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The posterior distribution:

$$\pi(a,\beta) = k^{-1}p(x|a)^{\beta}p(a)$$

For Simulated Tempering, we are considering $(a,\beta_1) \to (a,\beta_2)$, then acceptance probability is:

$$\alpha(1,2) = 1 \wedge \frac{p(x|a)^{\beta_2}/c(\beta_2)}{p(x|a)^{\beta_1}/c(\beta_1)}$$

For Parallel Tempering, we are considering $(a_1,\beta_1)(a_2,\beta_2) \to (a_2,\beta_1)(a_1,\beta_2)$, then swap probability is:

$$\begin{aligned}\alpha(1,2) &= 1 \wedge \frac{p(x|a_1)^{\beta_2}/c(\beta_2)}{p(x|a_1)^{\beta_1}/c(\beta_1)} \times \frac{p(x|a_2)^{\beta_1}/c(\beta_1)}{p(x|a_2)^{\beta_2}/c(\beta_2)} \\ &= 1 \wedge \left(\frac{p(x|a_2)}{p(x|a_1)}\right)^{\beta_1-\beta_2}\end{aligned}$$

In our case, we are considering $((a_1,b_1),\beta_1)((a_2,b_2),\beta_2) \to ((a_2,b_2),\beta_1)((a_1,b_1),\beta_2)$, then swap probability should still be:

$$\alpha(1,2) = 1 \wedge \left(\frac{p(x|a_2,b_2)}{p(x|a_1,b_1)}\right)^{\beta_1-\beta_2}$$

Because we are computing likelihood with the following codes, this should be fine.

```
res <- pg.residual - xa * Half.comb
log.likelihood <- sum(dnorm(res,0,sqrt(1/pg.draws),log=TRUE))
```

However, another question is: our model has the following form:

$$f(y|x) = \frac{\Phi\big(\sum_{k=1}^{K}(x^T\beta_k)P_k(y) - (x^T\alpha)_+Q(y;\lambda)I(y>0)\big)}{\int_{-1}^{1}\Phi\big(\sum_{k=1}^{K}(x^T\beta_k)P_k(y) - (x^T\alpha)_+Q(y;\lambda)I(y>0)\big)\mathrm{d}t}, \quad x \in \mathbb{R}^p, y \in (-1,1),$$

In `get.miss.reject.x()`, we have

```
w <- rowSums(x.remain * (pm %*% b)) + xa.remain*hk
u <- (runif(n.remain) < Phi(w))
w.miss <- c(w.miss, w[!u])
```

and

```
w.obs <- rowSums(x * (Poly.obs %*% b)) + xa.obs * Half.obs
w.miss <- missing.stuff$w.miss
w.comb <- c(w.obs, w.miss)
pg.draws <- pmax(1e-12, rpg.devroye(Ntot, h=1, z=w.comb))
pg.residual <- pg.resp - w.comb
```

Before, I only used `temp` when computing log.hastings.ratio for a parameter,

```
log.hastings.ratio <- (temp * ( sum(dnorm(res.new,0,sqrt(1/pg.draws),log=TRUE)
                                    - dnorm(res,0,sqrt(1/pg.draws),log=TRUE)) )
                       + sum(dnorm(a.new,0,sqrt(1/hprec),log=TRUE)
                             - dnorm(a,0,sqrt(1/hprec),log=TRUE)))
```

Should we add `temp` in the `get.miss.reject.x()` and `w.obs` to reflect different temperature configurations?

$$\alpha(1,2) = 1 \wedge \frac{p_1(y, y_2^*|\theta_2) * p_2(y, y_1^*|\theta_1)}{p_1(y, y_1^*|\theta_1) * p_2(y, y_2^*|\theta_2)}$$