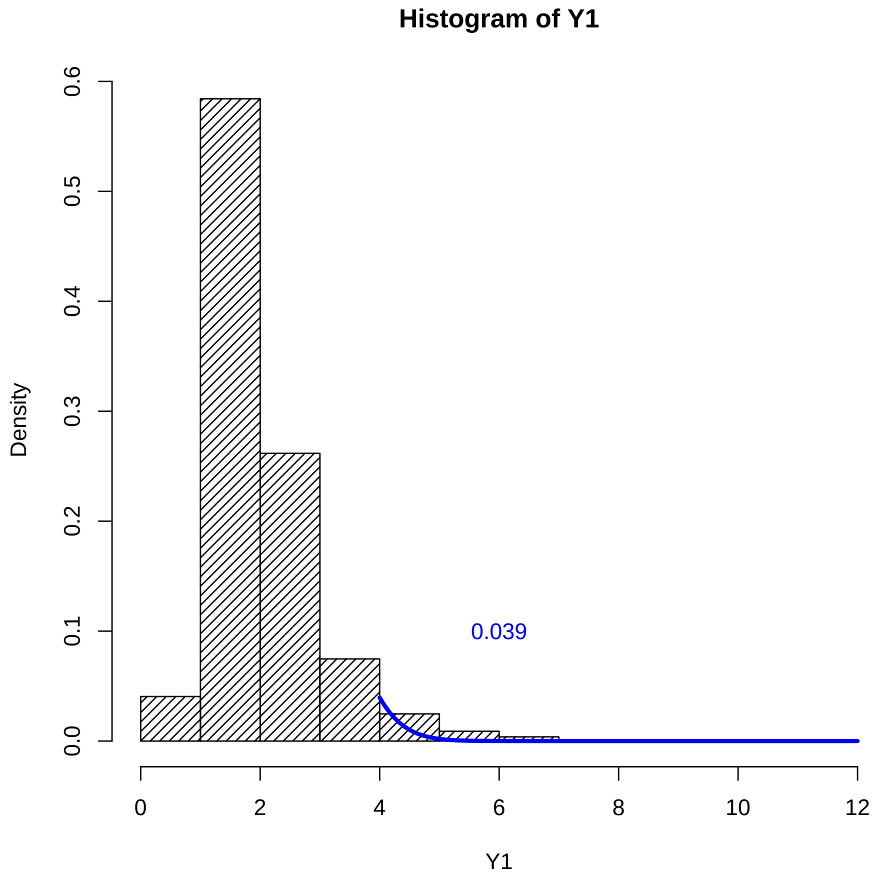

Figure 5: Histogram of $Y = T_1 + T_2$ with $\rho = -0,9$, $P(Y \geq 4) = 0.039$

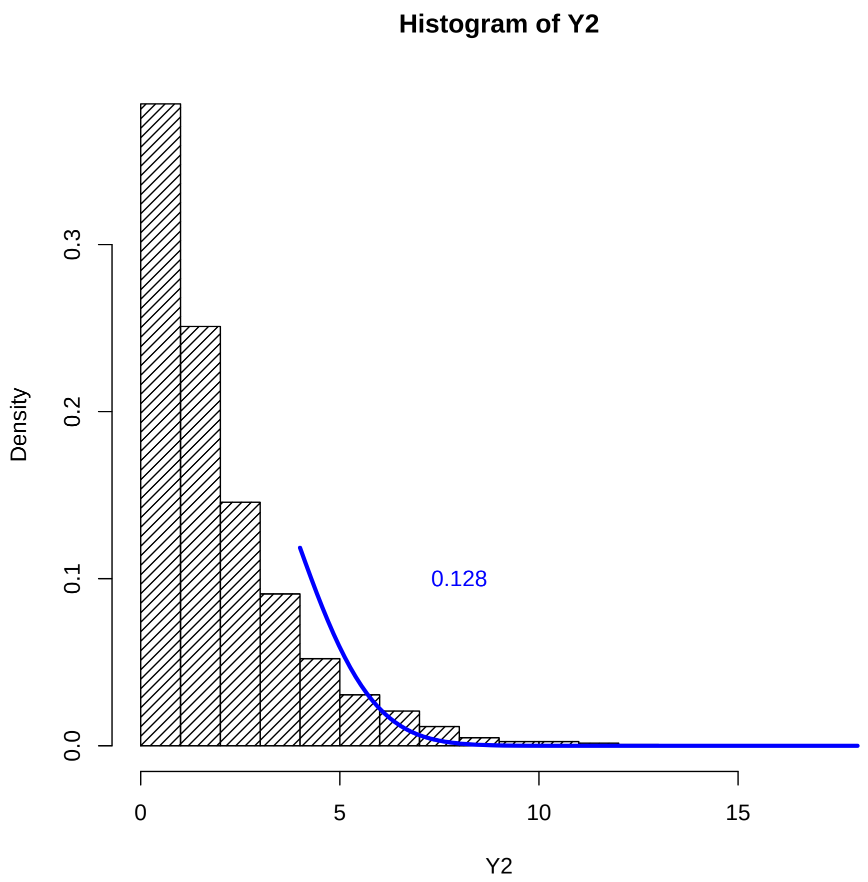

Figure 6: Histogram of $Y = T_1 + T_2$ with $\rho = 0, 9$, $P(Y \geq 4) = 0.128$

d)

The estimate of $P(Y \geq 4) = 0.04$ when $\rho = -0.9$ and $= 0.131$ when $\rho = 0.9$ see Figure 5 and 6

Corralation does not affect $E(Y) = E(T_1, T_2)$, $cor(X, Y) = \frac{cov(X,Y)}{sd(X)sd(Y)}$ where $cov(X, Y) = E([X - E(X)][Y - E(Y)])$

References

[1] Monty Hall problem, [cited 27.September 2016]. Available at `https://en.wikipedia.org/wiki/Monty_Hall_problem`